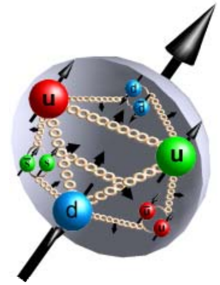


Pretzelocity &

Quark Angular Momentum

Bo-Qiang Ma (马伯强)

PKU (北京大学)

2nd Workshop on Hadron Physics in China
and Opportunities with 12 GeV JLab
July 28, 2010

Collaborators: Enzo Barone, Stan Brodsky, Jacques Soffer, Andreas Schafer, Ivan Schmidt, Jian-Jun Yang, Qi-Ren Zhang, and students

The Proton "Spin Crisis"

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.3$$

In contradiction with the naïve quark model expectation:

Naive Quark Model:

$$\Delta u = \tfrac{4}{3}; \qquad \Delta d = -\tfrac{1}{3}; \qquad \Delta s = 0$$

$$\Sigma = \Delta u + \Delta d + \Delta s = 1$$

The proton spin crisis
& the Melosh-Wigner rotation

- It is shown that the proton "spin crisis" or "spin puzzle" can be understood by the relativistic effect of quark transversal motions due to the Melosh-Wigner rotation.
- The quark helicity Δq measured in polarized deep inelastic scattering is actually the quark spin in the infinite momentum frame or in the light-cone formalism, and it is different from the quark spin in the nucleon rest frame or in the quark model.

B.-Q. Ma, J.Phys. G 17 (1991) L53

B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482

The Wigner Rotation

for a rest particle $(m,\vec{0}) = p^{\mu}$ $\quad (0,\vec{s}) = \mathrm{w}^{\mu}$

for a moving particle $\mathrm{L}(p)p = (m,\vec{0})$ $\quad (0,\vec{s}) = \mathrm{L}(p)w/m$

$\mathrm{L}(p)$ = ratationless Lorentz boost

Wigner Rotation

$\vec{s}, p_{\mu} \rightarrow \vec{s}', p'_{\mu}$

$\vec{s}' = R_w(\Lambda, p)\vec{s}$ $\qquad p' = \Lambda p$

$R_w(\Lambda, p) = \mathrm{L}(p')\Lambda \mathrm{L}^{-1}(p)$ $\quad$ a pure rotation

E.Wigner, Ann.Math.40(1939)149

Melosh Rotation for Spin-1/2 Particle

The connection between spin states in the rest frame and infinite momentum frame
Or between spin states in the conventional equal time dynamics and the light-front dynamics

$$\chi^{\uparrow}(T) = w[(q^{+} + m)\chi^{\uparrow}(F) - q^{R}\chi^{\downarrow}(F)];$$

$$\chi^{\downarrow}(T) = w[(q^{+} + m)\chi^{\downarrow}(F) + q^{L}\chi^{\uparrow}(F)].$$

What is Δq measured in DIS

- Δq is defined by $\Delta q\ s_\mu = \langle p, s | \bar{q}\gamma_\mu\gamma_5 q | p, s\rangle$

$$\Delta q = \langle p, s | \bar{q}\gamma^+\gamma_5 q | p, s\rangle$$

- Using light-cone Dirac spinors

$$\Delta q = \int_0^1 \mathrm{d}x \left[q^\uparrow(x) - q^\downarrow(x) \right]$$

- Using conventional Dirac spinors

$$\Delta q = \int \mathrm{d}^3\vec{p} M_q \left[q^\uparrow(\vec{p}) - q^\downarrow(\vec{p}) \right]$$

$$M_q = \frac{(p_0 + p_3 + m)^2 - \vec{p}_\perp^{\,2}}{2(p_0 + p_3)(p_0 + m)}$$

Thus Δq is the light-cone quark spin or quark spin in the infinite momentum frame, not that in the rest frame of the proton

Quark spin sum is not a Lorentz invariant quantity

Thus the quark spin sum equals to the proton in the rest frame does not mean that it equals to the proton spin in the infinite momentum frame

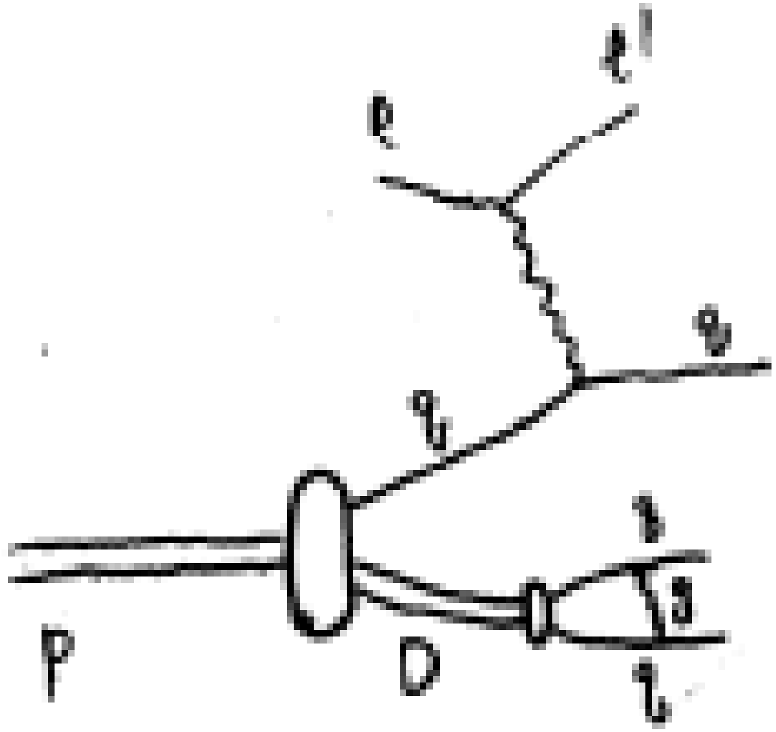
$$\sum_q \vec{s}_q = \vec{S}_p \quad \text{in the rest frame}$$

does not mean that

$$\sum_q \vec{s}_q = \vec{S}_p \quad \text{in the infinite momentum frame}$$

Therefore it is not a surprise that the quark spin sum measured in DIS does not equal to the proton spin

A relativistic quark-diquark model



B.-Q. Ma, Phys.Lett. B 375 (1996) 320-326.
B.-Q. Ma, I. Schmidt, J. Soffer, Phys.Lett. B 441 (1998) 461-467.

