Towards a global GPD analysis

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K. Kumerički, DM, K. Passek-Kumerički (KMP-K), hep-ph/0703179
GPD fits at NLO and NNLO of H1/ZEUS data

KMP-K, 0805.0152 [hep-ph]
constructive critics on ad hoc GPD model approach [lot of good news]
first applications of dispersion integral approach

KMP-K, 0807.0159 [hep-ph]; **KM 0904.0458 [hep-ph]**
flexible GPD model for small *x* and fits of H1/ZEUS data
dispersion integral fits of HERMES and JLAB data

C. Bechler, DM 0906.2571 [hep-ph]
a GPD LO-look at π^+ production

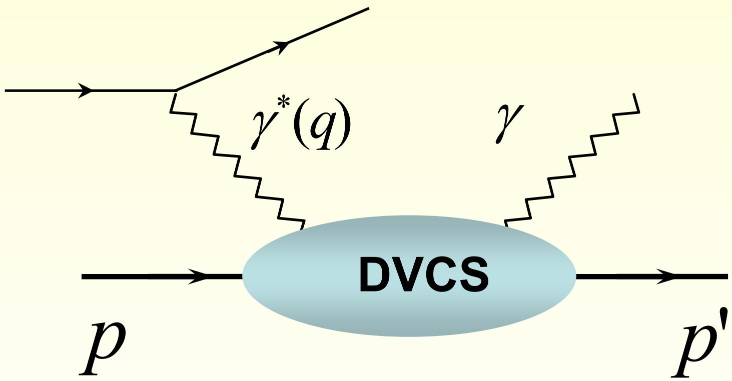
T. Lautenschlager et al. *(work in progress)*
GPD analysis of *ρ meson* production

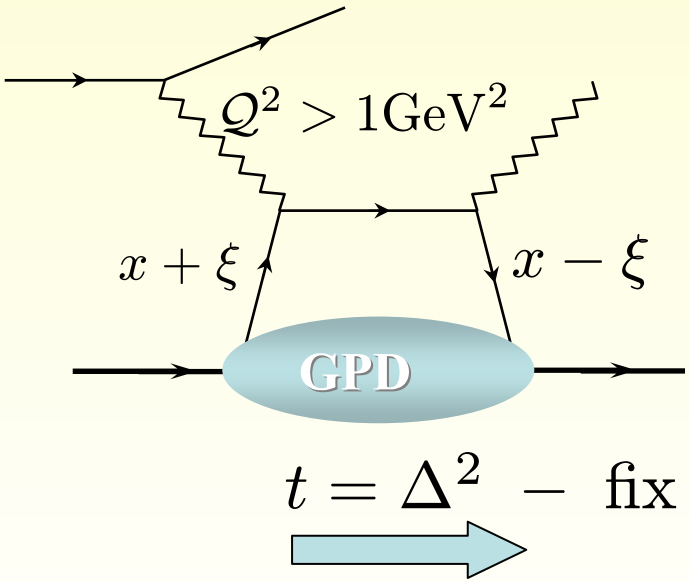
GPDs embed non-perturbative physics

[DM et. al (90/94)
Radyushkin (96)
Ji (96)]

GPDs appear in various hard exclusive processes,

e.g., hard electroproduction of photons (DVCS)





$$\mathcal{F}(\xi, \mathcal{Q}^2, t) = \int_{-1}^{1} dx\, C(x, \xi, \alpha_s(\mu), \mathcal{Q}/\mu) F(x, \xi, t, \mu) + O(\tfrac{1}{\mathcal{Q}^2})$$

CFF
Compton form factor
observable

hard scattering part
perturbation theory
(our conventions/microscope)

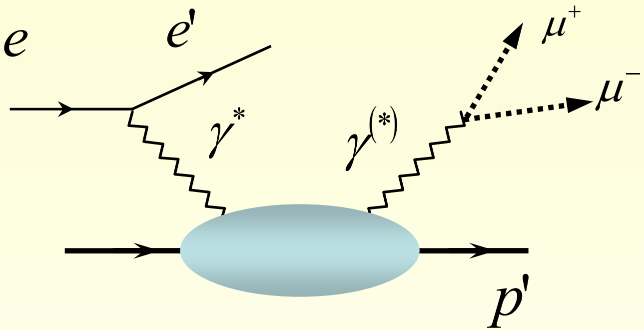
GPD
universal
(conventional)

higher twist
depends on
approximation

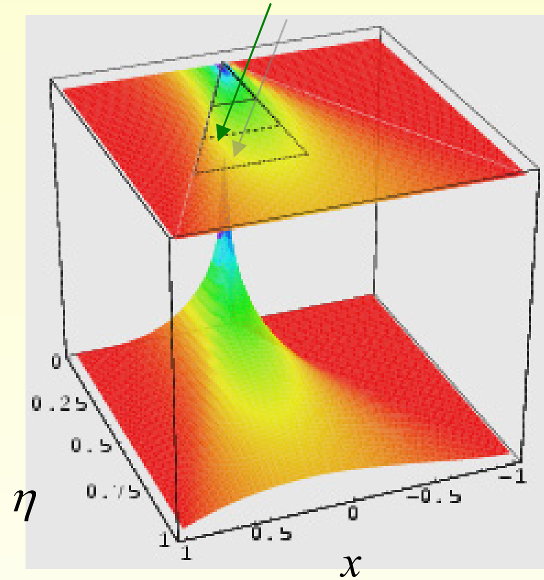
GPD related hard exclusive processes

- Deeply virtual Compton scattering (clean probe)

$$ep \to e'p'\gamma$$
$$ep \to e'p'\mu^+\mu^-$$
$$\gamma p \to p'e^+e^-$$



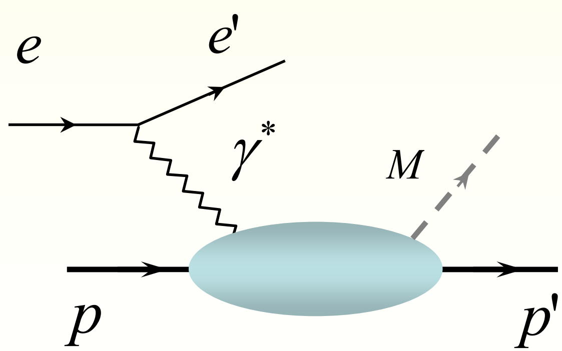
scanned area of the surface as a functions of lepton energy



$ep \to e'\, p'\, \mu^+ \mu^-$

- Hard exclusive meson production (flavor filter)

$$ep \to e'p'\pi$$
$$ep \to e'p'\rho$$
$$ep \to e'n\pi^+$$
$$ep \to e'n\rho^+$$



<u>twist-two observables:</u>

cross sections

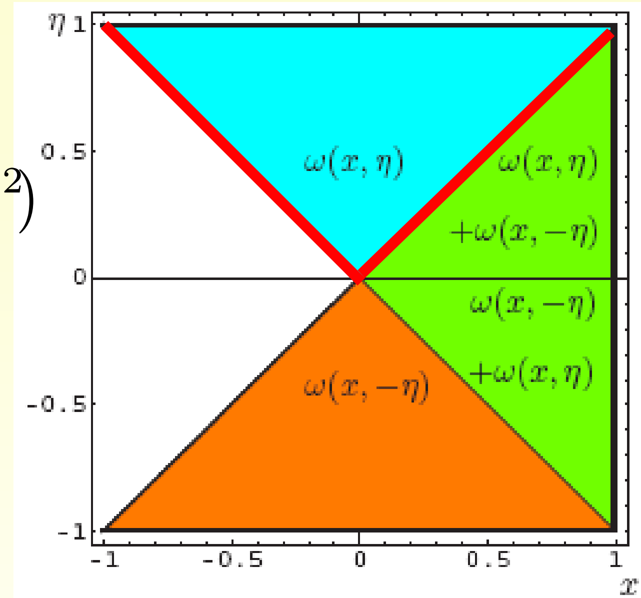
transverse target spin asymmetries

- etc.

A partonic duality interpretation

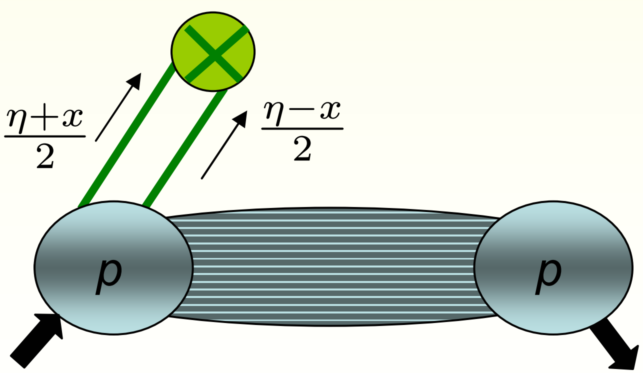
quark GPD (anti-quark $x \to -x$):

$$F = \theta(-\eta \le x \le 1)\, \omega\big(x, \eta, \Delta^2\big) + \theta(\eta \le x \le 1)\, \omega\big(x, -\eta, \Delta^2\big)$$



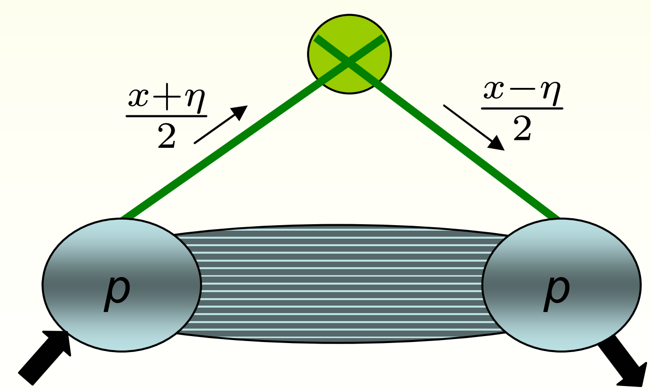
$$\omega\left(x, \eta, \Delta^2\right) = \frac{1}{\eta} \int_0^{\frac{x+\eta}{1+\eta}} dy\, x^p f(y, (x-y)/\eta, \Delta^2)$$

dual interpretation on partonic level:



support extension is unique [DM et al. 92]

ambiguous (*D*-term)
[DM, A. Schäfer (05)
KMP-K (07)]



central region $-\eta < x < \eta$

mesonic exchange in *t*-channel

outer region $\eta < x$

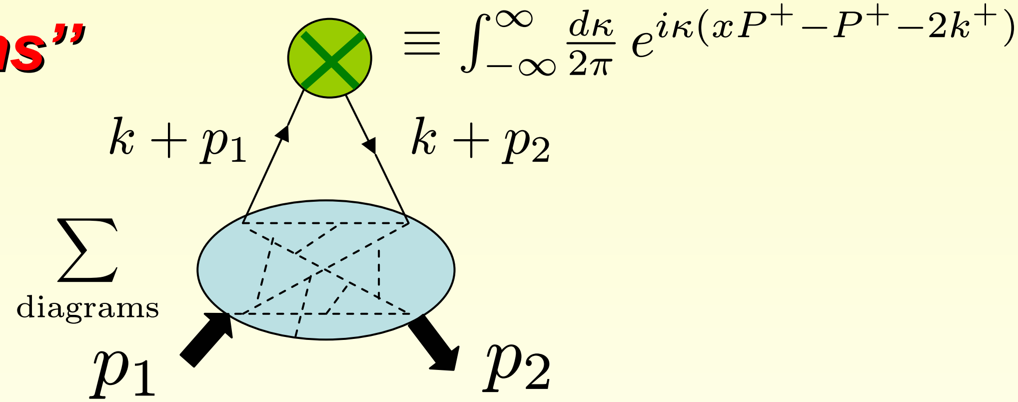
partonic exchange in *s*-channel

Overview: GPD representations

``light-ray spectral functions''

diagrammatic α-representation

DM, Robaschik, Geyer, Dittes, Hořejśi (88 (92) 94)



called ***double distributions***

A. Radyushkin (96)

light cone wave function overlap

(Hamiltonian approach in light-cone quantization)

