# Transversity in Hard Exclusive Electroproduction of Pions

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Jefferson Lab, May 2010

**Outline:**

- **Pion electroproduction to leading-twist accuracy**
- **Pion pole**
- **Transverse photon polarization**
- **Transversity**
- **Results**
- **Summary**

based on work done in collaboration with S. Goloskokov, arXiv:0906.0460 [hep-ph]

# The leading $\gamma^* p \to \pi^+ n$ amplitudes

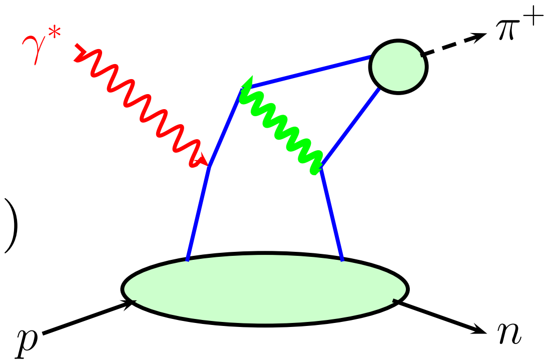
$$\mathcal{M}_{0+,0+}(\pi^+) = \sqrt{1-\xi^2}\frac{e_0}{Q}\left\{\langle\widetilde{H}^{(3)}\rangle - \frac{\xi^2}{1-\xi^2}\langle\widetilde{E}^{(3)}_{n.p.}\rangle - \frac{2m\xi Q}{1-\xi^2}\frac{\rho_\pi}{t-m_\pi^2}\right\},$$

$$\mathcal{M}_{0-,0+}(\pi^+) = \frac{e_0}{Q}\frac{\sqrt{-t'}}{2m}\left\{\xi\langle\widetilde{E}^{(3)}_{n.p.}\rangle + 2mQ^2\frac{\rho_\pi}{t-m_\pi^2}\right\},$$

$t' = t - t_0$, $\quad \xi \simeq x_{Bj}/2$, $\quad F^{(3)} = F^u - F^d$

convolution:

$$\langle F\rangle = \sum_\lambda \int_{-\xi}^{1} dx \mathcal{H}_{0\lambda,0\lambda}(x,\xi,Q^2,t=0)F(x,\xi,t)$$




GK: subprocess amplitudes worked out in modified pert. approach:
LO pQCD + quark trans. momenta and Sudakov suppressions
for $Q^2 \to \infty \Longrightarrow$ lead. twist (collinear approximation) (Sterman et al)

# Double distributions

integral representation ($i = u, d$ valence quarks)

$$\widetilde{H}^i(\bar{x}, \xi, t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \, \delta(\beta + \xi\alpha - \bar{x}) \, \tilde{f}_i(\beta, \alpha, t)$$

$\tilde{f}_i$ double distributions Mueller *et al* (94), Radyushkin (99)

advantage - polynomiality automatically satisfied

useful ansatz with relation to PDFs (reduction formula respected)

$$\tilde{f}_i(\beta, \alpha, t) \; = \; \Delta q_i(\beta) \, \Theta(\beta) \, \exp[(\tilde{b}_i + \tilde{\alpha}'_i \ln(1/\beta))t] \, \frac{3}{4} \, \frac{[(1 - |\beta|)^2 - \alpha^2]}{(1 - |\beta|)}$$

$\tilde{\alpha}'_h = 0.45\,\mathrm{GeV}^{-2}$ $\tilde{b}_h = 0$ $t$-dependence of Regge residue

Regge intercept and residue at $t = 0$ included in PDF

$\widetilde{E}_{n.p.}$ analogously, forward limit parameterized as

$\tilde{e}_u = -\tilde{e}_d = \tilde{N}_e \beta^{-0.48}(1 - \beta)^5$

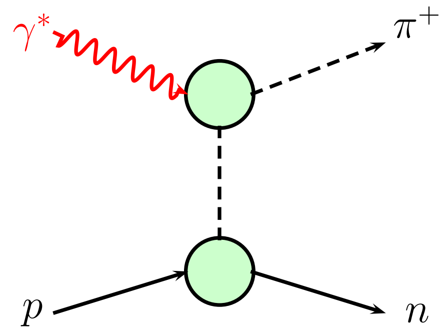
$\tilde{\alpha}'_e = 0.25\,\mathrm{GeV}^{-2}$ $\tilde{b}_e = 0$ values fitted to data

# The pion pole contribution

pion exchange (small $-t$, large $Q^2$)

$$\rho_\pi = \sqrt{2} g_{\pi NN} F_\pi(Q^2) F_{\pi NN}(t')$$
$$F_\pi = [1 + Q^2/(0.50\mathrm{GeV}^2)]^{-1}$$
$$F_{\pi NN} = (\Lambda_N^2 - m_\pi^2)/(\Lambda_N^2 - t')$$

$$\mathcal{M}^{\rm pole}_{0+,0+} = -e_0 \frac{2m\xi Q}{\sqrt{1-\xi^2}} \frac{\rho_\pi}{t-m_\pi^2}\,,$$
$$\mathcal{M}^{\rm pole}_{0-,0+} = +e_0 Q\sqrt{-t'}\, \frac{\rho_\pi}{t-m_\pi^2}\,,$$
$$\mathcal{M}^{\rm pole}_{0+,\pm+} = \pm 2\sqrt{2} e_0 \xi m \sqrt{-t'}\, \frac{\rho_\pi}{t-m_\pi^2}\,,$$
$$\mathcal{M}^{\rm pole}_{0-,\pm+} = \pm\sqrt{2} e_0 t' \sqrt{1-\xi^2}\, \frac{\rho_\pi}{t-m_\pi^2}\,.$$




