Fixed Angle Scattering and the Transverse Structure of Hadrons

Exclusive Reactions at High Momentum Transfer

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- Some classic results and
- Recalling some work with J. Botts and M. Sotiropoulos.
- Primarily on hadron-hadron reactions, but with implications for photon-induced processes.

I. Quark counting, the valence state and geometric counting

II. Splitting the hard scattering (Landshoff)

III. The return of (approximate) parton counting at wide angles

IV. Exchanging quarks and ratios of particle-antiparticle to particle-particle elastic scattering

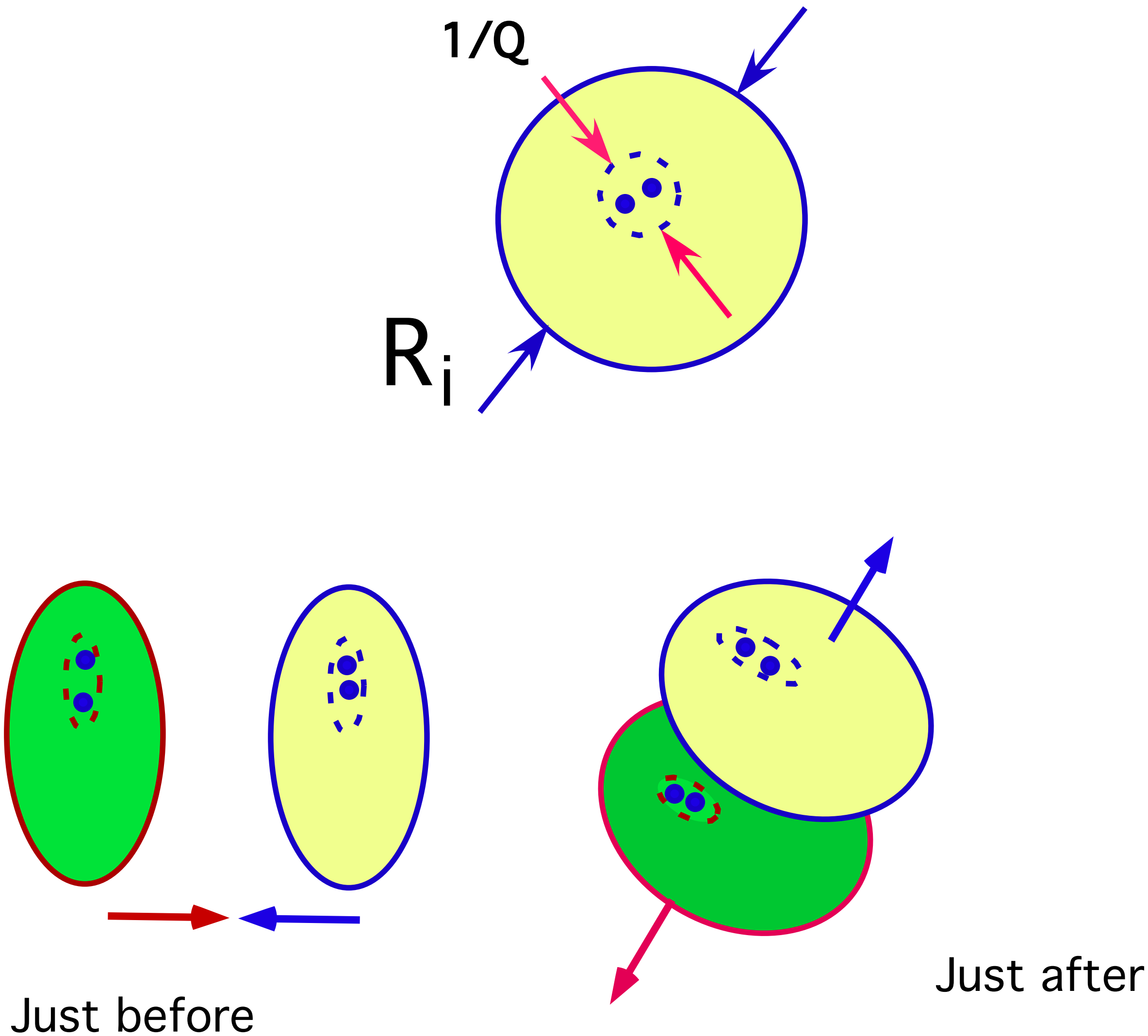
V. Conclusions

I. Quark counting, the valence state and geometric counting

- **Parton model applied to high-energy elastic scattering** (1973: Brodsky, Farrar; Matveev, Muradyan, Tavkhelidze)
- **Elastic scattering is through the valence state:**
  - Parton picture: in c.m., wave functions are Lorentz-contracted.
  - large t requires all n_i valence (anti-)quarks of hadron i in a region of area $1/Q^2$ for both incoming hadrons.