Diehl, Feldmann, Jakob, Kroll (98,00)

Diehl, Brodsky, Hwang (00)

SL(2,R) (conformal) expansion

(series of local operators)

Radyushkin (97); Belitsky, Geyer, DM, Schäfer (97); DM, Schäfer (05); Kirch et al (05)

one version is called Shuvaev transformation, used in `dual' (*t*-channel) GPD parameterization

Shuvaev (99); Noritzsch (00) Shuvaev, Polyakov (02) ; Polyakov 07; Semenov-Tian-Shansky (09)

➡ each representation has its own ***advantages***, however, they are ***equivalent*** **(clearly spelled out in [Hwang, DM 07])**

SL(2,R) representations for GPDs

- support is a consequence of Poincaré invariance (polynomiality)

$$H_j(\eta,t,\mu^2) = \int_{-1}^{1} dx\, c_j(x,\eta) H(x,\eta,t,\mu^2)\,, \qquad c_j(x,\eta) = \eta^j C_j^{3/2}(x/\eta)$$

- conformal moments evolve autonomous (to LO and beyond in a special scheme)

$$\mu\frac{d}{d\mu} H_j(\eta,t,\mu^2) = -\frac{\alpha_s(\mu)}{2\pi}\gamma_j^{(0)} H_j(\eta,t,\mu^2)$$

- inverse relation is given as series of mathematical distributions:

$$H(x,\eta,t) = \sum_{j=0}^{\infty}(-1)^j p_j(x,\eta) H_j(\eta,t)\,,\; p_j(x,\eta) \propto \theta(|x|\le\eta)\frac{\eta^2-x^2}{\eta^{j+3}} C_j^{3/2}(-x/\eta)$$

- various ways of resummation were proposed, we are using Sommerfeld-Watson transform, leading to a ***Mellin-Barnes integral***

  - *PDF and FF constraints are trivially implemented*
  - *flexible parameterization*
  - *positivity constraints can not be implemented*

Towards realistic GPD (TMD) models

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi - \frac{1}{2}\phi\left(\partial^2 + \lambda^2\right)\phi + g\bar{\psi}\psi\phi$$

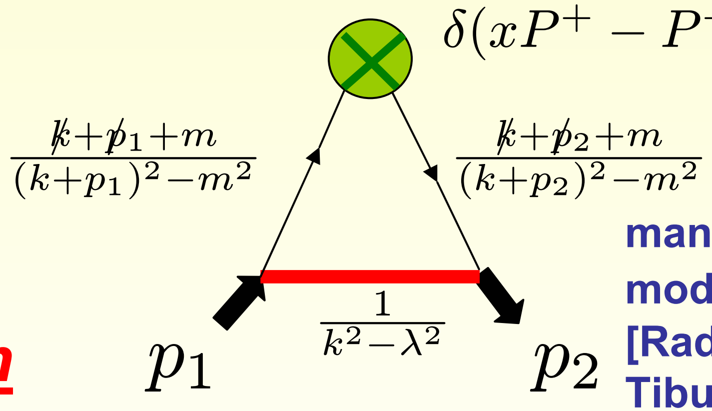
struck spin-*1/2* quark

collective scalar diquark spectator

coupling knows about spin

Diagrammatic approach:

via covariant time ordered perturbation theory



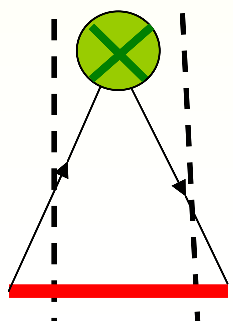
many spectator quark model studies [Radyuskin et.al (02); Tiburzi et.al (04); Hwang, DM (07)]

LC- Hamiltonian approach

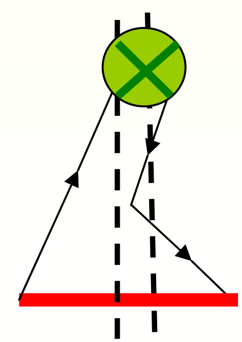
$$k^\mu \to (k^+, k^-, \mathbf{k}_\perp),\ k^\pm = k^0 \pm k^3,\ \mathbf{k}_\perp = (k^1, k^2).$$

integrate out minus component to find LCWF

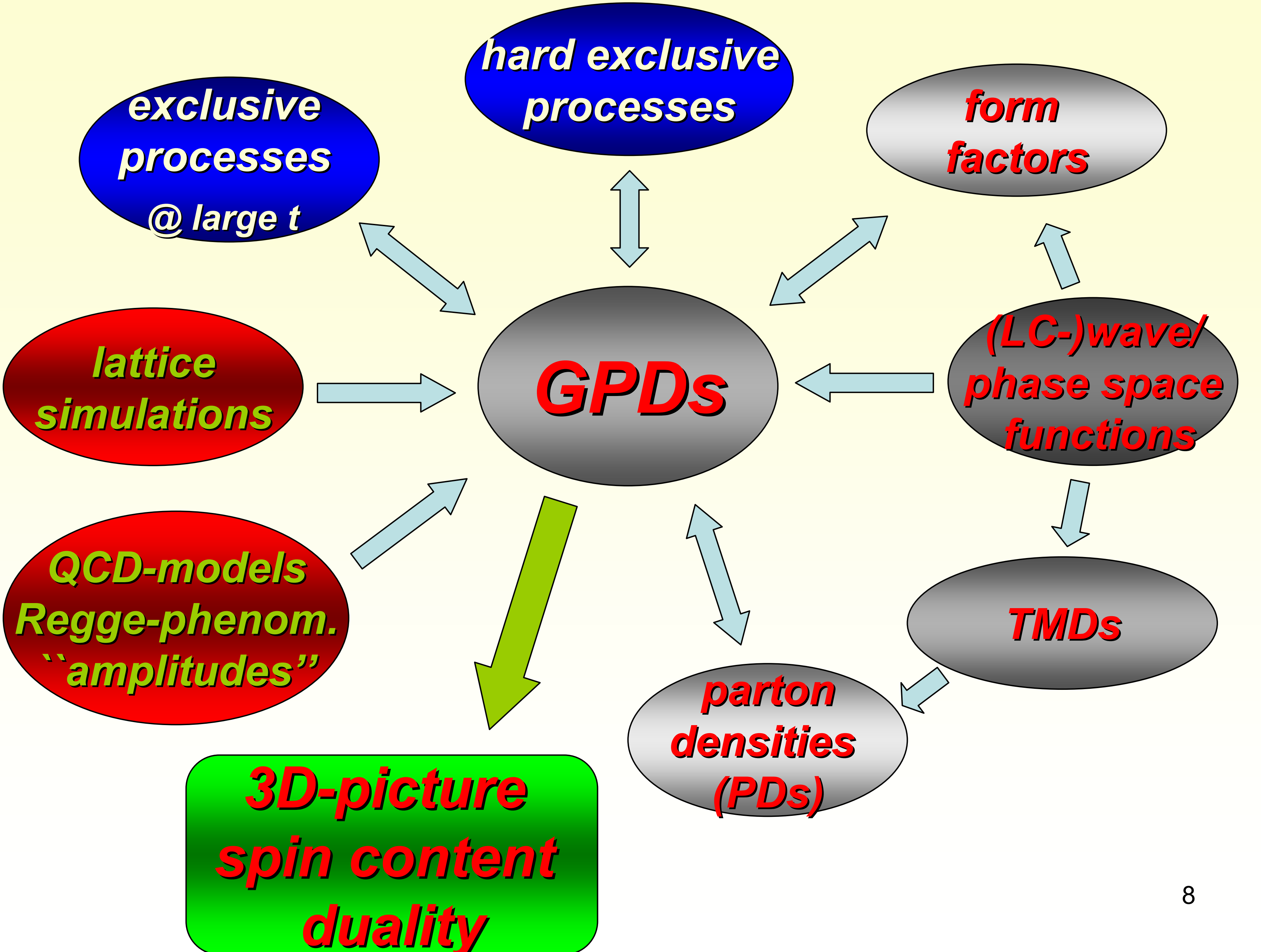
parton number conserved LCWF (outer region)



parton number violating LCWF (central region)



