Measurement of Generalized Form Factors near the Pion Threshold In high Q2

May. 18-21, 2010
Exclusive Reactions @ high Q2

Kijun Park

Perspective of soft pion in terms of Q^2 at threshold

$Q^2=0$ GeV^2

Low-Energy Theorem (LET) for $Q^2=0$
Restriction to the charged pion
Chiral symmetry + current algebra for electroproduction

1954 Kroll-Ruderman

1960s Nambu, Laurie, Schrauner

$Q^2 << \Lambda/m_\pi \sim 1$ GeV^2

Re-derived LETs
Current algebra + PCAC
Chiral perturbation theory

1970s Vainshtein, Zakharov

1990s Scherer, Koch

$Q^2 \sim 1 - 10$ GeV^2

???

$Q^2 >> \Lambda/m_\pi$

pQCD factorization methods
Brodsky, Lepage, Efremov, Radyunshkin, Pobylitsa, Polyakov, Strikman, et al

LCSR (Light Cone Sum Rule)

$$\langle N(P')\pi(k)|j_\mu^{em}(0)|p(P)\rangle = -\frac{i}{f_\pi}\bar{N}(P')\gamma_5\left\{(\gamma_\mu q^2 - q_\mu \not{q})\frac{1}{m_N^2}G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}G_2^{\pi N}(Q^2)\right\}p(P)$$

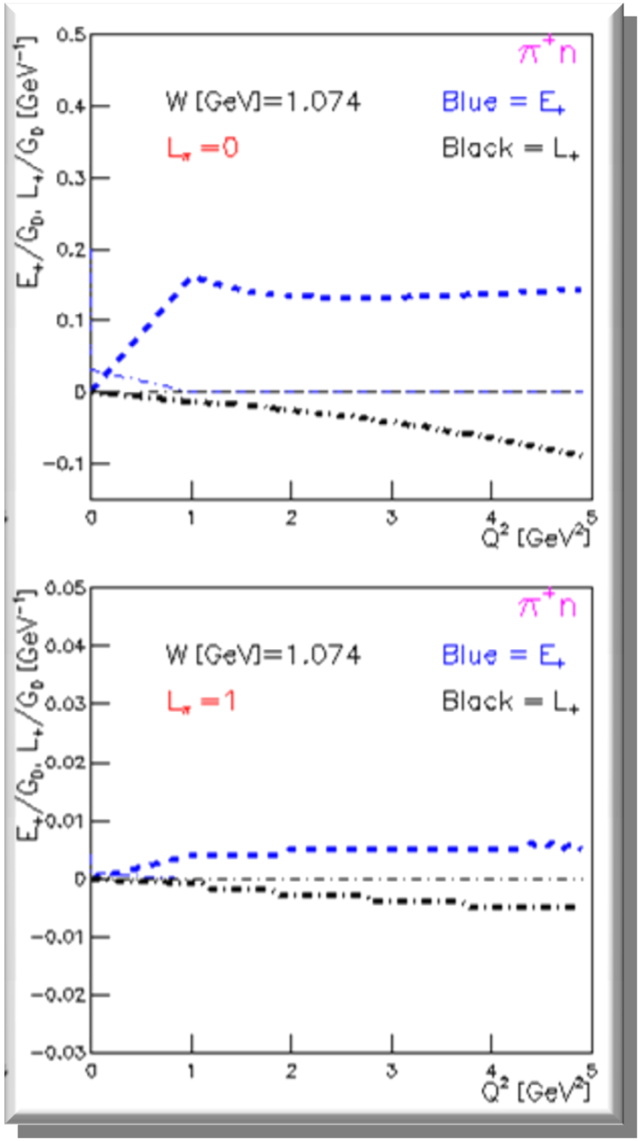
$$+\frac{ic_\pi g_A}{2f_\pi[(P'+k)^2 - m_N^2]}\bar{N}(P')\not{k}\gamma_5(\not{P}' + m_N)\left\{F_1^p(Q^2)\left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2}\right) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2^p(Q^2)\right\}p(P)$$

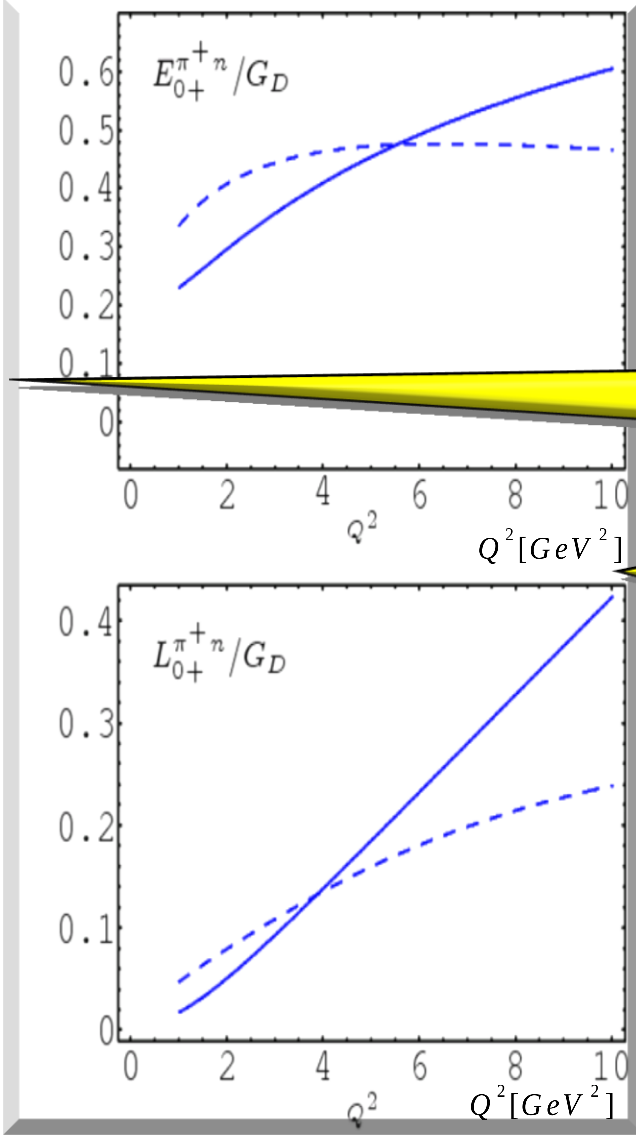
- S-wave: generalized form factors from LCSR ($G_1^{\pi N}$ and $G_2^{\pi N}$)
- P-wave: pion emission from final state nucleon

- Constructed relating the amplitude for the radiative decay of $\Sigma^+(p\gamma)$ to properties of the QCD vacuum in alternating magnetic field.
- An advantage of study because soft contribution to hadron form factor can be calculated in terms of DA's that enter pQCD calculation without other nonperturbative parameters.
- New technique : the expansion of the standard QCD sum rule approach to hadron properties in alternating external fields.

Prediction

LCSR vs. MAID





MAID2007
Bold = real part
Thin = imaginary

V. M. Braun et al., Phys. Rev. D 77:034016, 2008.

symbol index

Dashed Lines : pure LCSR

Solid Lines : LCSR using experimental EM form factor as input

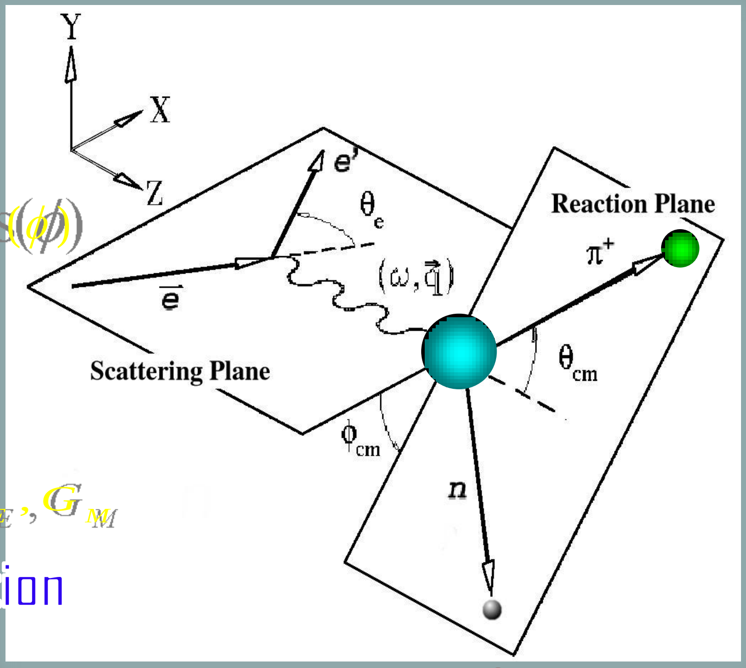
Thomas Jefferson National Accelerator Facility

CEBAF Large Acceptance Spectrometers

Kinematical Coverage — Differential Cross Section

$$d\sigma_{\gamma^*} = \frac{\alpha_{em}}{8\pi}\frac{k_f}{W}\frac{d\Omega_\pi}{W^2 - m_N^2}|\sigma|^2$$

$$|\sigma|^2 = \sigma_T + \varepsilon\sigma_L + \varepsilon\sigma_{TT}\cos(2\phi) + \sqrt{2\varepsilon(1+\varepsilon)}\sigma_{LT}\cos(\phi) + \lambda\sqrt{2\varepsilon(1-\varepsilon)}\sigma_{LT'}\sin(\phi)$$



$\sigma_T \rightarrow G_1^{\pi N}, G_M^2$ $\sigma_{LT} \rightarrow \mathrm{Re}\,G_1^{\pi N}, \mathrm{Re}\,G_2^{\pi N}, G_E, G_M$

$\sigma_L \rightarrow G_2^{\pi N}, G_E^2$ $\sigma_{TT} = 0$ No D-wave contribution