transverse amplitudes very small

full pion FF needed and non-pole $\widetilde{E}$

see Goloskokov-K(09),Bechler-Mueller (09) (analysis of $d\sigma/dt$ and $A_{UT}^{\sin(\phi-\phi_s)}$
using lead. twist with $\alpha_s^{\rm eff} = 0.8$)

lead. twist accuracy: only 'pert. contr.' to pion FF

(with $\alpha_s^{\rm eff} \simeq 0.3$ about 1/3 of exp. value measured in same reaction $F_\pi - 2$(08))

fails with cross section by order of magnitude

Mankiewicz et al (98), Frankfurt et al (99), Belitsky-Mueller (01),...

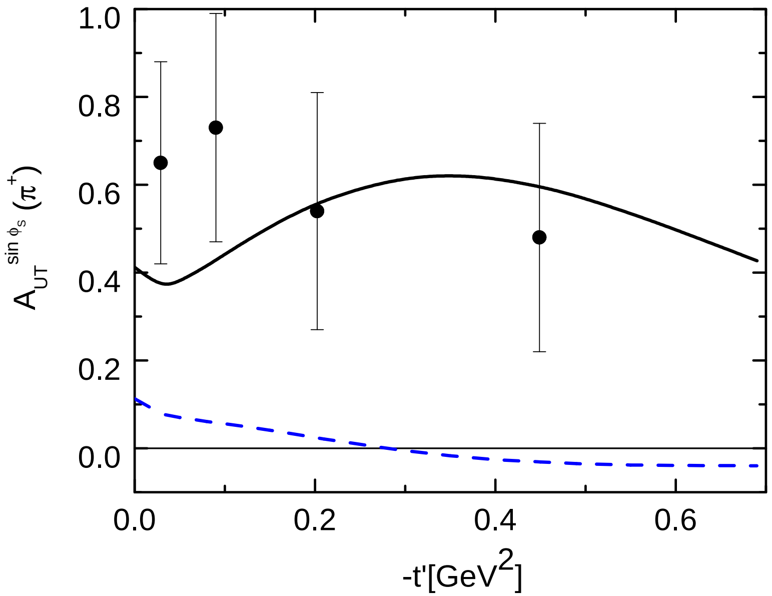
# Target asymmetries in electroproduction

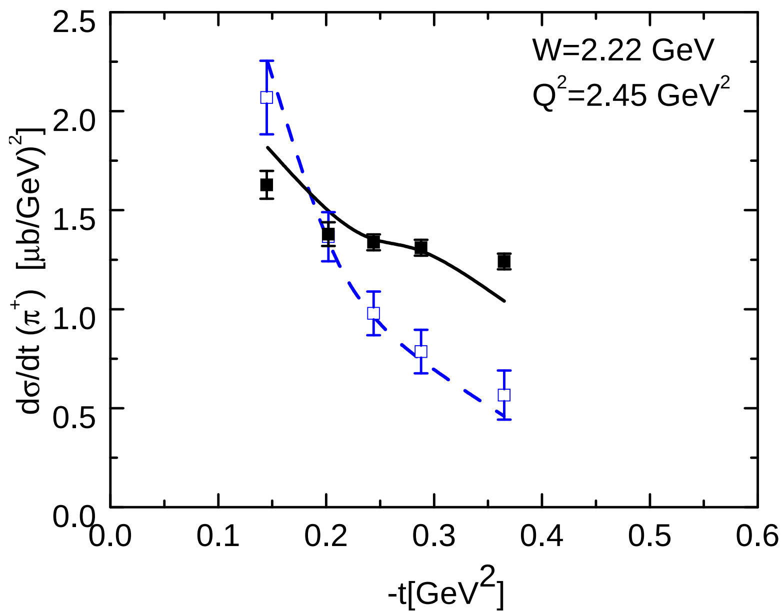
| observable | dominant interf. term | $\gamma^* p \to MB$ amplitudes | low $t'$ behavior |
|---|---|---|---|
| $A_{UT}^{\sin(\phi-\phi_s)}$ | LL | $\mathrm{Im}\big[\mathcal{M}^*_{0-,0+}\mathcal{M}_{0+,0+}\big]$ | $\propto \sqrt{-t'}$ |
| $A_{UT}^{\sin(\phi_s)}$ | LT | $\mathrm{Im}\big[\mathcal{M}^*_{0-,++}\mathcal{M}_{0+,0+}\big]$ | const. |
| $A_{UT}^{\sin(2\phi-\phi_s)}$ | LT | $\mathrm{Im}\big[\mathcal{M}^*_{0\mp,-+}\mathcal{M}_{0\pm,0+}\big]$ | $\propto t'$ |
| $A_{UT}^{\sin(\phi+\phi_s)}$ | TT | $\mathrm{Im}\big[\mathcal{M}^*_{0-,++}\mathcal{M}_{0+,++}\big]$ | $\propto \sqrt{-t'}$ |
| $A_{UT}^{\sin(2\phi+\phi_s)}$ | TT | $\propto \sin\theta_\gamma$ | $\propto t'$ |
| $A_{UT}^{\sin(3\phi-\phi_s)}$ | TT | $\mathrm{Im}\big[\mathcal{M}^*_{0-,-+}\mathcal{M}_{0+,-+}\big]$ | $\propto (-t')^{(3/2)}$ |
| $A_{UL}^{\sin(\phi)}$ | LT | $\mathrm{Im}\big[\mathcal{M}^*_{0-,++}\mathcal{M}_{0-,0+}\big]$ | $\propto \sqrt{-t'}$ |

