# Refined analysis of DVCS

Andrei Belitsky

*Arizona State University*

Based on AB and D. Mueller
0809.2890 and (to appear)

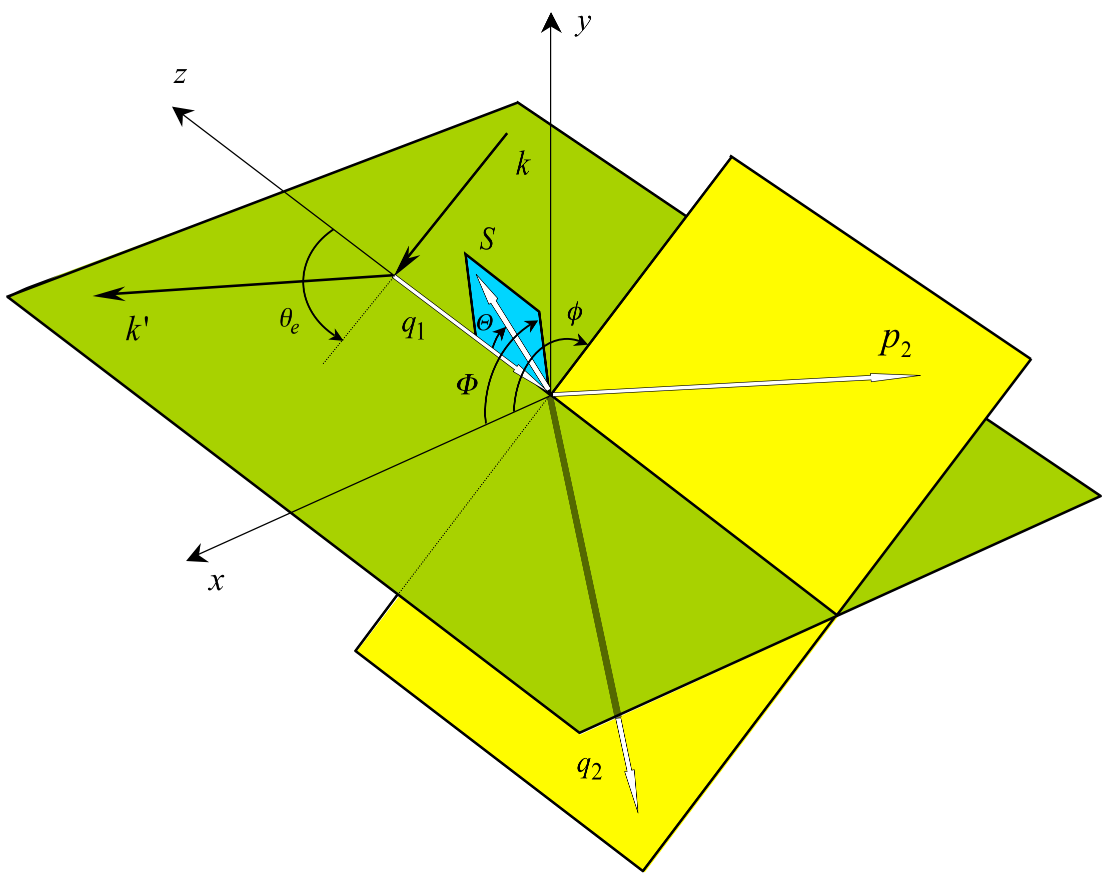
# Electroproduction of photons

$$e(k)h(p_1) \to e(k')h(p_2)\gamma(q_2)$$

Differential cross section:

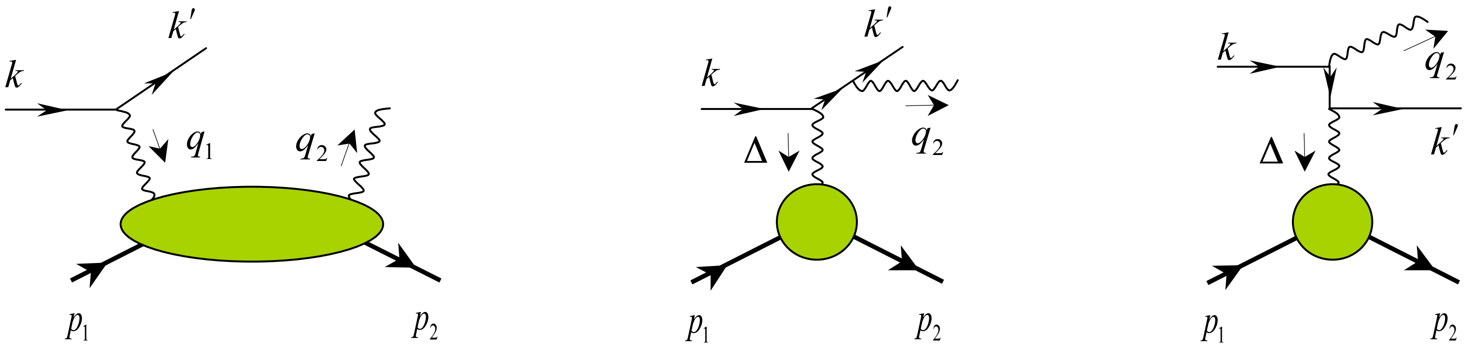
$$d\sigma = \frac{\alpha^3 x_B y}{8\pi\, Q^2 \sqrt{1+\epsilon^2}} \left|\frac{\mathcal{T}}{e^3}\right|^2 dx_B dy d|t| d\phi$$