  - Likelihood is $\sim \left(\frac{1}{Q^2} \times \frac{1}{\pi R_H^2}\right)^{n_H - 1}$ for each hadron.
  - Geometric picture: Must be true of both incoming and outgoing states, for overlap of wave functions.
  - Scaling: assume that otherwise the amplitude is a function only of the scattering angle.
  - The result, at fixed s/t (c.m. scattering angle):

$$\frac{d\sigma}{dt} = \frac{f(s/t)}{s^2} \left(\frac{m^2}{s}\right)^{\Sigma_{i=1}^{4}(n_i - 1)}$$

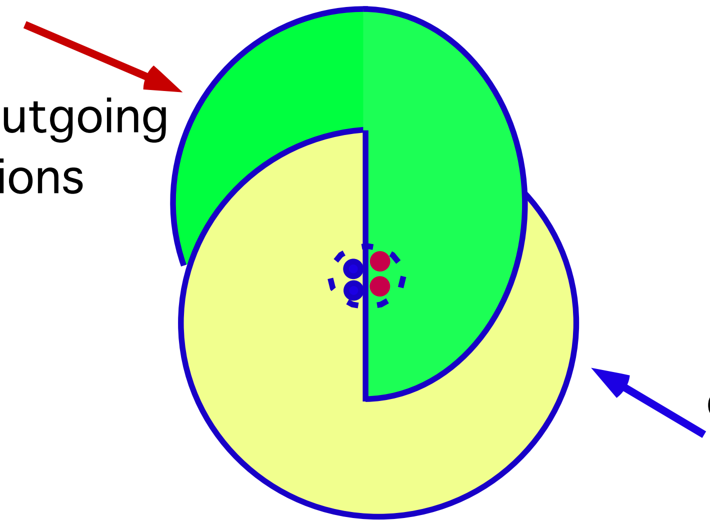
How it looks:



And also:

Quark counting picture just at the moment of collision
for mesons

overlap of outgoing
wave functions



overlap of incoming
wave functions

- **The corresponding elastic amplitude**

(1979: Brodsky and Lepage, Efremov and Radyushkin)

$$\mathcal{M}(s,t;h_i) = \int \prod_{i=1}^{4} [dx]\, \phi(x_{m,i}, \lambda_{m,i}, h_i; \mu) \times M_H\left(\frac{x_{n,i} x_{m,j} p_i \cdot p_j}{\mu^2}; \lambda_{n,i}, h_i\right)$$

with factorized & evolved valence (light-cone) wave functions $\phi(x_{m,i}, \lambda_{m,i}, h_i; \mu)$, and with

$$[dx] = dx_{1,i} dx_{2,i} dx_{3,i}\, \delta\left(1 - \sum_{n=1}^{3} x_{n,i}\right)$$