$\phi$ azimuthal angle between lepton and hadron plane; $\phi_s$ orientation of target spin vector; $\theta_\gamma$ rotation from direction of incoming lepton to virtual photon one

$\pi^+$: all measured; detailed info. on amplitudes

# Transverse photon polarization matters




HERMES(09)

$Q^2 \simeq 2.5\,\mathrm{GeV}^2$, $W = 3.99\,\mathrm{GeV}$

$\sin\phi_s$ moment very large

does not seem to vanish for $t' \to 0$

$A_{UT}^{\sin\phi_S} \propto \mathrm{Im}\Big[M^*_{0-,++} M_{0+,0+}\Big]$

n-f. ampl. $\mathcal{M}_{0-,++}$ required

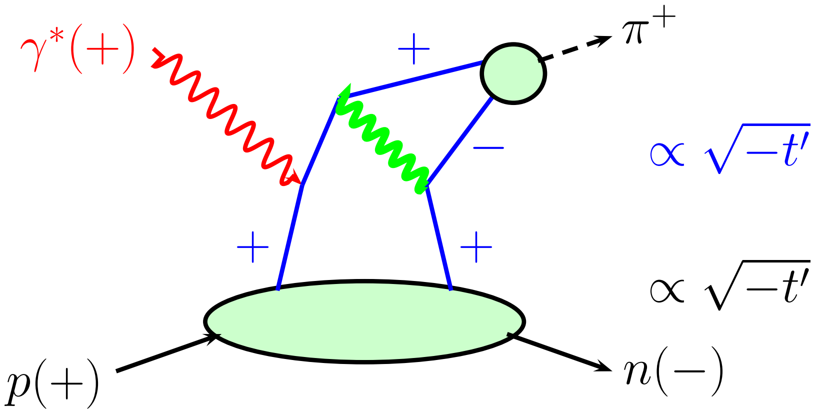
JLab $F_\pi - 2$

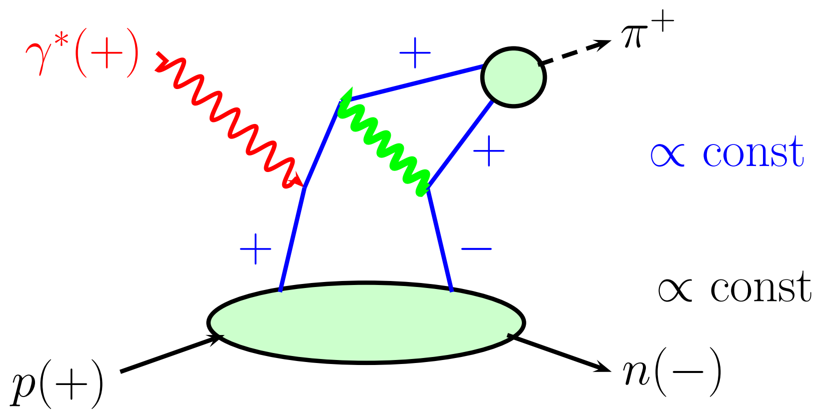
black: $\sigma_T$ blue: $\sigma_L$

$\sigma_T$ large at large $-t$

**$\gamma^*_T \to P$ transitions substantial**

# Can $\mathcal{M}_{0-,++}$ be fed by ordinary GPDs?




lead. twist pion wave fct. $\propto q' \cdot \gamma\gamma_5$
(perhaps including $\mathbf{k}_\perp$)

twist-3 w.f.

$\mathcal{M}_{0-,++} \propto t'$

$\mathcal{M}_{0-,++} \propto$ const

helicity flip GPDs ($H_T, E_T, \widetilde{H}_T, \widetilde{E}_T$) required
Hoodbhoy-Ji (98), Diehl (01)

# A twist-3 contribution

$$\mathcal{M}^{\text{twist}-3}_{0-,\mu+} = e_0\sqrt{1-\xi^2}\int_{-\xi}^{1} d\bar{x}\,\Big\{\mathcal{H}_{0-,\mu+}\big[H_T^{(3)} - \frac{\xi}{1-\xi^2}(\xi E_T^{(3)} - \widetilde{E}_T^{(3)})\big] + (\mathcal{H}_{0+,\mu-} - \mathcal{H}_{0-,\mu+})\frac{t'}{4m^2}\widetilde{H}_T^{(3)}\Big\}$$

$$\mathcal{M}^{\text{twist}-3}_{0+,\mu+} = e_0\frac{\sqrt{-t'}}{2m}\int_{-\xi}^{1} d\bar{x}\,\Big\{(\mathcal{H}_{0+,\mu-} - \mathcal{H}_{0-,\mu+})\widetilde{H}_T^{(3)} + \big[(1-\xi)\mathcal{H}_{0+,\mu-} - (1+\xi)\mathcal{H}_{0-,\mu+}\big]E_T^{(3)}/2 + [(1-\xi)\mathcal{H}_{0+,\mu-} + (1+\xi)\mathcal{H}_{0-,\mu+}]\widetilde{E}_T^{(3)}/2\Big\}$$

at small $\xi$ and small $t'$: $H_T$ dominant in $\mathcal{M}_{0-,++}$

in $\mathcal{M}_{0-,-+}$ suppressed by $t'/Q^2$

twist-3 in long. amplitudes suppressed by $\sqrt{-t'}/Q$

subprocess amplitudes to be evaluated with twist-3 pion wave function $\Longrightarrow$