$$\epsilon \equiv 2x_B \frac{M}{Q}$$




- Bjorken variable: $x_B = \frac{Q^2}{2p_1 \cdot q_1}$
- Lepton energy loss: $y = \frac{p_1 \cdot q_1}{p_1 \cdot k}$
- Momentum transfer: $t = \Delta^2$
- Photon mass: $Q^2 = -q_1^2$
- Scattering angle: $\phi$

# Building blocks




$$\mathcal{T}^2 = |\mathcal{T}^{\mathrm{BH}}|^2 + |\mathcal{T}^{\mathrm{DVCS}}|^2 + \mathcal{I}$$

- Squared Bethe-Heitler:

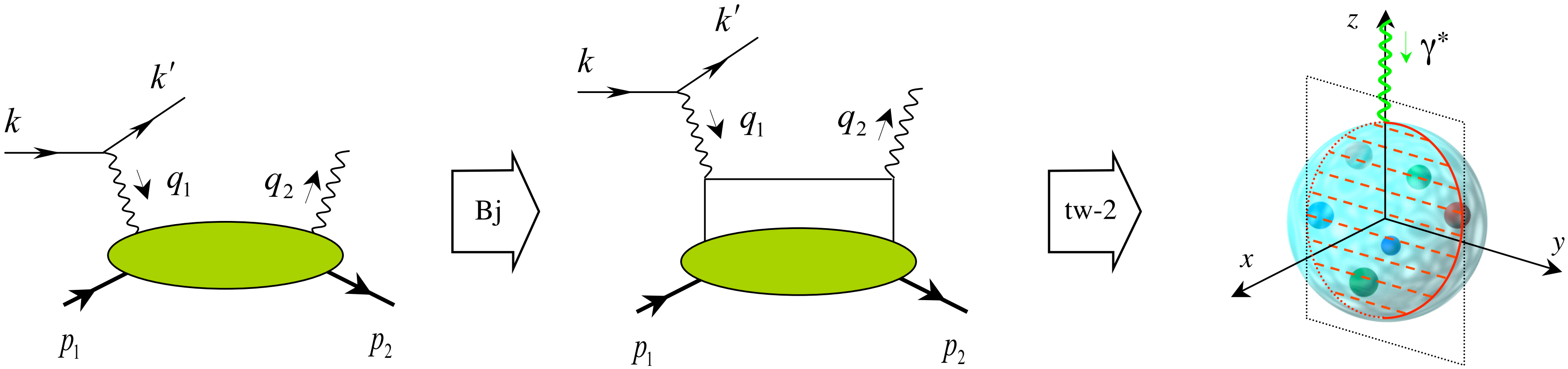
  calculated exactly

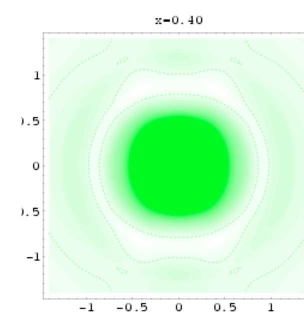
- Squared DVCS and interference amplitude:

  calculated to twist-3 accuracy

# Light-cone dominance

Quantum mechanical incoherence of physical processes at short and large distance scales imply factorization:



Hadronic part of the Compton amplitude is computed to twist-3 accuracy:

$$T = \mathrm{CFF}_{\tau=2} + \frac{1}{Q}\mathrm{CFF}_{\tau=3} + \ldots$$
$$= C_{\tau=2} * \mathrm{GPD}_{\tau=2} + \frac{1}{Q} C_{\tau=3} * \mathrm{GPD}_{\tau=3} + \ldots$$

BKM'01 framework: approximation of the leptonic tensor to leading and first subleading terms in $1/Q$ expansion; this yields matching expansions for both leptonic and hadronic parts of the amplitude in inverse powers of the hard scale.

# CFFs and Fourier harmonics

Within the systematic $1/Q$ expansion, there is a one-to-one correspondence between Fourier harmonics and twist of contributing CFFs.