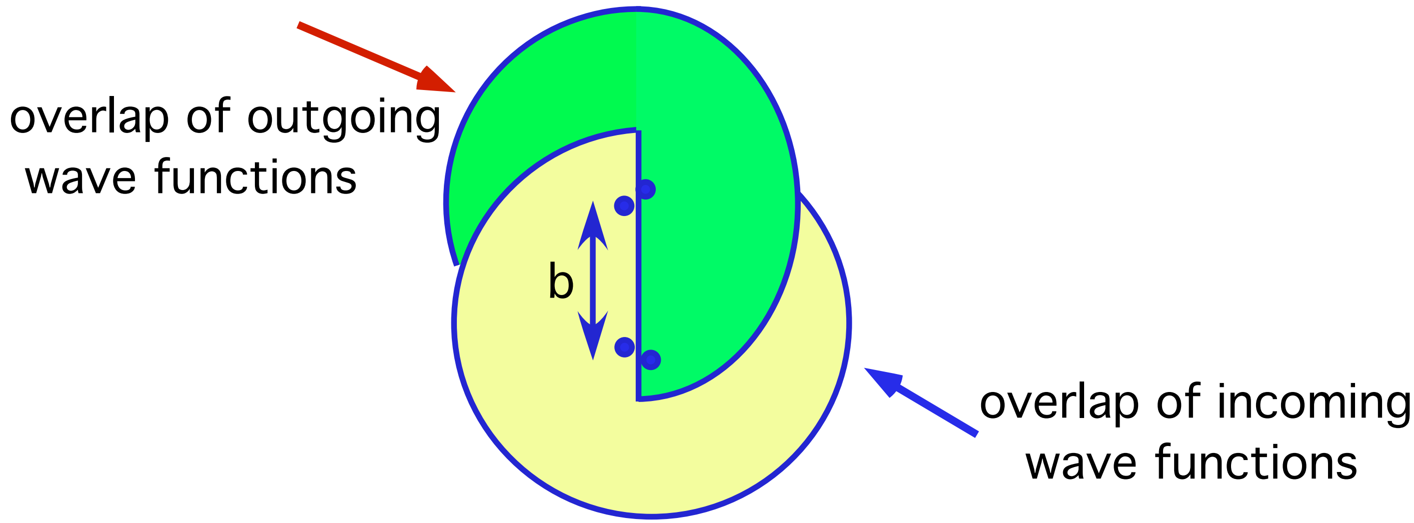
for helicities: h_i (hadron) $\lambda_{n,i}$ (quarks)

- **In principle straightforward, but:**
  - **For NN scattering, M_H is thousands of diagrams even at tree level, although with recent advances, $3 \to 3$ should be manageable.**
  - **Knowledge of the wave functions is incomplete.**
  - **Soft effects at higher orders are not under full control here.**

    (Duncan and Mueller, 1979 and see below)

II. Splitting the hard scattering (Landshoff)

Two independent scatterings for meson-meson scattering



- $1/Q \to R_H$ in amplitude $\Rightarrow 1/s \to R_H^2$ in cross section.
- This geometric configuration gives for NN at fixed angle (s/t) (1974: Landshoff)

$$\frac{d\sigma}{dt} = \frac{f(s/t)}{s^2} \left(\frac{1}{s\,\pi R_H^2}\right)^6$$

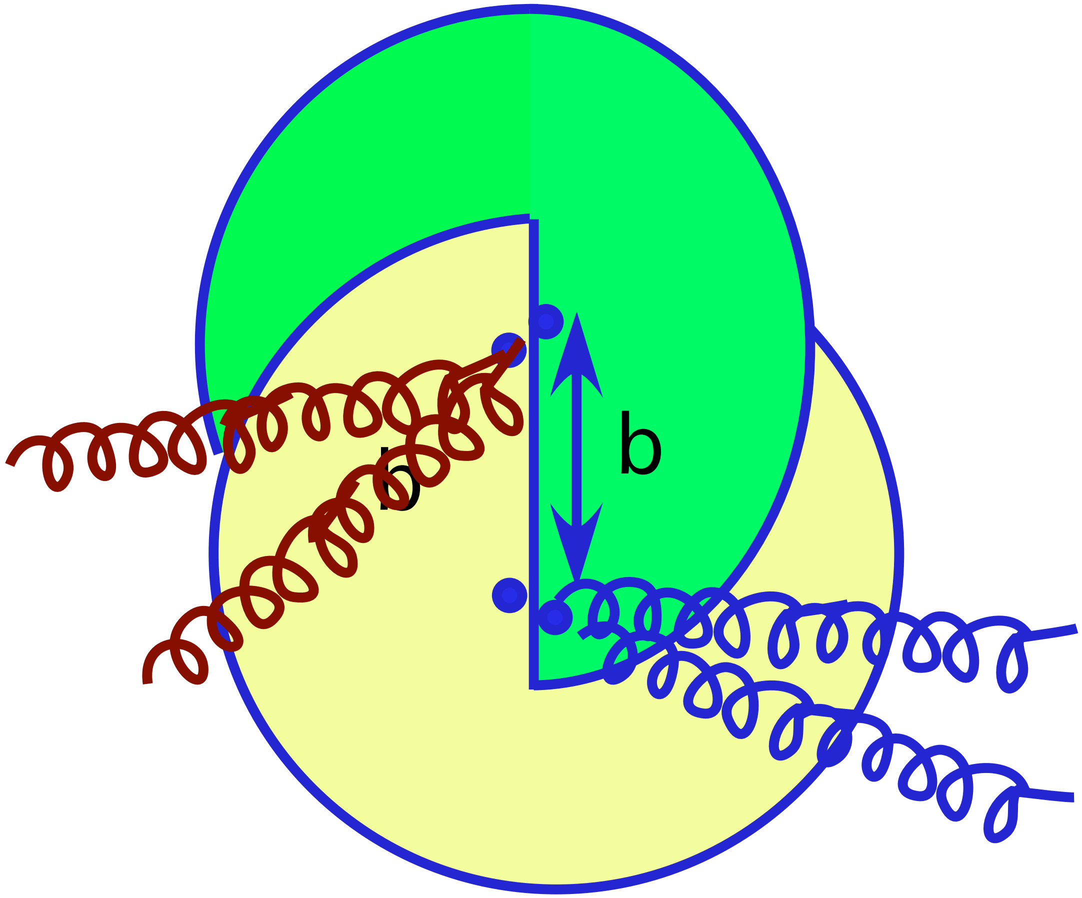
- And for $s \gg -t \gg \Lambda_{\rm QCD}$ it gives