# The twist-3 pion distr. amplitude

projector $q\bar{q} \to \pi$ (3-part. $q\bar{q}g$ contr. neglected) Beneke-Feldmann (01)

$\sim q' \cdot \gamma\gamma_5\Phi + \mu_\pi\gamma_5\Big[\Phi_P - \imath\sigma_{\mu\nu}(\ldots\Phi'_\sigma + \ldots\Phi_\sigma\partial/\partial\mathbf{k}_{\perp\nu})\Big]$

definition: $\langle\pi^+(q') \mid \bar{d}(x)\gamma_5 u(-x) \mid 0\rangle = f_\pi\mu_\pi \int d\tau \mathrm{e}^{q'x\tau}\Phi_P(\tau)$

local limit $x \to 0$ related to divergency of axial vector current

$\Longrightarrow \mu_\pi = m_\pi^2/(m_u + m_d) \simeq 2\,\mathrm{GeV}$ at scale $2\,\mathrm{GeV}$ (conv. $\int d\tau\Phi_P(\tau) = 1$)

Eq. of motion: $\tau\Phi_P = \Phi_\sigma/N_c - \tau\Phi'_\sigma/(2N_c)$

solution: $\Phi_P = 1, \quad \Phi_\sigma = \Phi_{AS}$ Braun-Filyanov (90)

$\mathcal{H}_{0-,++} \neq 0$, $\Phi_P$ dominant, $\Phi_\sigma$ contr. $\propto t'/Q^2$

in coll. appr.: $\mathcal{H}_{0-,++}$ infr. sing. and double pole $1/(x-\xi)^2$ m.p.a. regular

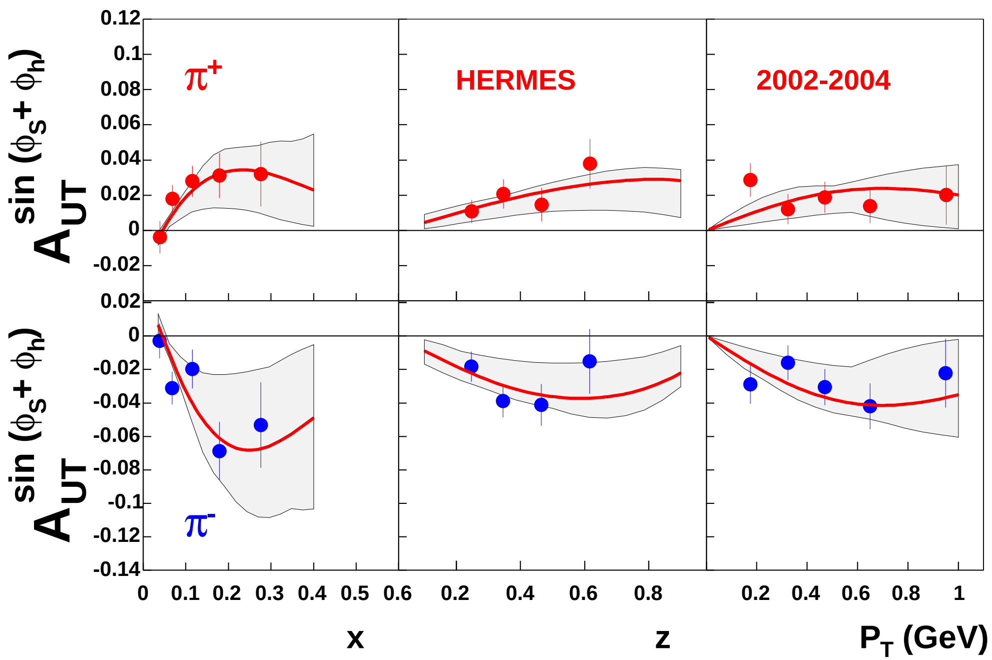
twist-3 mechanism applied to wide-angle photo- and electroproduction
no problem in coll. approx. Jakob-Huang-K-Passek (04)

# Modeling $H_T$

small $\xi$: $H_T$ should dominate; use double distr. ansatz; $H_T^a(x,0,0) = \delta^a(x)$

take transversity PDF from Anselmino et al (07)(08) large errors

$\delta^a = 7.46 N_T^a x(1-x)^5[q_a(x) + \Delta q_a(x)]$ $\quad N_T^u = 0.5 \quad N_T^d = -0.6$



