Hadron Form Factors at Large Momentum Transfer from Lattice QCD

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Outline

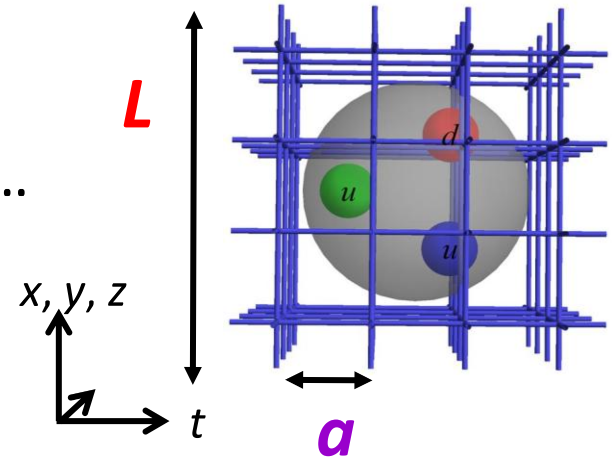
~~§ Why Higher-Q^2 form factors?~~

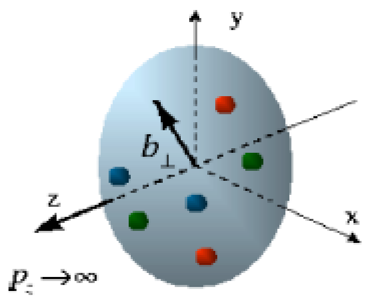
Talks at this workshop: G. Huber, K. de Jager…

~~§ The tool = Lattice Gauge Theory~~

$$\frac{1}{Z}\int [dA][d\overline{\psi}][d\psi]\, O(\overline{\psi},\psi,A)\, e^{i\int d^4x\, \mathcal{L}^{\mathrm{QCD}}(\overline{\psi},\psi,A)}$$

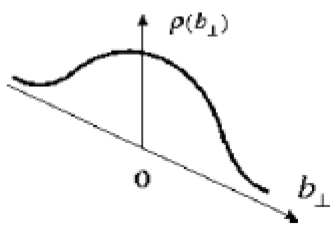
Ph. Hagler (Tue), B. Musch (Fri)





§ Lattice Form Factor Calculations

- What's been done in the past
- What's new in this talk
- Some results (nucleon and pion)



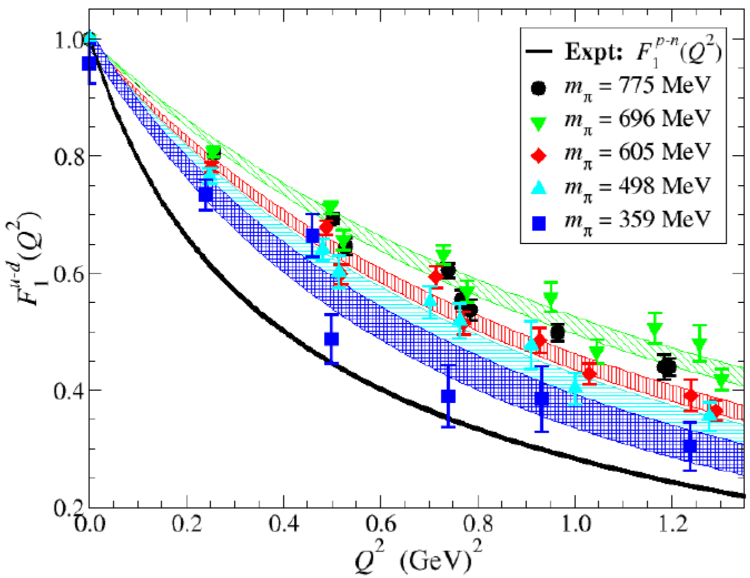
§ Summary and Outlook

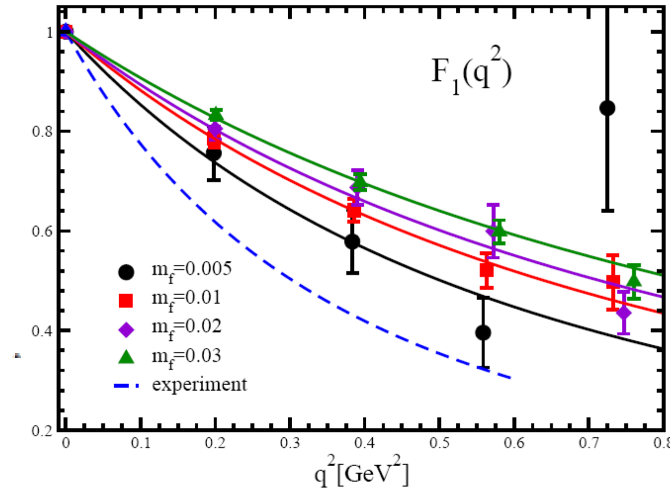
Conventional Calculation

§ Challenge for lattice-QCD calculations

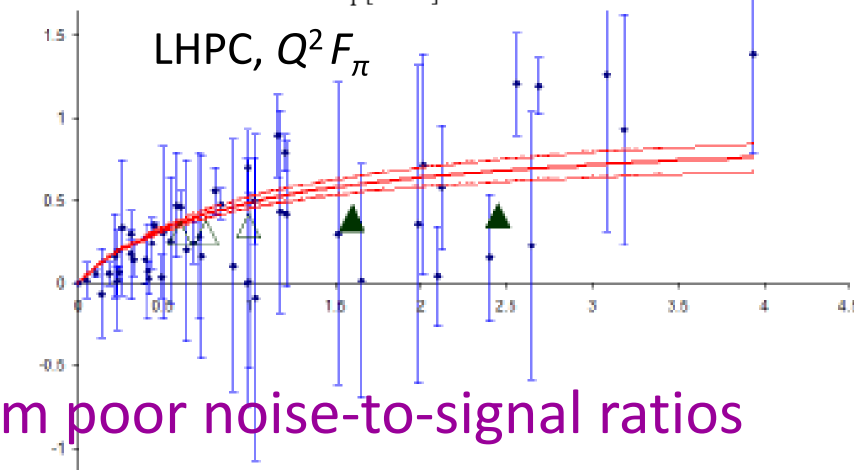
- Typical Q^2 range for nucleon form factors is < 3.0 GeV^2
- Examples from 2+1f cases

LHPC, Dirac FF





RBC/UKQCD Dirac FF



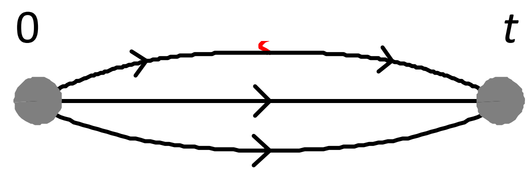
LHPC, $Q^2 F_\pi$

§ Higher-Q^2 calculations suffer from poor noise-to-signal ratios

Conventional Calculation

§ Problem: traditional approach

- Study hadron properties by looking at 2-point function
  $\langle J_N J_N \rangle = \Sigma_n \langle J|n\rangle \langle n|J\rangle e^{-E_n t}$



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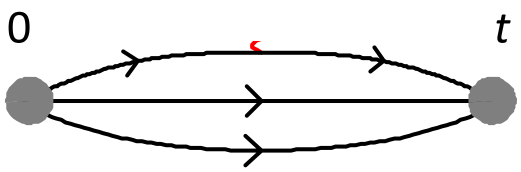
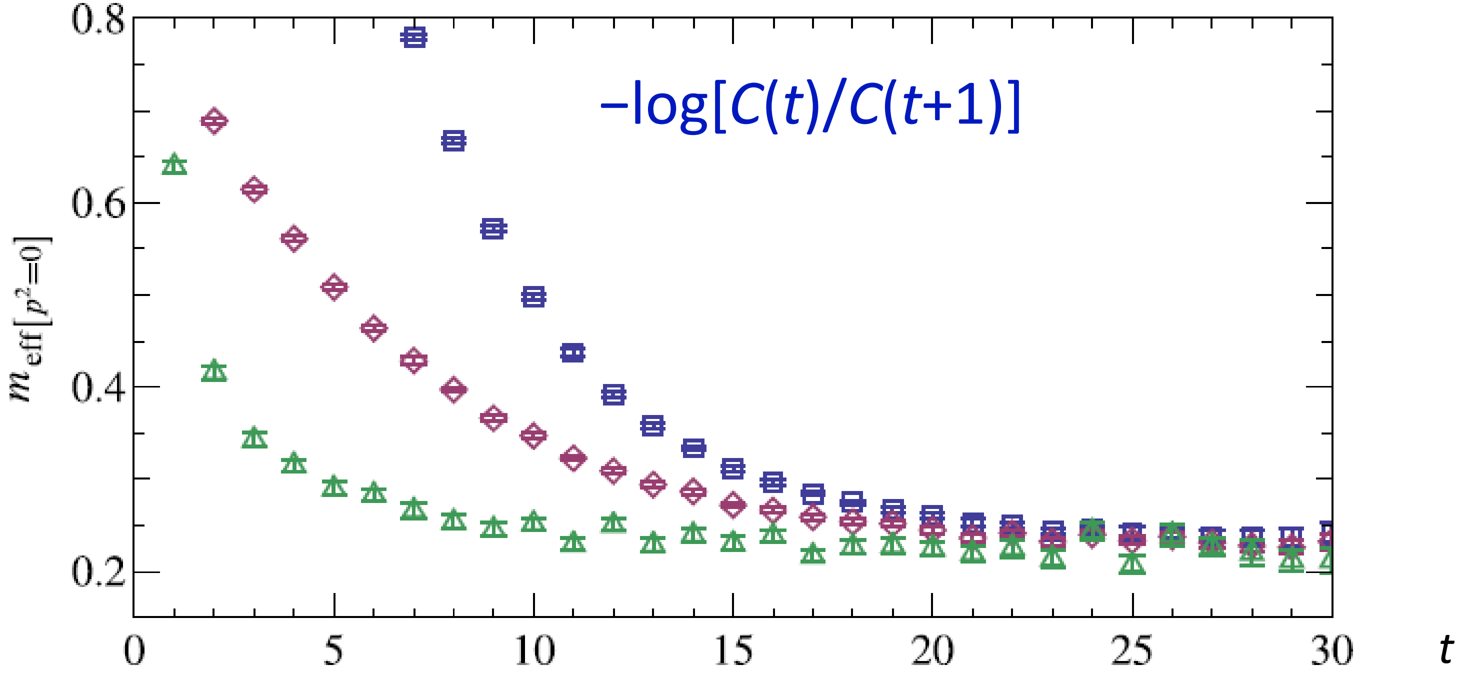
Conventional Calculation