$$\frac{d\sigma}{dt} = \frac{F(s)}{t^2} \left(\frac{1}{t\,\pi R_H^2}\right)^6$$

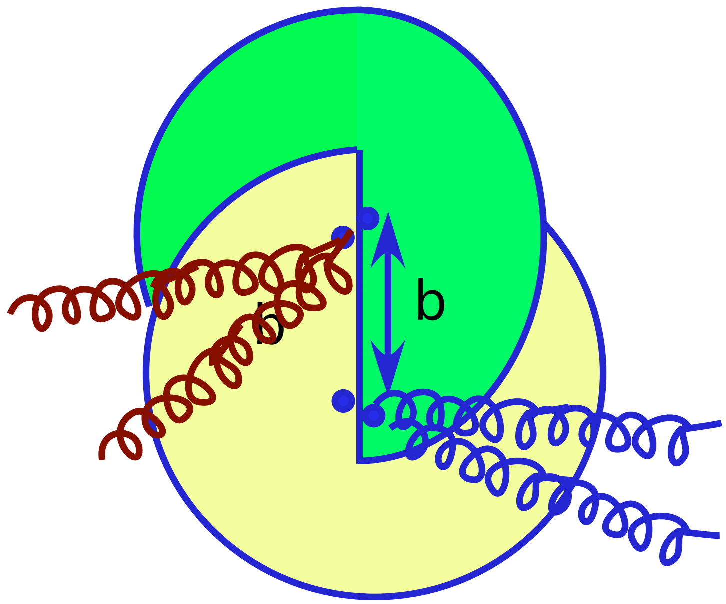
- Experiment: the latter works, the former doesn't.

