

# Azimuthal asymmetry in Semi-Inclusive Deep-Inelastic Scattering

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#### Introduction

Higher twist contributions and collinear expansion in semi-inclusive DIS

> Azimuthal asymmetries in the unpolarized Semiinclusive DIS process  $e + N \rightarrow e + q + X$  up to twist-4

#### Summary

### **Azimuthal asymmetry in unpolarized SIDIS**



1977: Georgi & Politzer, "Clean test to pQCD", Phys. Rev. Lett. 40,3 (77).



# **Azimuthal asymmetry in unpolarized SIDIS**



#### 1978: Cahn, Intrinsic momentum effects. Phys. Lett. B78B, 269 (1978).

Generalized parton model to include an intrinsic transverse momentum  $\vec{k}_{\perp}$  but no gluon radiation in the final state:

$$\langle \cos \varphi \rangle = -\frac{|\vec{k}_{\perp}|}{Q} \frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2} \qquad \langle \cos 2\varphi \rangle = -\frac{|\vec{k}_{\perp}|^2}{Q^2} \frac{2(1-y)}{1+(1-y)^2}$$
  
twist 3 twist 4

"Realistic studies": Monte-Carlo, experiments etc.

small  $p_{h\perp}$ , moderate Q, intrinsic  $k_{\perp}$  contribution dominates; large  $p_{h\perp}$ , large Q, Georgi-Politzer mechanism dominates.

# □ In DIS at intermediate energies: intrinsic transverse momentum $k_{\perp}$ effects have to be taken into account.

# Azimuthal asymmetry in polarized SIDIS

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#### Motivation: Single-spin asymmetry in pp collisions \_\_\_\_\_ azimuthal asymmetry in singly polarized pp collisions



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# **Azimuthal asymmetry in polarized SIDIS**



History: theoretical studies on single-spin asymmetry in  $p(\uparrow)+p\rightarrow\pi+X$ 

Factorization  $d\sigma_{p(\uparrow)+p\to\pi+X} \propto (p.d.f.) \otimes d\hat{\sigma}_{q(\uparrow)+q\to q+X} \otimes (frag.fun.)$ 

1978, Kane, Pumplin, Repko: pQCD  $a_N[q(\uparrow)+q\rightarrow q+q]=0$ .

**1991**, **Sivers**: asymmetric quark distribution (Sivers effect)

1993, Boros, Liang, Meng: quark orbital angular momentum & "surface effect"

**Collins:** Proof of non-existence of Sivers effect, asymmetric fragmentation function (Collins effect).

2002, Brodsky, Hwang, Schmidt: quark orbital angular momentum & "final state interaction".

Ji, Yuan: "final state interaction" = "gauge link" Collins: 1993's proof is wrong because forgot gauge link.



# **Conclusion:**

both Sivers and Collins effects can exist when gauge link is taken into account.

intrinsic transverse momentum and gauge link are important in studying azimuthal asymmetries in SIDIS.

# **Question:**

# Where does the gauge come from?



# **DIS without QCD interaction**

#### **Inclusive DIS with QCD interaction**



$$W_{\mu\nu}(q,p,S) = W^{(0)}_{\mu\nu}(q,p,S) + W^{(1)}_{\mu\nu}(q,p,S) + W^{(2)}_{\mu\nu}(q,p,S) + \dots$$



$$\hat{H}_{\mu\nu}^{(0)}(k,q) = \gamma_{\mu}(k+q)\gamma_{\nu}(2\pi)\delta_{+}\left((k+q)^{2}\right); \quad \hat{H}_{\mu\nu}^{(1,L;si)}(k,q) = \gamma_{\mu}\frac{(k_{1}+q)\gamma^{\rho}(k_{2}+q)}{(k_{2}+q)^{2}-i\varepsilon}\gamma_{\nu}(2\pi)\delta_{+}\left((k_{1}+q)^{2}\right)$$

#### Parton distribution/correlation: $\hat{\phi}^{(0)}(k,p,S) = \int d^4z e^{ikz} \langle p, S | \overline{\psi}(0)\psi(z) | p, S \rangle$

 $\hat{\phi}_{\rho}^{(1)}(k_1,k_2,p,S) = \int d^4z d^4y e^{ik_1y+ik_2(z-y)} \langle p,S | \overline{\psi}(0)gA_{\rho}(y)\psi(z) | p,S \rangle$  Not gauge invariant!