- Problem: traditional approach
  - Study hadron properties by looking at 2-point function $\langle J_N J_N \rangle = \Sigma_n \langle J|n\rangle\langle n|J\rangle e^{-E_n t}$
  - Simplify to a one-state problem



Nucleon "effective mass"



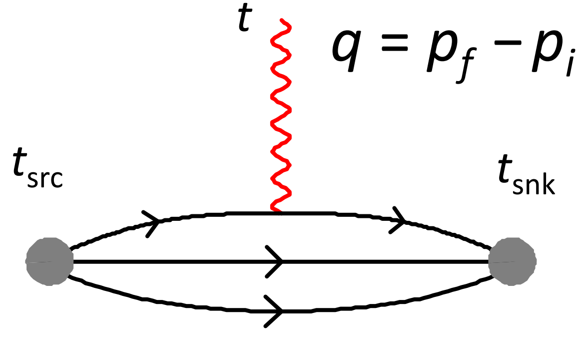
Conventional Calculation

§ Problem: traditional approach

- Simplify to one-state problem

3pt correlator

$$\sum_{\alpha,\beta} \Gamma^{\alpha,\beta}\langle J_\beta\left(\vec{p},t\right)\mathcal{O}(\tau)\overline{J}_\alpha\left(\vec{p},0\right)\rangle$$



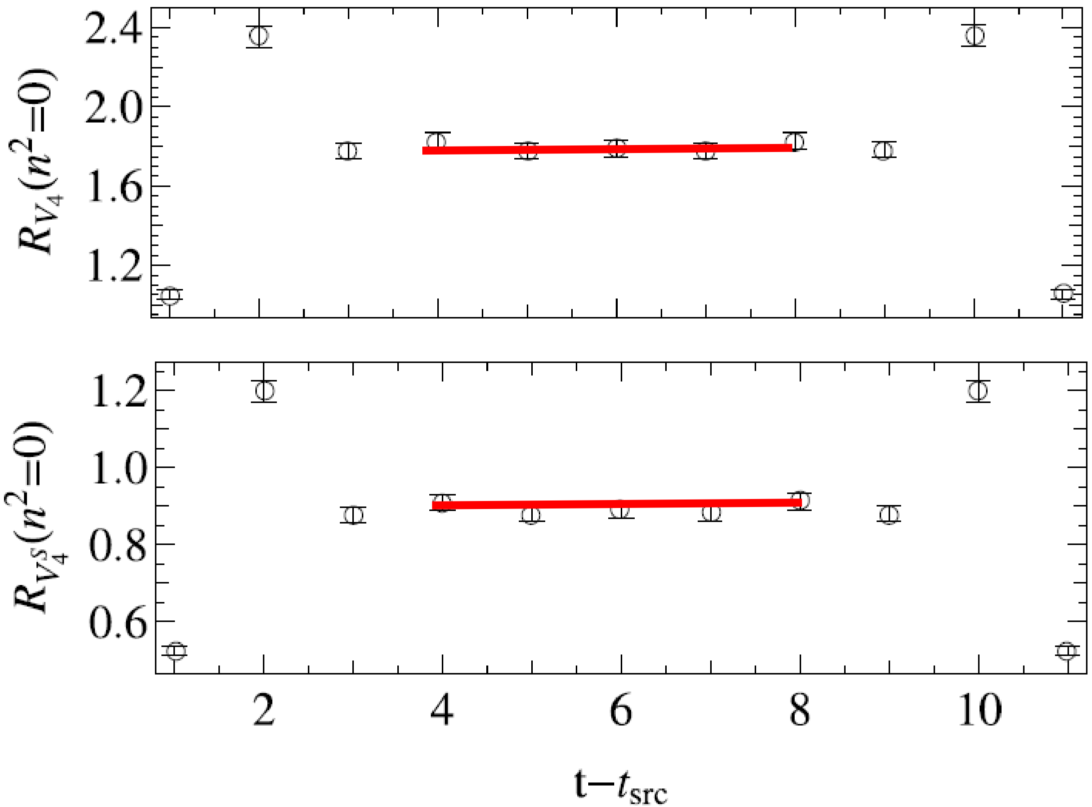
$p_{i,f} = (2\pi/L)\, n_{i,f}\ a^{-1}$

Conventional Calculation

§ Problem: traditional approach

- Simplify to one-state problem

Example of Σ @ 600 MeV pion



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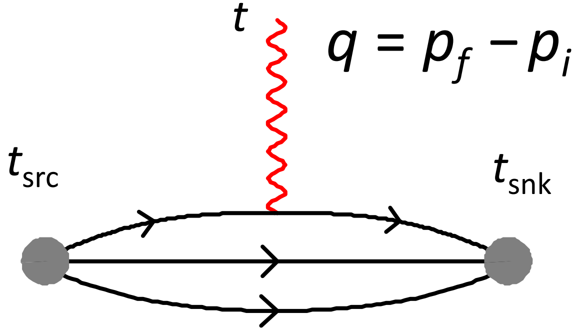
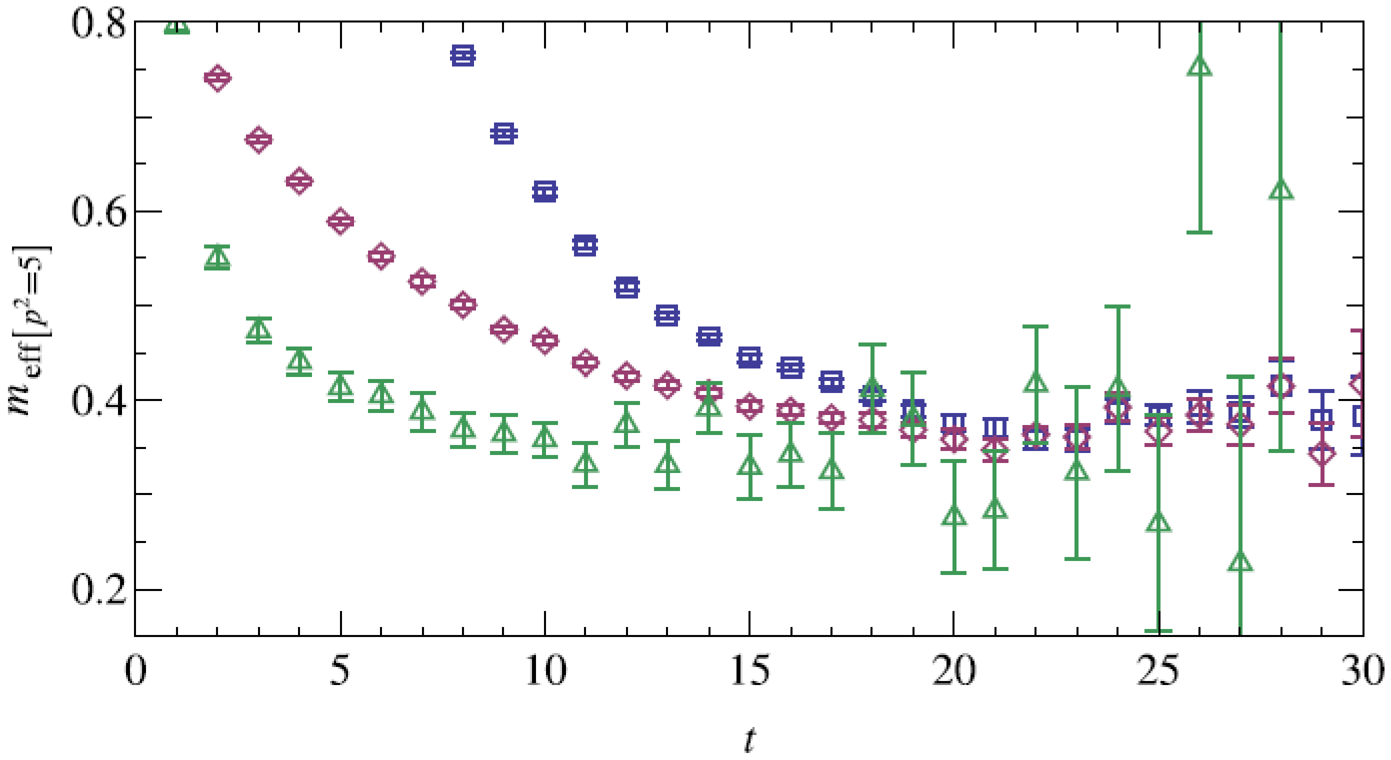


$p_{i,f} = (2\pi/L)\, n_{i,f}\, a^{-1}$

Conventional Calculation

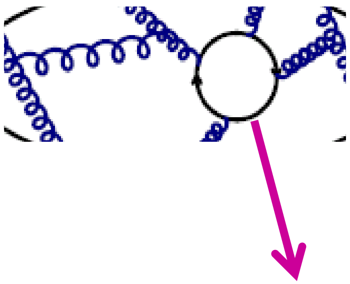
- Problem: traditional approach fails at large Q^2



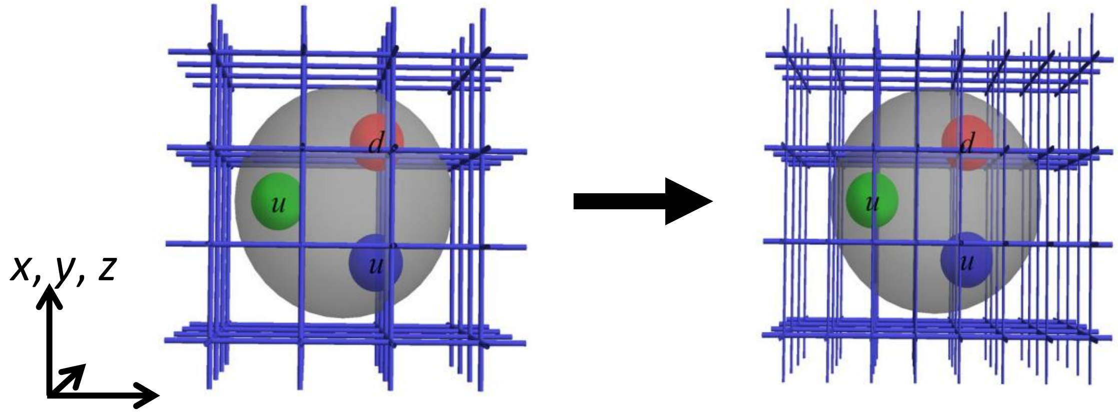
- Solution: confront excited states directly and allow operators to couple to excited states