Variable	Unit	Range	# Bin	Width
Q^2	GeV^2	2.05 ~ 4.16	5	various
W	GeV	1.11 ~ 1.15	3	0.02
$\cos\theta^*_\pi$		-1.0~ 1.0	10	0.2
φ^*_π	Deg.	0. ~ 360.	12	30

* E=5.754GeV(pol.), LH2 target (unpol.)
* I_B=3375/6000A, Oct. 2001-Jan. 2002

Legendre moments vs. Form Factors

V. Braun PRD 77(2008)

$$G_1^{\pi N} \qquad G_2^{\pi N} \qquad\qquad G_M \quad G_E$$

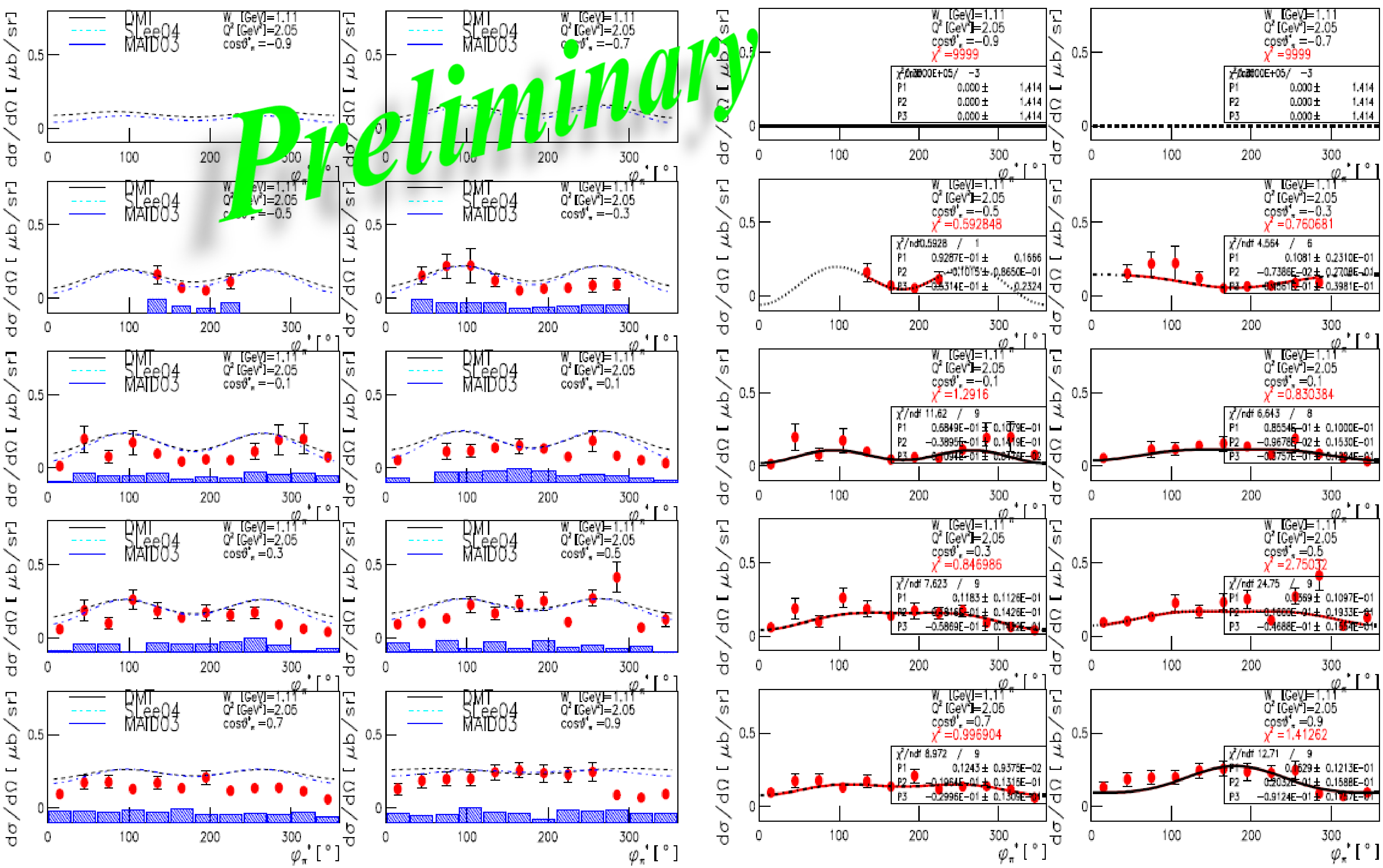
$$A_0 = D_0^{T+L} = \frac{1}{f_\pi^2}\left[\frac{4\vec{k}_i^2 Q^2}{m_N^2}\left|G_1^{\pi N}\right|^2 + \frac{c_\pi^2 g_A^2 \vec{k}_f^2}{W^2 - m_N^2} Q^2 m_N^2 G_M^2 + \varepsilon_L\left(\vec{k}_i^2\left|G_2^{\pi N}\right|^2 + \frac{4 c_\pi^2 g_A^2 \vec{k}_f^2}{W^2 - m_N^2} m_N^4 G_E^2\right)\right]$$

$$A_1 = D_1^{T+L} = \frac{1}{f_\pi^2}\frac{4 c_\pi g_A |k_i||k_f|}{W^2 - m_N^2}\left(Q^2 G_M \,\mathrm{Re}\left(G_1^{\pi N}\right) - \varepsilon_L m_N^2 G_E \,\mathrm{Re}\left(G_2^{\pi N}\right)\right)$$

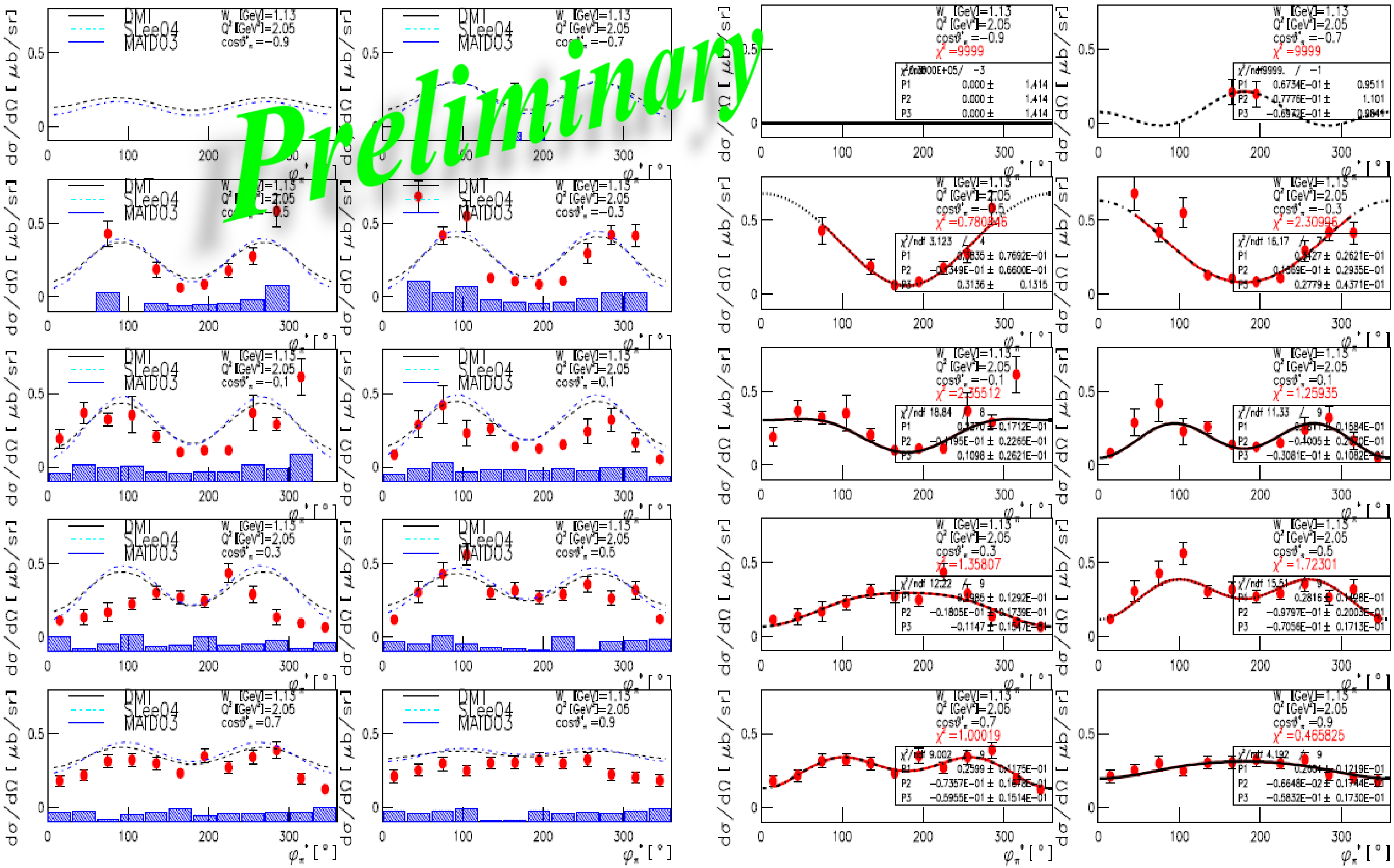
$$C_0 = C_0^{TT} = 0$$

$$D_0 = D_0^{LT} = -\frac{1}{f_\pi^2}\frac{c_\pi g_A |k_i||k_f|}{W^2 - m_N^2} Q m_N\left(G_M \,\mathrm{Re}\left(G_2^{\pi N}\right) + 4 G_E \,\mathrm{Re}\left(G_1^{\pi N}\right)\right)$$