### Inclusive DIS with QCD interaction

#### **Collinear expansion:**

• Expanding the hard parts around k = xp:

$$\hat{H}_{\mu\nu}^{(0)}(k,q) = \hat{H}_{\mu\nu}^{(0)}(x) + \frac{\partial \hat{H}_{\mu\nu}^{(0)}(x)}{\partial k^{\rho}} \omega_{\rho}^{\rho'} k_{\rho'} + \dots$$
$$\hat{H}_{\mu\nu}^{(1)\rho}(k,q) = \hat{H}_{\mu\nu}^{(1)\rho}(x_1,x_2) + \frac{\partial \hat{H}_{\mu\nu}^{(1)\rho}(x_1,x_2)}{\partial k_1^{\sigma}} \omega_{\sigma}^{\sigma'} k_{1\sigma'} + \dots$$

Decomposition of the gluon field:

$$A_{\rho}(y) = n \cdot A(y) \frac{p_{\rho}}{n \cdot p} + \omega_{\rho}^{\rho'} A_{\rho'}(y)$$

Using the identities such as,

$$\frac{\partial \hat{H}_{\mu\nu}^{(0)}(x)}{\partial k^{\rho}} = -\hat{H}_{\mu\nu}^{(1)\rho}(x,x), \qquad p_{\rho}\hat{H}_{\mu\nu}^{(1,L)\rho}(x_1,x_2) = \frac{\hat{H}_{\mu\nu}^{(0)}(x_1)}{x_2 - x_1 - i\varepsilon}$$

#### to replace the derivatives etc.



Qiu, Sterman (90,91)  $x = k^{+} / p^{+}$   $\omega_{\rho}^{\rho'} \equiv g_{\rho}^{\rho'} - \overline{n}_{\rho} n^{\rho'}$   $\omega_{\rho}^{\rho'} k_{\rho'} = (k - xp)_{\rho}$   $k^{\pm} = \frac{1}{\sqrt{2}} (k_{0} \pm k_{3})$ 

$$k^{\pm} = \frac{-}{\sqrt{2}} (k_0 \pm k_3)$$
$$n = (0, 1, \vec{0}_{\perp})$$
$$\overline{n} = (1, 0, \vec{0}_{\perp})$$

#### **Inclusive DIS with QCD interaction**



$$W_{\mu\nu}(q,p,S) = \tilde{W}_{\mu\nu}^{(0)}(q,p,S) + \tilde{W}_{\mu\nu}^{(1)}(q,p,S) + \tilde{W}_{\mu\nu}^{(2)}(q,p,S) + \dots$$

$$\begin{split} \widetilde{W}_{\mu\nu}^{(0)}(q,p,S) &= \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr} \Big[ \hat{\Phi}^{(0)}(k,p,S) \; \hat{H}_{\mu\nu}^{(0)}(x) \; \Big] \\ \hat{\Phi}^{(0)}(k,p,S) &= \int d^4z e^{ikz} \langle p,S \; | \; \overline{\psi}(0) \mathcal{L}(0,z) \psi(z) \; | \; p,S \rangle \\ \mathcal{L}(0,z) &= P e^{i \frac{b}{0}^{\int_0^2 d^{-A^+(0,y^-,\bar{0}_\perp)}} = 1 + ig \int_0^{z^-} dy^- A^+(0,y^-,\bar{0}_\perp) + (ig)^2 \int_0^{z^-} dy^- A^+(0,y^-,\bar{0}_\perp) + ...} \\ \widetilde{W}_{\mu\nu}^{(1)}(q,p,S) &= \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \; \mathrm{Tr} \Big[ \hat{\Phi}_{\rho'}^{(1)}(k_1,k_2,p,S) \; \hat{H}_{\mu\nu'}^{(1)\rho}(x_1,x_2) \; \boldsymbol{\varpi}_{\rho}^{\rho'} \; \Big] \\ \hat{\Phi}_{\rho}^{(1)}(k_1,k_2,p,S) &= \int d^4z d^4y e^{ik_1y+ik_2(z-y)} \langle p,S \; | \; \overline{\psi}(0) \mathcal{L}(0,y) D_{\rho}(y) \mathcal{L}(y,z) \psi(z) \; | \; p,S \rangle \\ D_{\rho}(y) &= -i\partial_{\rho} + gA_{\rho}(y) \end{split}$$



#### **Conclusion:**

Gauge link comes from the multiple gluon scattering and collinear expansion is the necessary procedure to obtain the correct form of gauge invariant parton distributions.

