Near Threshold π^0 Electroproduction at High Q^2

Puneet Khetarpal

Rensselaer Polytechnic Institute
Troy, NY

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P. Stoler · V. Kubarovsky · V. Braun
&
The CLAS Collaboration

Outline

- Motivation
- Historical background
- Theoretical predictions
- Results from CLAS
- Summary

Motivation: Theoretical and Experimental

Processes

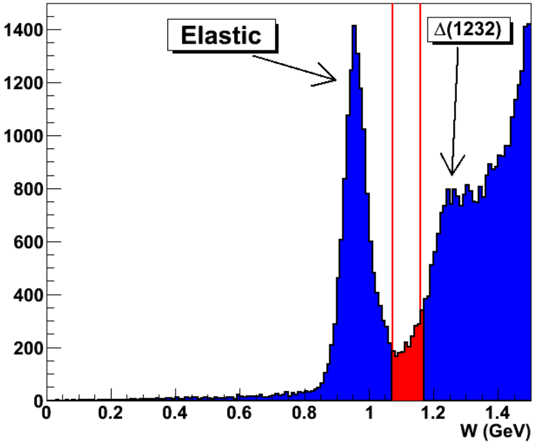
$$e + p \to e + \pi^0 + p$$
$$e + p \to e + \pi^+ + n$$

- Theoretical:
  - New extensions of low energy theorems at high Q^2 (Braun *et al*)
  - New generalized form factors $G_1(Q^2)$ and $G_2(Q^2)$
  - Axial form factor $G_A(Q^2)$: Fourier transform of the axial charge distribution in the proton that is probed via axial current

$$J^a_{5\mu} \sim \bar{q}\gamma_\mu\gamma_5\lambda^a q$$

- Experimental:
  - Previous experiments limited to $Q^2 < 1\,\mathrm{GeV}^2$
  - No data exists for Q^2 between $1 - 10\,\mathrm{GeV}^2$

Threshold



$W \in (W_{th}, 1.16)\,\text{GeV}$
$Q^2 \sim 2 - 5\,\text{GeV}^2$ (CLAS)

Historical Background: Threshold Physics

- For $Q^2 = 0\,\mathrm{GeV}^2$
  - Low-Energy Theorems (LETs) (Kroll-Ruderman, 1954)
  - *Axial form factor* - G_A

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  - Perturbative QCD (pQCD) factorization methods (Pobylitsa *et al*, 2001)

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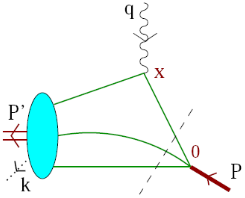
  - *Axial form factor* - G_A
- For $Q^2 \gtrsim 10\,\mathrm{GeV}^2$
  - Perturbative QCD (pQCD) factorization methods (Pobylitsa *et al*, 2001)
- For $Q^2 \sim 1 - 10\,\mathrm{GeV}^2$
  - Light Cone Sum Rule (LCSR) approach
  - Reproduce LET predictions for $Q^2 \sim 1\,\mathrm{GeV}^2$ and pQCD predictions for $Q^2 \to \infty$
  - Predictions of spatial distribution of the axial charge (G_A) and two new generalized form factors $G_1^{\pi N}$ and $G_2^{\pi N}$ (Braun *et al*, 2008)

At Threshold

Correlation function:

$$\int dx\, e^{-iqx} \langle N(P')\pi(k)|j_\mu^{\rm em}(0)|p(P)\rangle$$



At threshold, only S-wave contribution

$$\langle N(P')\pi(k)|j_\mu^{\rm em}(0)|p(P)\rangle \propto \bar{N}(P')\gamma_5 \left\{ (\gamma_\mu q^2 - q_\mu \not{q}) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^\nu}{2m_N} G_2^{\pi N}(Q^2) \right\} p(P)$$

- S-wave: generalized form factors from LCSR ($G_1^{\pi N}$ and $G_2^{\pi N}$)
- Related to S-wave multipoles: E_{0+} and L_{0+}

Braun *et al.* Phys. Rev. D (2008).