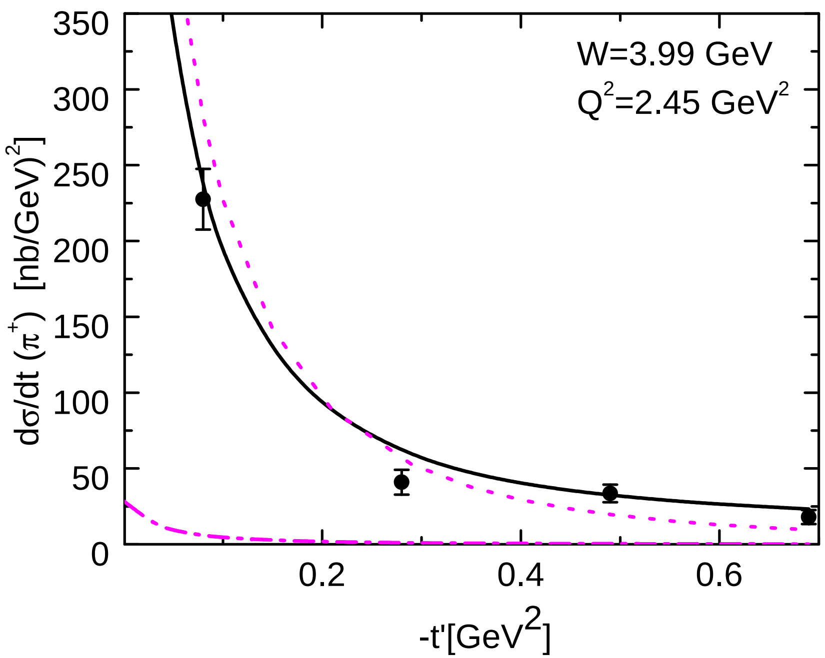

combined analysis of transv. PDFs and Collins fcts. HERMES and COMPASS SIDIS and BELLE $e^+e^- \to h_1 h_2 X$ data

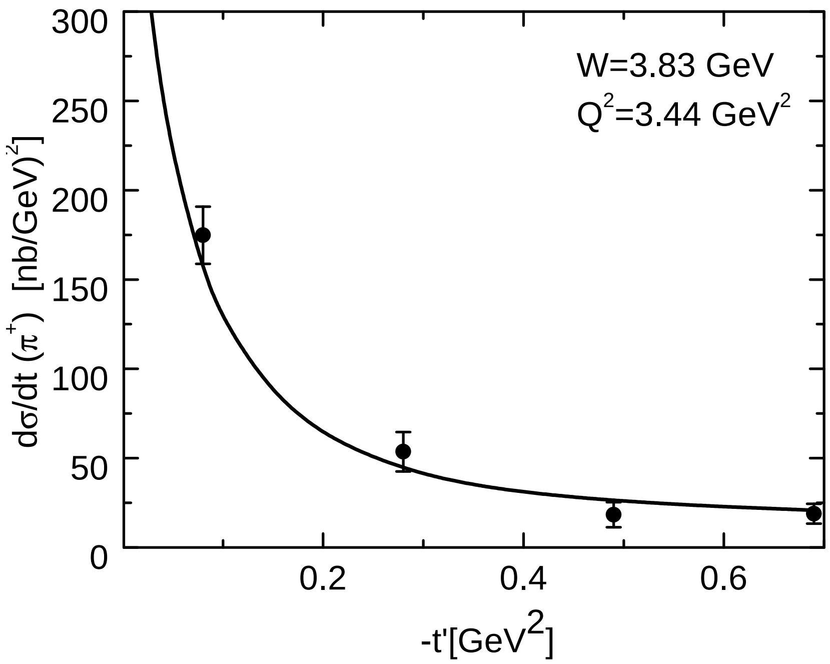
$A_{UT}^{\sin(\phi+\phi_s)} \propto \delta \otimes \Delta\hat{\sigma} \otimes \Delta^N D_{\pi/q\uparrow}$

fixes scale of $H_T$, $t$-dependence free ($\alpha'_T = 0.45\,\mathrm{GeV}^{-2}$, $b_T = 0.9\,\mathrm{GeV}^{-2}$)

