

Transverse Momentum Dependent Distributions in hard scattering

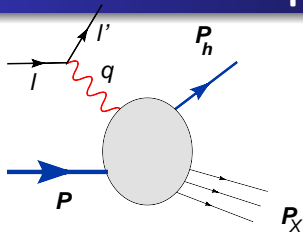
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Thomas Jefferson National Accelerator Facility

4th Workshop
Exclusive Reactions at High Momentum Transfer
May 18-21, 2010



Semi Inclusive Deep Inelastic Scattering



$$l + P \rightarrow l' + h + X$$

Kinematical variables:

$$Q^2 = -q^2 = -(l - l')^2,$$

$$x_B = \frac{Q^2}{2P \cdot q},$$

$$y = \frac{P \cdot q}{P \cdot l},$$

$$z_h = \frac{P \cdot P_h}{P \cdot q}$$

Cross section

$$\frac{d^6 \sigma^{\ell p \rightarrow \ell' h X}}{dx_B dy dz_h d^2 \mathbf{P}_T d\Phi_S} \propto \frac{\alpha^2}{Q^4} L_{\mu\nu} W^{\mu\nu}$$

Leptonic tensor

$$L_{\mu\nu} = \sum_{\lambda'} (\bar{u}_{\lambda'}(l') \gamma_\mu u_\lambda(l))^* (\bar{u}_{\lambda'}(l') \gamma_\nu u_\lambda(l)) = 2l_\mu l'_\nu + 2l'_\nu l_\mu - Q^2 g_{\mu\nu} + 2i\lambda \epsilon_{\mu\nu\rho\sigma} l^\rho l'^\sigma$$

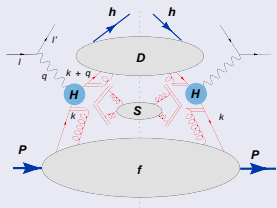
Hadronic tensor

$$2MW^{\mu\nu} = \frac{1}{2\pi} \sum_X \int \frac{d^3 \mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta^{(4)}(q + P - P_X - P_h) \langle PS | J^\mu(0) | h, X \rangle \langle h, X | J^\nu(0) | PS \rangle$$

TMD and collinear factorizations

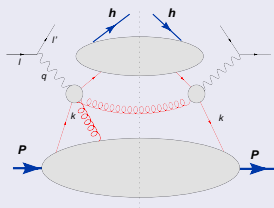
TMD factorization describes the structure of the hadron in terms of Transverse Momentum Dependent distribution functions and valid for $P_T \sim \Lambda_{QCD}$ while Collinear factorization employs multiparton correlators, in particular twist-three Qiu-Sterman matrix elements, and $P_T \sim Q$.

TMD factorization



Ji, Ma, Yuan 2005

Collinear factorization



Collins, Soper, Sterman 1982; Qiu, Sterman 2000

$$d\sigma \propto L_{\mu\nu} W^{\mu\nu} = \sum_q e_q^2 \int d^2\mathbf{k}_\perp d^2\mathbf{p}_\perp d^2\mathbf{l}_\perp f_{q/p}(x, \mathbf{k}_\perp, \dots)$$

$$S(\mathbf{l}_\perp, \dots) H H^*(Q^2) D_q^h(z, \mathbf{p}_\perp, \dots) \delta^{(2)}(z\mathbf{k}_\perp + \mathbf{p}_\perp + \mathbf{l}_\perp - \mathbf{P}_{Th})$$

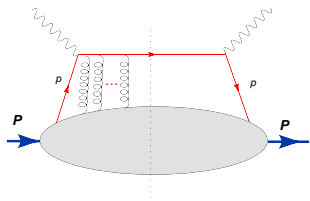
Quark-quark Correlator and TMDs



$$\Phi_{ij}(p, P, S) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip \cdot \xi} \langle P, S | \bar{\psi}_j(0) \mathcal{W}(0, \xi | n^-) \psi_i(\xi) | P, S \rangle$$

Mulders, Tangerman 95; Goeke, Metz, Belitsky, Ji, Yuan 2002, Schlegel 05, Bacchetta et al 07

Gauge link $\mathcal{W}(0, \xi | n^-)$ (Belitsky, Ji, Yuan 2002) ensures gauge invariance of the correlator. In SIDIS gauge link will be formed by summing over gluon exchanges between struck quark and the remnant of the proton (Final State Interactions) :



$$\mathcal{W}(\xi^-, \xi_\perp; \infty | n^-) =$$

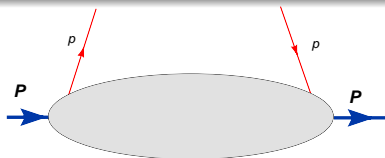
$$\mathcal{P} \exp[-ig \int_{\xi^-}^{\infty} A_+(\xi^-, \xi_\perp) dx^-]$$

$$\mathcal{W}(0, \xi | n^-) =$$

$$\mathcal{W}^\dagger(0^-, 0_\perp; +\infty | n^-) \mathcal{W}(\xi^-, \xi_\perp; +\infty | n^-)$$