LET Form Factors: At Threshold

At threshold, the differential cross section in terms of S-wave multipoles

$$\left.\frac{d\sigma_{\gamma^*}}{d\Omega_\pi}\right|_{\rm th} \propto \left[(E_{0+}^{\pi N})^2 + \epsilon \frac{Q^2}{(\omega_{\gamma^*}^{\rm th})^2} (L_{0+}^{\pi N})^2 \right]$$

Relate the multipoles to the generalized form factors

$$E_{0+}^{\pi N} \propto \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_1^{\pi N} \qquad L_{0+}^{\pi N} \propto \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_2^{\pi N}$$

LET Form Factors: At Threshold

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LET expressions for the form factors at threshold can be related to the elastic form factors and the axial form factor:

$$\frac{Q^2}{m_N^2} G_1^{\pi^0 p} = \frac{g_A}{2} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^p \qquad \frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^n + \frac{G_A}{\sqrt{2}}$$

$$G_2^{\pi^0 p} = \frac{2 g_A m_N^2}{Q^2 + 2m_N^2} G_E^p \qquad G_2^{\pi^+ n} = \frac{2\sqrt{2} g_A m_N^2}{Q^2 + 2m_N^2} G_E^n$$

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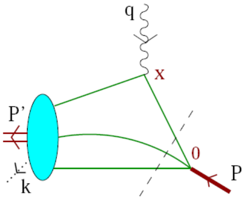
- Obtained in the chiral limit $m_\pi = 0$
- Only valid at threshold for $Q^2 \sim 1\,\text{GeV}^2$

Correlation function:

$$\int dx\, e^{-iqx} \langle N(P')\pi(k)|j_\mu^{\rm em}(0)|p(P)\rangle$$



Near threshold, S and P wave contributions

$$\langle N(P')\pi(k)|j_\mu^{\rm em}(0)|p(P)\rangle \propto \bar{N}(P')\gamma_5 \left\{ (\gamma_\mu q^2 - q_\mu \not{q}) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^\nu}{2m_N} G_2^{\pi N}(Q^2) \right\} p(P)$$
$$+ \bar{N}(P')\not{k}\gamma_5(\not{P}' + m_N) \left\{ F_1^p(Q^2) \left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2} \right) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N} F_2^p(Q^2) \right\} p(P)$$

- S-wave: generalized form factors from LCSR ($G_1^{\pi N}$ and $G_2^{\pi N}$)
- P-wave: electromagnetic form factors (F_1^p and F_2^p)
- Both S and P-wave multipoles are involved ($l = 0, 1, \cdots$)

Braun *et al.* Phys. Rev. D (2008).

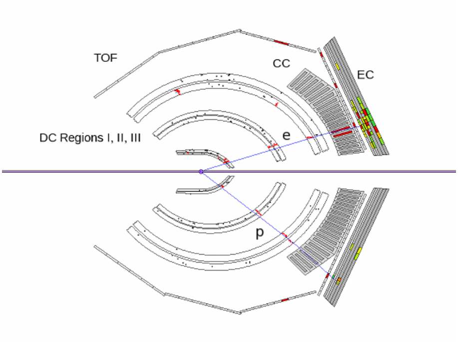
CLAS Experiment $e1$-6: Analysis

Experimental difficulties:

- Very small cross sections
- High Bethe-Heitler contamination
- Poor ϕ_π resolution because $\theta_\pi \approx \theta_{\gamma^*}$

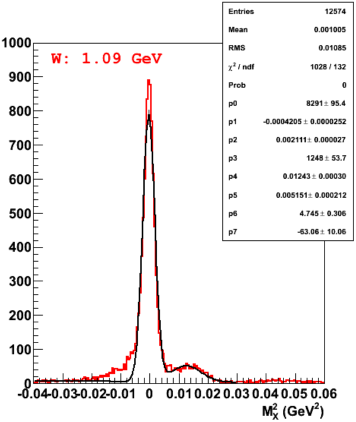
Analysis Steps:

- Electron and proton identification
- Corrections - Kinematic and Acceptance
- Pion Identification
  - Missing Mass Technique (epX)
  - Bethe-Heitler Subtractions