III. The return of (approximate) parton counting at wide angles

- **Resolution of the single/triple scattering ambiguity in radiation:**

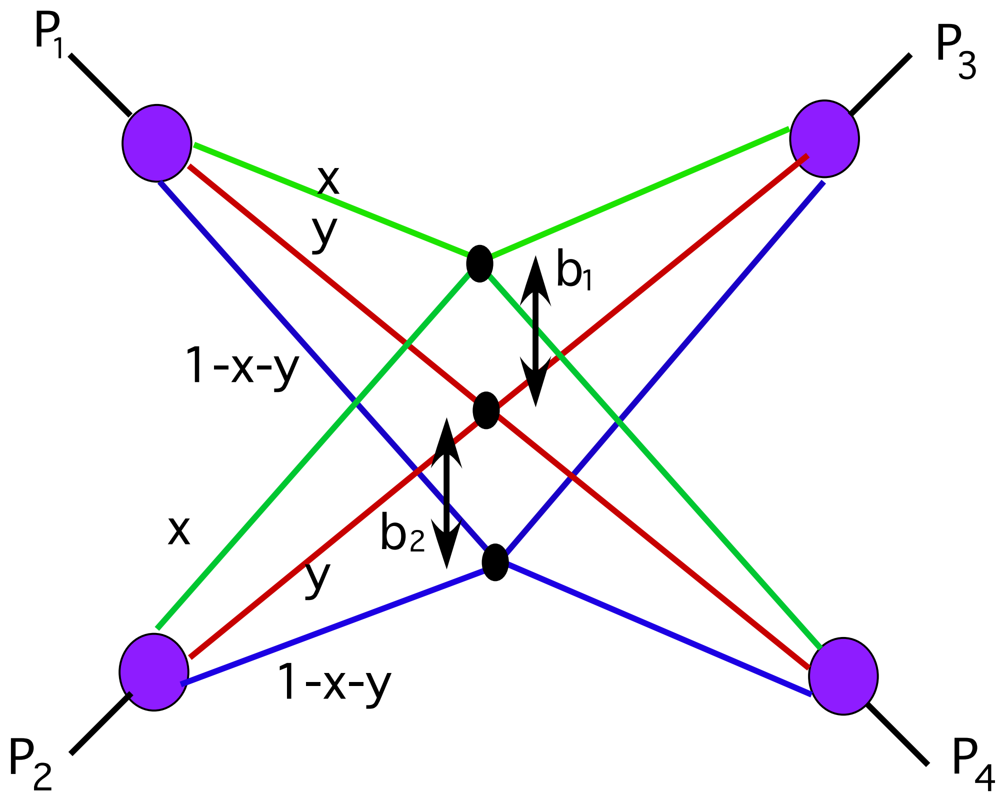


Scattering of isolated color charges wants to produce radiation in the incoming and outgoing directions. Configurations without such radiation are suppressed unless b is small. The full amplitude is the result of a competition between geometric enhancement and radiation suppression.



- **b is conjugate to $Q = \sqrt{-t}$. At $-t$ increases toward s, radiative corrections force the b's to $1/\sqrt{s}$ and geometric picture should be recovered approximately.**

  ($\sim$ 1980: Brodsky, Lepage; Mueller; Landshoff, Pritchard)



- Color-singlet hadrons $\Rightarrow$

$$\mathcal{M}(s,t) = \frac{N}{stu} \sum_f \int_0^1 \frac{dx\,dy}{x^2y^2(1-x-y)^2}$$
$$\times \int db_1 db_2 \, \mathrm{Tr}_{\mathrm{color}} \left[U(b_i Q) M^1 M^2 M^3 \right]$$
$$\times \prod_{i=1,2,3,4} \Psi_{H_i}(x, y, b_1, b_2)$$

- The Trace $\left[U(b_i Q) M^1 M^2 M^3 \right]$ ties color together and includes ϵ_{abc} for colors of three quark, with possible color exchange in each hard scattering $M^i(x_i p_j)$.

- **The wave functions behave as**

$$\Psi(x,y,b_i) \sim \Psi_{NP}(x,y,b_i)\exp[-\ln^2(1/Qb_i)]$$

$$\to \phi_{asy}(x_j)\exp[-\ln^2(1/Qb_i)]$$

- **This gives the asymptotic amplitude, an example of "Sudakov resummation":**

(1979: Botts and Sterman)

$$\mathcal{M}(s,t) = \frac{N}{stu}\sum_f \int_0^1 \frac{dxdy}{x^2y^2(1-x-y)^2} \prod_{i=1,2,3,4} \phi_{i,asy}(x,y)$$

$$\times \int db_1 db_2 \, \mathrm{Tr}_{\mathrm{color}}\left[U(b_iQ)M^1M^2M^3\right]$$

$$\times\ e^{-S_1(b_iQ)-S_2(b_iQ)-S_3(b_iQ)}$$

- At large Q for each scattering, radiation suppression drives the hard scatterings back together.

- At moderate $(xQ)^2$, $(yQ)^2$, amplitude is dominated by the "boundary conditions," $\Psi_{NP}(x, y, b_i)$ rather than asymptotic behavior.

