Deep-Inelastic Scattering and Parton Distribution and Correlation Functions

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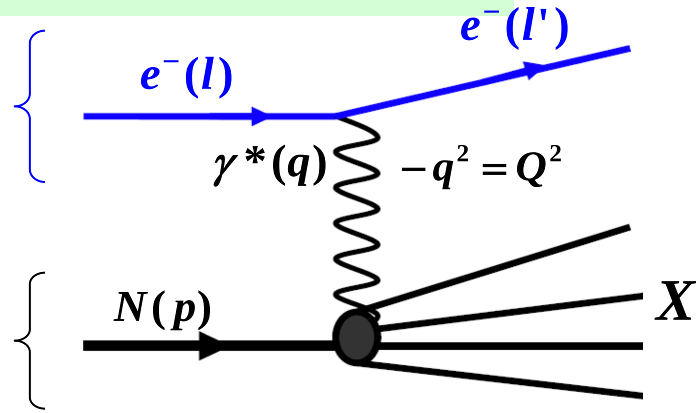
Deep-inelastic lepton-nucleon scattering $e^- + N \to e^- + X$

The differential cross section:

$$d\sigma = \frac{\alpha_{em}^2}{sQ^4} L^{\mu\nu}(l,l')\, W_{\mu\nu}(q,p) \frac{d^3 l'}{2E'}$$

leptonic tensor

hadronic tensor



电子：（电磁相互作用 点结构） ⟹ $L^{\mu\nu}(l,l')$ 完全确定 ⟹ 核子结构理想的探针

$W_{\mu\nu}$的算符表达式：$W_{\mu\nu}(q,p) = \sum_X \langle p | J_\mu(0) | X \rangle \langle X | J_\nu(0) | p \rangle (2\pi)^4 \delta^4(p+q-p_X)$

electromagnetic current：$J_\mu(x) = e\overline{\psi}(x)\gamma_\mu \psi(x)$, $\quad | p \rangle$: wave function of nucleon

一般形式：$W_{\mu\nu}(q,p) = (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) F_1(x,Q^2) + \frac{1}{4x^2 p\cdot q}(q_\mu + 2xp_\mu)(q_\nu + 2xp_\nu) F_2(x,Q^2)$

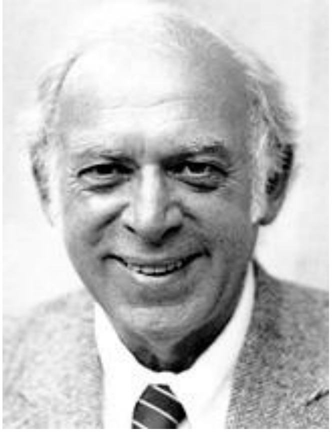
structure function of nucleon

Introduction: DIS and Nucleon Structure

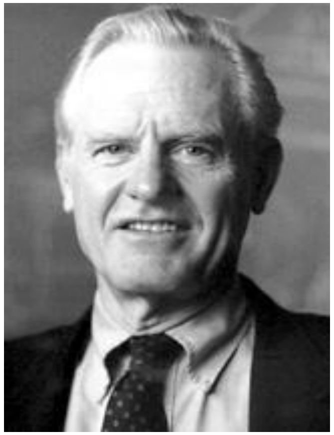
几十年实验与理论研究，取得了巨大成就：

Bjorken Scaling	$F_i(x,Q^2)=F_i(x)$	⟹	质子由更小的点粒子构成
Callen-Gross关系	$F_2(x)=2xF_1(x)$	⟹	构成质子的点粒子自旋为1/2
Gottfried求和规则	$\int\frac{dx}{x}[F_2^p(x)-F_2^n(x)]=e_u^2-e_d^2=1/3$	⟹	构成质子的点粒子带分数电荷

Nobel Prize 1990



Jerome I. Friedman



Henry W. Kendall

Richard E. Taylor

“We see quarks through deeply inelastic lepton-nucleon scattering.”

⟹ **部分子模型：目前核子结构最成功的描述。**

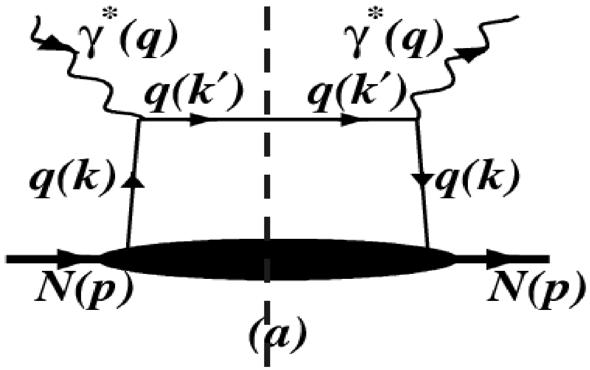
Parton model:

electron scatter with free parton

$$|\mathcal{M}(eN \to eX)|^2 = \sum_q f_q(x) |\mathcal{M}(eq \to eq)|^2$$

$f_q(x)$: number density of parton *q* in nucleon.

$$\Rightarrow \quad F_1(x,Q^2) = \sum e_q^2 f_q(x)$$



Field operator expression of the parton distribution function

$$f_q(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p | \overline{\psi}(0) \frac{\gamma^+}{2} \psi(z) | p \rangle$$

not gauge invariant !

Gauge invariant form:

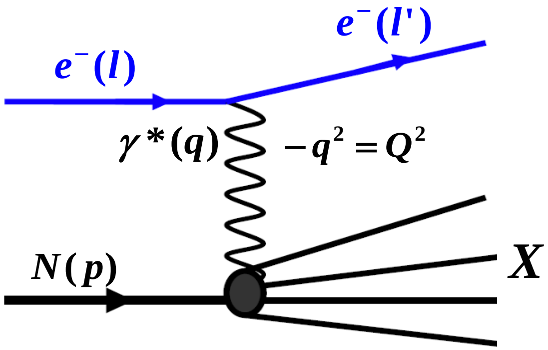
$$f_q(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p | \overline{\psi}(0) \frac{\gamma^+}{2} \mathcal{L}(0,z) \psi(z) | p \rangle, \quad \mathcal{L}(0,z) = P e^{ig\int_0^z dy^- A^+(0,y^-,\vec{0}_\perp)}$$

gauge link

Questions: How do we get these expressions?
Where does the gauge link come from?