# **Question:** How about semi-inclusive DIS?

#### **Semi-Inclusive DIS with QCD interaction**



#### Consider first $e + p \rightarrow e + q + X$ , i.e., NO fragmentation

$$W_{\mu\nu}^{(si)}(q,p,k',S) = \sum_{X} \langle p, S | J_{\mu}(0) | k', X \rangle \langle k', X | J_{\nu}(0) | p, S \rangle (2\pi)^{4} \delta^{4}(p+q-p_{X})$$
  
=  $W_{\mu\nu}^{(0,si)}(q,p,S,k') + W_{\mu\nu}^{(1,si)}(q,p,S,k') + W_{\mu\nu}^{(2,si)}(q,p,S,k') + \dots$ 



$$W_{\mu\nu}^{(0,si)}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4} \operatorname{Tr}[\hat{\phi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0,si)}(k,q)]$$

$$\hat{H}_{\mu\nu}^{(0,si)}(k,q) = \gamma_{\mu}(k+q)\gamma_{\nu}(2\pi)^{4}\delta^{4}(k'-k-q)$$

C.f.: 
$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \gamma_{\mu}(k+q)\gamma_{\nu}(2\pi)\delta_{+}((k+q)^{2})$$

#### **Semi-Inclusive DIS with QCD interaction**

#### Using the mathematical identity:

$$(2\pi)^4 \delta^4(k'-k-q) = (2\pi)\delta_+ \left((k-q)^2\right)(2\pi)^3(2E_{k'})\delta^3(\vec{k'}-\vec{k}-\vec{q})$$

We obtain:  $\hat{H}^{(0,si)}_{\mu\nu}(k,q) = \hat{H}^{(0)}_{\mu\nu}(k,q)(2E_{k'})(2\pi)^3 \delta^3(\vec{k'}-\vec{k}-\vec{q})$ 

$$\hat{H}_{\mu\nu}^{(1,\rho,c,si)}(k_1,k_2,q) = \hat{H}_{\mu\nu}^{(1,\rho,c)}(k_1,k_2,q)(2E_{k'})(2\pi)^3\delta^3(\vec{k'}-\vec{k_c}-\vec{q})$$

$$W_{\mu\nu}^{(0,si)}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4} \operatorname{Tr}[\hat{\phi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(k,q)] (2E_{k'})(2\pi)^3 \delta^3(\vec{k'}-\vec{k}-\vec{q}) \\ W_{\mu\nu}^{(0)}(q,p,S) \\ W_{\mu\nu}^{(1,si)}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \sum_{c=\mathrm{L},\mathrm{R}} \operatorname{Tr}[\hat{\phi}_{\rho}^{(1)}(k_1,k_2,p,S)\hat{H}_{\mu\nu}^{(1,c)\rho}(k_1,k_2,q)] \\ \times (2E_{k'})(2\pi)^3 \delta^3(\vec{k'}-\vec{k_c}-\vec{q}) \\ W_{\mu\nu}^{(1)}(q,p,S)$$

#### **Semi-Inclusive DIS with QCD interaction**



# The same collinear expansion leads to

 $W_{\mu\nu}^{(si)}(q,p,S,k') = \tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k') + \dots$ 

$$\begin{split} \widetilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') &= \int \frac{d^4k}{(2\pi)^4} \operatorname{Tr}[\hat{\varPhi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(x)] (2E_{k'})(2\pi)^3 \delta^3(\vec{k'}-\vec{k}-\vec{q}) \\ \widetilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') &= \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \sum_{c=\mathrm{L,R}} \operatorname{Tr}[\hat{\varPhi}_{\rho'}^{(1)}(k_1,k_2,p,S)\hat{H}_{\mu\nu}^{(1,c)\rho}(x_1,x_2)\omega_{\rho}^{\rho'}] \\ &\times (2E_{k'})(2\pi)^3 \delta^3(\vec{k'}-\vec{k_c}-\vec{q}) \\ \widetilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k') &= \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \sum_{c=\mathrm{L,C,R}} \operatorname{Tr}[\hat{\varPhi}_{\rho'\sigma'}^{(2)}(k_1,k_2,k,p,S)\hat{H}_{\mu\nu}^{(2,c)\rho\sigma}(x_1,x_2,x)\omega_{\rho}^{\rho'}\omega_{\sigma'}^{\gamma'}] \\ &\times (2E_{k'})(2\pi)^3 \delta^3(\vec{k'}-\vec{k_c}-\vec{q}) \end{split}$$