Actions

- $N_f = 2+1$ anisotropic clover fermions



2+1f : u/d + s

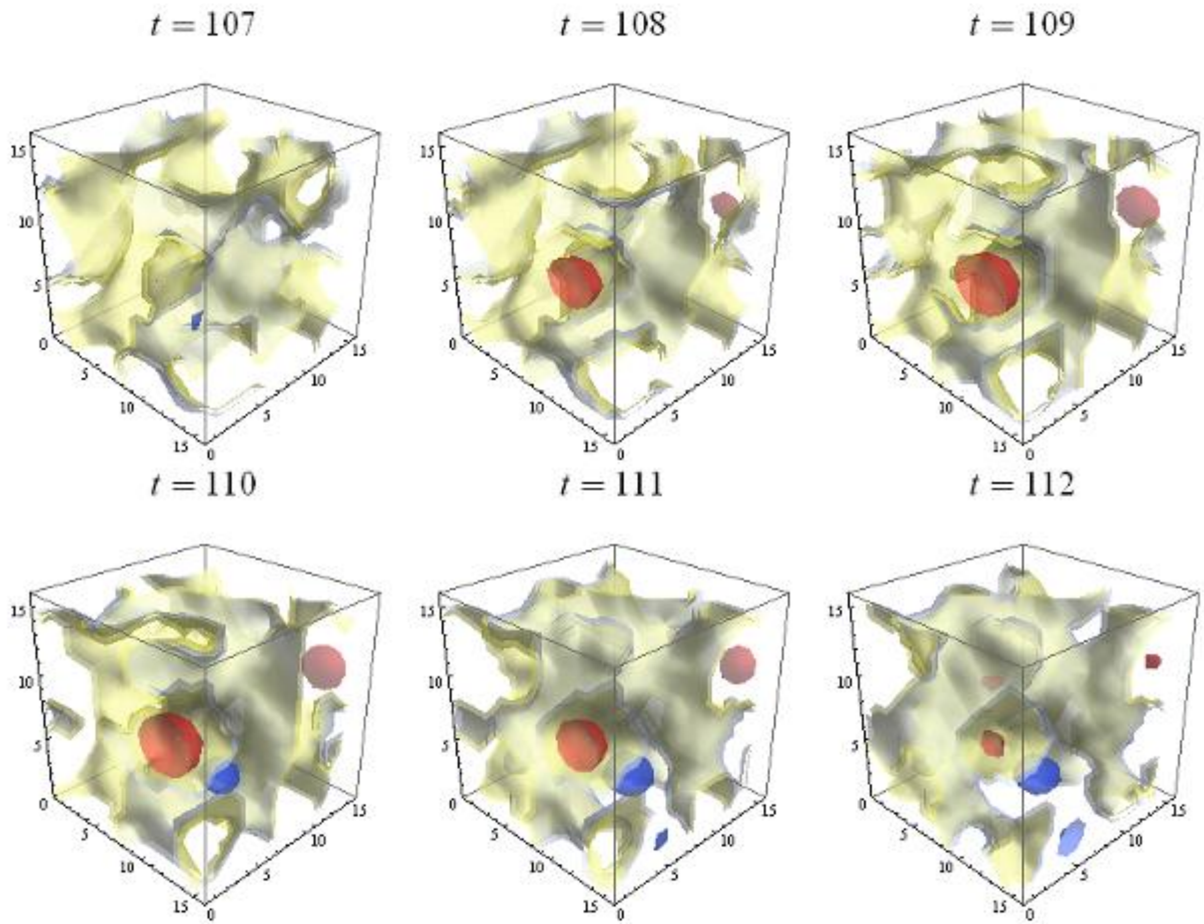


- Renormalized anisotropy $a_s/a_t = 3.5$
- Better resolution in temporal direction
  - Correlators have time-dependent form $A\,e^{-Et}$
- $a_s \approx 0.1227(8)$ fm (using m_Ω)

R. Edwards, B. Joo, HWL, Phys. Rev. D 78, 014505 (2008)
HWL et al., Phys. Rev. D 79, 034502 (2009)

Dynamical Anisotropic Lattices

§ N_f = 2+1 **an**isotropic clover vacuum structure $(M_\pi \approx 380\ \mathrm{MeV})$



http://www.phys.washington.edu/users/hwlin/visQCD.html

Form Factors

- Consider multiple-state 3pt correlators…

$$\Gamma^{(3),T}_{\mu,AB}(t_i,t,t_f,\vec{p}_i,\vec{p}_f)$$
$$= a^3 \sum_n \sum_{n'} \frac{1}{Z_j} \frac{Z_{n',B}(p_f) Z_{n,A}(p_i)}{4E'_n(\vec{p}_f)E_n(\vec{p}_i)} e^{-(t_f-t)E'_n(\vec{p}_f)} e^{-(t-t_i)E_n(\vec{p}_i)}$$
$$\times \sum_{s,s'} T_{\alpha\beta} u_{n'}(\vec{p}_f,s')_\beta \langle N_{n'}(\vec{p}_f,s') \,|j_\mu(0)|\, N_n(\vec{p}_i,s)\rangle \overline{u}_n(\vec{p}_i,s)_\alpha$$

Wanted

Form Factors

- Consider multiple-state 3pt correlators…

$$\Gamma^{(3),T}_{\mu,AB}(t_i,t,t_f,\vec{p}_i,\vec{p}_f)$$
$$= a^3 \sum_n \sum_{n'} \frac{1}{Z_j} \frac{Z_{n',B}(p_f) Z_{n,A}(p_i)}{4E'_n(\vec{p}_f)E_n(\vec{p}_i)} e^{-(t_f-t)E'_n(\vec{p}_f)} e^{-(t-t_i)E_n(\vec{p}_i)}$$
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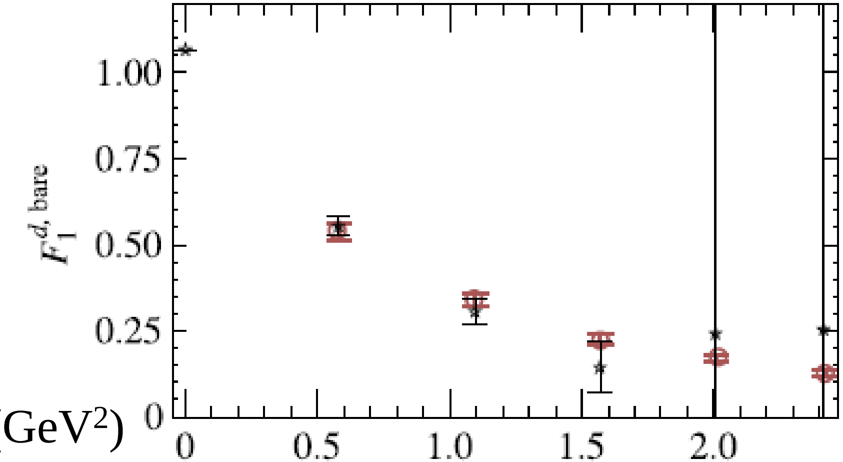
- “Variational method” for better determined Z’s and E’s

Form Factors

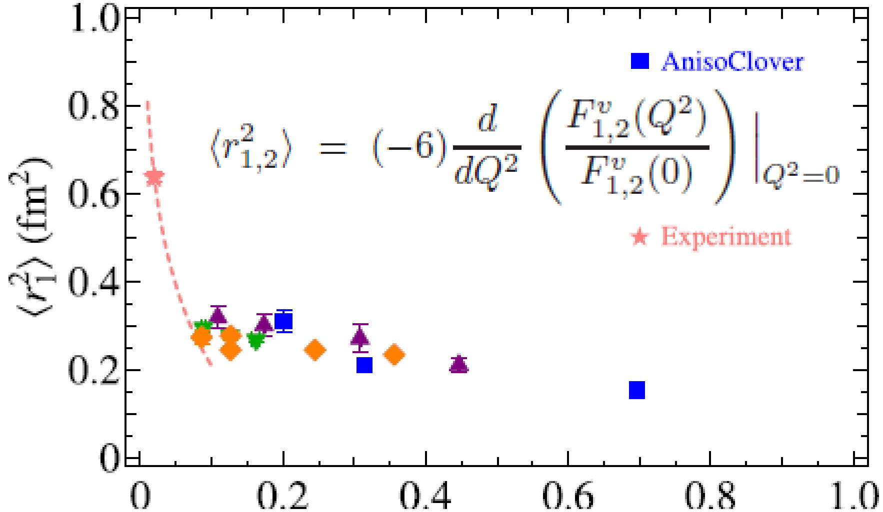
- Consider multiple-state 3pt correlators…