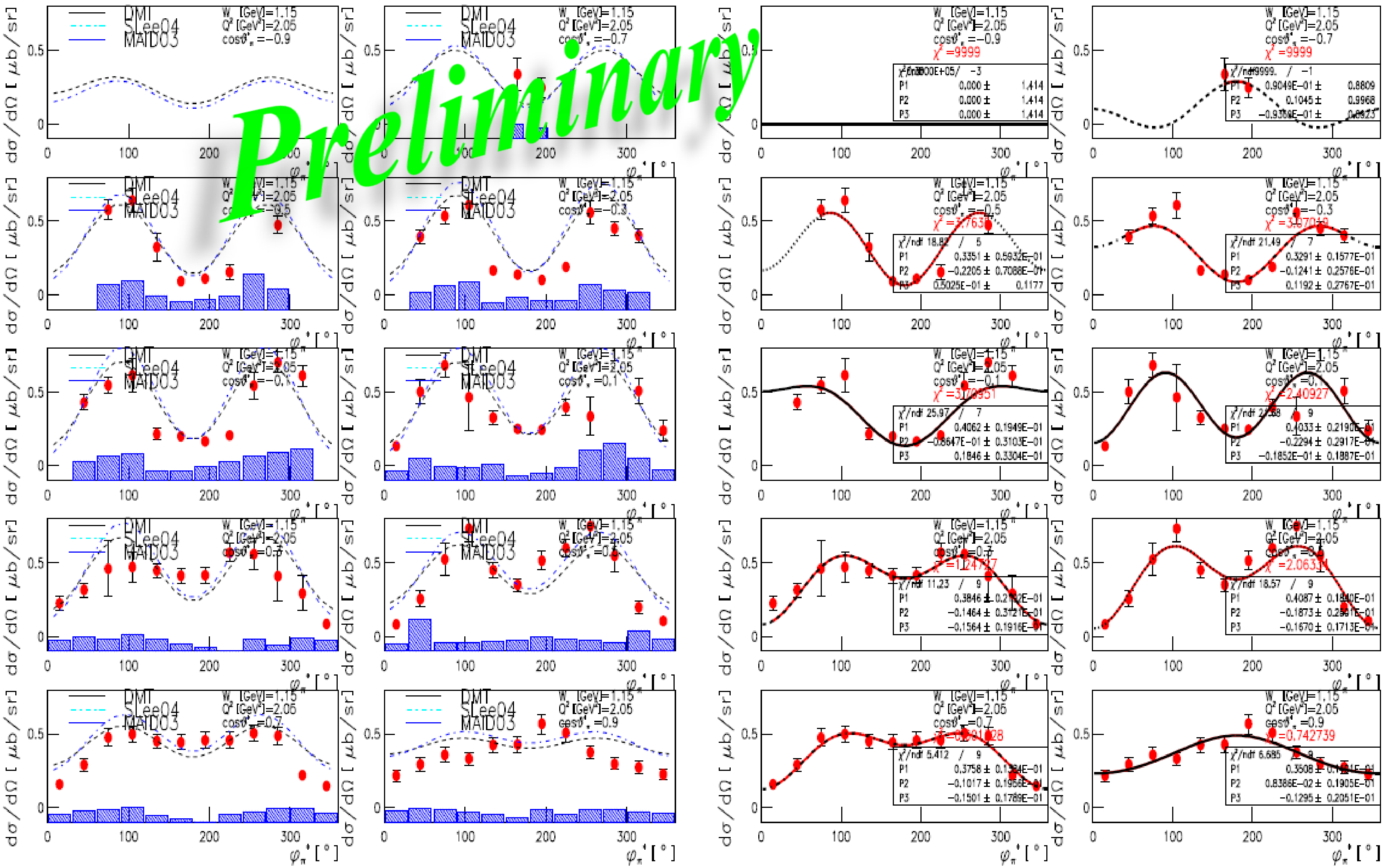
Preliminary differential cross sections



Preliminary differential cross sections

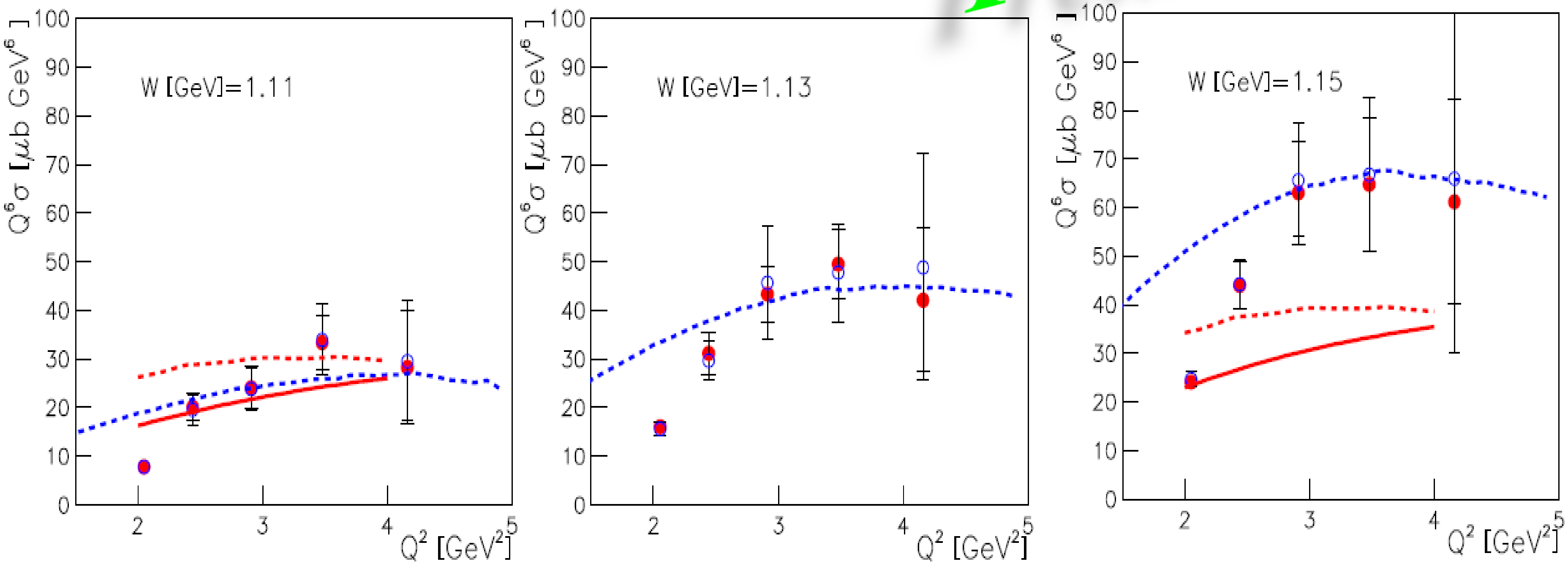


Preliminary differential cross sections



Mutipole extraction

Preliminary



Red solid line = LCSR pure

Red dash line = LCSR + exp. F.F.

Blue dash line = MAID2007

Red lines : LCSR
solid line : pure calc.
dash line :exp. F. F. input

Blue line : MAID07, E0+
Black MAId07 L0+

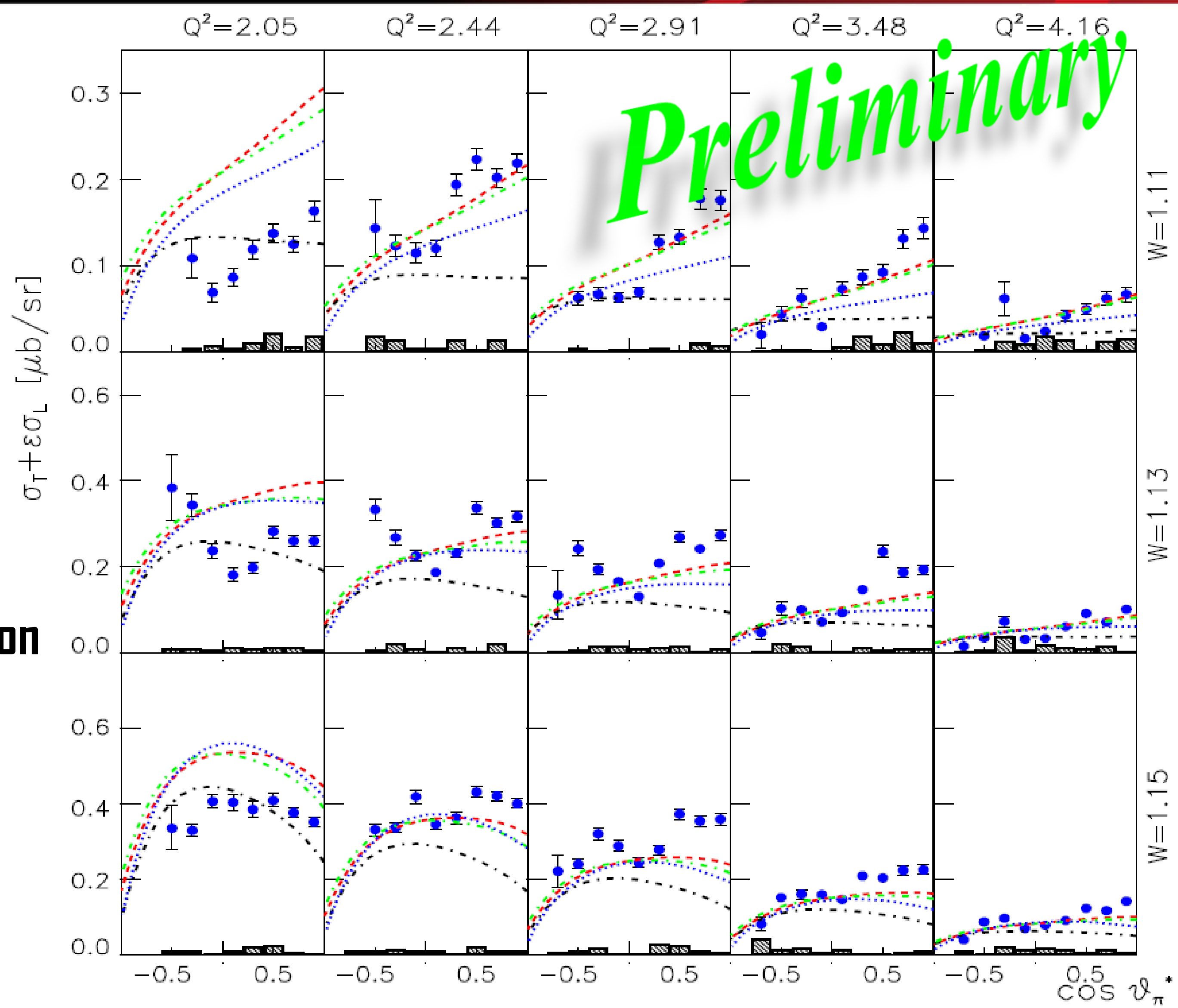
Blue open circle (o) = RC corr. w/ SLee04

Red solid circle (o) = RC corr. w/ MAID03

Structure functions

$\sigma_T + \varepsilon \sigma_L$

E_{0+} sensitive !