Inclusive DIS: where does the gauge link come from?

Consider $e^- + N \to e^- + X$



$$d\sigma = \frac{1}{2sTV}\sum_X \int \frac{d^3 p_X}{(2\pi)^3 2E_X} |\mathcal{M}(eN \to eX)|^2 \frac{d^3 l'}{(2\pi)^3 2E'}$$

The scattering matrix element:

$$\mathcal{M}(eN \to eX) = \langle l' X | \hat{S} | l\, p\rangle = \langle l' X | T\int d^4x d^4y\, j_l^\mu(x) A_\mu(x) J^\nu(y) A_\nu(x) | l\, p\rangle$$

$$= \int d^4x d^4y e^{i(l-l')\cdot x}\, \bar{u}(l')\gamma^\mu u(l) \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)} \frac{g_{\mu\nu}}{q^2}\; e^{i(p-p_X)\cdot y} \langle X | J^\nu(0) | p\rangle$$

$$= \bar{u}(l')\gamma^\mu u(l) \frac{ig_{\mu\nu}}{q^2 + i\varepsilon} \langle X | J^\nu(0) | p\rangle\; (2\pi)^4 \delta^4(p + l - p_X - l')$$

$$j_l^\mu(x) = \bar{\psi}(x)\gamma^\mu \psi(x) \quad \langle l' | j^\mu(x) | l\rangle = e^{-i(l-l')\cdot x}\bar{u}(l')\gamma^\mu u(l), \; \langle X | J^\nu(y) | p\rangle = e^{-i(p-p_X)\cdot y}\langle X | J^\nu(0) | p\rangle$$

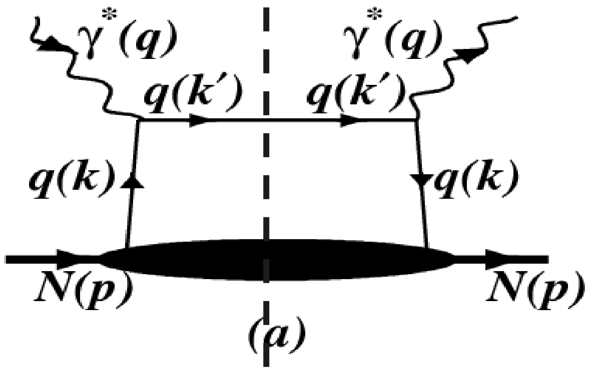
$$\langle 0 | T A_\mu(x) A_\nu(y) | 0\rangle = \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)} D_{F,\mu\nu}(q), \quad D_{F,\mu\nu}(q) = \frac{ig_{\mu\nu}}{q^2 + i\varepsilon}$$

$$\Rightarrow \quad d\sigma = \frac{e^4}{2sq^4} L^{\mu\nu}(l,l') W_{\mu\nu}(q,p) \frac{d^3 l'}{(2\pi)^3 2E'},$$

$$L^{\mu\nu}(l,l') = \bar{u}(l')\gamma^\mu u(l)\bar{u}(l)\gamma^\nu u(l'), \qquad W_{\mu\nu}(q,p) = \sum_X \langle p | J_\mu(0) | X\rangle\langle X | J_\nu(0) | p\rangle (2\pi)^4 \delta^4(p + q - p_X)$$

Inclusive DIS: where does the gauge link come from?

Parton model $|X\rangle = |X', k'\rangle,\quad \sum_X = \sum_{X'} \int \frac{d^3k'}{(2\pi)^3 2E_k'}$



The hadronic tensor:

$$W_{\mu\nu}(q,p) = \sum_X \langle p | J_\mu(0) | X\rangle\langle X | J_\nu(0) | p\rangle (2\pi)^4 \delta^4(p+q-p_X)$$

$$= \sum_{X'} \int \frac{d^3k'}{(2\pi)^3 2E_k'} \langle p | \bar\psi(0)\gamma_\mu\psi(0) | X', k'\rangle\langle X', k' | \bar\psi(0)\gamma_\nu\psi(0) | p\rangle (2\pi)^4 \delta^4(p+q-p_{X'}-k')$$

$$= \int d^4z \sum_{X'} \int \frac{d^3k'}{(2\pi)^3 2E_k'} e^{i(p+q-p_{X'}-k')\cdot z} \langle p | \bar\psi(0)\gamma_\mu | X'\rangle \not{k}' \langle X' | \gamma_\nu\psi(0) | p\rangle$$

$$= \int d^4z \sum_{X'} \int \frac{d^3k'}{(2\pi)^3 2E_k'} e^{i(q-k')\cdot z} \langle p | \bar\psi(0)\gamma_\mu | X'\rangle \not{k}' \langle X' | \gamma_\nu\psi(z) | p\rangle$$

$$= \int d^4z \frac{d^4k}{(2\pi)^4} e^{ik\cdot z} \langle p | \bar\psi(0)\gamma_\mu(\not{q}+\not{k})\gamma_\nu\psi(z) | p\rangle (2\pi)\delta_+\left((k+q)^2\right)$$

$$\psi(x) | X', k'\rangle = u(k')e^{-ik'\cdot x} | X'\rangle$$

$$u(k')\bar u(k') = \not{k}'$$

$$\int \frac{d^3k'}{(2\pi)^3 2E_k'} = \int \frac{d^4k'}{(2\pi)^4} \delta_+(k'^2)$$

$$\Longrightarrow\quad W_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\hat\varphi(k,p,S)\hat H_{\mu\nu}(k,q)]$$

$$\hat H_{\mu\nu}(k,q) = \gamma_\mu(\not{k}+\not{q})\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)$$

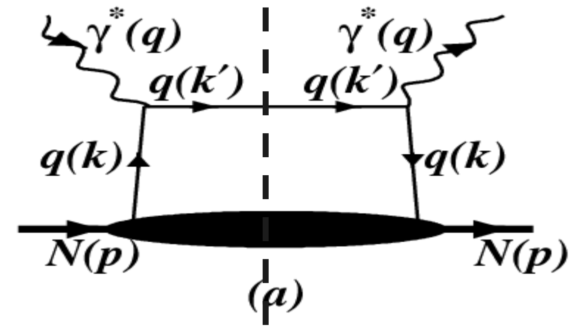
No QCD interactions.
Not (color) gauge invariant!

$$\hat\varphi(k,p,S) = \int d^4z\, e^{ikz} \langle p, S | \bar\psi(0)\psi(z) | p, S\rangle$$

Inclusive DIS: where does the gauge link come from?