See talk of Igor Cherednikov on evolution and definition

Quark-quark Correlator and TMDs



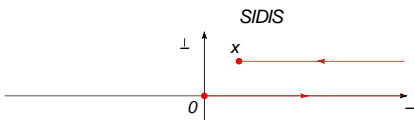
$$\Phi_{ij}(p, P, S) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip \cdot \xi} \langle P, S | \bar{\psi}_j(0) \mathcal{W}(0, \xi | n^-) \psi_i(\xi) | P, S \rangle$$

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Gauge link $\mathcal{W}(0, \xi | n^-)$ ensures gauge invariance of the correlator.

$$\mathcal{W}(0, \xi | n^-) = \mathcal{W}^\dagger(0^-, 0_\perp; +\infty | n^-) \mathcal{W}(\xi^-, \xi_\perp; +\infty | n^-)$$

this link connects two fields which are positioned at “0” and “ ξ ”. Process “defines” direction of the gauge link. In different processes the direction of the link changes thus not trivial relations among TMDs appear [Collins 2002](#). Separation in transverse direction is crucial.



Quark-quark Correlator and TMDs



$$\Phi_{ij}(p, P, S) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip\cdot\xi} \langle P, S | \bar{\psi}_j(0) \mathcal{W}(0, \xi | n^-) \psi_i(\xi) | P, S \rangle$$

Mulders, Tangerman 95; Goeke, Metz, Belitsky, Ji, Yuan 2002, Schlegel 05, Bacchetta et al 07

TMD distribution functions can be found via p^- integration

$$\Phi(x, \mathbf{p}_T, S) = \int dp^- \Phi(p, P, S) \Big|_{p^+ = xP^+}$$

Dirac decomposition is done by projecting onto the basis of Dirac matrices

$$\Phi^{[\Gamma]}(x, \mathbf{p}_T, S) = \frac{1}{2} \text{Tr}(\Phi(x, \mathbf{p}_T, S) \Gamma)$$

Quark-quark Correlator and TMDs



$$\Phi_{ij}(p, P, S) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip \cdot \xi} \langle P, S | \bar{\psi}_j(0) \mathcal{W}(0, \xi | n^-) \psi_i(\xi) | P, S \rangle$$

Mulders, Tangerman 95; Goeke, Metz, Belitsky, Ji, Yuan 2002, Schlegel 05, Bacchetta et al 07

Twist-2 decomposition (= leading terms in P^+ expansion) contains **8 functions**:

$$\Phi^{[\gamma^+]}(x, \mathbf{p}_T, S) = f_1(x, \mathbf{p}_T^2) - \frac{\epsilon_T^{ij} \mathbf{p}_{T i} S_{T j}}{M} f_{1T}^\perp(x, \mathbf{p}_T^2)$$

$$\Phi^{[\gamma^+ \gamma_5]}(x, \mathbf{p}_T, S) = S_L g_{1L}(x, \mathbf{p}_T^2) - \frac{\mathbf{p}_T \cdot S_T}{M} g_{1T}^\perp(x, \mathbf{p}_T^2)$$

$$\Phi^{[i\sigma^{i+} \gamma_5]}(x, \mathbf{p}_T, S) = S_T^i h_1 + S_L \frac{\mathbf{p}_T^i}{M} h_{1L}^\perp - \frac{\mathbf{p}_T^i \mathbf{p}_T^j - 1/2 \mathbf{p}_T^2 g_T^{ij}}{M^2} S_{T j} h_{1T}^\perp - \frac{\epsilon_T^{ij} \mathbf{p}_{T j}}{M} h_1^\perp$$

"Amsterdam notation" is used for the TMDs

Kotzinian 95; Mulders, Tangerman 96; Barone, Drago, Ratcliffe 02; Bacchetta et al 07; Anselmino et al 06

talk of Peter Schweitzer

Polarised Semi Inclusive Deep Inelastic Scattering

Asymmetry in γ^*p cm frame of $lp^\uparrow \rightarrow \ell'hX$

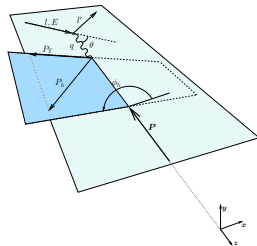
TMD functions can be studied in asymmetries

$$A_{UT} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{\frac{1}{2}(d\sigma^\uparrow + d\sigma^\downarrow)}$$

Unpolarised electron beam, Transversely polarised proton. Azimuthal dependence on Φ_h and Φ_S singles out different combinations.