Color index

Red : full MAID calculation

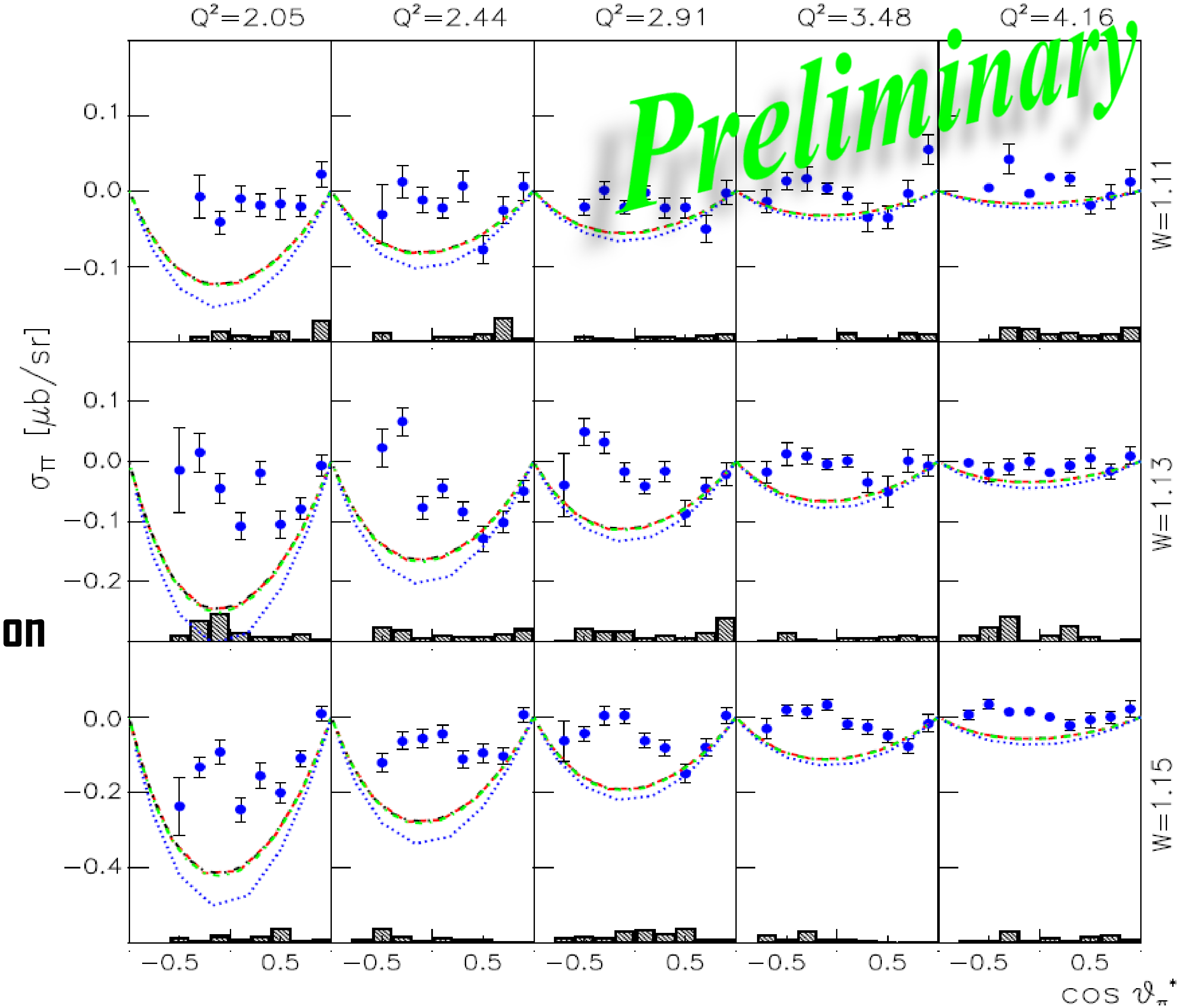
Green : S0+ absence

Black : E0+ absence

Structure functions

σ_{TT}

E_{0+} insensitive !



Color index

Red : full MAID calculation

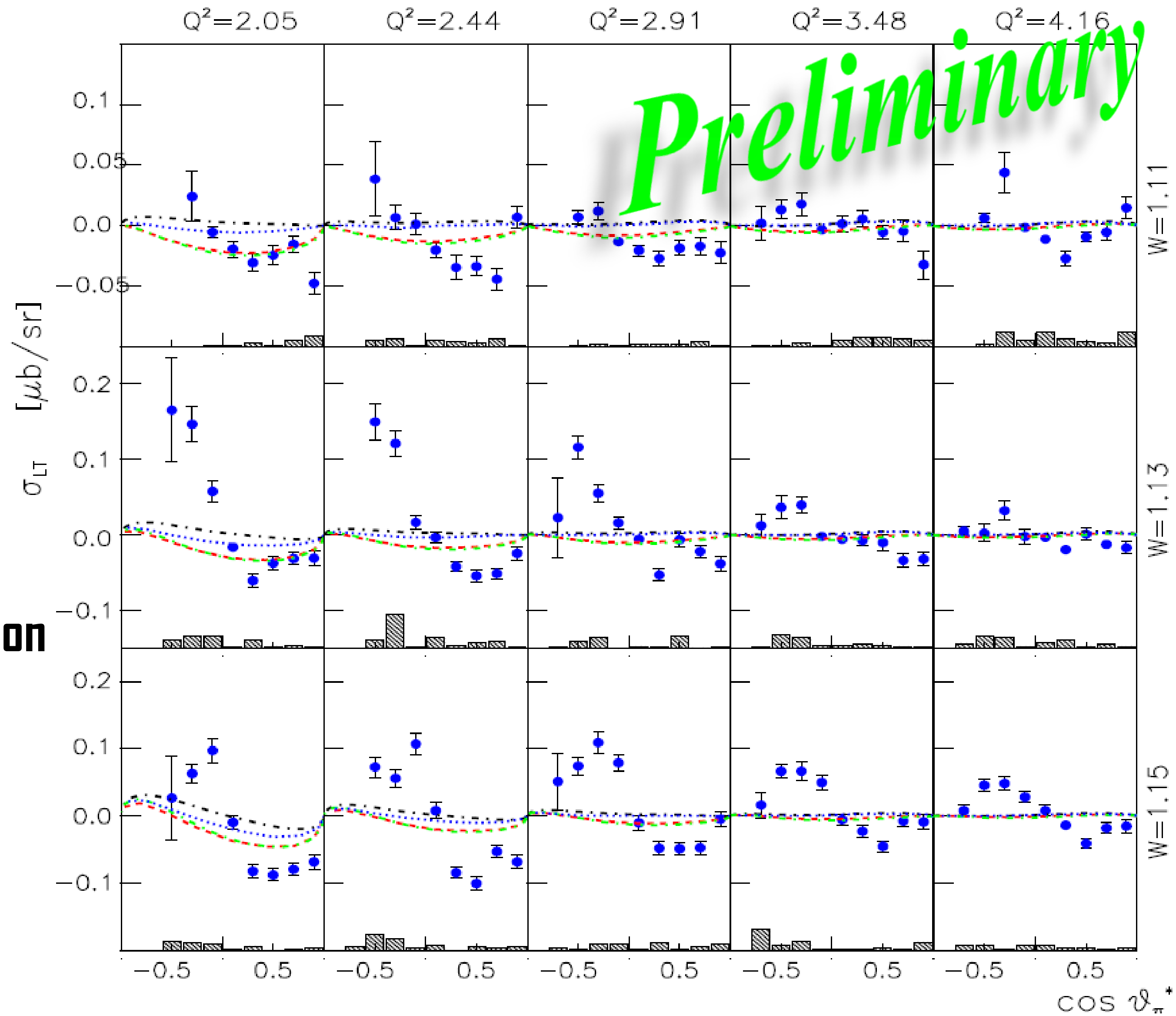
Green : S0+ absence

Black : E0+ absence

Structure functions

σ_{LT}

E_{0+} sensitive !