Collinear approximation $\hat{H}_{\mu\nu}(k,q) \approx \hat{H}_{\mu\nu}(k,q)|_{k=xp} \equiv \hat{H}_{\mu\nu}(x)$

The hadronic tensor:



$$W_{\mu\nu}(q,p,S) \approx \int dx \mathrm{Tr}[\hat{\varphi}(x;p,S)\hat{H}_{\mu\nu}(x)]$$

$$\hat{H}_{\mu\nu}(x) = \gamma_\mu(\not{q} + x\not{p})\gamma_\nu \delta(x - x_B)$$

$$\hat{\varphi}(x;p,S) \equiv \int \frac{d^4k}{(2\pi)^4}\delta(x - \frac{k^+}{p^+})\,\hat{\varphi}(k;p,S) = \int \frac{p^+ dz^-}{2\pi} e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)\psi(0,z^-,\vec{0}_\perp)|p,S\rangle$$

$$\hat{\varphi}(x;p,S) = [\gamma^\alpha \varphi_\alpha(x;p,S) + \gamma_5\gamma^\alpha \varphi_\alpha(x;p,S)]/2 + \ldots$$

$n = (0,1,\vec{0}_\perp)$

$\bar{n} = (1,0,\vec{0}_\perp)$

$k^\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$

$x = k^+/p^+$

Unpolarized case: $W_{\mu\nu}(q,p) \approx \frac{1}{2}\int dx \mathrm{Tr}[\gamma^\alpha \hat{H}_{\mu\nu}(x)]\varphi_\alpha(x;p)$

Lorentz structure: $\varphi_\alpha(x;p) = p_\alpha f(x) + Mn_\alpha f_{(-)}(x)$

$$f(x) = \frac{1}{p^+} n^\alpha \varphi_\alpha(x;p) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-}\langle p|\bar{\psi}(0)\frac{\gamma^+}{2}\psi(0,z^-,\vec{0}_\perp)|p\rangle$$

Leading twist: $W_{\mu\nu}(q,p) \approx \int dx \mathrm{Tr}[\not{p}\hat{H}_{\mu\nu}(x)] f(x) = \mathrm{Tr}[\not{p}\gamma_\mu(\not{q} + x\not{p})\gamma_\nu] f(x)$

Inclusive DIS with QCD interaction

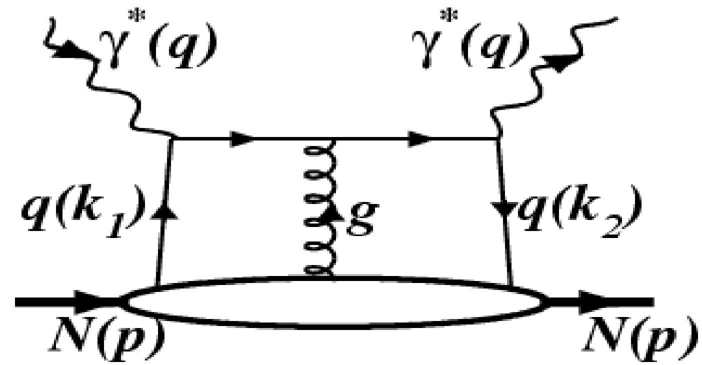
Consider QCD interaction: first order

$$\mathcal{H}_I(y) = \mathcal{H}_I^{QED}(y) + \mathcal{H}_I^{QCD}(y)$$

$$\mathcal{H}_I^{QED}(y) = e\bar{\psi}(y)\gamma_\mu\psi(y)A^\mu_{em}(y)$$

$$\mathcal{H}_I^{QCD}(y) = g\bar{\psi}(y)\gamma^\rho\psi(y)A_\rho(y)$$

$$J_\mu(x) \to T\int d^4y\, \mathcal{H}_I^{QCD}(y)\bar{\psi}(x)\gamma_\mu\psi(x),$$



(b)

$$W^{(1,R)}_{\mu\nu}(q,p) = T\int d^4y \sum_{X'} \frac{d^3k'}{(2\pi)^3 2E'_k} \langle p|\bar{\psi}(0)\gamma_\mu\psi(0)|X',k'\rangle\langle X',k'|\mathcal{H}_I^{QCD}(y)\bar{\psi}(0)\gamma_\nu\psi(0)|p\rangle(2\pi)^4\delta^4(p+q-p_{X'}-k')$$

$$= g\int d^4y \sum_{X'} \frac{d^3k'}{(2\pi)^3 2E'_k} \langle p|\bar{\psi}(0)\gamma_\mu\psi(0)|X',k'\rangle\langle X',k'|T\bar{\psi}(y)\gamma^\rho\psi(y)A_\rho(y)\bar{\psi}(0)\gamma_\nu\psi(0)|p\rangle(2\pi)^4\delta^4(p+q-p_{X'}-k')$$

$$= g\int d^4y \sum_{X'} \frac{d^3k'}{(2\pi)^3 2E'_k} \langle p|\bar{\psi}(0)\gamma_\mu\psi(0)|X',k'\rangle\langle X',k'|\bar{\psi}(y)\gamma^\rho\psi(y)A_\rho(y)\bar{\psi}(0)\gamma_\nu\psi(0)|p\rangle(2\pi)^4\delta^4(p+q-p_{X'}-k')$$

$$= g\int d^4y\, d^4z \frac{d^4k'}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} e^{-i(q-k')\cdot z} e^{-i(k-k')\cdot y} \langle p|\bar{\psi}(0)\gamma_\mu \not{k}'\gamma^\rho S_F(k)A_\rho(y+z)\gamma_\nu\psi(z)|p\rangle$$

