4th Workshop on Exclusive Reactions at High Momentum Transfer

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Quark orbital angular momentum (OAM): can we learn about it from GPDs and TMDs?

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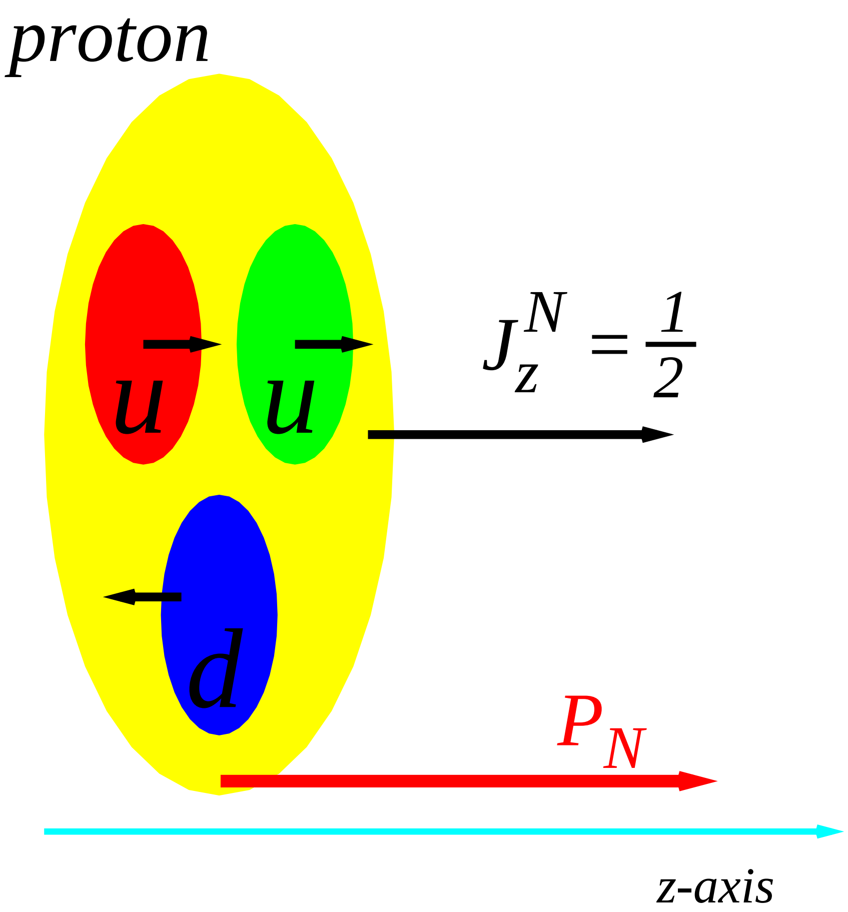
based on works with H.Avakian, A.Efremov, O.Teryaev, F.Yuan, P.Zavada

Overview:

- GPDs $\stackrel{!!!}{\longrightarrow}$ spin structure of nucleon $\stackrel{!!}{\longrightarrow}$ OAM!!
- TMDs $\stackrel{!}{\longrightarrow}$ transverse parton motion $\stackrel{??}{\longrightarrow}$ OAM???
- how? pretzelosity? only in quark models? why possible at all?
- in any case interesting function! can we access it? where?
- conclusions

1. Spin Structure of the nucleon

consider longitudinally polarized nucleon moving very fast in z-direction:



very naive picture!
sea-quarks, gluons, OAM!?

What we would like to know:

$$
\begin{aligned}
\frac{1}{2} &= J_z^N \\
&= S_z^Q + L_z^Q + J_z^{\text{glue}} \quad ?
\end{aligned}
$$

2. GPDs and orbital angular momentum *

* in principle (in practice, see talks by D. Müller, ...)

Exclusive reactions: $H^a(x,\xi,t)$, $E^a(x,\xi,t)$

$\Rightarrow$ form factors of energy momentum tensor

- $\int \mathrm{d}x\, x \left(H^a(x,\xi,t) + E^a(x,\xi,t) \right) = J^a(t)$ ("polynomiality")
- $\lim_{t\to 0} J^a(t) = J^a(0)$ Ji,1997

Deeply inelastic scattering: $g_1^a(x)$ $\Leftrightarrow$ Exclusive reactions: $\lim_{\xi,t\to 0} \tilde{H}^a(x,\xi,t)$

- $\int \mathrm{d}x\, g_1^q(x) \;\to\; S^q$

Combine:

- $J^q - S^q = L^q$

(issues, decomposition schemes, etc.)