Color index

Red : full MAID calculation

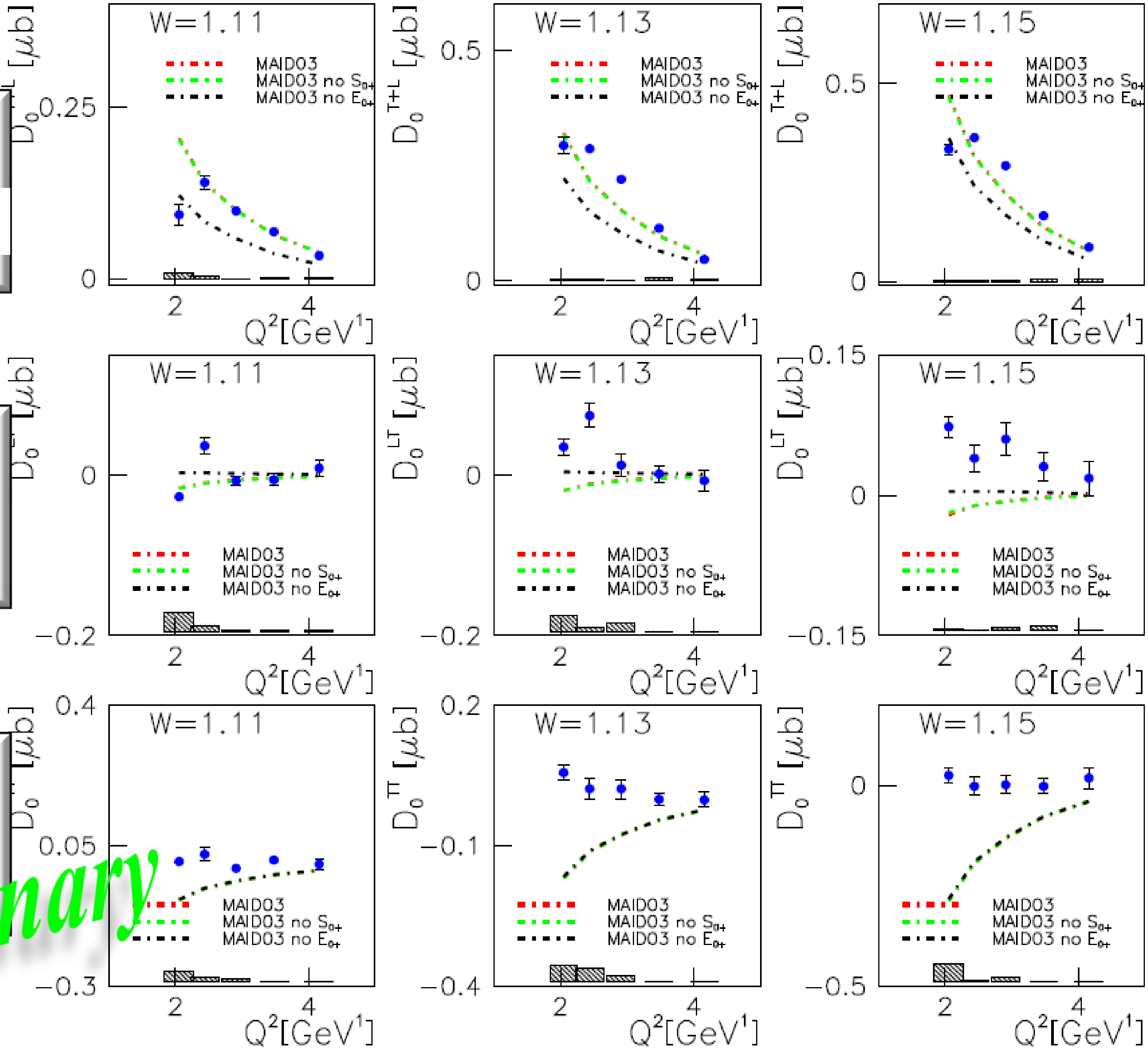
Green : S0+ absence

Black : E0+ absence

$$\sigma_T + \varepsilon_L \sigma_L = D_0^{T+L} + D_1^{T+L} P_1(\cos\theta_\pi^*)$$

$$\sigma_{LT} = D_0^{LT} + D_1^{LT} P_1(\cos\theta_\pi^*)$$

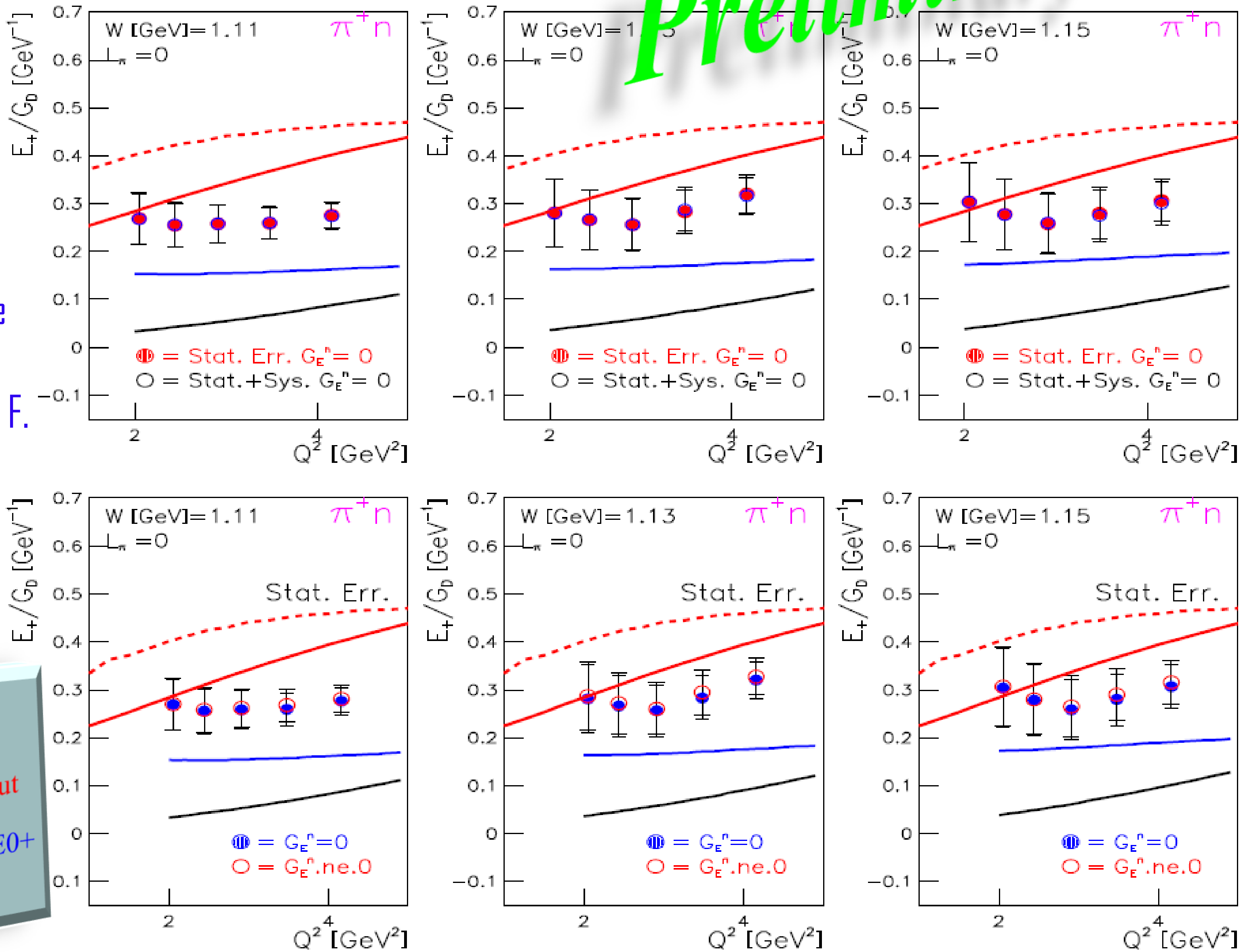
$$\sigma_{TT} = D_0^{TT}$$



Legendre –moments

Mutipole extraction

Preliminary



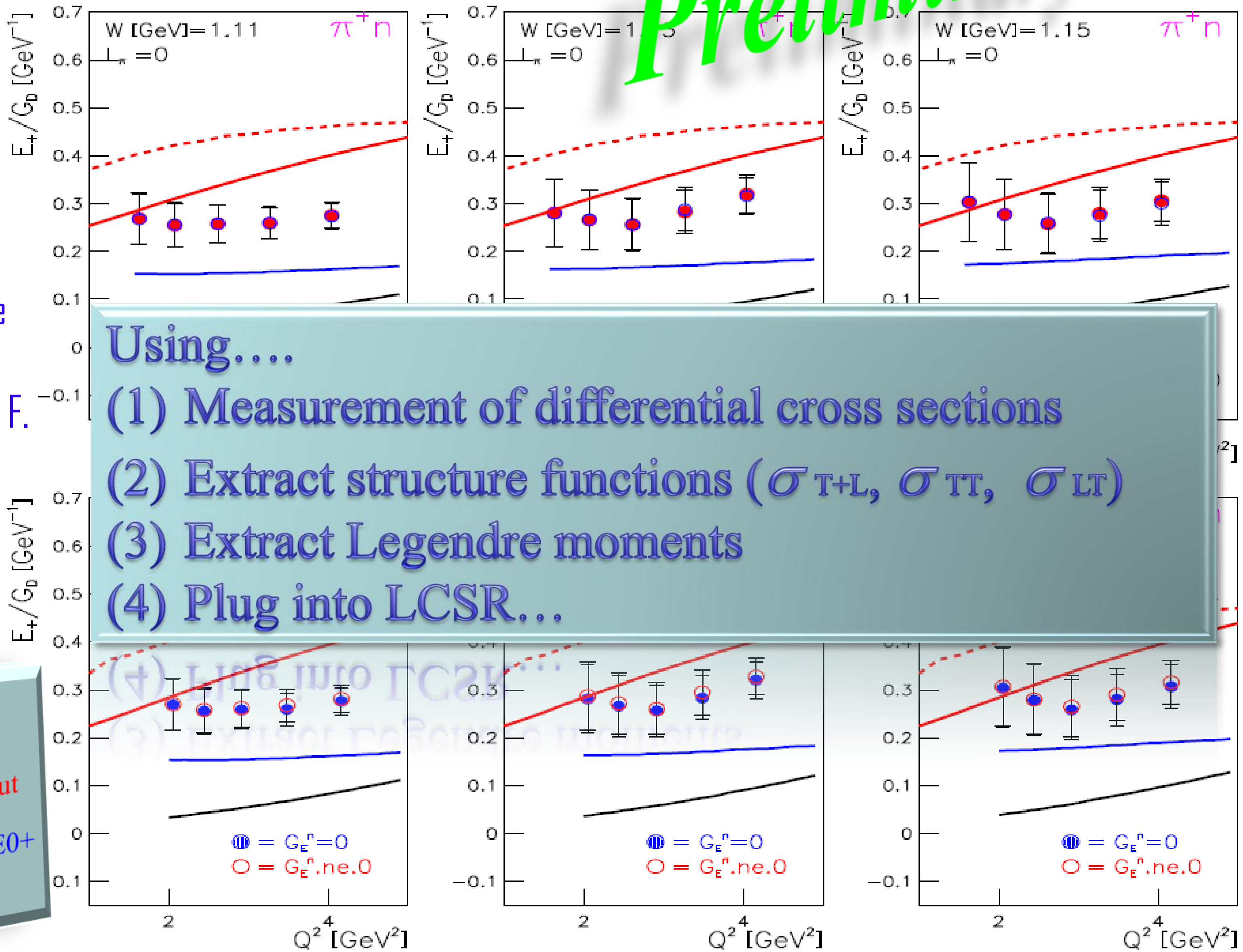
Q^2 dependence of the Normalized E_{0+} Multipole by dipole F. F.