Inclusive DIS with QCD interaction

Consider QCD interaction: first order

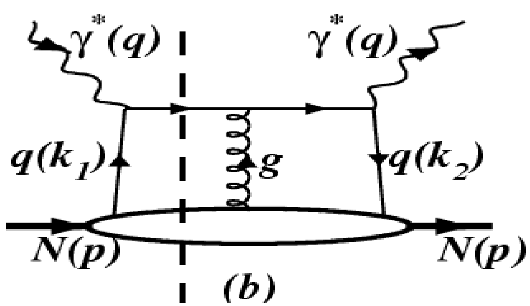
$$\Longrightarrow \quad W_{\mu\nu}^{(1,R)}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}[\hat{\phi}_\rho^{(1)}(k_1,k_2;p,S)\hat{H}_{\mu\nu}^{(1,R)\rho}(k_1,k_2,q)]$$

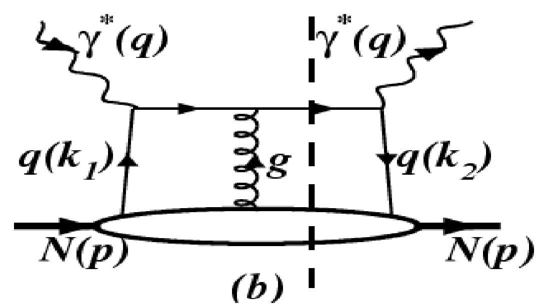
$$\hat{H}_{\mu\nu}^{(1,R)\rho}(k,q) = \gamma_\mu \frac{(\not{k}_1+\not{q})\gamma^\rho(\not{k}_2+\not{q})}{(k_1+q)^2+i\varepsilon}\gamma_\nu(2\pi)\delta_+\left((k_2+q)^2\right)$$

$$\hat{\phi}_\rho^{(1)}(k_1,k_2;p,S) = \int d^4z d^4y e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)gA_\rho(y)\psi(z)|p,S\rangle$$

Similarly: $$W_{\mu\nu}^{(1,L)}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}[\hat{\phi}_\rho^{(1)}(k_1,k_2;p,S)\hat{H}_{\mu\nu}^{(1,L)\rho}(k_1,k_2,q)]$$

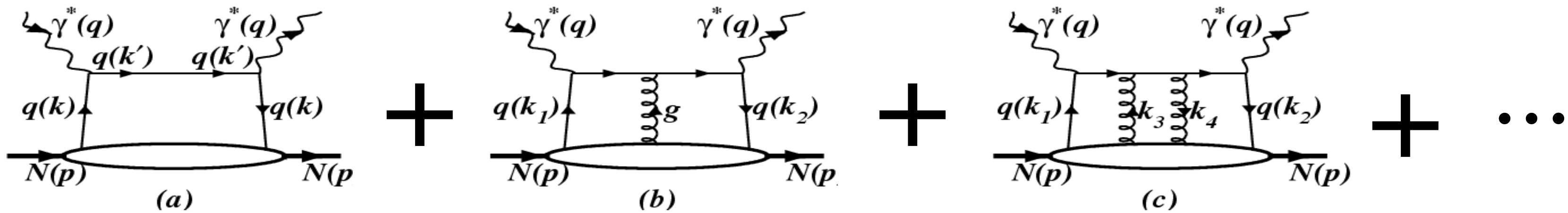
$$\hat{H}_{\mu\nu}^{(1,L)\rho}(k,q) = \gamma_\mu \frac{(\not{k}_1+\not{q})\gamma^\rho(\not{k}_2+\not{q})}{(k_2+q)^2-i\varepsilon}\gamma_\nu(2\pi)\delta_+\left((k_1+q)^2\right)$$





Inclusive DIS with QCD interaction

$$W_{\mu\nu}(q,p,S)=W^{(0)}_{\mu\nu}(q,p,S)+W^{(1)}_{\mu\nu}(q,p,S)+W^{(2)}_{\mu\nu}(q,p,S)+\dots$$



$$W^{(0)}_{\mu\nu}(q,p,S)=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{\phi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(k,q)\right]$$

$$W^{(1)}_{\mu\nu}(q,p,S)=\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}\left[\hat{\phi}^{(1)}_{\rho}(k_1,k_2,p,S)\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q)\right]$$

Parton distribution/correlation:

$$\hat{\phi}^{(0)}(k,p,S)=\int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\psi(z)|p,S\rangle$$

Not gauge invariant!

$$\hat{\phi}^{(1)}_{\rho}(k_1,k_2,p,S)=\int d^4z d^4y e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)gA_{\rho}(y)\psi(z)|p,S\rangle$$

Inclusive DIS with QCD interaction

Ellis, Furmanski, Petronzio, (82)
Qiu, Sterman (90,91)

Collinear expansion:

- Expanding the hard parts around $k = xp$:

$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \hat{H}^{(0)}_{\mu\nu}(x) + \frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^{\rho}} \omega_{\rho}{}^{\rho'} k_{\rho'} + \ldots\ldots$$

$$\hat{H}^{(1)\rho}_{\mu\nu}(k,q) = \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2) + \frac{\partial \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)}{\partial k_1^{\sigma}} \omega_{\sigma}{}^{\sigma'} k_{1\sigma'} + \ldots\ldots$$

- Decomposition of the gluon field:

$$A_{\rho}(y) = n \cdot A(y) \frac{p_{\rho}}{n \cdot p} + \omega_{\rho}{}^{\rho'} A_{\rho'}(y)$$

- Using the identities such as,

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^{\rho}} = -\hat{H}^{(1)\rho}_{\mu\nu}(x,x), \qquad p_{\rho} \hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\varepsilon}$$

to replace the derivatives etc.

$$x = k^+ / p^+$$

$$\omega_{\rho}{}^{\rho'} \equiv g_{\rho}{}^{\rho'} - \bar{n}_{\rho} n^{\rho'}$$

$$\omega_{\rho}{}^{\rho'} k_{\rho'} = (k - xp)_{\rho}$$

$$k^{\pm} = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$$

$$n = (1,-1,\vec{0}_{\perp})/\sqrt{2}$$

$$n = (1,1,\vec{0}_{\perp})/\sqrt{2}$$

Inclusive DIS with QCD interaction

$$W_{\mu\nu}(q,p,S)=\tilde{W}^{(0)}_{\mu\nu}(q,p,S)+\tilde{W}^{(1)}_{\mu\nu}(q,p,S)+\tilde{W}^{(2)}_{\mu\nu}(q,p,S)+\ldots$$