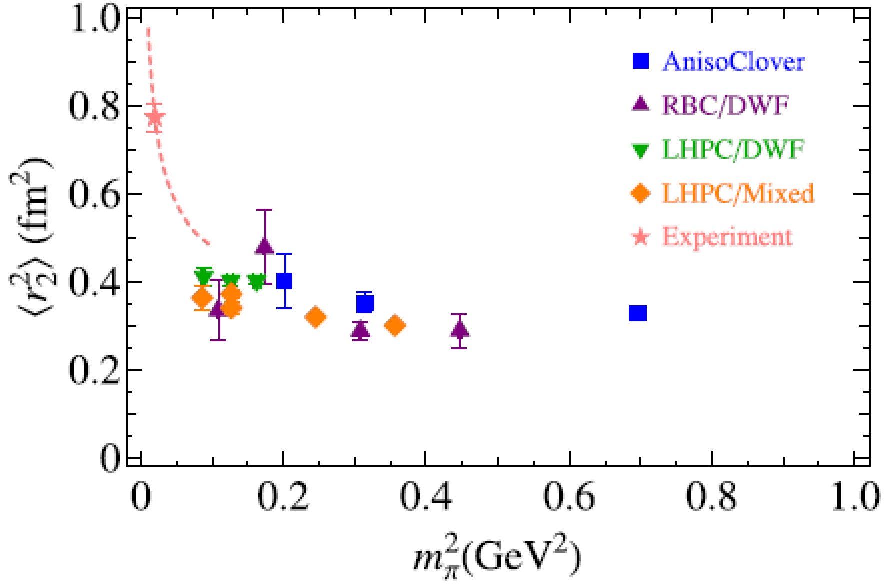
$$\Gamma^{(3),T}_{\mu,AB}(t_i,t,t_f,\vec{p}_i,\vec{p}_f) = a^3\sum_n\sum_{n'}\frac{1}{Z_j}\frac{Z_{n',B}(p_f)Z_{n,A}(p_i)}{4E'_n(\vec{p}_f)E_n(\vec{p}_i)}e^{-(t_f-t)E'_n(\vec{p}_f)}e^{-(t-t_i)E_n(\vec{p}_i)} \times\sum_{s,s'}T_{\alpha\beta}u_{n'}(\vec{p}_f,s')_\beta\langle N_{n'}(\vec{p}_f,s')\,|j_\mu(0)|\,N_n(\vec{p}_i,s)\rangle\overline{u}_n(\vec{p}_i,s)_\alpha$$

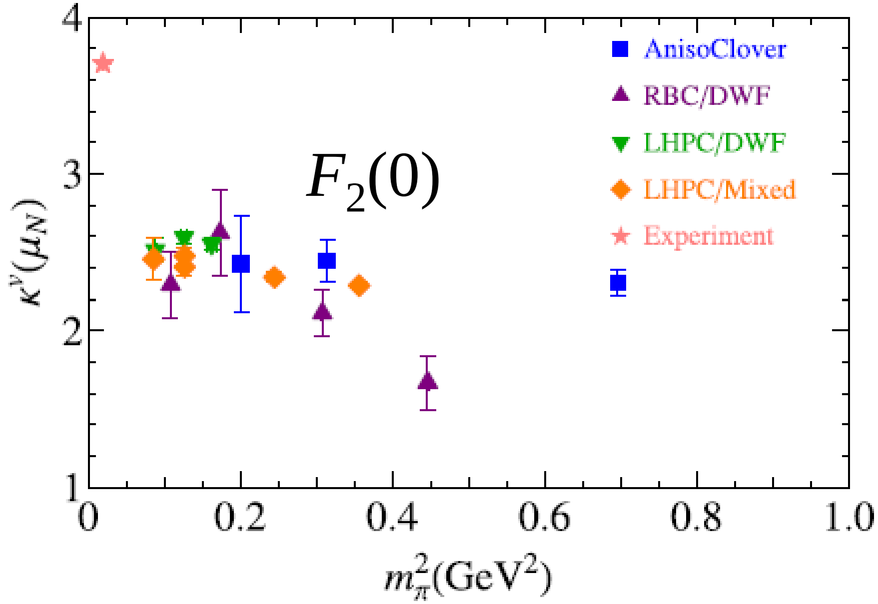
- "Variational method" for better determined Z's and E's
- $n = n' = 0$ gives us nucleon
  Matrix Element $\langle N\,|V_\mu|\,N\rangle(q)$
  and solve linear equations
  for form factors