Red lines : LCSR
solid line : pure calc.
dash line :exp. F. F. input

Blue line : MAID07, E0+
Black MAId07 L0+

Mutipole extraction

Preliminary



Q^2 dependence of the Normalized E_{0+} Multipole by dipole F. F.

Using….

(1) Measurement of differential cross sections

(2) Extract structure functions (σ_{T+L}, σ_{TT}, σ_{LT})

(3) Extract Legendre moments

(4) Plug into LCSR…

Red lines : LCSR
solid line : pure calc.
dash line :exp. F. F. input

Blue line : MAID07, E0+
Black MAId07 L0+

Form factors and Multipole for $n\pi^+$ channel

$$G_1^{\pi N} = G_1^{\pi^+ n} \quad G_M = G_M^n \approx \mu_n G_D(Q^2)$$

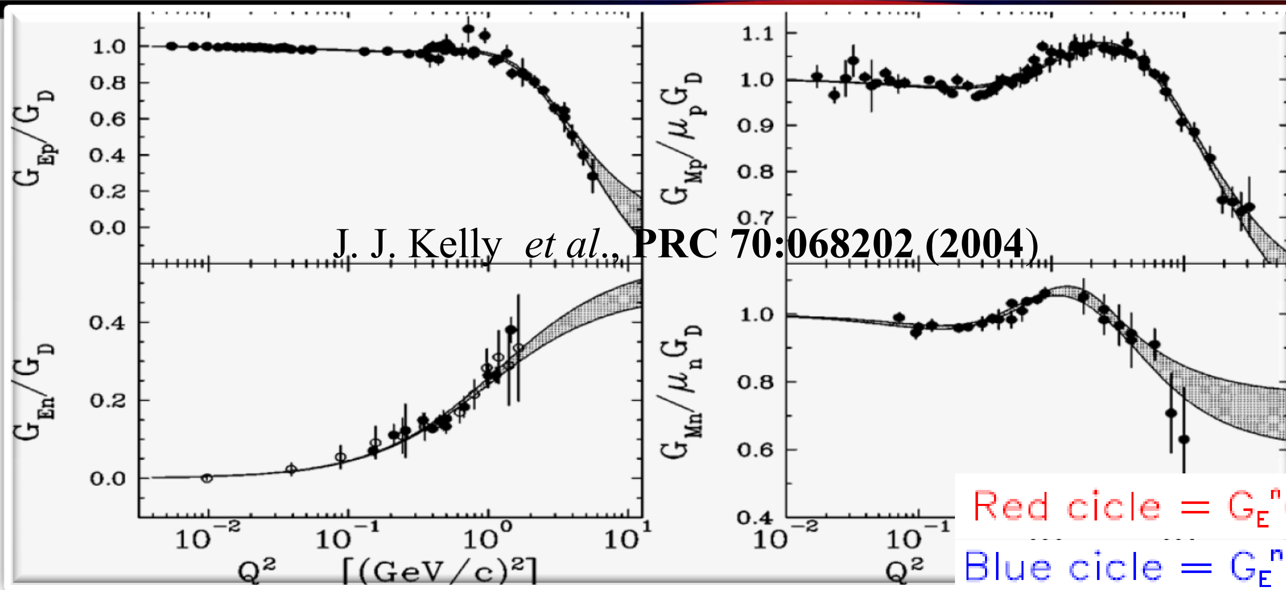
$$G_2^{\pi N} = G_2^{\pi^+ n} \quad G_E = G_E^n \approx 0 \quad G_E = G_E^n \neq 0$$

P.E. Bosted
Phys. Rev. C 51 (1995)

S. Platchekov
Nucl.Phys. A 70 (1990)

J.J. Kelly
Phys. Rev. C 70 (2004)

Form factors and Multipole for $n\pi^+$ channel



P.E. Bosted
Phys. Rev. C 51 (1995)

S. Platchekov
Nucl.Phys. A 70 (1990)

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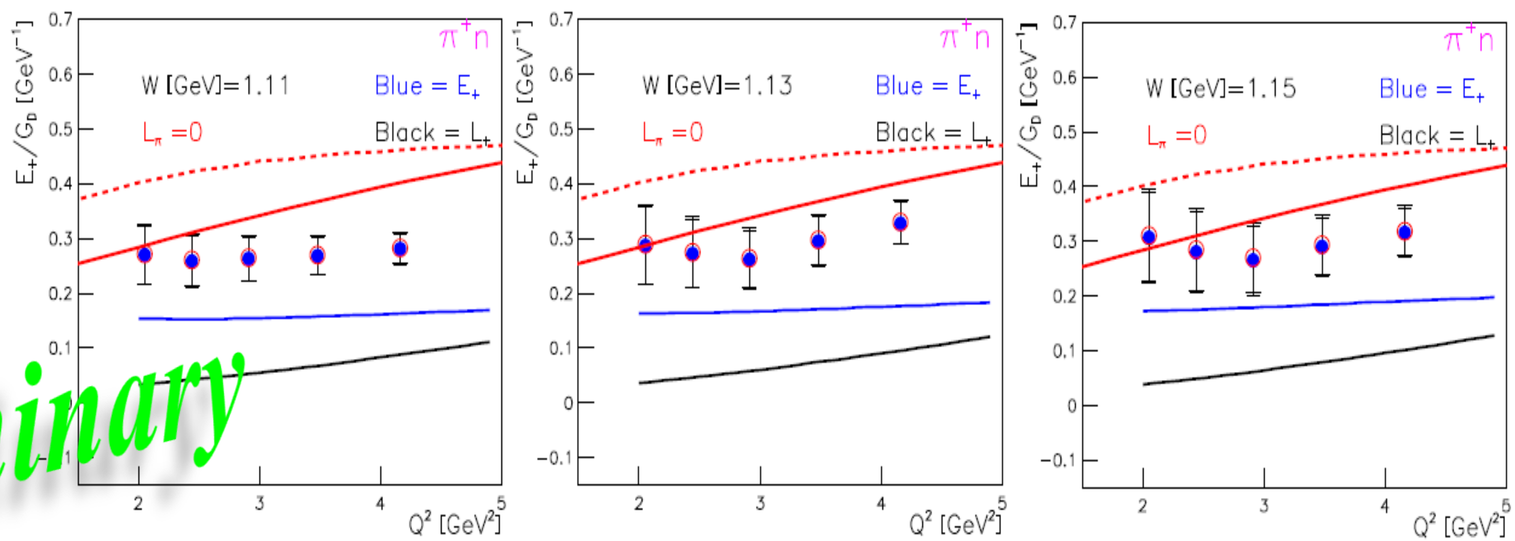
Red cicle = $G_E^n(Q^2) = -a\mu_n\tau G_D(Q^2)/(1+b\tau)$

Blue cicle = $G_E^n(Q^2) = A\tau/(1+B\tau)\ G_D(Q^2)$



Preliminary

Form factors and Multipole for $n\pi^+$ channel

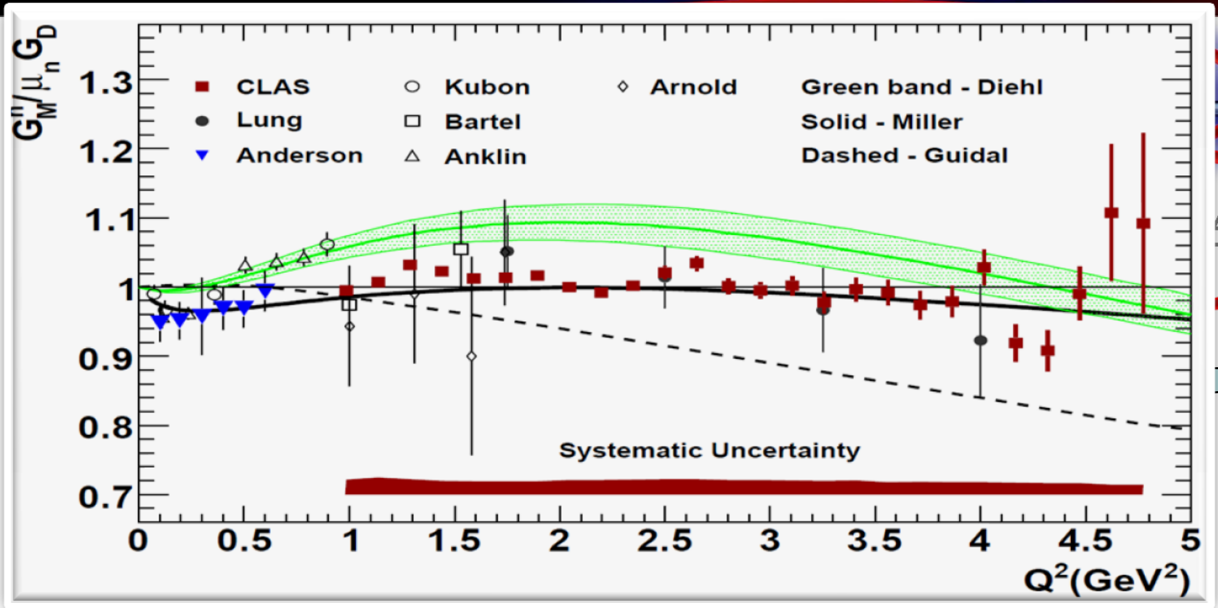
$G_1^{\pi N} = G_1^{\pi^+ n}$ G_M = CLAS DATA