Gauge invariant parton distributions/correlations (with gauge link)

#### SIDIS with $k_{\perp}$ : Direct consequence I



# Consider the contribution from $\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k')$



(with transverse momentum)

#### SIDIS with $k_{\perp}$ : Direct consequence II



**A simplification of**  $\widetilde{W}^{(1,si)}_{\mu\nu}(q,p,S,k')$ 



$$\hat{H}_{\mu\nu}^{(1,L)\rho}(x_1, x_2)\omega_{\rho}^{\rho'}$$

$$= \frac{\pi}{2q \cdot p} \delta(x_1 - x_B)\omega_{\rho}^{\rho'} \gamma_{\mu} \hbar \gamma^{\rho} \overline{h} \gamma_{\nu}$$

$$\equiv \hat{H}_{\mu\nu}^{(1)\rho'}(x_1) \qquad \text{independent of } x_2$$



$$\hat{H}_{\mu\nu}^{(1,R)\rho}(x_1, x_2)\omega_{\rho}^{\rho'}$$

$$= \frac{\pi}{2q \cdot p} \delta(x_2 - x_B)\omega_{\rho}^{\rho'} \gamma_{\mu} \overline{m} \gamma^{\rho} m \gamma_{\nu}$$

$$= \gamma_0 \hat{H}_{\mu\nu}^{(1)\rho'+}(x_2) \gamma_0 \quad \text{independent of } x_1!$$



### This leads to

$$\widetilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \sum_{c=L,R} \text{Tr}[\hat{\varPhi}_{\rho'}^{(1)}(k_1,k_2,p,S) \hat{H}_{\mu\nu}^{(1,c)\rho}(x_1,x_2) \omega_{\rho}^{\rho'}] \times (2E_{k'})(2\pi)^3 \delta^3(\vec{k'}-\vec{k_c}-\vec{q})$$

$$= 2 \operatorname{Re} \int \frac{d^4 k}{(2\pi)^4} \operatorname{Tr} [\hat{\varphi}_{\rho}^{(1)}(k,p,S) \hat{H}_{\mu\nu}^{(1)\rho}(xp)] (2E_{k'}) (2\pi)^3 \delta^3 (\vec{k'} - \vec{k} - \vec{q})$$

Only 
$$\hat{\varphi}_{\rho}^{(1)}(k, p, S) = \int \frac{d^4 k_2}{(2\pi)^4} \hat{\varphi}_{\rho}^{(1)}(k, k_2, p, S)$$
  
=  $\int d^4 z e^{ikz} \langle p, S | \overline{\psi}(0) \mathcal{L}(0, z) D_{\rho}(z) \psi(z) | p, S \rangle$ 

contributes in semi-inclusive deep-inelastic lepton-nucleon scattering.

Similar for 
$$\widetilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k')$$

#### SIDIS with $k_{\perp}$ : differential cross-section to $1/Q^2$



A complete twist-4 result for  $e+N \rightarrow e+q+X$ 

$$\frac{d\sigma}{dxdyd^{2}k_{\perp}} = \frac{2\pi o_{\ell_{em}}^{2} e_{q}^{2} \mathbf{1}}{Q^{2}y} \left\{ \left[ \mathbf{1} + (\mathbf{1} - y)^{2} \right] f_{q} (x, k_{\perp}) - 4(2 - y)\sqrt{1 - y} \frac{|\vec{k}_{\perp}|}{Q} x f_{q\perp}(x, k_{\perp}) \cos \phi - 4(1 - y) \frac{|\vec{k}_{\perp}|^{2}}{Q^{2}} x \left[ \varphi_{\perp 2}^{(1)}(x, k_{\perp}) - \tilde{\varphi}_{\perp 2}^{(1)}(x, k_{\perp}) \right] \cos 2\phi + 8(1 - y) \left( \frac{|\vec{k}_{\perp}|^{2}}{Q^{2}} x \left[ \varphi_{\perp 2}^{(1)}(x, k_{\perp}) - \tilde{\varphi}_{\perp 2}^{(1)}(x, k_{\perp}) \right] + \frac{2x^{2}M^{2}}{Q^{2}} f_{q(-)}(x, k_{\perp}) \right) - 2 \left[ \mathbf{1} + (1 - y)^{2} \right] \frac{|\vec{k}_{\perp}|^{2}}{Q^{2}} x \varphi_{\perp 2}^{(2,L)}(x, k_{\perp}) \right\}$$