- Squared DVCS amplitudes:

$$|\mathcal{T}^{\mathrm{DVCS}}|^2 = \frac{e^6}{y^2 Q^2}\left\{ c_0^{\mathrm{DVCS}} + \sum_{n=1}^{2}\left[ c_n^{\mathrm{DVCS}}\cos(n\phi) + s_n^{\mathrm{DVCS}}\sin(n\phi)\right]\right\}$$

$$c_0^{\mathrm{DVCS}} \sim (\mathrm{tw}-2)^2, \qquad c_1^{\mathrm{DVCS}}, s_1^{\mathrm{DVCS}} \sim \frac{\Delta}{Q}(\mathrm{tw}-2)(\mathrm{tw}-3), \qquad c_2^{\mathrm{DVCS}}, s_2^{\mathrm{DVCS}} \sim \alpha_s(\mathrm{tw}-2)(\mathrm{tw}-2)_{\mathrm{gluon}}$$

- Interference amplitude:

$$\mathcal{I} = \frac{\pm e^6}{x_{\mathrm{B}} y^3 t \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}\left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3}\left[ c_n^{\mathcal{I}}\cos(n\phi) + s_n^{\mathcal{I}}\sin(n\phi)\right]\right\}$$

$$c_0^{\mathrm{I}} \sim \frac{\Delta^2}{Q^2}(\mathrm{tw}-2), \qquad c_1^{\mathrm{I}}, s_1^{\mathrm{I}} \sim \frac{\Delta}{Q}(\mathrm{tw}-2), \qquad c_2^{\mathrm{I}}, s_2^{\mathrm{I}} \sim \frac{\Delta^2}{Q^2}(\mathrm{tw}-3), \qquad c_3^{\mathrm{I}}, s_3^{\mathrm{I}} \sim \alpha_s \frac{\Delta}{Q}(\mathrm{tw}-2)_{\mathrm{gluon}}$$

# Sources of 1/*Q* corrections

- Kinematical:
  - Choice of scaling variables
  - Exact vs. expanded form of process kinematics in lepton amplitudes
- "Dynamical":
  - Target mass corrections (recovery of trace effects due to nonzero hadron mass/t-channel momentum)

$$\langle p_2 | \overline{\psi} \gamma_{\{\mu} D_{\nu\}} \psi | p_1 \rangle = A(p_\mu p_\nu + \ldots) - B g_{\mu\nu}$$

  - High-twist parton correlations

$$\langle p_2 | \overline{\psi} G \ldots G \psi | p_1 \rangle$$

# Scaling variables

- Generalized scaling variables (skewness/Bjorken):

$$\eta = \frac{(q_1+q_2)\cdot(q_2-q_1)}{(q_1+q_2)\cdot(p_1+p_2)}, \qquad \xi = -\frac{(q_1+q_2)^2}{2(q_1+q_2)\cdot(p_1+p_2)}$$

$$\delta = \left(M^2 - \tfrac{1}{4}\Delta^2\right)/Q^2$$

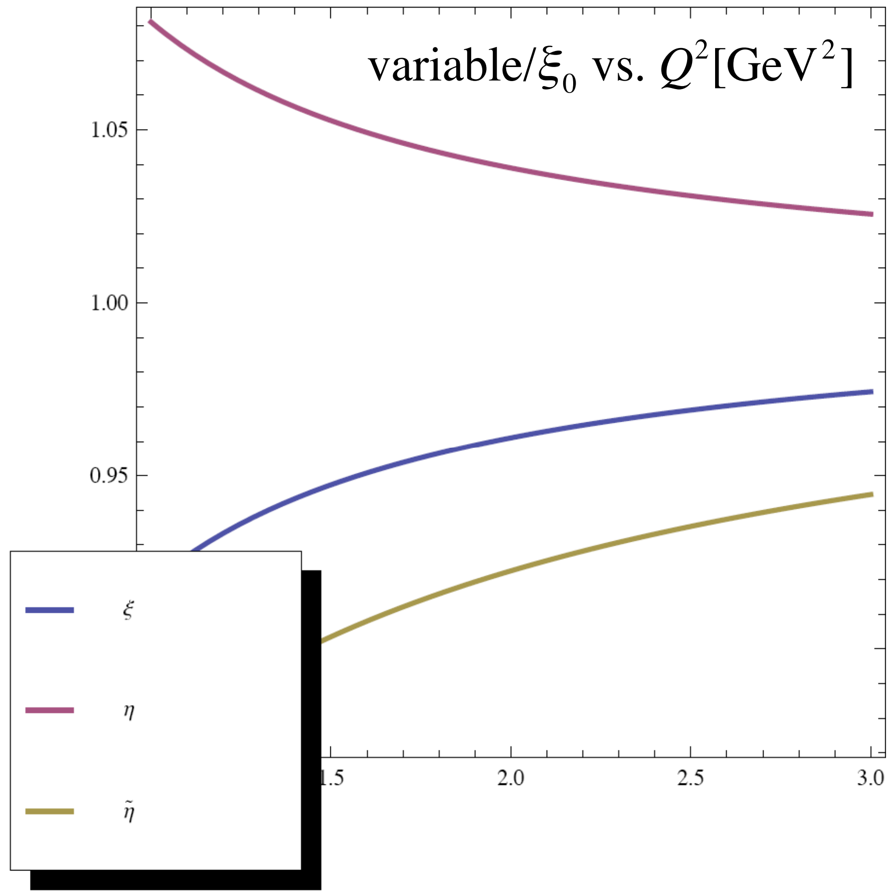
- Light-cone scaling variables (skewness):

$$\tilde{\eta} = \frac{(q_2-q_1)_+}{(p_1+p_2)_+} = \frac{\eta}{\sqrt{1+4(\xi\delta)^2}}$$