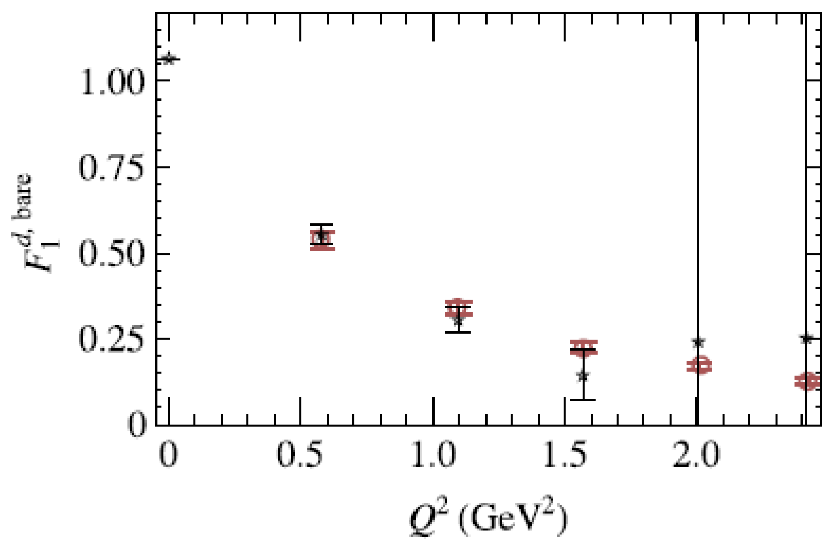


Consistency Checks



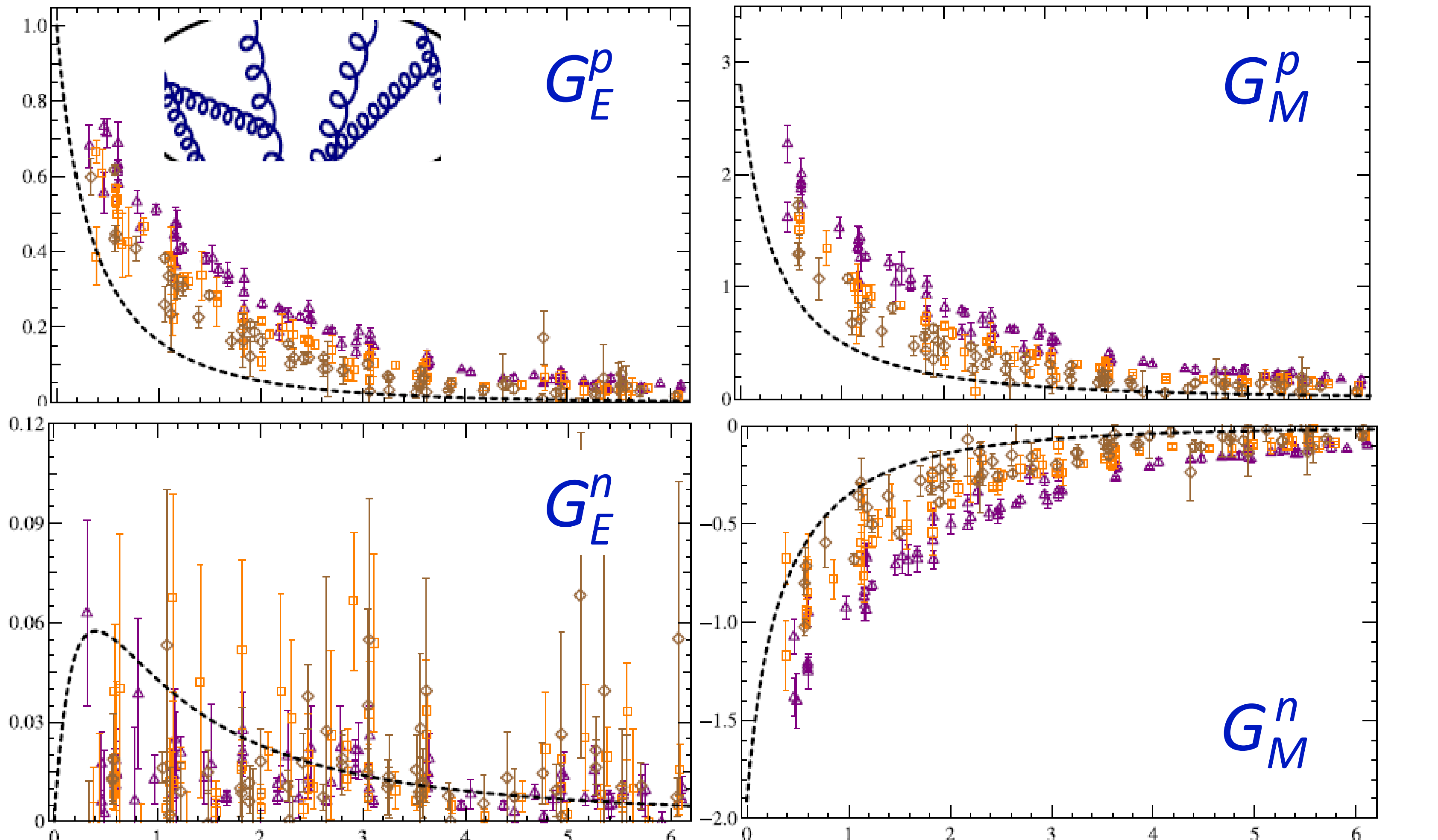






Our Results

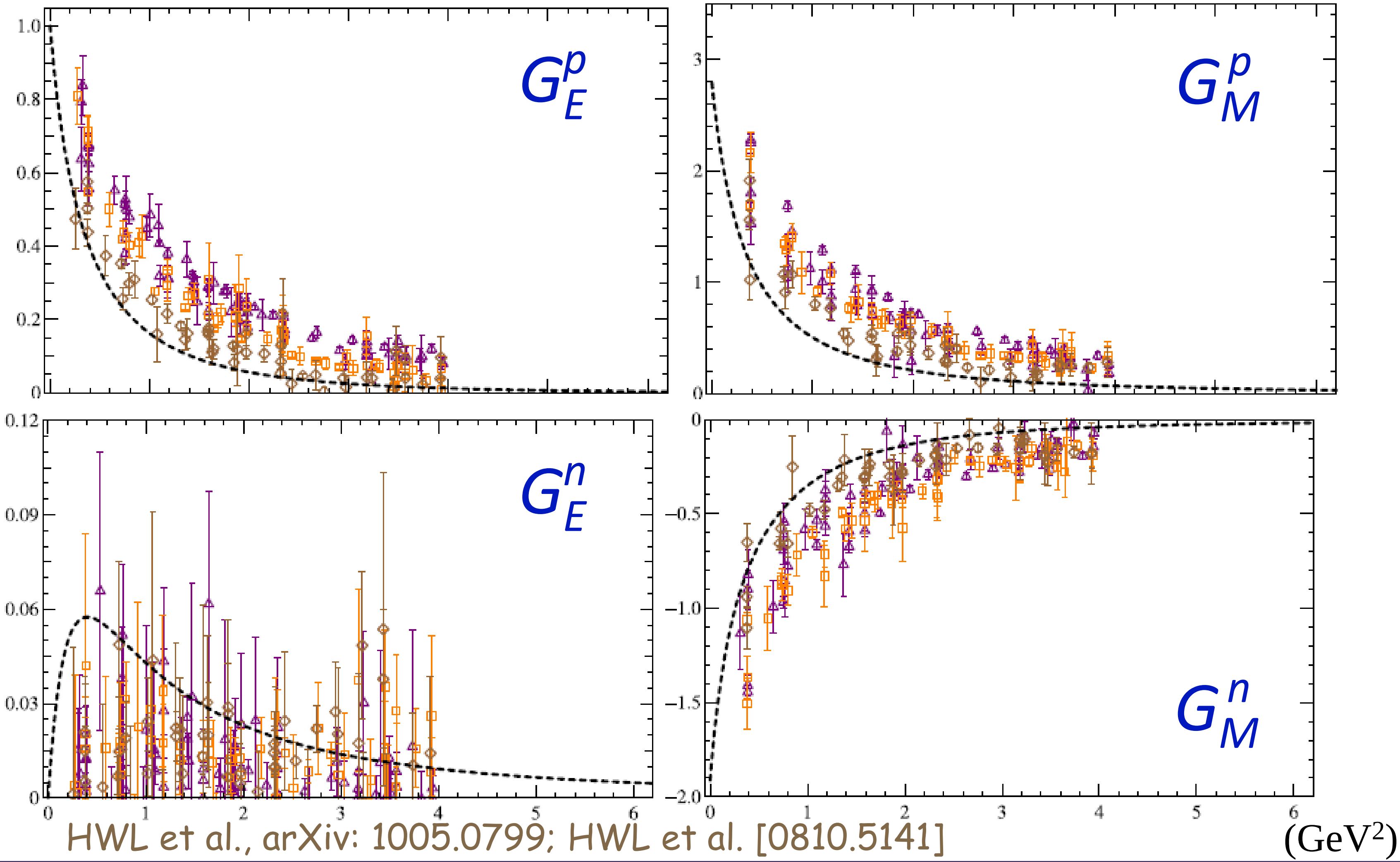
- $N_f = 0$ anisotropic lattices, $M_\pi \approx$ **480**, **720**, **1080** MeV



HWL et al., arXiv: 1005.0799; HWL et al. [0810.5141]

Our Results

§ $N_f = 2+1$ anisotropic lattices, $M_\pi \approx$ **450**, **580**, **875** MeV



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Parametrization

§ Phenomenological choice with dimensionless parameter

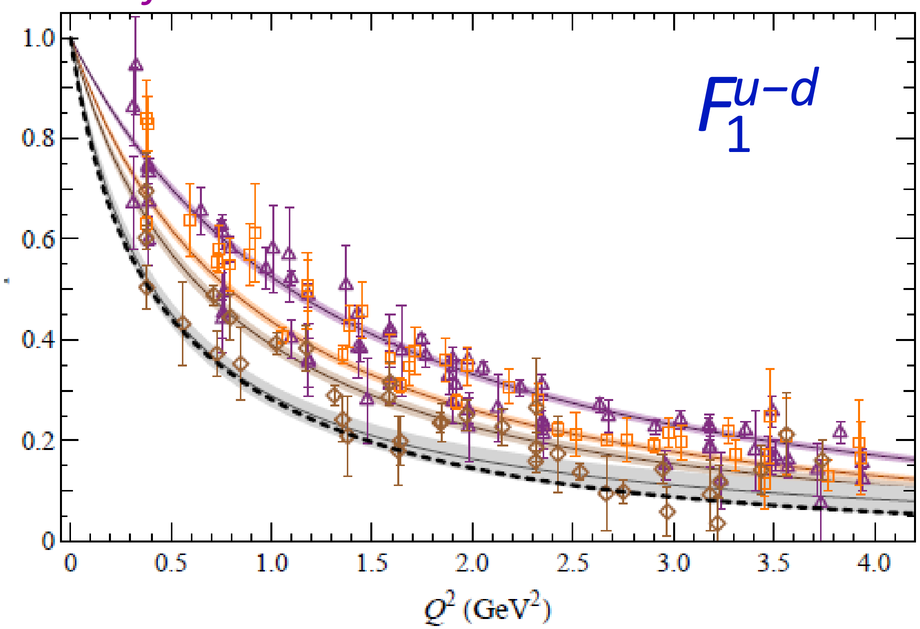
$$F_1 = \frac{a_0 + \sum_{i=1}^{k-2} a_i \tau^i}{1 + \sum_{i=1}^{k} b_i \tau^i}$$

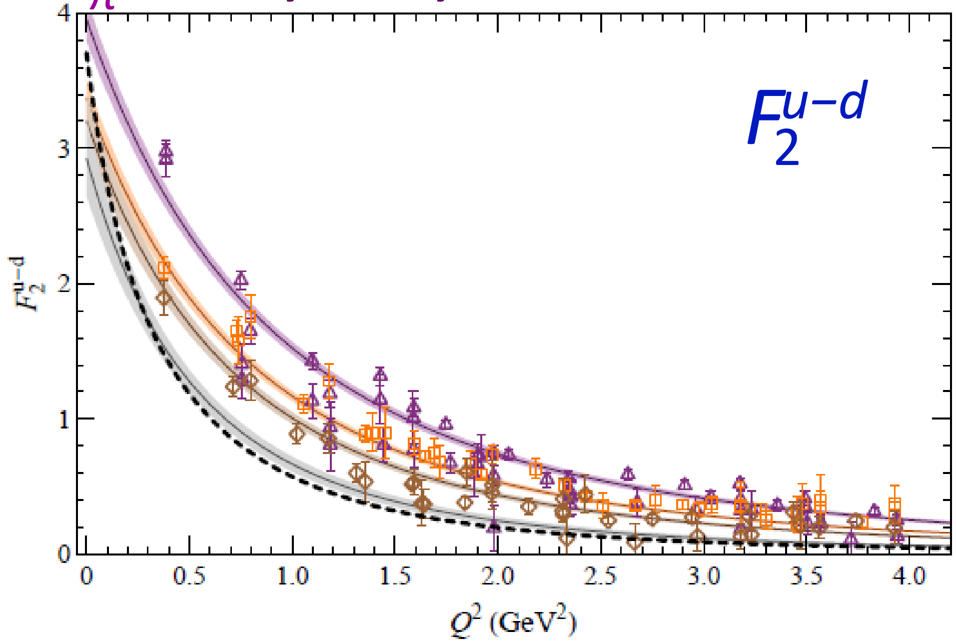
$$\left(\frac{F_2}{\kappa}\right) = \frac{1 + \sum_{i=1}^{k-3} a_i \tau^i}{1 + \sum_{i=1}^{k} b_i \tau^i}.$$

$$\tau = \frac{Q^2}{4m_N^2}$$

HWL et al., arXiv: 1005.0799

§ N_f = 2+1 anisotropic lattices, M_π ≈ **450**, **580**, **875** MeV





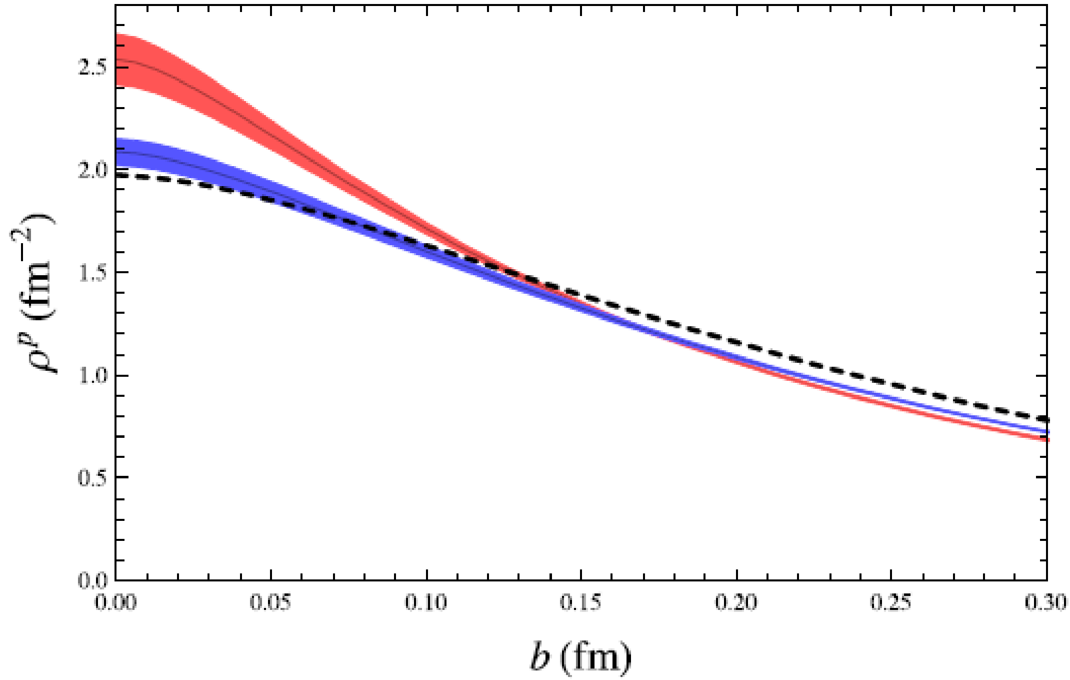
Transverse Charge Density

G. A. Miller, arXiv: 1002.0355

- Infinite-momentum frame

$$\rho(\mathbf{b}) \equiv \int \frac{d^2\mathbf{q}}{(2\pi)^2} F_1(\mathbf{q}^2) e^{i\mathbf{q}\cdot\mathbf{b}} = \int_0^\infty \frac{Q\,dQ}{2\pi} J_0(bQ) F_1(Q^2)$$

- How does high-Q^2 affect charge density?