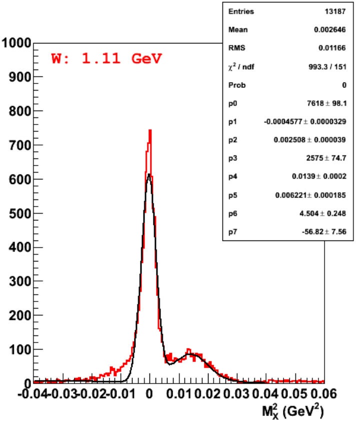


$$ep \to ep\pi^0$$

epX



Entries 12574
Mean 0.001005
RMS 0.01085
χ² / ndf 1028 / 132
Prob 0
p0 8291 ± 95.4
p1 -0.0004205 ± 0.0000252
p2 0.002111 ± 0.000027
p3 1248 ± 53.7
p4 0.01243 ± 0.00030
p5 0.005151 ± 0.000212
p6 4.745 ± 0.306
p7 -63.06 ± 10.06
W: 1.09 GeV
M²X (GeV²)

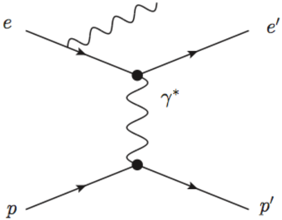


Entries 13187
Mean 0.002646
RMS 0.01166
χ² / ndf 993.3 / 151
Prob 0
p0 7618 ± 98.1
p1 -0.0004577 ± 0.0000329
p2 0.002508 ± 0.000039
p3 2575 ± 74.7
p4 0.0139 ± 0.0002
p5 0.006221 ± 0.000185
p6 4.504 ± 0.248
p7 -56.82 ± 7.56
W: 1.11 GeV
M²X (GeV²)

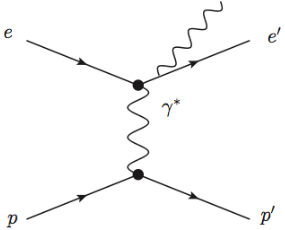
Bethe-Heitler Subtraction

Assuming elastic scattering, we can compute θ_{proton} independent of incident and scattered electron energies.

Pre-Radiation



Post-Radiation



$$\tan\theta_p^1 = \frac{1}{\left(1 + \frac{E'}{M - E' \cos\theta_e'}\right)\tan\frac{\theta_e'}{2}}$$

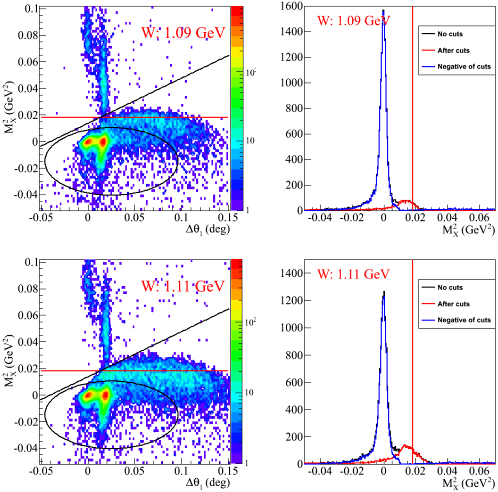
$$\tan\theta_p^2 = \frac{1}{\left(1 + \frac{E}{M}\right)\tan\frac{\theta_e'}{2}}$$

Calculate deviation of measured proton angle from elastic scattering process

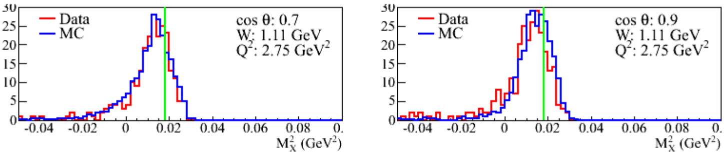
$$\Delta\theta_{1,2}^P \equiv \theta_{calc} - \theta_{meas}$$

Bethe-Heitler Subtraction (continued)

epX



After Bethe-Heitler Subtraction



- Select pions with $|M_X^2 - \mu| < 3\sigma$
- Lose pions with BH subtraction cuts
- Use simulation to estimate % of lost pions
- Correct for this loss for each kinematic bin at the cross section level

Cross Section Descriptions

Measure differential cross section for $ep \rightarrow ep\pi^0$:

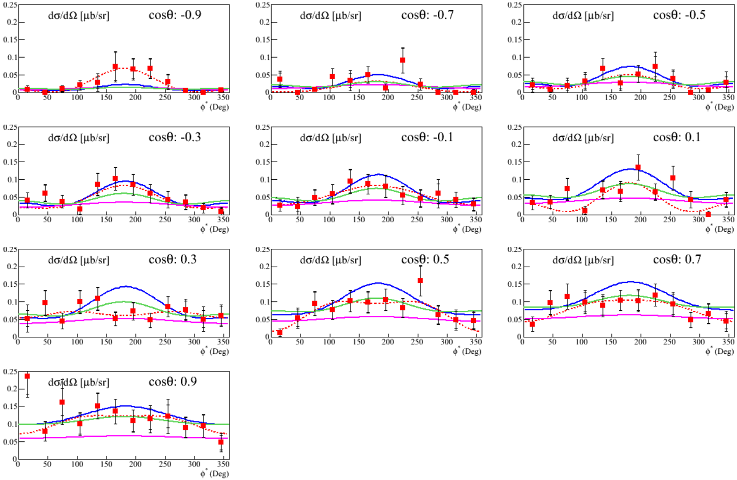
$$\frac{d\sigma}{dE' d\Omega'_e d\Omega^*_\pi} = \Gamma \frac{d\sigma}{d\Omega^*_\pi}$$

Extract structure functions from reduced differential cross section:

$$\frac{d\sigma_{\gamma^* p \rightarrow p\pi^0}}{d\Omega^*_\pi} = \frac{p^*_\pi}{k^*_\gamma} \left(\frac{d\sigma_T}{d\Omega^*} + \varepsilon_L \frac{d\sigma_L}{d\Omega^*} + \varepsilon \frac{d\sigma_{TT}}{d\Omega^*} \cos 2\phi^*_\pi + \sqrt{2\varepsilon_L(\varepsilon+1)} \frac{d\sigma_{LT}}{d\Omega^*} \cos \phi^*_\pi \right)$$

- Obtain multipoles E_{0+} and L_{0+}
- Obtain form factors $G_1^{\pi N}$ and $G_2^{\pi N}$

Differential Cross Sections - $d\sigma/d\Omega$



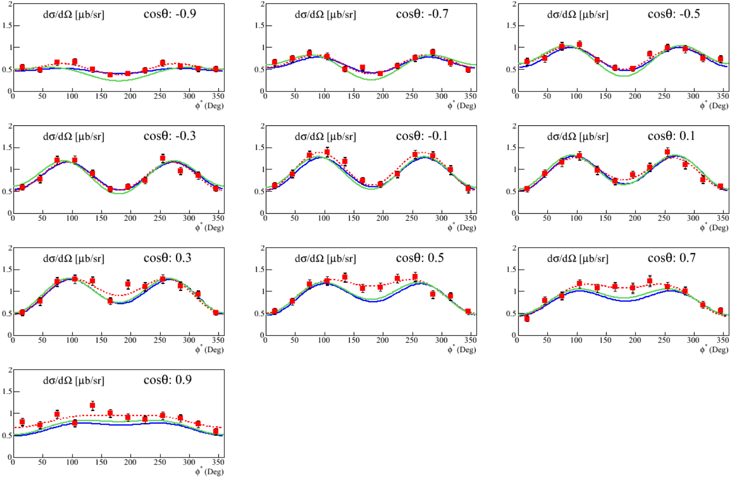
$W = 1.09$ GeV, $Q^2 = 2.75\,\mathrm{GeV}^2$

Red points: experiment (dashed curve fit) · Blue curve: MAID 2007

Magenta curve: Braun 2008 · Green curve: Aznauryan 2009

Preliminary

Differential Cross Sections - $d\sigma/d\Omega$ (continued)



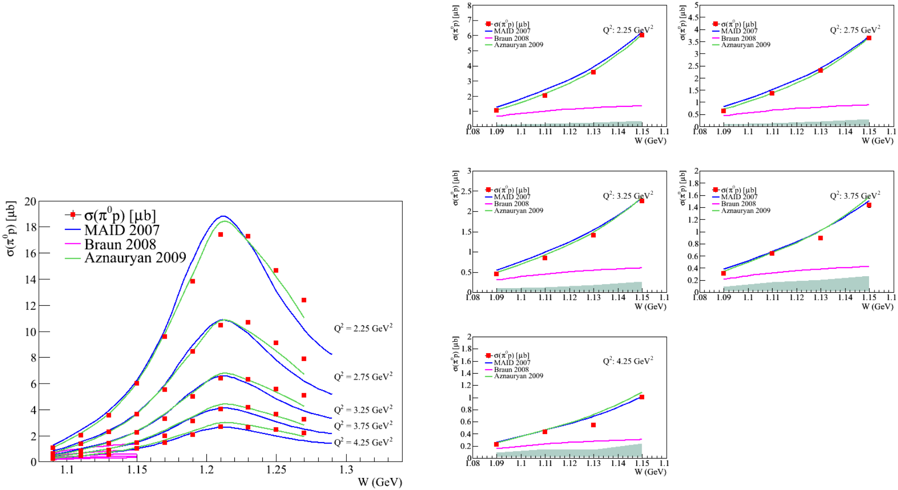
$W = 1.23$ GeV, $Q^2 = 2.75\,\text{GeV}^2$