A relativistic quark-diquark model

- longitudinally polarized quark distribution

$$\Delta q(x) = \tilde{c}_q^S \tilde{a}_S(x) + \tilde{c}_q^V \tilde{a}_V(x)$$

where

$$\tilde{a}_D(x) = \int [\mathrm{d}^2 \mathbf{k}_\perp] W_D(x, \mathbf{k}_\perp) |\phi(x, \mathbf{k}_\perp)|^2 \quad (D = S \text{ or } V)$$

- Melosh-Winger rotation factor

Longitudinally polarized

$$W_D(x, \mathbf{k}_\perp) = \frac{(k^+ + m_q)^2 - \mathbf{k}_\perp^2}{(k^+ + m_q)^2 + \mathbf{k}_\perp^2}$$

where $k^+ = x\mathcal{M}$, $\mathcal{M}^2 = \frac{m_q^2 + \mathbf{k}_\perp^2}{x} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x}$.

pQCD counting rule

$$q_{\rm h}^{\pm} \propto (1-x)^{p}$$

$$p = 2n-1+2\,|\,\Delta s_z\,| \qquad\qquad \Delta s_z = s_q - s_N$$

- **Based on the minimum connected tree graph of hard gluon exchanges.**
- **"Helicity retention" is predicted -- The helicity of a valence quark will match that of the parent nucleon.**

Parameters in pQCD counting rule analysis

In leading term

$$q_i^+ = \frac{\tilde{A}_{q_i}}{B_3} x^{-\frac{1}{2}} (1-x)^3$$

$$q_i^- = \frac{\tilde{C}_{q_i}}{B_5} x^{-\frac{1}{2}} (1-x)^5$$

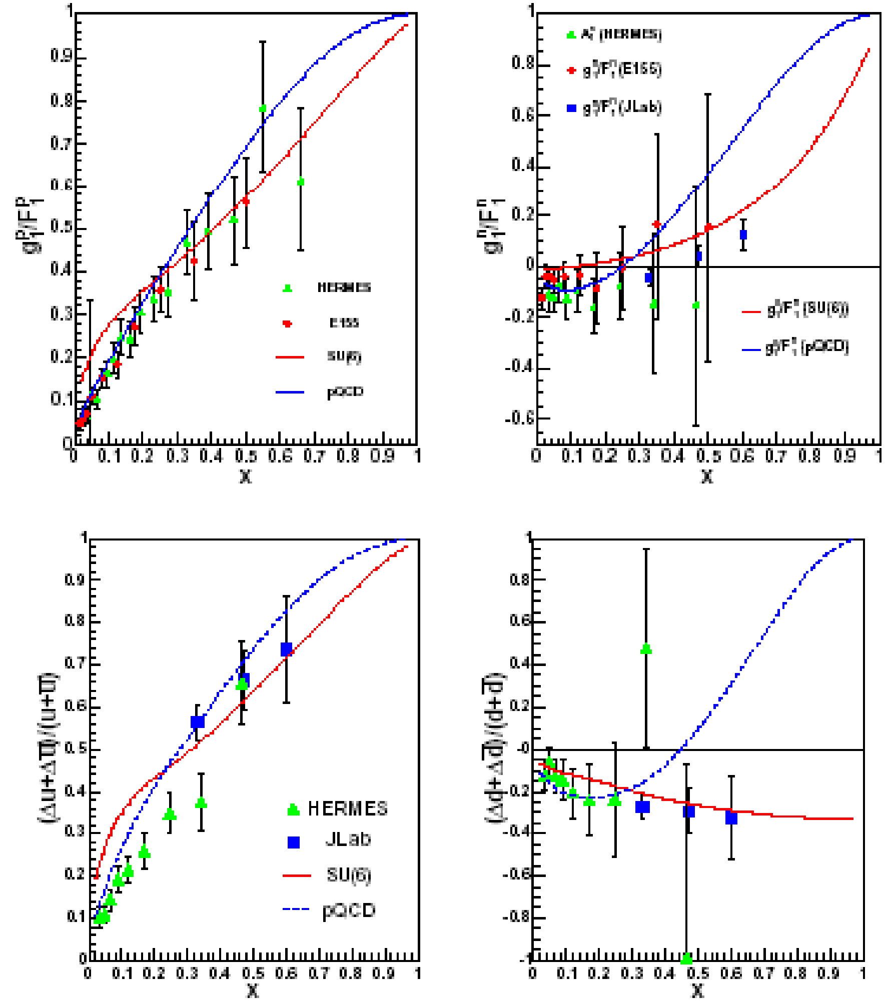
Baryon	q_1	q_2	$\tilde{A}_{q_1}$	$\tilde{C}_{q_1}$	$\tilde{A}_{q_2}$	$\tilde{C}_{q_2}$
p	u	d	1.375	0.625	0.275	0.725

B.-Q. Ma, I. Schmidt, J.-J. Yang, Phys.Rev.D63(2001) 037501.

New Development: H. Avakian, S.J.Brodsky, D.Boer, F.Yuan, Phys.Rev.Lett.99:082001,2007.

Different predictions in two models

- Helicity distribution
  - SU(6) quark-diquark model: $\Delta u(x)/u(x) \to 1$ as $x \to 1$. $\Delta d(x)/d(x) \to -\frac{1}{3}$ as $x \to 1$.
  - pQCD based counting rule analysis: $\Delta u(x)/u(x) \to 1$ as $x \to 1$. $\Delta d(x)/d(x) \to 1$ as $x \to 1$.



The Melosh-Wigner Rotation in Transversity

$$2\,\delta q = \langle\, p,\uparrow |\bar{q}_{\lambda}\gamma^{\perp}\gamma^{+}q_{-\lambda}| p,\downarrow \rangle$$

$$\delta q(x) = \int [\mathrm{d}^2 \boldsymbol{k}_{\perp}]\, \tilde{M}_q(x, \boldsymbol{k}_{\perp})\, \Delta q_{\mathrm{RF}}(x, \boldsymbol{k}_{\perp})$$

$$\tilde{M}_q(x, \boldsymbol{k}_{\perp}) = \frac{(k^{+} + m)^2}{(k^{+} + m)^2 + \boldsymbol{k}_{\perp}^2}$$

I.Schmidt&J.Soffer, Phys.Lett.B 407 (1997) 331

Transversity with Melosh-Wigner rotation in the quark-diquark model

$$\delta u_v(x) = \left[u_v(x) - \frac{1}{2} d_v(x) \right] \hat{W}_S(x) - \frac{1}{6} d_v(x) \hat{W}_V(x),$$

$$\delta d_v(x) = -\frac{1}{3} d_v(x) \hat{W}_V(x),$$

$\hat{W}_V(x)$ $w_S(x)$

B.-Q. Ma, I. Schmidt, J. Soffer, Phys.Lett. B 441 (1998) 461-467.

The transversity in pQCD, in similar to helicity distributions

$$\delta q(x) = \frac{\widetilde{A}_q}{B_3} x^{(-1/2)} (1-x)^3 - \frac{\widetilde{C}_q}{B_5} x^{(-1/2)} (1-x)^5$$

$B_3 = 32/35$ $B_3 = 512/693$

Baryon	q_1	q_2	$\tilde{A}_{q1}$	$\tilde{C}_{q1}$	$\tilde{A}_{q2}$	$\tilde{C}_{q2}$	$\hat{A}_{q1}$	$\hat{C}_{q1}$	$\hat{A}_{q2}$	$\hat{C}_{q2}$
p	u	d	1.375	0.625	0.275	0.725	1.52	0.48	0.305	0.695

B.-Q. Ma, I. Schmidt, J.-J. Yang, Phys.Rev.D63(2001) 037501.