Bjorken limit

$$\xi_0 = \frac{x_B}{2-x_B}$$

~ O(5%) difference from scaling limit



# Photon helicity amplitudes

- Efficient separation of power suppressed effects emerging in the leptonic part from corrections induced due to different choices of parametrization of the hadronic tensor

- The choice of target rest frame with $z$-axis along the virtual photon allows one to localize azimuthal angle dependence in leptonic helicity amplitudes

- Concise and systematic calculational scheme

- Straightforward reduction to previously used harmonic expansion

# “Uncertainties” in hadronic tensor

Dependence of the hadronic helicity amplitudes on the choice of parametrization

- Jlab kinematics ($E$ = 5.7 GeV): $t' = -0.3\,\mathrm{GeV}^2\,, \quad x_{\mathrm{B}} = 0.3\,, \quad Q^2 = 1.5\,\mathrm{GeV}^2$

$$\mathcal{T}_{++}^{\mathrm{DVCS}} = \left\{ \begin{matrix} 0.996 \\ 1.003 \end{matrix} \right\} \mathcal{H} + \left\{ \begin{matrix} 0.011 \\ 0.008 \end{matrix} \right\} \mathcal{H}_3 + \left\{ \begin{matrix} 0.019 \\ 0.000 \end{matrix} \right\} \mathcal{H}_T$$

← BKM
← VGG

$$\frac{(2 - x_{\mathrm{B}}) Q \mathcal{T}_{0+}^{\mathrm{DVCS}}}{\sqrt{2\widetilde{K}}} = \left\{ \begin{matrix} 1.34 \\ 0.91 \end{matrix} \right\} \mathcal{H}_3^{\mathrm{eff}} + \left\{ \begin{matrix} -0.17 \\ 0.02 \end{matrix} \right\} \mathcal{H} - \left\{ \begin{matrix} 0.34 \\ 0.03 \end{matrix} \right\} \mathcal{H}_T$$

- HERMES kinematics ($E$ = 27.5 GeV): $t' = -0.3\,\mathrm{GeV}^2\,, \quad x_{\mathrm{B}} = 0.1\,, \quad Q^2 = 2.5\,\mathrm{GeV}^2$

$$\frac{(2 - x_{\mathrm{B}}) Q \mathcal{T}_{0+}^{\mathrm{DVCS}}}{\sqrt{2\widetilde{K}}} = \left\{ \begin{matrix} 1.03 \\ 1.01 \end{matrix} \right\} \mathcal{H}_3^{\mathrm{eff}} + \left\{ \begin{matrix} -0.01 \\ 0.00 \end{matrix} \right\} \mathcal{H} + \left\{ \begin{matrix} -0.04 \\ -0.02 \end{matrix} \right\} \mathcal{H}_T$$

Our consideration assumes validity of a hierarchy of hadronic scales associated with hadronic matrix elements of higher twist operators, i.e.,

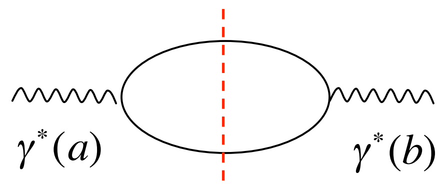
$$\varepsilon^2\,\mathrm{tw}\text{ - }2 \gg \frac{\mathrm{tw}\text{ - }4}{Q^2}$$

# Squared DVCS amplitude

Expansion of squared DVCS amplitude:

$$|\mathcal{T}^{\rm DVCS}|^2 = \frac{1}{Q^2} \sum_{a=-,0,+} \sum_{b=-,0,+} \mathcal{L}_{ab}(\lambda,\phi)\mathcal{W}_{ab}\,, \quad \mathcal{W}_{ab} = \mathcal{T}^{\rm DVCS}_{a+}\left(\mathcal{T}^{\rm DVCS}_{b+}\right)^* + \mathcal{T}^{\rm DVCS}_{a-}\left(\mathcal{T}^{\rm DVCS}_{b-}\right)^*$$

$$\mathcal{L}_{ab}(\lambda,\phi) = \varepsilon_1^{\mu *}(a)\mathcal{L}_{\mu\nu}(\lambda)\varepsilon_1^{\nu}(b) \;=$$




