Transverse structure of the nucleon

19 May 2010   Jefferson Lab

Leonard Gamberg
Penn State University

Outline

- **Transverse spin Effects in TSSAs**
- **Gauge links-Color Gauge Inv.-"T-odd" TMDs**
- **Transverse Distortion and TSSAs**
- **Unifying structure GTMDs/Wigner Functions**

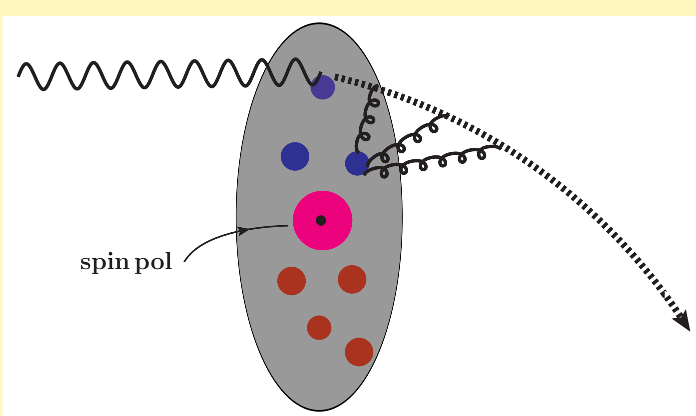
"QCD calc" FSIs Gauge Links-Color Gauge Inv. "T-odd" TMDs

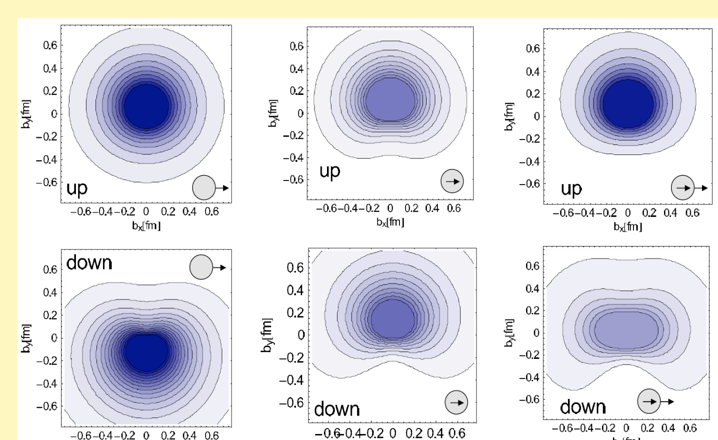
"Pheno" -Transverse Structure TMDs and TSSAs-**b** and **k** asymm

An improved dynamical approach for FSIs & model building

$f_{1T}^{\perp}(x, \mathbf{k}_{\perp}^{2})$

$\mathcal{E}(x, \mathbf{b}_{\perp}^{2})$





Conclusions

- **EIC in conjunction w/ Drell Yan can test fundamental factorization theorem of QCD: predicted sign change of Sivers function**
- **Crucial to have Q^2 range to pin down TMDs in particular Sivers function**
- **Transverse Distortion/Structure and TSSAs and unintegrated PDFs --- "Wigner functions" are there exclusive processes where they come in?**
- **Unifying structure GTMDs/Wigner Functions**
- **Pheno-Transverse Structure TMDs and TSSAs *b* and *k* asymm. An improved dynamical approach for FSI & model building**

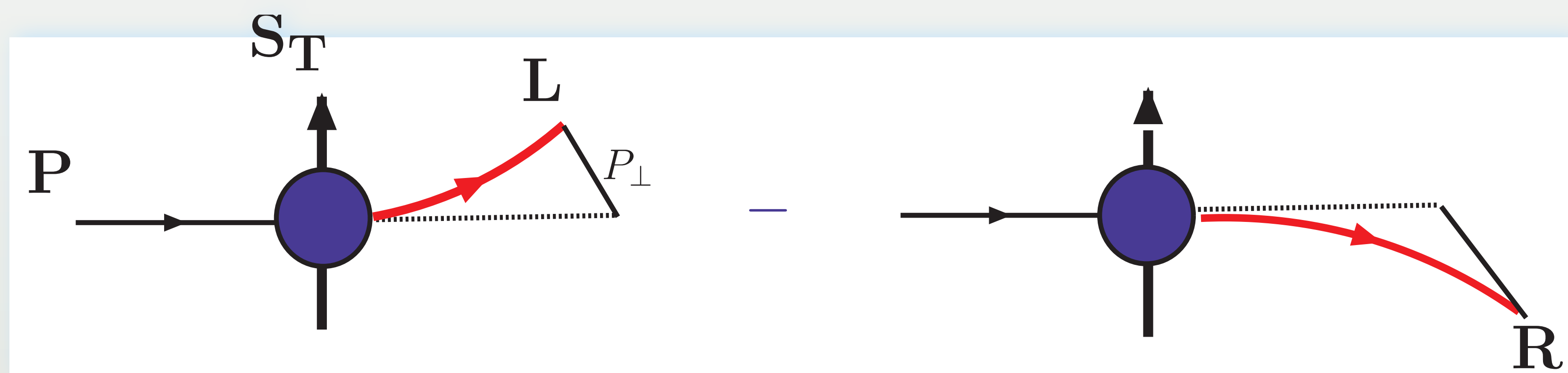
"QCD calc " FSIs Gauge Links-Color Gauge Inv. "T-odd" TMDs

MORE .....

- **Jet SIDIS**
- **Extracting weighted TSSAs**
- **Connection bwtn. gluonic and fermionic poles--twist 3 ETQS approach to TSSAs and the TMD description**
- **Opportunities to further explore angular momentum sum rule(s)**

Transverse SPIN Observables SSA (TSSA) $P^{\uparrow}P \to \pi X$

- **Single Spin Asymmetry**

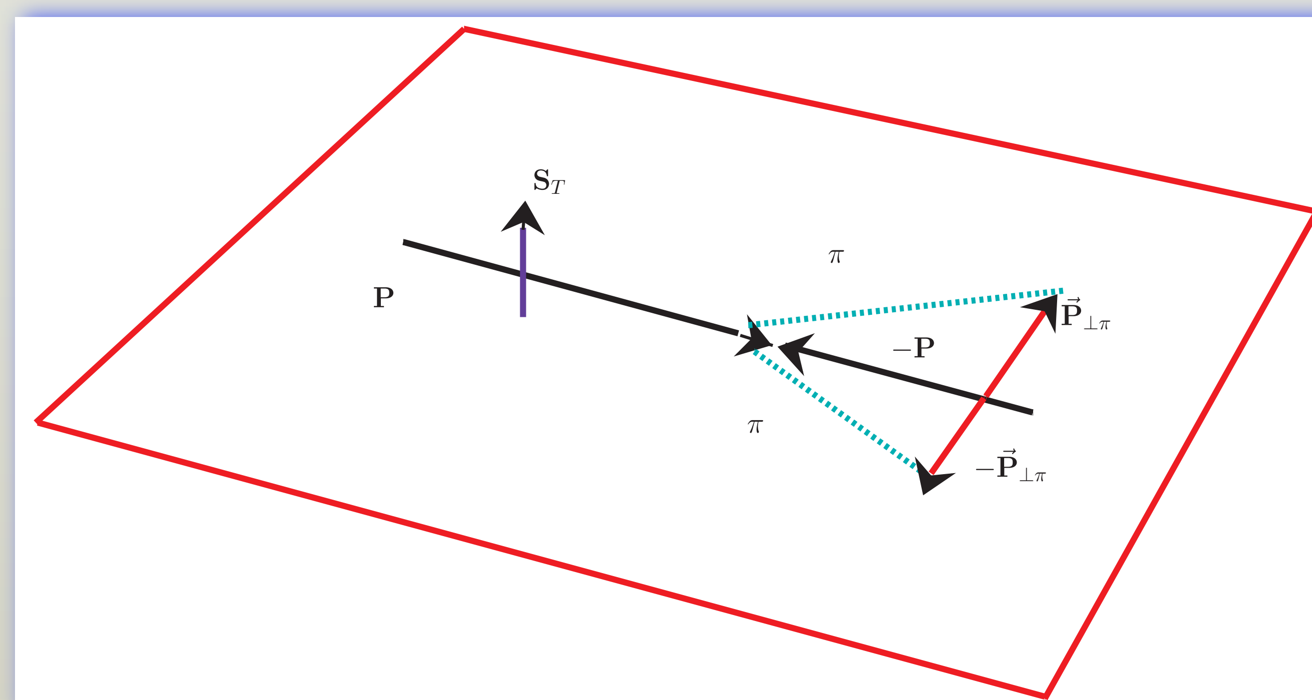


Parity Conserving interactions: SSAs Transverse Scattering plane

$$\Delta\sigma \sim iS_T \cdot (\mathbf{P} \times P_{\perp}^{\pi})$$

- **Rotational invariance** $\sigma^{\downarrow}(x_F, \boldsymbol{p}_\perp) = \sigma^{\uparrow}(x_F, -\boldsymbol{p}_\perp)$
  $\Rightarrow$ ***Left-Right Asymmetry***

$$\boldsymbol{A}_N = \frac{\sigma^{\uparrow}(x_F,\boldsymbol{p}_\perp) - \sigma^{\uparrow}(x_F,-\boldsymbol{p}_\perp)}{\sigma^{\uparrow}(x_F,\boldsymbol{p}_\perp) + \sigma^{\uparrow}(x_F,-\boldsymbol{p}_\perp)} \equiv \Delta\sigma$$



Reaction Mechanism

⋆ **Co-linear factorized QCD-parton dynamics**

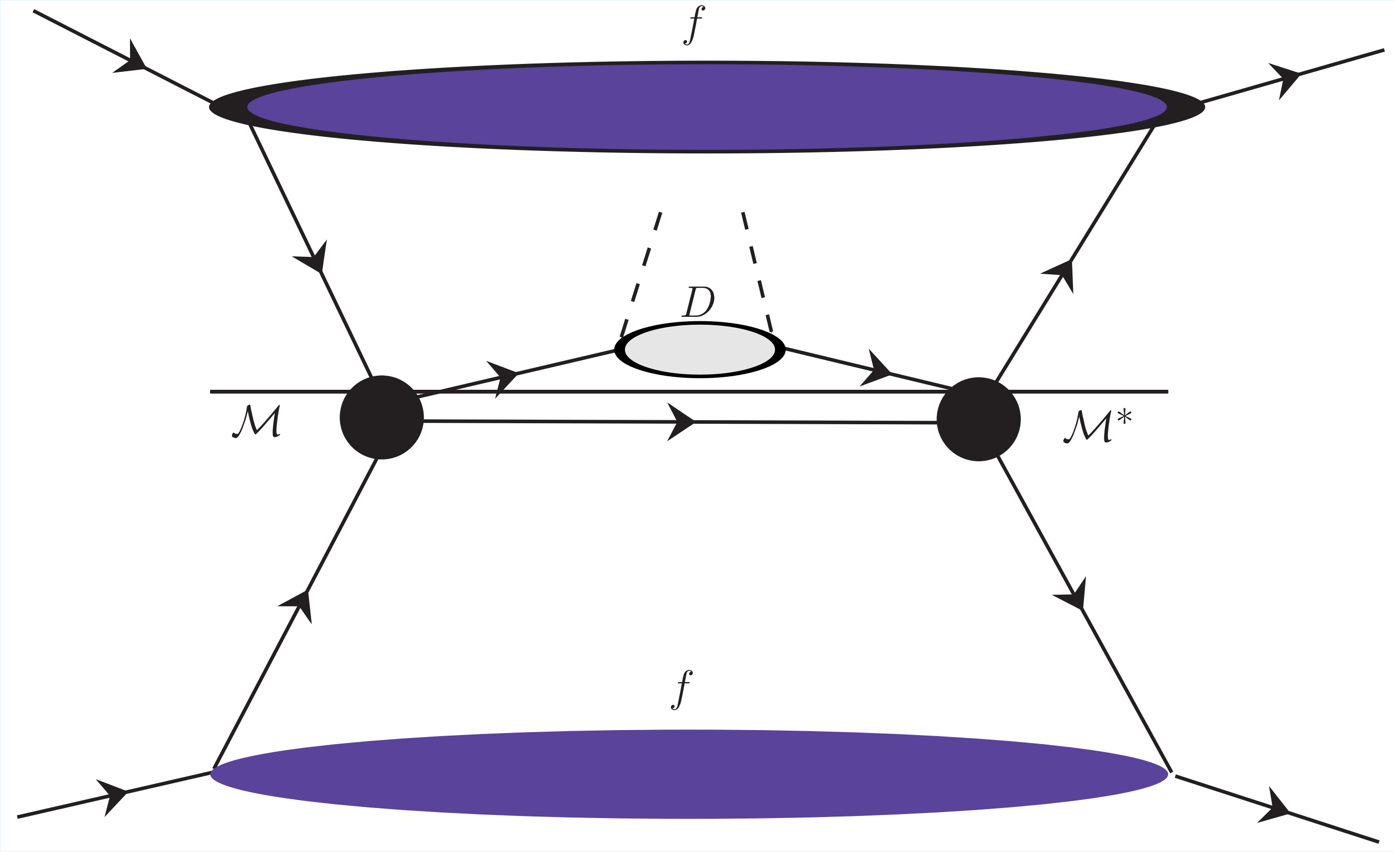
$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$

Requires helicity flip-hard part $\Delta\hat{\sigma} \equiv \hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}$

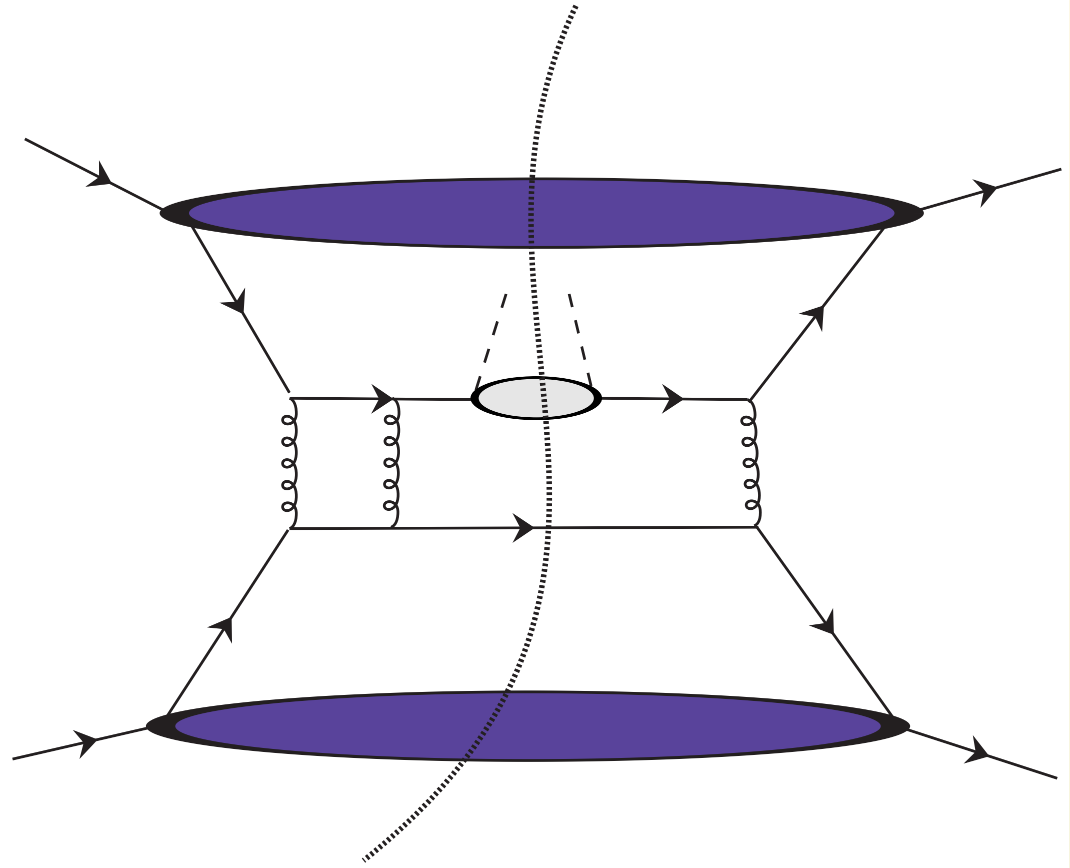
⋆ **TSSA requires relative phase btwn** ***different*** **helicity amps**

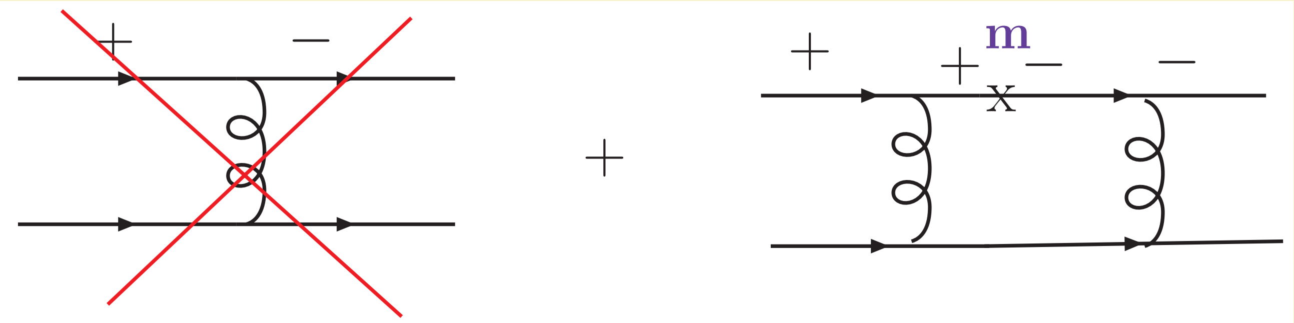
$$\hat{a}_N = \frac{\hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}}{\hat{\sigma}^{\uparrow} + \hat{\sigma}^{\downarrow}} \sim \frac{\mathrm{Im}\left(\mathcal{M}^{+*}\mathcal{M}^{-}\right)}{|\mathcal{M}^{+}|^2 + |\mathcal{M}^{-}|^2}$$

$$|\uparrow / \downarrow\rangle = (|+\rangle \pm i|-\rangle)$$



Factorization Theorem in QCD Helicity limit....triviality.....





- QCD interactions conserve helicity
  $m_q \to 0$ and Born amplitudes real

$\star$ $A_N \sim \frac{m_q \alpha_s}{E}$ Kane, Repko, PRL:1978

Twist three and trival?!

Not the full story.... Twist 3 approach ETQS approach

Phases in *soft* poles of propagator in hard subprocess Efremov & Teryaev :PLB 1982

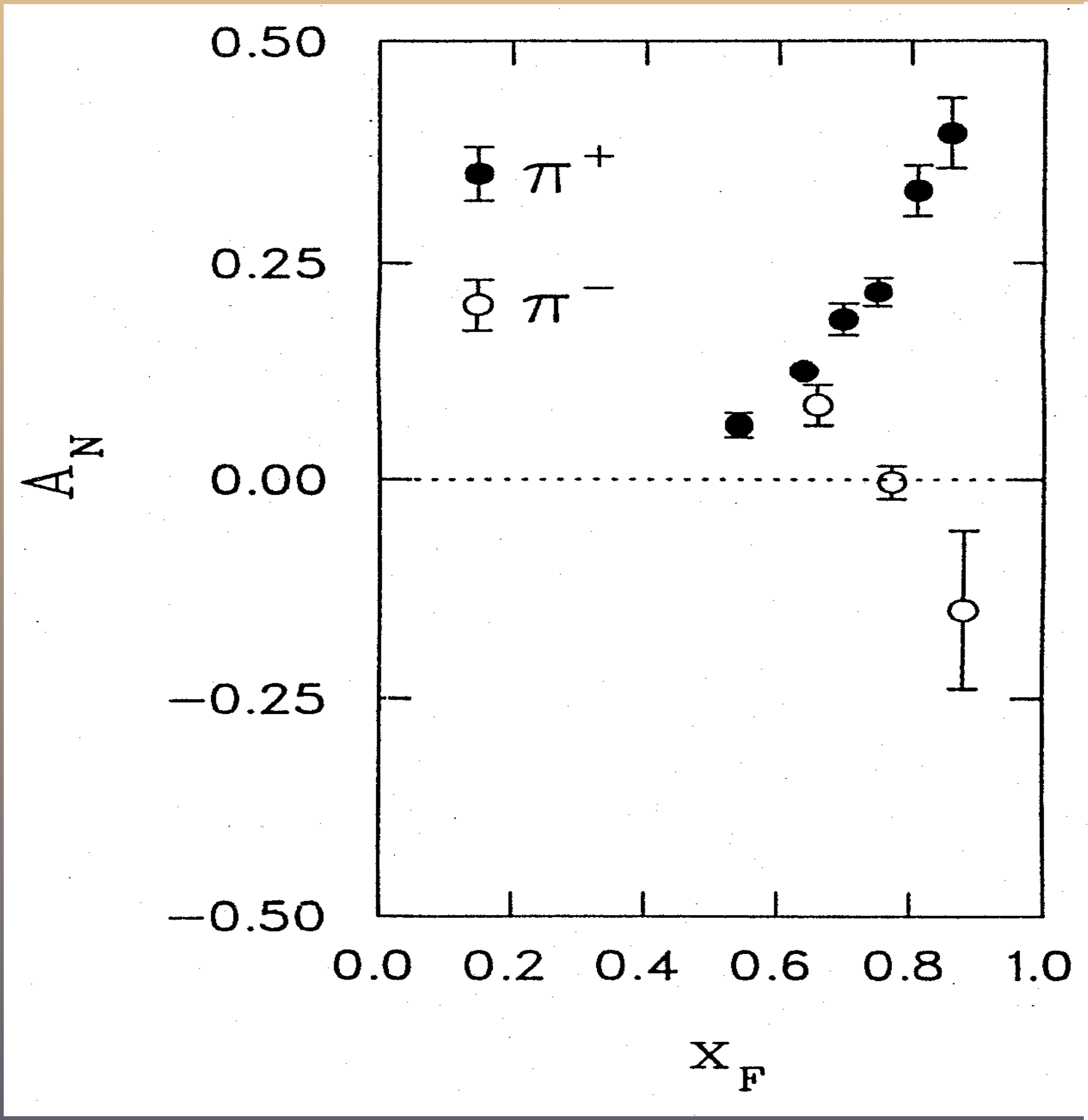
Qiu-Sterman:PLB 1991, 1999, Koike et al. PLB 2000. . . 2007,

Ji,Qiu,Vogelsang,Yuan:PR 2006,2007. . .

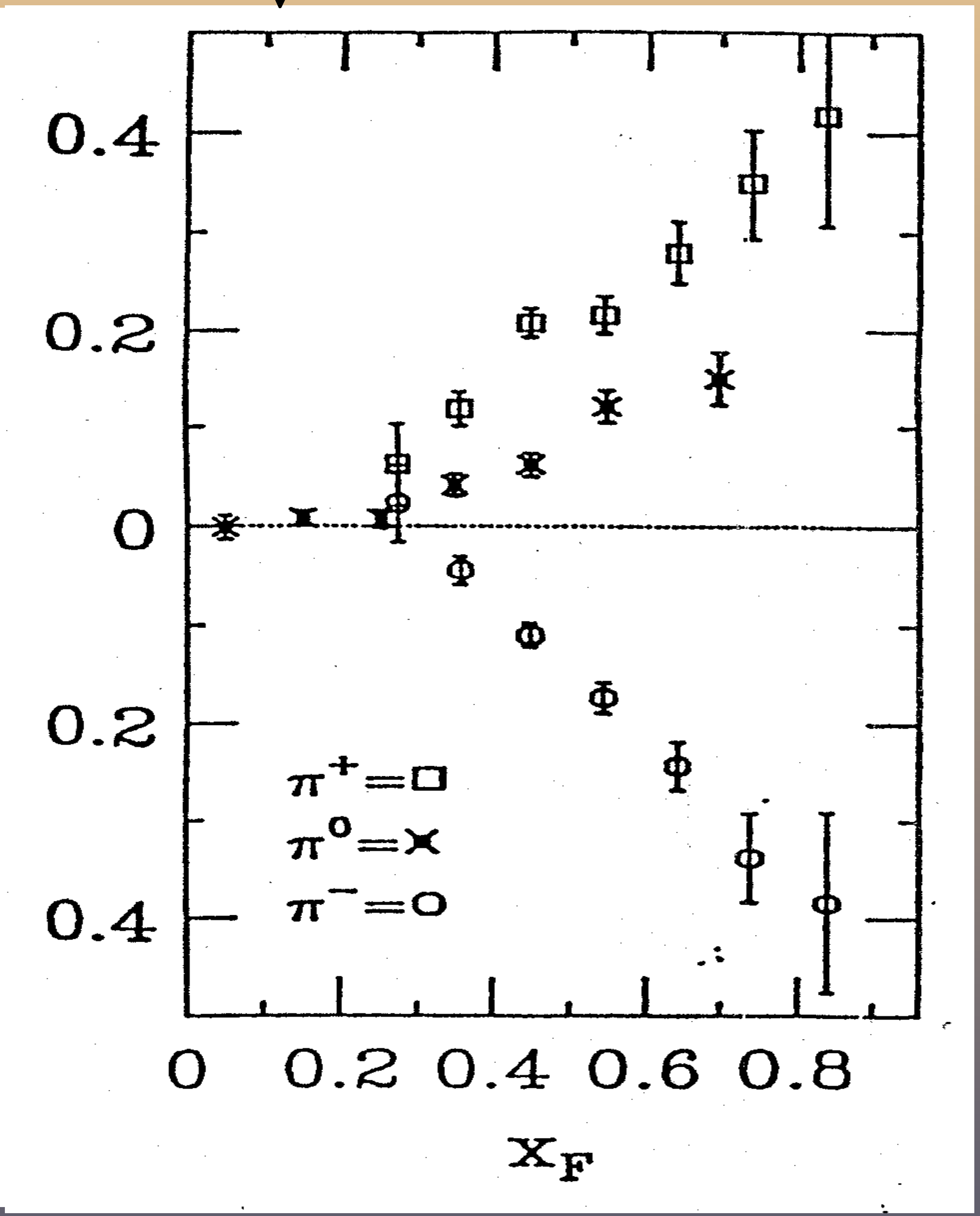
Large Transverse Polarization in Inclusive Reactions $P^{\uparrow} P \to \pi X$

W.H. Dragoset et al.,
PRL 36, 929 (1976)

Argonne ZGS, p_{beam} = 12 GeV/c



FNAL-E704
PLB261, 201 (1991), PLB264, 462 (1991)
200 GeV Beam
$\sqrt{s}$ =20 GeV



Polarization in inclusive Λ and $\bar{\Lambda}$ production at large p_T

B. Lundberg,* R. Handler, L. Pondrom, M. Sheaff, and C. Wilkinson†
Physics Department, University of Wisconsin, Madison, Wisconsin 53706