Contributions at leading twist

$$d\sigma^\uparrow - d\sigma^\downarrow \propto \underbrace{f_{1T}^\perp \otimes d\hat{\sigma} \otimes D_{h/q} \sin(\phi_h - \phi_S)}_{\text{Sivers effect}} + \underbrace{h_1 \otimes \Delta\hat{\sigma}^\uparrow \otimes H_1^\perp \sin(\phi_h + \phi_S)}_{\text{Collins effect}} + \dots$$



Kotzinian 1995;

Mulders, Tangerman 1995; Boer and

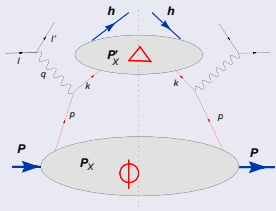
Mulders 1997; Bacchetta et al 2007

see talks of
Heiner Wollny,
Luciano Pappalardo,
and Patrizia Rossi on
experimental results

TMDs in other processes

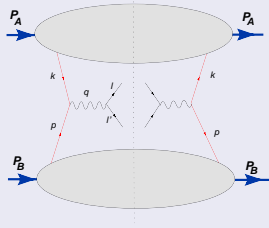
Transverse Momentum Dependent distributions are present in other processes, such as Drell Yan $A^\uparrow B \rightarrow l^+ l^- X$

SIDIS



Ji, Ma, Yuan 2005

Drell Yan



Collins, Soper, Sterman 1985; Ji, Ma, Yuan, 2004

$$d\sigma^{SIDIS} \propto f_{q/P^\uparrow}^{SIDIS}(x, \mathbf{k}_\perp) \otimes \sigma \otimes D_q^h(z, \mathbf{p}_\perp)$$

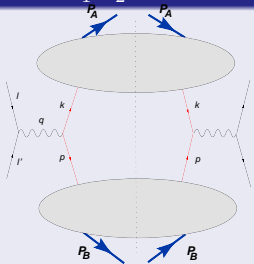
$$d\sigma^{DY} \propto f_{q/P_A^\uparrow}^{DY}(x, \mathbf{k}_\perp) \otimes \sigma \otimes f_{\bar{q}/P_B}^{DY}(x, \mathbf{p}_\perp)$$

Universality of distributions is proven [Belitsky, Ji, Yuan 2002](#); [Collins 2002](#). Sivers and Boer-Mulders distributions have *modified universality* \rightarrow they *change sign*. This is a unique property of TMDs.

TMDs in other processes

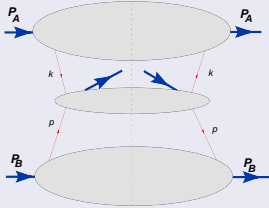
e^+e^- annihilation $e^+e^- \rightarrow H_1 H_2 X$ and inclusive hadron production in hadron-hadron scattering $A^\dagger B \rightarrow HX$

$e^+e^- \rightarrow H_1 H_2 X$



Collins, Soper 1981

Hadron-hadron scattering



TMD factorization is not proven, "counterexamples" exist

Collins, Qiu 2007; Rogers, Mulders 2010

$$d\sigma^{e^+e^-} \propto D_{q/H_1}(z_1, \mathbf{k}_\perp) \otimes \sigma \otimes D_{\bar{q}/H_2}(z_2, \mathbf{p}_\perp)$$

$$d\sigma \propto f_{q_1/P_A^\dagger}(x_1, \mathbf{k}_{1\perp}) \otimes f_{q_2/P_B}(x_2, \mathbf{k}_{2\perp}) \otimes \sigma \otimes D_{q_3/H}(z, \mathbf{p}_\perp) ???$$

See talk of Les Bland on A_N in hadron-hadron scattering

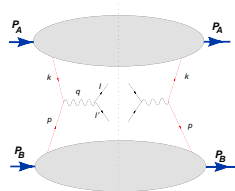
Sivers function: process dependence

Sivers function [Sivers 1990](#) can be measured in both SIDIS and DY processes.

$$f_{q/P^\uparrow}(x, \mathbf{k}_\perp, S) = f_1(x, \mathbf{k}_\perp^2) - \frac{S \cdot (\hat{P} \times \mathbf{k}_\perp)}{M} f_{1T}^\perp(x, \mathbf{k}_\perp^2)$$

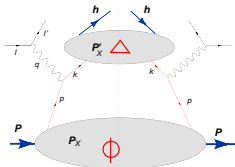
Drell Yan $A^\uparrow B \longrightarrow l^+ l^- X$

$$A_{UT}^{sin(\phi_\gamma - \phi_S)} \sim f_{1T}^{\perp DY}(x, k_\perp) \otimes f_{\bar{q}/B}(x, p_\perp)$$



SIDIS $\ell P^\uparrow \longrightarrow \ell' h X$

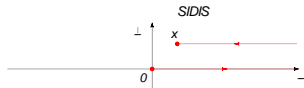
$$A_{UT}^{sin(\phi_H - \phi_S)} \sim f_{1T}^{\perp SIDIS}(x, k_\perp) \otimes D_{h/q}(z, p_\perp)$$