- BKM approximation is improved by exact account for kinematically suppressed contributions in leptonic helicity amplitudes.
- One-to-one correspondence between helicity amplitudes and Fourier harmonics (no mixture!)
- Exact amplitudes are built from mass-corrected QED "splitting functions" (of lepton energy loss $y$):

$$2-2y+y^2 \;\Rightarrow\; \frac{2-2y+y^2+\frac{\epsilon^2}{2}y^2}{1+\epsilon^2}\,,$$

$$\begin{Bmatrix} 2-y \\ -\lambda y \end{Bmatrix} \;\Rightarrow\; \frac{1}{1+\epsilon^2}\begin{Bmatrix} 2-y \\ -\lambda y\sqrt{1+\epsilon^2} \end{Bmatrix}$$

# Numerical estimates I

Jlab kinematics ($E$ = 5.7 GeV, $t$=-0.3 GeV$^2$, $x_B$=0.3, $Q^2$=2GeV$^2$):

- $\mathcal{H}$ only (admixture of higher harmonics arises from hadronic tensor):

$$\left|\mathcal{T}_{\rm DVCS}\right|^2 = [2.99 - 0.53\cos\phi + 0.01\cos(2\phi)]\mathcal{H}\mathcal{H}^* \quad \longleftarrow \text{ BKM hadron.}$$
$$= [2.97 - 0.35\cos\phi + 0.01\cos(2\phi)]\mathcal{H}\mathcal{H}^* \quad \longleftarrow \text{ VGG hadron.}$$

- twist-3 contamination of twist-2 (tiny):

$$2.99[\mathcal{H}\mathcal{H}^* + 0.003(\mathcal{H}_3\mathcal{H}^* + \mathcal{H}_3^*\mathcal{H})] \quad \longleftarrow \text{ exact}$$

- twist-2 contamination of twist-3 (strong):

$$0.24[\mathcal{H}\mathcal{H}_3^* + \mathcal{H}^*\mathcal{H}_3 - 2.19\mathcal{H}\mathcal{H}^* + 0.05\mathcal{H}_3\mathcal{H}_3^*]\cos(\phi) \quad \longleftarrow \text{ exact}$$

- cf. BKM approximation:

$$\left|\mathcal{T}_{\rm DVCS}\right|^2 = 3.34\mathcal{H}\mathcal{H}^* + 0.29(\mathcal{H}\mathcal{H}_3^* + \mathcal{H}_3\mathcal{H}^*)\cos\phi \quad \longleftarrow \text{ BKM approx.}$$

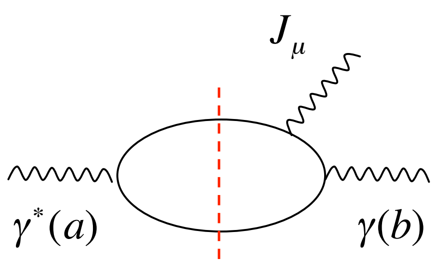
# Interference

Spinless target as an example:

$$\mathcal{I} = \frac{\pm e^6 F(t)}{t\,\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}\Big[\left(\mathcal{L}^P_{++} + \mathcal{L}^P_{--}\right)T_{++} + \left(\mathcal{L}^P_{0+} + \mathcal{L}^P_{0-}\right)T_{0+} + \left(\mathcal{L}^P_{-+} + \mathcal{L}^P_{+-}\right)T_{-+} + \text{c.c.}\Big]$$

with leptonic helicity amplitudes

$$\mathcal{L}^P_{+a\,+b} + \mathcal{L}^P_{-a\,-b}$$
$$= -\frac{1}{2x_{\rm B}y^3}\left\{\sum_{n=0}^{3} C_{ab}(n)\cos(n\phi) + i\lambda\sum_{n=1}^{2} S_{ab}(n)\sin(n\phi)\right\} =$$




- BKM approximation is improved by exact account for kinematically suppressed contributions in leptonic helicity amplitudes.
- One-to-one correspondence between helicity amplitudes and Fourier harmonics is lost!
- Treatment of hadronic amplitudes is plagued by uncertainties in the choice of the Lorentz tensor decomposition (e.g., exact vs. light-cone parametrization)

$$\mathcal{T}^{\rm DVCS}_{++} = \mathcal{H} + \mathcal{O}(1/\mathcal{Q}^2)\,,$$
$$\mathcal{T}^{\rm DVCS}_{0+} = \frac{\sqrt{2}}{2 - x_{\rm B}}\frac{\widetilde{K}}{\mathcal{Q}}\mathcal{H}_3^{\rm eff} + \mathcal{O}(1/\mathcal{Q}^3)$$

# Numerical estimates IIa

Jlab kinematics ($E$ = 5.7 GeV, $t$=-0.3 GeV$^2$, $x_B$=0.3, $Q^2$=2GeV$^2$):

- $\mathcal{H}$ only:

$$\mathcal{I} = [-2.34 - 7.54\cos\phi + 1.21\cos(2\phi)]\mathrm{Re}\,\mathcal{H}$$ ← BKM hadron.

$$= [-2.36 - 7.56\cos\phi + 0.93\cos(2\phi)]\mathrm{Re}\,\mathcal{H}$$ ← VGG hadron.