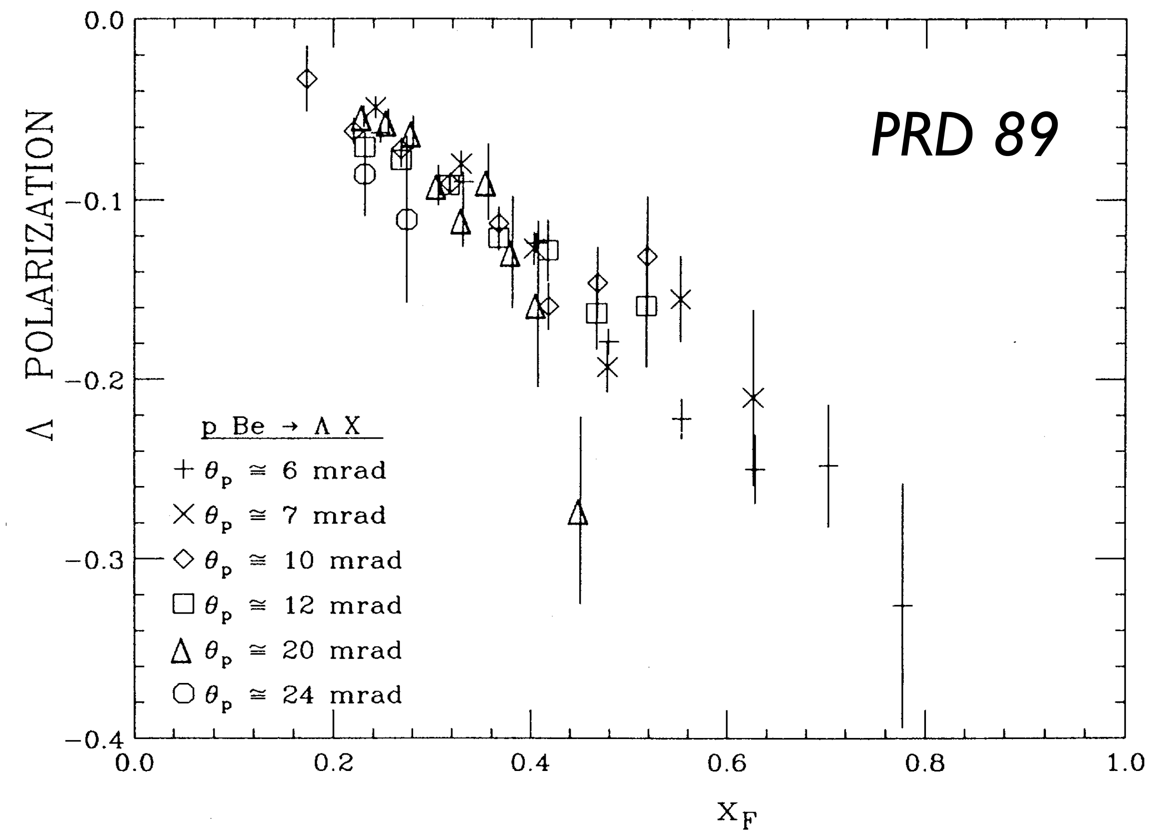
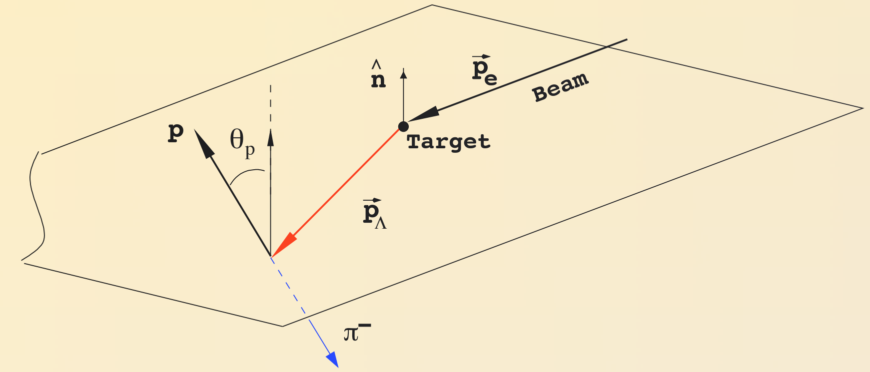


FIG. 4. The Λ polarization is shown as a function of x_F for all production angles. Over this range of production angles and within experimental uncertainties, the polarization is angle (or p_T) independent.



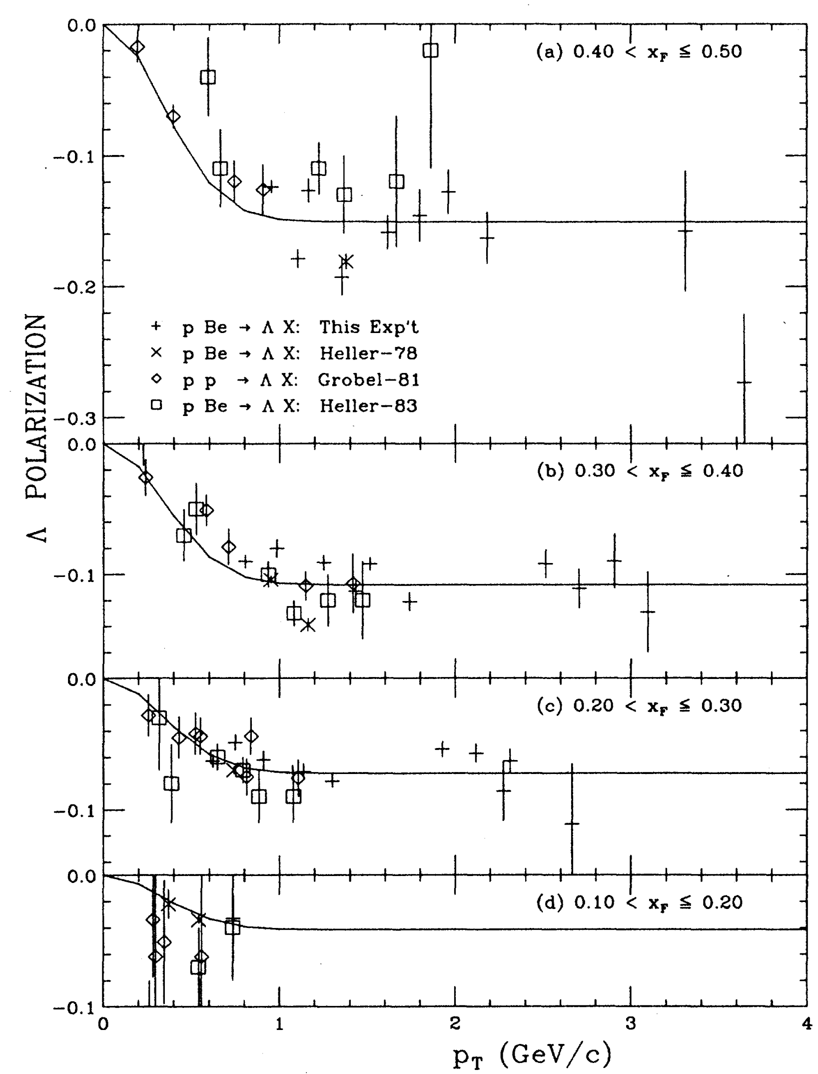
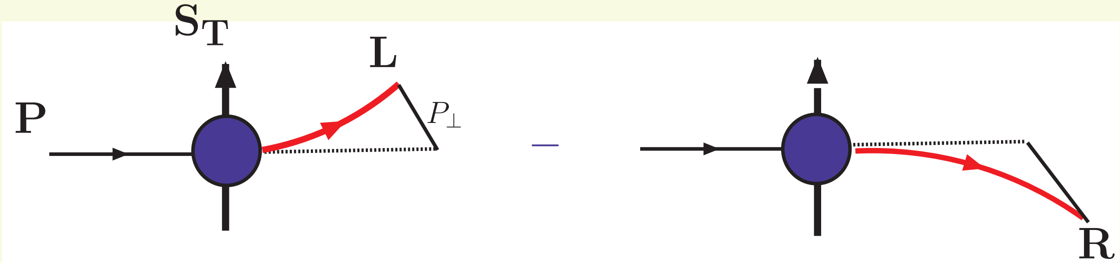
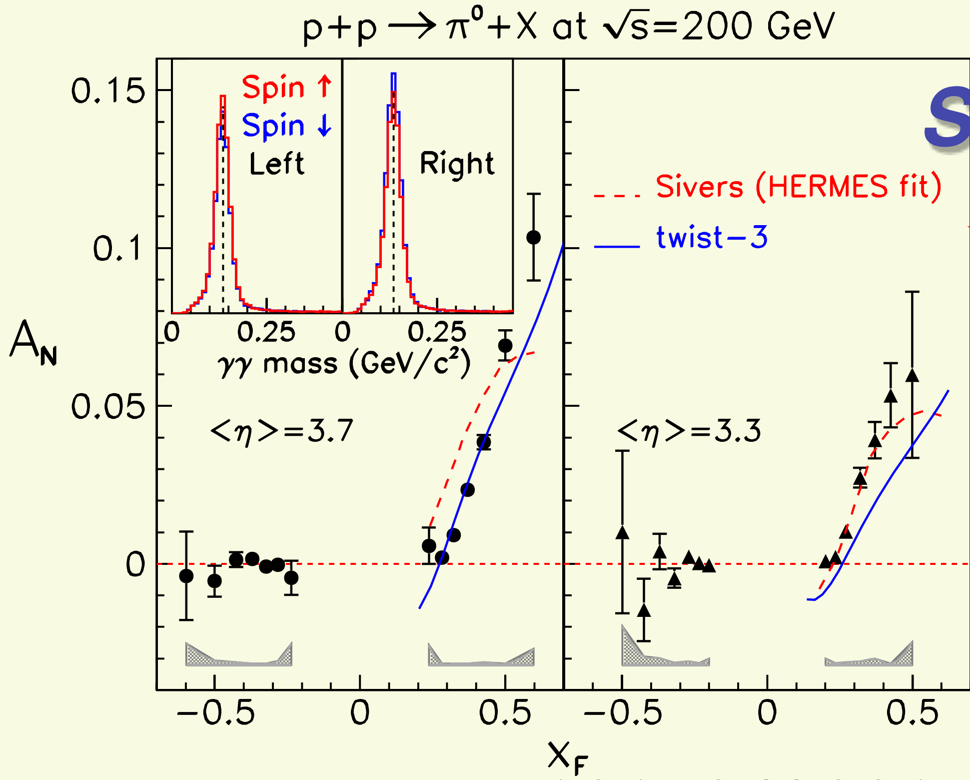


FIG. 5. Inclusive Λ polarization as a function of p_T with x_F restricted to each of the four ranges indicated in (a)–(d). The data plotted are from this experiment and Refs. 3, 23, and 24. All four experiments used the same spectrometer and measurement techniques. Errors when not shown are smaller than the points. The lines are a fit to the $p+$Be data using Eq. (9). Note that some of the scatter in the points is due to differences in the values of x_F at which they were measured.

$$P_\Lambda = \frac{\sigma^{pp\to\Lambda^\uparrow X} - \sigma^{pp\to\Lambda^\downarrow X}}{\sigma^{pp\to\Lambda^\uparrow X} + \sigma^{pp\to\Lambda^\downarrow X}}$$

Transverse SSA's at $\sqrt{s}$ = 62.4 & 200 GeV at RHIC

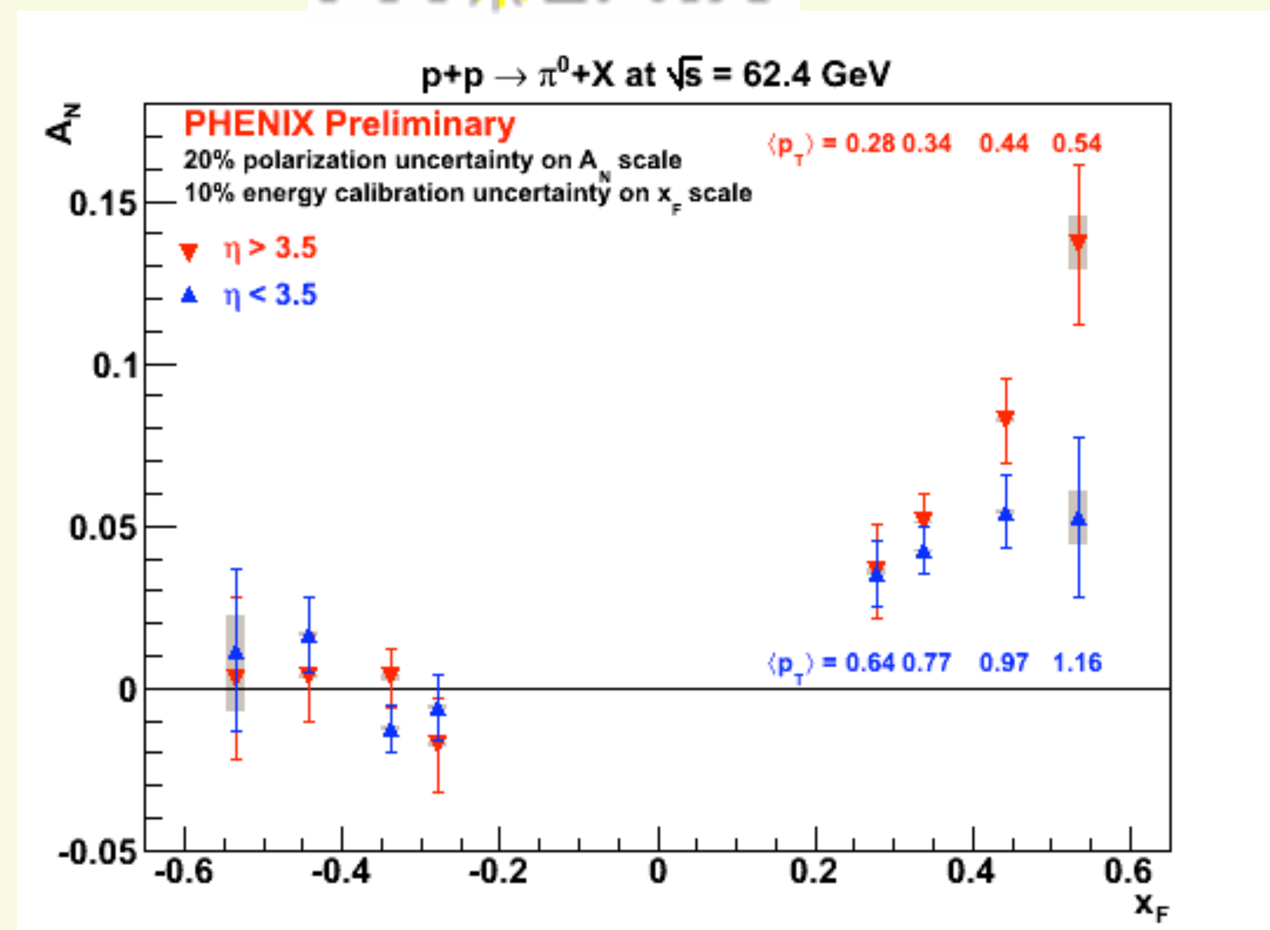




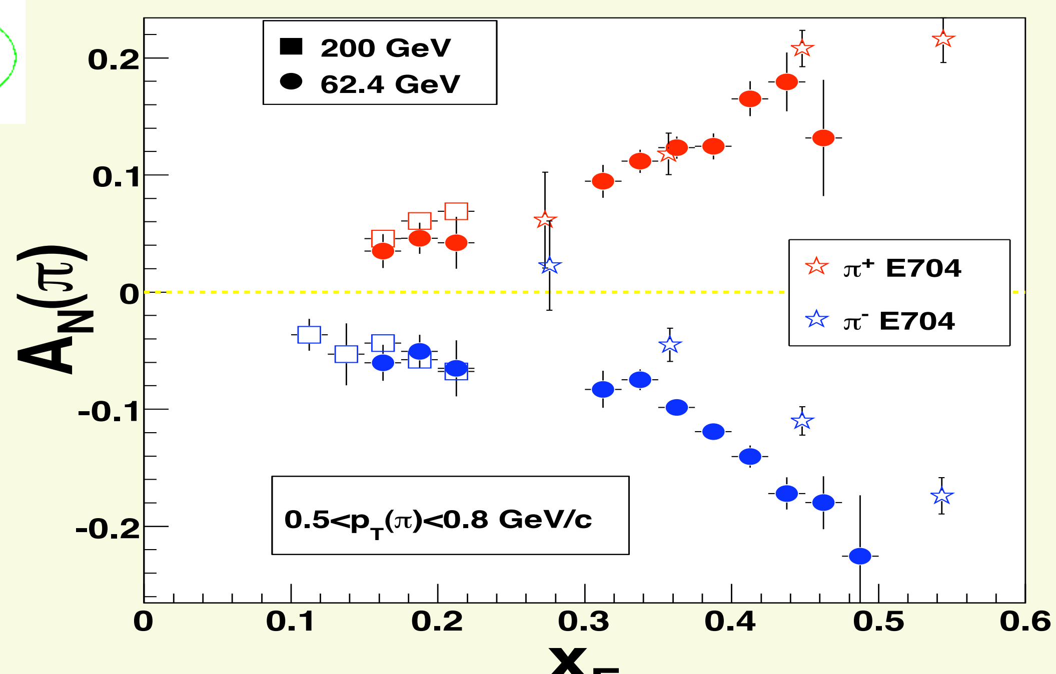
STAR

PRL101, 042001 (2008)