! PDF and FF constraints can not be simply implemented



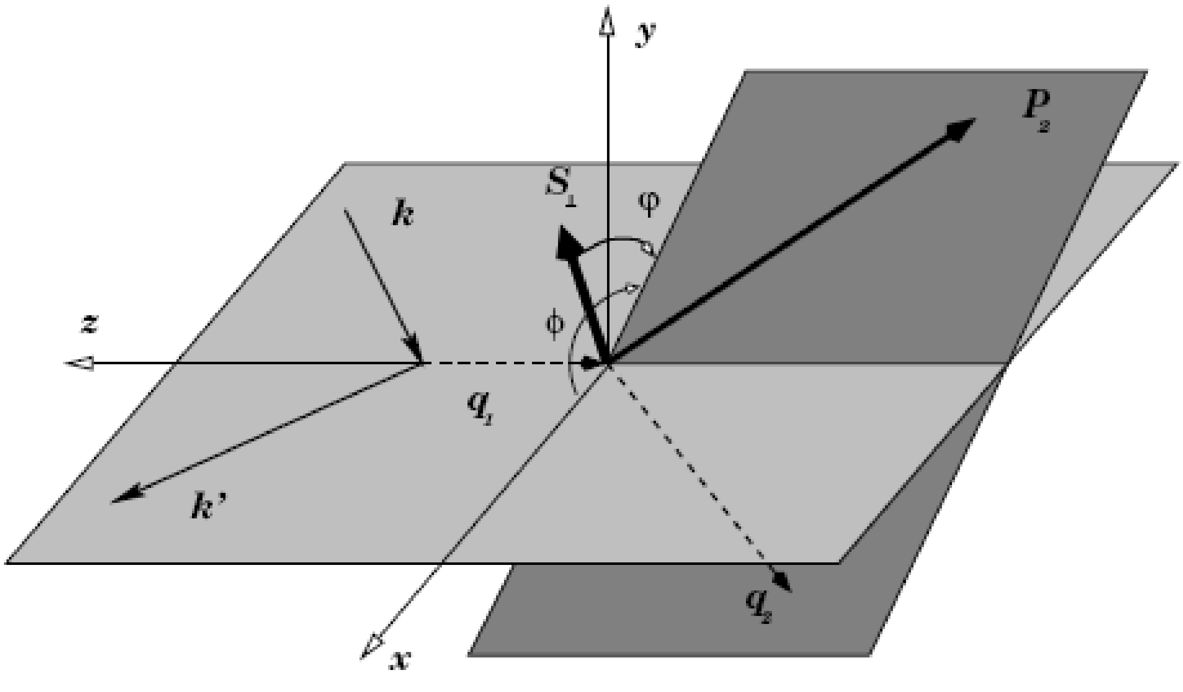
hard exclusive processes
exclusive processes @ large t
form factors
lattice simulations
GPDs
(LC-)wave/ phase space functions
QCD-models Regge-phenom. ``amplitudes''
TMDs
parton densities (PDs)
3D-picture spin content duality

Photon leptoproduction $e^{\pm}N \to e^{\pm}N\gamma$

measured by ***H1, ZEUS, HERMES, CLAS, HALL A*** collaborations

planed at ***COMPASS, JLAB@12GeV***, perhaps at ?? EIC,

$$\frac{d\sigma}{dx_{\rm Bj}dyd|\Delta^2|d\phi d\varphi} = \frac{\alpha^3 x_{\rm Bj} y}{16\,\pi^2\,\mathcal{Q}^2}\left(1+\frac{4M^2x_{Bj}^2}{\mathcal{Q}^2}\right)^{-1/2}\left|\frac{\mathcal{T}}{e^3}\right|^2,$$



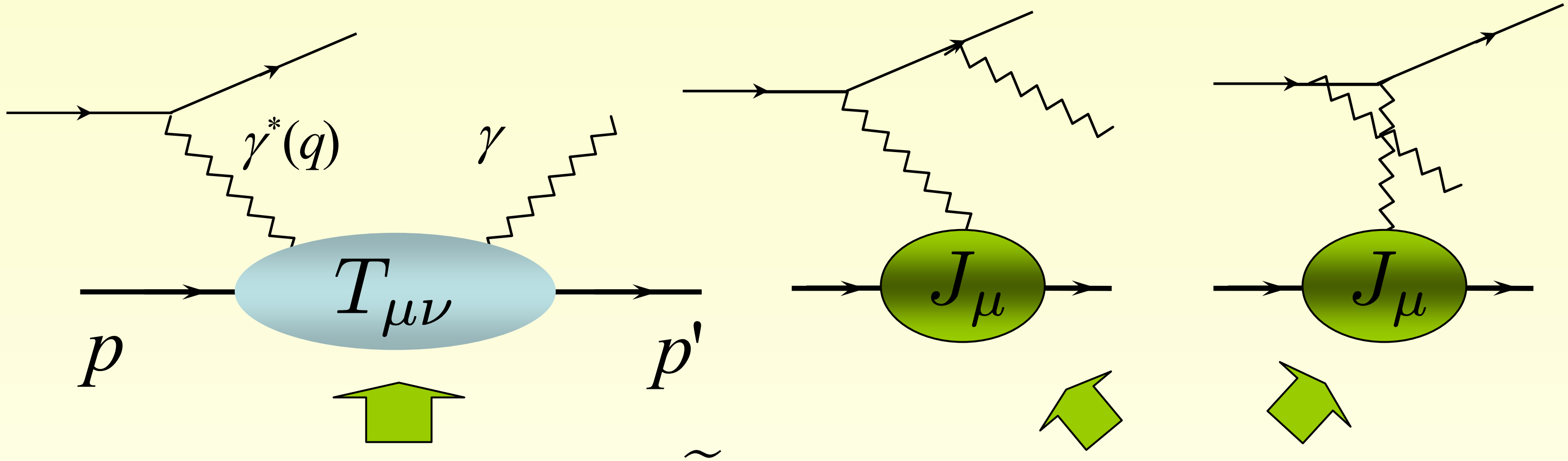
$$x_{\rm Bj} = \frac{\mathcal{Q}^2}{2P_1\cdot q_1} \approx \frac{2\xi}{1+\xi},$$

$$y = \frac{P_1\cdot q_1}{P_1\cdot k},$$

$$\Delta^2 = t \quad (\text{fixed, small}),$$

$$\mathcal{Q}^2 = -q_1^2 \quad (> 1\text{GeV}^2),$$

interference of *DVCS* and *Bethe-Heitler* processes



12 Compton form factors $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}} \cdots$ (helicity amplitudes)

elastic form factors F_1, F_2

$$|\mathcal{T}_{\rm BH}|^2 = \frac{e^6(1+\epsilon^2)^{-2}}{x_{\rm Bj}^2 y^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\rm BH} + \sum_{n=1}^{2} c_n^{\rm BH} \cos(n\phi) \right\},$$

exactly known (LO, QED)

$$|\mathcal{T}_{\rm DVCS}|^2 = \frac{e^6}{y^2 \mathcal{Q}^2} \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2} \left[c_n^{\rm DVCS} \cos(n\phi) + s_n^{\rm DVCS} \sin(n\phi) \right] \right\},$$

harmonics ⇕ <u>1:1</u> helicity ampl.

$$\mathcal{I} = \frac{\pm e^6}{x_{\rm Bj} y^3 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\}.$$

harmonics ⇕ ~~1:1~~ 10 helicity ampl.

Can one `measure' GPDs?

- ***CFF*** given as ***GPD*** **convolution**:

$$\mathcal{H}(\xi,t,\mathcal{Q}^2) \overset{\text{LO}}{=} \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right) H(x,\eta=\xi,t,\mathcal{Q}^2)$$

$$\overset{\text{LO}}{=} i\pi H^{-}(x=\xi,\eta=\xi,t,\mathcal{Q}^2) + \text{PV}\int_{0}^{1} dx \frac{2x}{\xi^2 - x^2} H^{-}(x,\eta=\xi,t,\mathcal{Q}^2)$$

- *H(x,x,t,$\mathcal{Q}^2$)* viewed as **"spectral function"** (*s*-channel cut):

$$H^{-}(x,x,t,Q^2) \equiv H(x,x,t,Q^2) - H(-x,x,t,Q^2) \overset{\text{LO}}{=} \frac{1}{\pi}\Im\mathrm{m}\mathcal{F}(\xi = x,t,Q^2)$$

- ***CFFs*** satisfy `**dispersion relations**'
  (not the physical ones, threshold ξ_0 set to 1)

**[Frankfurt et al (97)
Chen (97)
Terayev (05)
KMP-K (07)
Diehl, Ivanov (07)]**

$$\Re\mathrm{e}\mathcal{F}(\xi,t,Q^2) = \frac{1}{\pi}\text{PV}\int_{0}^{1} d\xi' \left(\frac{1}{\xi - \xi'} \mp \frac{1}{\xi + \xi'}\right) \Im\mathrm{m}\mathcal{F}(\xi',t,Q^2) + \mathcal{C}(t,Q^2)$$

[Terayev (05)]

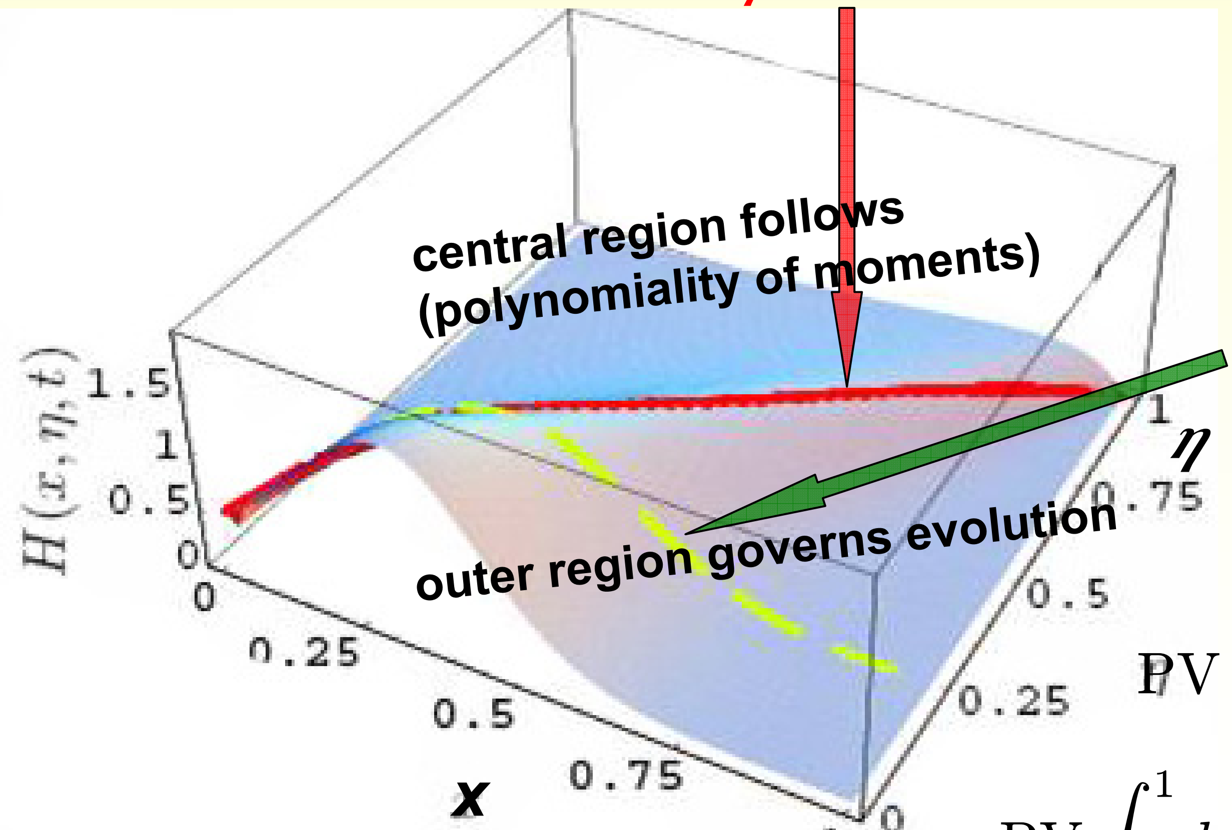
access to the ***GPD*** on the **cross-over line** $\eta = x$ (at LO)