- Red band uses lattice data ≤ 2.0 GeV^2
- Blue band uses lattice data ≤ 4.0 GeV^2

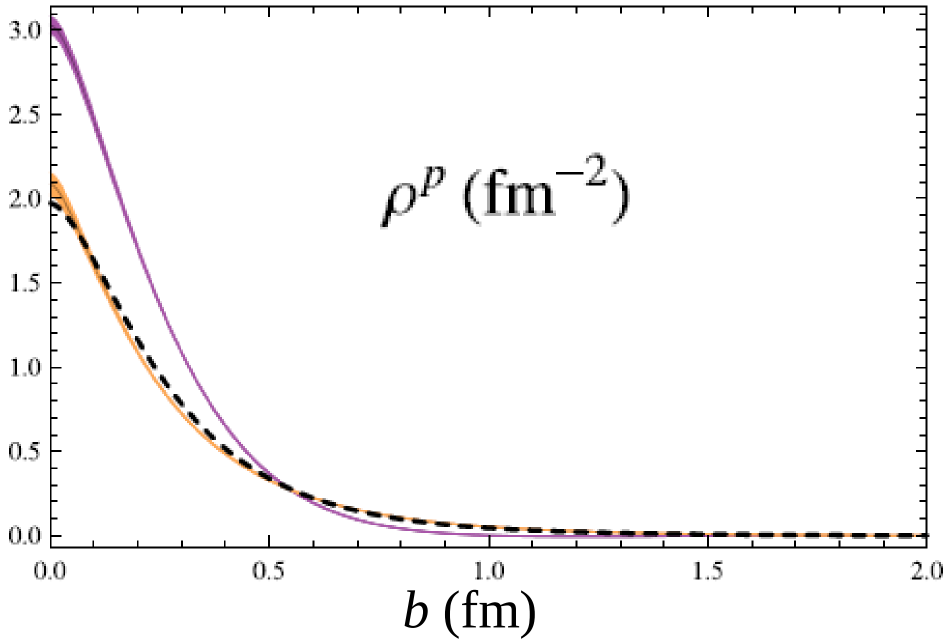
HWL et al., arXiv: 1005.0799

Transverse Charge Density

§ Transverse charge density in infinite-momentum frame

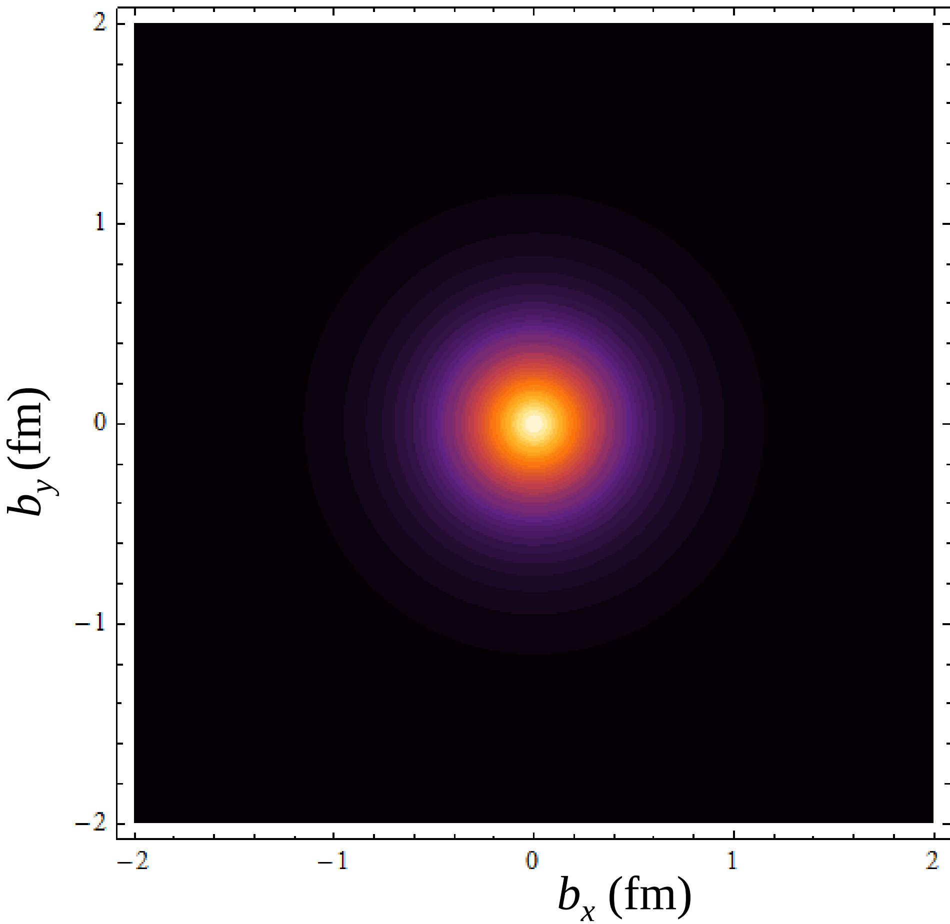
$$\rho(b) = \int_0^\infty \frac{Q\,dQ}{2\pi} J_0(bQ) F_1(Q^2)$$

Proton



HWL et al., arXiv: 1005.0799

G. A. Miller, arXiv: 1002.0355



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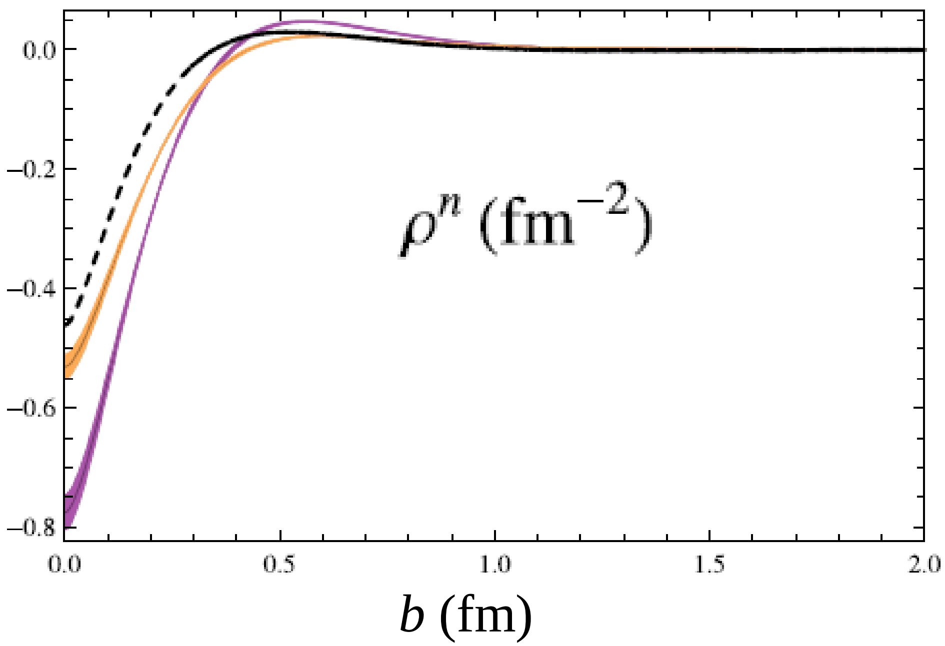
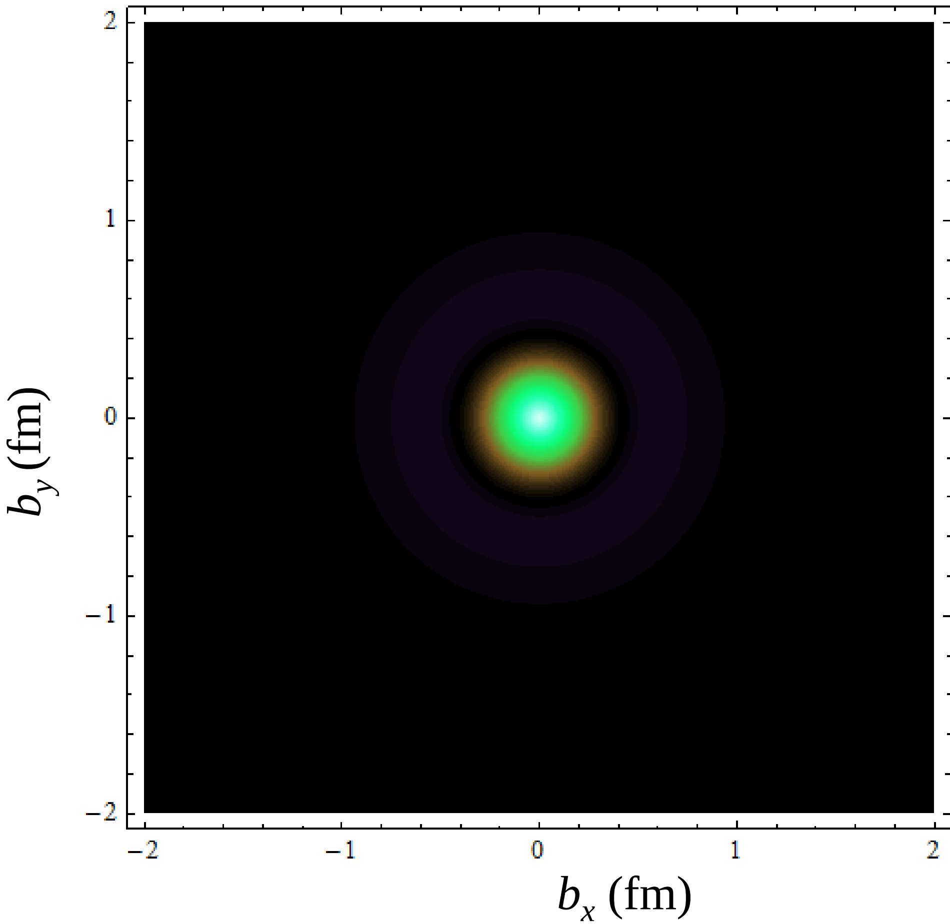
Neutron