PHENIX



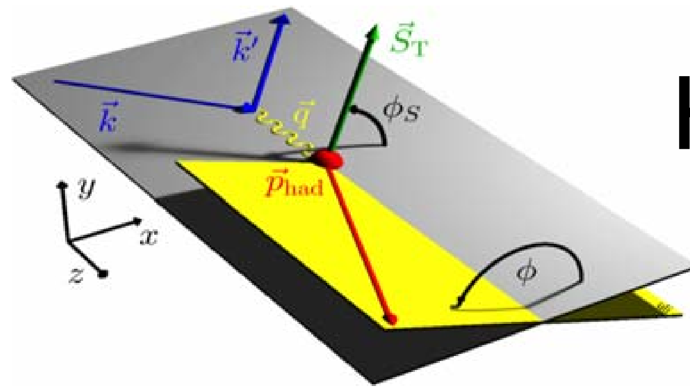
BRAHMS

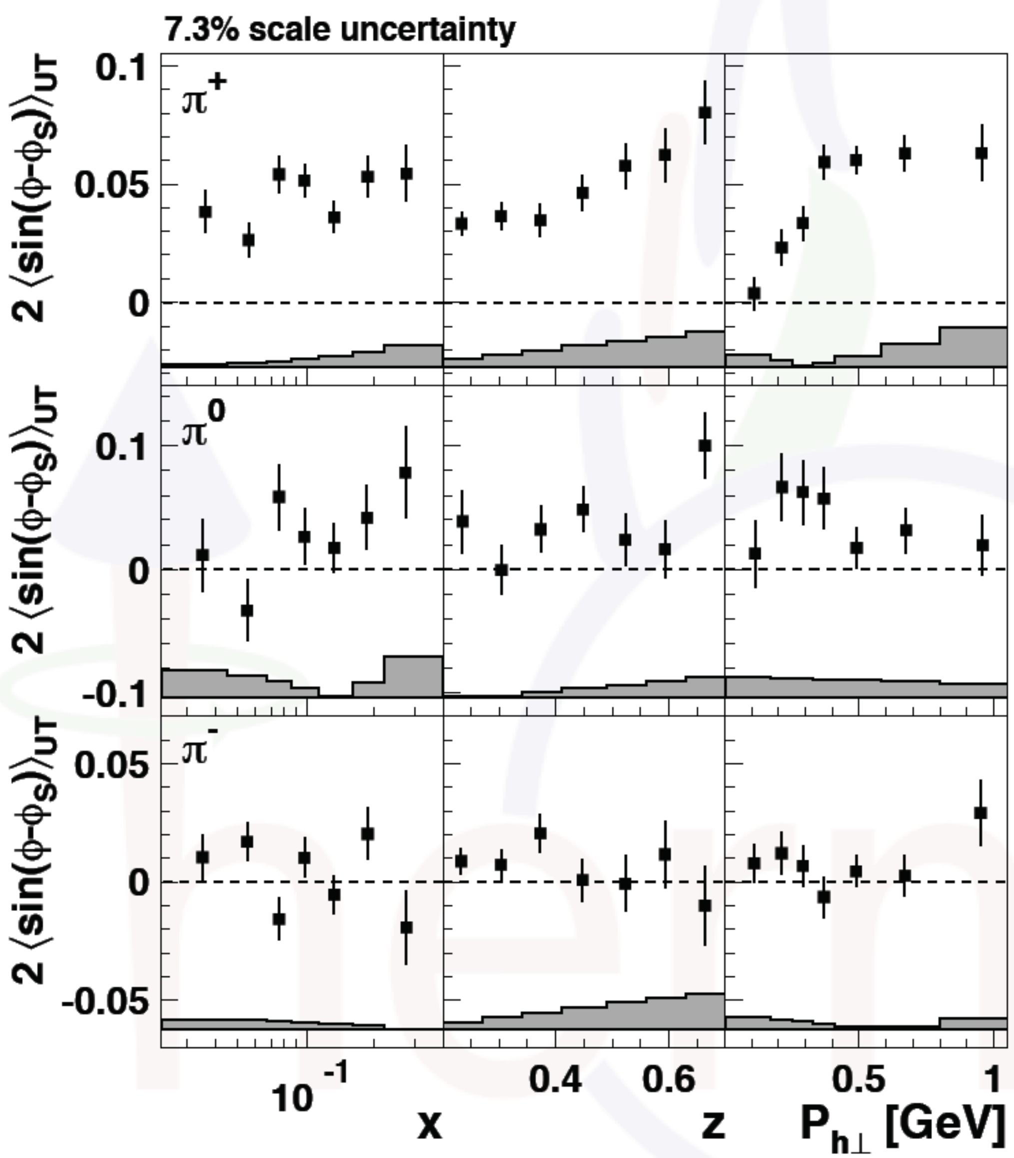


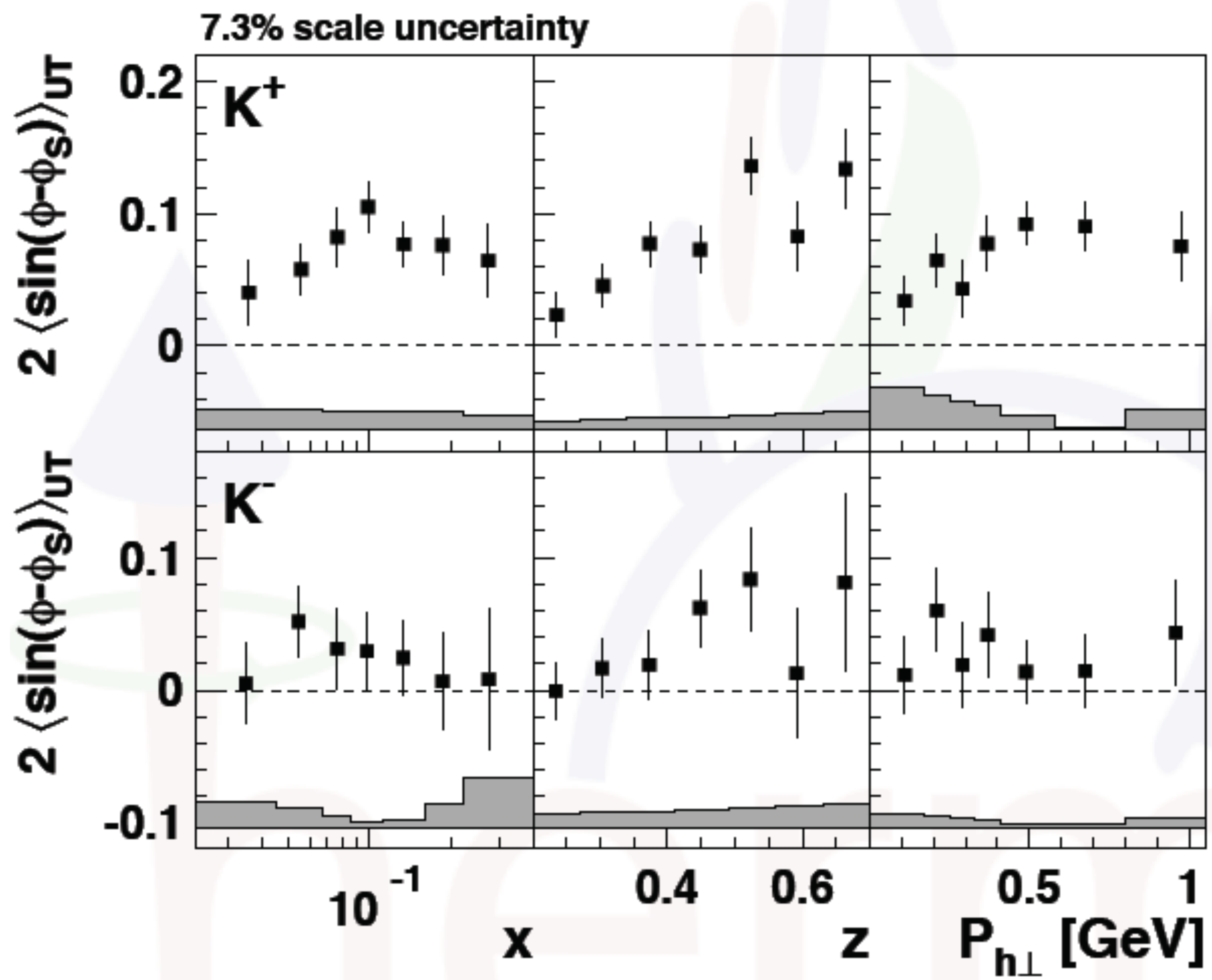
See talk of Les Bland

Hermes PRL 2009

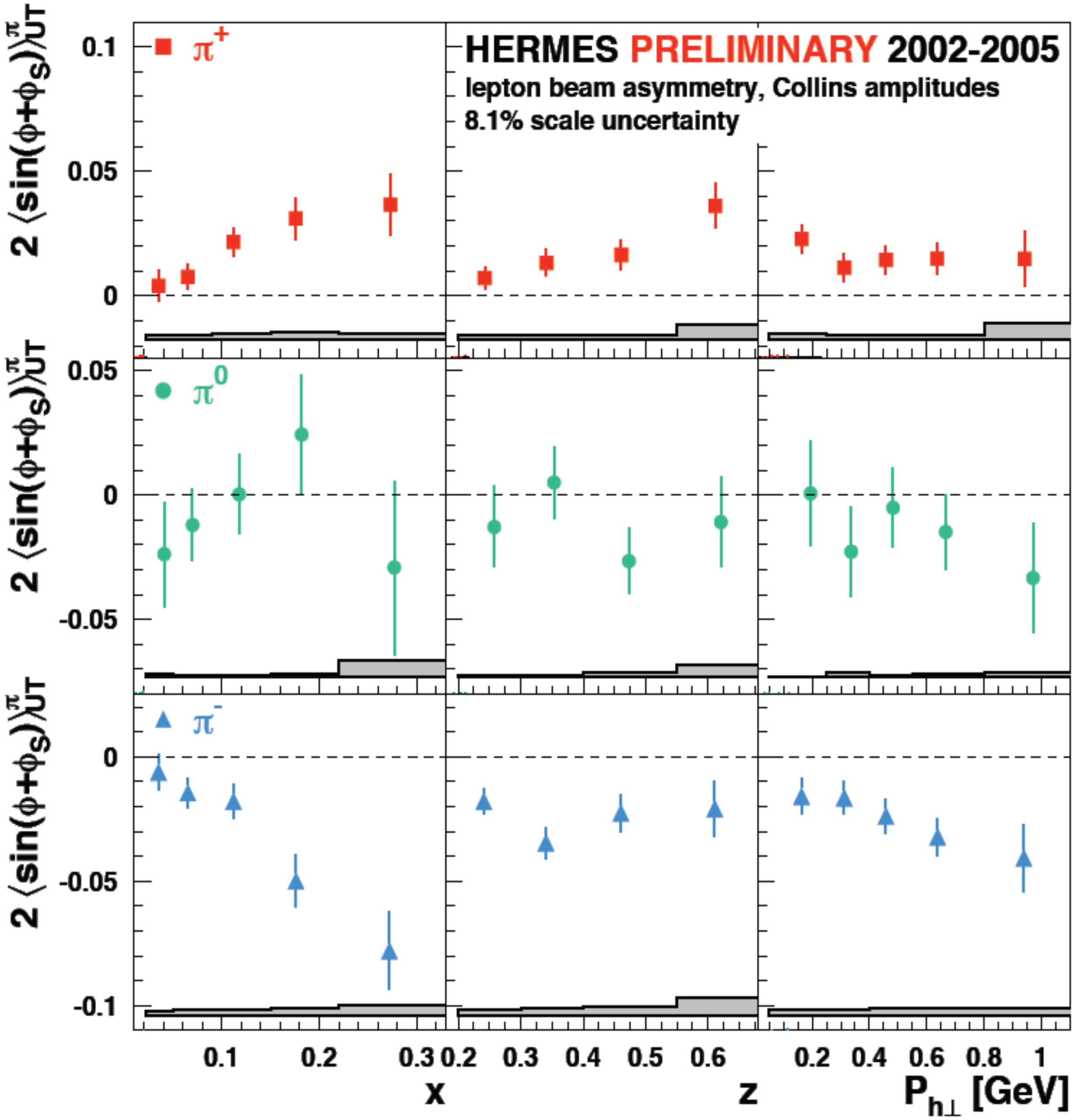
$$\ell p \to \ell' \pi X$$

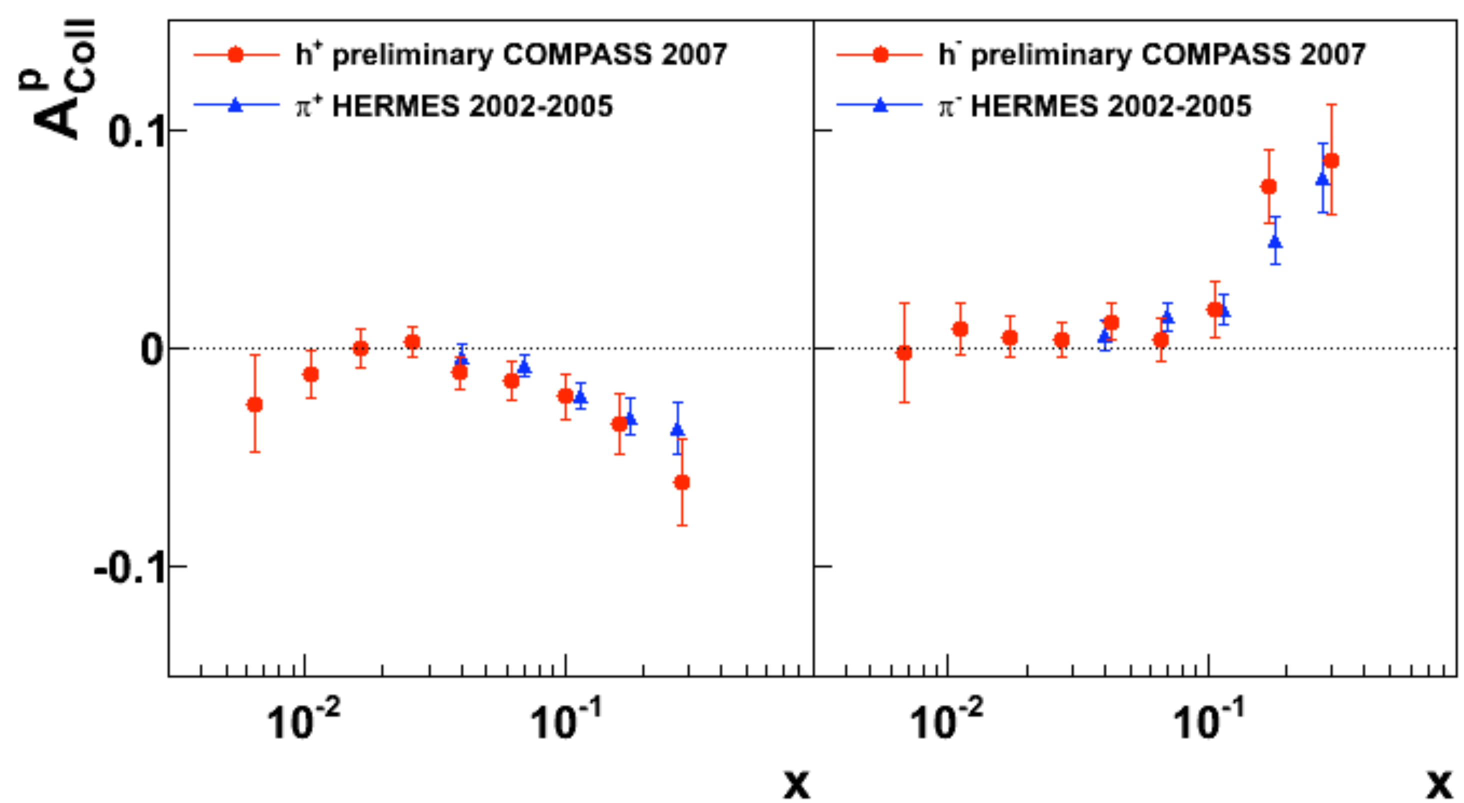






Collins Asymmetry
Compass-proton data 2007 comparison w/ HERMES-Collins
D. Hasch INT-12 GeV

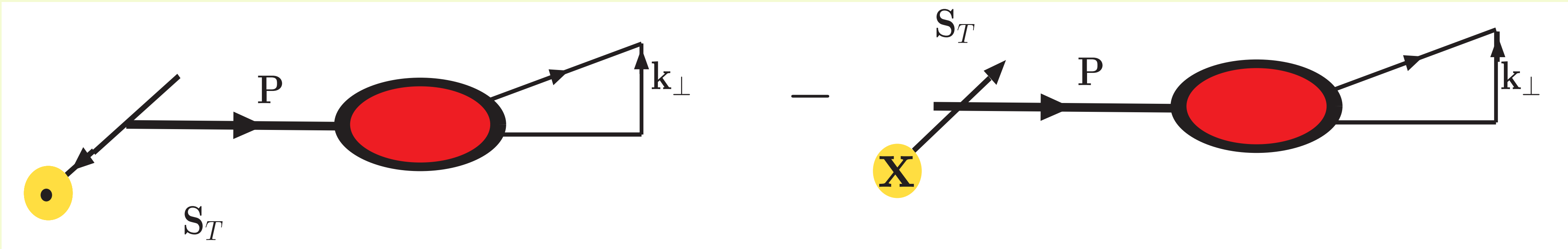




TSSAs thru "T-odd" non-pertb. **spin-orbit** *correlations....*

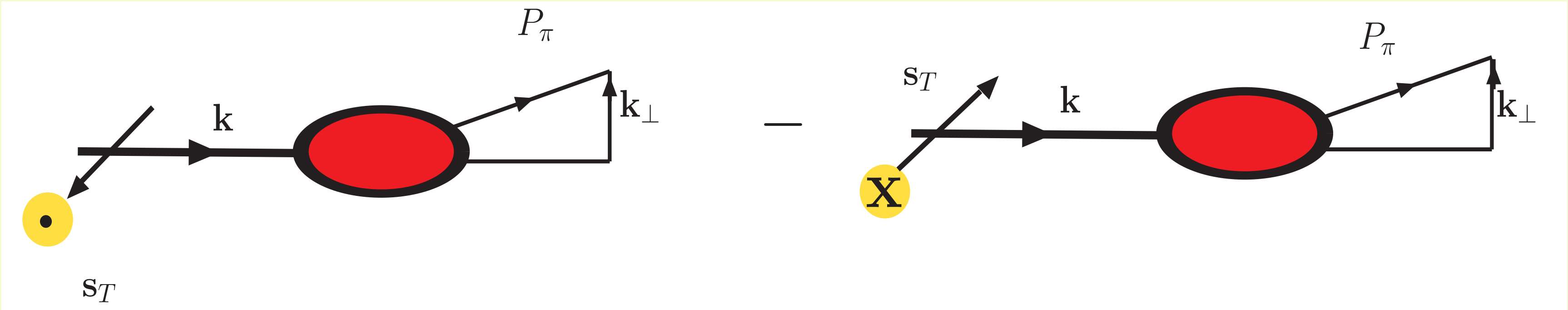
Sensitivity to $p_T \sim \mathbf{k}_T << \sqrt{Q^2}$

- Sivers PRD: 1990 TSSA is associated w/ correlation *transverse* spin and momenta in initial state hadron



$$\Delta\sigma^{pp^{\uparrow}\rightarrow\pi X} \sim D \otimes f \otimes \Delta f^{\perp} \otimes \hat{\sigma}_{Born} \Longrightarrow \boldsymbol{\Delta f^{\perp}(x, k_{\perp}) = i S_T \cdot (P \times k_{\perp})}\, f_{1T}^{\perp}(x, \boldsymbol{k}_{\perp})$$

- Collins NPB: 1993 TSSA is associated with *transverse* spin of fragmenting quark and transverse momentum of final state hadron



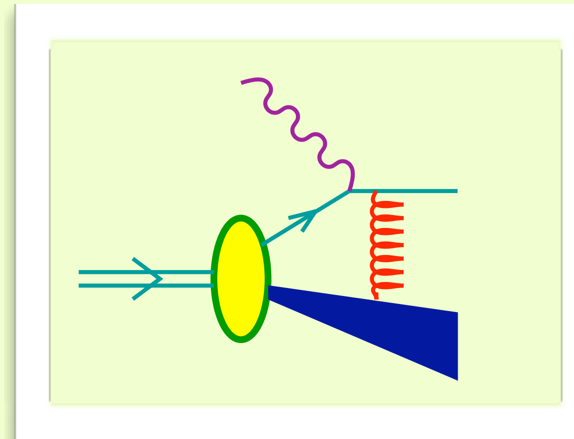
$$\Delta\sigma^{ep^{\uparrow}\rightarrow e\pi X} \sim \Delta D^{\perp} \otimes f \otimes \hat{\sigma}_{Born} \Longrightarrow \boldsymbol{\Delta D^{\perp}(x, p_{\perp}) = i s_T \cdot (P \times p_{\perp})}\, H_1^{\perp}(x, \boldsymbol{p}_{\perp})$$

Mechanism-FSI produce phases in TSSAs at Leading Twist

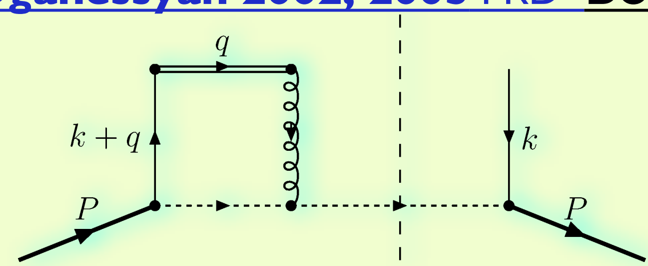
- Brodsky, Hwang, Schmidt PLB: 2002

SIDIS w/ transverse polarized nucleon target

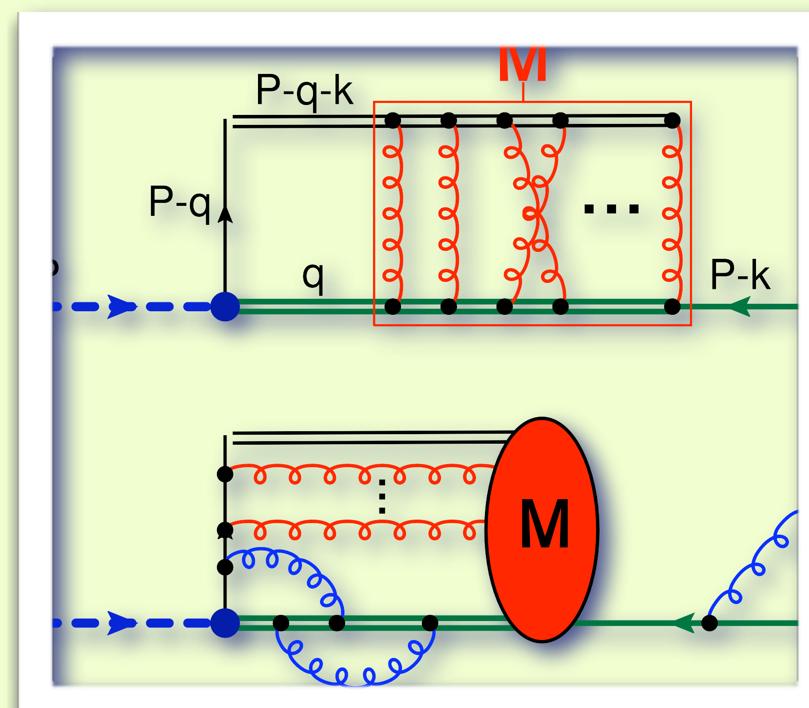
$$e\ p^{\uparrow} \rightarrow e\pi X$$



- *Collins PLB 2002- Gauge link Sivers function doesn't vanish*
- Ji, Yuan PLB: 2002 -Sivers fnct. FSI emerge from Color Gauge-links
- **LG, Goldstein, Oganessyan 2002, 2003** PRD Boer-Mulders Fnct, and Sivers -spectator model



- **LG, M. Schlegel, PLB 2010** Boer-Mulders Fnct, and Sivers beyond summing the FSIs through the gauge link



TMD Factorization .....

Factorization Sensitivity to $P_T \sim k_\perp \longrightarrow$ TMDs

John Collins   Nuclear Physics B396 (1993) 161–182

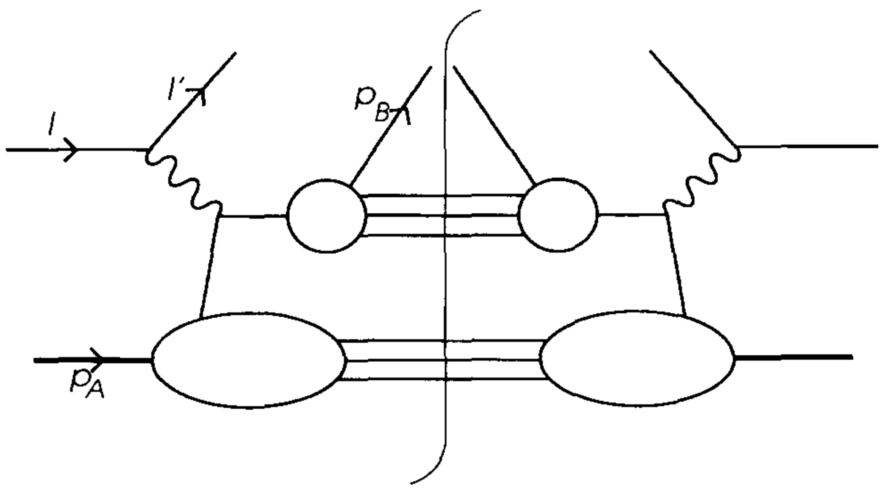


Fig. 2. Parton model for semi-inclusive deeply inelastic scattering.

3.4. FACTORIZATION WITH INTRINSIC TRANSVERSE MOMENTUM AND POLARIZATION

We now explain factorization for the semi-inclusive deep inelastic cross section when the incoming hadron A is transversely polarized but the lepton remains unpolarized. (It is left as an exercise to treat the most general case.) The factorization theorems, eq. (12) and eq. (14), continue to apply when we include polarization for the incoming hadron, but with the insertion of helicity density matrices for in and out quarks; this is a simple generalization of the results in refs. [10,23].

....

Ralston Soper NPB 1979, Collins NPB 1993

$$E'E_B \frac{\mathrm{d}\sigma}{\mathrm{d}^3 l' \, \mathrm{d}^3 p_B} = \sum_a \int \mathrm{d}\xi \int \frac{\mathrm{d}\zeta}{\zeta} \int \mathrm{d}^2 k_{a\perp} \int \mathrm{d}^2 k_{b\perp} \, \hat{f}_{a/A}(\xi, k_{a\perp})$$

$$\times E'E_{k_b} \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^3 l' \, \mathrm{d}^3 k_b} \hat{D}_{B/a}(\zeta, k_{b\perp}) + Y(x_{\mathrm{Bj}}, Q, z, q_\perp/Q)$$

Collins Soper NPB 1981, & Sterman NPB 1985

The function $\hat{f}_{a/A}$ defined earlier gives the intrinsic transverse-momentum dependence of partons in the initial-state hadron. Similarly, $\hat{D}_{B/a}$ gives the distribution of hadrons in a parton, with $k_{b\perp}$ being the transverse momentum of the parton relative to the hadron.

Factorization parton model, P_T of the hadron is small!

$$W^{\mu\nu}(q,P,S,P_h) \approx \sum_a e^2 \int \frac{d^2\mathbf{p}_T dp^- dp^+}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T dk^- dk^+}{(2\pi)^4} \delta(p^+ - x_B P^+)\delta(k^- - \frac{P_h^-}{z})\delta^2(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T)$$

integrate out small momenta components

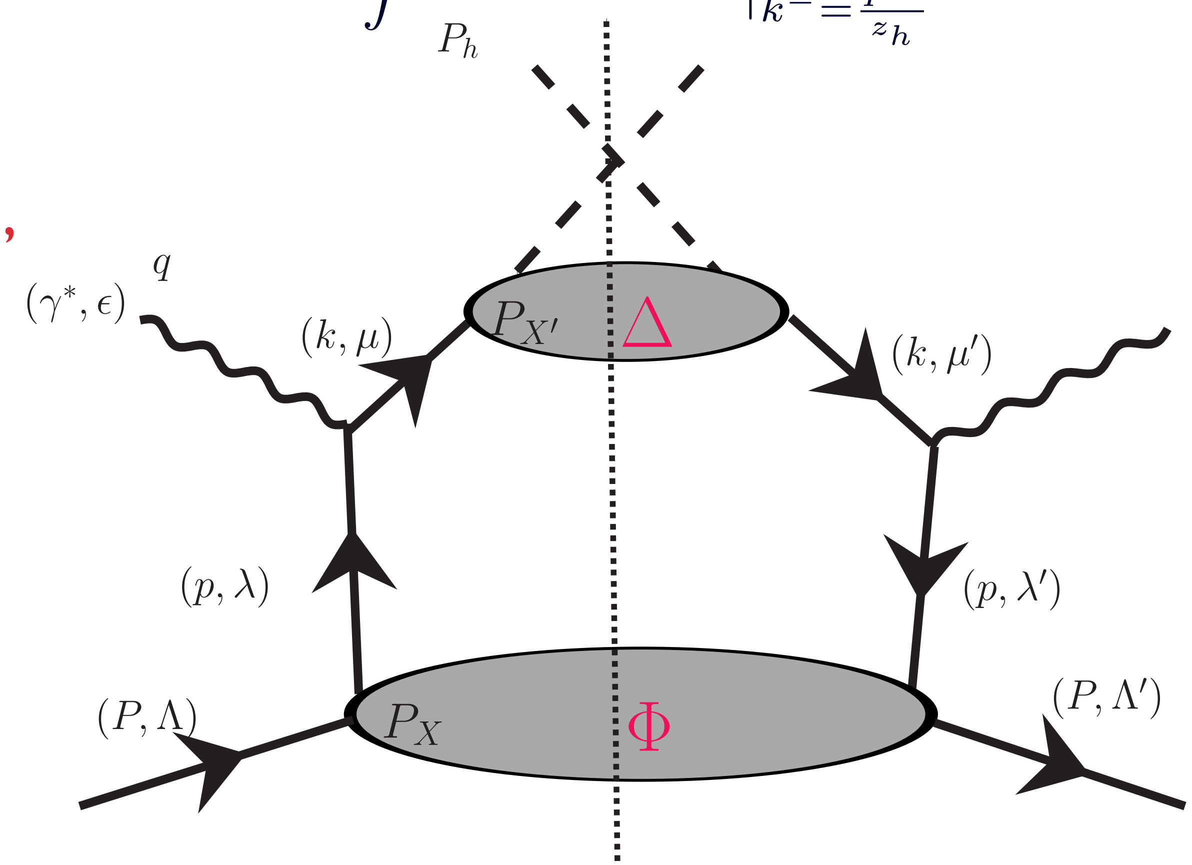
$$\times \mathrm{Tr}\left[\Phi(p,P,S)\gamma^\mu \Delta(k,P_h)\gamma^\nu\right]$$

$$W^{\mu\nu}(q,P,S,P_h) = \int \frac{d^2\mathbf{p}_T}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T}{(2\pi)^4} \delta^2(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z} - \mathbf{k}_T) \mathrm{Tr}\left[\left(\int dp^-\Phi\right)\gamma^\mu \left(\int dk^+\Delta\right)\gamma^\nu\right]$$

Small transverse momentum

$$\Phi(x,\mathbf{p}_T,S) \equiv \int dp^- \Phi(p,P,S)\Big|_{p^+=x_B P^+}, \qquad \Delta(z,\mathbf{k}_T) \equiv \int dk^+ \Delta(k,P_h)\Big|_{k^-=\frac{P^-}{z_h}}$$

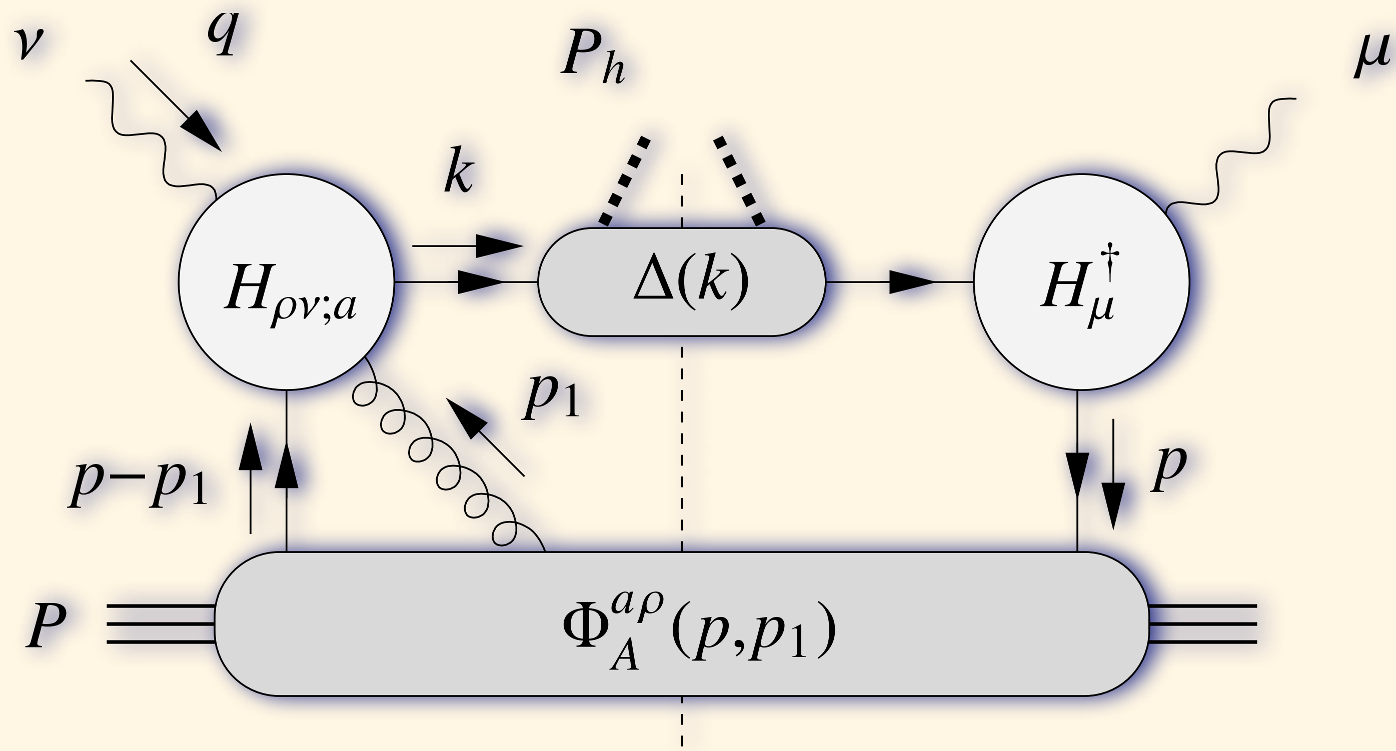
Integration support for integrals is where transverse momentum is small-"cov parton model"
e.g. Landshoff Polkinghorne NPB28, 1971



Extend Parton Model result-Gauge Links

- What are the "leading order" gluons that implement color gauge invariance?
- How is the correlator modified?