Modeling & Evolution

outer region governs the evolution at the cross-over trajectory

$$\mu^2 \frac{d}{d\mu^2} H(x,x,t,\mu^2) = \int_x^1 \frac{dy}{x} V(1, x/y, \alpha_s(\mu)) H(y,x,\mu^2)$$

GPD at $\eta = x$ is `measurable' (LO)



net contribution of outer + central region is governed by a sum rule:

$$\mathrm{PV}\int_0^1 dx\, \frac{2x}{\eta^2 - x^2} H^-(x,\eta,t)$$

$$= \mathrm{PV}\int_0^1 dx\, \frac{2x}{\eta^2 - x^2} H^-(x,x,t) + C(t)$$

Strategies to analyze DVCS data

ad hoc modeling: **VGG** code [**Goeke et. al (01)** based on **Radyuskin's** DDA]
(first decade) BKM model [**Belitsky, Kirchner, DM (01)** based on RDDA]
`aligned jet' model [**Freund, McDermott, Strikman (02)**]
minimalist "dual" model [**Polyakov,Shuvaev 02;Guzey,Teckentrup 06**]
" -- " [**KMP-K (07)** in MBs-representation]

Kroll/Goloskokov (05,09) based on RDDA [handbag approach to meson production]

dynamical models: not applied **[Radyuskin et.al (02); Tiburzi et.al (04); Hwang DM (07)]…**

flexible models: in any representation by including ***unconstrained*** degrees of freedom
! expansion in polynomials [**Belitsky et al. (00), Liuti et. al (07), Moutarde (09)**]

? physical/partonic content of ***`invisible'*** (***unconstrained***) degrees of freedom

Extracting CFFs from data: real and imaginary part

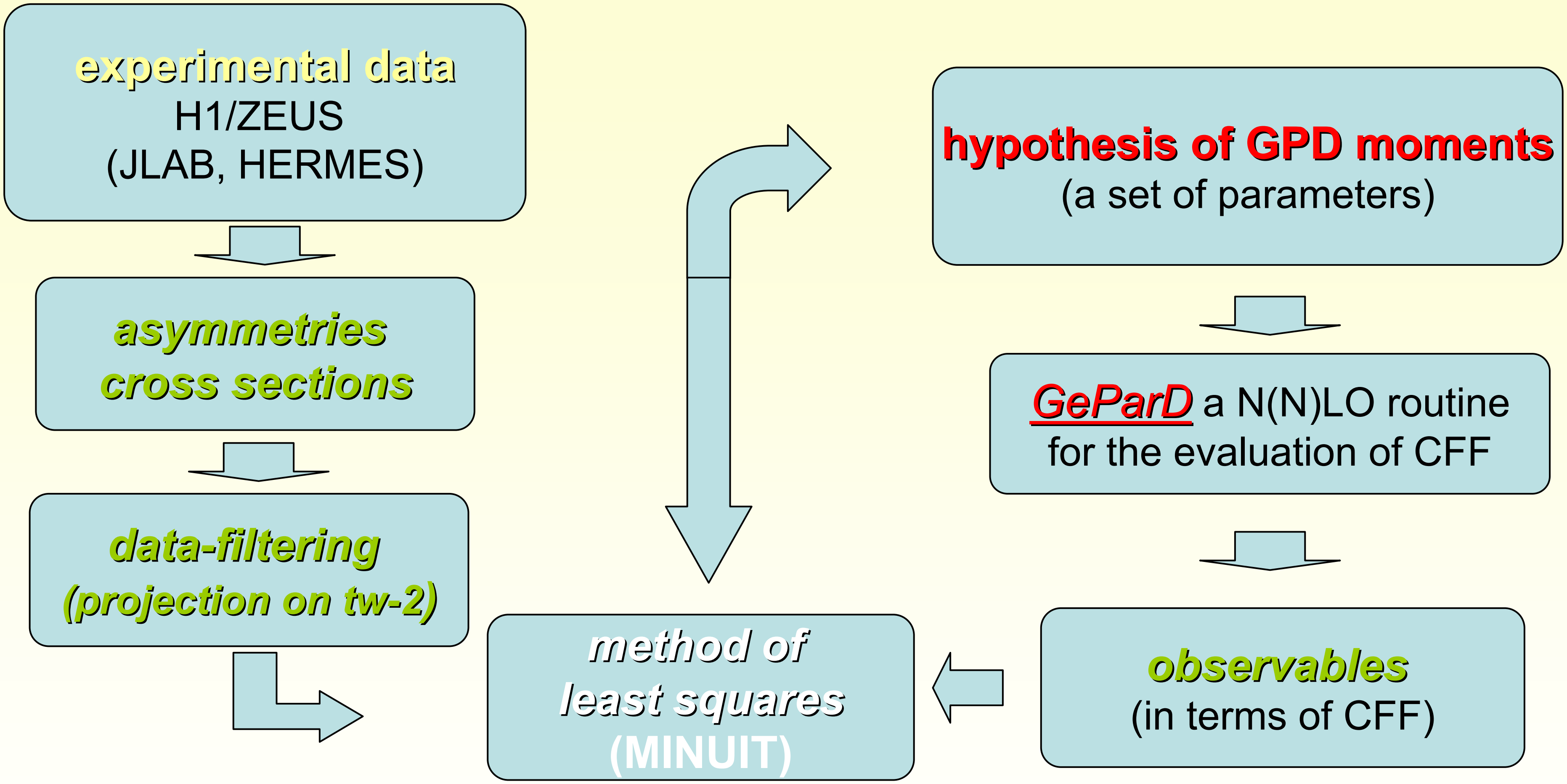
0. analytic formulae [BMK 01]

i. (almost) without modeling **[Guidal, Moutarde (08/09)]**
[KM, Vlah in preparation]

ii. dispersion integral fits **[KMP-K (08),KM (08/09)]**

iii. flexible GPD modeling **[KM (08/09)]**

Getting ready for flexible GPD model fits



- reasonable well motivated hypotheses of GPDs (moment) must be implemented
- switching to x-space representation to implement dynamical models
- many parameters – Is a least square fit an appropriate strategy?
- some technical, however, straightforward work is left (see Andrei`s talk)

DVCS fits for H1 and ZEUS data

DVCS cross section measured at small $x_{\rm Bj} \approx 2\xi = \frac{2\mathcal{Q}^2}{2W^2+\mathcal{Q}^2}$

$$40\mathrm{GeV} \lesssim W \lesssim 150\mathrm{GeV}, \quad 2\mathrm{GeV}^2 \lesssim \mathcal{Q}^2 \lesssim 80\mathrm{GeV}^2, \quad |t| \lesssim 0.8\mathrm{GeV}^2$$

predicted by

$$\frac{d\sigma}{dt}(W,t,\mathcal{Q}^2) \approx \frac{4\pi\alpha^2}{\mathcal{Q}^4}\frac{W^2\xi^2}{W^2+\mathcal{Q}^2}\left[|\mathcal{H}|^2 - \frac{\Delta^2}{4M_{\rm p}^2}|\mathcal{E}|^2 + \left|\widetilde{\mathcal{H}}\right|^2\right](\xi,t,\mathcal{Q}^2)\Big|_{\xi=\frac{\mathcal{Q}^2}{2W^2+\mathcal{Q}^2}}$$

suppressed contributions *<<0.05>>* relative *O(ξ)*

- @LO data could not be described before **2008**
- NLO works with ad hoc GPD models **[Freund, McDermott (02)]**

  results strongly depend on employed PDF parameterization

⇨ ***do a simultaneous fit to DIS and DVCS*** **[KMP-K (07)]**

➡ ***use flexible GPD models in a two-step fit*** **[KMP-K (08)]** *effective* functional form at small x:

PDFs: $$q^{\rm sea}(\xi,\mathcal{Q}) = n(\mathcal{Q})\xi^{-\alpha(\mathcal{Q})}, \quad \alpha\sim 1, \quad F^{\rm sea}(0)=1$$

GPDs: $$H = r(\eta/x=1,\mathcal{Q})F^{\rm sea}(t)\xi^{\alpha'(t,\mathcal{Q})}q^{\rm sea}(\xi,\mathcal{Q})$$

skewness **transverse distribution**

?

$$E(\xi,\xi,t,\mathcal{Q})$$

mostly not seen in Regge phenomenology, evidence **[Donnachie 05]**

chromo-magnetic "pomeron" might be sizeable (instantons) **[Diakonov 02]**

pQCD suggests `pomeron' intercept

$$J_q(\mathcal{Q}^2) = \frac{1}{2}(A+B)(\mathcal{Q}^2), \quad \left\{\begin{matrix} A \\ B \end{matrix}\right\} = \int_0^1 dx\, x \left\{\begin{matrix} H \\ E \end{matrix}\right\}(x,\eta,t=0,\mathcal{Q}^2)$$

Is the emerging angular momentum picture *with* $B_{u+d}\sim 0$ reliable?