HWL et al., arXiv: 1005.0799

Transverse Magnetization Density

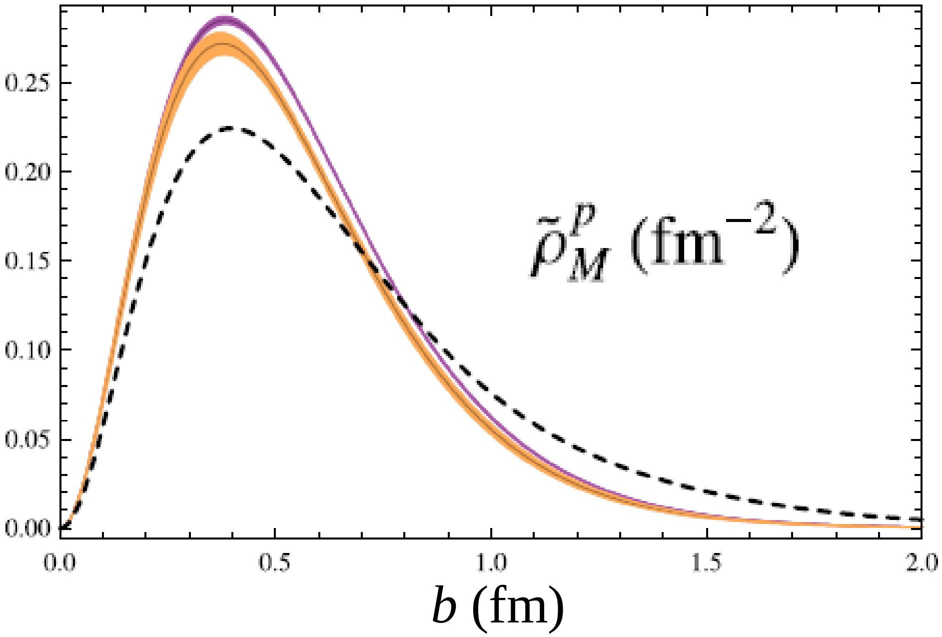
- Magnetization density in infinite-momentum frame

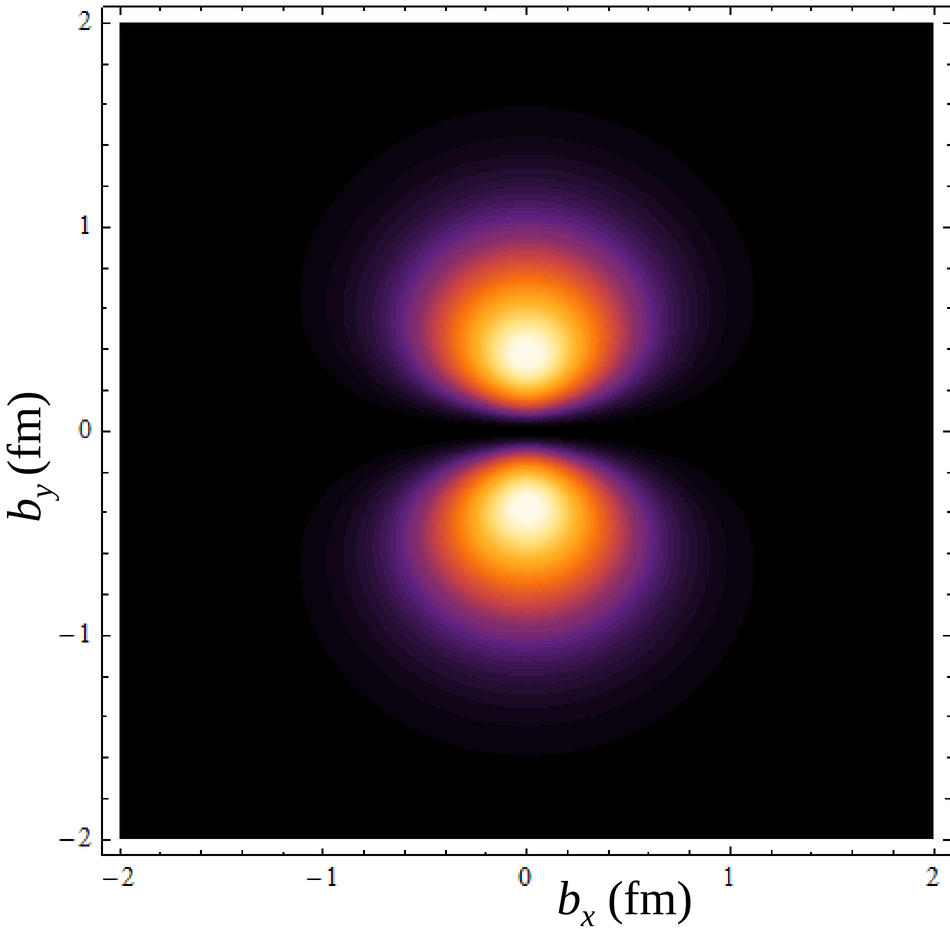
$$b \sin^2 \phi \int_0^\infty \frac{Q^2\, dQ}{2\pi} J_1(bQ) F_2(Q^2)$$

G. A. Miller, arXiv: 1002.0355

- $\varphi = \pi/2$

Proton





HWL et al., arXiv: 1005.0799

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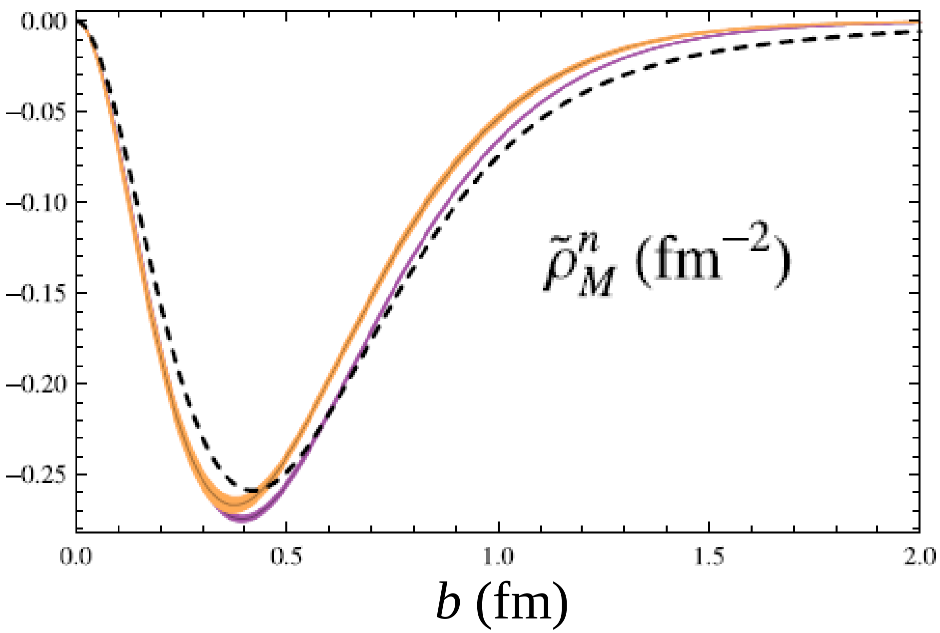
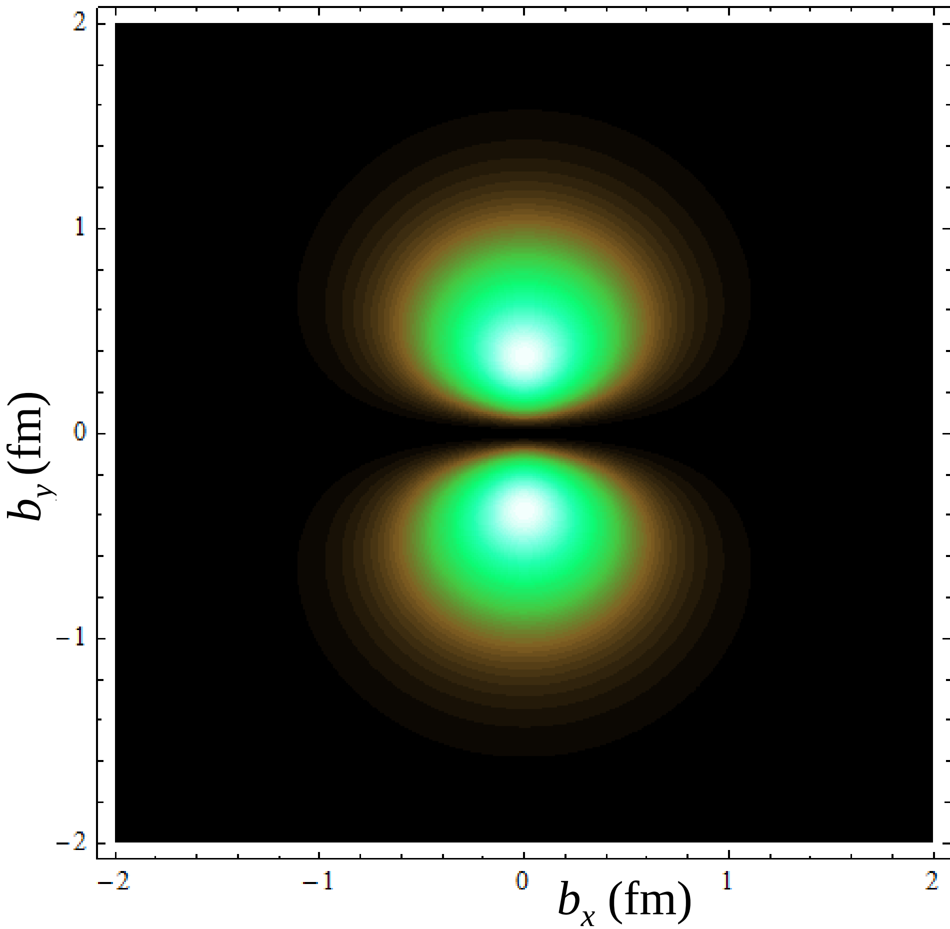
G. A. Miller, arXiv: 1002.0355