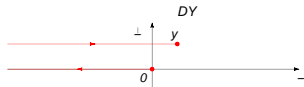
Sivers function: process dependence

Operator definitions of distributions in SIDIS and DY:

$$f_{q/p^\uparrow}^{SIDIS}(x, k_\perp, \vec{S}) = \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{ixP^+ y^- - i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \cdot \langle P, S | \bar{\psi}(0) \mathcal{W}^{SIDIS}(0, y | n^-) \psi(y) | P, S \rangle$$



$$f_{q/p^\uparrow}^{DY}(x, k_\perp, \vec{S}) = \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{ixP^+ y^- - i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \cdot \langle P, S | \bar{\psi}(0) \mathcal{W}^{DY}(0, y | n^-) \psi(y) | P, S \rangle$$



From parity and time reversal invariance one can show (Collins 2002; Qiu, Kang 2008)

$$f_{q/p^\uparrow}^{SIDIS}(x, k_\perp, \vec{S}) = f_{q/p^\uparrow}^{DY}(x, k_\perp, -\vec{S})$$

and

$$f_{1T}^{\perp SIDIS}(x, k_\perp) = -f_{1T}^{\perp DY}(x, k_\perp)$$

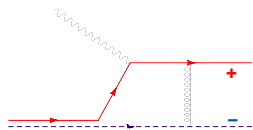
Modified universality

Sivers function is process dependent. Collins 2002

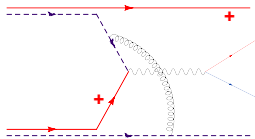
$$f_{1T}^{\perp DY} = -f_{1T}^{\perp SIDIS}$$

Let's consider a simple model of Final State Interactions as in Brodsky, Hwang, Schmidt 2002,

proton = quark⁺ + antiquark⁻



SIDIS - attractive



DY - repulsive

- Experimental test of this relation is fundamental for our understanding of the origin of the correlation between parton angular momentum and the spin of the proton and the gauge link formalism itself.

Experimental DY data are not available, experiments are planned.

See talk of Les Bland

The fundamental distributions of partons inside a nucleon

Unpolarised Distribution

$f_1(x)$ or $q(x)$



Distribution of unpolarised partons in an unpolarised nucleon.

Well known

Helicity Distribution

$g_1(x)$ or $\Delta q(x)$

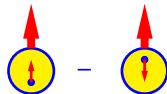


Distribution of longitudinally polarised partons in a longitudinally polarised nucleon.

Known

Transversity Distribution

$h_1(x)$ or $\Delta_T q(x)$



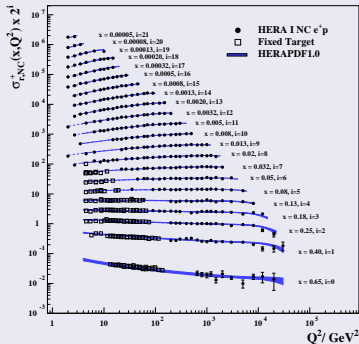
Distribution of transversely polarised quarks in a transversely polarised nucleon.

Little known!
HERMES and COMPASS
experimental measurements

The fundamental distributions of partons inside a nucleon

Unpolarised

H1 and ZEUS



D
p
nucleon.

Well known

Helicity Distribution

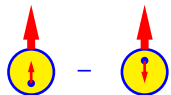
$\Delta q(x)$



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linally polarised
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Distribution of transversely
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The fundamental distributions of partons inside a nucleon

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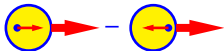


Distribution of unpolarised partons in an unpolarised nucleon.

Well known

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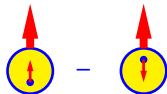


Distribution of longitudinally polarised partons in a longitudinally polarised nucleon.

Known

Transversity Distribution

$h_1(x)$ or $\Delta_T q(x)$



Distribution of transversely polarised quarks in a transversely polarised nucleon.

Little known!
HERMES and COMPASS
experimental measurements

The fundamental distributions of partons inside a nucleon

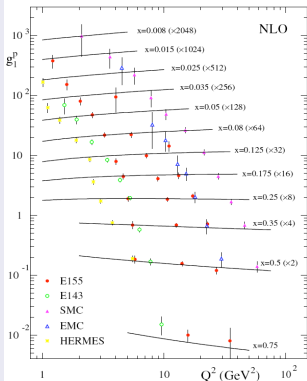
Unpolarised Distribution

$f_1(x)$ or $q(x)$



Distribution of unpolarised partons in an unpolarised nucleon.

Well known

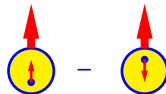


partons in a longitudinally polarised nucleon.

Known

Transversity Distribution

$h_1(x)$ or $\Delta_T q(x)$



Distribution of transversely polarised quarks in a transversely polarised nucleon.

Little known!

HERMES and COMPASS
experimental measurements

The fundamental distributions of partons inside a nucleon

Unpolarised Distribution

$f_1(x)$ or $q(x)$



Distribution of unpolarised partons in an unpolarised nucleon.

Well known

Helicity Distribution

$g_1(x)$ or $\Delta q(x)$

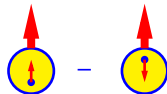


Distribution of longitudinally polarised partons in a longitudinally polarised nucleon.