Transversity in two models

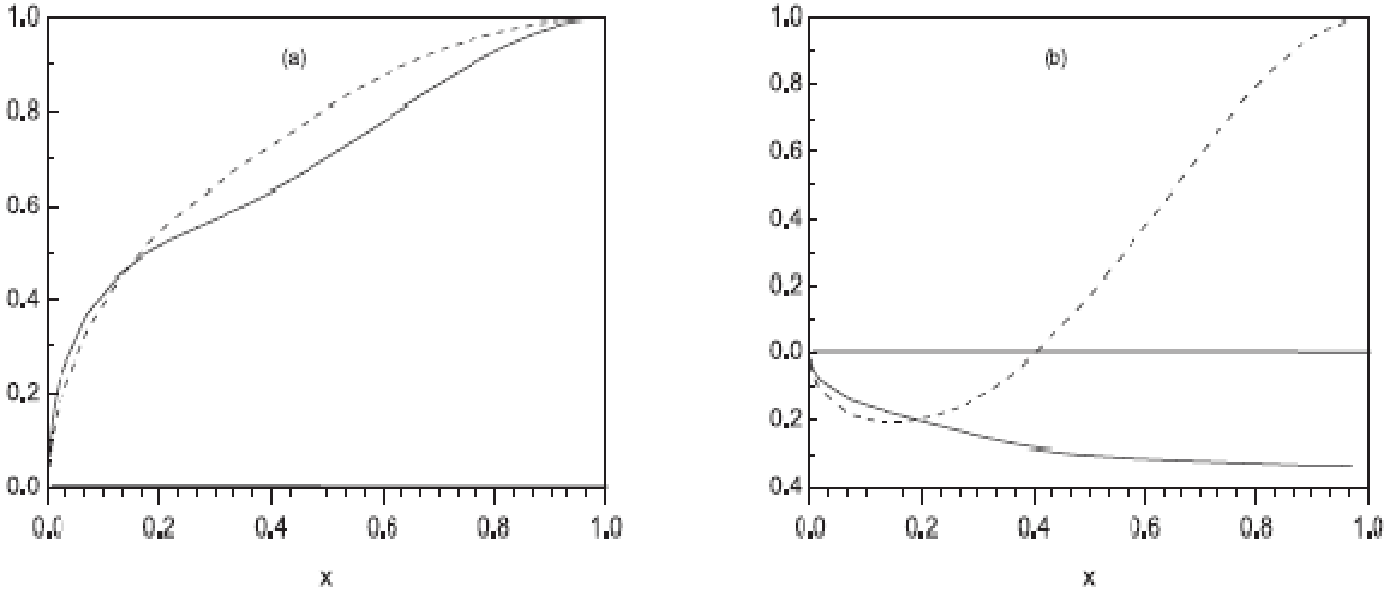
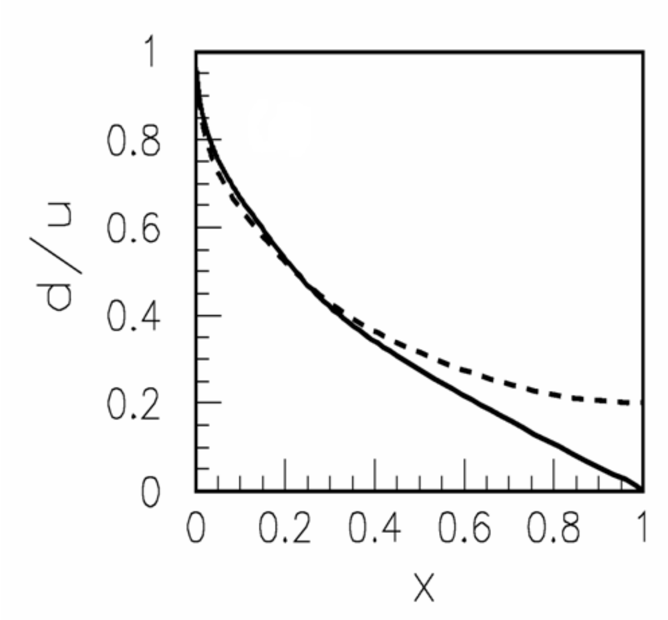
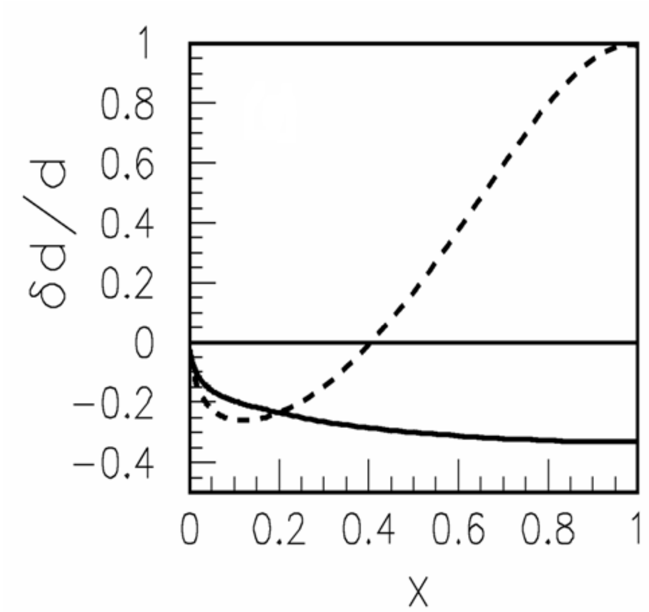


图 3.1 $\delta u/u$ (a) 和$\delta d/d$ (b) 的曲线示意图，$Q^2 = 2$ GeV^2, 实线代表的是quark-diquark 模型，虚线代表的是pQCD 理论.

SU(6) quark-diquark model VS pQCD based analysis

Ma, Schmidt and Yang, PRD 65, 034010 (2002)





solid curve for SU(6) and dashed curve for pQCD

The Melosh-Wigner Rotation in Quark Orbital Angular Moment

$$\hat{L}_q = -i\left(k_1 \frac{\partial}{\partial k_2} - k_2 \frac{\partial}{\partial k_1}\right).$$

$$L_q(x) = \int [d^2 k_\perp] M_L(x, k_\perp) \Delta q_{QM}(x, k_\perp)$$

$$M_L(x, k_\perp) = \frac{k_\perp^2}{(k^+ + m)^2 + k_\perp^2}$$

Ma&Schmidt, Phys.Rev.D 58 (1998) 096008

Three QCD spin sums for the proton spin

$$\vec{J}_{QCD} = \int d^3x \psi^\dagger \frac{\vec{\Sigma}}{2} \psi + \int d^3x \psi^\dagger \vec{x} \times (-i\nabla)\psi$$
$$+ \int d^3x \vec{E}^a \times \vec{A}^a - \int d^3x E^{ai} \vec{x} \times \nabla A^{ai}$$
$$\equiv \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g,$$

$$\vec{J}_{QCD} = \int d^3x \psi^\dagger \frac{\vec{\Sigma}}{2} \psi + \int d^3x \psi^\dagger \vec{x} \times (-i\vec{D})\psi + \int d^3x \vec{x} \times (\vec{E} \times \vec{B})$$
$$\equiv \vec{S}_q + \vec{L}_q + \vec{J}_g,$$

$$\vec{J}_{QCD} = \int d^3x \psi^\dagger \frac{\vec{\Sigma}}{2} \psi + \int d^3x \psi^\dagger \vec{x} \times (-i\vec{D}_{pure})\psi$$
$$+ \int d^3x \vec{E}^a \times \vec{A}^a_{phys} + \int d^3x E^{ai} \vec{x} \times \nabla A^{ai}_{phys}$$
$$\equiv \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g,$$