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S)=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Phi}^{(0)}(k,p,S)\,\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

$$\hat{\Phi}^{(0)}(k,p,S)=\int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

$$\mathcal{L}(0,z)=Pe^{ig\int_0^z dy^- A^+(0,y^-,\vec{0}_\perp)}=1+ig\int_0^{z^-}dy^- A^+(0,y^-,\vec{0}_\perp)+(ig)^2\int_0^{z^-}dy^- A^+(0,y^-,\vec{0}_\perp)\int_0^{y^-}dy'^- A^+(0,y'^-,\vec{0}_\perp)+\ldots$$

$$\tilde{W}^{(1)}_{\mu\nu}(q,p,S)=\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Phi}^{(1)}_{\rho'}(k_1,k_2,p,S)\quad\hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'}\right]$$

$$\hat{\Phi}^{(1)}_{\rho}(k_1,k_2,p,S)=\int d^4z d^4y e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle$$

$$D_\rho(y)=-i\partial_\rho+gA_\rho(y)$$

Contain QCD interactions.
(Color) gauge invariant !

Inclusive DIS with QCD interaction

$$W_{\mu\nu}(q,p,S) = \tilde{W}^{(0)}_{\mu\nu}(q,p,S) + \tilde{W}^{(1)}_{\mu\nu}(q,p,S) + \tilde{W}^{(2)}_{\mu\nu}(q,p,S) + \ldots$$

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\hat{\Phi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(x)] = \int dx \mathrm{Tr}[\hat{\Phi}^{(0)}(x,p,S)\hat{H}^{(0)}_{\mu\nu}(x)]$$

$$\hat{\Phi}^{(0)}(x,p,S) = \int \frac{d^4k}{(2\pi)^4} \delta\left(x - \frac{k^+}{p^+}\right) \hat{\Phi}^{(0)}(k,p,S)$$

$$= \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p,S|\bar{\psi}(0)\mathcal{L}(0,z^-)\psi(0,z^-,\vec{0}_\perp)|p,S\rangle$$

$$\hat{\Phi}^{(0)}(x,p,S) = \gamma^\alpha \Phi^{(0)}_\alpha(x,p,S)/2 + \ldots \qquad \Phi^{(0)}_\alpha(x,p,S) = p_\alpha f_1(x) + \ldots$$

$$f_1(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p,S|\bar{\psi}(0)\mathcal{L}(0,z^-)\frac{\gamma^+}{2}\psi(0,z^-,\vec{0}_\perp)|p,S\rangle$$

Definition of parton distributions

$$\Longrightarrow \quad \tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \frac{1}{2}\int dx\, f_1(x) \mathrm{Tr}[\not{p}\hat{H}^{(0)}_{\mu\nu}(xp)] + \ldots$$

Conclusion:

Collinear expansion is the necessary procedure to obtain the gauge invariant parton distributions.

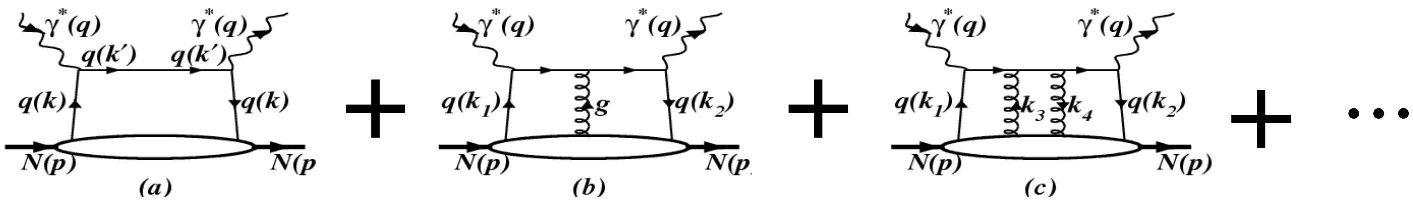
Question: **How about semi-inclusive DIS?**

Semi-Inclusive DIS with QCD interaction

Consider first $e + p \to e + q + X$, i.e., NO fragmentation

$$W_{\mu\nu}^{(si)}(q,p,k') = \sum_X \langle p | J_\mu(0) | X, k' \rangle \langle X, k' | J_\nu(0) | p \rangle (2\pi)^4 \delta^4(p+q-k'-p_X)$$

$$W_{\mu\nu}(q,p) = \sum_{X'} \int \frac{d^3k'}{(2\pi)^3 2E_k'} \langle p | J_\mu(0) | X', k' \rangle \langle X', k' | J_\nu(0) | p \rangle (2\pi)^4 \delta^4(p+q-p_{X'}-k')$$



$$W_{\mu\nu}^{(si)}(q,p,S,k') = W_{\mu\nu}^{(0,si)}(q,p,S,k') + W_{\mu\nu}^{(1,si)}(q,p,S,k') + W_{\mu\nu}^{(2,si)}(q,p,S,k') + \ldots$$

$$W_{\mu\nu}^{(0,si)}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\hat{\phi}^{(0)}(k,p,S) \hat{H}_{\mu\nu}^{(0,si)}(k,q)]$$

$$W_{\mu\nu}^{(1,si)}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \sum_{c=L,R} \mathrm{Tr}[\hat{\phi}_\rho^{(1)}(k_1,k_2,p,S) \hat{H}_{\mu\nu}^{(1,c;si)\rho}(k_1,k_2,q)]$$

Semi-Inclusive DIS with QCD interaction

The hard parts

$$\hat{H}^{(0,si)}_{\mu\nu}(k,q)=\gamma_\mu(\not{k}+\not{q})\gamma_\nu(2\pi)^4\delta^4(k'-k-q)$$

$$\hat{H}^{(1,L;si)}_{\mu\nu}(k_1,k_2,q)=\gamma_\mu\frac{(\not{k}_1+\not{q})\gamma^\rho(\not{k}_2+\not{q})}{(k_2+q)^2-i\varepsilon}\gamma_\nu(2\pi)^4\delta^4(k'-k_1-q)$$