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Transversity Distribution

$h_1(x)$ or $\Delta_T q(x)$



Distribution of transversely polarised quarks in a transversely polarised nucleon.

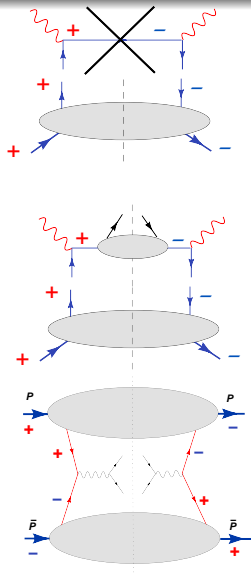
Little known!
HERMES and COMPASS
experimental measurements

TRANSVERSITY

Transversity cannot be studied in DIS as QED and QCD interactions conserve helicity up to corrections $\mathcal{O}(m_q/E)$.

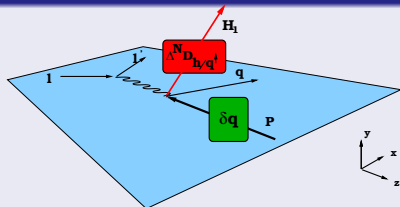
Transversity can be measured if coupled with another chiral-odd function. This can be done in Semi Inclusive DIS (SIDIS), quark fragments into unpolarised hadron. It couples to so called Collins Fragmentation function that describes how a polarised quark fragments into unpolarised hadron.

Golden channel to study transversity is proton - antiproton double spin asymmetry at GSI
 $A_N \propto h_{q/P}(x)h_{\bar{q}/\bar{P}}(x)$.



How to measure transversity? SIDIS and e^+e^- annihilation

SIDIS $lN \rightarrow l'H_1X$



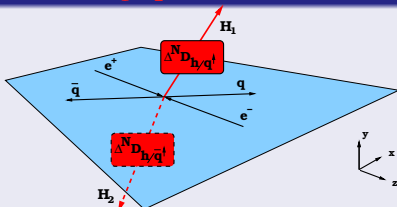
Collins effect gives rise to azimuthal Single Spin Asymmetry

$$\begin{array}{c} \uparrow \\ \circ \end{array} - \begin{array}{c} \uparrow \\ \circ \\ \downarrow \end{array} = \Delta_T q(x, Q^2)$$

$$\begin{array}{c} \uparrow \\ \circ \end{array} - \begin{array}{c} \downarrow \\ \circ \end{array} = \Delta^N D_{h/q^\uparrow}(z, Q^2)$$

J. C. Collins, *Nucl. Phys.* **B396** (1993) 161

$e^+e^- \rightarrow H_1 H_2 X$



Collins effect gives rise to azimuthal asymmetry, q and \bar{q} Collins functions are present in the process:

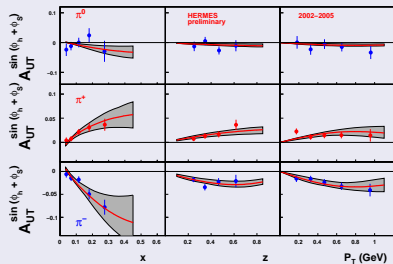
$$\Delta^N D_{h/q^\uparrow}(z_1, Q^2)$$

$$\Delta^N D_{h/\bar{q}^\uparrow}(z_2, Q^2)$$

D. Boer, R. Jacob and P. J. Mulders *Nucl. Phys.* **B504** (1997) 345

Experimental data

HERMES $A_{UT}^{\sin(\phi_h + \phi_S)}$



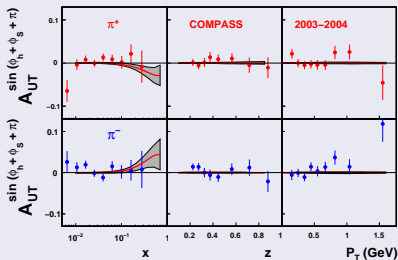
$ep \rightarrow e\pi X$, $p_{lab} = 27.57$ GeV.

HERMES, M. Dieffenthaler, (2007), arXiv:0706.2242

COMPASS, M. Alekseev et al., (2008), Phys.Lett.B673:127-135,2009

see talks of Heiner Wollny, Luciano Pappalardo, and Patrizia Rossi on experimental results