X.-S.Chen, X.-F.Lu, W.-M.Sun, F.Wang, T.Goldman, PRL100(2008)232002

Spin and orbital sum in light-cone formalism

$$\frac{1}{2}M_q + M_L = \frac{1}{2}$$

$$M_q(x,k_\perp) = \frac{(k^+ + m)^2 - k_\perp^2}{(k^+ + m)^2 + k_\perp^2} \qquad M_L(x,k_\perp) = \frac{k_\perp^2}{(k^+ + m)^2 + k_\perp^2}$$

$$\frac{1}{2}\Delta q(x) + L_q(x) = \frac{1}{2}\Delta q_{QM}(x)$$

Ma&Schmidt, Phys.Rev.D 58 (1998) 096008

The Melosh-Wigner Rotation in "Pretzelosity"

$$g_1^q(x,k_\perp) - h_1^q(x,k_\perp) = h_{1T}^{\perp(1)q}(x,k_\perp)\ .$$

$$\frac{(k^+ + m)^2 - \mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} - \frac{(k^+ + m)^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} = -\frac{\mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2}$$

$$\mathrm{Pretzelosity} - \Delta \mathrm{q} - \delta \mathrm{q} - -\mathrm{L_q}$$

$$\mathrm{Pretzelosity} = -\int [\mathrm{d}^2\mathbf{k}\_] \frac{\mathbf{k}_\perp^2}{(\mathrm{k}^+ + \mathrm{m})^2 + \mathbf{k}_\perp^2} \Delta \mathrm{q_{QM}}(\mathrm{x}, \mathbf{k}_\perp)$$

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

“Pretrel” or “Brezel”

“Pretrel” or “Brezel”

“Mahua(麻花)”: the Chinese Preztel

What is “Pretzelosity” ?

- Pretzelosity: one of the eight leading twist transverse dependent parton distributions (TMDs).
- The quark-quark correlator up to the leading twist

$$
\begin{aligned}
\Phi(x, \mathbf{p}_\perp) = \frac{1}{2}\{ & f_1 \not{n}_+ - f_{1T}^\perp \frac{\epsilon_\perp^{ij} p_\perp^i S_\perp^j}{M_N} \not{n}_+ \\
& + (S_\| g_{1L} + \frac{\mathbf{p}_\perp \cdot \mathbf{S}_\perp}{M_N} g_{1T}) \gamma_5 \not{n}_+ + h_{1T} \frac{[\not{S}_\perp, \not{n}_+]\gamma_5}{2} \\
& + (S_\| h_{1L}^\perp + \frac{\mathbf{p}_\perp \cdot \mathbf{S}_\perp}{M_N} h_{1T}^\perp) \frac{[\not{p}_\perp, \not{n}_+]\gamma_5}{2M_N} + i h_1^\perp \frac{[\not{p}_\perp, \not{n}_+]}{2M_N} \}. (7)
\end{aligned}
$$

P.J. Mulders and R.D. Tangerman, Nucl. Phys. **B 461**, 197 (1996), Erratum-ibid. **B 484**, 538 (1997). K. Goeke, A. Metz, and M. Schlegel, Phys. Lett. **B 618**, 90 (2005).

What is "Pretzelosity" ?

$$\frac{p_\perp^x p_\perp^y}{M_N^2} h_{1T}^\perp(x, p_\perp^2) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{16\pi^3} e^{i(xP^+\xi^- - \mathbf{p}_\perp\cdot\boldsymbol{\xi}_\perp)}$$
$$\times \langle PS^y|\bar{\psi}(0) i\sigma^{1+}\gamma_5\psi(0,\xi^-,\xi_\perp)|PS^y\rangle, \quad (12)$$

$|PS^y\rangle$: the hadronic state with a polarization in the y direction.

- Some properties of pretzelosity:
  1. It is chiral-odd, and needs a chiral-odd partner in the SIDIS.
  2. There is no gluon analog of pretzelosity.
  3. In a large class of models, it is the difference of helicity and transversity, and hence a measure for relativistic effects.

H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, arXiv:0805.3355.

A Simple Relation

- The difference of helicity and transversity is the first moment of pretzelosity.

$$h_{1T}^{\perp(1)qv}(x, \mathbf{p}_\perp) \equiv \frac{p_\perp^2}{2M_N^2} h_{1T}^{\perp qv}(x, \mathbf{p}_\perp) = g_1^{qv}(x, \mathbf{p}_\perp) - h_1^{qv}(x, \mathbf{p}_\perp),$$

- This relation has already been obtained in
  H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, arXiv:0805.3355. B. Pasquini, S. Cazzaniga and S. Boffi, Phys. Rev. **D 78**, 034025 (2008).
- But this relation is not fully satisfied in
  A. Bacchetta, F. Conti, and M. Radici, Phys. Rev. **D 78**, 074010 (2008).

Connection with Quark Orbital Angular Momentum

- The rotation factor for $\vec{x} \times -i\nabla$ is $\frac{p_\perp^2}{(x\mathscr{M}_D+m_q)^2+p_\perp^2}$
  B.-Q. Ma, I. Schmidt, Phys. Rev. **D 58**, 096008 (1998).
- a simple relation between the pretzelosity and the quark orbital angular momentum

$$L^{qv}(x,\mathbf{p}_\perp) = -h_{1T}^{\perp(1)qv}(x,\mathbf{p}_\perp) = h_1^{qv}(x,\mathbf{p}_\perp) - g_1^{qv}(x,\mathbf{p}_\perp), (21)$$

or at the integration level

$$L^{qv}(x) = \int d^2\mathbf{p}_\perp L^{qv}(x,\mathbf{p}_\perp) = -h_{1T}^{\perp(1)qv}(x) = h_1^{qv}(x) - g_1^{qv}(x).$$

- A measurement of pretzelosity may reveal the information on the quark orbital angular momentum.

Pretzelosity in SIDIS

- Pretzelosity can be measured through $\sin(3\phi_h - \phi_S)$ asymmetry in the SIDIS process, where the cross section can be written as

$$\frac{d^6\sigma_{UT}}{dxdyd\phi_S dzd^2\mathbf{P}_{h\perp}} = \frac{2\alpha^2}{sxy^2}\{(1-y+\frac{1}{2}y^2)F_{UU} + S_\perp \sin(3\phi_h - \phi_S)(1-y)F_{UT}^{\sin(3\phi_h-\phi_S)} + \ldots\}, \quad (23)$$

with $F_{UU} = \mathcal{F}[\omega_1 f_1 D_1]$, $F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{F}[\omega_2 h_{1T}^\perp H_1^\perp]$

- The $\sin(3\phi_h - \phi_S)$ asymmetry

$$A_{UT}^{\sin(3\phi_h-\phi_S)} = \frac{\frac{2\alpha^2}{sxy^2}(1-y)F_{UT}^{\sin(3\phi_h-\phi_S)}}{\frac{2\alpha^2}{sxy^2}(1-y+\frac{1}{2}y^2)F_{UU}}. \quad (24)$$

Quantities in Calculation

- DFs and FFs to be parametrized:

	x dependence	z dependence	TM dependence
f_1	well known	—	not so clear
$h_{1T}^{\perp}$	not known	—	not known
D_1	—	known	not so clear
$H_1^{\perp}$	—	a little known	not clear

- Theoretical understanding: non-perturbative, model calculation, cannot give the exact value so far.
- Transverse momentum dependence: not so clearly yet, usually parametrized in a Gaussian form.
- D_1 and $H_1^{\perp}$: Gaussian parametrization given by
  S. Kretzer, *et al.*, Eur. Phys. J. **C 22**, 269 (2001).
  M. Anselmino, *et al.*, arXiv:0807.0173.