$G_2^{\pi N} = G_2^{\pi^+ n}$ $G_E = G_E^n \neq 0$

J. Lachniet (2009)
Phys. Rev. Lett. 102

J.J. Kelly
Phys. Rev. C 70 (2004)

Form factors and Multipole for $n\pi^+$ channel



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Phys. Rev. Lett. 102

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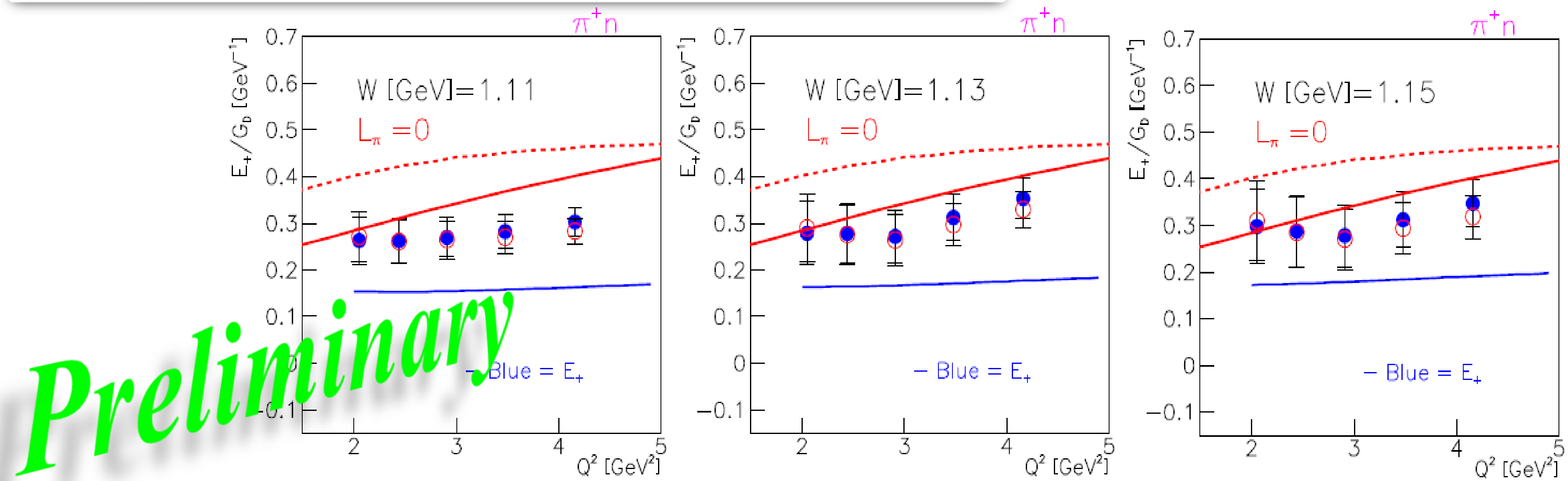


Multipoles Analysis

I. G. Aznauryan, PRD 57, 2727 (1998)

Using six amplitudes (F_i) :
** if $I_\pi = 1$

$F_1 = E_{0+} + 3*\cos(\theta)*(E_{1+} + M_{1+})$

$F_2 = 2*M_{1+} + M_{1-}$

$F_3 = 3*(E_{1+} - M_{1+})$

$F_4 = 0$

$F_5 = S_{0+} + 6*\cos(\theta)*S_{1+}$

$F_6 = S_{1-} - 2*S_{1+}$

Helicity amplitudes (H_i) :

$H_1 = (-1/\mathrm{sqrt}(2))*\cos(\theta/2)*\sin(\theta)*(F_3 + F_4)$

$H_2 = -1*\mathrm{sqrt}(2)*\cos(\theta/2)*(F_1 - F_2 - \sin(\theta)*(F_3 - F_4))$

$H_3 = (1/\mathrm{sqrt}(2))*\sin(\theta/2)*\sin(\theta)*(F_3 - F_4)$

$H_4 = \mathrm{sqrt}(2)*\sin(\theta/2)*(F_1 + F_2 +(\cos(\theta/2))**2*(F_3 + F_4))$

$H_5 = -1*(\mathrm{sqrt}(Q2)/\mathrm{abs}(k\_cm))*\cos(\theta/2)*(F_5 + F_6)$

$H_6 = (\mathrm{sqrt}(Q2)/\mathrm{abs}(k\_cm))*\sin(\theta/2)*(F_5 - F_6)$

Structure functions vs. Helicity amplitudes (H_i) :

$\sigma_{T+L} = (1/2)*(H_1^2 + (H_2^2)*(H_3^2) + H_4^2) + \varepsilon*(H_5^2 + H_6^2)$

$\sigma_{TT} = H_3*H_2 - H_4*H_1$

$\sigma_{LT} = (-1/\mathrm{sqrt}(2))*(H_5*(H_1 - H_4) + H_6*(H_2 + H_3))$

Constraints :

* E_{0+}, S_{0+} are dominated in this regime.

** M_{1-}, S_{1-} were used from MAID2007 model prediction.

→ $G_{D'}$ =(1+Q2/mu_02)^2

→ GM = 3.*exp(-0.21*Q2)/(1. +0.0273*Q2 -0.0086*Q2^2)/$G_{D'}$

→ M_{1+} = (Y_0/52.437)* GM * sqrt(((2.3933+Q2)/2.46)**2-0.88)* 6.786

→ E_{1+} = -0.02 * M_{1+}

→ R_sm =-6.066 -8.5639*Q2 +2.3706*Q2^2 +5.807* sqrt(Q2) -0.75445* Q2^2* sqrt(Q2)

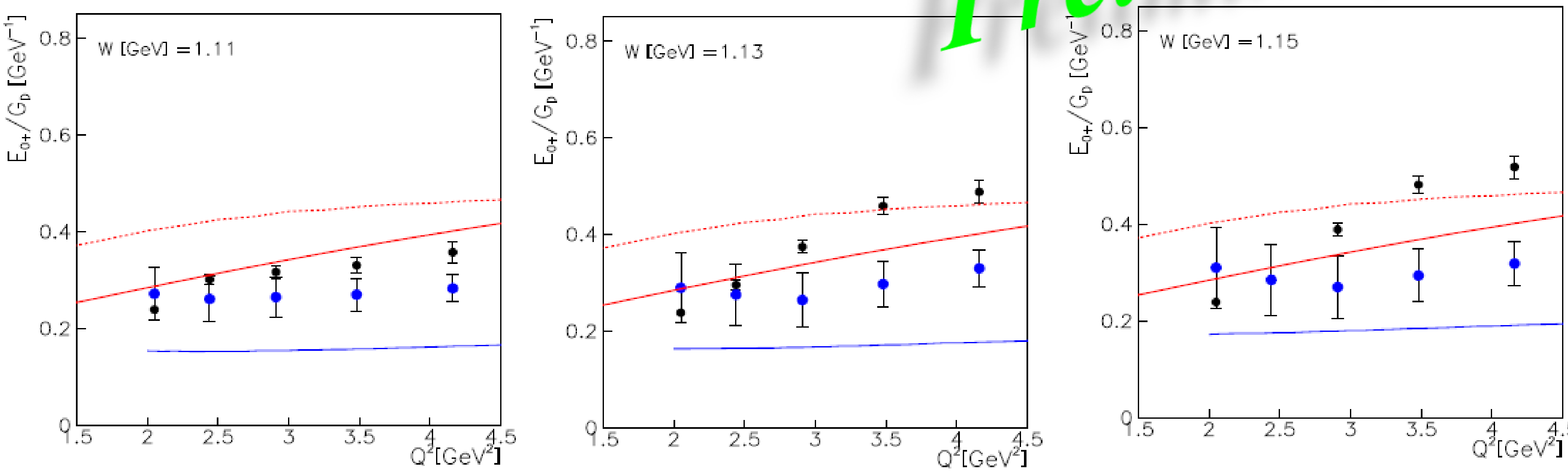
→ S_{1+} = R_sm*M_{1+}/100.

where, mu_02=0.71, Y_0 is the interpolation value from SAID model.

Multipoles extraction

Q^2 dependence of the Normalized E_{0+} , L_{0+} and E_{1+} Multipole by dipole F. F.