COMPASS $A_{UT}^{\sin(\phi_h + \phi_S + \pi)}$

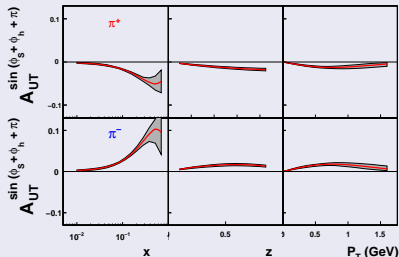


$\mu D \rightarrow \mu\pi X$, $p_{lab} = 160$ GeV

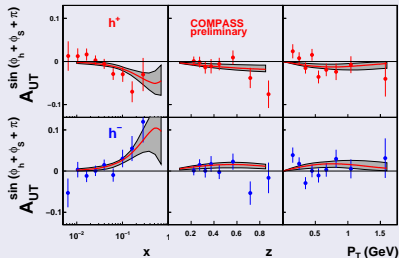
Description of the data

Predictions for COMPASS operating on PROTON target

COMPASS $A_{UT}^{\sin(\phi_h + \phi_S + \pi)}$



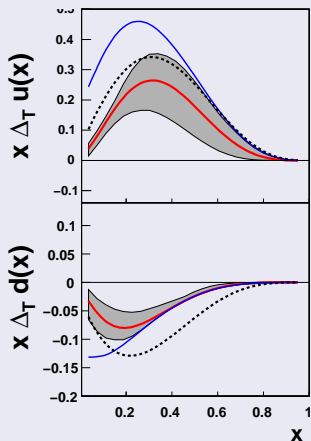
COMPASS $A_{UT}^{\sin(\phi_h + \phi_S + \pi)}$



Comparison with preliminary
COMPASS data arXiv:0808.0086

Anselmino et al 2009

Transversity vs. helicity



- 1 Solid red line – transversity distribution

$$\Delta_T q(x)$$

this analysis at $Q^2 = 2.4 \text{ GeV}^2$.

- 2 Solid blue line – Soffer bound

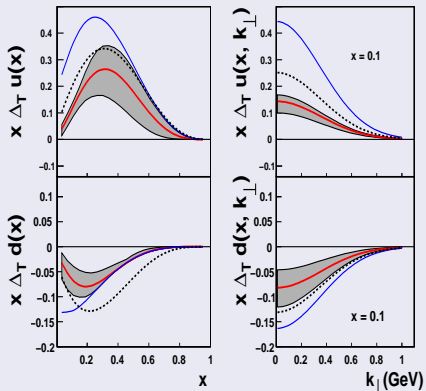
$$|\Delta_T q(x)| < \frac{q(x) + \Delta q(x)}{2}$$

GRV98LO + GRSV98LO

- 3 Dashed line – helicity distribution

$$\Delta q(x)$$

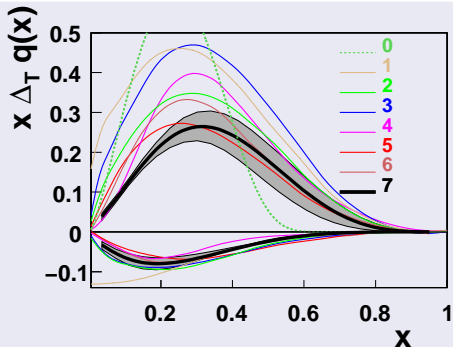
GRSV98LO



- This is the extraction of **transversity** from existing experimental data. Anselmino et al 2009
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- $|\Delta_T q(x)| < |\Delta q(x)|$.
- JLab @ 12 GeV will provide wider region of x for tensor charge extraction.

Transversity, comparison with models

New extraction is close to most models.



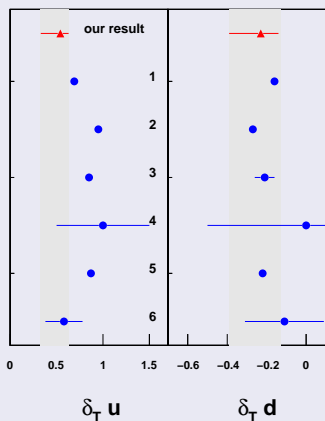
- ① Barone, Calarco, Drago PLB 390 287 (97)
- ① Soffer et al. PRD 65 (02)
- ② Korotkov et al. EPJC 18 (01)
- ③ Schweitzer et al. PRD 64 (01)
- ④ Wakamatsu, PLB B653 (07)
- ⑤ Pasquini et al., PRD 72 (05)
- ⑥ Cloet, Bentz and Thomas PLB 659 (08)
- ⑦ Anselmino et al 2009.

see talk of Barbara Pasquini on model calculations

Tensor charges

$$\delta_T q = \int_0^1 dx (h_{1q} - h_{1\bar{q}}) = \int_0^1 dx h_{1q}$$

$$\delta_T u = 0.54_{-0.22}^{+0.09}, \delta_T d = -0.23_{-0.16}^{+0.09} \text{ at } Q^2 = 0.8 \text{ GeV}^2$$