- twist-3 contamination of twist-2 (small):

$$-2.43\,\mathrm{Re}[\mathcal{H} - 0.06\,\mathcal{H}_3]$$
$$-7.54\,\mathrm{Re}[\mathcal{H} + 0.02\,\mathcal{H}_3]\cos\phi$$ ← exact

- twist-2 contamination of twist-3 (strong):

$$-0.77\,\mathrm{Re}[\mathcal{H}_3 - 1.57\,\mathcal{H}]\cos(2\phi)$$ ← exact

- cf. BKM approximation:

$$\mathcal{I} = [-2.3 - 12.9\cos\phi]\mathrm{Re}\,\mathcal{H} - 1.1\cos(2\phi)\mathrm{Re}\,\mathcal{H}_3$$ ← BKM approx.

# Hot fix

An approximation to exact results which accounts for the most significant source of power suppressed effects.

- Replace BKM harmonics by exact ones.

- Ignore admixture of harmonics induced by power suppressed effects for the same hadron helicity amplitude.

$$s_1 \sin\phi \overset{\text{exact}}{\longrightarrow} S_1^{++} \sin\phi + S_2^{++} \sin(2\phi) \overset{\text{hot fix}}{\longrightarrow} S_1^{++} \sin\phi$$

$$c_1 \cos\phi \overset{\text{exact}}{\longrightarrow} C_1^{++} \cos\phi + C_2^{++} \cos(2\phi) + C_3^{++} \cos(3\phi) \overset{\text{hot fix}}{\longrightarrow} C_1^{++} \cos\phi$$