**link between transversity in inclusive and exclusive reactions**

# Results on unseparated $\pi^+$ cross section

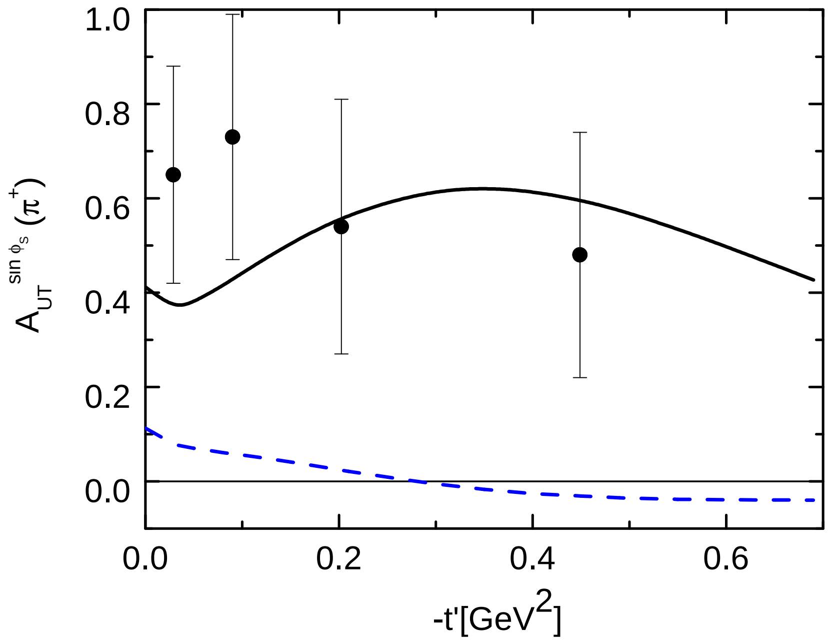


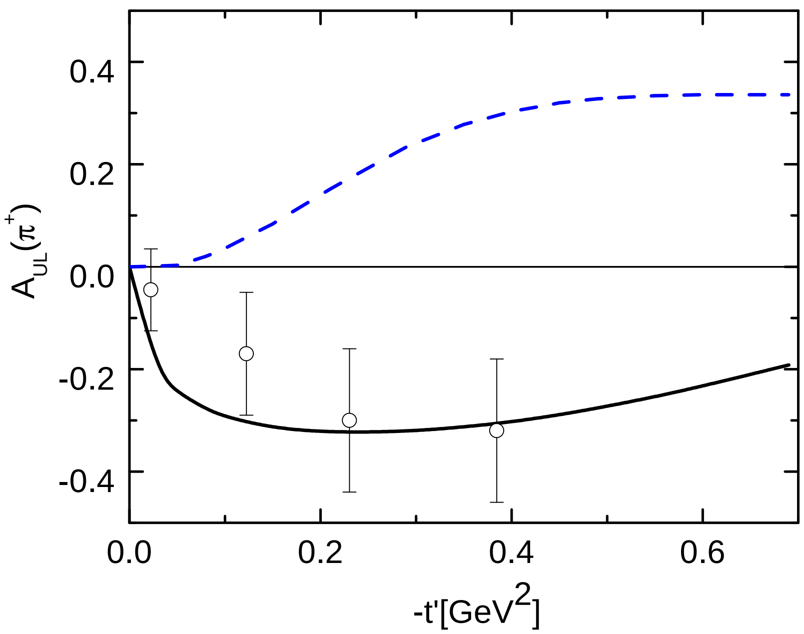




data from HERMES 07

magenta lines: pion pole contr. (unseparated and transverse cross sections)

# Results on target asymetries



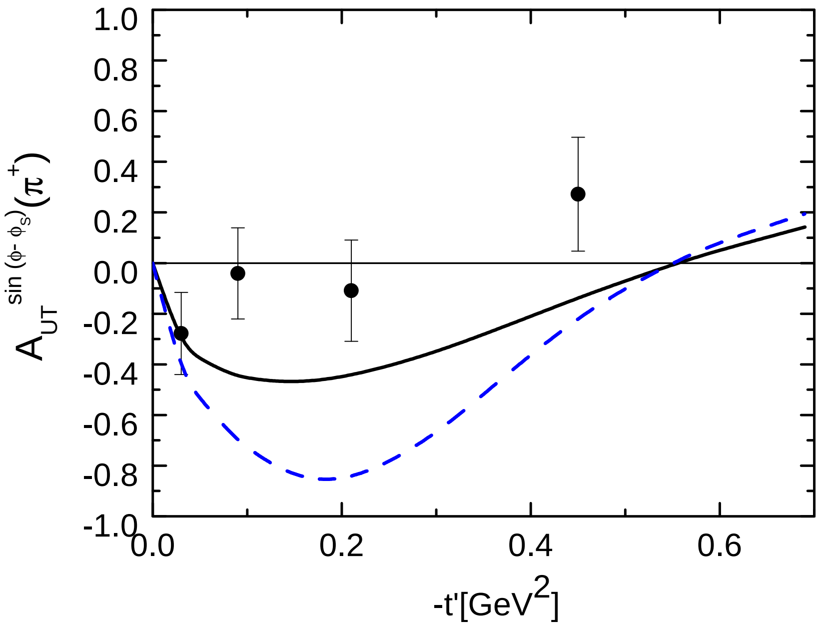

$Q^2 = 2.5\,\text{GeV}^2 \;\; W = 3.99\,\text{GeV}$

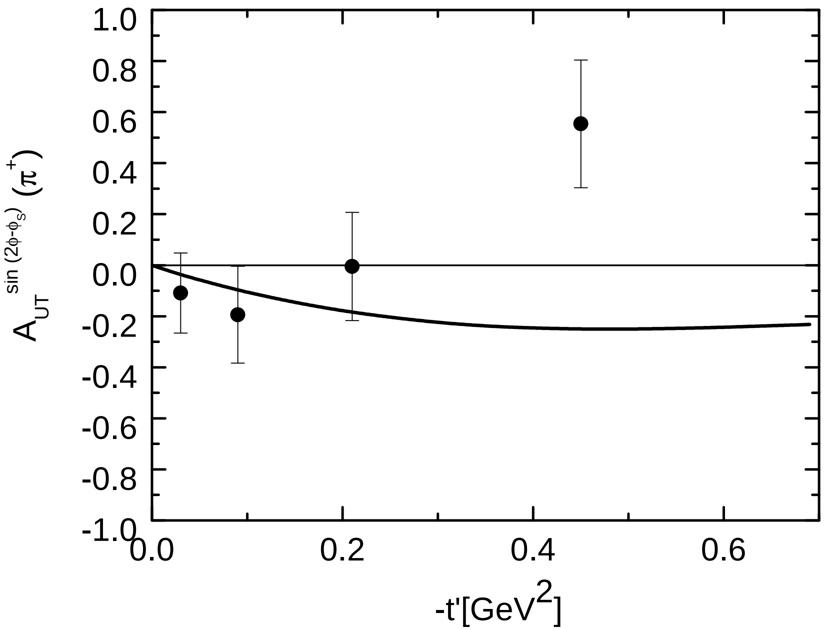
data on $A_{UT}$ HERMES (08); $A_{UL}$ HERMES(02)

blue lines: without twist-3 contr.