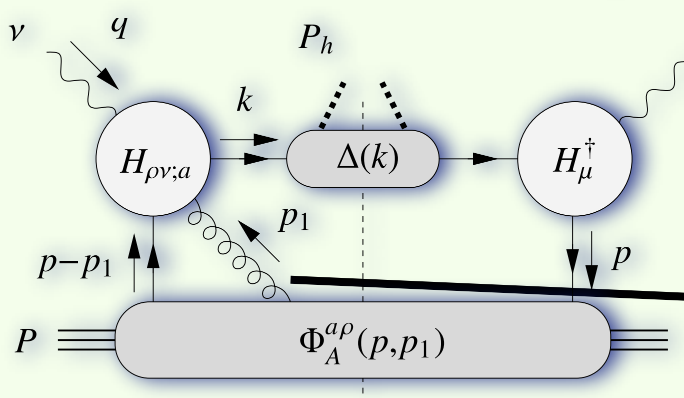
$$H_{\rho,\nu} = \gamma^{\nu}$$

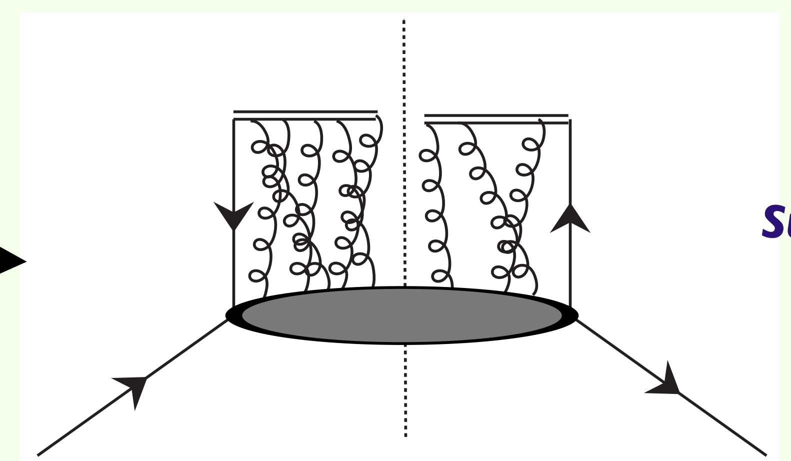


“T-Odd” Effects From Color Gauge Inv.Via Gauge links

Gauge link determined re-summing gluon interactions btwn soft and hard
Efremov,Radyushkin *Theor. Math. Phys. 1981*
Belitsky, Ji, Yuan NPB 2003,
Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD

$$\Phi^{[\mathcal{U}[\mathcal{C}]]}(x,p_T)=\int\frac{d\xi^- d^2\xi_T}{2(2\pi)^3}e^{ip\cdot\xi}\langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-,\xi_T)|P\rangle|_{\xi^+=0}$$

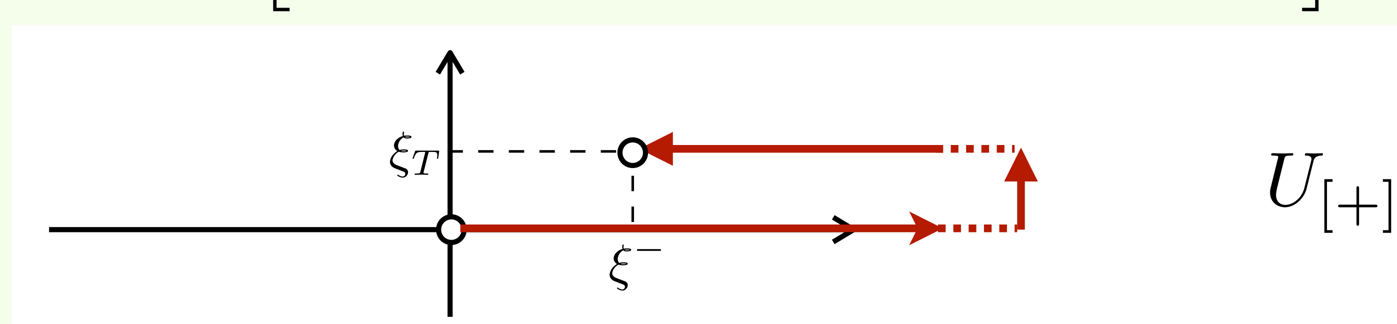




Summing gauge link with color
LG, M. Schlegel PLB 2010

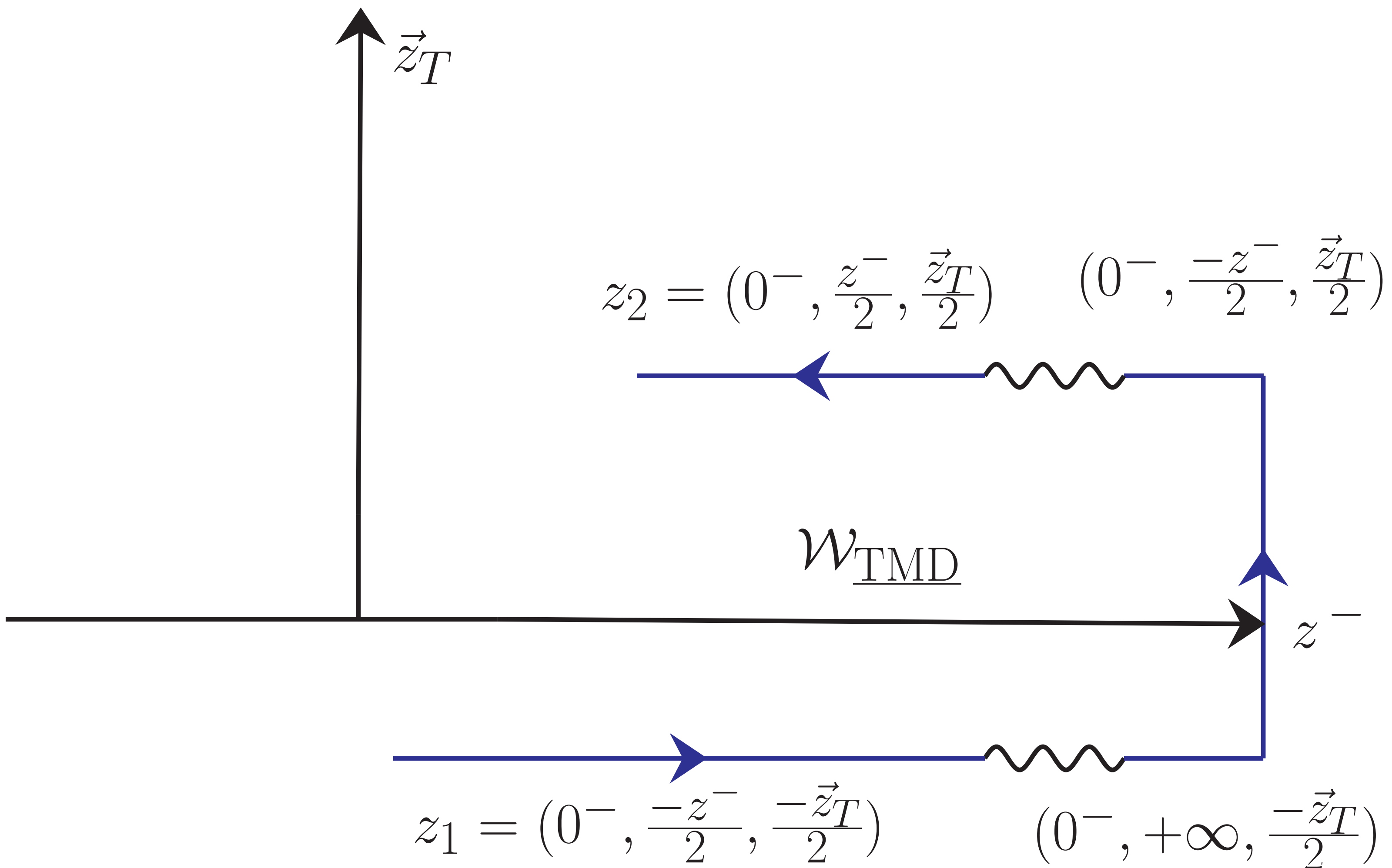
- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

$$\int d^4p\,d^4k\,\delta^4(p+q-k)\mathrm{Tr}\left[\Phi^{[U^{C}_{[\infty;\xi]}]}(p)H^\dagger_\mu(p,k)\Delta(k)H_\nu(p,k)\right]$$

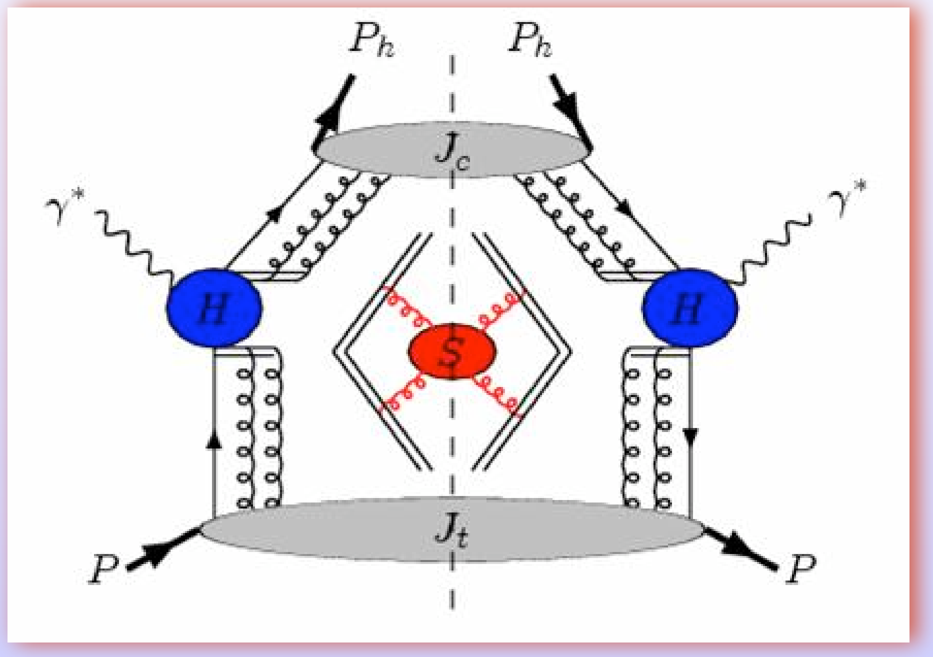


Wilson Line = Gauge links

$$U^s_{[z_1,z_2]} = \mathcal{W}[z_1;z_2] = [z_1;z_2] = \mathcal{P}\mathrm{e}^{-ig\int_{z_1}^{z_2} ds\cdot A(s)}$$



Ji, Ma, Yuan: PLB, PRD 2004, 2005 Extend factorization of CS-NPB: 81



$$F_{UU,T}(x,z,P_{h\perp}^2,Q^2) = \mathcal{C}\big[f_1 D_1\big]$$
$$= \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T - \boldsymbol{P}_{h\perp}/z\big)$$
$$x \sum_a e_a^2\, f_1^a(x,p_T^2,\mu^2)\, D_1^a(z,k_T^2,\mu^2)\, U(l_T^2,\mu^2) H(Q^2,\mu^2)$$

TMD PDF TMD FF Soft factor Hard part

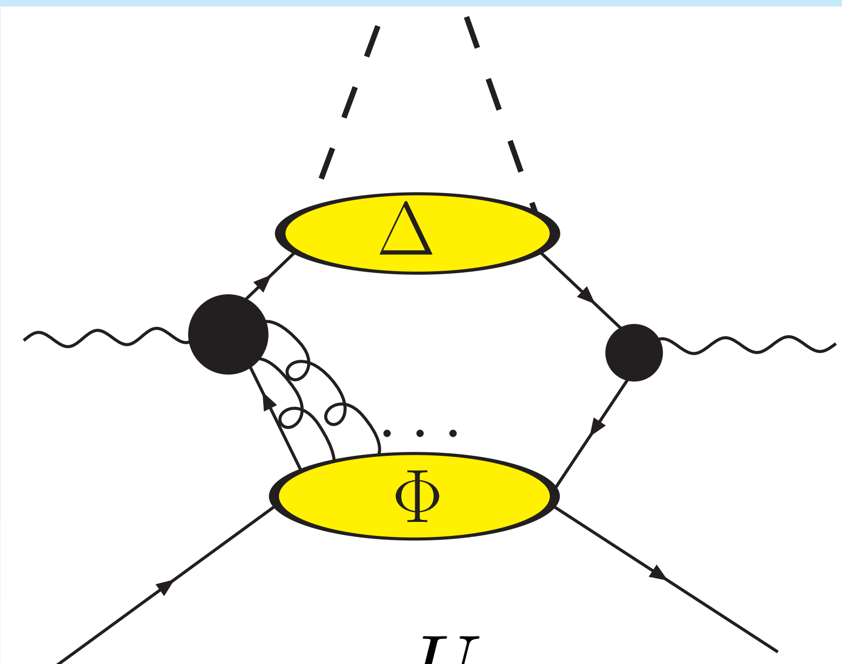
Collins, Soper, NPB 193 (81)
Ji, Ma, Yuan, PRD 71 (05)

“Generalized Universality” Fund. Prediction of QCD Factorization

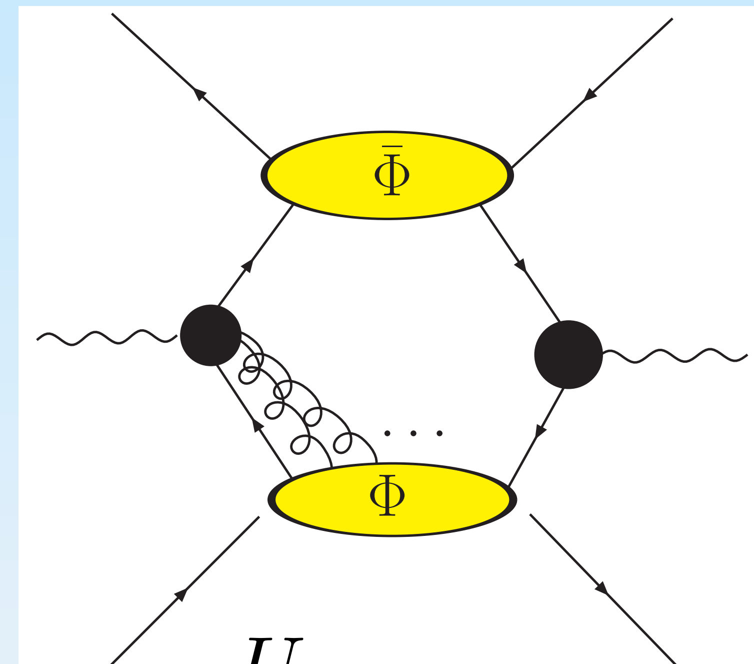
$$f^{\perp}_{1T\,sidis}(x, k_T) = -f^{\perp}_{1T\,DY}(x, k_T) \qquad p_T \sim \mathbf{k}_T << \sqrt{Q^2}$$

EIC conjunction with DY exp. E906-Fermi, RHIC II, Compass, JPARC

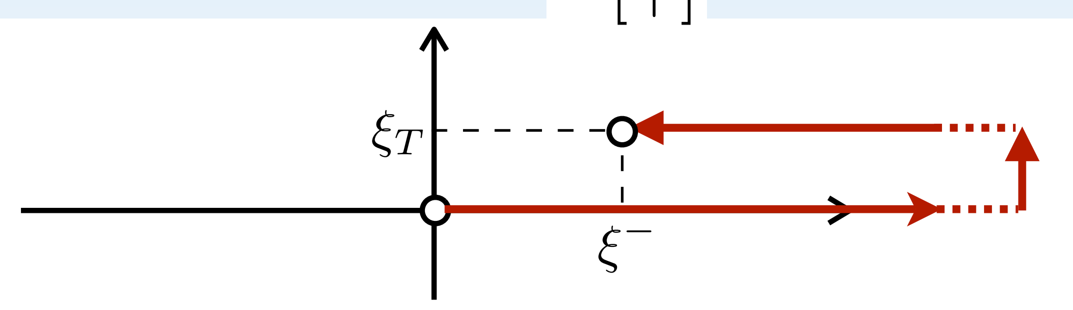
Process Dependence, Collins plb 02, Brodsky et al. NPB 02, Boer Mulders Pijlman Bomhoff 03, 04 ...



$$d\sigma = L_{\mu\nu} W^{\mu\nu} \Rightarrow$$

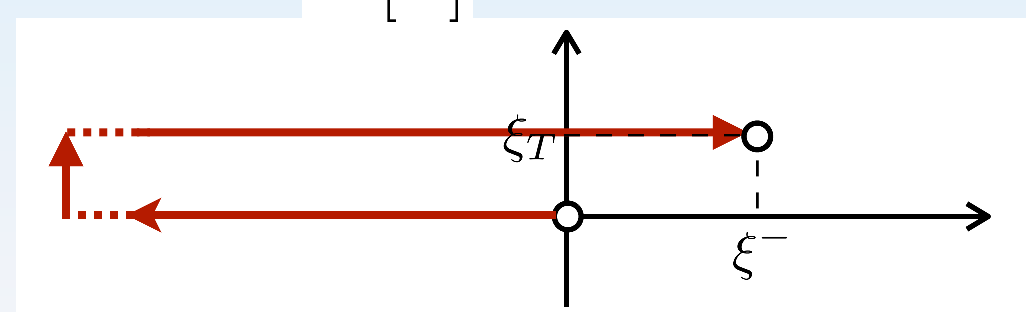


$U_{[+]}$



P&T

$U_{[-]}$



$$\Phi^{[+]*}(x, p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x, p_T) i\gamma^1\gamma^3$$

Correlator is Matrix in Dirac space

$$\Phi_{ji}(p;P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ip\cdot\xi}\langle PS|\bar{\psi}_i(0)\psi_j(\xi)|PS\rangle$$

$$\Phi_{ji}(x,\mathbf{p}_T) = \int \frac{dp^-}{2}\Phi_{ji}(p,P,S)|_{p^+=xP^+}$$

$$\Phi_{ji}(x,\mathbf{p}_T) = \int \frac{d\xi^- d^2\xi}{2(2\pi)^3} e^{ip\cdot\xi}\langle PS|\bar{\psi}_i(0)\psi_j(\xi)|PS\rangle|_{x^+=0}$$

Decompose into basis of Dirac matricies

$$\mathbf{1},\ \gamma_5,\ \gamma^\mu,\ \gamma^\mu\gamma_5,\ \mathrm{i}\sigma^{\mu\nu}\gamma_5$$

Hermiticity: $\Phi(p,P,S) = \gamma^0\,\Phi^\dagger(p,P,S)\,\gamma^0,$

parity: $\Phi(p,P,S) = \gamma^0\,\Phi(\tilde{p},\tilde{P},-\tilde{S})\,\gamma^0$

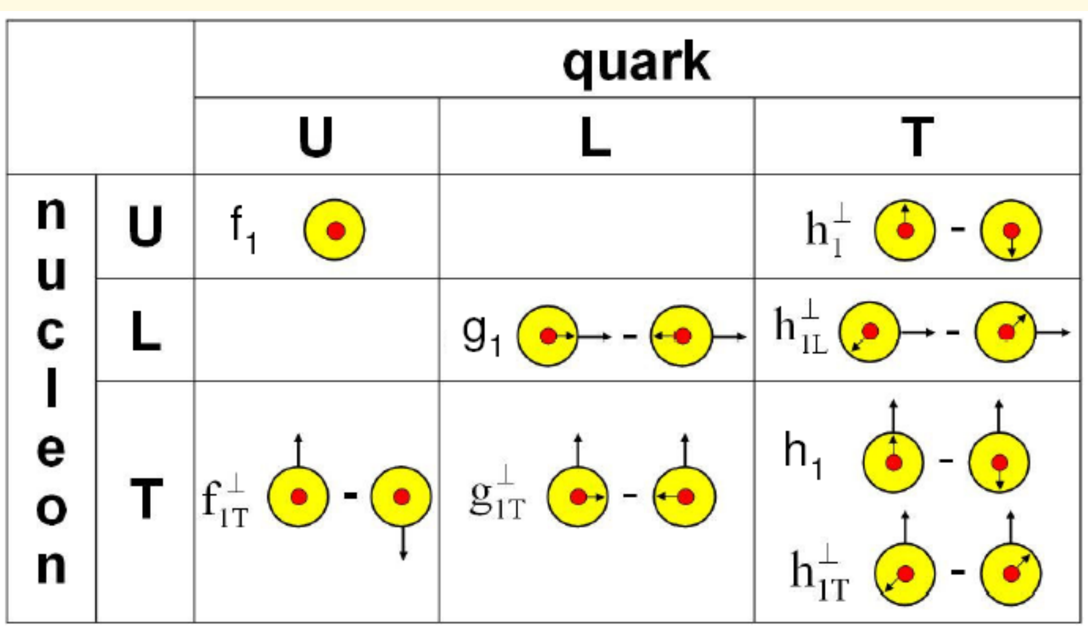
Leading Twist TMDs from Correlator

$$\Phi^{[\gamma^+]}(x,\boldsymbol{p}_T) \equiv f_1(x,\boldsymbol{p}_T^2) + \frac{\epsilon_T^{ij} p_{Ti} S_{Tj}}{M} f_{1T}^{\perp}(x,\boldsymbol{p}_T^2)$$

$$\Phi^{[\gamma^+\gamma_5]}(x,\boldsymbol{p}_T) \equiv \lambda\, g_{1L}(x,\boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M} g_{1T}(x,\boldsymbol{p}_T^2)$$

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x,\boldsymbol{p}_T) \equiv S_T^i\, h_{1T}(x,\boldsymbol{p}_T^2) + \frac{p_T^i}{M}\left(\lambda\, h_{1L}^{\perp}(x,\boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M} h_{1T}^{\perp}(x,\boldsymbol{p}_T^2)\right)$$

$$+ \frac{\epsilon_T^{ij} p_T^j}{M} h_1^{\perp}(x,\boldsymbol{p}_T^2)$$



"Avakian Mulders-tableau"

Integrated pdfs

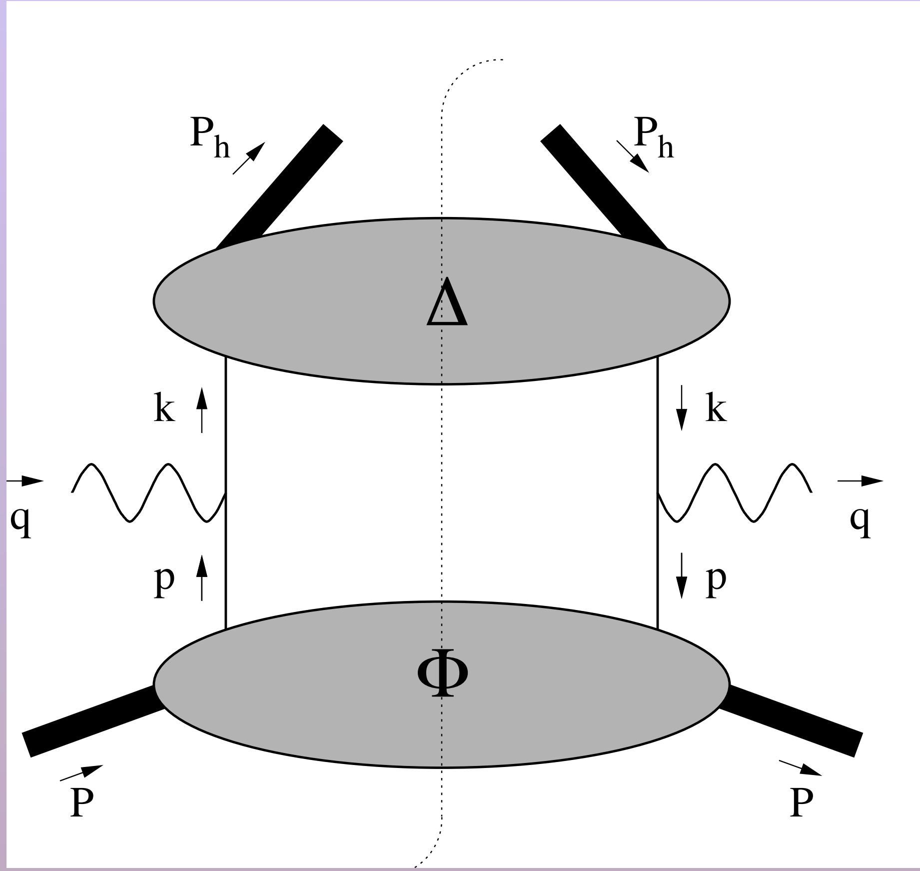
$$f(x) = \int \mathrm{d}^2\boldsymbol{p}_T \; f(x, \boldsymbol{p}_T^2)$$

Transversity

$$h_1(x) = \int \mathrm{d}^2\boldsymbol{p}_T \; \left(h_{1T}(x, \boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T^2}{2M^2} \, h_{1T}^{\perp}(x, \boldsymbol{p}_T^2) \right)$$

TSSAs in SIDIS

$$d^6\sigma = \hat{\sigma}_{\rm hard}\,\mathcal{C}[w\,f\,D]$$

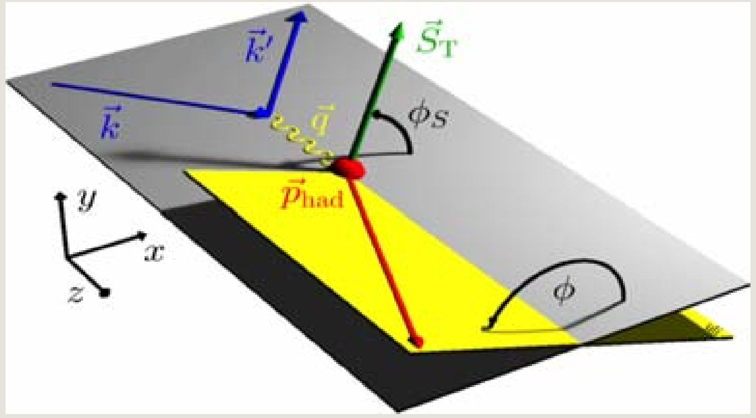


Structure functions that are extracted

$$\mathcal{F}_{AB} = \mathcal{C}[w\,f\,D]$$

$$\mathcal{C}\big[wfD\big] = \sum_a x\,e_a^2 \int d^2\boldsymbol{p}_T\,d^2\boldsymbol{k}_T\,\delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\,w(\boldsymbol{p}_T,\boldsymbol{k}_T)\,f^a(x,p_T^2)\,D^a(z,k_T^2)$$

Transverse Spin Observables and TMD Correlators in SIDIS



$$\Phi(x,\boldsymbol{p}_T)=\frac{1}{2}\Big\{f_1(x,\boldsymbol{p}_T)\not{P}+ih_1^\perp(x,\boldsymbol{p}_T)\frac{[\not{p}_T,\not{P}]}{2M}-f_{1T}^\perp(x,\boldsymbol{p}_T)\frac{\epsilon_T^{ij}p_{Ti}S_{Tj}}{M}\not{P}\cdots\Big\}$$

$$\Delta(z,\boldsymbol{k}_T)=\frac{1}{4}\Big\{zD_1(z,\boldsymbol{k}_T)\not{P}_h+izH_1^\perp(z,\boldsymbol{k}_T)\frac{[\not{k}_T,\not{P}_h]}{2M_h}-zD_{1T}^\perp(z,\boldsymbol{k}_T)\frac{\epsilon_T^{ij}k_{Ti}S_{Tj}}{M_h}\not{P}_h+\cdots\Big\}$$