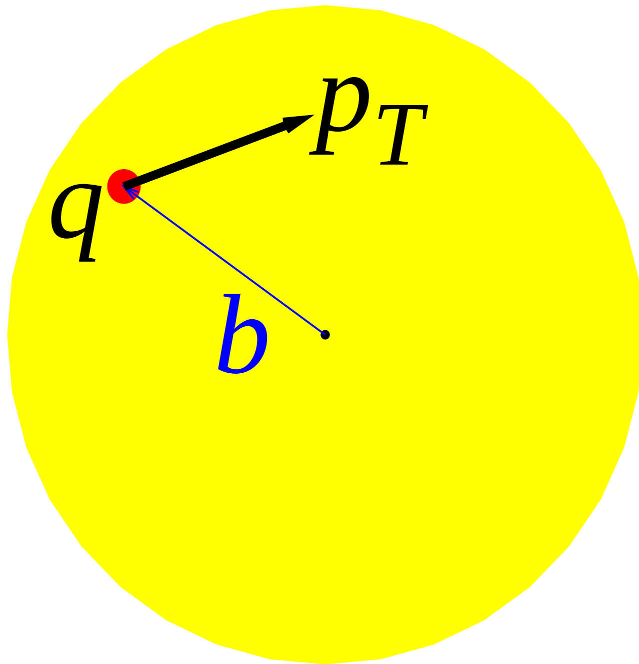
3. OAM, GPDs, and TMDs

- quarks are in transverse plane

GPDs $\stackrel{!!!}{\longrightarrow}$ b $\stackrel{!!!}{\longrightarrow}$ OAM

- quarks move in transverse plane

TMDs $\stackrel{!!!}{\longrightarrow}$ p_T $\stackrel{???}{\longrightarrow}$ OAM



nucleon moving towards us

we expect a connection:

TMDs $\leftrightarrow$ OAM

But how?

4. Pretzelosity

- Definition: (j transverse to +)

$$\frac{1}{2}\,\mathrm{tr}\left[i\sigma^{+j}\gamma_5\,\phi(x,\vec{p}_T)\right] \;=\; S_T^j\,h_1 + S_L\,\frac{p_T^j}{M_N}\,h_{1L}^\perp + \frac{(p_T^j p_T^k - \frac{1}{2}\,\vec{p}_T^{\,2}\delta^{jk})S_T^k}{M_N^2}\,h_{1T}^\perp + \frac{\varepsilon^{jk}p_T^k}{M_N}\,h_1^\perp$$

- inequalities $|h_{1T}^{\perp q}(x,p_T)| + |h_1^q(x,p_T)| \leq f_1^q(x,p_T)$ (Bacchetta et al. 1999)

- describes non-sphericity of "transverse spin distribution" (G. Miller, Burkhardt)

- requires nucleon wave-function components with $\Delta L = 2$ (M. Burkhardt, 2007)

- some (not all) quark models: (Avakian et al, Bacchetta et al, Efremov et al, Jakob et al, Pasquini et al, She et al)

$$h_{1T}^{\perp(1)q}(x,p_T) = g_1^q(x,p_T) - h_1^q(x,p_T)$$

"measure-of-relativity"

notation $h_{1T}^{\perp(1)q}(x,p_T) \equiv \frac{p_T^2}{2M^2}\,h_{1T}^{\perp q}(x,p_T)$, $h_{1T}^{\perp(1)q}(x) = \int \mathrm{d}p_T h_{1T}^{\perp(1)q}(x,p_T)$

relation model-dependent . . .

- not valid in quark-target model $h_{1T}^{\perp q} = 0$, $h_1^q - g_1^q \neq 0$ (Meissner, Metz, Goeke, 2007)

- not supported in some versions of spectator models (Bacchetta et al 2008)

. . . but inspiring

- known in light-cone SU(6) quark-diquark model (Ma and Schmidt, 1998)

$$h_1^q(x) - g_1^q(x) = L_z^q(x), \quad \int \mathrm{d}x L_z^q(x) = L_z^q$$

direct calculation in light-cone SU(6) quark-diquark model She, Zhu, Ma, 2009

$$h_{1T}^{\perp(1)q}(x, p_T) = g_1^q(x, p_T) - h_1^q(x, p_T) \quad \text{pretzelosity-relation!}$$