Approach 0 to TMDs

- Starting with the equation

$$
\begin{aligned}
h_{1T}^{\perp(uv)}(x) &= \left[f_1^{(uv)}(x) - \frac{1}{2} f_1^{(dv)}(x)\right] \hat{W}_S(x) - \frac{1}{6} f_1^{(dv)}(x) \hat{W}_V(x), \\
h_{1T}^{\perp(dv)}(x) &= -\frac{1}{3} f_1^{(dv)}(x) \hat{W}_V(x), \qquad (25)
\end{aligned}
$$

  where $\hat{W}_D(x) = \int d^2\mathbf{p}_\perp \varphi^2(x, \mathbf{p}_\perp) W_D(x, \mathbf{p}_\perp) / \int d^2\mathbf{p}_\perp \varphi^2(x, \mathbf{p}_\perp)$
- $f_1(x)$: CTEQ6L as an input. $h_{1T}^{\perp}(x)$: from Eq. 25
- Transverse momentum dependence: Gaussian form.
- How to fit the Gaussian width? $p_{av}/k_{av} \approx 2$?
  H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, arXiv:0805.3355.

Approach 1 to TMDs

- Model calculation.

$$f_1^{(uv)}(x,\mathbf{p}_\perp) = \frac{1}{16\pi^3}\times(\frac{1}{3}\sin^2\theta\varphi_V^2+\cos^2\theta\varphi_S^2),$$
$$f_1^{(dv)}(x,\mathbf{p}_\perp) = \frac{1}{8\pi^3}\times\frac{1}{3}\sin^2\theta\varphi_V^2.$$

$$h_{1T}^{\perp(uv)}(x,\mathbf{p}_\perp) = -\frac{1}{16\pi^3}\times(\frac{1}{9}\sin^2\theta\varphi_V^2 W_V-\cos^2\theta\varphi_S^2 W_S),$$
$$h_{1T}^{\perp(dv)}(x,\mathbf{p}_\perp) = -\frac{1}{8\pi^3}\times\frac{1}{9}\sin^2\theta\varphi_V^2 W_V.$$

- $\varphi_D(x,\mathbf{p}_\perp)$: adopting the BHL form:

$$\varphi_D(x,\mathbf{p}_\perp)=A_D\exp\{-\frac{1}{8\alpha_D^2}[\frac{m_q^2+p_\perp^2}{x}+\frac{m_D^2+p_\perp^2}{1-x}]\},$$

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

Approach 2 to TMDs

- Staring with the equation (an unintegrated version)

$$h_{1T}^{\perp(uv)}(x,\mathbf{p}_\perp) = \left[f_1^{(uv)}(x,\mathbf{p}_\perp) - \frac{1}{2}f_1^{(dv)}(x,\mathbf{p}_\perp)\right]W_S(x,\mathbf{p}_\perp) - \frac{1}{6}f_1^{(dv)}(x,\mathbf{p}_\perp)W_V(x,\mathbf{p}_\perp),$$

$$h_{1T}^{\perp(dv)}(x,\mathbf{p}_\perp) = -\frac{1}{3}f_1^{(dv)}(x,\mathbf{p}_\perp)W_V(x,\mathbf{p}_\perp). \qquad (27)$$

- $f_1(x,\mathbf{p}_\perp)$: a Gaussian form

$$f_1(x,\mathbf{p}_\perp) = f_1(x)\frac{\exp(-p_\perp^2/p_{av}^2)}{\pi p_{av}^2}, \qquad (28)$$

with CTEQ6L parametrization for $f_1(x)$.

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

$h_{1T}^{\perp(1)}(x)$ and $f_1(x)$

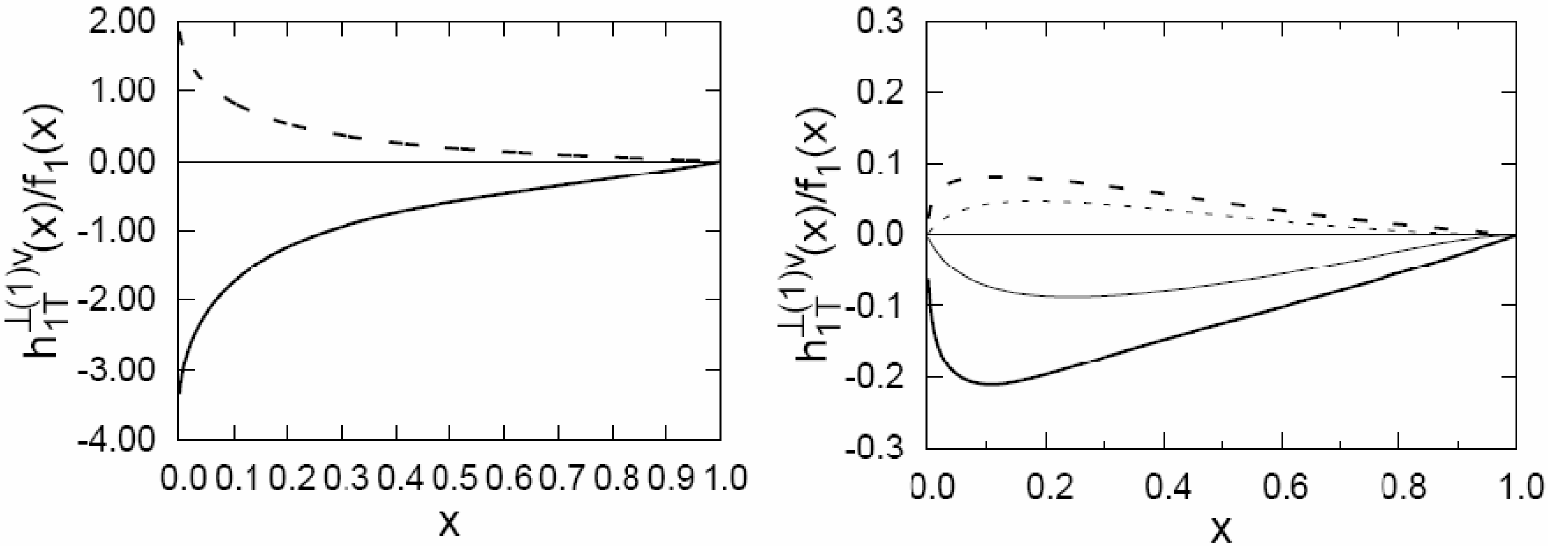


Figure: The ratio $h_{1T}^{\perp(1)(x)}/f_1(x)$. Left panel for approach 0 and right panel for approach 1 (thin curves) and approach 2 (thick curves). Solid curves for the u quark, and dashed curves for the d quark. Only valence quarks are considered.

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

Results at HERMES kinematics.