Red points: experiment (dashed curve fit) · Blue curve: MAID 2007

Green curve: Aznauryan 2009

Preliminary

Integrated Cross Sections $\sigma(\pi^0 p)$



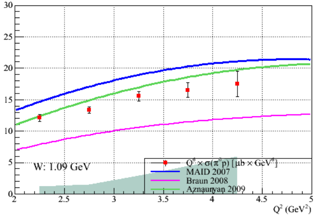
Preliminary

Red: experiment, Blue: MAID 2007, Magenta: Braun 2008, Green: Aznauryan 2009

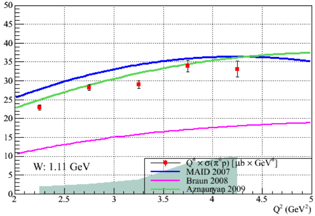
Integrated Cross Sections - $Q^6\sigma(\pi^0 p)$

$Q^6 \times \sigma(\pi^0 p)$ [$\mu b \times$ GeV^6]

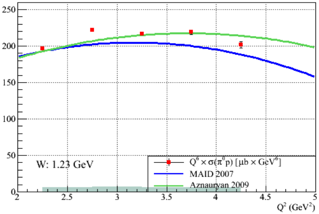
$W = 1.09$ GeV



$W = 1.11$ GeV



$W = 1.23$ GeV



Preliminary

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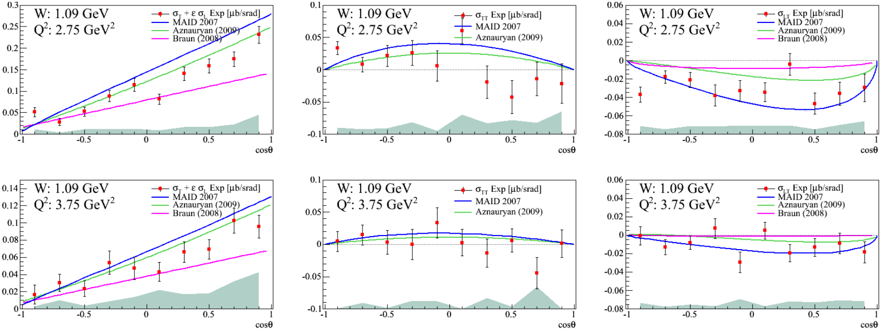


$\frac{d\sigma_T}{d\Omega} + \varepsilon \frac{d\sigma_L}{d\Omega}$ $\frac{d\sigma_{TT}}{d\Omega}$ $\frac{d\sigma_{LT}}{d\Omega}$

$$W = 1.09 \text{ GeV}$$

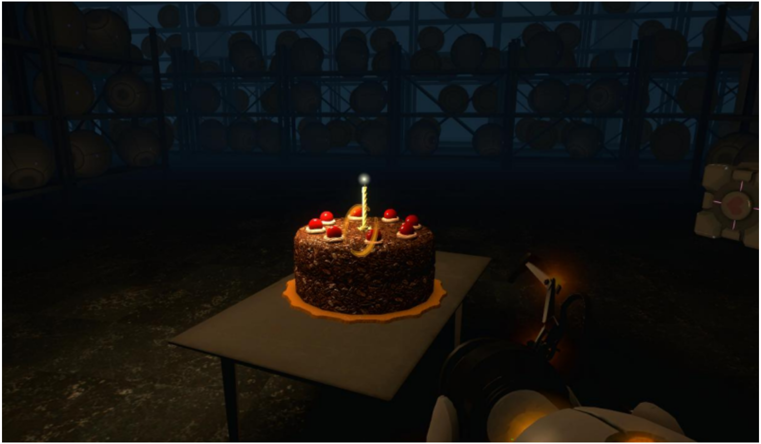
Preliminary

Red: experiment, Blue: MAID 2007, Magenta: Braun 2008, Green: Aznauryan 2009

Summary

- Pion electroproduction near threshold
- Test the applicability of LETs in $Q^2 \sim 1 - 10\,\mathrm{GeV}^2$
- Very low statistics and small cross sections - difficult
- Differential and integrated cross sections have been obtained
- Structure functions have been extracted
- Extract S-wave multipoles E_{0+} and L_{0+} that are directly related to the generalized form factors, $G_1^{\pi N}$ and $G_2^{\pi N}$, near threshold