- light-cone SU(6) quark-diquark model She, Zhu, Ma, 2009

$$L_z^q = -\int \mathrm{d}x\, h_{1T}^{\perp(1)q}(x)$$

first connection of TMDs and OAM! **But model!**

take different model: you get different result **(?)** let's see:

- bag model uses SU(6)

$$L_z^q = -\int \mathrm{d}x\, h_{1T}^{\perp(1)q}(x)$$

Avakian, Efremov, PS, Yuan, 2010

- covariant parton model no SU(6)-symmetry,

$$L_z^q = -\int \mathrm{d}x\, h_{1T}^{\perp(1)q}(x)$$

Efremov, PS, Teryaev, Zavada, 2010

- non-relativistic limit $\lim_{\text{non-rel}} h_{1T}^{\perp q}(x, p_T) = -\frac{N_c^2}{2}\, P_q\, \delta\left(x - \frac{1}{N_c}\right)\, \delta^{(2)}(\vec{p}_T)$

$$\mathbf{0} = -\mathbf{0}$$

trivial but consistent byproduct in op. cit.

Questions arise (some answers here, some answers elsewhere)

- How can chiral-even and chiral-odd be related?

$\psi = \psi_L + \psi_R, \quad \psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi$

$$\psi^\dagger \Gamma \psi = \psi_R^\dagger \Gamma \psi_L + \psi_L^\dagger \Gamma \psi_R$$ pretzelosity, chiral odd

$$\psi^\dagger \hat{L}_z \psi = \psi_R^\dagger \hat{L}_z \psi_R + \psi_L^\dagger \hat{L}_z \psi_L$$ OAM, chiral even

kind of "chiral symmetry breaking" (chirality-flip)?

simple answer in bag model: $\psi = \begin{pmatrix} s\text{-wave} \\ p\text{-wave} \end{pmatrix} \Rightarrow \langle \hat{L}_z \rangle \propto |p\text{-wave}|^2$

pretzelosity $\propto |p\text{-wave}|^2$ (interference of $L_z = \pm 1 \Rightarrow$ needed $\Delta L = 2$, op. cit.)

$$\Rightarrow \quad \text{chiral-even} = \psi^* \hat{L}_z \psi = \psi^* \begin{pmatrix} * & 0 \\ 0 & 1 \end{pmatrix} \hat{L}_z \psi = -\psi^* \gamma^0 \hat{L}_z \psi$$

$$= \text{chiral-odd} \equiv -\text{pretzelosity}$$

- How can we have relations with S_L and S_T?

$$\begin{aligned} \text{OAM} &= \langle N(S_L)| \ldots |N(S_L)\rangle \\ \text{pretzelosity} &= \langle N(S_T)| \ldots |N(S_T)\rangle \end{aligned}$$

Why not? Simple rotation $|N(S_L)\rangle = U_{90^\circ}|N(S_T)\rangle$

But: no operator identity, $\nexists \;\; \hat{O}_{\text{OAM}} = \hat{O}_{\text{pretzelosity}}$
at best: relation at the level of matrix-elements

- Does the result depend on choice of OAM definition?

Here (no-gauge-field theory) for L_z^q no ambiguity

(Jaffe-Manohar = Ji, M. Burkardt and H. BC, 2009)

- What are model limitations? Valid in models with $L \geq 2$ (d-wave, ...)?
  $\longrightarrow$ Cédric Lorcé, Barbara Pasquini, ...

- What happens when we have gluons?
  No relation! (Meissner, Metz, Goeke, 2007)
  Jaffe-Manohar vs. Ji matters (Burkardt, BC, 2009)

- What do we know from lattice? Lattice-sign of L_z^q "opposite to all quark-models on the planet" (M. Burkardt, on Monday)

- Not quite true! Chiral quark-soliton model $\longrightarrow$ sea-quarks! (Wakamatsu)

  resolutions to puzzles (?)
  Matthias-puzzle: (other) quark models on planet vs. lattice
  Dieter-puzzle: how can CQSM (model quarks) and lattice (real quarks) agree?

  sea-quarks in model, but model reasonable! Based on (relevant!):
  chiral symmetry breaking from instanton-picture of QCD-vacuum!
  (Diakonov, Petrov 1984, ...)

OAM and sea quarks?