IV. Exchanging quarks and ratios of particle-antiparticle to particle-particle elastic scattering

- All this applies to NN, $\bar{N}N$, etc.
- Computations are simpler for the "triple scattering" picture, and can be compared.
- Early on, contrast was made between gluon and quark exchange processes. pQCD factorization has both.

  (Ramsey and Sivers, 1992, after Gunion, Blankenbecler, Brodsky, 1973)

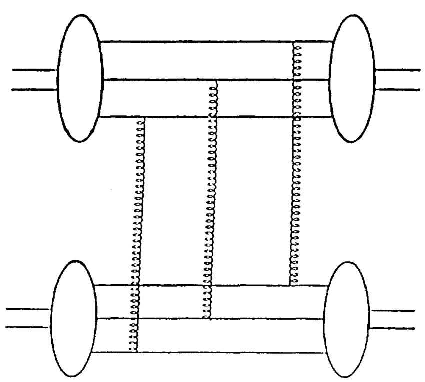


FIG. 2. Landshoff diagram for fixed-angle large-s NN scattering.

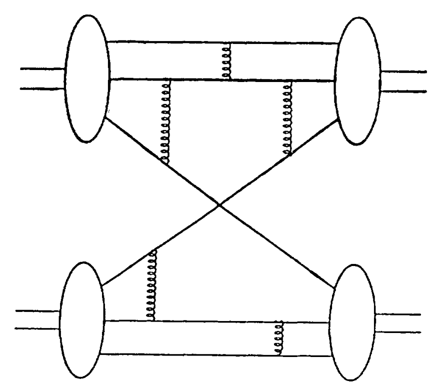


FIG. 3. Typical diagram for the quark-interchange mechanism in exclusive NN scattering.

- Quark exchange, of course, is not possible for $p\bar{p} \to p\bar{p}$. For pp there are 2^3 ways of connecting incoming and outgoing quarks compared to only one for $p\bar{p}$.
- The BNL experiments: ratios seems roughly consistent with this counting!

$$R_N = \frac{\frac{d\sigma_{N\bar{N}}}{dt}}{\frac{d\sigma_{NN}}{dt}}|_{90 \text{ deg}} \sim \frac{1}{40}$$

- Caveat in any pQCD picture – how to reform an antisymmetric color combination of quarks when they are exchanged?
- Sotiropoulos (1996) studied this issue in the Landshoff scattering picture. He found that it works qualitatively only with a "color randomization" picture in which the factor $\left[U(bQ)\,\Pi_i\,M^i\right]$ is independent of the flavor flow. He found $R_p \sim 1/30$ at ninety degrees with color randomization, $\sim 1/3$ without.