The End



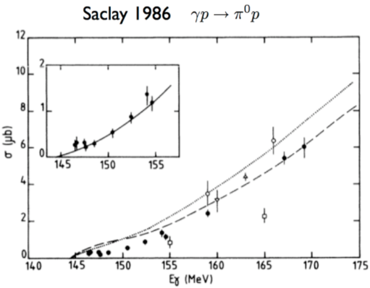
"The cake is a lie." - *Portal*

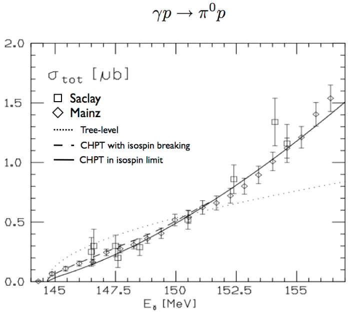
Back up slides

Backup slides

Historical background: @ threshold (continued)

Bernard *et al.* Int. J. Mod. Phys. E4 (1995) 193-346





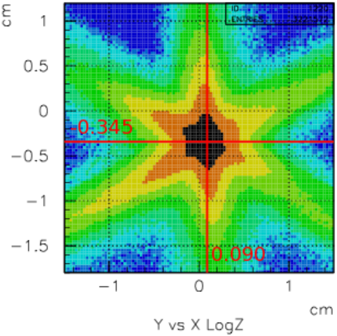
- rederived low energy theorems to $O(m_\pi^2)$
- used chiral perturbation theory
- yields better results at $Q^2 = 0$
  (Vainshtein, Zakharov, 1970s; Scherer, Koch, 1990s)

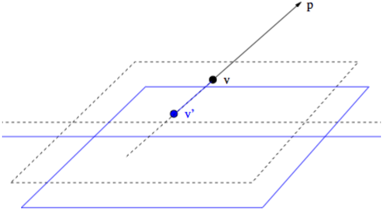
LCSR: Differential cross section

$$\frac{d\sigma_{\gamma^*}}{d\Omega_\pi} = \sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT}\cos(\phi_\pi) + \epsilon\sigma_{TT}\cos(2\phi_\pi) + \lambda\sqrt{2\epsilon(1-\epsilon)}\sigma'_{LT}\sin(\phi_\pi)$$

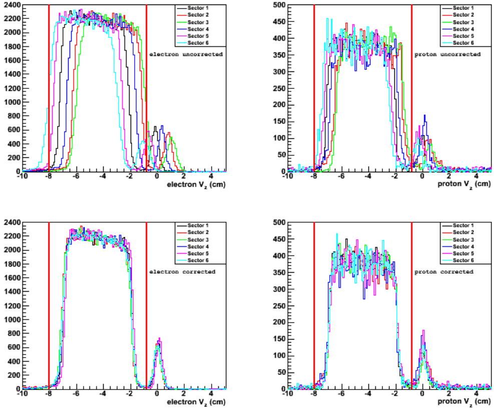
$\sigma_T \rightarrow G_1^{\pi N}, G_M^2$
$\sigma_L \rightarrow G_2^{\pi N}, G_E^2$
$\sigma_{LT} \rightarrow G_M, G_E, ReG_1^{\pi N}, ReG_2^{\pi N}$
$\sigma_{TT} = 0$
$\sigma'_{LT} \rightarrow G_M, G_E, ImG_1^{\pi N}, ImG_2^{\pi N}$

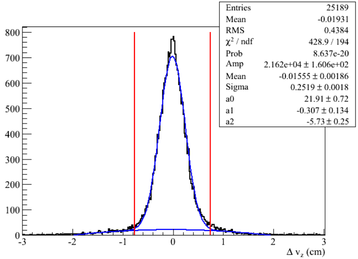
- $\sigma_{TT} = 0$: D-wave contribution neglected
- σ'_{LT}: related to single-spin symmetry