1. if you find something at large b: likely sea-quark $\in$ "pion-cloud" ("valence-quark" wave-function vanishes exponentially with b)

2. $$\langle \boldsymbol{p_T^2} \rangle_{\mathbf{sea}} = \frac{(-1)\,\langle\bar{\psi}\psi\rangle\,M}{2F_\pi^2} = (\mathbf{2}\text{–}\mathbf{3})\langle \boldsymbol{p_T^2}\rangle_{\mathbf{val}}, \quad \langle p_T^2\rangle_{\text{val}} \approx 0.2\,\text{GeV}^2, \quad \mu \sim \rho_{\text{av}}^{-1}$$
(Wakamatsu; PS, Strikman, Weiss)

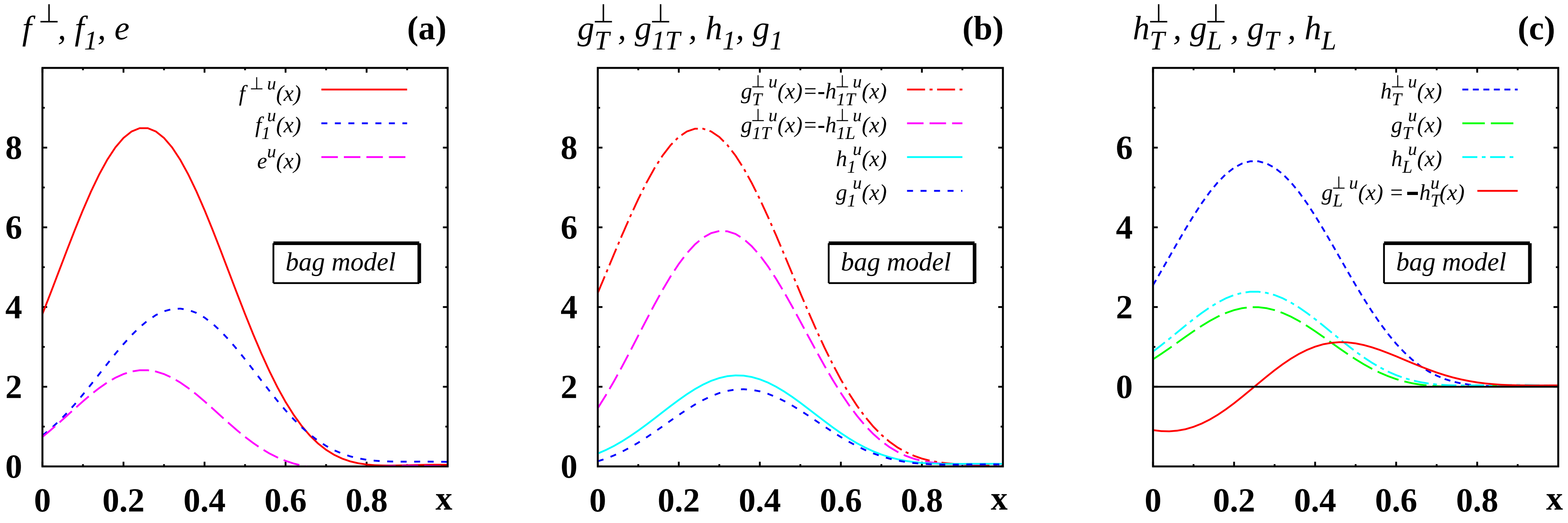
   How to see?

   – DY with pp vs. $p\bar{p}$: $$\langle q_T^2\rangle = \begin{cases} \langle p_T^2\rangle_{\text{val}} + \langle p_T^2\rangle_{\text{sea}} & \text{in } pp \\ \langle p_T^2\rangle_{\text{val}} + \langle p_T^2\rangle_{\text{val}} & \text{in } p\bar{p} \end{cases}$$ ($\exists$ some data, GSI and PAX)

   – JLab 12, EIC: $\frac{d\sigma(P_{h\perp})}{dP_{h\perp}}$ of $K^+ = u\bar{s}$ vs. $K^- = \bar{u}s$

3. Add 1 + 2! Larger b + larger p_T = more $L_z^{\bar{q}}$! (intuitive but classic) to be studied in (tractable, effective) quantum field theory (model)!
(e.g. chiral quark-soliton model)

Look on pretzelosity: bag model (Avakian, Efremov, PS, Yuan 2009)