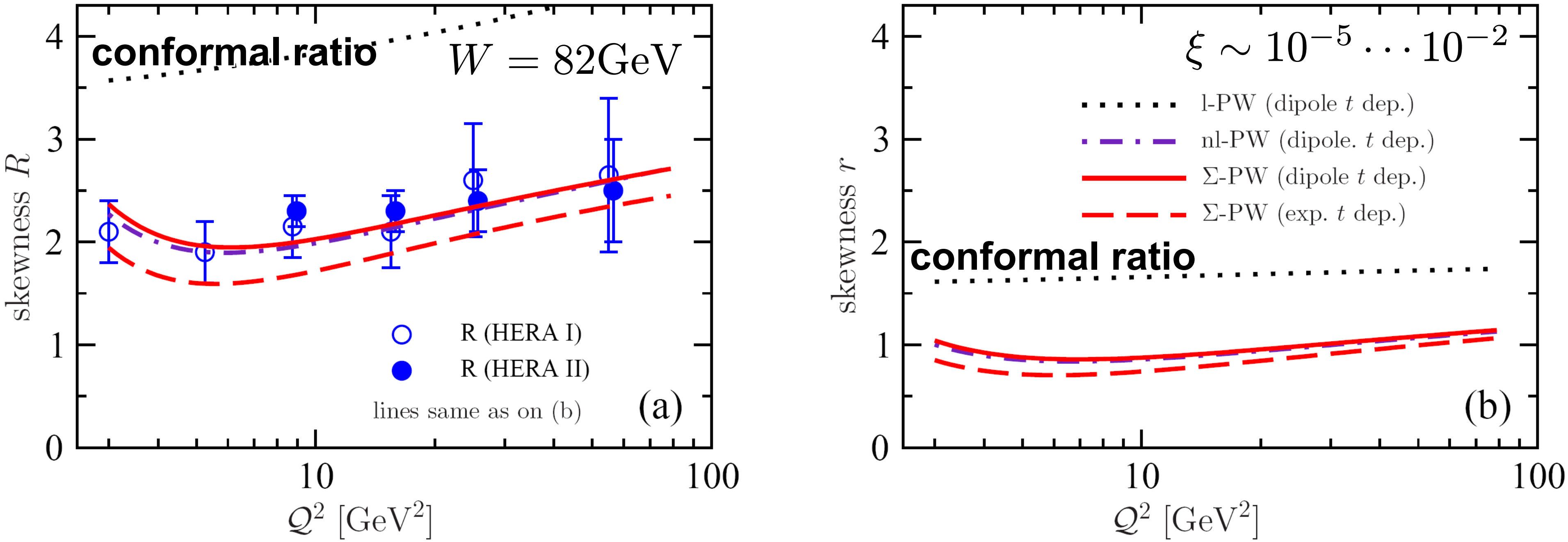
(? lattice contributions of disconnected diagrams, evolution, models are not dealing with partonic degrees of freedom)

qualitative understanding of *E* is needed (not only forJi`s spin sum rule)

$$B = \int_0^1 dx\, x E(x,\eta,t,\mathcal{Q})$$

quark skewness ratio from DVCS fits @ LO

$$R = \frac{\Im\mathrm{m}A_{\mathrm{DVCS}}}{\Im\mathrm{m}A_{\mathrm{DIS}}} \stackrel{LO}{=} \frac{H(\xi,\xi)}{H(2\xi,0)} \approx 2^{\alpha} r \qquad r = \frac{H(\xi,\xi)}{H(\xi,0)}$$



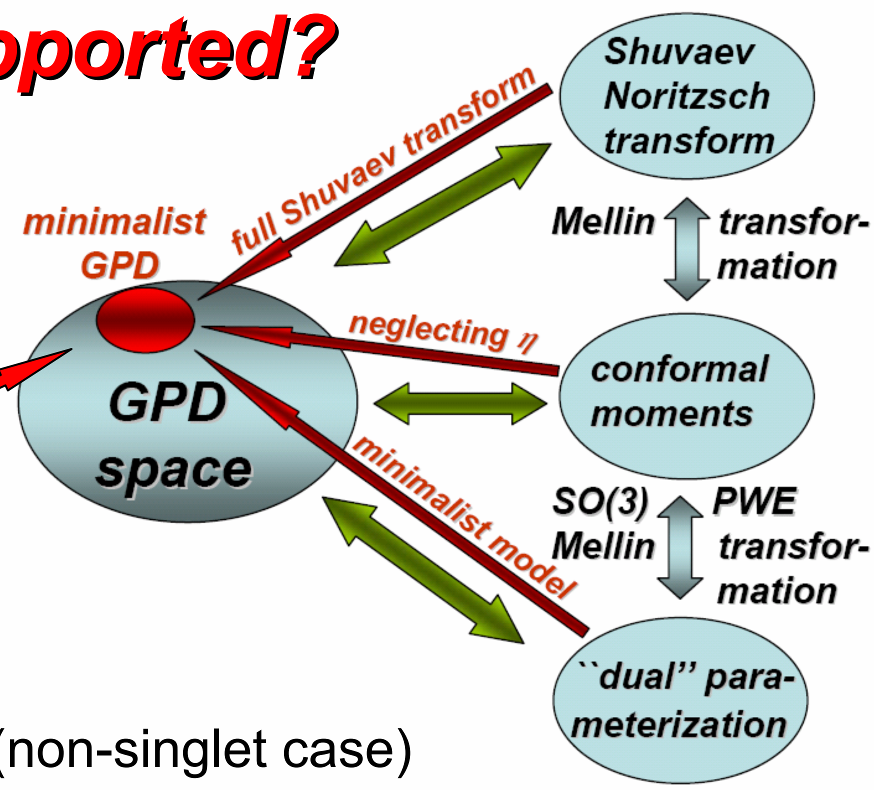
- @LO the conformal ratio $r_{\mathrm{con}} = \frac{2^{\alpha}\Gamma(3/2+\alpha)}{\Gamma(3/2)\Gamma(2+\alpha)}$ is ruled out for sea quark GPD
- a generic zero-skewness effect over a large Q^2 lever arm
- scaling violation consistent with pQCD prediction
- this zero-skewness effect is non-trivial to realize in conformal space (SO(3) sibling poles are required)

Is the conformal ratio supported?

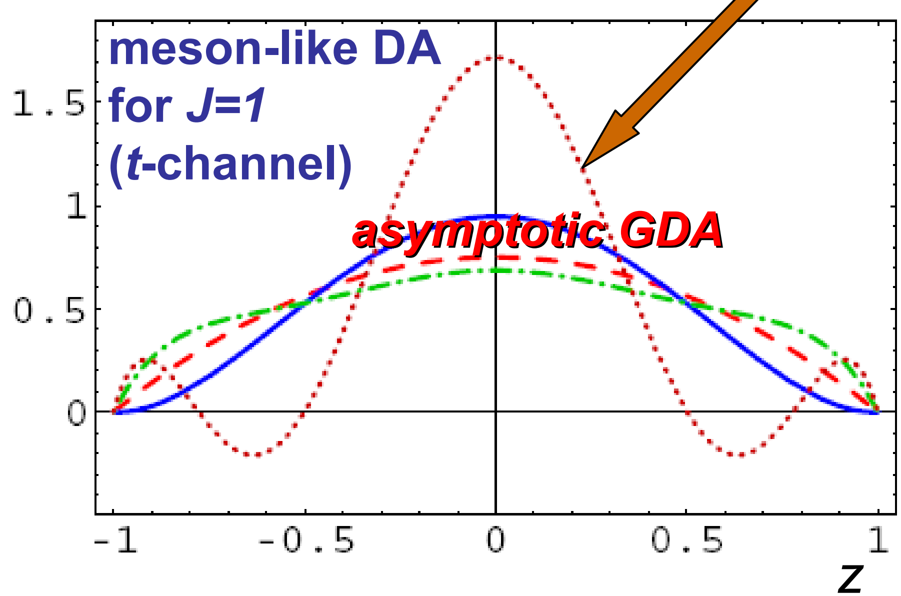
$$r = \frac{H(x,x,t=0,Q^2)}{q(x,Q^2)}$$

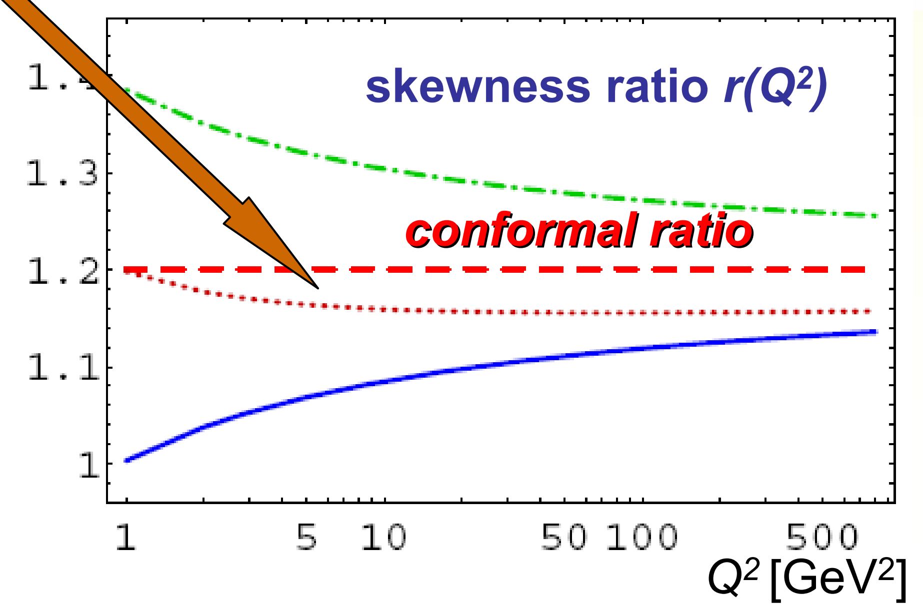
``erroneous small *x*-claim''

$$r_{\rm con} = \frac{2^{\alpha}\Gamma(3/2+\alpha)}{\Gamma(3/2)\Gamma(2+\alpha)}$$



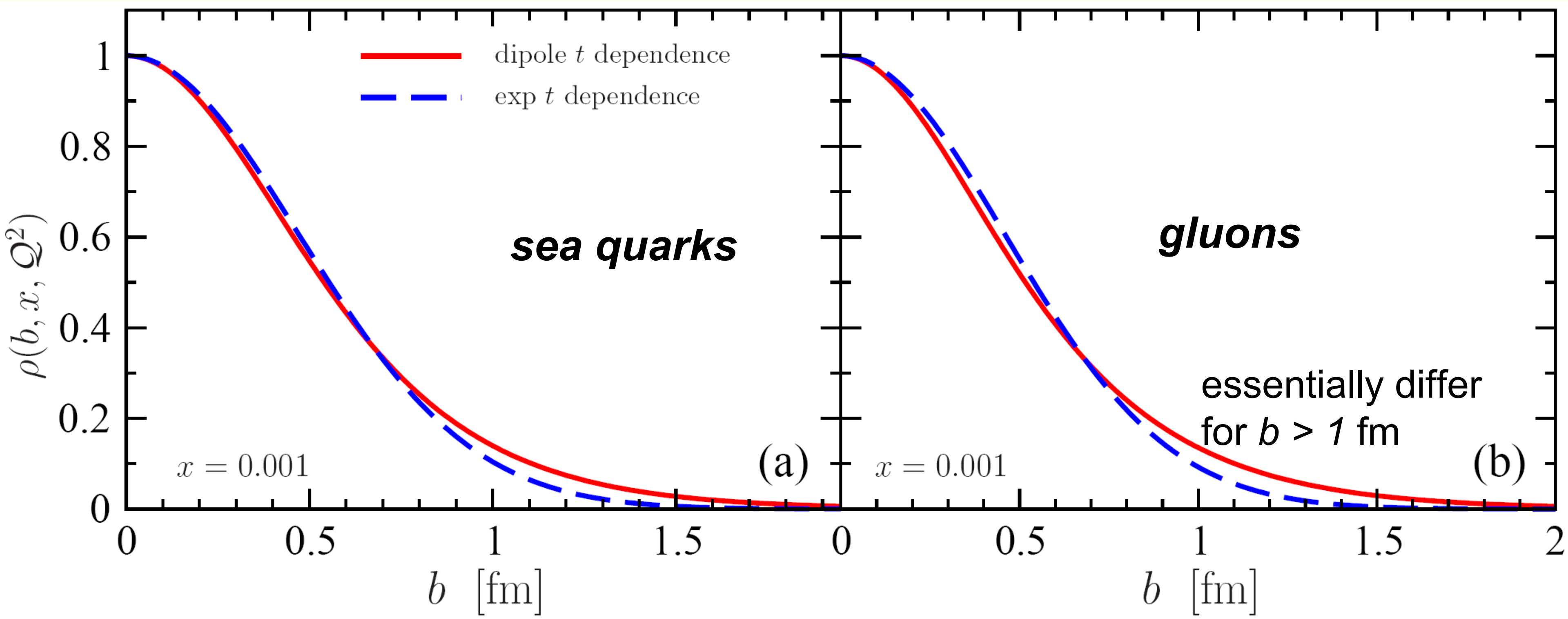
a **counter example** (non-singlet case)