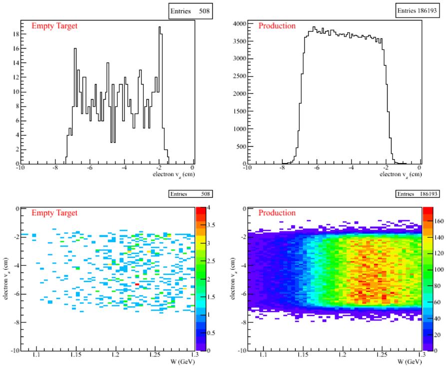
- Beam line = $(0.09, -0.345, z)$ cm
- All sector midplanes are corrected to line up with the beamline

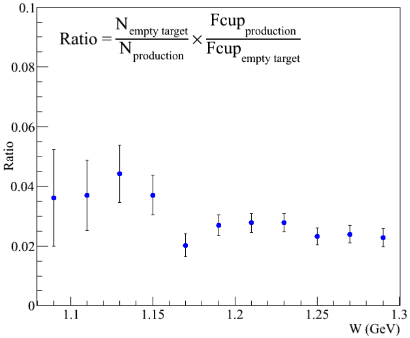




- Beam line $= (0.09, -0.345, z)$ cm
- All sector midplanes are corrected to line up with the beamline
- Cut on $V_z \in (-8.0, -0.8)$ cm
- Cut on $|\Delta V_z(e-p)| < 0.74$ cm

e1-6: Empty Target Comparison



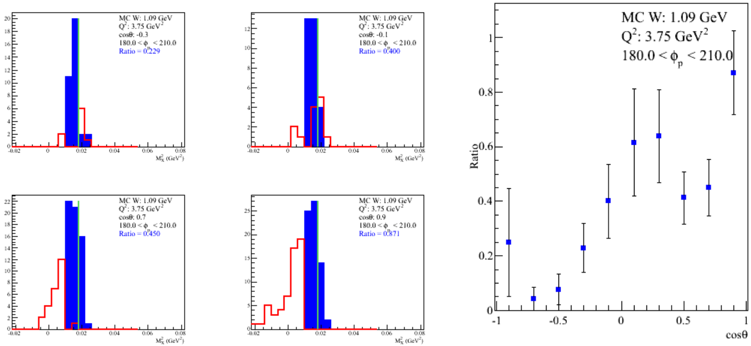


- Very loose cut on vertex $v_z \in (-8.0, -0.8)$ cm
- Include all cuts from analysis
- Total charge collected

$$\begin{aligned} \text{FCUP}_{\text{empty target}} &= 2.214\,\text{mC} \\ \text{FCUP}_{\text{production}} &= 21.287\,\text{mC} \end{aligned}$$

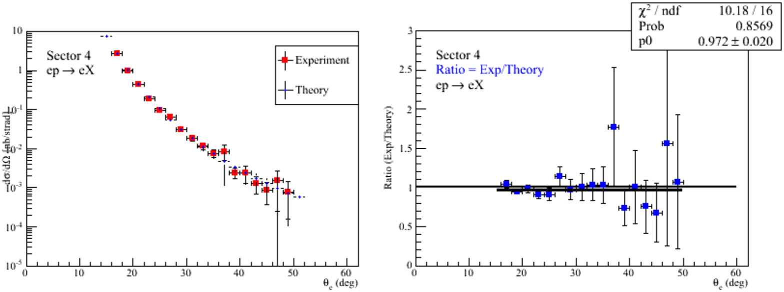
- Total contamination $\sim 2 - 5\%$

e1-6: Pion Loss Estimation

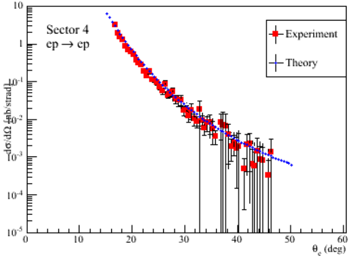


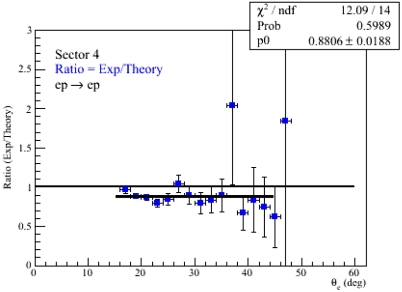
- BH cleanup cuts throw away good pions
- Estimate the pion loss using simulation: MAID 2007 model
- Ratio = Thrown/ Kept
- Ratio is highest at $\cos\theta^*_\pi \to 1$ and $\phi^*_{proton} \approx 180°$ ($i.e.$, $\phi^*_\pi \approx 0°$)
- Apply correction to each kinematic bin near threshold