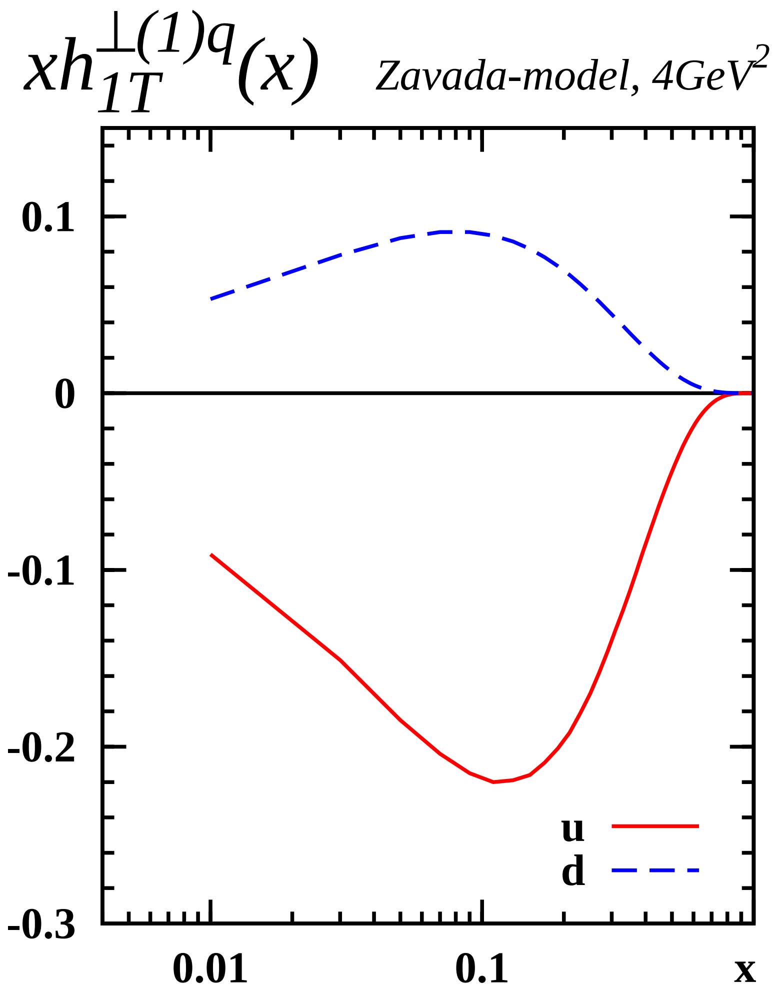
Striking: $h_{1T}^{\perp}(x)$ large! But in cross sections $\dfrac{p_T^i p_T^j}{M^2}\, h_{1T}^{\perp}(x, p_T)$

$\dfrac{\langle p_T^2 \rangle}{M^2} \sim \dfrac{1}{3}$ at $s = 50\,\text{GeV}^2$ (HERMES) (PS, Teckentrup, Metz 2010)

notice $\langle p_T^2 \rangle = \langle p_T^2(s) \rangle$. Important for JLab, HERMES, COMPASS $\rightarrow$ EIC!

Look on pretzelosity: covariant parton (Zavada) model

(Efremov, PS, Teryaev, Zavada 2009)



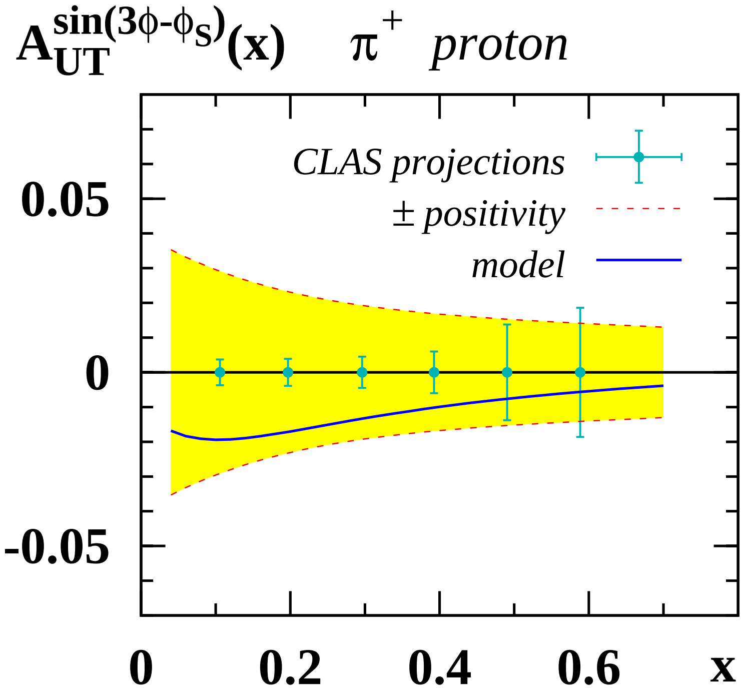
Glimpse (trough Zavada-model-glasses) on $(-1)\times$ OAM ??? Will see ...