 $\begin{aligned} \text{TMD quark distribution:} \quad & f_q(x,k_\perp) = \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ixp^+ z^- - i\bar{k}_\perp \cdot \bar{z}_\perp} \langle p \,|\, \overline{\psi}(0) \mathcal{L}(0,z) \frac{\gamma^+}{2} \psi(z) \,|\, p \rangle \\ \text{Quark correlation} \\ \text{functions such as :} \quad & f_{q\perp}(x,k_\perp) = \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ixp^+ z^- - i\bar{k}_\perp \cdot \bar{z}_\perp} \langle p \,|\, \overline{\psi}(0) \mathcal{L}(0,z) \frac{\gamma \cdot k_\perp}{2k_\perp^2} \psi(z) \,|\, p \rangle \\ & k_\perp^2 \varphi_{\perp 2}^{(1)}(x,k_\perp) = -(2\hat{k}_\perp^\rho \hat{k}_\perp^\alpha + d^{\rho\alpha}) \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ixp^+ z^- - i\bar{k}_\perp \cdot \bar{z}_\perp} \langle p \,|\, \overline{\psi}(0) \mathcal{L}(0,z) \frac{\gamma_\alpha}{2} D_\rho(z) \psi(z) \,|\, p \rangle \end{aligned}$ 

#### SIDIS with $k_{\perp}$ : Azimuthal asymmetries



Up to twist-4 
$$\langle \cos \phi \rangle = -\frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2} \frac{|\vec{k}_{\perp}|}{Q} \frac{xf_{q\perp}(x,k_{\perp})}{f_q(x,k_{\perp})}$$

$$\langle \cos 2\phi \rangle = -\frac{2(1-y)}{1+(1-y)^2} \frac{|\vec{k}_{\perp}|^2}{Q^2} \frac{x[\varphi_{\perp 2}^{(1)}(x,k_{\perp}) - \tilde{\varphi}_{\perp 2}^{(1)}(x,k_{\perp})]}{f_q(x,k_{\perp})}$$

#### If g=0, i.e., no multiple gluon scattering (results by Cahn):

$$\begin{aligned} \left<\cos\phi\right>|_{g=0} &= -\frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2} \frac{|\vec{k}_{\perp}|}{Q}, \quad \text{proportional to} \quad \frac{|\vec{k}_{\perp}|}{Q}, \\ \left<\cos 2\phi\right>|_{g=0} &= -\frac{2(1-y)}{1+(1-y)^2} \frac{|\vec{k}_{\perp}|^2}{Q^2} \quad \text{proportional to} \quad \frac{|\vec{k}_{\perp}|^2}{Q^2}, \end{aligned}$$

#### A good place to study such correlation functions and effects of multiple gluon scattering.



#### Gauge link comes from:



Replace *N* by *A*, the gluons can connect to different nucleons in *A*.



Nuclear enhancement Transverse momentum broadening

Transverse momentum broadening should be contained in the gauge link

With "maximal two gluon approximation":

$$f_{q}^{A}(x,\vec{k}_{\perp}) = \frac{A}{\pi \Delta_{2F}} \int d^{2}l_{\perp} e^{-(\vec{k}_{\perp}-\vec{l}_{\perp})^{2}/\Delta_{2F}} f_{q}^{N}(x,\vec{l}_{\perp})$$

talk by Jian-hua Gao on 30th

# Summary



- Gauge link is result of multiple gluon scattering and collinear expansion is a necessary procedure to obtain the correct form.
- > Collinear expansion can be extended to SIDIS  $e+p \rightarrow e+q+X$ .
- ➢ Naïve extension of TMD parton distributions convoluting with eq → eq cross section to include intrinsic transverse momentum is incorrect.
- Many other consequences.

# Summary





# Thank you for your attention!

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