SIDIS cross section

$$d\sigma^{\ell N\to\ell\pi X}_{\{\lambda,\Lambda\}} \propto f_1\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1+\frac{k_\perp}{Q}f_1\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1\cdot\cos\phi$$

$$+\left[\frac{k_\perp^2}{Q^2}f_1\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1+h_1^\perp\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes H_1^\perp\right]\cdot\cos 2\phi$$

$$+|S_T|\cdot h_1\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes H_1^\perp\cdot\sin(\phi+\phi_S)\quad \text{Collins}$$

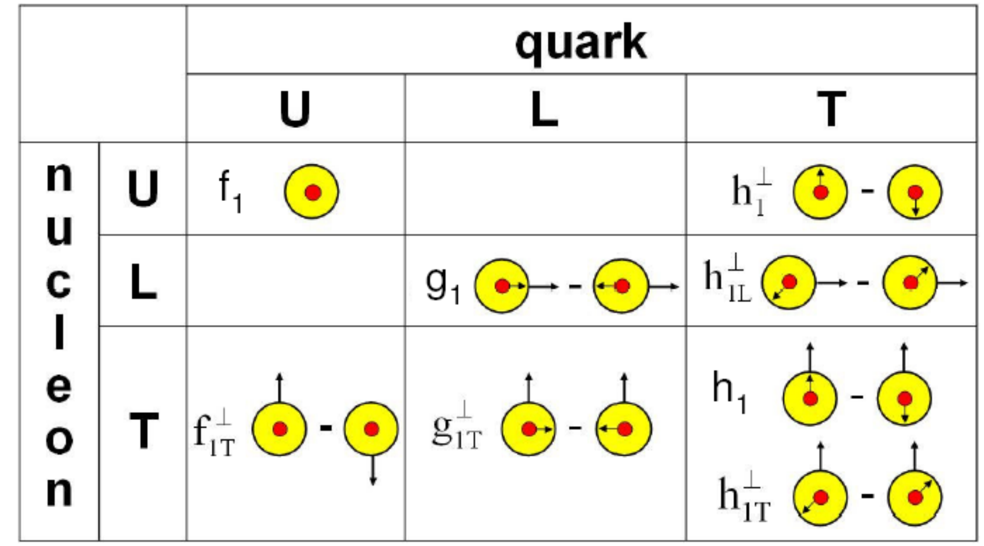
$$+|S_T|\cdot f_{1T}^\perp\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1\cdot\sin(\phi-\phi_S)\quad \text{Sivers}$$

$$+|S_L|\cdot h_{1L}^\perp\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes H_1^\perp\cdot\sin(2\phi)\quad \text{Kotzinian-MuldersPLB}$$

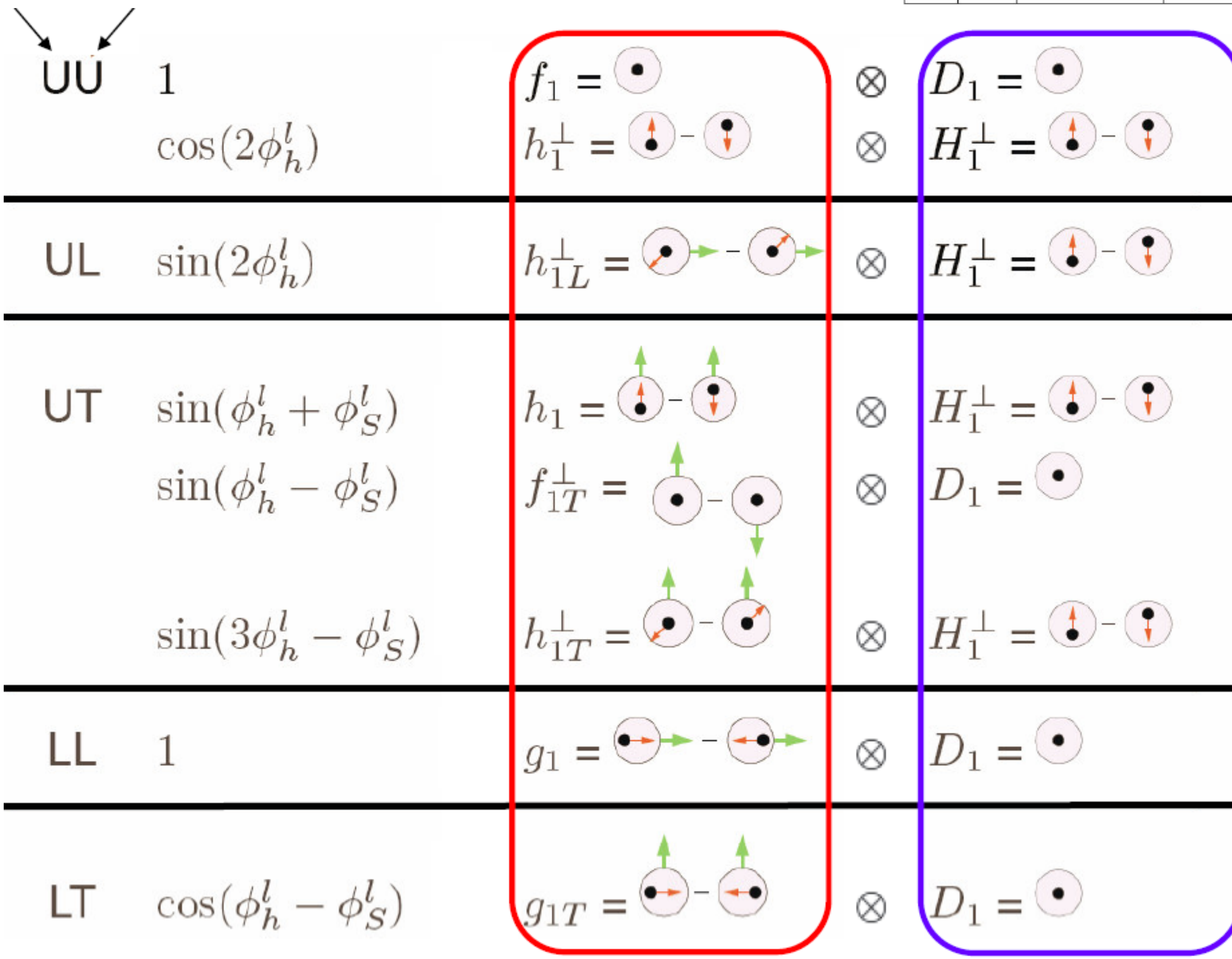
Boer-Mulders

transversity

Leading Twist Contributions



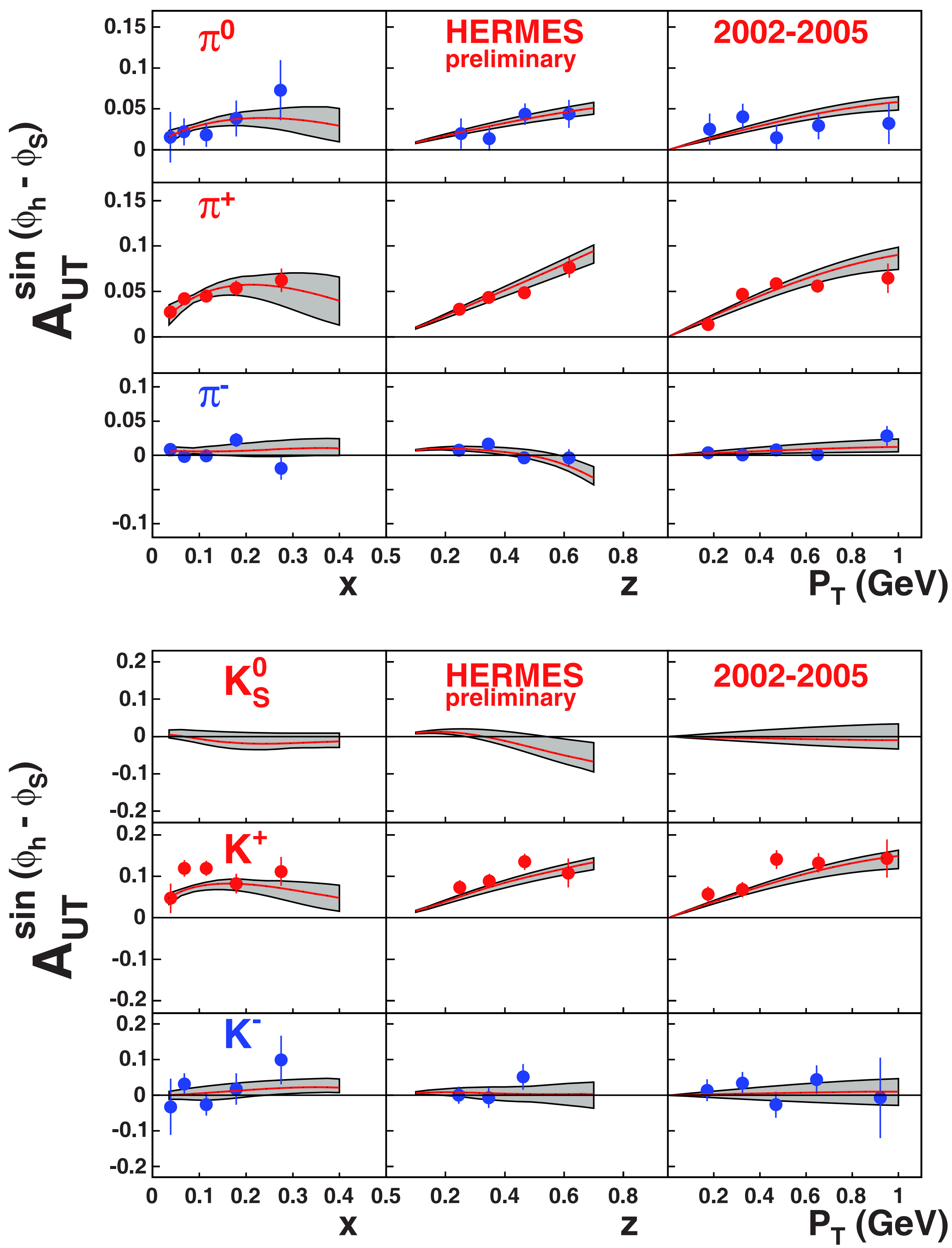
$$\mathcal{F}_{AB} = \mathcal{C}[w \otimes f \otimes D]$$



Simultaneous fit of pion and kaon data from HERMES and COMPASS

See Talk of Alexei Prokudin

Anselmino et al. PRD 05, EPJA 08



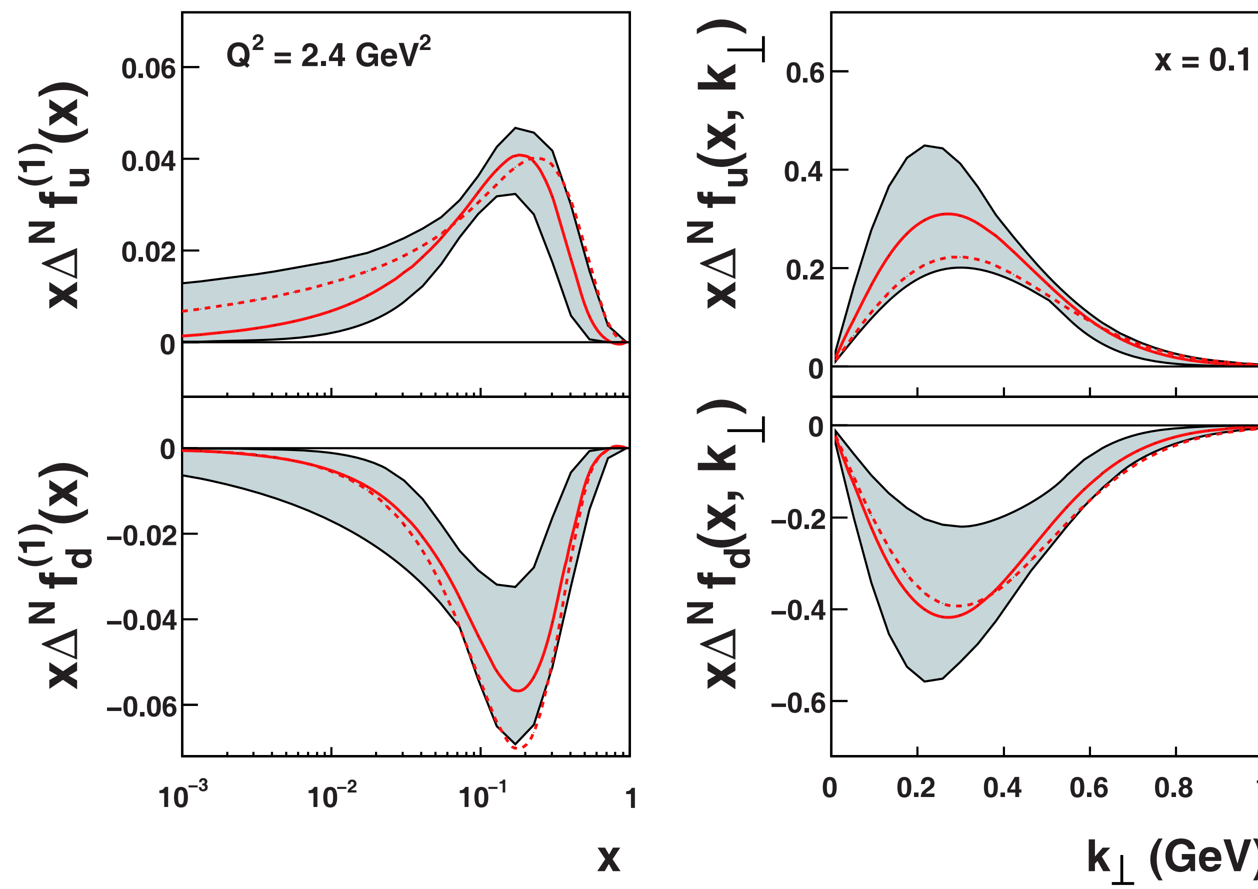
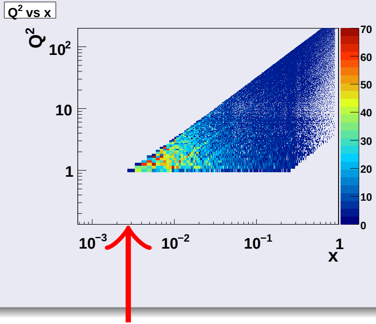


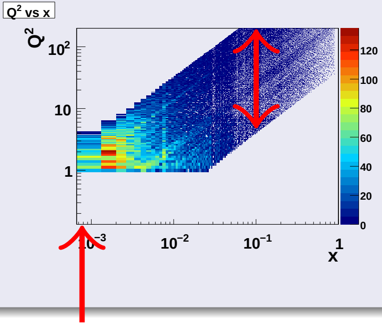
Fig. 7. The Sivers distribution functions for u and d flavours, at the scale $Q^2 = 2.4\,(\mathrm{GeV}/c)^2$, as determined by our present fit (solid lines), are compared with those of our previous fit [2] of SIDIS data (dashed lines), where π^0 and kaon productions were not considered and only valence quark contributions were taken into account. This plot clearly shows that the Sivers functions previously found are consistent, within the statistical uncertainty bands, with the Sivers functions presently obtained.

Some snapshots of EIC

EIC @ $\sqrt{s} = 20$ GeV



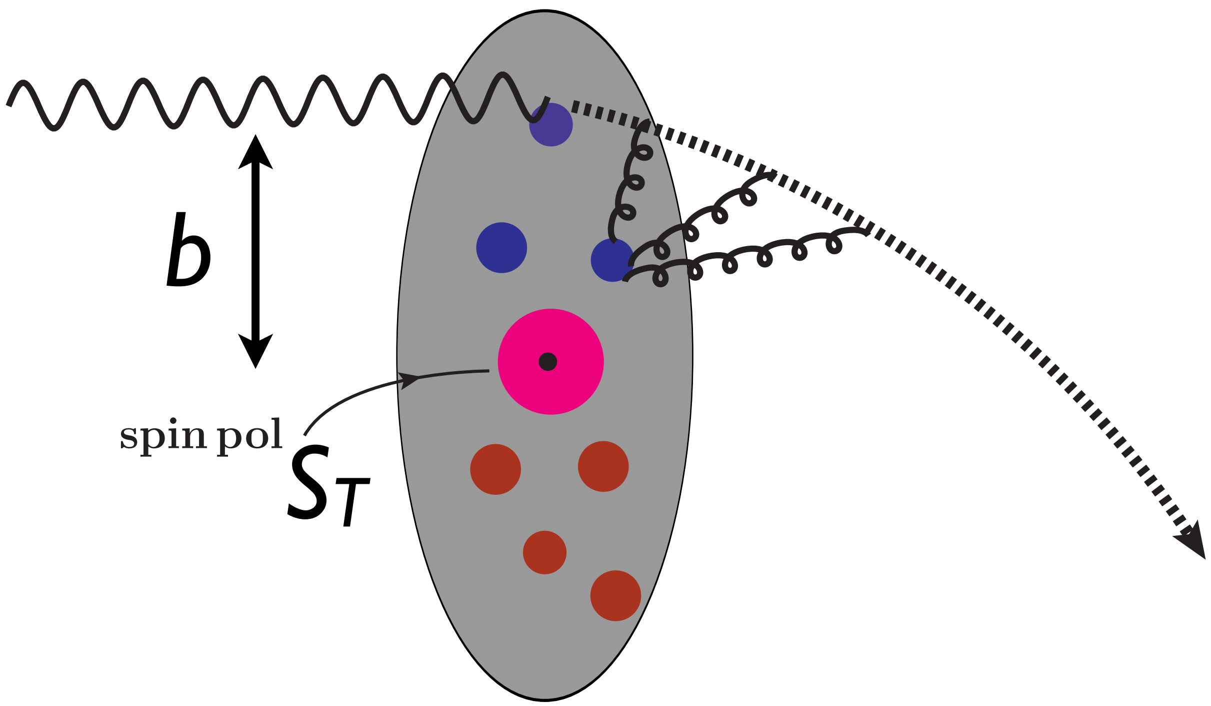
EIC @ $\sqrt{s} = 65$ GeV



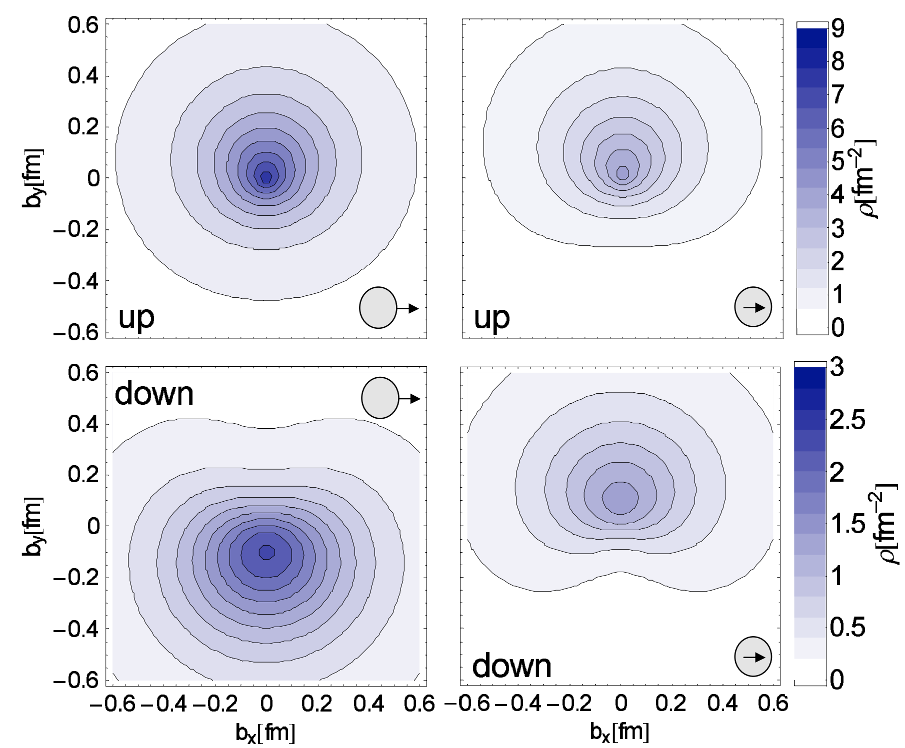
Biggest asymmetries are at $x \sim 0.2$.
Wide range of Q^2 at some fixed x is plausible.
Increasing the energy we go to the low-x region, but loose Q^2 range at moderate x. One of the advantages of EIC is a possibility to vary the energy and to accommodate appropriate $x - Q^2$ range.

Intuitive picture of Sivers asymmetry:
Spatial distortion in transverse plane due to polarization
+ FSI leads to observable effect
non-zero Left Right (Sivers) momentum asymmetry

M. Burkardt [Nucl.Phys. A735, 185], [PRD66, 114005]



Gockler et al. PRL07 x-moments of IP-GPDs



$$\vec{S} \cdot (\hat{P} \times \vec{k}_\perp)\, f_{1T}^{\perp}(x, \vec{k}_\perp^2)$$

$$\vec{S} \cdot (\hat{P} \times \vec{b}) \left(\mathcal{E}(x, \vec{b}^2)\right)'$$

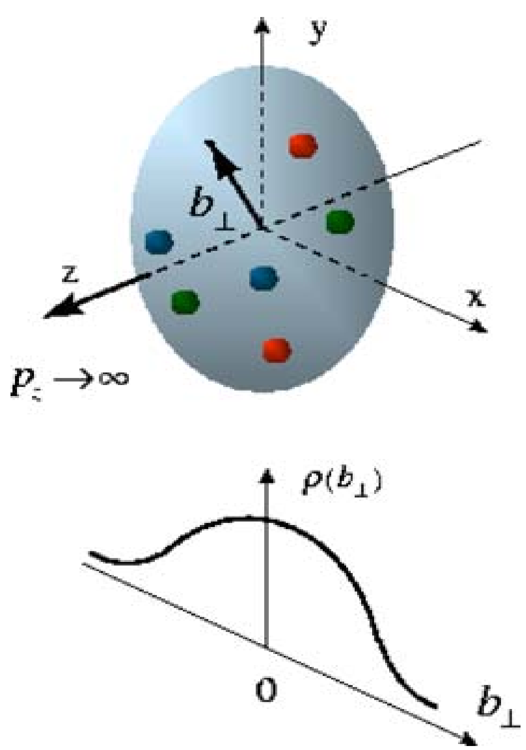
Transverse Structure-Consider "3-D" Parton Structure

Uncertainty Princ. doesn't forbid simultaneous info
longitudinal momentum and transverse position of partons
"Impact Parameter PDFs"

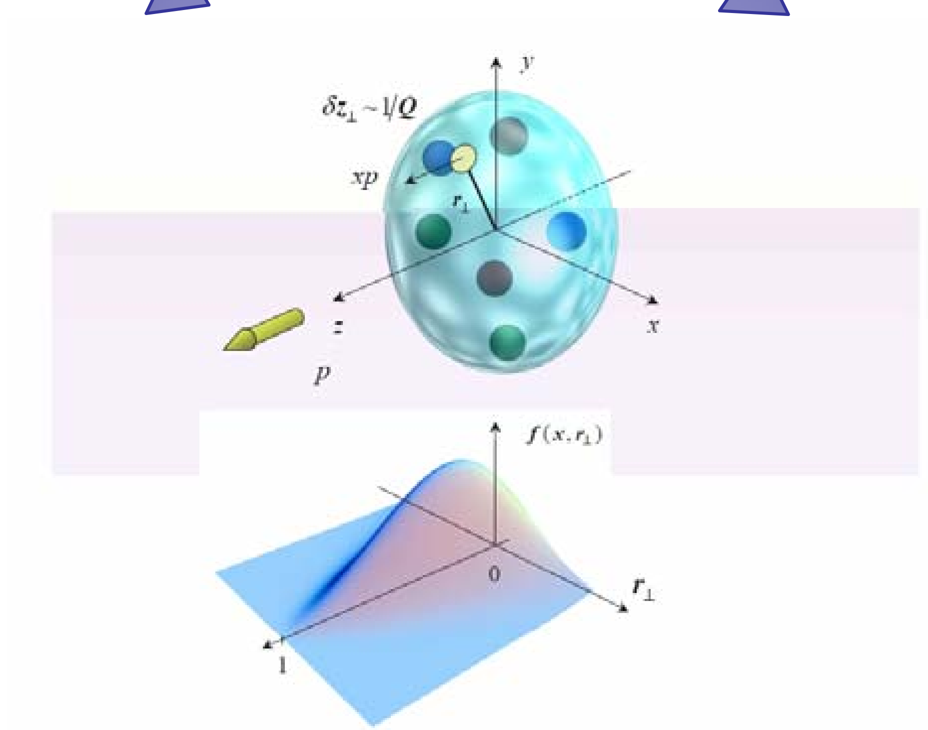
$$(\vec{\mathbf{b}}_\perp \ \& \ x) \longleftrightarrow f(x, \mathbf{b}_\perp)$$

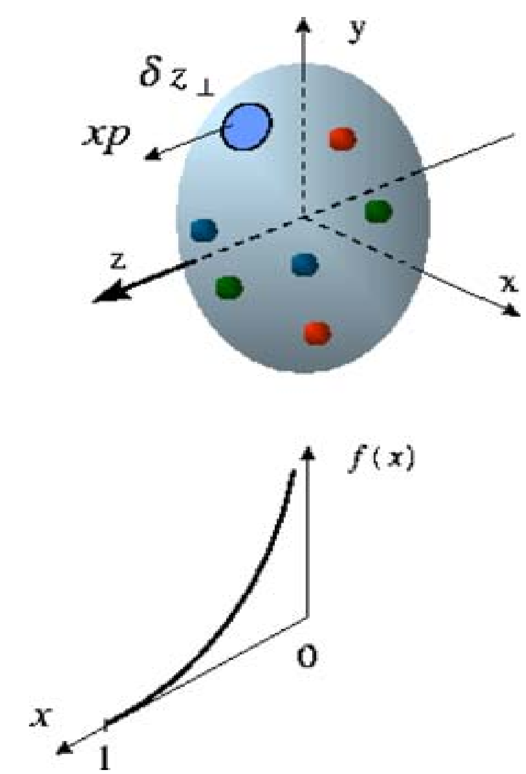
form factors
location of partons in nucleon

parton distributions
longitudinal momentum fraction x



D. Hasch's talk
M. Vanderhaeghen
Pictures of Andre Belitsky





Remind ourselves of Some simple relations for FFs and forward PDFs

$$\int_{-1}^{1} dx\, H^q(x,\xi,t) = F_1^q(t)$$

$$\int_{-1}^{1} dx\, E^q(x,\xi,t) = F_2^q(t)$$

$$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) = G_A^q(t)$$

$$\int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) = G_P^q(t)$$

Trivial Relations are well-known:

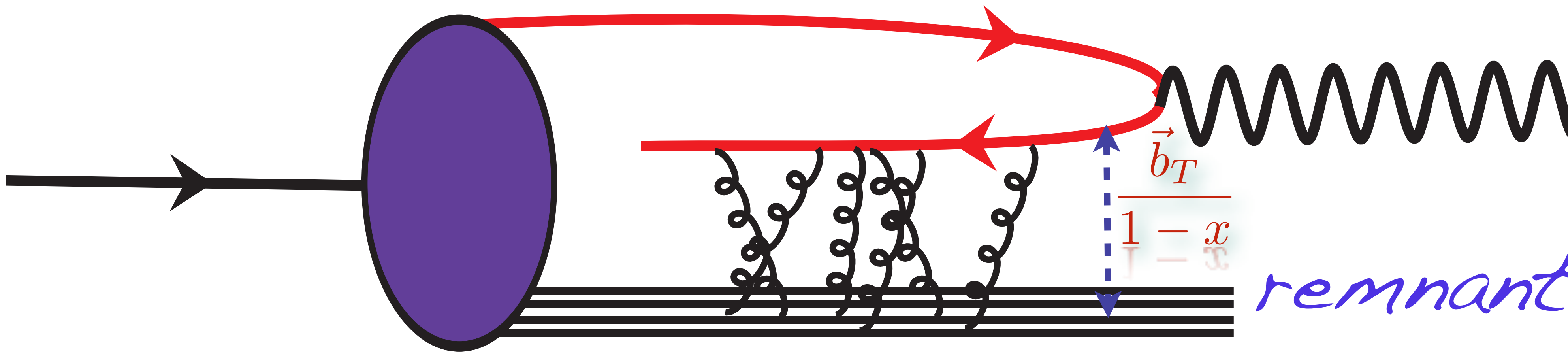
$$f_1(x) = H(x,0,0) = \int d^2k_T\, f_1(x,\vec{k}_T^2) = \int d^2b_T \mathcal{H}(x,\vec{b}_T^2)$$

$$g_1(x) = \tilde{H}(x,0,0) = \int d^2k_T\, g_{1L}(x,\vec{k}_T^2)$$

$$h_1(x) = H_T(x,0,0) = \int d^2k_T\, h_1(x,\vec{k}_T^2)$$

model-independent, integrated relations

Explore connection FSIs-Links & Transv. Distortion



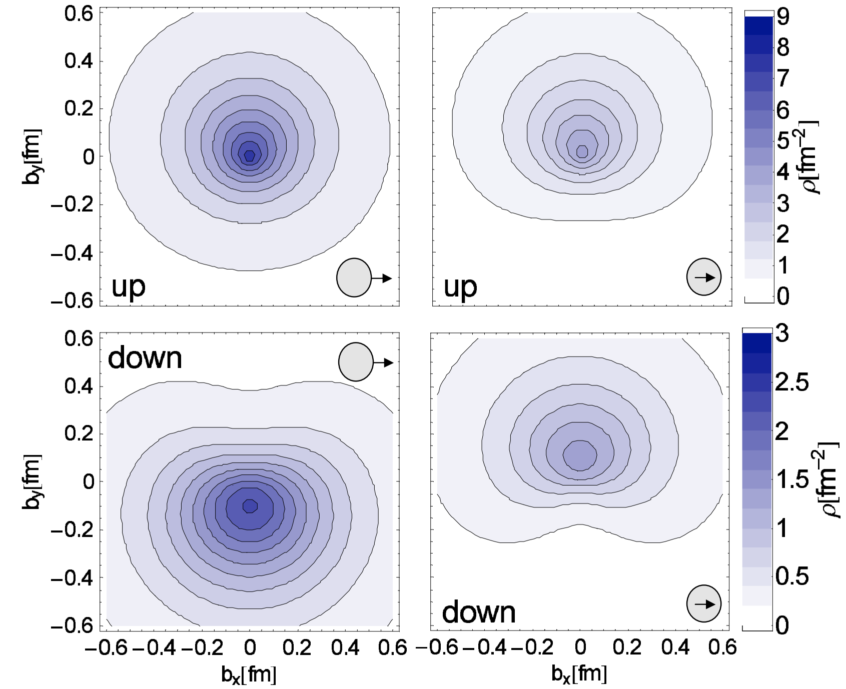
Non-perturbative calculation of FSIs

L.G. & Marc Schlegel
Phys.Lett.B685:95-103, 2010 &
Mod.Phys.Lett.A24:2960-2972,2009

W~

Used to predicting sign of TSSA-Sivers

Burkardt 02,04 NPA PRD



$$d_q^y = \frac{1}{2M}\int dx \int d^2\mathbf{b}_\perp \mathcal{E}_q(x,\mathbf{b}_\perp)$$

$$= \frac{1}{2M}\int dx E_q(x,0,0) = \frac{F_{2,q}(0)}{2M} = \frac{\kappa}{2M}$$

$$\kappa^p = 1.79\,, \quad \kappa^n = -1.91$$

$$\longrightarrow \kappa^{u/p} = 1.67\,, \quad \kappa^{d/p} = -2.03$$ w/ attractive interactions

$$f_{1T}^{\perp(u)} = \text{neg} \quad \& \quad f_{1T}^{\perp(d)} = \text{pos}$$

Anselmino et al. PRD 05, EPJA 08

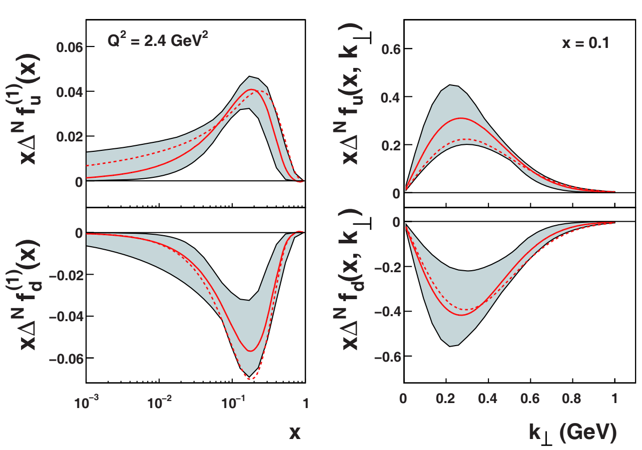


Fig. 7. The Sivers distribution functions for u and d flavours, at the scale $Q^2 = 2.4\,(\text{GeV}/c)^2$, as determined by our present fit (solid lines), are compared with those of our previous fit [2] of SIDIS data (dashed lines), where π^0 and kaon productions were not considered and only valence quark contributions were taken into account. This plot clearly shows that the Sivers functions previously found are consistent, within the statistical uncertainty bands, with the Sivers functions presently obtained.

Sivers

Gamberg, Goldstein, Schlegel PRD 77, 2008

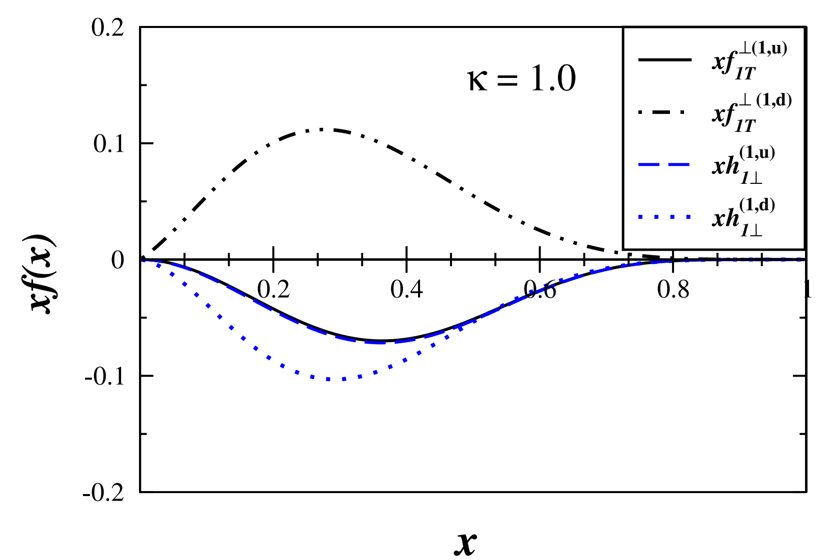


FIG. 5 (color online). The first moment of the Boer-Mulders and Sivers functions versus x for $\kappa = 1.0$.

"Spin-Orbit kinematics"

Analysis of correlators for TMDs and IP-GPDs similar forms

Burkhardt-02 PRD & ...
Diehl Hagler-05 EPJC,
Meissner, Metz, Goeke 07 PRD

$$\Phi^q(x, \vec{k}_T; S) = f_1^q(x, \vec{k}_T^2) - \frac{\epsilon_T^{ij} k_T^i S_T^j}{M} f_{1T}^{\perp q}(x, \vec{k}_T^2) ;$$

$$\mathcal{F}^q(x, \vec{b}_T; S) = \mathcal{H}^q(x, \vec{b}_T^2) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M} \left(\mathcal{E}^q(x, \vec{b}_T^2)\right)' ;$$

$\mathbf{k}_T \leftrightarrow \mathbf{b}_T$ Not conjugates (!) and ...

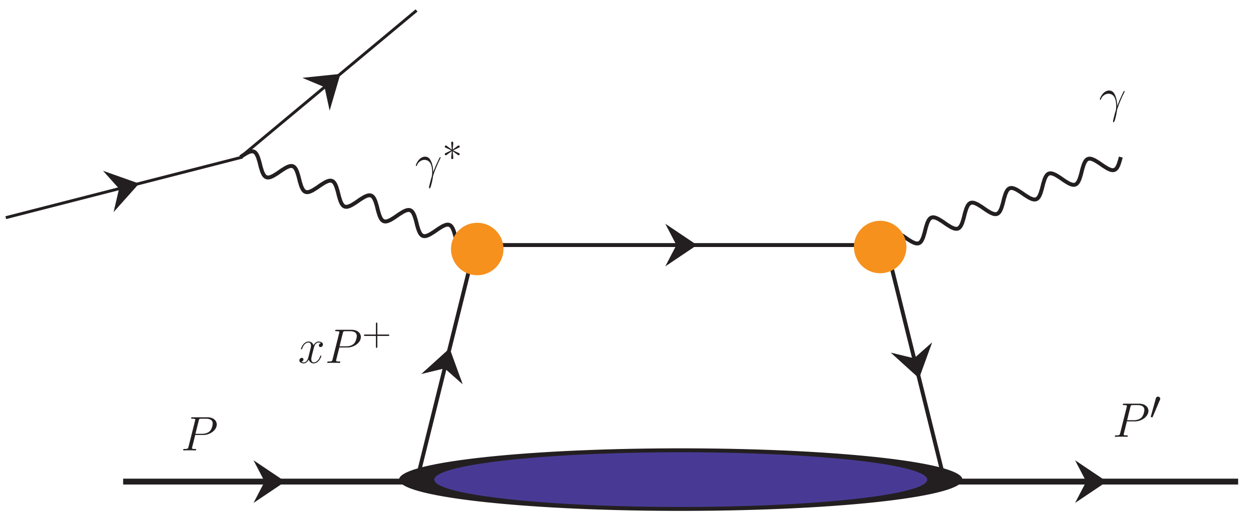
$f_{1T}^{\perp}(x, \vec{k}_T^2)$ "Naive T-odd"

$\left(\mathcal{E}(x, \vec{b}_T^2)\right)'$ "Naive T-even"

FSIs needed.... Burkardt PRD 02 & NPA 04

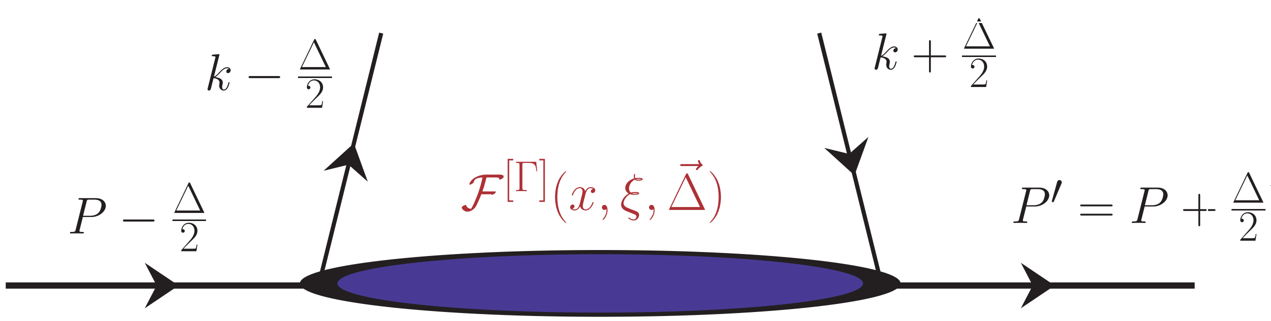
How do we test this further?

DVCS Factorizes into hard and soft ⟶ GPDs



Collins & Freund PRD (1999)
Collins Frankfurt Strikman (1997) DVMP
&
X. Ji, PRL (1997); PRD(1997)
A.V. Radyushkin, PLB (1996); PRD (1997)

$$F^{[\Gamma]}(x,\xi,t;\lambda,\lambda') = \int \frac{dz^-}{(4\pi)} e^{ixP^+z^-} \langle P';\lambda'|\bar{q}\left(\frac{-z}{2}\right)\Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2})q\left(\frac{z}{2}\right)|P;\lambda\rangle|_{z^+=\mathbf{z}_\perp=0}$$



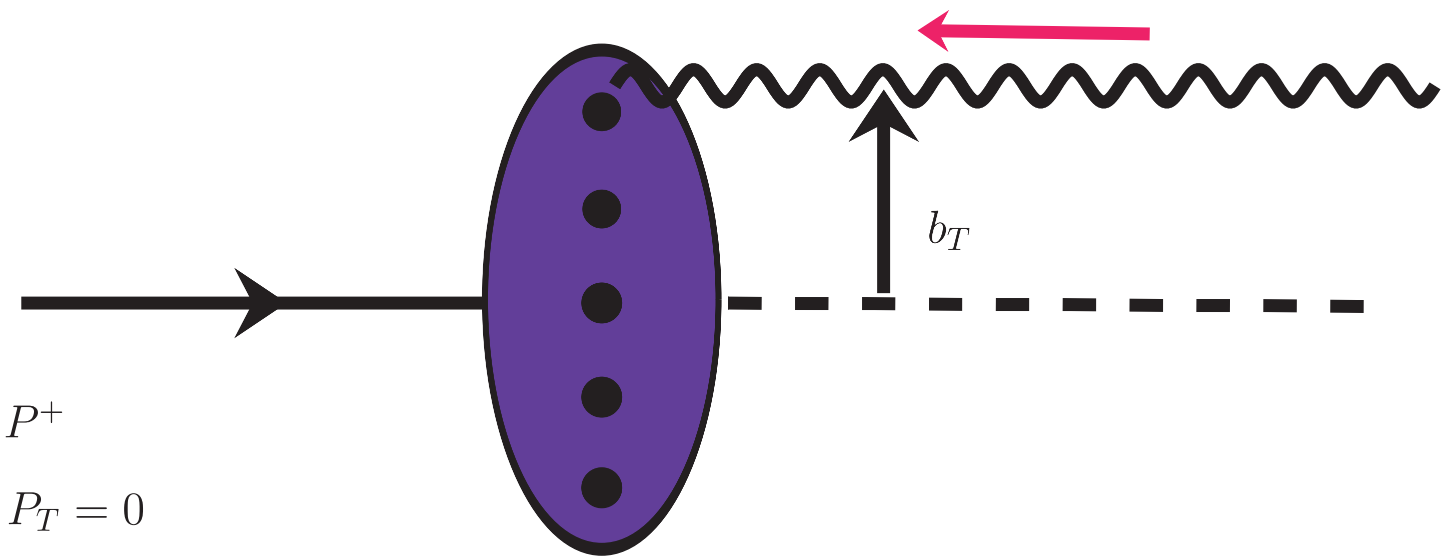
$$P=\frac{p+p'}{2} \quad \Delta = p'-p$$

$$k^+ = xP^+ \,, \Delta^+ = -2\xi P^+$$

Eight GPDs *H unpol* & *E*-helicity flip

$$F^{q[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\bar{u}(p',\lambda')\left(\gamma^+ H^q(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E^q(x,\xi,t)\right)u(p,\lambda),$$

Fourier transform of GPD $F(x,0,\vec{\Delta}_T)$ @ $\xi = 0$



Burkardt PRD 00, 02, 04...

Localizing partons: impact parameter

- states with definite light-cone momentum p^+ and transverse position (impact parameter):

Soper PRD1977 $|p^+, \boldsymbol{b}\rangle = \int d^2\boldsymbol{p}\, e^{-i\boldsymbol{b}\boldsymbol{p}}\, |p^+, \boldsymbol{p}\rangle$

$$\mathcal{F}(x,\vec{b}) = \int \frac{dz^-}{(2\pi)^2} e^{ixP^+z^-} \langle P^+; \vec{0}_T | \bar{q}(z_1)\, \mathcal{W}(z_1,z_2) q(z_2)\, | P^+; \vec{0}_T \rangle$$

$$z_{1/2} = \pm \frac{z^-}{2} n_- + \frac{\vec{b}_T}{2}$$

$$\begin{aligned}\mathcal{F}(x,\vec{b}) &= \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\vec{\Delta}_T\cdot\vec{b}}\, F(x,0,\vec{\Delta}_T) \\ &= \mathcal{H}(x,\vec{b}) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M} \left(\mathcal{E}(x,\vec{b})\right)'\end{aligned}$$

F.T.

$\vec{b} \leftrightarrow \vec{\Delta}_T$

Prob. of finding unpol. quark w/ long momentum x at position b_T in trans. polarized S_T nucleon: spin independent $\mathcal{H}$ and spin flip part $\mathcal{E}'$

What observable to test this possible connection btnw TMD and Impact par. picture?

Gluonic Pole ME

$$\langle k_T^i \rangle_T(x) = \int d^2k_T\, k_T^i\, \tfrac{1}{2}\Big[\mathrm{Tr}[\gamma^+\Phi(\vec{S}_T)] - \mathrm{Tr}[\gamma^+\Phi](-\vec{S}_T)\Big]$$

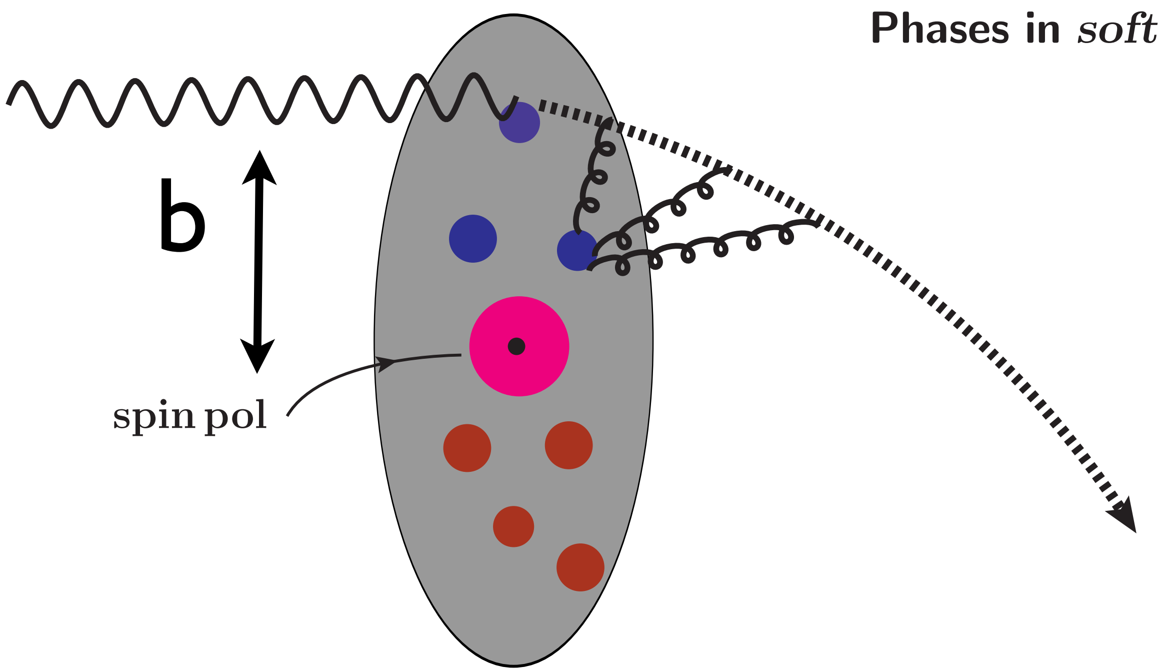
$$\langle k_T^i \rangle(x) = \int d^2b_T \int \frac{dz^-}{2(2\pi)} e^{ixP^+z^-} \langle P^+;\vec{0}_T;S_T|\bar{\psi}(z_1)\gamma^+[z_1;z_2] I^i(z_2) \psi(z_2)|P^+;\vec{0}_T;S_T\rangle$$

$$z_{1/2} = \mp\frac{z^-}{2}n_- + b_T$$

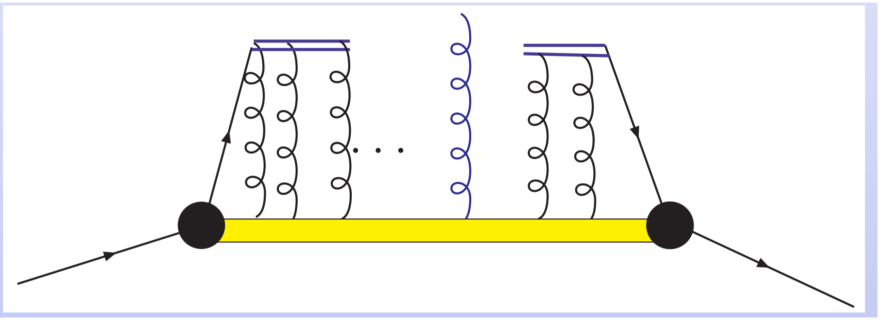
Impact parameter rep for GPD E

$$I^i(z^-) = \int dy^-[z^-;y^-]gF^{+i}(y^-)[y^-;z^-]$$

Soft gluonic pole op



Phases in *soft* poles of propagator in hard subprocess Efremov & Teryaev :PLB 1982



Conjecture: factorization of FSI and spatial distortion:

$$\langle k_T^i \rangle(x) = M \epsilon_T^{ij} S_T^i f_{1T}^{\perp(1)} \approx \int d^2 b_T \mathcal{I}^i(x, \vec{b}_T^2) \frac{\vec{b}_T \times \vec{S}_T}{M} \frac{\partial}{\partial b_T^2} \mathcal{E}(x, \vec{b}_T^2)$$

$\mathcal{I}^i(x, \vec{b}_T^2)$ Lensing Function

Boer Mulders as well ...

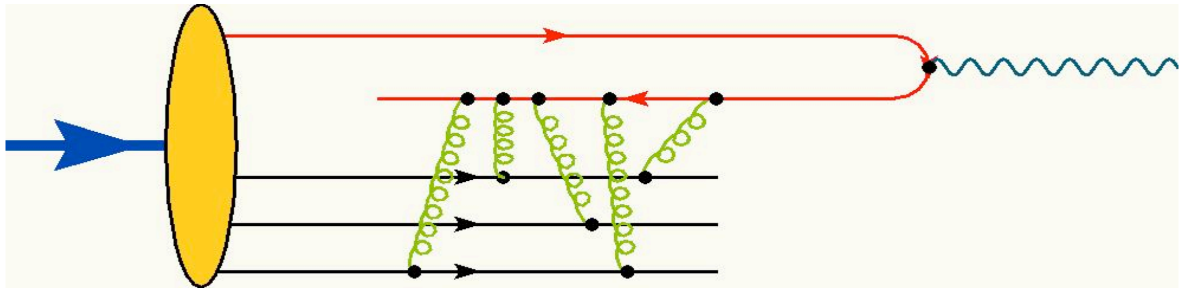
- Av. transv. momentum of transv. pol. partons in an unpol. hadron:

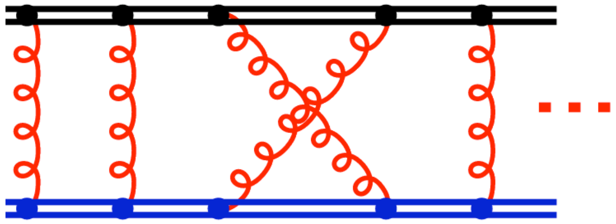
$$\langle k_T^i \rangle^j(x) = \int d^2 k_T \, k_T^i \, \frac{1}{2} \left(\Phi^{[i\sigma^{i+}\gamma^5]}(S) + \Phi^{[i\sigma^{i+}\gamma^5]}(-S) \right)$$

$$\Rightarrow \quad -2M^2 h_1^{\perp,(1)}(x) \simeq \int d^2 b_T \, \vec{b_T} \cdot \vec{\mathcal{I}}(x, \vec{b}_T) \frac{\partial}{\partial b_T^2} \left(\mathcal{E}_T + 2\tilde{\mathcal{H}}_T \right)(x, \vec{b}_T^2)$$

Diehl & Hagler EJPC (05), Burkardt PRD (04)

Sivers Function in this approach





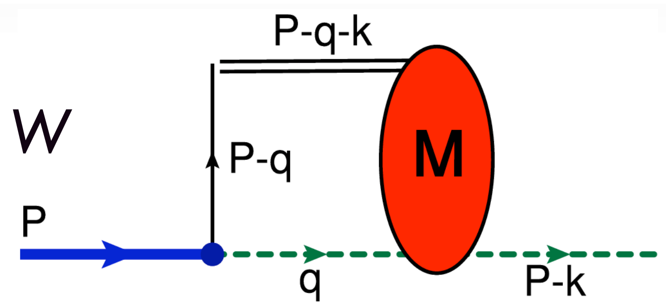
- **Relativistic Eikonal models: Treat FSI non-perturbatively.**

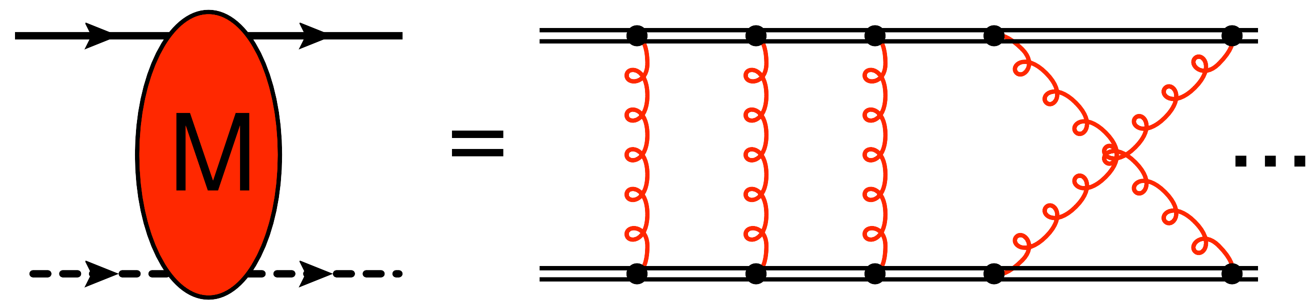
For Details see extra slides and

L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 *&* in prep for Sivers...