$A_{LU}^{\sin\phi} \propto \text{Im}[\mathcal{M}^*_{0-,++}\mathcal{M}_{0-,0+}]$ $\qquad A_{LL}^{\cos\phi} \propto \text{Re}[\mathcal{M}^*_{0-,++}\mathcal{M}_{0-,0+}]$

# Results continued



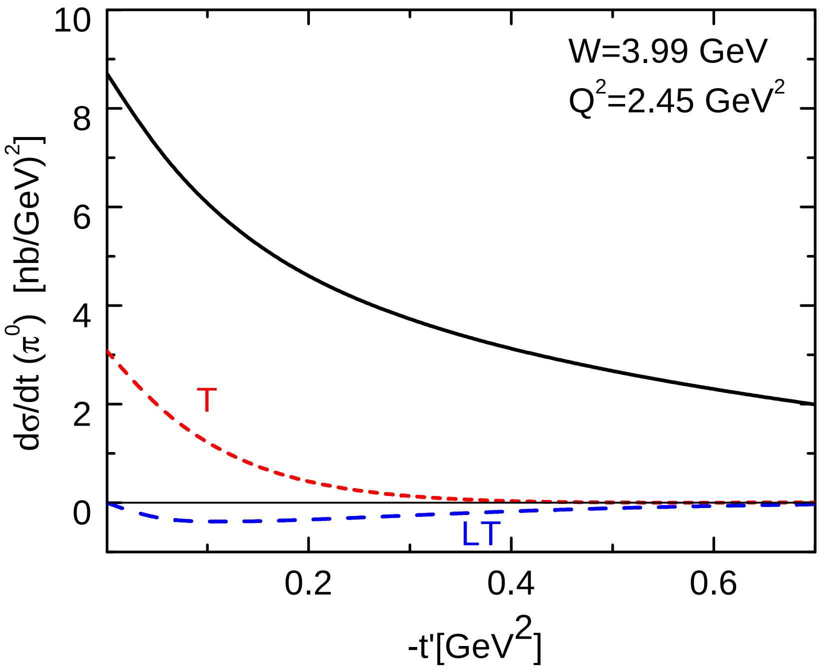

$Q^2 = 2.5\,\mathrm{GeV}^2$ $W = 3.99\,\mathrm{GeV}$

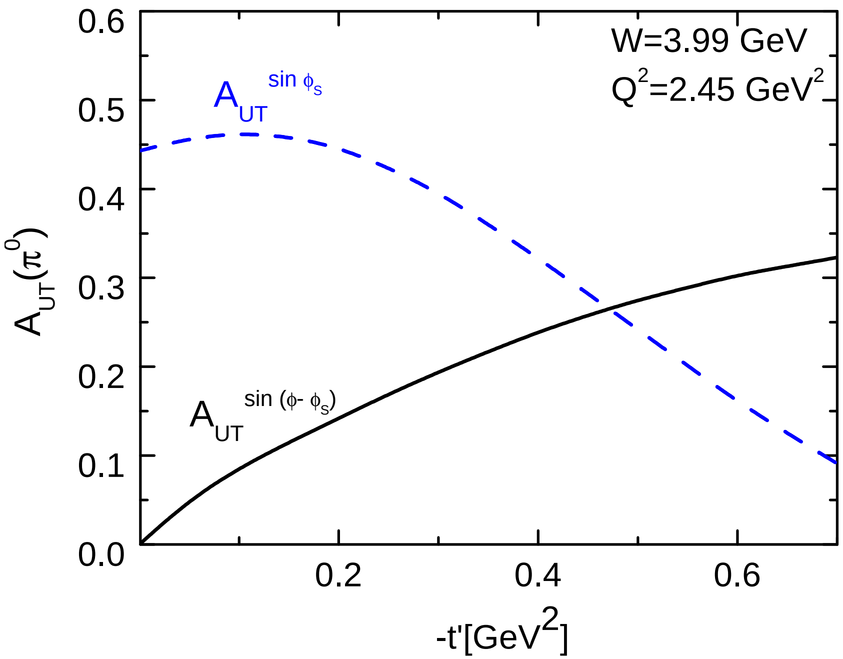
data on $A_{UT}$ HERMES (08)

blue: only LL contr.

other asymmetries: $|A_{UT}| < 0.1$ in agreement with exp.

# Results on $\pi^0$ electroproduction




pion exchange absent

important to have data

independent check of GPDs

implicit check of pion f.f.

Other analysis of $\pi^0$ prod.: Ahmad et al (08)

(subprocess viewed as form factors for $\gamma - \pi$ transitions under the action of vector and axial-vector currents)

# Comparison with $F_\pi - 2$ data

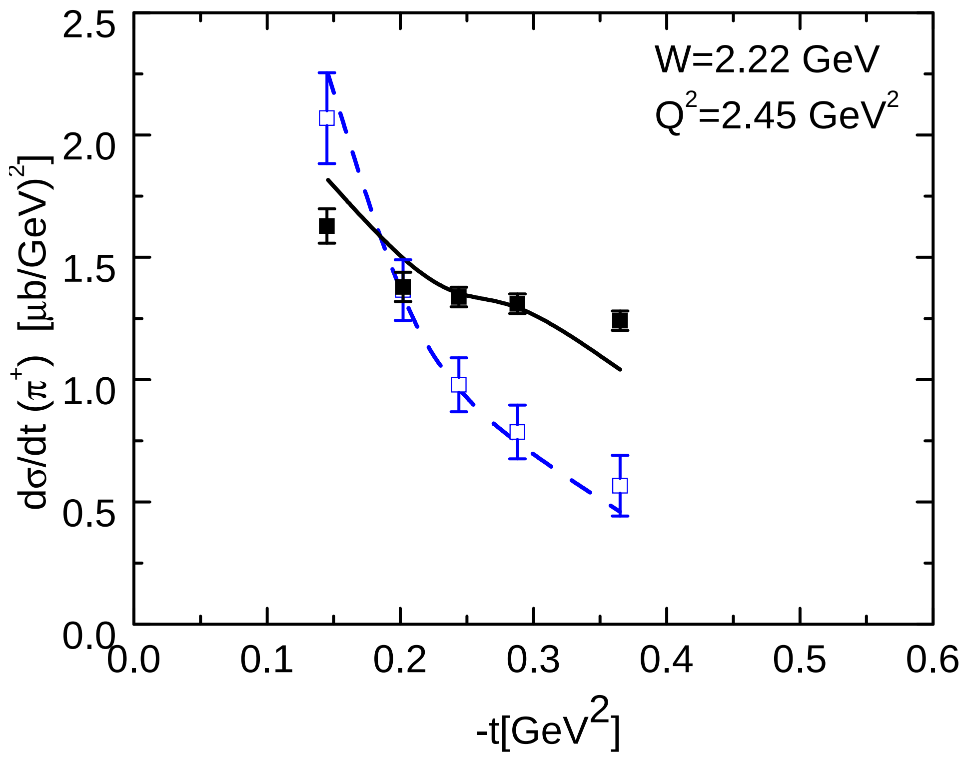
Can we apply our approach for $W \leq 4\,\mathrm{GeV}$?