Here

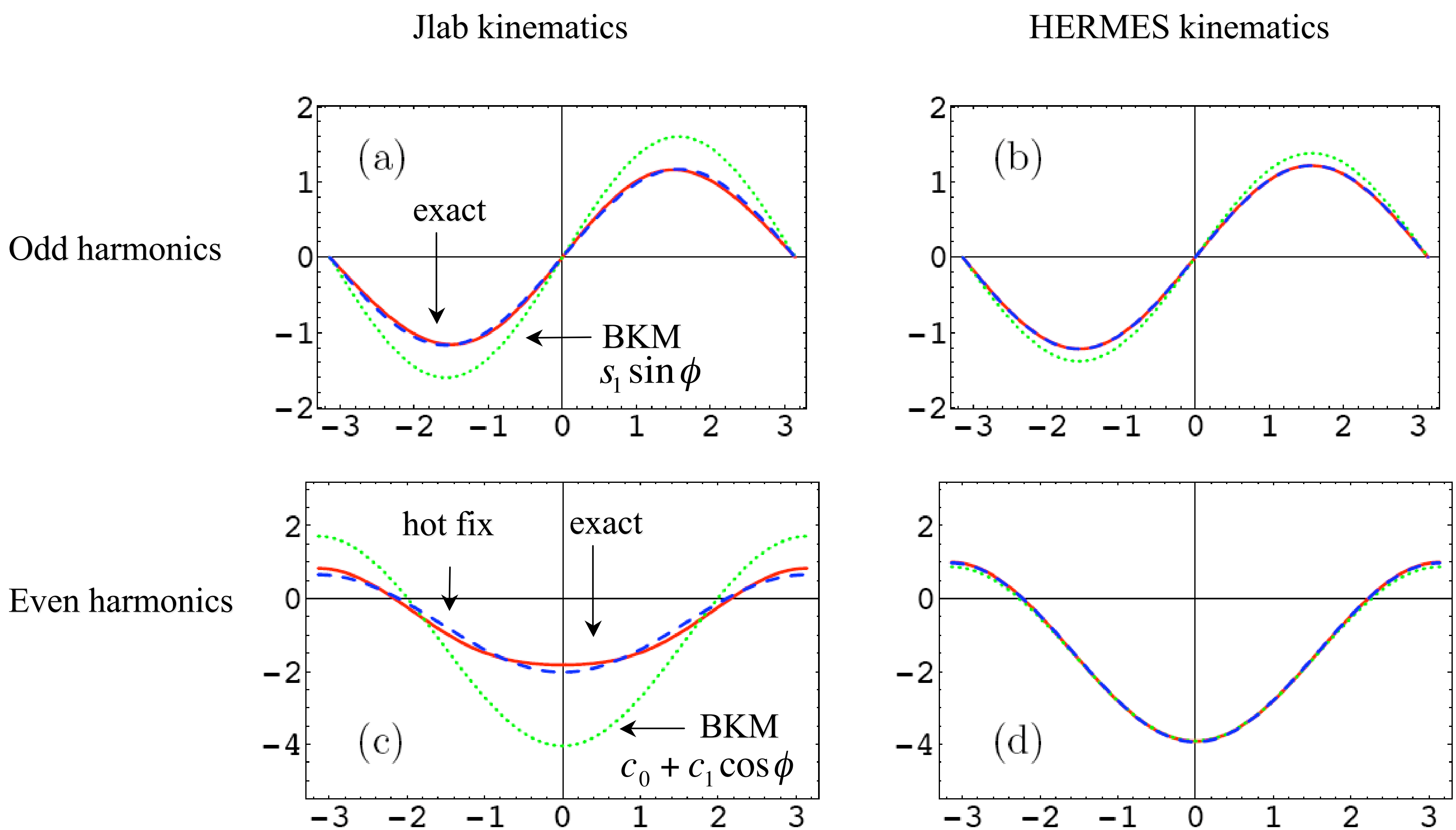
BKM

$$S_{++}(n=1) = \frac{8K(2-y)y}{1+\epsilon^2}\left\{1 + \frac{1 - x_{\mathrm{B}} + \frac{\sqrt{1+\epsilon^2}-1}{2}}{1+\epsilon^2}\frac{t'}{Q^2}\right\}$$

$$C_{++}(n=1) = \frac{-16K\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{(1+\epsilon^2)^{5/2}}\left\{\left(1+(1-x_{\mathrm{B}})\frac{\sqrt{\epsilon^2+1}-1}{2x_{\mathrm{B}}}+\frac{\epsilon^2}{4x_{\mathrm{B}}}\right)\frac{x_{\mathrm{B}}t}{Q^2}-\frac{3\epsilon^2}{4}\right\}$$
$$-4K\left(2-2y+y^2+\frac{\epsilon^2}{2}y^2\right)\frac{1+\sqrt{1+\epsilon^2}-\epsilon^2}{(1+\epsilon^2)^{5/2}}\left\{1-(1-3x_{\mathrm{B}})\frac{t}{Q^2} + \frac{1-\sqrt{1+\epsilon^2}+3\epsilon^2}{1+\sqrt{1+\epsilon^2}-\epsilon^2}\frac{x_{\mathrm{B}}t}{Q^2}\right\},$$

BKM

# Numerical estimates IIb

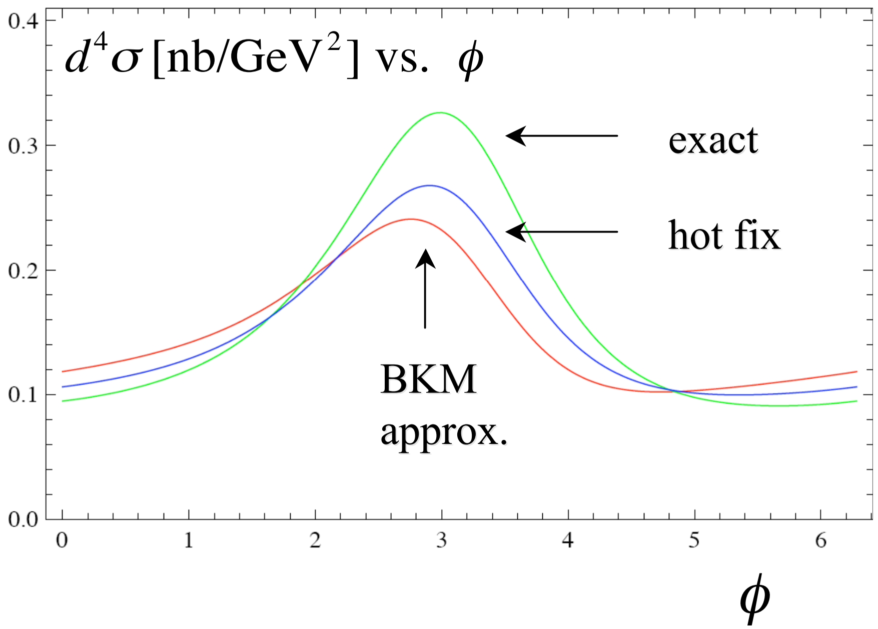



Significant deviations from BKM'01 for Jlab kinematics.