• **Relativistic Eikonal models: Treat FSI non-perturbatively.**





We calc "W" again....

$$\epsilon_T^{ij} k_T^i S_T^j f_{1T}^{\perp}(x, \vec{k}_T^2) = -\frac{M}{8(2\pi)^3(1-x)P^+}\left(\bar{W}\gamma^+ W\Big|_{S_T} - \bar{W}\gamma^+ W\Big|_{-S_T}\right)$$

$$\Delta W(P,k) = \int \frac{d^4q}{(2\pi)^4} g_N[(P-q)^2] \frac{[(\not{P} - \not{q} + m_q)u(P,S)]_i \mathcal{M}_{bc}^{ab}(q, P-k)}{[n\cdot(P-k-q)+i\varepsilon][(P-q)^2+m_q^2+i\varepsilon][q^2-m_s^2+i\varepsilon]}$$

• **Step 1: Integration over q^-:**

color indices suprsd.

Assume no q^- & q^+ poles in M.

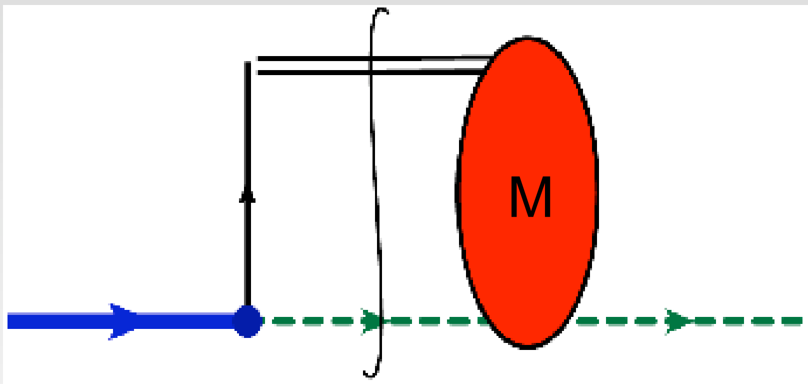
q^- - poles at one loop for higher twist T-odd TMDs [Gamberg, Hwang, Metz, MS, PLB 639, 508]

• **Step 2: Integration over q^+:**

$$\frac{1}{(1-x)P^+ - q^+ + i\varepsilon} = \mathrm{P}\frac{1}{(1-x)P^+ - q^+} - i\pi\delta((1-x)P^+ - q^+)$$

Fix the q+ - pole
emphasizes a "natural" picture of FSI
equivalent to Cutkosky cut, assumptions of Step 1 valid in Eikonal models

Lensing Function



Assume a non-perturbative scattering amplitude M +

Separate GPD and FSI via contour integration

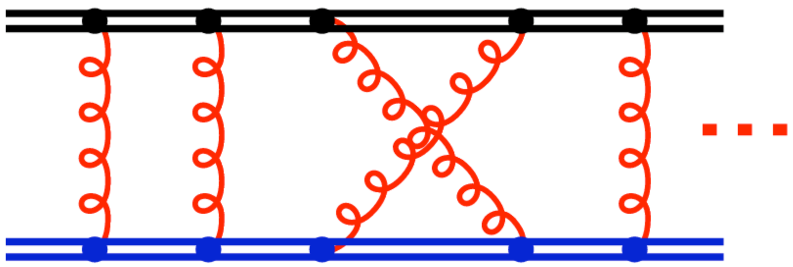
Contour integration → cut diagram → enforces *"natural"* picture of FSI

$$f_{1T}^{\perp,(1)u}(x) = -\frac{1}{2(1-x)M^2}\int \frac{d^2q_T}{(2\pi)^2}\, q_T^y\, I^y(x,\vec{q}_T)\, E^u\left(x,0,-\frac{\vec{q}_T^{\,2}}{(1-x)^2}\right)$$

$$I^i(x,\vec{q}_T) = \int \frac{d^2p_T}{(2\pi)^2}\,(2p_T-q_T)^i\, \Im M_{bc}^{ab}(|\vec{p}_T|)\Big((2\pi)^2\delta^{ac}\delta^{(2)}(\vec{p}_T-\vec{q}_T)+\Re M_{da}^{cd}(|p_T-q_T|)\Big)$$

- **More or less "realistic" model for M → allows for numerical comparison**
- **Sivers function from HERMES/COMPASS data,**
  GPD E from models or parameterizations

Eikonal Color calculation



Abarabanel Itzykson PRL 69
Gamberg Milton PRD 1999
Fried et al. 2000

$$G^{ab}_{\mathrm{eik}}(x,y|A) = -i\int_0^\infty ds\, \mathrm{e}^{-is(m_q - i0)}\delta^{(4)}(x-y-sv)\left(\mathrm{e}^{-ig\int_0^s d\beta\, v\cdot A^\alpha(y+\beta v)\, t^\alpha}\right)^{ab}_+$$

Trick to disentangle the A-field and the color matrices t: Functional FT

$$\left(\mathrm{e}^{-ig\int_0^s d\beta\, v\cdot A^\alpha(y+\beta v)\, t^\alpha}\right)^{ab}_+ = \mathcal{N}'\int \mathcal{D}\alpha \int \mathcal{D}u\, \mathrm{e}^{i\int d\tau\, \alpha^\beta(\tau) u^\beta(\tau)}\mathrm{e}^{ig\int d\tau\, \alpha^\beta(\tau)\, v\cdot A^\beta(y+\tau v)}\left(\mathrm{e}^{i\int_0^s d\tau\, t^\beta u^\beta(\tau)}\right)^{ab}_+$$

FLOW CHART for calculation of Boer Mulders

L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

$$2m_\pi^2 h_1^{\perp(1)}(x) \simeq \int d^2b_T\, \vec{b}_T \cdot \vec{\mathcal{I}}(x,\vec{b}_T)\, \frac{\partial}{\partial \vec{b}_T^2} \mathcal{H}_1^\pi(x,\vec{b}_T^2),$$

$$I^i(x,\vec{q}_T) = \frac{1}{N_c}\int \frac{d^2p_T}{(2\pi)^2}(2p_T - q_T)^i \left(\Im[\bar{\mathrm{M}}^{\mathrm{eik}}]\right)_{\delta\beta}^{\alpha\delta}(|\vec{p}_T|)$$
$$\left((2\pi)^2\delta^{\alpha\beta}\delta^{(2)}(\vec{p}_T - \vec{q}_T) + \left(\Re[\bar{\mathrm{M}}^{\mathrm{eik}}]\right)_{\gamma\alpha}^{\beta\gamma}(|\vec{p}_T - \vec{q}_T|)\right).$$

$$\left(\mathrm{M}^{\mathrm{eik}}\right)_{\delta\beta}^{\alpha\delta}(x,|\vec{q}_T + \vec{k}_T|) = \frac{(1-x)P^+}{m_s}\int d^2z_T\, \mathrm{e}^{-i\vec{z}_T\cdot(\vec{q}_T+\vec{k}_T)} \quad (20)$$
$$\times\left[\int d^{N_c^2-1}\alpha \int \frac{d^{N_c^2-1}u}{(2\pi)^{N_c^2-1}}\,\mathrm{e}^{-i\alpha\cdot u}\left(\mathrm{e}^{i\chi(|\vec{z}_T|)t\cdot\alpha}\right)_{\alpha\delta}\left(\mathrm{e}^{it\cdot u}\right)_{\delta\beta} - \delta_{\alpha\beta}\right].$$

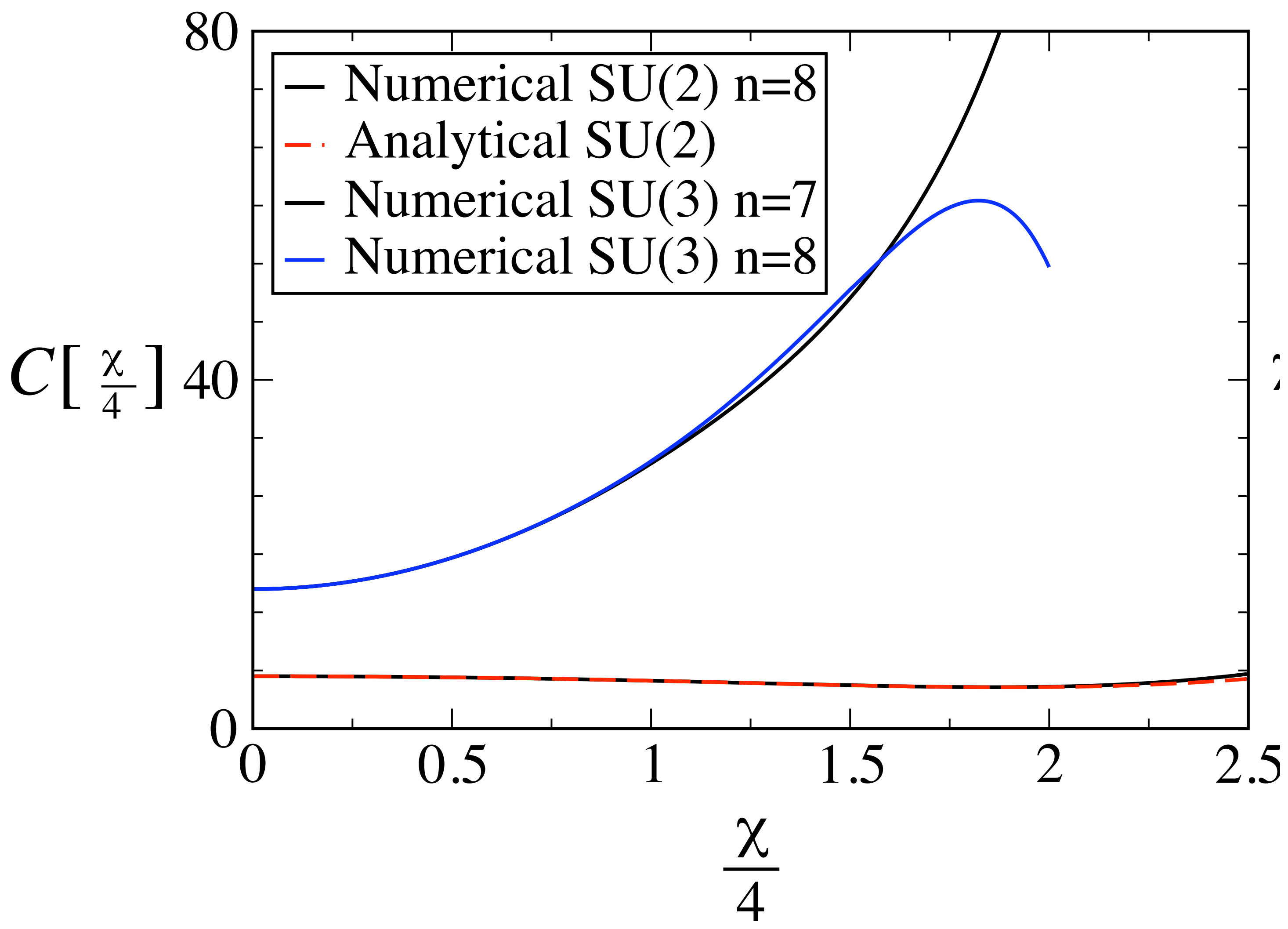
COLOR Integral

$$f_{\alpha\beta}(\chi) \equiv \int d^{N_c^2-1}\alpha \int \frac{d^{N_c^2-1}u}{(2\pi)^{N_c^2-1}}\,\mathrm{e}^{-i\alpha\cdot u}\left(\mathrm{e}^{i\chi(|\vec{z}_T|)t\cdot\alpha}\right)_{\alpha\delta}\left(\mathrm{e}^{it\cdot u}\right)_{\delta\beta} - \delta_{\alpha\beta}$$

$$f_{\alpha\beta}(\chi) = \sum_{n=1}^{\infty}\frac{(i\chi)^n}{(n!)^2}\sum_{a_1=1}^{N_c^2-1}\cdots\sum_{a_n=1}^{N_c^2-1}\sum_{P_n}\left(t^{a_1}\ldots t^{a_n}t^{a_{P_n(1)}}\ldots t^{a_{P_n(n)}}\right)_{\alpha\beta}.$$

COLOR FACTOR!

$$\mathcal{I}^i(x,\vec{b}_T) = \frac{(1-x)}{2N_c}\frac{b_T^i}{|\vec{b}_T|}\frac{\chi'}{4}C\left[\frac{\chi}{4}\right],$$



Eikonal Phase

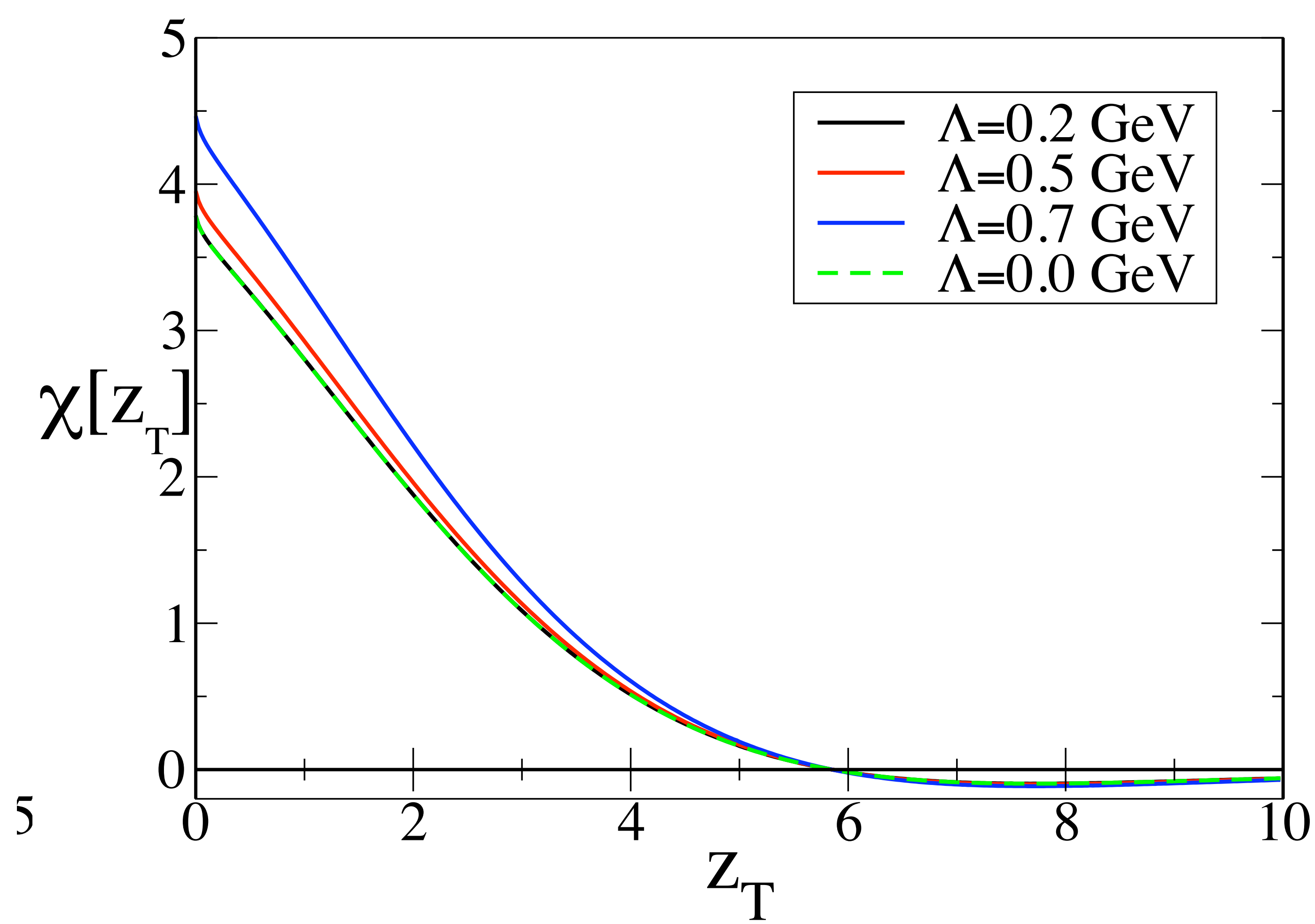
$$\chi^{DS}(|\vec{z}_T|) = 2\int_0^\infty dk_T\, k_T \alpha_s(k_T^2) J_0(|\vec{z}_T| k_T) Z(k_T^2, \Lambda_{QCD}^2)/k_T^2.$$

$$\alpha_s(\mu^2) = \frac{\alpha_s(0)}{\ln\left[e + a_1(\mu^2/\Lambda^2)^{a_2} + b_1(\mu^2/\Lambda^2)^{b_2}\right]}. \quad (35)$$

The values for the fit parameters are $\Lambda = 0.71$ GeV, $a_1 = 1.106$, $a_2 = 2.324$, $b_1 = 0.004$ and $b_2 = 3.169$. These calculations

$$Z(p^2,\mu^2) = p^2 \mathcal{D}^{-1}(p^2,\mu^2)$$
$$= \left(\frac{\alpha_s(p^2)}{\alpha_s(\mu^2)}\right)^{1+2\delta} \left(\frac{c\left(\frac{p^2}{\Lambda^2}\right)^{\kappa} + d\left(\frac{p^2}{\Lambda^2}\right)^{2\kappa}}{1 + c\left(\frac{p^2}{\Lambda^2}\right)^{\kappa} + d\left(\frac{p^2}{\Lambda^2}\right)^{2\kappa}}\right)^2, \quad (36)$$

with the parameters $c = 1.269$, $d = 2.105$, and $\delta = -\frac{9}{44}$.



Lensing Function

Express Lensing Function in terms of Eikonal Phase:

$$\mathcal{I}^i_{(N=1)}(x,\vec{b}_T) = \frac{1}{4}\frac{b^i_T}{|\vec{b}_T|}\chi'\left(\frac{|\vec{b}_T|}{1-x}\right)\left[1+\cos\chi\left(\frac{|\vec{b}_T|}{1-x}\right)\right]$$

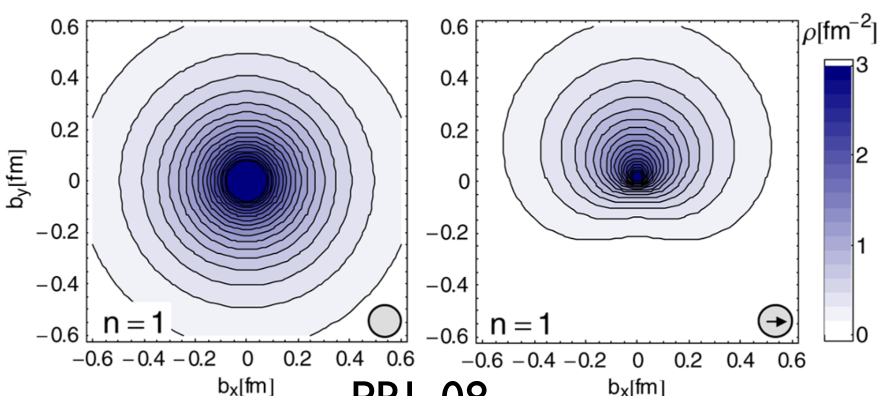
$$\mathcal{I}^i_{(N=2)}(x,\vec{b}_T) = \frac{1}{8}\frac{b^i_T}{|\vec{b}_T|}\chi'\left(\frac{|\vec{b}_T|}{1-x}\right)\left[3\left(1+\cos\frac{\chi}{4}\right)+\left(\frac{\chi}{4}\right)^2-\sin\frac{\chi}{4}\left(\frac{\chi}{4}-\sin\frac{\chi}{4}\right)\right]\left(\frac{|\vec{b}_T|}{1-x}\right)$$

$$\mathcal{I}^i_{(N=3)}(x,\vec{b}_T) = \text{numerics}$$

L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

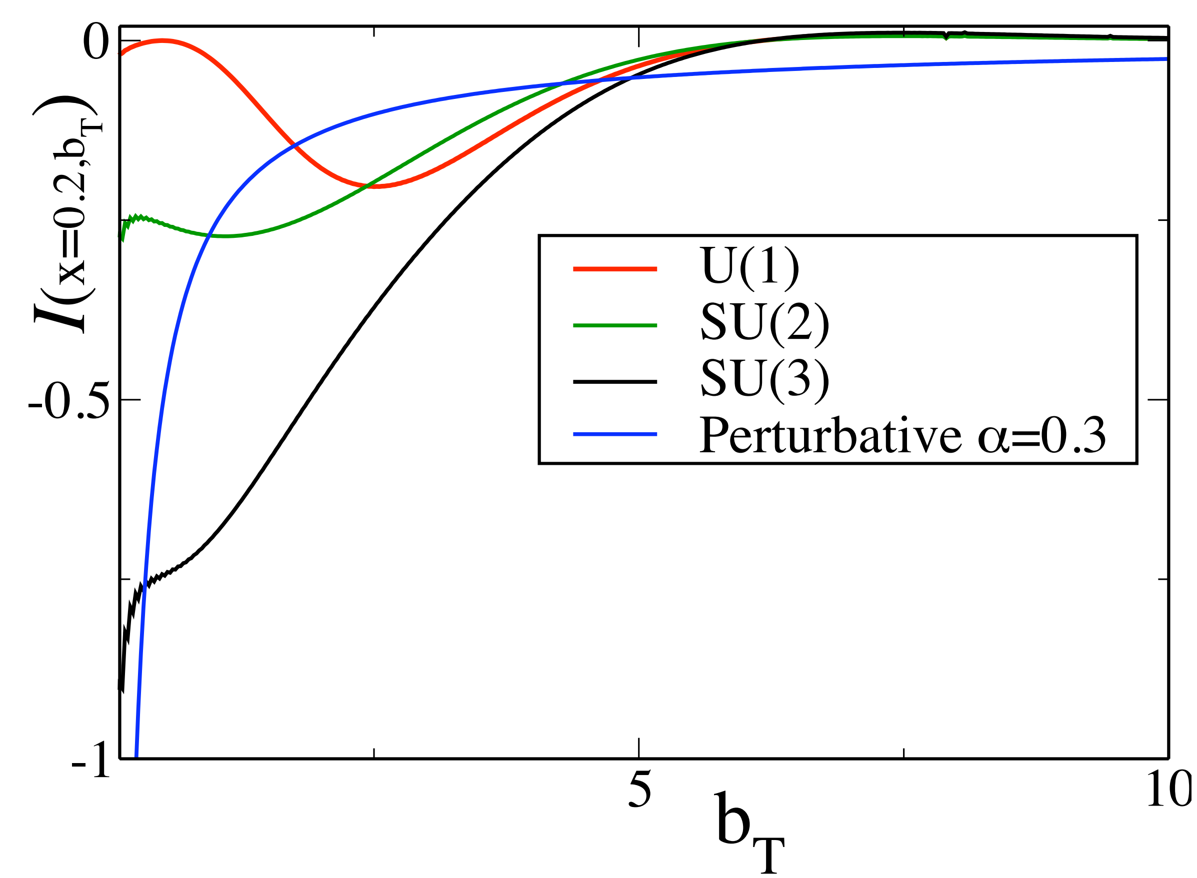
Spin Structure of the Pion



PRL 08

D. Brömmel,[1,2] M. Diehl,[1] M. Göckeler,[2] Ph. Hägler,[3]

FSI + distortion

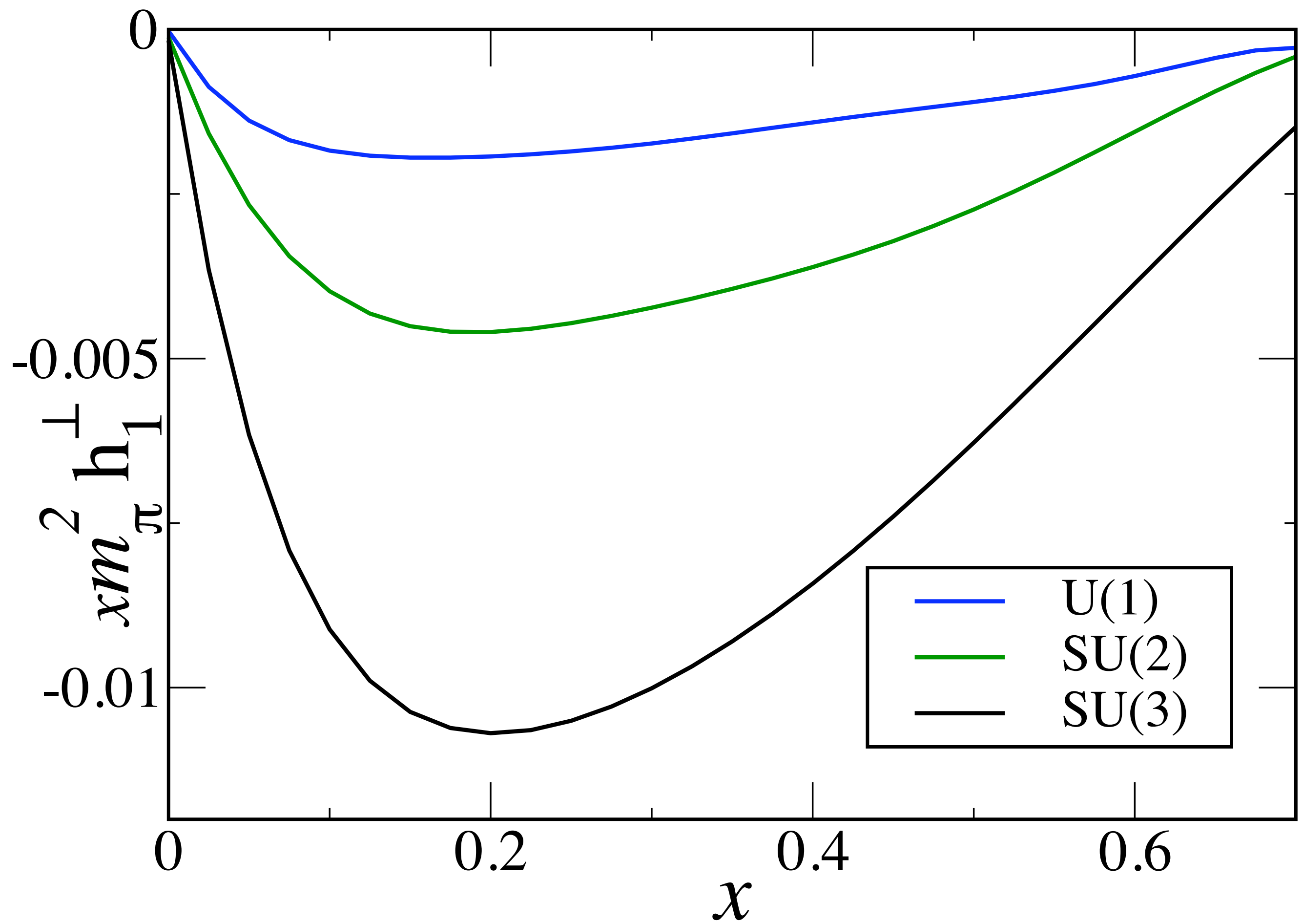


FSIs are negative and "grow" with Color!