$e1$-6: Elastic Cross Section ($ep \to eX$)

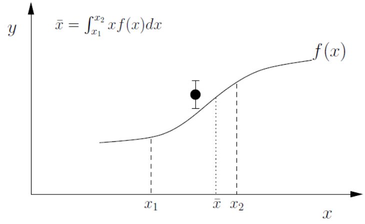


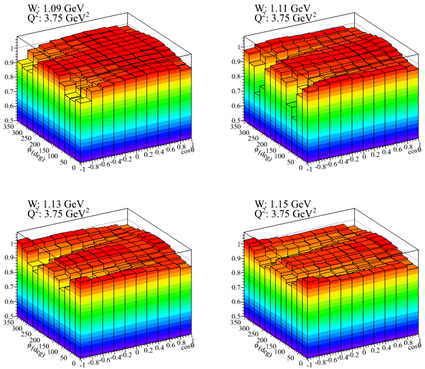
- Inclusive reaction: $ep \to eX$
- Compare with Bosted parameterization of form factors - Phys. Rev. C 51, 409 (1995)
- Good agreement within $\pm 5\%$





- Exclusive reaction: $ep \to ep$
- Proton detection inefficiencies
- Ad hoc overall correction $\sim 10\%$

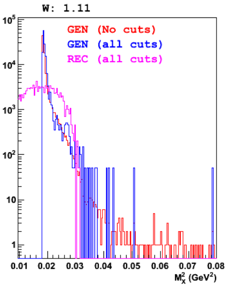


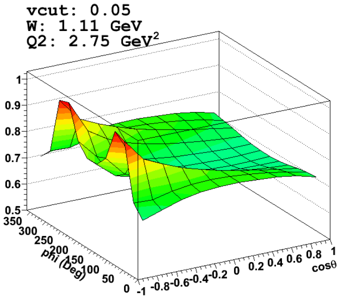


- The cross sections vary inside bin
- Center of bin is not true center
- Correction factor:

$$R_{W,Q^2,\cos\theta,\phi} = \frac{\sigma_{center}}{\sigma_{average}}$$

- Average overall correction $\sim 10\%$



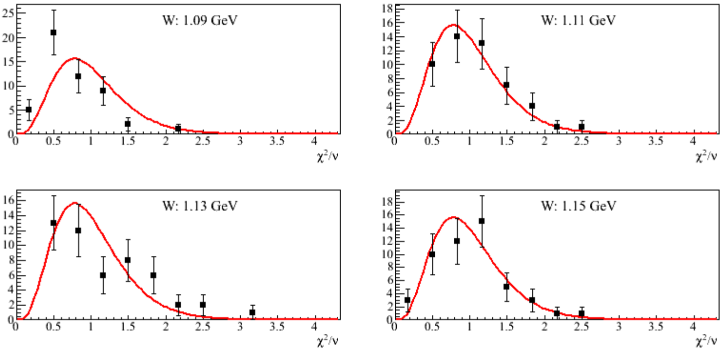


Preliminary

$$\sigma_{born} = \frac{\sigma_{meas}}{RC} = \sigma_{meas}\left(\frac{RAD_{gen}}{RAD_{rec}}\right) \times \left(\frac{NORAD_{gen}}{RAD_{gen}}\right) \quad (1)$$

- Corrections obtained from EXCLURAD using MAID 2007 model
- Radiative correction independent of vcut near threshold
- Correction is largest at $\cos\theta \sim -0.9$ because pion cross section goes to zero and BH dominates; is ϕ dependent
- About 20% overall correction

e1-6: χ^2 Distributions



$$\frac{d\sigma}{d\Omega_\pi^*} = A + B\varepsilon\cos 2\phi_\pi^* + C[2\varepsilon_L(\varepsilon+1)]^{1/2}\cos\phi_\pi^*$$

12 points in ϕ_π^* $-$ 3 parameters = 9 d.o.f.
Total number of fits = 5 Q^2 bins $\times$ 10 $\cos\theta_\pi^*$ bins = 50.