- CFF H posses ``pomeron behavior'' $\xi^{-\alpha(Q) - \alpha'(Q)t}$
  - ✓ **α increases with growing** Q^2
  - ✓ **α' decreases growing** Q^2
- t-dependence: exponential — shrinkage is disfavored ($\alpha' \approx 0$)
  dipole — shrinkage is visible ($\alpha' \approx 0.15$ at $Q^2=4$ GeV^2)
- (normalized) profile functions

$$\rho \propto \int d^2\vec{\Delta}_\perp \, e^{i\vec{b}\cdot\vec{\Delta}_\perp} H(x,0,t=-\vec{\Delta}_\perp^2)$$



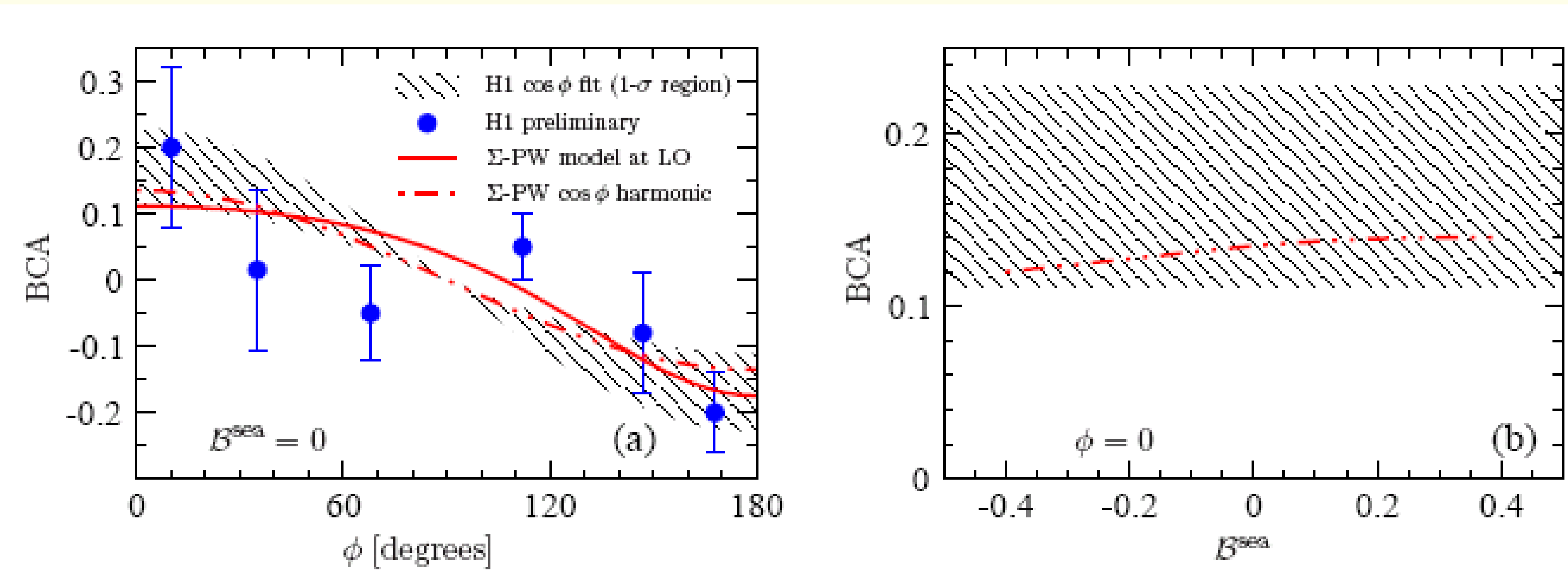
Beam charge asymmetry

$$BCA = \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} = \frac{\mathcal{T}_{\text{Interference}}}{|\mathcal{T}_{\text{BH}}|^2 + |\mathcal{T}_{\text{DVCS}}|^2}$$

$$\propto F_1(t)\Re\mathrm{e}\mathcal{H} + \frac{|t|}{4M^2} F_2(t)\Re\mathrm{e}\mathcal{E}$$

the unknown in Ji's nucleon spin sum rule

- set $E_{\text{sea}} \propto H_{\text{sea}}$, use *anomalous gravitomagnetic moment* $B_{\text{sea}} = \int_0^1 dx\, x E_{\text{sea}}$ as parameter



unfortunately, H1 data do not allow to access B_{sea}

Dispersion relation fits to unpolarized DVCS

- model of GPD *H(x,x,t) within DD motivated ansatz at* $Q^2=2$ GeV^2

fixed: *PDF normalization*, *eff. Reage pole*, *large t-counting rules*

$$H(x,x,t) = \frac{n\,r\,2^{\alpha}}{1+x}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}\,.$$

free: *r-ratio at small x*, *large x-behavior*, *p-pole mass*

sea quarks (taken from LO fits)

$$n=0.68,\quad r=1,\quad \alpha(t)=1.13+0.15t/\mathrm{GeV}^2,\quad m^2=0.5\mathrm{GeV}^2,\quad p=2$$

valence quarks

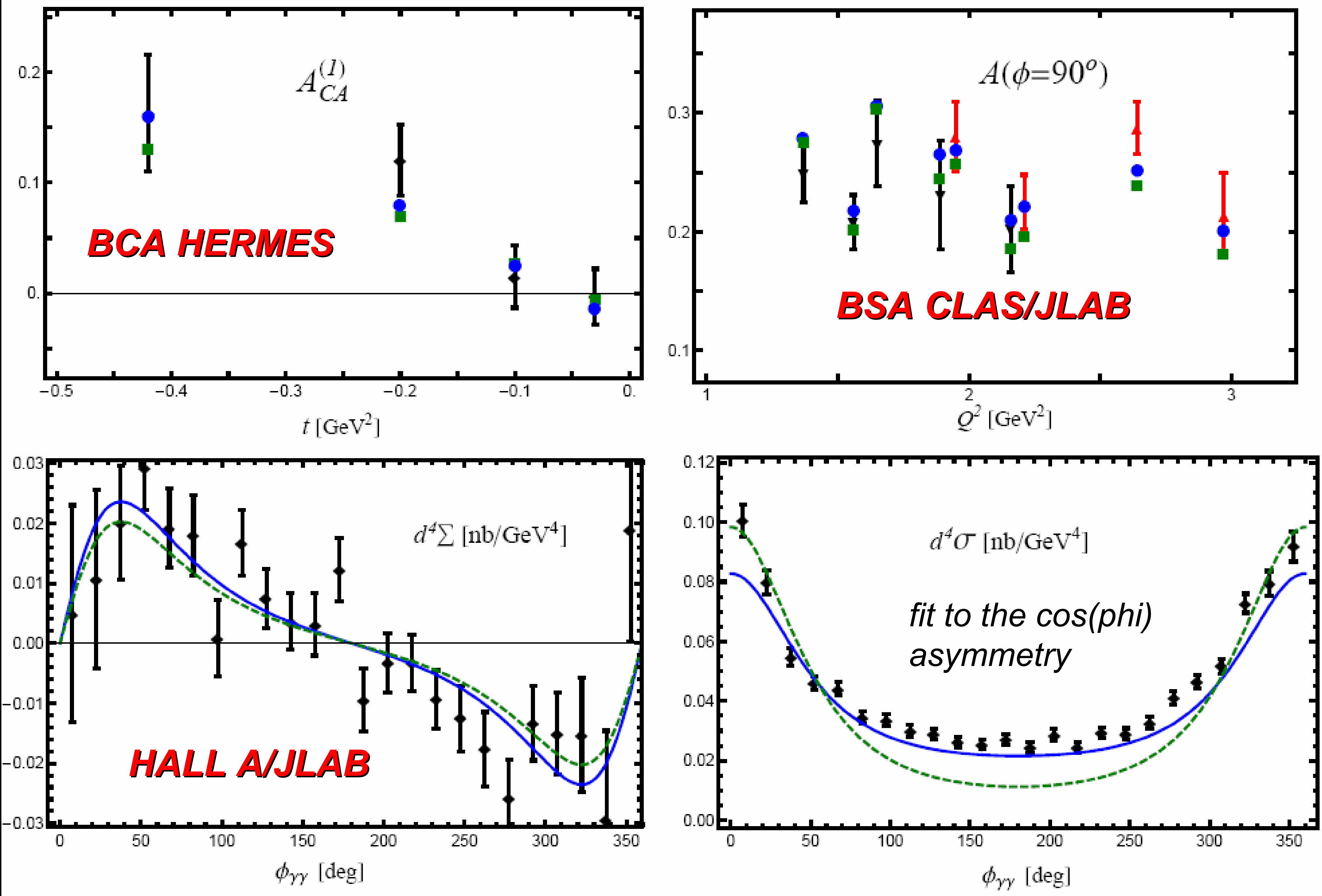
$$n=1.0,\quad \alpha(t)=0.43+0.85t/\mathrm{GeV}^2,\quad p=1$$

flexible parameterization of subtraction constant $\mathcal{D}(t)=\frac{-C}{(1-t/M_c^2)^2}$

+ pion-pole contribution

36 + 4 data points quality of ***global fit*** is good $\chi^2/\mathrm{d.o.f.}\approx 1$