To compare

$$\hat{H}^{(0)}_{\mu\nu}(k,q)=\gamma_\mu(\not{k}+\not{q})\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)$$

$$\hat{H}^{(1,L)}_{\mu\nu}(k_1,k_2,q)=\gamma_\mu\frac{(\not{k}_1+\not{q})\gamma^\rho(\not{k}_2+\not{q})}{(k_2+q)^2-i\varepsilon}\gamma_\nu(2\pi)\delta_+\left((k_1+q)^2\right)$$

Semi-Inclusive DIS with QCD interaction

Using the mathematical identity:

$$(2\pi)^4\delta^4(k'-k-q)=(2\pi)\delta_+\left((k+q)^2\right)(2\pi)^3(2E_{k'})\delta^3(\vec{k}'-\vec{k}-\vec{q})$$

We obtain:

$$\hat{H}^{(0,si)}_{\mu\nu}(k,q)=\hat{H}^{(0)}_{\mu\nu}(k,q)(2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}-\vec{q})$$

$$\hat{H}^{(1,\rho,c,si)}_{\mu\nu}(k_1,k_2,q)=\hat{H}^{(1,\rho,c)}_{\mu\nu}(k_1,k_2,q)(2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}_c-\vec{q})$$

$$\Rightarrow W^{(0,si)}_{\mu\nu}(q,p,S,k')=\underbrace{\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\hat{\phi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(k,q)]}_{W^{(0)}_{\mu\nu}(q,p,S)}(2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}-\vec{q})$$

$$W^{(1,si)}_{\mu\nu}(q,p,S,k')=\underbrace{\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\sum_{c=L,R}\mathrm{Tr}[\hat{\phi}^{(1)}_\rho(k_1,k_2,p,S)\hat{H}^{(1,c)\rho}_{\mu\nu}(k_1,k_2,q)]}_{W^{(1)}_{\mu\nu}(q,p,S)}\times(2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}_c-\vec{q})$$

Semi-Inclusive DIS with QCD interaction

The same collinear expansion technique leads to

$$W_{\mu\nu}^{(si)}(q,p,S,k')=\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k')+\tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k')+\tilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k')+\dots$$

$$\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k')=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\hat{\Phi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(x)]\,(2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}-\vec{q})$$

$$\tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k')=\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\sum_{c=L,R}\mathrm{Tr}[\hat{\Phi}_{\rho'}^{(1)}(k_1,k_2,p,S)\hat{H}_{\mu\nu}^{(1,c)\rho}(x_1,x_2)\omega_\rho^{\ \rho'}]\times(2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}_c-\vec{q})$$

$$\tilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k')=\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\sum_{c=L,C,R}\mathrm{Tr}[\hat{\Phi}_{\rho'\sigma'}^{(2)}(k_1,k_2,k,p,S)\hat{H}_{\mu\nu}^{(2,c)\rho\sigma}(x_1,x_2,x)\omega_\rho^{\ \rho'}\omega_\sigma^{\ \sigma'}]\times(2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}_c-\vec{q})$$

Gauge invariant parton distributions/correlations (with gauge link)

Consider the contribution from $\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k')$

Cross section for $ep \to eqX$ $d\sigma(e+p\to e+q+X)$

$=$ Parton distributions $q(x)$ $\otimes$ Cross section for $eq\to eq$ without $\vec{k}_\perp$ $d\sigma(eq\to eq)|_{k=xp}$

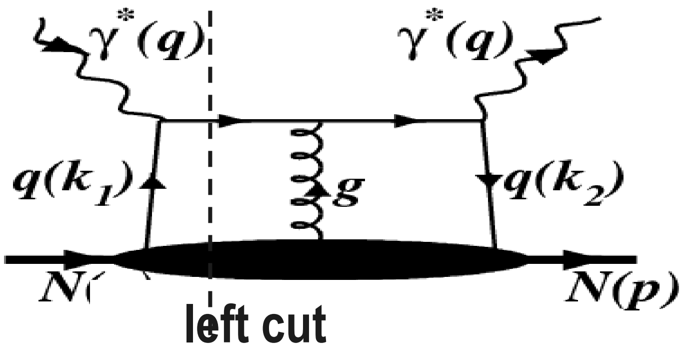
(without transverse momentum)

$\Longrightarrow$ $q(x,\vec{k}_\perp)$ $\otimes$ $d\sigma(eq\to eq)|_{k=xp}$

(with transverse momentum)

SIDIS with $k_\perp$: Direct consequence II

A simplification of $\tilde{W}^{(1,si)}_{\mu\nu}(q,p,S,k')$

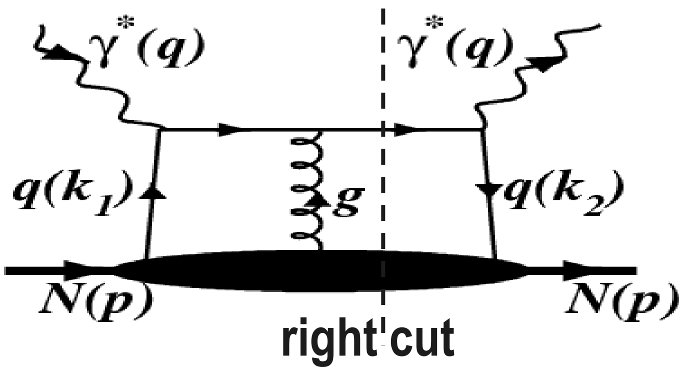


$$\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'}$$

$$=\frac{\pi}{2q\cdot p}\delta(x_1-x_B)\omega_\rho^{\ \rho'}\gamma_\mu \not{n}\gamma^\rho \not{\bar{n}}\gamma_\nu$$

$$\equiv \hat{H}^{(1)\rho'}_{\mu\nu}(x_1)$$

independent of x_2!



$$\hat{H}^{(1,R)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'}$$

$$=\frac{\pi}{2q\cdot p}\delta(x_2-x_B)\omega_\rho^{\ \rho'}\gamma_\mu \not{\bar{n}}\gamma^\rho \not{n}\gamma_\nu$$

$$=\gamma_0 \hat{H}^{(1)\rho'+}_{\mu\nu}(x_2)\gamma_0$$

independent of x_1!