- 1 Quark-diquark model:
Cloet, Bentz and Thomas
PLB **659**, 214 (2008), $Q^2 = 0.4 \text{ GeV}^2$
- 2 CQSM:
M. Wakamatsu, PLB **653** (2007) 398.
 $Q^2 = 0.3 \text{ GeV}^2$
- 3 Lattice QCD:
M. Gockeler et al.,
Phys.Lett.B627:113-123,2005 ,
 $Q^2 = 4 \text{ GeV}^2$
- 4 QCD sum rules:
Han-xin He, Xiang-Dong Ji,
PRD 52:2960-2963,1995, $Q^2 \sim 1 \text{ GeV}^2$
- 5 Constituent quark model:
B. Pasquini, M. Pincetti, and S. Boffi,
PRD72(2005)094029 and PRD76(2007)034020,
 $Q^2 \sim 0.8 \text{ GeV}^2$
- 6 Spin-flavour SU(6) symmetry
L. Gamberg, G. Goldstein,
Phys.Rev.Lett.87:242001,2001 $Q^2 \sim 1 \text{ GeV}^2$

See talk of Bernhard Musch on
lattice QCD calculations of TMDs

Sivers effect

The azimuthal asymmetry $A_{UT}^{\sin(\phi_h - \phi_S)}$ arises due to Sivers function (Sivers 90)

Torino notations are used

$$f_{q/p^\uparrow}(x, \mathbf{k}_\perp) = f_{q/p}(x, \mathbf{k}_\perp) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, \mathbf{k}_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp)$$

Spin sum rule:

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + \langle L_z^{q, \bar{q}} \rangle + \langle L_z^G \rangle$$

EMC result on $\Delta \Sigma = \sum_{q, \bar{q}} \Delta q \simeq 0.3$ triggered so called "Spin crisis" – only 30% of the spin of the proton is carried by quarks.

Leader, Anselmino 'A Crisis In The Parton Model: Where, Oh Where Is The Proton's Spin?'
Z.Phys.C41:239,1988

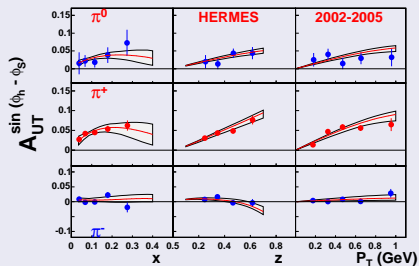
$\mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp)$ – correlation between the spin (\mathbf{S}_T) and angular momentum (\mathbf{L}_q) implies non zero contribution $\langle L_z^{q, \bar{q}} \rangle \neq 0$

Data are available from HERMES and COMPASS. u and d Sivers functions are non zero thus $\mathbf{L}_{u,d} \neq 0$. See talk of Peter Schweitzer on connection of \mathbf{L}_q and TMDs

HERMES and COMPASS DATA.

HERMES

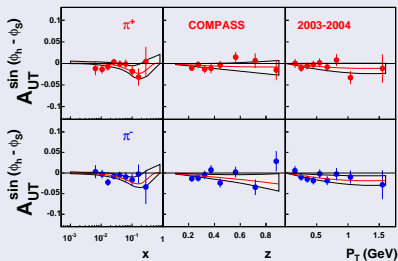
$ep \rightarrow e\pi X$, $p_{lab} = 27.57$ GeV.



M. Anselmino et al 2009

COMPASS

$\mu D \rightarrow \mu\pi X$, $p_{lab} = 160$ GeV.



M. Anselmino et al 2009

$$lp^\uparrow \rightarrow l\pi^+ X \simeq \Delta^N u \otimes D_{u/\pi^+} > 0$$

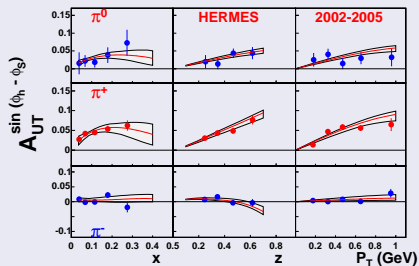
$$lp^\uparrow \rightarrow l\pi^- X \simeq 4\Delta^N u \otimes D_{u/\pi^-} + \Delta^N d \otimes D_{d/\pi^-} \simeq 0$$

$$lD^\uparrow \rightarrow l\pi^+ X \simeq (\Delta^N u + \Delta^N d) \otimes D_{u/\pi^+} \simeq 0$$

HERMES and COMPASS DATA.

HERMES

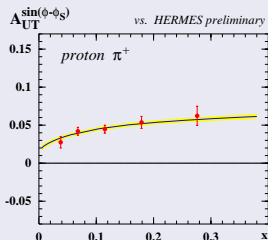
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HERMES

$ep \rightarrow e\pi X$, $p_{lab} = 27.57$ GeV.