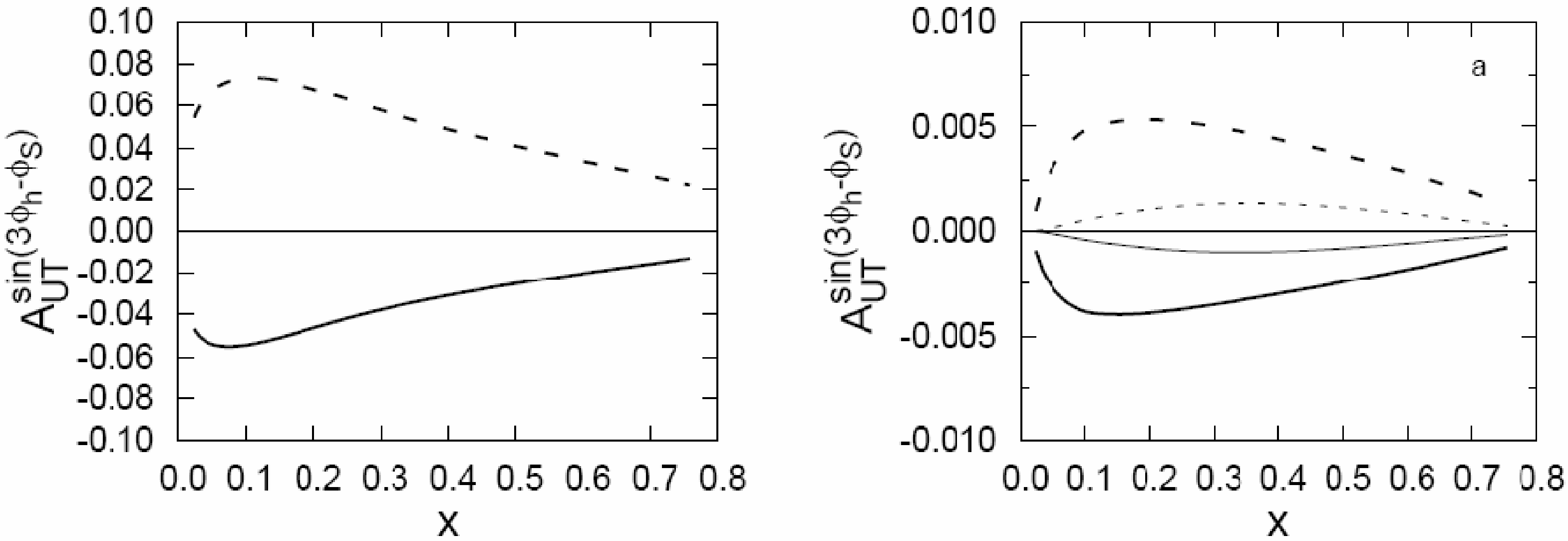


Figure: The results for HERMES kinematics with a proton target. Left panel for approach 0 and right panel for approach 1 (thin curves) and approach 2 (thick curves). Solid curves for the π^+ production, and dashed curves for the π^- production.

Results at COMPASS kinematics.

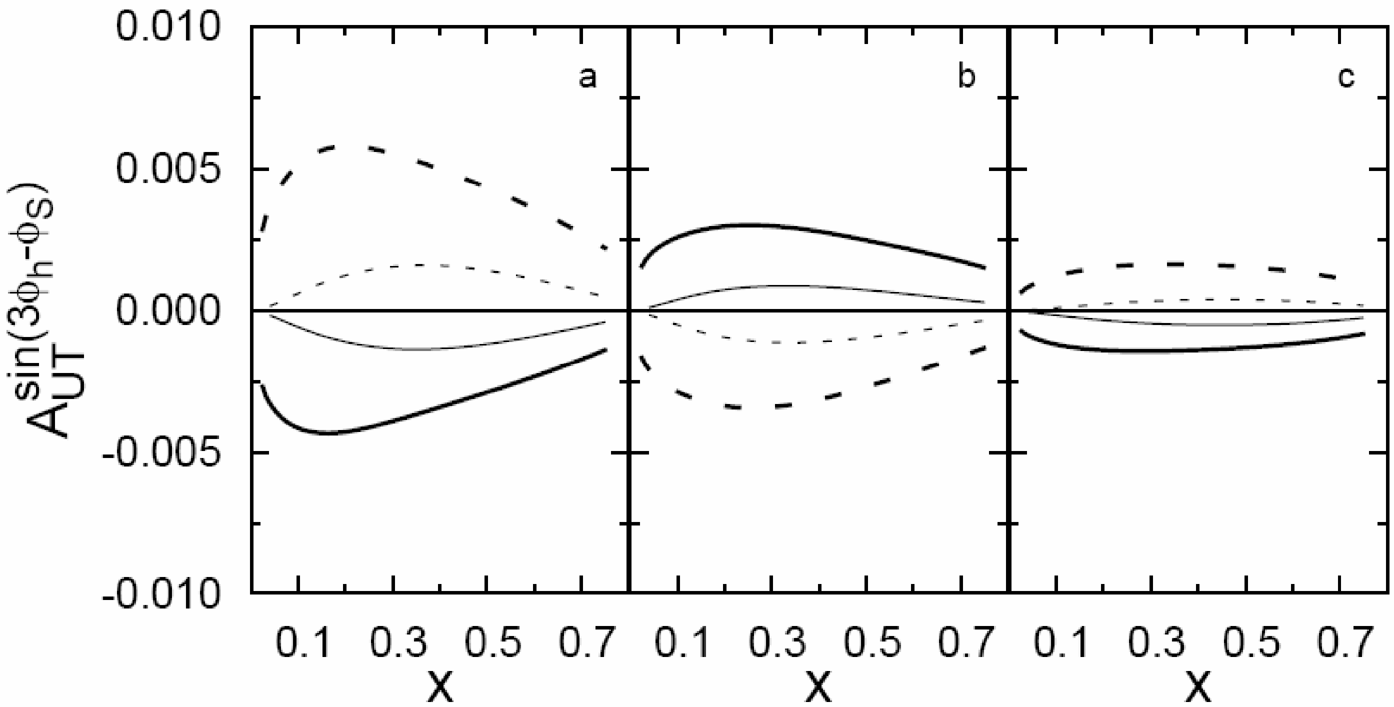


Figure: The results for COMPASS kinematics. a) proton target, b) neutron target, and c) deuteron target.

Results at JLab kinematics.