Global GPD fit example: HERMES & JLAB



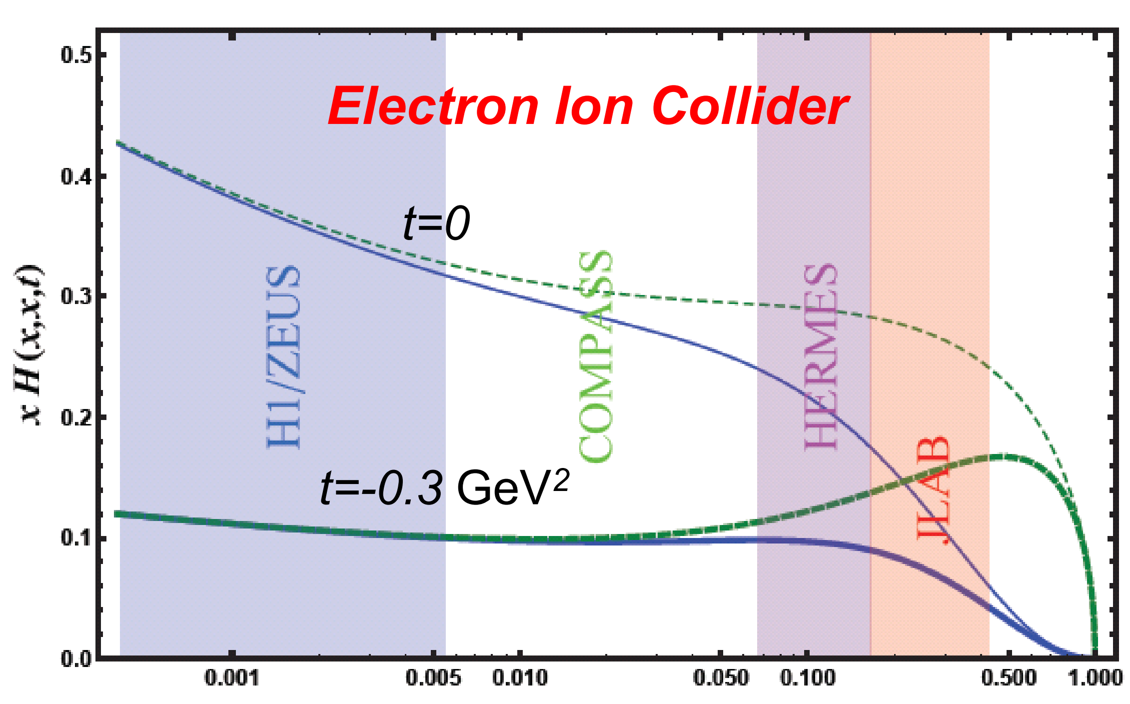
- extracting GPD from present collider and fixed target DVCS data

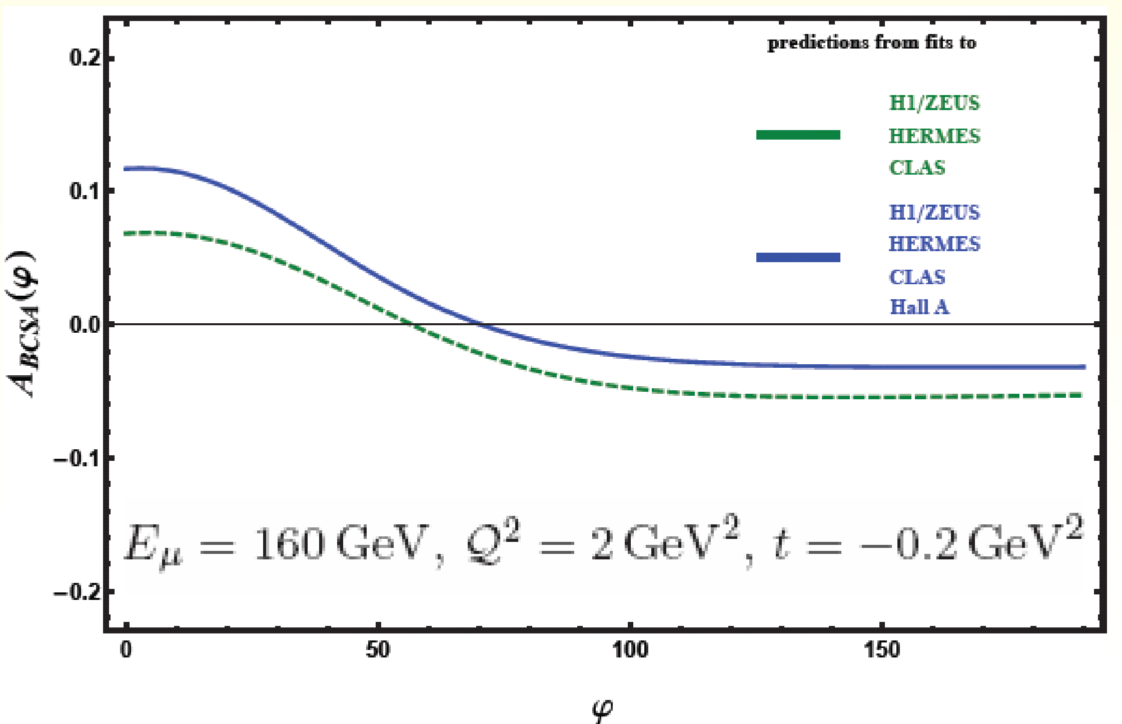
H(x,x,t,Q²=2 GeV²)

- subtraction constant/*D*-term is negative (as expected)

- prediction for COMPASS

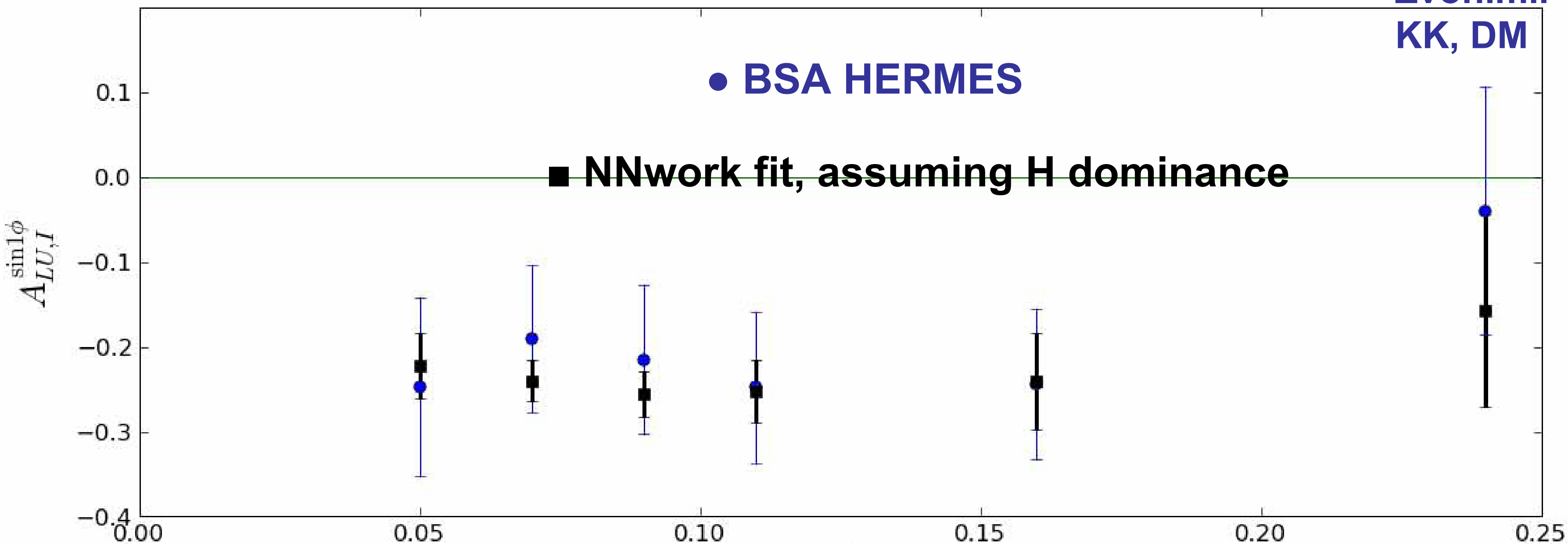
$$A_{\mathrm{BCSA}} = \frac{d\sigma^{\uparrow +} - d\sigma^{\downarrow -}}{d\sigma^{\uparrow +} + d\sigma^{\downarrow -}}$$

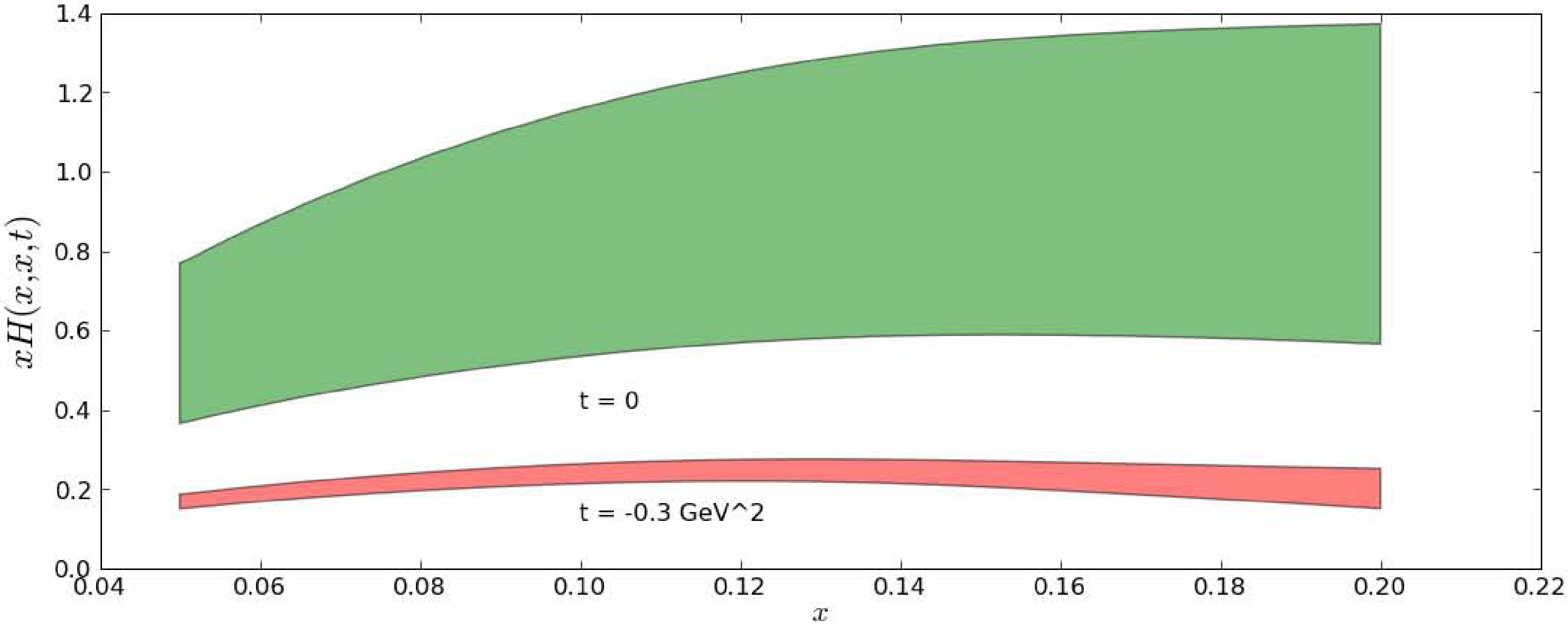




neural network: extraction of *H(x,x,t)* and error estimate

Zvonimir Vlah,
KK, DM





Hard exclusive meson production

vector meson production (σ_L/σ_T separation)

$$\frac{d\sigma_L^{\gamma^\star p \to V N}}{dt} \propto \frac{x_{\rm Bj}^2}{\mathcal{Q}^6}\left(|\mathcal{H}|^2 - \frac{t}{4M^2}|\mathcal{E}|^2 + \cdots\right)$$