Arnold et al 2008

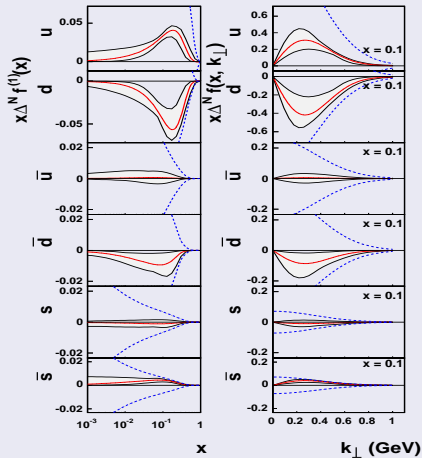
$$lp^\uparrow \rightarrow l\pi^+ X \simeq \Delta^N u \otimes D_{u/\pi^+} > 0$$

$$lp^\uparrow \rightarrow l\pi^- X \simeq 4\Delta^N u \otimes D_{u/\pi^-} + \Delta^N d \otimes D_{d/\pi^-} \simeq 0$$

$$lD^\uparrow \rightarrow l\pi^+ X \simeq (\Delta^N u + \Delta^N d) \otimes D_{u/\pi^+} \simeq 0$$

Sivers functions

$$\Delta^N f_q^{(1)}(x) \equiv \int d^2 \mathbf{k}_\perp \frac{k_\perp}{4m_p} \Delta^N f_{q/p\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x).$$



Sivers functions for u , d and sea quarks are extracted from **HERMES** and **COMPASS** data. $\Delta^N f_u > 0$, $\Delta^N f_d < 0$, first hints on nonzero sea quark Sivers functions.

Sivers function comparison with models

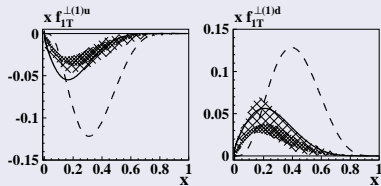
There is a number of model calculations of Sivers function

Light-cone quark model Barbara Pasquini and Feng Yuan 2010

Diquark model Alessandro Bacchetta et al 2010, Leonard Gamberg, Gary Goldstein, and Marc Schlegel 2008 etc

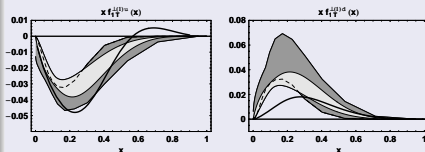
MIT bag model Feng Yuan 2003, H. Avakian, A.V. Efremov, P. Schweitzer, F. Yuan 2010 etc

Pasquini and Yuan 2010



Pasquini and Yuan arXiv:1001.5398

Alessandro Bacchetta et al 2010



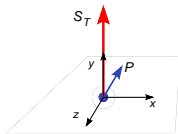
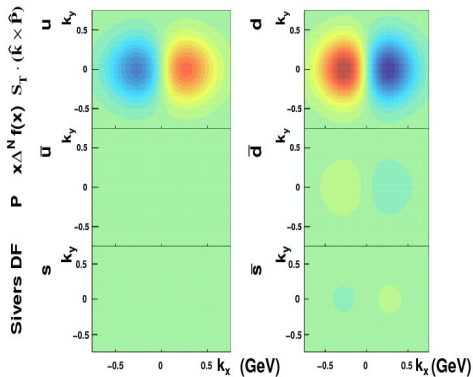
Bacchetta et al arXiv:1003.1328

Reasonable agreement of the extracted Sivers functions [Anselmino et al 2009](#) and [Collins et al 2005](#) and model calculations.

See talk of Barbara Pasquini for model calculation results

Three dimensional picture of the proton

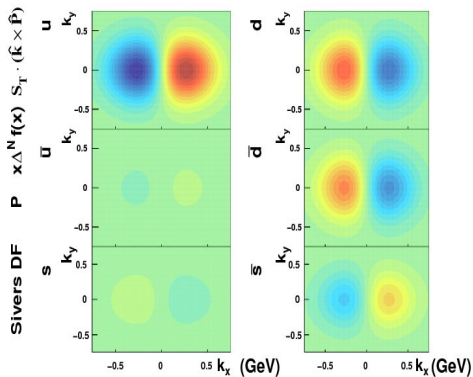
The proton moves along $-Z$ direction (into the screen) and S_T is along Y .



This is the three dimensional view of the proton as “seen” by the virtual photon.
Red color – more quarks. **Blue color** – less quarks. Distributions of quarks are not symmetrical and shifted due to final state interactions.
 $x = 0.2$

Three dimensional picture of the proton

The proton moves along $-Z$ direction (into the screen) and S_T is along Y .



Sivers functions for u , d and sea quarks are extracted from **HERMES** and **COMPASS** data. Red color – more quarks. Blue Color – less quarks. Sivers functions is a left – right asymmetry of quark distribution. $x = 0.01$

More information on sea quarks. Future Electron Ion Collider and JLab will contribute.

CONCLUSIONS

- **8** Transverse Momentum Dependent functions describe spin structure of the proton at twist-2.
- Spin Asymmetries are used to study TMDs experimentally.
- T-odd TMDS: Sivers and Boer-Mulders functions have *modified universality*, they change sign from SIDIS to DY.
- HERMES, COMPASS, JLAB, RHIC, and BELLE provide lots of experimental data for TMD extraction.
- Model and lattice QCD calculations of TMDs are possible and match well with TMDs extracted from the experimental data.
- Future facilities such as JLab @ 12 GeV, Electron Ion Collider and GSI will contribute to unravel three dimensional structure of the proton.

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THANK YOU!

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