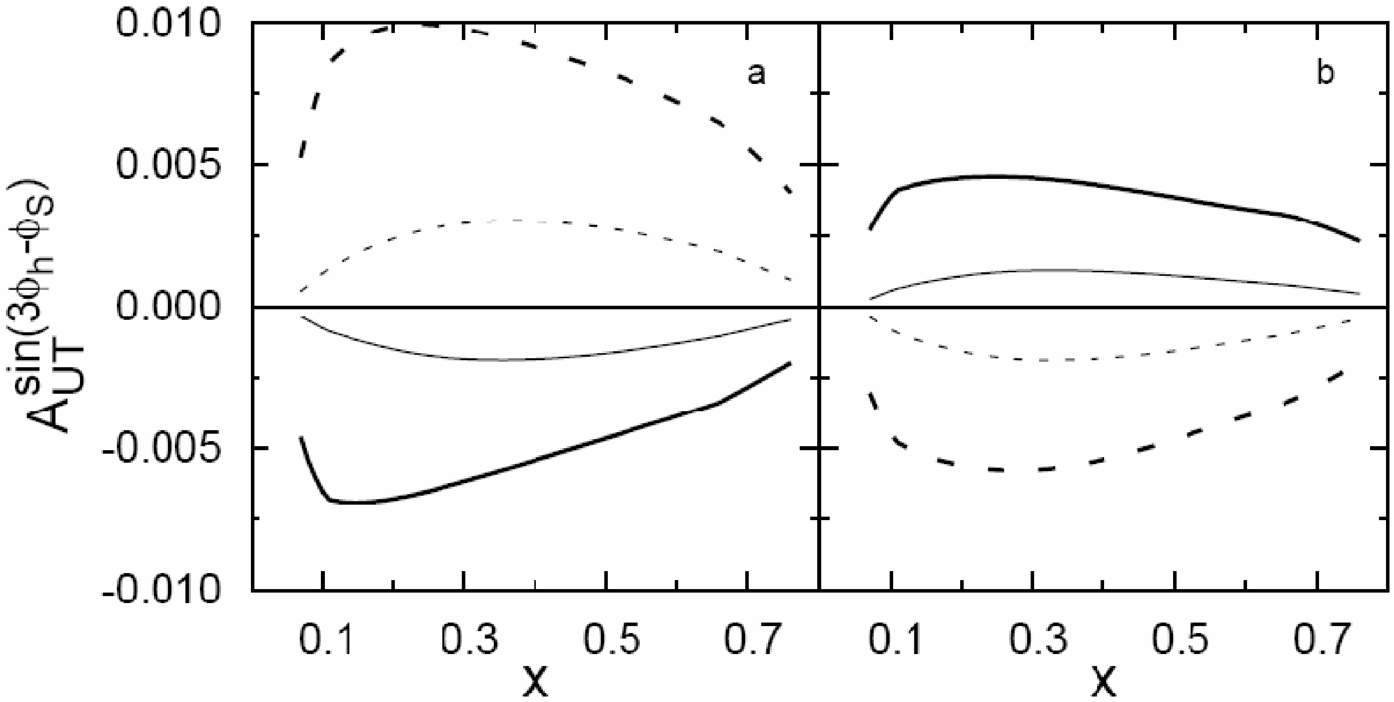


Figure: The results for JLab kinematics. a) proton target and b) neutron target.

Avakian's work

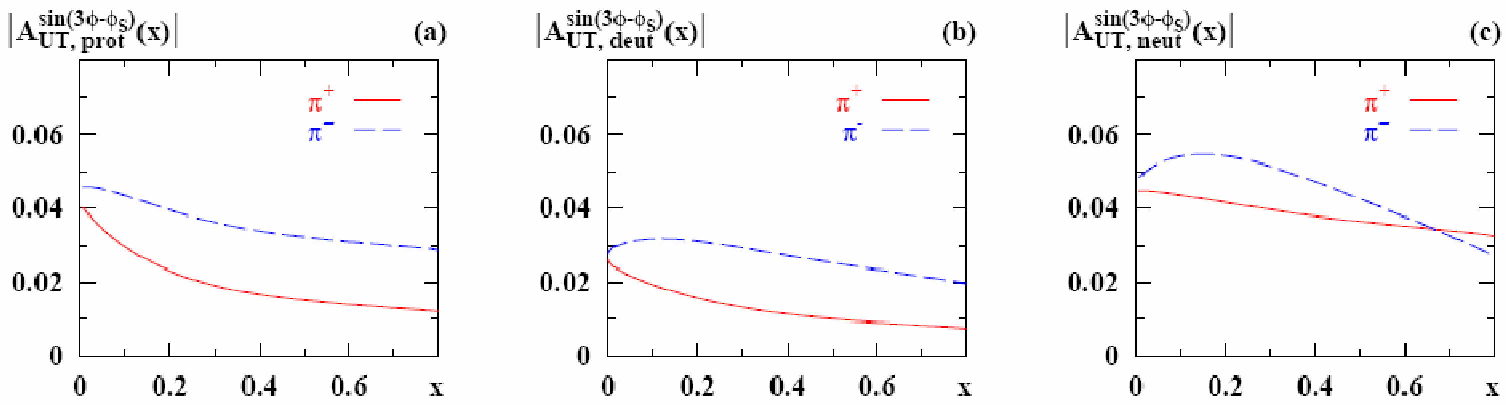


Figure: The predictions on the $\sin(3\phi_h - \phi_S)$ asymmetry at JLab kinematics. a) proton target, b) deuteron target and c) neutron target.

Much larger than our result!

H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, PRD 78, 114024 (2008) .

Boffi's work

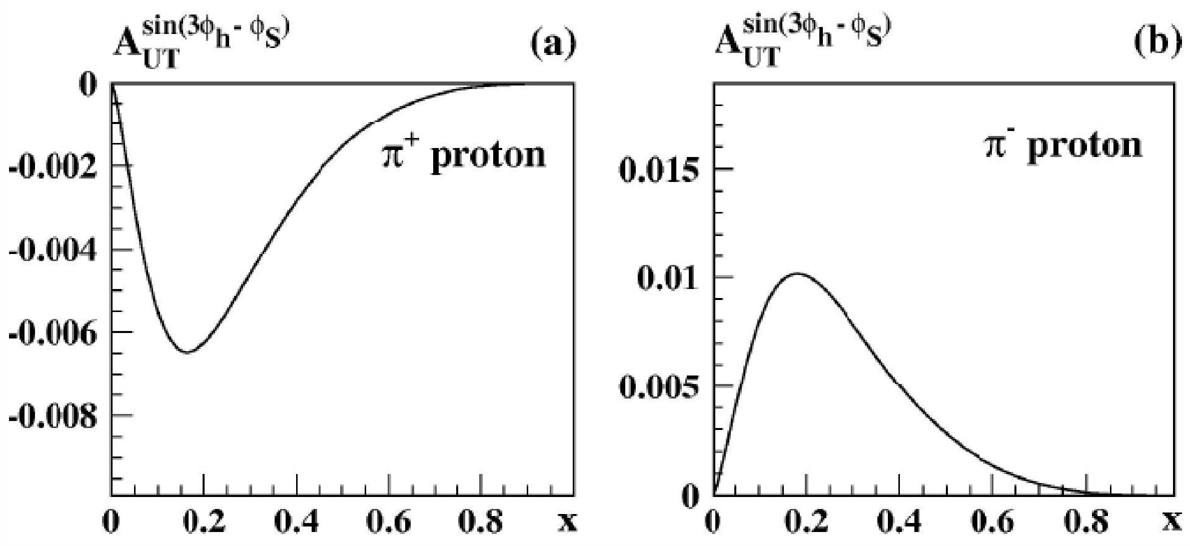


Figure: The $\sin(3\phi_h - \phi_S)$ asymmetry on a proton target.

Much smaller than Aviank's result, but a little larger than ours.
S. Boffi, A. V. Efremov, B. Pasquini, and P. Schweitzer, arXiv:0903.1271.

A short summary

- Results are sensitive to different transverse momentum approaches.
- The asymmetry is not an increasing function of x.
- The asymmetry is too small, up to a maximum less than 1%. A great challenge for a direct measurement.
- Can we enhance the asymmetry? We observe that the asymmetry is an increasing function of $\mathbf{p}_\perp^2$, but $\mathbf{p}_\perp^2$ cannot be manipulated directly.
- A compromise method is to select large $P_{h\perp}$ events instead, $\mathbf{P}_{h\perp} = z(\mathbf{p}_\perp - \mathbf{k}_\perp)$. We can exclude most small $p_\perp$ events.
- We will recalculate our results with a cutoff $P_{h\perp} > 1.0\mathrm{GeV}$.

HERMES results with a cutoff.