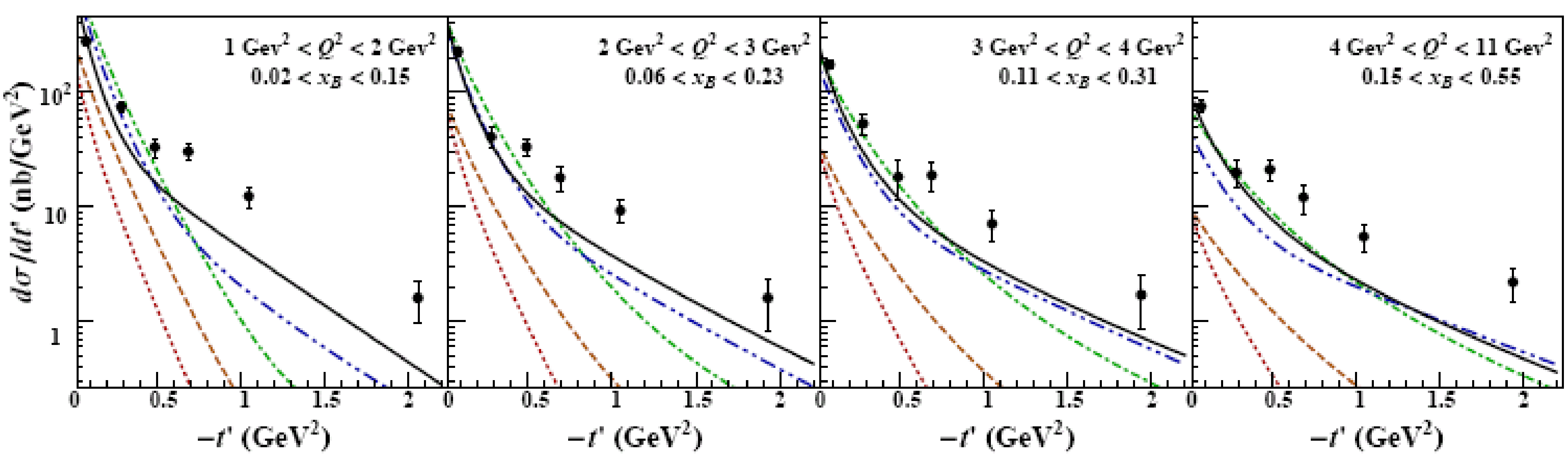
$$\mathcal{H} \sim x_{Bj}^{-1\cdots}$$
$$\mathcal{E} \sim x_{Bj}^{-?}$$

hard exclusive pion production (GPD $\hat{H}$ related to $\Delta q^{(3)}$)

$$\frac{d\sigma_L^{\gamma^* p \to \pi N}}{dt} \propto \frac{x_{\rm Bj}^2}{\mathcal{Q}^6}\left(|\widetilde{\mathcal{H}}|^2 - \frac{t}{4M^2}|\xi\widetilde{\mathcal{E}}|^2 + \cdots\right)$$

$$\widetilde{\mathcal{H}} \sim x_{Bj}^{-?}$$
$$\widetilde{\mathcal{E}} \sim \pi - \text{pole}$$

HERMES: differential cross section versus various GPD models



Summary

GPDs are intricate and (thus) a promising tool

- to reveal the transverse distribution of partons
- to address the spin content of the nucleon
- providing a bridge to non-perturbative methods (e.g., lattice)

hard exclusive leptoproduction

- possesses a rich structure, allowing to access various CFFs/GPDs
- it is elaborated in NLO and offers a new insight in QCD
- DVCS is widely considered as a theoretical clean process
- covering the kinematical region between HERA/HERMES/COMPASS and JLAB and future experiments (high luminosity and dedicated detectors) is needed to quantify exclusive and inclusive QCD phenomena

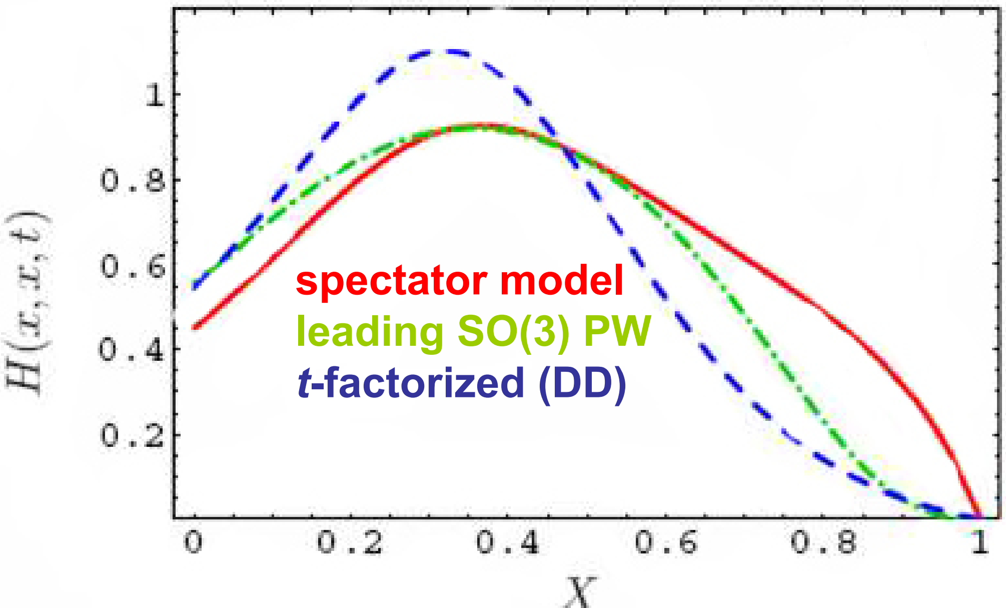
"next generation" tools/technology for global fits are desired:

to quantify the partonic picture and to get a better QCD understanding

Back up slides are coming

(partonic) `quantum' numbers in GPD representations

name	's-channel' variable	't-channel' variable
GPD	PMF x	PMF ratio η
DD	PMF y	PMF z
CPWE	conformal spin $j+2$	PMF ratio η
'forward-like' CPWE	forward-like PMF z	PMF ratio η
Mellin-Barnes CPWE	conformal spin $j+2$	PMF ratio η
'dual' CPWE	forward-like PMF z	$\rho = j+2-J$
'dual' Mellin-Barnes CPWE	conformal spin $j+2$	t-channel AM J
SO(3)-PWE	PMF x	t-channel AM J

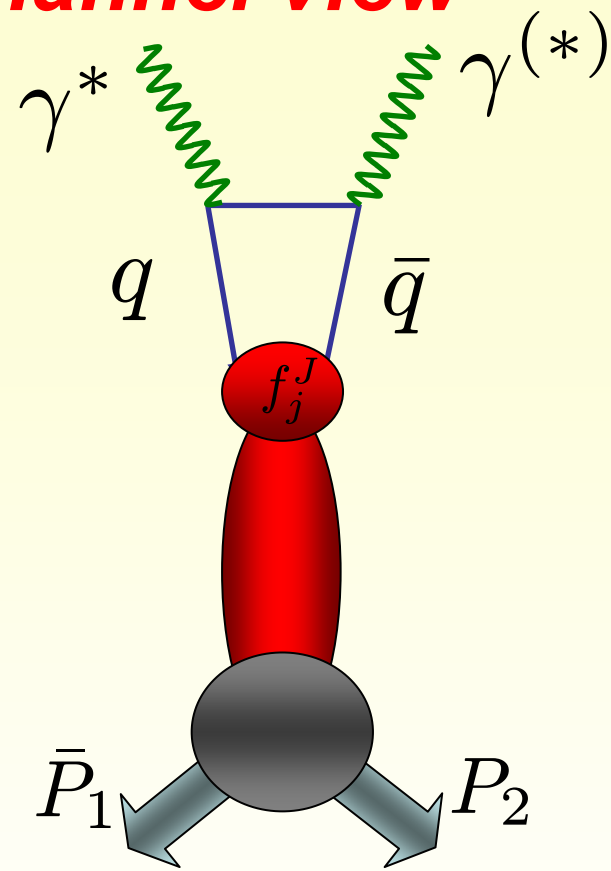


? about representation is not so essential

should be replaced by

How a GPD looks like on its cross-over trajectory ?

GPD ansatz at small x from t-channel view



- at short distance a quark/anti-quark state is produced, labeled by ***conformal spin*** *j*+2
- they form an intermediate mesonic state with total angular momentum ***J***
  strength of ***coupling*** is $f_j^J,\ J \leq j+1$
- mesons propagate with $\frac{1}{m^2(J)-t} \propto \frac{1}{J-\alpha(t)}$
- decaying into a nucleon anti-nucleon pair with given angular momentum *J*, described by an ***impact form factor***

$$F_j^J(t) = \frac{f_j^J}{J-\alpha(t)} \frac{1}{(1-\frac{t}{M^2(J)})^p}$$

! GPD *E* is zero if chiral symmetry holds
(partial waves are Gegenbauer polynomials with index 3/2)

D-term arises from the SO(3) partial wave *J*=*j*+1 (*j* → -1)