SIDIS with $k_\perp$: Direct consequence II

This leads to

$$\tilde{W}^{(1,si)}_{\mu\nu}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \sum_{c=L,R} \mathrm{Tr}[\hat{\Phi}^{(1)}_{\rho'}(k_1,k_2,p,S)\,\hat{H}^{(1,c)\rho}_{\mu\nu}(x_1,x_2)\,\omega_\rho{}^{\rho'}]$$

$$\times(2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}_c-\vec{q})$$

$$= 2\mathrm{Re}\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\hat{\varphi}^{(1)}_\rho(k,p,S)\hat{H}^{(1)\rho}_{\mu\nu}(x)](2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}-\vec{q})$$

Only $\hat{\varphi}^{(1)}_\rho(k,p,S) \equiv \int \frac{d^4k_2}{(2\pi)^4}\hat{\Phi}^{(1)}_\rho(k,k_2,p,S)$

$$= \int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)D_\rho(z)\psi(z)|p,S\rangle$$

is relevant in semi-inclusive deep-inelastic lepton-nucleon scattering.

SIDIS with $k_\perp$: differential cross-section to 1/Q

Unpolarized SIDIS: $e^- + p \rightarrow e^- + q(jet) + X$

- Expanding the matrices φ's in terms of γ's:

$$\hat{\Phi}^{(0)}(k,p) = \Phi_\alpha^{(0)}(k,p)\gamma^\alpha/2, \qquad \hat{\varphi}_\rho^{(1)}(k,p) = [\varphi_{\rho\alpha}^{(1)}(k,p)\gamma^\alpha + \tilde{\varphi}_{\rho\alpha}^{(1)}(k,p)\gamma_5\gamma^\alpha]/2$$

- The general Lorentz-structure of the coefficient functions are:

$$\Phi_\alpha^{(0)}(k,p) = p_\alpha f_1 + \omega_\alpha^{\alpha'} k_{\alpha'} f_\perp,$$

un-integrated parton distribution/correlation functions

$$\varphi_{\rho\alpha}^{(1)}(k,p) = k_\alpha p_\rho \varphi_\perp^{(1)} + \ldots, \qquad \tilde{\varphi}_{\rho\alpha}^{(1)}(k,p,S) = i p_\alpha \varepsilon_{\perp\rho\gamma} k^\gamma \tilde{\varphi}_\perp^{(1)} + \ldots$$

E.g.: $$f_1(k) = \int \frac{d^4 z}{(2\pi)^4} e^{ikz} \langle p | \bar{\psi}(0) \mathcal{L}(0,z) \frac{\gamma^+}{2} \psi(z) | p \rangle$$

- Equation of motion ⟹ $xf_\perp = -\varphi_\perp^{(1)} + \tilde{\varphi}_\perp^{(1)},$

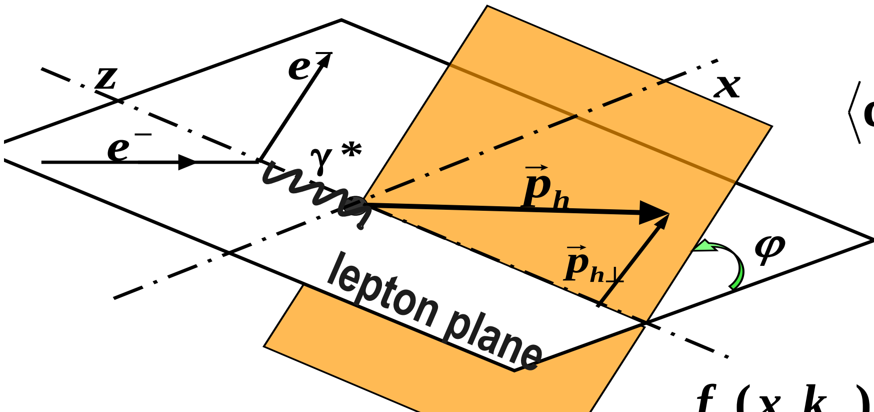
$$i\gamma \cdot D(y)\psi(y) = 0$$

SIDIS with $k_\perp$: differential cross-section to 1/Q

Unpolarized SIDIS: $e^- + p \to e^- + q(jet) + X$

$$d\sigma = \frac{2\alpha_{em}^2 e_q^2}{Q^4}\frac{dx_B dQ^2 d^2k_\perp}{(2\pi)^3}\left[[1+(1-y)^2]f_q - 2(2-y)\sqrt{1-y}\frac{|\vec{k}_\perp|}{Q}x_B f_{\perp q}\cos\phi\right]$$

Azimuthal asymmetry:



$$\langle\cos\phi\rangle = -\frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2}\frac{|\vec{k}_\perp|}{Q}\frac{x_B f_{\perp q}(x_B,k_\perp)}{f_q(x_B,k_\perp)}$$

$$f_q(x,k_\perp) = \int\frac{dz^- d^2z_\perp}{(2\pi)^3}e^{ixp^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp}\langle p|\bar{\psi}(0)\mathcal{L}(0,z)\frac{\gamma^+}{2}\psi(z)|p\rangle$$

$$f_{q\perp}(x,k_\perp) = \int\frac{dz^- d^2z_\perp}{(2\pi)^3}e^{ixp^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp}\langle p|\bar{\psi}(0)\mathcal{L}(0,z)\frac{\gamma\cdot k_\perp}{2k_\perp^2}\psi(z)|p\rangle$$

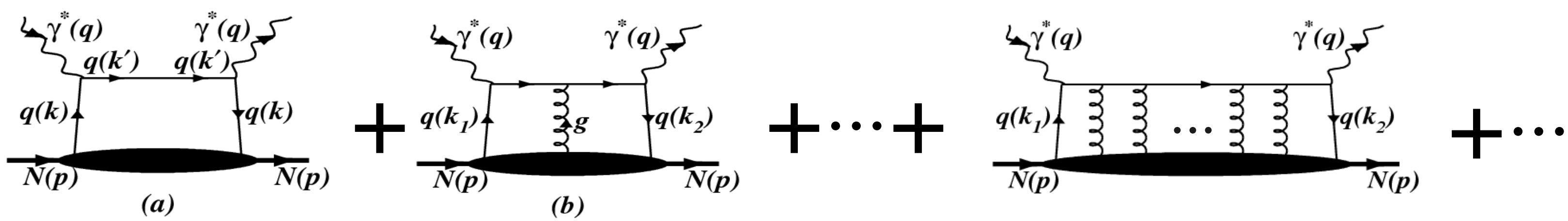
⇨ **A practical way to study the parton correlation function $f_{\perp q}(x_B,k_\perp)$**