Can we access pretzelosity? in semi-inclusive DIS

$$A_{UT}^{\sin(3\phi-\phi_S)} = \frac{h_{1T}^{\perp} H_1^{\perp}}{f_1\, D_1} \sim 0 \text{ within error bars}$$

preliminary COMPASS (deuteron)
HERMES (proton)

one prediction: light-front constituent model $\rightarrow$ talk by Barbara
another prediction Zavada-model: Efremov, PS, Teryaev, Zavada



covariant parton model with
rotationally symmetric parton motion
$G(Pp/M) = G(p^0)$ in rest frame,
Interesting because $h_1^u > g_1^q$

$\rightarrow$ sizeable $h_{1T}^{\perp(1)q}(x)$

positivity bound Bacchetta et al, 1999

projections CLAS12 H.Avakian

Will we get a weakly(?) model-dependent glimpse on OAM from pretzelosity!?

Why should we believe in quark models? Could be better than we think.

- quark-models have $>$ 30 years of successful phenomenology!
  Have limitations, have model-accuracy, but we know this
  (Boffi, Efremov, Pasquini, PS 2009)

- LIRs

$$
\begin{aligned}
g_T(x) &\stackrel{\text{LIR}}{=} g_1(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_{1T}^{(1)}(x), \\
h_L(x) &\stackrel{\text{LIR}}{=} h_1(x) - \frac{\mathrm{d}}{\mathrm{d}x} h_{1L}^{\perp(1)}(x), \\
h_T(x) &\stackrel{\text{LIR}}{=} -\frac{\mathrm{d}}{\mathrm{d}x} h_{1T}^{\perp(1)}(x) \\
g_L^{\perp}(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_T^{\perp(1)}(x) &\stackrel{\text{LIR}}{=} 0 \\
\underbrace{h_T(x,p_T^2) - h_T^{\perp}(x,p_T^2)}_{\text{twist-3}} &\stackrel{\text{LIR}}{=} \underbrace{h_{1L}^{\perp}(x,p_T^2)}_{\text{twist-2}}
\end{aligned}
$$

Mulders, Tangerman 1995, Kundu, Metz 2001, Goeke, Metz, Pobylitsa, Polyakov 2003

LIRs must hold in all relativistic quark models without gluons
$\Leftrightarrow$ Wandzura-Wilczek (type) approximations $\langle \bar{q}gq \rangle \ll \langle \bar{q}q \rangle$
Metz, PS, Teckentrup 2009

classic Wandzura-Wilczek approximation (Wandzura, Wilczek, 1977)

$$g_T^q(x) = \int_x^1 \frac{dy}{y} g_1^q(y) + \tilde{g}_T^q(x)$$

$\tilde{g}_T^q(x) = \langle \bar{q}gq \rangle$ + current quark mass-terms
in instanton vacuum suppressed Balla, Polyakov, Weiss 1997
in experiment $\tilde{g}_T^q(x)$ small! SLAC, JLab (review by Accardi et al, 2009)
on the lattice also small Göckeler *et al.* 2001

Does not imply that other quark-model relations hold with similar accuracy.
Have to be careful and check case by case. But it motivates
to have a closer look on such relations in QCD.

In view of the many novel functions:
would be welcome to have (approximate) relations among TMDs!

Conclusions

- **dual picture of OAM**

  at least in (naive, happy) quark-model world:

$$L_z^q = \int \mathrm{d}x \int \mathrm{d}^2 b \, \{H^q(x,b),\ E^q(x,b),\ \tilde{H}^q(x,b)\} \quad \text{(Ji sum rule)}$$
$$= -\int \mathrm{d}x \int \mathrm{d}^2 p_T \, h_{1T}^{\perp(1)q}(x, p_T) \quad \text{(“pretzelosity sum rule”)}$$

- first explicit connection of **OAM and TMDs** in quark models

- quark-model relations **might work** reasonably well (WW, valence-x)

- **future data** (JLab, EIC) on exclusive + deeply inelastic reactions will decide

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\begin{aligned}
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&= -\int \mathrm{d}x \int \mathrm{d}^2 p_T \, h_{1T}^{\perp(1)q}(x, p_T) && \text{("pretzelosity sum rule")}
\end{aligned}
$$

- first explicit connection of **OAM and TMDs** in quark models
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Thank you!!