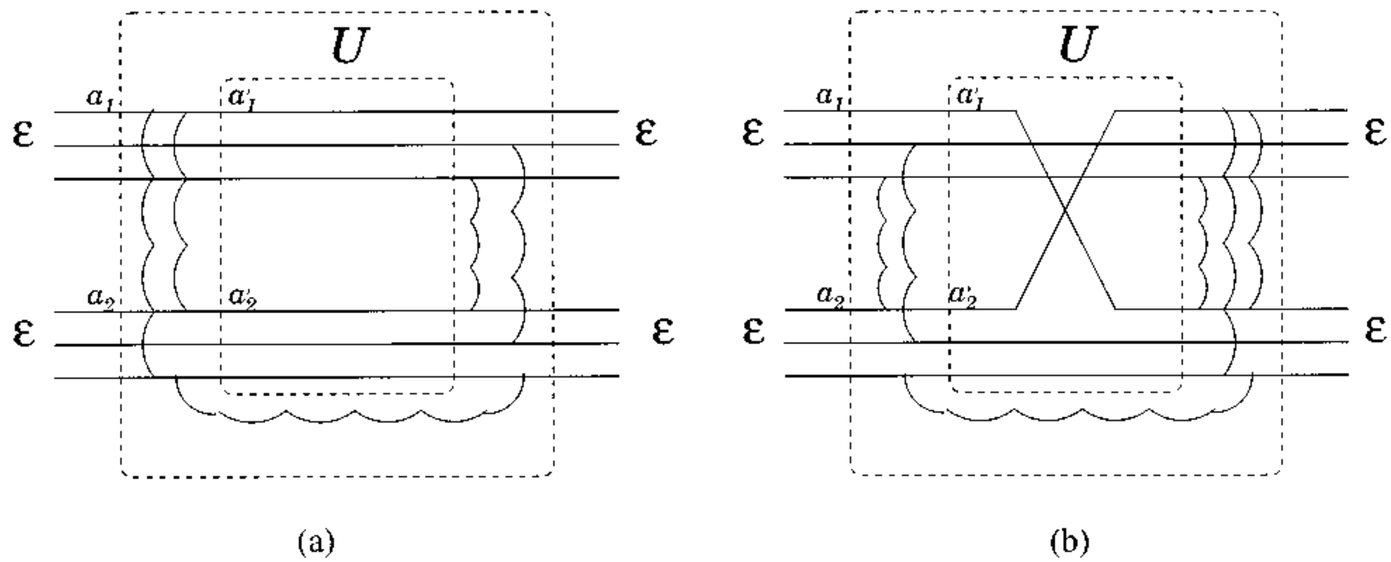


FIG. 3. Soft gluon exchange and color mixing for the direct (ttt), (a), and the single interchange (utt) channel, (b), in baryon-baryon elastic scattering. Hard gluons are not shown. Interpreted as color graphs, these diagrams represent contributions to U_{222}, (a), and U_{211}, (b).

- **Randomization possibly natural at moderate BNL energies, $\sqrt{s} \sim 3.5$ GeV^2, (Blazey *et al.*, Carroll *et al.* 1998, White *al.* 1994) and easy to picture in the context of quark exchange.**

- **A decrease of R_{pp} with energy would be a compelling signal for an emerging role for color.**
- **The Landshoff/Sudakov model with or without randomizaiton gives explicit predictions for angular and helicity dependence.**
- **Asymptotic, color-randomized and "data fit" curves**

(Sotiropoulos, 1996, data fit Farrar, Wu, 1975)

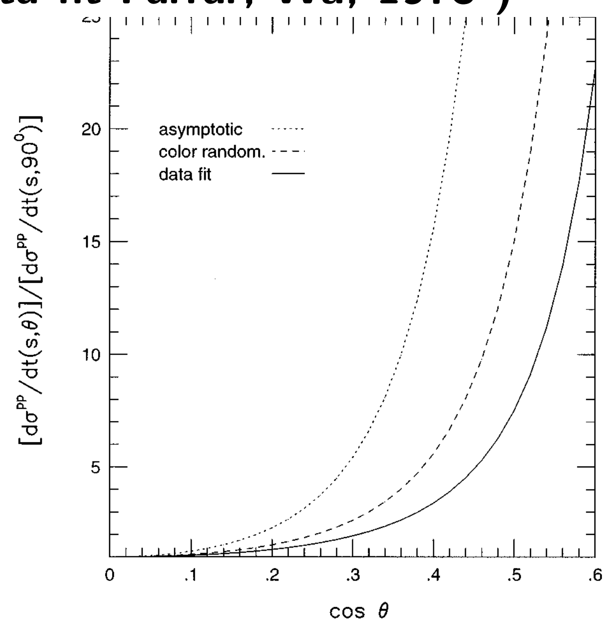


FIG. 4. Angular distribution for proton-proton elastic scattering. The data fit is from Ref. [19].

Conclusions

- **Wide angle elastic scattering is well-understood at "asymptotic" energies. Its energy-dependence reflects exchange of relevant degrees of freedom.**
- **We can learn of the applicability of the formalism, and much more, by comparing NN to $\bar{N}N$ elastic *and* to the production of hyperons over a wider range of energies and angles.**
- **Similarly for the comparison of analogous patterns for meson-meson, meson-baryon and photon-hadron reactions.**
- **Recent advances in tree-level scattering amplitudes may make previously unthinkable calculations possible.**
- **Duality-based insights may have shed new light on valence light-cone wave functions.**

  (Brodsky, de Teremond; Grigoryan, Radyushkin recent)

- **It's a good time to revisit large momentum transfer elastic scattering in a time of expanding capabilities in experiment and theory.**