Neutron



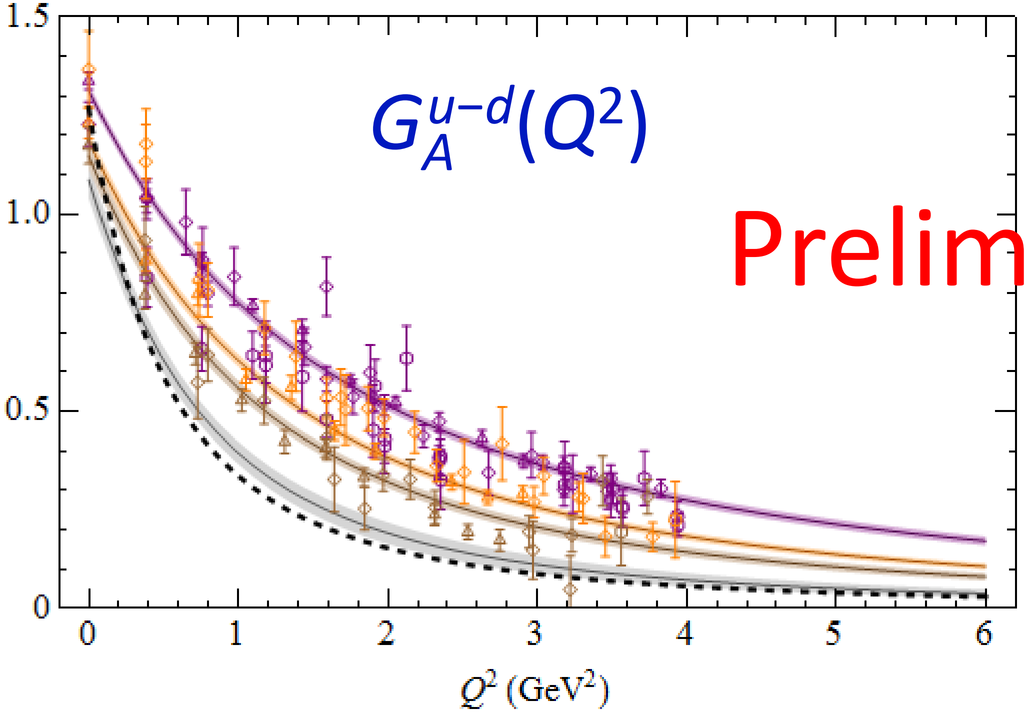


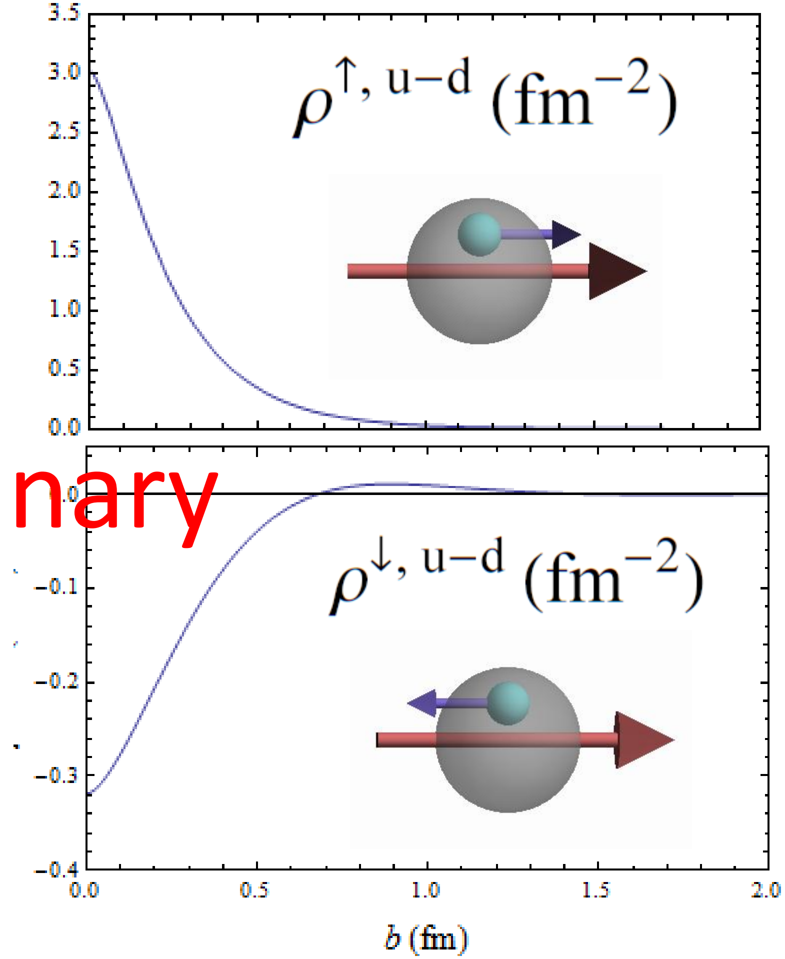
HWL et al., arXiv: 1005.0799

Nucleon Axial Form Factors

- $N_f = 2+1$ anisotropic lattices, $M_\pi \approx$ **450**, **580**, **875** MeV

$$\overline{u}_B(p')\left[\gamma_\mu\gamma_5 G_A(q^2) + \gamma_5 q_\nu \frac{G_P(q^2)}{2M_B}\right]u_B(p)$$

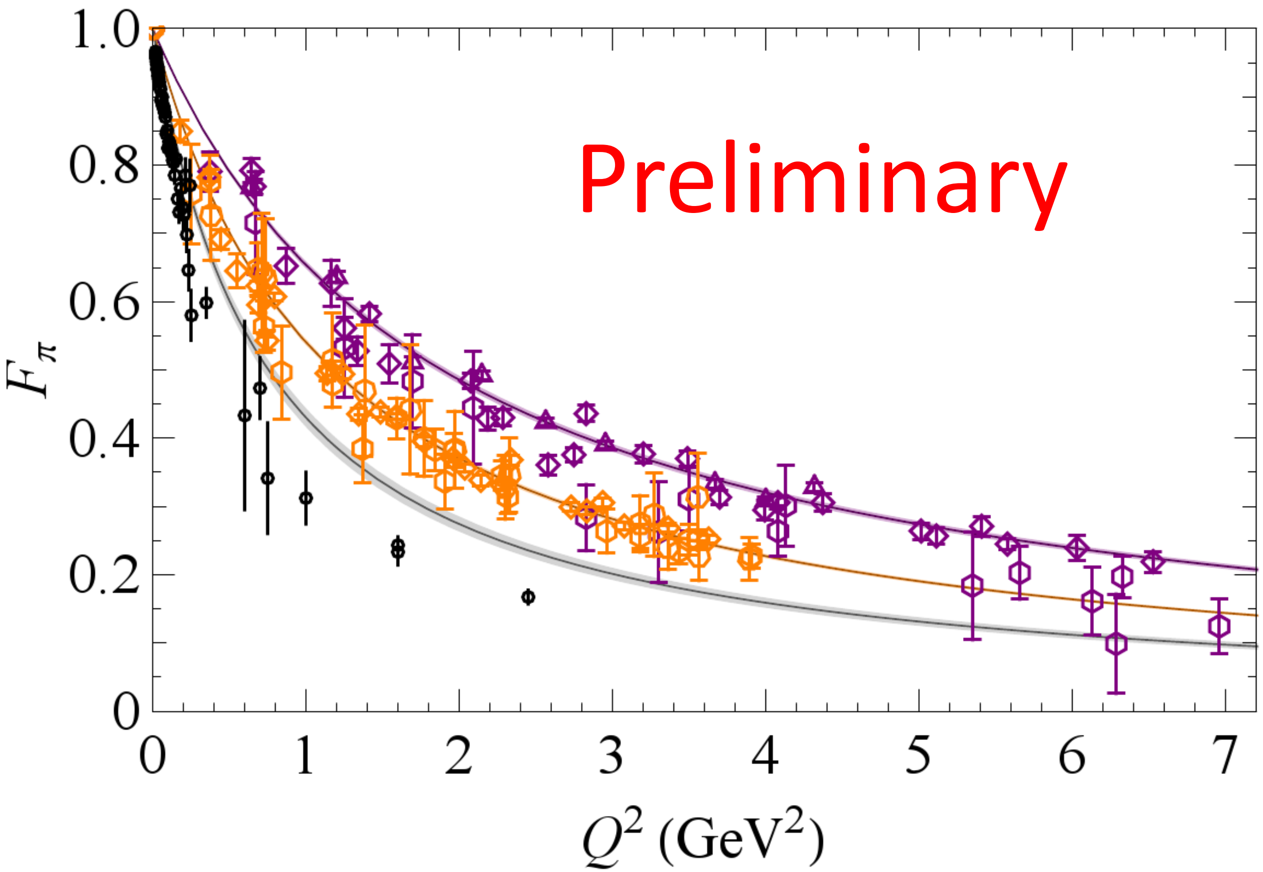




Preliminary

Pion Form Factors

- § $N_f = 2+1$ anisotropic lattices, $M_\pi \approx$ **875**, **1350** MeV



G.M. Huber et al., Phys.Rev.C78:045203,2008.

Miscellany

§ Disconnected contribution O(10^{-2}) for EM form factor

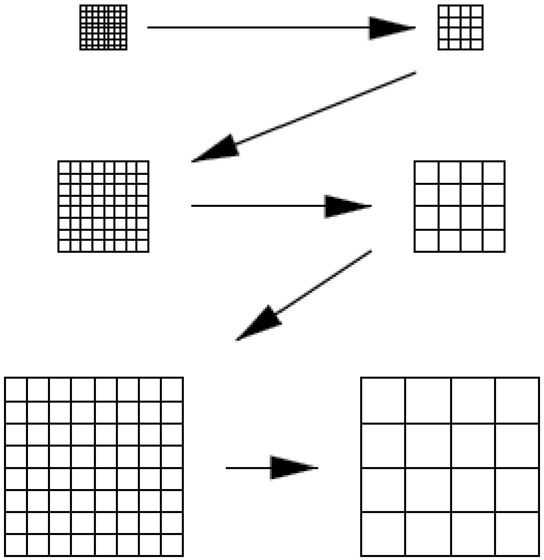
- Small for most of the form factors but
  could be significant for neutron electric form factor

§ To get larger momentum, we use O(*ap*) ≈ 1

- Rome was not built in a day…
- Methodology for improving a traditional lattice calculation

§ Possible future improvement

- Step-scaling through
  multiple lattice spacings and volumes
- Higher momentum transfer



Summary

Exploratory study of large momentum transfer on the lattice

- A Novel Strategy
  - Include operators that couple to high-momentum and excited states
  - Explicitly analyze excited states to get better ground-state signal
- What We Show
  - Demonstrated results for heavier pions
  - Transverse densities
- Future Work
  - Smaller src-snk separation for better signal
  - Extend to other hadrons or isotropic lattices
  - Multiple lattice spacings to study/reduce systematic error