Prediction for Boer-Mulders Function of PION

L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

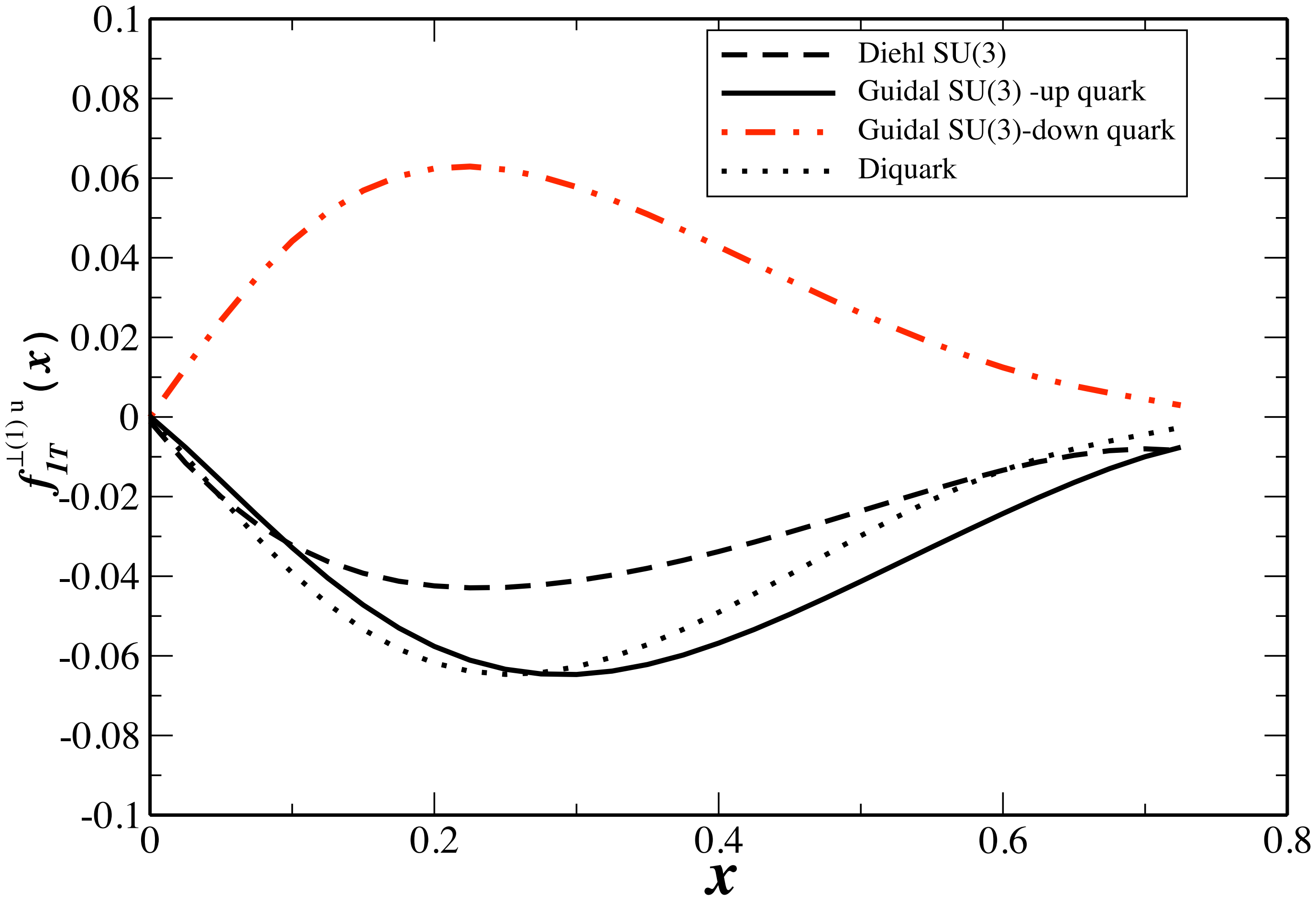


Relations produce a BM funct. approx equiv. to Sivers from HERMES

Expected sign i.e. FSI are negative

Answer will come from pion BM from COMPASS πN Drell Yan

Results for u&d-quark Sivers



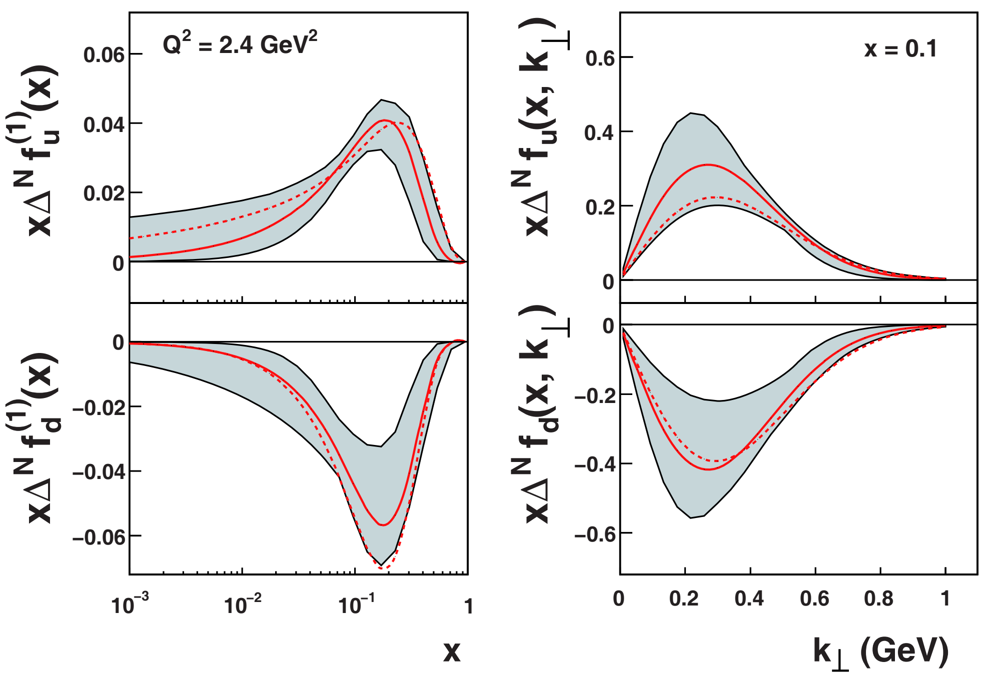


Fig. 7. The Sivers distribution functions for u and d flavours, at the scale $Q^2 = 2.4\,(\mathrm{GeV}/c)^2$, as determined by our present fit (solid lines), are compared with those of our previous fit [2] of SIDIS data (dashed lines), where π^0 and kaon productions were not considered and only valence quark contributions were taken into account. This plot clearly shows that the Sivers functions previously found are consistent, within the statistical uncertainty bands, with the Sivers functions presently obtained.

o 3

- Torino extraction ~ 0.05 SU(3) ! agrees with Chromodynamic LENSING
- Sivers effect increases with color: Color tracing in summing gauge link goes like
- Color tracing gives result of N_c counting of Pobylitsa

Unifying Transverse Structure of Nucleon GTMDs

GTMD--Meissner Metz Schlegel 07, 08

$$W^{[\Gamma]}_{\lambda,\lambda'}(P,x,\mathbf{k}_T,\Delta,n) = \int dk^- \, W^{[\Gamma]}_{\lambda,\lambda'}(P,k,\Delta,n)$$

Integ. small component !!!

$$\mathcal{FT} : \Delta \Longleftrightarrow \vec{b}$$

$$W^{[\Gamma]}_{\lambda,\lambda'}(P,k,\Delta;n) \Longleftrightarrow W^{[\Gamma]}_{\lambda,\lambda'}(P,k,\vec{b};n)$$

Wigner functions--Belitsky Ji Yuan, 04

Reduce to TMDs, GPDs, Impact GPDs

Relations among them?

TMDs & Impact GPDs Project from GTMDs

$$W^{[\Gamma]}_{\lambda,\lambda'}(P,x,\mathbf{k}_T,\Delta,n) = \int dk^- \, W^{[\Gamma]}_{\lambda,\lambda'}(P,k,\Delta,n)$$

Integ. small component,

GTMD--Meissner Metz Schlegel, 07

EIC??

$\int d^2\mathbf{k}_T$

$\Delta = 0$

$$F^{[\Gamma]}_{\lambda,\lambda'}(x,\xi,t) = \int d^2\mathbf{k}_T \, W^{[\Gamma]}_{\lambda,\lambda'}(P,x,\mathbf{k}_T,\Delta,n)$$

$\xi = 0$

$$\mathcal{FT} : \Delta_T \Longleftrightarrow \vec{b}_T$$

$$W^{[\Gamma]}_{\lambda,\lambda'}(P,x,\mathbf{k}_T,0,n) = \Phi(x,\mathbf{k}_T)$$

$$\mathcal{F}_{\lambda,\lambda'}(x,\vec{b}) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\vec{\Delta}_T\cdot\vec{b}} F_{\lambda,\lambda'}(x,0,\vec{\Delta}_T)$$

TMD

Impact-GPD

Exclusive Inclusive Relations

$$\int_{-1}^{1} dx\, H^q(x,\xi,t) = F_1^q(t)$$
$$\int_{-1}^{1} dx\, E^q(x,\xi,t) = F_2^q(t)$$
$$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) = G_A^q(t)$$
$$\int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) = G_P^q(t)$$

Trivial Relations are well-known:

$$f_1(x) = H(x,0,0) = \int d^2k_T\, f_1(x,\vec{k}_T^2) = \int d^2b_T \mathcal{H}(x,\vec{b}_T^2)$$

$$g_1(x) = \tilde{H}(x,0,0) = \int d^2k_T\, g_{1L}(x,\vec{k}_T^2)$$

$$h_1(x) = H_T(x,0,0) = \int d^2k_T\, h_1(x,\vec{k}_T^2)$$

→ model-independent, integrated relations

Reality Check

Parm. of GTMD correlator hermiticity parity time-reversal

from Andreas Metz INT talk

$$(x, \xi, \vec{k}_T, \vec{\Delta}_T)$$

$$W^q = \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2\vec{z}_T}{(2\pi)^2} e^{ik\cdot z} \langle p'; \lambda' | \bar{\psi}\Big(-\frac{z}{2}\Big) \gamma^+ \mathcal{W}_{GTMD}\, \psi\Big(\frac{z}{2}\Big) |p; \lambda\rangle \Big|_{z^+=0}$$

- Projection onto GPDs and TMDs

$$F^q = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p'; \lambda' | \bar{\psi}\Big(-\frac{z}{2}\Big) \gamma^+ \mathcal{W}_{GPD}\, \psi\Big(\frac{z}{2}\Big) |p; \lambda\rangle \Big|_{z^+=z_T=0}$$

$$= \int d^2\vec{k}_T\, W^q$$

$$\Phi^q = \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2\vec{z}_T}{(2\pi)^2} e^{ik\cdot z} \langle p; \lambda' | \bar{\psi}\Big(-\frac{z}{2}\Big) \gamma^+ \mathcal{W}_{TMD}\, \psi\Big(\frac{z}{2}\Big) |p; \lambda\rangle \Big|_{z^+=0}$$

$$= W^q \Big|_{\Delta=0}$$

GTMD-Wigner Function Correlator

Miessner Metz & Schlegel JHEP 2008 & 2009

- Parameterization of GTMD-correlator

  Example:

$$W^{q\,[\gamma^+]} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[F_{1,1} + \frac{i\sigma^{i+}k_T^i}{P^+}\,F_{1,2} + \frac{i\sigma^{i+}\Delta_T^i}{P^+}\,F_{1,3} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}\,F_{1,4}\right]u(p,\lambda)$$

  $\longrightarrow$ GTMDs are complex functions: $F_{1,n} = F_{1,n}^e + iF_{1,n}^o$

- Implications for potential nontrivial relations
  - Relations of second type

$$E(x,0,\vec{\Delta}_T^2) = \int d^2\vec{k}_T\left[-F_{1,1}^e + 2\left(\frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\,F_{1,2}^e + F_{1,3}^e\right)\right]$$

$$f_{1T}^{\perp}(x,\vec{k}_T^2) = -F_{1,2}^o(x,0,\vec{k}_T^2,0,0)$$

These Have Different Mothers

$$\int d^2\vec{b}_T\,\mathcal{H}^q(x,\vec{b}_T^2) = \int d^2\vec{k}_T\,f_1^q(x,\vec{k}_T^2) = \int d^2\vec{k}_T\,\mathrm{Re}\Big[F_1^q(x,0,\vec{k}_T^2,0,0)\Big]$$

$$f_{1T}^{\perp}(x,\vec{k}_T^2;\eta) = -F_{1,2}^{o}(x,0,\vec{k}_T^2,0,0;\eta)$$

$$E(x,\xi,t) = \int d^2\vec{k}_T\left[-F_{1,1}^e + 2(1-\xi^2)\left(\frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}F_{1,2}^e + F_{1,3}^e\right)\right]$$

$\longrightarrow$ No model-independent nontrivial relation between E and $f_{1T}^{\perp}$ possible

$\longrightarrow$ Relation in spectator model due to simplicity of the model

$\longrightarrow$ No information on numerical violation of relation

$\longrightarrow$ Likewise for nontrivial relation involving $h_1^{\perp}$

However is approximate relation good for phenomenological approach for model builders

Conclusions

- **EIC in conjunction w/ Drell Yan can test fundamental factorization theorem of QCD: predicted sign change of Sivers function**
- **Crucial to have Q^2 range to pin down TMDs in particular Sivers function**
- **Transverse Distortion/Structure and TSSAs and unintegrated PDFs --- "Wigner functions" are there exclusive processes where they come in?**
- **Unifying structure GTMDs/Wigner Functions**
- **Pheno-Transverse Structure TMDs and TSSAs *b* and *k* asymm. An improved dynamical approach for FSI & model building**

"QCD calc" FSIs Gauge Links-Color Gauge Inv. "T-odd" TMDs

MORE .....

- **Jet SIDIS**
- **Extracting weighted TSSAs**
- **Connection bwtn. gluonic and fermionic poles-- twist 3 ETQS approach to TSSAs and the TMD description**
- **Opportunities to further explore angular momentum sum rule(s)**

Spect. model workbench ISI/FSI in AA & TMDs $h_1^\perp$, $f_{1T}^\perp$, $H_1^\perp$ gluonic poles

Has also been used to study Universality of PDFs and FFs

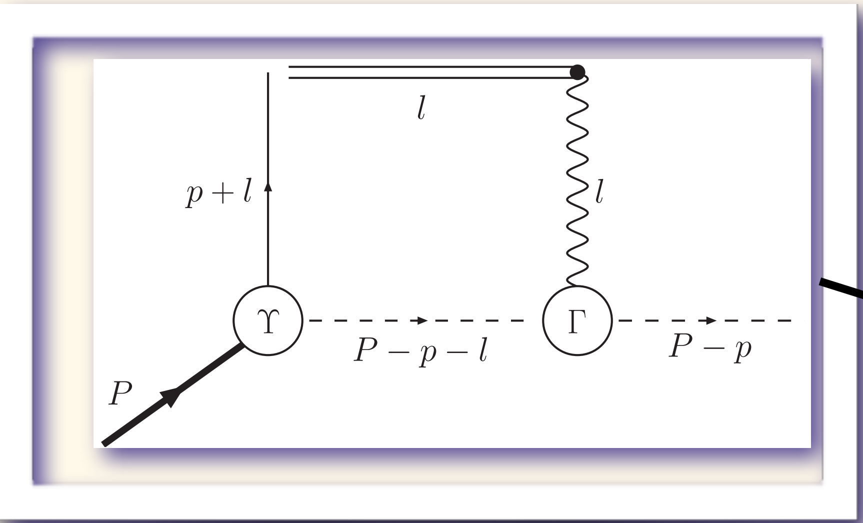
- *$\nexists$ calculation Quark-Quark Correlator in Full QCD*

$$\Phi^{[\mathcal{U}[\mathcal{C}]]}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-, \xi_T)|P\rangle|_{\xi^+=0}$$

- Use Spectator Framework Develop a QFT to explore and estimates these effects with gauge links
  - ⋆ BHS FSI/ISI Sivers fnct, -PLB 2002, NPB 2002
  - ⋆ Ji, Yuan PLB 2002 - Sivers Function
  - ⋆ Metz PLB 2002 - Collins Function
  - ⋆ L.G. Goldstein, 2002 ICHEP- Boer Mulders Function
  - ⋆ L.G. Goldstein, Oganessyan TSSA & AAS PRD 2003-SIDIS
  - ⋆ Boer Brodsky Hwang PRD 2003-Drell Yan Boer Mulders
  - ⋆ Bacchetta Jang Schafer 2004- PLB, Flavor-Sivers, Boer Mulders
  - ⋆ Lu Ma Schmidt PLB, PRD, 2004/2005 Pion Boer Mulders
  - ⋆ L.G. Goldstein DY and higher twist, PLB 2007
  - ⋆ LG, Goldstein, Schlegel PRD 2008-Flavor dep. Boer Mulders $\cos 2\phi$ SIDIS
  - ⋆ Conti, Bacchetta, Radici, Ellis, Hwang, Kotzinian 2008 hep-ph . . . !
- Spectator Model "Field Theoretic" used study Universality of T-odd Fragmentation Δ_{ij}
  - ⋆ Metz PLB 2002, Collins Metz PRL 2004
  - ⋆ Bacchetta, Metz, Yang, PBL 2003, Amrath, Bacchetta, Metz 2005,
  - ⋆ Bacchetta, L.G. Goldstein, Mukherjee, PLB 2008
  - ⋆ Collins Qui, Collins PRD 2007,2008
  - ⋆ Yuan 2-loop Collins function PRL 2008
  - ⋆ L.G., Mulders, Mukherjee Gluonic Poles PRD 2008

Studies FSIs in 1-gluon exchange approx.

LG, G. Goldstein, M. Schlegel PRD 77 2008, Bacchetta Conti Radici PRD 78, 2008



Build the T-odd TMD PDF with Final State Interactions-- one gluon exchange approx of Gauge link

$$W_i(P,k,S) = -ie_q e_{dq}\int \frac{d^4 l}{(2\pi)^4}\frac{g_{ax}((p+l)^2)}{\sqrt{3}}\varepsilon^*_\sigma(P-p,\lambda)\mathcal{D}^{ax}_{\rho\eta}(P-p-l)$$
$$\times\frac{[g^{\sigma\rho}v\cdot(2P-2p-l)+(1+\kappa)(v^\sigma(P-p+l)^\rho+v^\rho(P-p-2l)^\sigma)]}{[l\cdot v+i0][l^2+i0][(l+p)^2-m_q^2+i0]}$$
$$\times\left[(\not p+\not l+m_q)\gamma_5\left(\gamma^\eta - R_g\frac{P^\eta}{M}\right)u(P,S)\right]_i,$$

$$\epsilon_T^{ij}k_T^i S_T^j f_{1T}^\perp(x,\vec{k}_T^2) = -\frac{M}{8(2\pi)^3(1-x)P^+}\left(\bar W\gamma^+W\Big|_{S_T} - \bar W\gamma^+W\Big|_{-S_T}\right)$$

Many model calculations studying dynamics of FSIs
Brodsky, Hwang et al, Bacchetta & Radici, et al,
Pasquini et al, Courtoy et al,
....

Sivers Parameterizations and studies from FSIs

Anselmino et al. PRD 05, EPJA 08

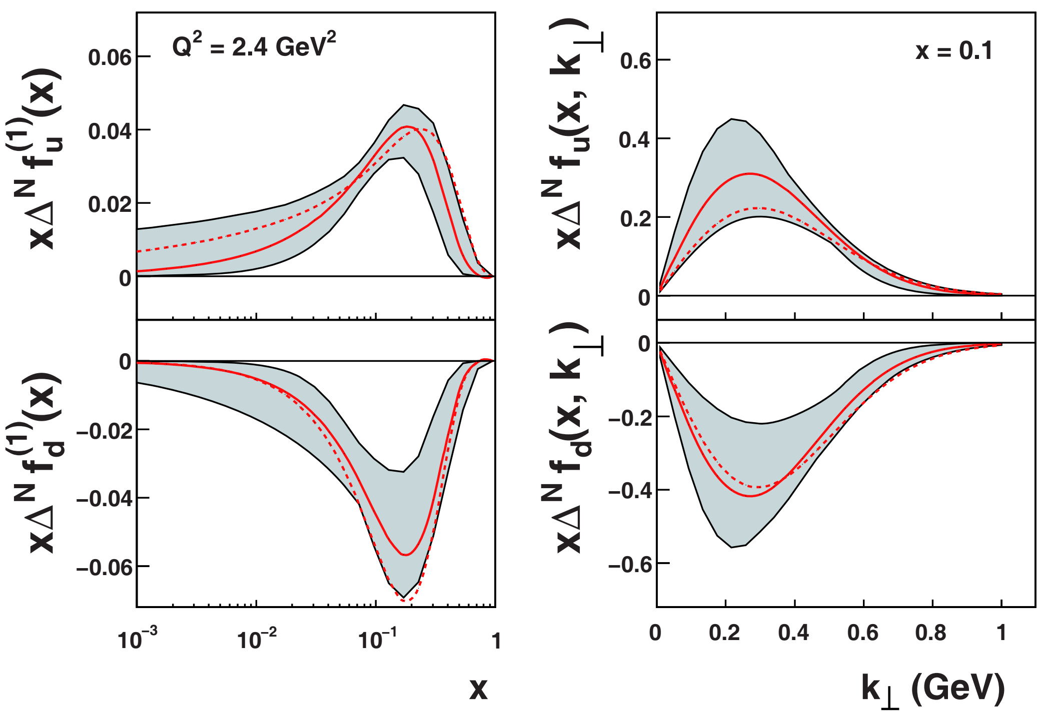


Fig. 7. The Sivers distribution functions for u and d flavours, at the scale $Q^2 = 2.4\,(\mathrm{GeV}/c)^2$, as determined by our present fit (solid lines), are compared with those of our previous fit [2] of SIDIS data (dashed lines), where π^0 and kaon productions were not considered and only valence quark contributions were taken into account. This plot clearly shows that the Sivers functions previously found are consistent, within the statistical uncertainty bands, with the Sivers functions presently obtained.

Gamberg, Goldstein,Schlegel PRD 77, 2008

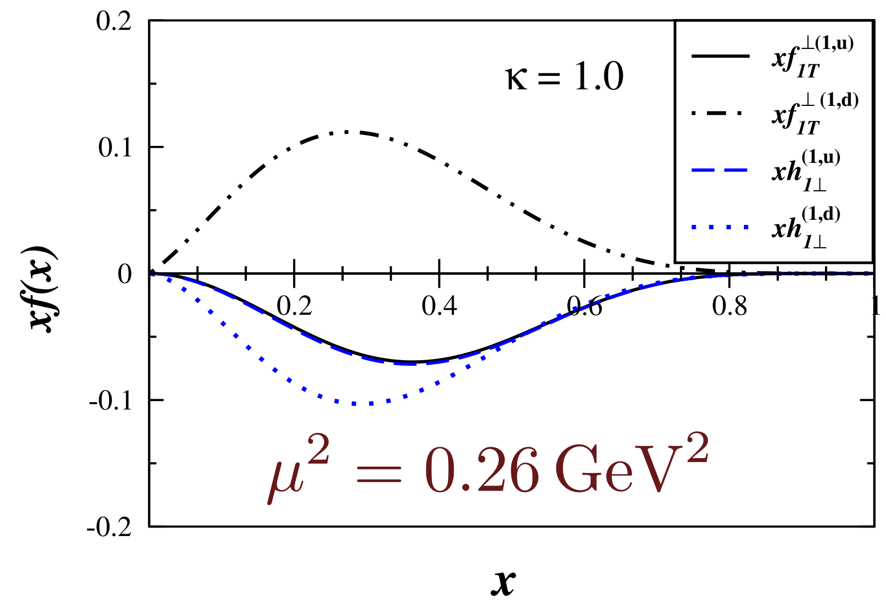


FIG. 5 (color online). The first moment of the Boer-Mulders and Sivers functions versus x for $\kappa = 1.0$.

"Factorization" of Distortion and FSIs

M. Burkardt [Nucl.Phys. A735, 185], [PRD66, 114005]

$$\langle k_T^{q,i}(x)\rangle_{UT} = \int d^2k_T\, k_T^i\, \frac{1}{2}\Big[\mathrm{Tr}[\gamma^+\Phi(\vec{S}_T)] - \mathrm{Tr}[\gamma^+\Phi](-\vec{S}_T)\Big]$$

Manipulate gauge link and trnsfm to $\vec{b}$ space

1) $\langle k_T^{q,i}(x)\rangle_{UT} = \frac{1}{2}\int d^2\vec{b}_T \int \frac{dz^-}{2\pi} e^{ixP^+z^-}\langle P^+, \vec{0}_T; S|\bar{\psi}(z_1)\gamma^+\mathcal{W}(z_1;z_2)I^{q,i}(z_2)\psi(z_2)|P^+, \vec{0}_T; S\rangle$

2) $\mathcal{F}^{q[\Gamma]}(x, \vec{b}_T; S) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-}\langle P^+, \vec{0}_T; S|\bar{\psi}(z_1)\Gamma\mathcal{W}(z_1;z_2)\psi(z_2)|P^+, \vec{0}_T; S\rangle,$ $\quad \Gamma \equiv \gamma^+$

Comparing expressions difference is additional factor,
$I^{q,i}$ and integration over $\vec{b}$