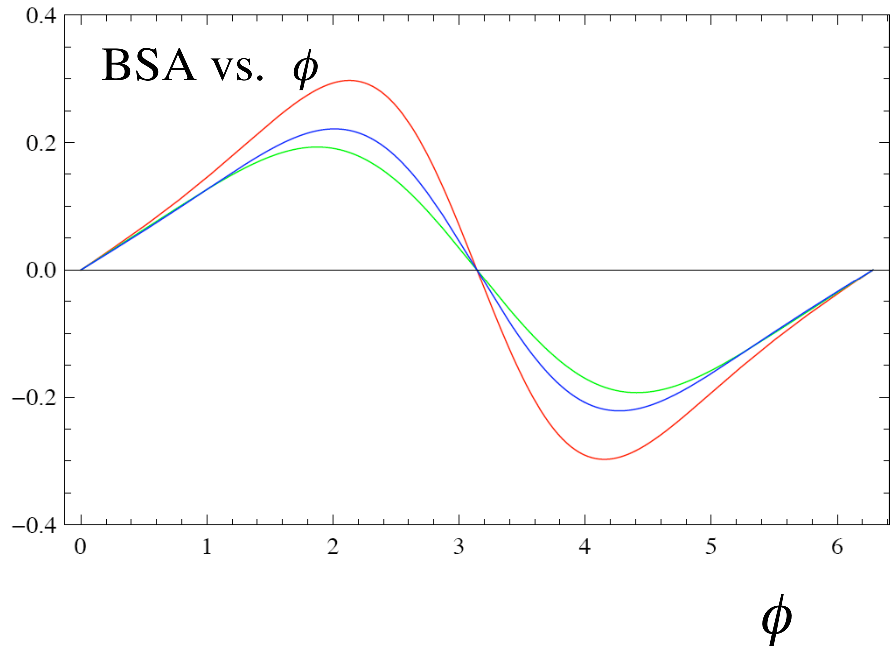
# DVCS on the proton

Jlab kinematics ($E$ = 5.7 GeV, $t$=-0.2 GeV$^2$, $x_B$=0.3, $Q^2$=1.5GeV$^2$):

Cross section




Beam spin asymmetry

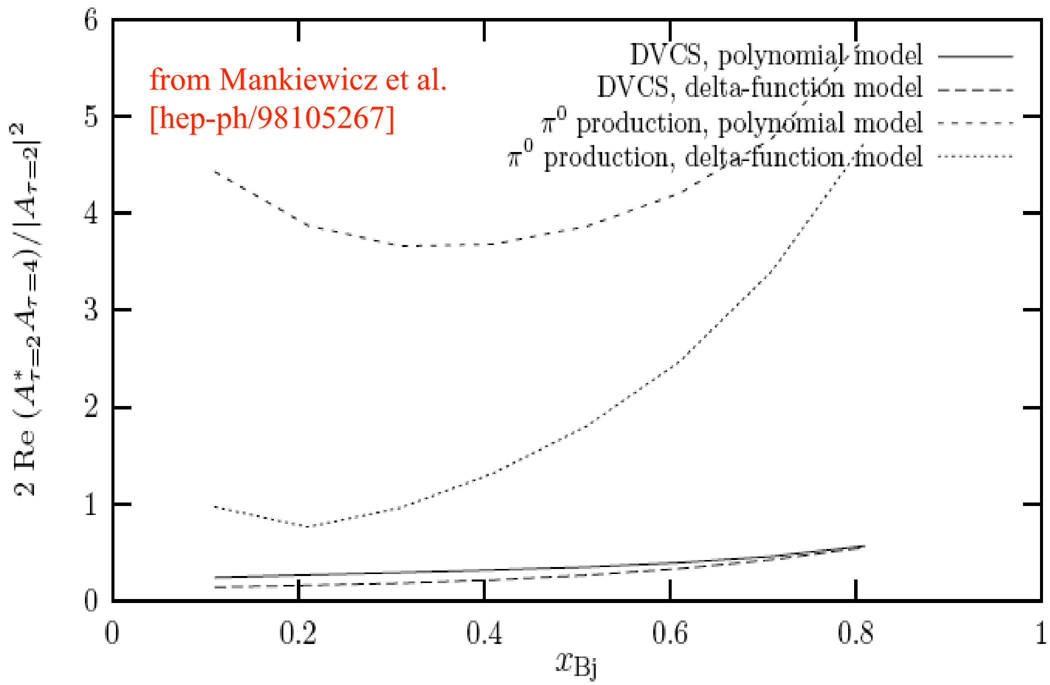


# Dynamical higher twists

$$A = \left|A_{\tau=2} + A_{\tau=4} + \ldots\right|^2 \approx \left|A_{\tau=2}\right|^2 \left(1 + 2\mathrm{Re}(A^*_{\tau=2} A_{\tau=4}) \big/ \left|A_{\tau=2}\right|^2\right)$$

Renormalon estimates of
twist-four effects:

$$A_{\tau=4} = \frac{\Lambda^2}{Q^2} C * \mathrm{GPD}_{\tau=2}$$




Moderate effects assuming that the scale $\Lambda^2$ of the twist four contribution is identical to the one in DIS.

# Conclusion

- Approach provides analytical framework for analysis of elecroproduction observables
- Exact treatment of kinematical effects is crucial for current Jlab kinematics
- Theory of dynamical higher twist effects is needed