Preliminary



Red lines : LCSR
solid line : pure calc.
dash line : exp. F. F. input

Blue line : MAID07, E0+

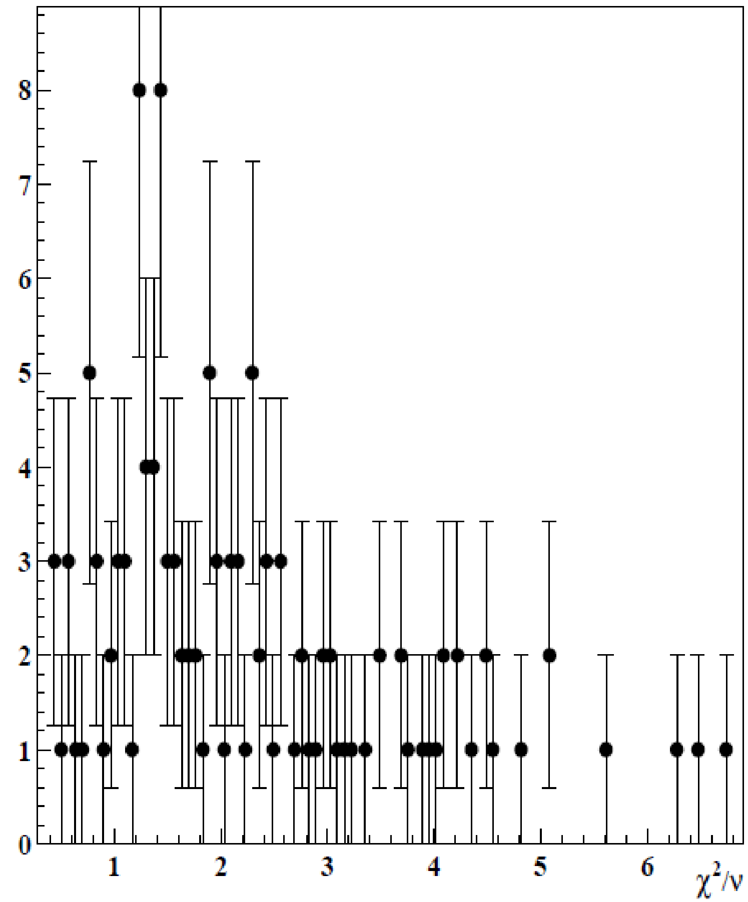
- Blue : E0+ using LCSR w/ zero pion mass
- Black : E0+ from multipole analysis

Summary

- As first time, E0+ multipole comparison near pion threshold between two methods (LCSR, multipole fit) was performed.
- Multipole analysis gives us same answer for extracting E0+ multipole with LCSR method.
- Direct use of neutron magnetic form factor from CLAS publication gives consistent result with F.F. parametrization.
- E0+ plays an important role in forward angle, which is consistent with models prediction

BACKUP SLIDES

Fit qualities



Systematic errors

Sources	Criteria	Avg.Sys.Error
e^- PID	width of sampling fraction cut in EC ($3\sigma_{SF} \rightarrow 3.5\sigma_{SF}$)	$\sim 4\%$
e^- fiducial cut	Width (10% reduced)	2.2%
π^+ PID	β resolution change ($2\sigma_{TOF} \rightarrow 2.5\sigma_{TOF}$)	1.3%
π^+ fiducial cut	Width (10% reduced)	$\sim 3\%$
MMx cut (n)	neutron missing mass resolution ($3\sigma_{MMx} \rightarrow 3.5\sigma_{MMx}$)	$\sim 1\%$
vertex cut	width (5% reduced)	$\sim 1\%$
Acceptance correction	event generator dependence between AAO and GENEV	$\sim 4\%$
radiative correction	physics model dependence between SLee04 and MAID03	$\sim 0.5\%$
Total		$\sim 7.05\%$

Theoretical imrovement plans

- Enrgy dependent generalized form factors generated by FSI
- Adding D-wave contributio model
- Tune calculation with low Q^2 and high W experimental data
- Systematic approach in the global PWA analysis framework in Np and g*N scattering under QCD S-, P- and D partial waves.

Multipoles vs. F. F. for $n\pi^+$channel

$$G_1^{\pi^+ n} \quad G_2^{\pi^+ n}$$

$$E_{0+}^{\pi^+ n} = \frac{\sqrt{4\pi\alpha_{em}}}{8\pi} \frac{Q^2\sqrt{Q^2+4m_p^2}}{m_p^3 f_\pi} G_1^{\pi^+ n} \quad \times 1/G_D$$

$$L_{0+}^{\pi^+ n} = \frac{\sqrt{4\pi\alpha_{em}}}{32\pi} \frac{Q^2\sqrt{Q^2+4m_p^2}}{m_p^3 f_\pi} G_2^{\pi^+ n} \quad \times 1/G_D$$

Legendre –moment vs. F. F. for $n\pi^+$ channel

$$G_1^{\pi N} = G_1^{\pi^+ n} \qquad G_M = G_M^n \approx \mu_n G_D(Q^2)$$

P.E. Bosted
Phys. Rev. C 51 (1995)

$$G_2^{\pi N} = G_2^{\pi^+ n} \qquad G_E = G_E^n \approx 0$$

Assumption in LCSR
V.Braun PRD77(2008)

Due to low-energy theorem(LET) relates the S-wave multipoles or equivalently, the form factor G_1, G_2 @ threshold $m_\pi = 0$

$$\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{Q^2 + 2m_N^2} G_M^n + \frac{1}{2} G_A$$

$$G_2^{\pi^+ n} = \frac{2\sqrt{2} g_A m_N^2}{Q^2 + 2m_N^2} G_E^n = 0$$

Scherer, Koch,
NPA534(1991)
Vainshtein, Zakharov
NPB36(1972)

Legendre moments vs. Form Factors

$$G_1^{\pi^+ n} \quad G_2^{\pi^+ n}$$

$$G_1^{\pi^+ n} = x_1 + iy_1$$

$$G_2^{\pi^+ n} = x_2 + iy_2$$

$$A_0 = D_0^{T+L} = \frac{1}{f_\pi^2}\left[\frac{4\vec{k}_i^2 Q^2}{m_p^2}\left|G_1^{\pi^+ n}\right|^2 + \frac{c_\pi^2 g_A^2 \vec{k}_f^2}{W^2 - m_p^2} Q^2 m_p^2 G_M^{n\,2}\right]$$

$$A_1 = D_1^{T+L} = \frac{1}{f_\pi^2}\frac{4 c_\pi g_A \left|k_i\right|\left|k_f\right|}{W^2 - m_p^2}\left(Q^2 G_M^n \,\mathrm{Re}\left(G_1^{\pi^+ n}\right)\right)$$

$$C_0 = C_0^{TT} = 0$$

$$D_0 = D_0^{LT} = 0$$

$$g_A = 1.267$$

$$c_{\pi^+} = \sqrt{2}$$

$$f_\pi = 93\,MeV$$

l-moments vs. F. F. for $n\pi^+$channel

$$G_1^{\pi N} = G_1^{\pi^+ n} \qquad G_M = G_M^n \approx \mu_n G_D(Q^2)$$

$$G_2^{\pi N} = G_2^{\pi^+ n} \qquad G_E = G_E^n \neq 0$$

P.E. Bosted
Phys. Rev. C 51 (1995)

Due to low-energy theorem(LET) relates the S-wave multipoles or equivalently, the form factor G_1, G_2 @ threshold $m_\pi = 0$

$$\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{Q^2 + 2m_N^2} G_M^n + \frac{1}{2} G_A$$

$$G_2^{\pi^+ n} = \frac{2\sqrt{2} g_A m_N^2}{Q^2 + 2m_N^2} G_E^n$$

Legendre-moments vs. F. F.

$$G_1^{\pi^+ n} \quad G_2^{\pi^+ n}$$

$$G_1^{\pi^+ n} = x_1 + iy_1$$
$$G_2^{\pi^+ n} = x_2 + iy_2$$

$$A_0 = D_0^{T+L} = \frac{1}{f_\pi^2}\left[\frac{4\vec{k}_i^2 Q^2}{m_N^2}\left|G_1^{\pi N}\right|^2 + \frac{c_\pi^2 g_A^2 \vec{k}_f^2}{W^2 - m_N^2} Q^2 m_N^2 G_M^2 + \varepsilon_L\left(\vec{k}_i^2\left|G_2^{\pi N}\right|^2 + \frac{4c_\pi^2 g_A^2 \vec{k}_f^2}{W^2 - m_N^2} m_N^4 G_E^2\right)\right]$$

$$A_1 = D_1^{T+L} = \frac{1}{f_\pi^2}\frac{4c_\pi g_A |k_i||k_f|}{W^2 - m_N^2}\left(Q^2 G_M \,\mathrm{Re}\left(G_1^{\pi N}\right) - \varepsilon_L m_N^2 G_E \,\mathrm{Re}\left(G_2^{\pi N}\right)\right)$$

$$C_0 = C_0^{TT} = 0$$

$$D_0 = D_0^{LT} = -\frac{1}{f_\pi^2}\frac{c_\pi g_A |k_i||k_f|}{W^2 - m_N^2} Q m_N\left(G_M \,\mathrm{Re}\left(G_2^{\pi N}\right) + 4G_E \,\mathrm{Re}\left(G_1^{\pi N}\right)\right)$$

$$g_A = 1.267$$
$$c_{\pi^+} = \sqrt{2}$$
$$f_\pi = 93\,MeV$$

Legendre moments vs. Form Factors

$$G_1^{\pi^+ n} = x_1 + iy_1$$

$$G_2^{\pi^+ n} = x_2 + iy_2$$

* 3 Eqs. 4 parameter should be determined
* Real parts x1, x2 can be determined by A1, D0 legendre coeff.
* Imaginary parts y1, y2 can be determined in 2 cases
* Asymmetry helps to determine complete form factor

$$D_0^{/} = D_0^{LT\,/} = -\frac{1}{f_\pi^2}\frac{c_\pi g_A |k_i||k_f|}{W^2 - m_N^2} Q m_N \left(G_M \,\mathrm{Im}\left(G_2^{\pi N}\right) - 4G_E \,\mathrm{Im}\left(G_1^{\pi N}\right)\right)$$

- Historically, threshold pion in the photo- and electroproduction is the very old subject that has been receiving continuous attention from both experiment and theory sides for many years.

- Pion mass vanishing approximation in Chiral Symmetry allows us to make an exact prediction for threshold cross section known as LET

- The LET established the connection between charged pion electroproduction and axial form factor in nucleon.

- Therefore, It is very interesting to extracting Axial Form Factor which is dominated by S- wave transverse multipole E_{0+} in LCSR

LCSR (Light Cone Sum Rule)

- Constructed relating the amplitude for the radiative decay of $\Sigma^{+}(p\gamma)$ to properties of the QCD vacuum in alternating magnetic field.

- An advantage of study because soft contribution to hadron form factor can be calculated in terms of DA's that enter pQCD calculation without other nonperturbative parameters.

- New technique : the expansion of the standard QCD sum rule approach to hadron properties in alternating external fields.