(cf. $\rho^0, \omega, \rho^+$ production - difficulties with val. quarks)

our approach is designed for small $\xi (\lesssim 0.1)$

large $x (\gtrsim 0.6)$ behaviour of GPDs not probed

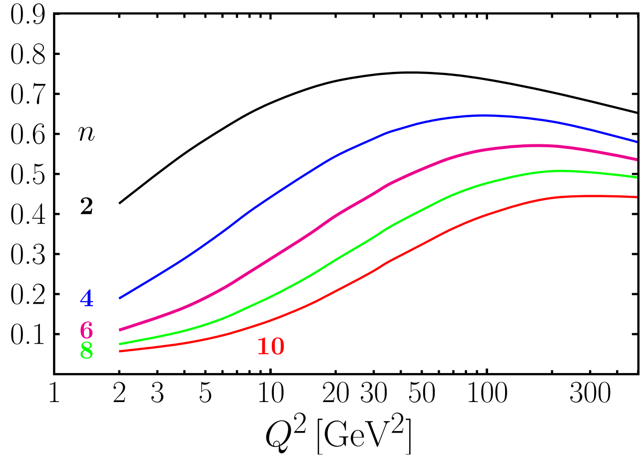
with little modifications of large $x$ behavior of $\widetilde{E}$ and $H_T$:




work in progress

# Implications of the $\pi\gamma$ transition form factor

surprising new data from Babar(09)



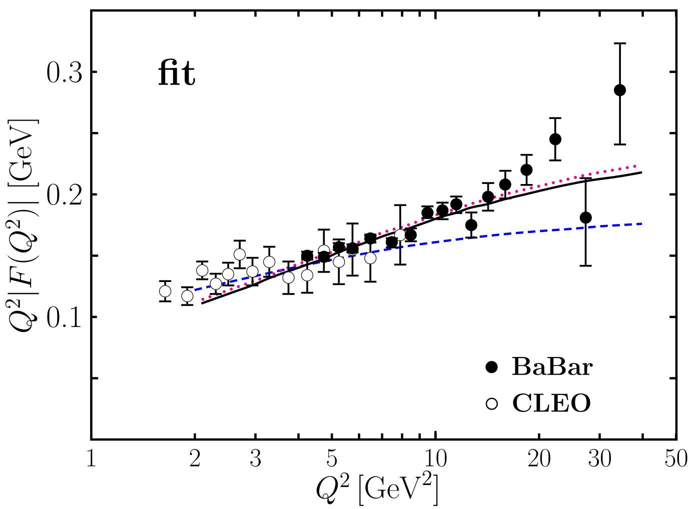

within m.p.a.:

$$Q^2 F_{\pi\gamma} = \sqrt{2} f_\pi \mathcal{C}_0 \left[1 + \sum_n a_n(\mu_0)\mathcal{C}_n/\mathcal{C}_0\right]$$

strong supp. of high. Gegenb. terms at low $Q^2$ and at large $Q^2$ due to evolution

low $Q^2$: $\Phi_{AS}$ suffices (see fit to CLEO data)

increasing $Q^2$: higher Gegenb. terms become gradually more important




Braun-Diehl-K.

Fit to CLEO and Babar data (at scale $1\,\mathrm{GeV}$):

$a_2 = 0.25$ (fixed from lattice Braun(06))

$a_4 = 0.07 \pm 0.10$

$\sigma = 0.42 \pm 0.07\,\mathrm{GeV}^{-1}$ trans. size parameter

solid line blue K.-Raulfs $\Phi_{AS}$

# Summary

- analysis of $\pi^+$ electroproduction within handbag approach full pion form factor taken into account; $\widetilde{H}$, $\widetilde{E}$ modeled with double distr. ansatz
- clear indication in data for contr. from $\gamma_T^* \to \pi$ transitions amplitude $\mathcal{M}_{0-,++}$ particular important
- within handbag approach $\mathcal{M}_{0-,++}$ is fed by transversity (chiral odd) GPDs accompanied by a twist-3 pion wave fct.
- $H_T$ modeled with double distr., forward limit - transversity PDFs - taken from analysis of $A_{UT}$ for SIDIS
- relates transversity in excl. processes to transversity in inclusive data
- fair description of data
- interesting check: $\pi^- p \to \mu^+ \mu^- n$ Compass? Pire et al