Transverse momentum broadening in nucleus

Transverse momentum dependent (TMD) quark distribution:

$$f_q^N(x,k_\perp)=\int\frac{dz^- d^2z_\perp}{(2\pi)^3}e^{ixp^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp}\langle p|\bar{\psi}(0)\mathcal{L}(0,z)\frac{\gamma^+}{2}\psi(0,z^-,\vec{z}_\perp)|p\rangle$$

Gauge link comes from:



Replace *N* by *A*, the gluons can connect to different nucleons in *A*.

$$f_q^A(x,k_\perp)=\int\frac{dz^- d^2z_\perp}{(2\pi)^3}e^{ixp^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp}\langle A|\bar{\psi}(0)\mathcal{L}(0,z)\frac{\gamma^+}{2}\psi(0,z^-,\vec{z}_\perp)|A\rangle$$

⇨ **Nuclear enhancement or transverse momentum broadening**

Transverse momentum broadening should be contained in the gauge link.

Transverse momentum dependent PDF in nucleus

With "maximal two gluon approximation",

i.e., only maximal two gluons are connected to one nucleon

$$f_q^A(x,k_\perp) \approx \frac{A}{\pi \Delta_{2F}} \int d^2 l_\perp e^{-(\vec{k}_\perp - \vec{l}_\perp)^2/\Delta_{2F}} f_q^N(x,\vec{l}_\perp)$$

$$\Delta_{2F} = \int d\xi_N^- \hat{q}_F(\xi_N)$$ transverse momentum broadening squared

nucleon density in nucleus *A*

$$\hat{q}_F(\xi_N) = \frac{2\pi^2 \alpha_s}{N_c} \rho_N^A(\xi_N)[x f_g^N(x)]|_{x=0}$$

gluon distribution function in nucleon *N*

Nuclear dependence of the azimuthal asymmetry

Azimuthal asymmetry in $e^- + N \to e^- + q(jet) + X$

$$\langle \cos\phi \rangle_{eN} = -\frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2} \frac{|\vec{k}_\perp|}{Q} \frac{x_B f_{\perp q}^N(x_B, k_\perp)}{f_q^N(x_B, k_\perp)}$$

Azimuthal asymmetry in $e^- + A \to e^- + q(jet) + X$

$$\langle \cos\phi \rangle_{eA} = -\frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2} \frac{|\vec{k}_\perp|}{Q} \frac{x_B f_{\perp q}^A(x_B, k_\perp)}{f_q^A(x_B, k_\perp)}$$

Nuclear dependence of TMD parton distribution/correlation functions

$$f_q^A(x, k_\perp) \approx \frac{A}{\pi \Delta_{2F}} \int d^2 l_\perp e^{-(\vec{k}_\perp - \vec{l}_\perp)^2/\Delta_{2F}} f_q^N(x, l_\perp)$$

$$|\vec{k}_\perp| f_{\perp q}^A(x, k_\perp) \approx -\frac{A}{\pi \Delta_{2F}} \int d^2 l_\perp e^{-(\vec{k}_\perp - \vec{l}_\perp)^2/\Delta_{2F}} \left(l_\perp \cdot \hat{k}_\perp\right) f_{\perp q}^N(x, l_\perp)$$

Nuclear dependence of the azimuthal asymmetry

An example: take a Gaussian for the transverse momentum dependence

$$f_q^N(x,k_\perp)=\frac{1}{\pi\alpha}f_q^N(x)e^{-\vec{k}_\perp^2/\alpha},\qquad f_{\perp q}^N(x,k_\perp)=\frac{1}{\pi\beta}f_{\perp q}^N(x)e^{-\vec{k}_\perp^2/\beta}$$

$$\Longrightarrow\quad f_q^A(x,k_\perp)\approx\frac{A}{\pi(\alpha+\Delta_{2F})}f_q^N(x)e^{-\vec{k}_\perp^2/(\alpha+\Delta_{2F})}$$

$$f_{\perp q}^A(x,k_\perp)\approx\frac{A\beta}{\pi(\beta+\Delta_{2F})^2}f_{\perp q}^N(x)e^{-\vec{k}_\perp^2/(\beta+\Delta_{2F})}$$

$$\frac{\langle\cos\phi\rangle_{eA}}{\langle\cos\phi\rangle_{eN}}\approx\frac{\beta^2(\alpha+\Delta_{2F})}{\alpha(\beta+\Delta_{2F})^2}\exp\left\{\frac{(\alpha-\beta)\Delta_{2F}(\alpha+\beta+\Delta_{2F})}{\alpha\beta(\alpha+\Delta_{2F})(\beta+\Delta_{2F})}\vec{k}_\perp^2\right\}$$

$$=\frac{\alpha}{\alpha+\Delta_{2F}}$$ **for the case that** $\alpha=\beta$

suppressed!

Summary

- Field operator expressions of parton distribution and/or correlation functions are derived from the basic principles of QFT.
- Gauge link comes from multiple gluon scattering and collinear expansion is a necessary procedure to obtain the correct form of gauge link and that of the corresponding hard parts.
- Collinear expansion applies in inclusive and semi-icluisve DIS $e+p \to e+q+X$ as well.
- In "maximal two gluon approximation",

$$f_q^A(x,\vec{k}_\perp) \approx \frac{A}{\pi \Delta_{2F}} \int d^2 l_\perp e^{-(\vec{k}_\perp - \vec{l}_\perp)^2/\Delta_{2F}} f_q^N(x,\vec{l}_\perp), \qquad \Delta_{2F} = \frac{2\pi^2 \alpha_s}{N_c} \int d\xi_N^- \rho_N^A(\xi_N^-)[x f_g^N(x)]|_{x=0}$$

- Many other consequences.