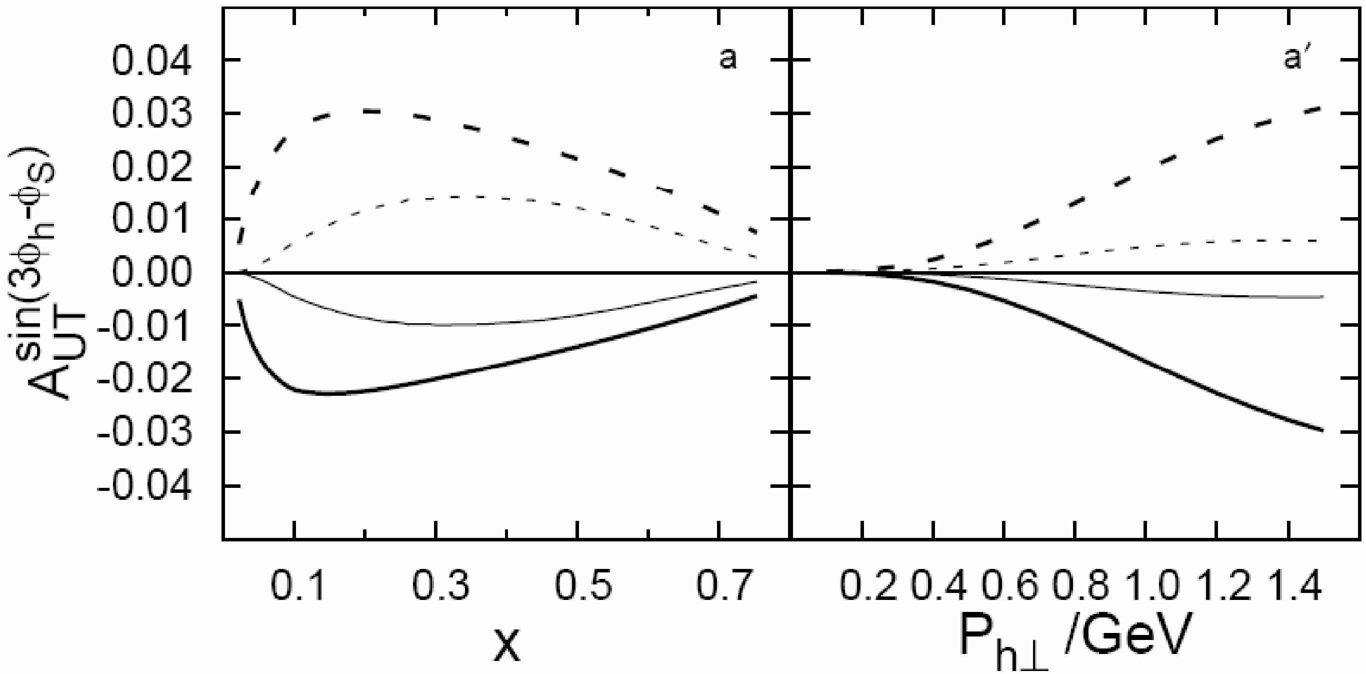


Figure: The results for HERMES kinematics on a proton target with a cutoff $P_{h\perp} > 1.0\,\mathrm{GeV}$, while the right panel shows the $P_{h\perp}$ dependence of the asymmetry after integrating all the other kinematic variables.

COMPASS results with a cutoff.

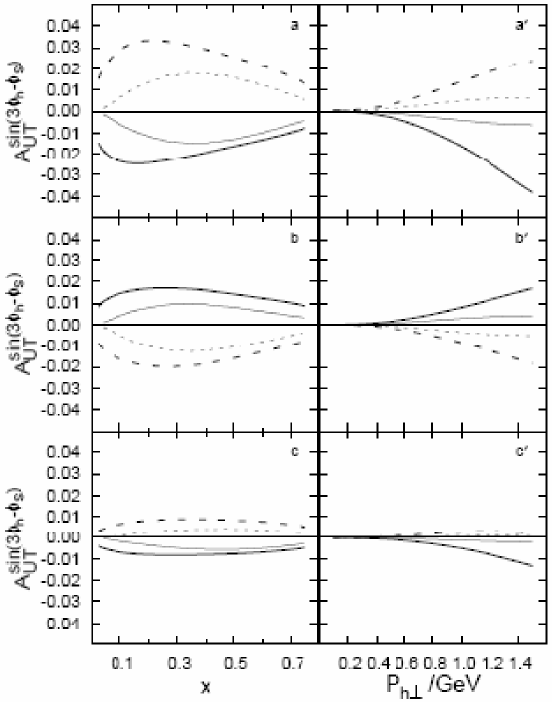


Figure: The results for COMPASS kinematics with a cutoff $P_{h\perp} > 1.0\,\mathrm{GeV}$. The upper, middle, and lower panels correspond to the proton, neutron, and deuteron target, respectively.

JLab results with a cutoff.

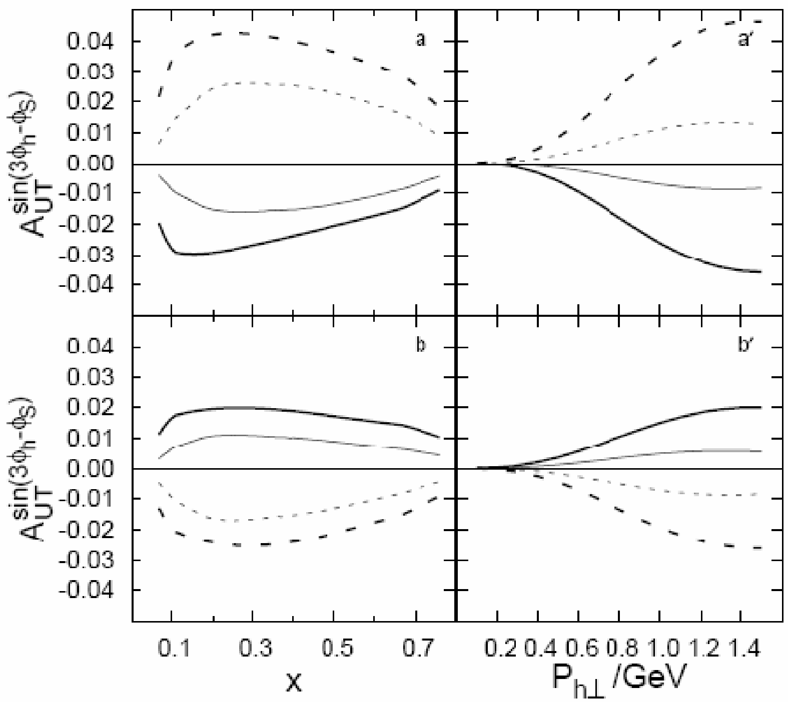


Figure: The results for JLab kinematics with a cutoff $P_{h\perp} > 1.0\ \mathrm{GeV}$. The upper and lower panels correspond to the proton and neutron target, respectively.

Caution

- The TMD factorization was proved to be valid only in the region $\Lambda_{\rm QCD} \ll P_{h\perp} \ll Q$.
- If $P_{h\perp} \sim \Lambda_{\rm QCD}$ and Q^2 is too large, a higher order pQCD correction (the gluon radiation) will be important.
- This transition point is around $P_{h\perp} \approx 1{\rm GeV}$.
- We must be careful and we assume that our results at a little larger $P_{h\perp}$ but not too larger than 1GeV are still acceptable.
- Another problem: the events will be exponentially suppressed at $P_{h\perp} \gg \Lambda_{\rm QCD}$, a challenge for the experiments to collect more data.

Conclusions

- **Relativistic effect of Melosh-Winger rotation is important in hadron spin physics.**
- **The pretzelosity is an important quantity for the spin-orbital correlation.**
- **New way to access quark orbital angular momentum is suggested.**