Timelike Form Factors

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Introduction

On May 18, 2002, almost exactly five years ago, in this same place at JLab, I gave a talk on **Timelike Form Factors**, and I had two additional words in the title — **of Protons** [1]. Today, my talk has those additional words removed, for I am going to talk about more than protons. I will also talk about **Pions and Kaons**, for which I have some beautiful results to report.

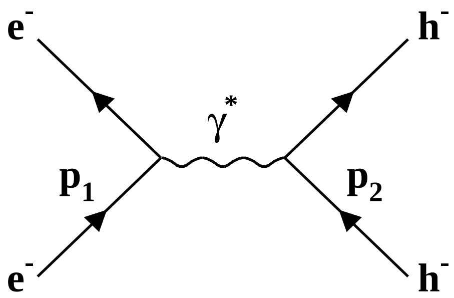
In my 2002 talk, I pointed out that "ONLY PROTONS ARE FOREVER", and therefore we must try to understand them. Today, I make a different pitch. All hadronic reactions end up in **pions**, therefore we must try to understand them. **Kaons**, well, they are strange, therefore we must understand them too.

So, here we go!

Form Factors

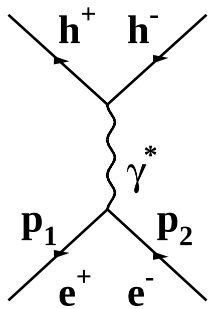
Electromagnetic form factors of a hadron are the most direct link to the structure of the hadron in terms of its constituents. They describe the coupling of a photon with a certain four–momentum to the distribution of charges and currents in the hadron.

The four–momentum transfer Q^2 in the collision of two particles with four-momenta p_1 and p_2 can be positive or space-like (in scattering) or negative or time-like (in annihilation/production).



Scattering, Spacelike

positive $Q^2 = t$



Annihilation, Production

negative $Q^2 = s$

Form Factors for Space-like Momentum Transfers

The form factor measurements done at SLAC and JLab with electron beams scattered from targets of p, d, ..., etc., and for electroproduction of pions (essentially electron scattering from the pion cloud) are **exclusively for spacelike momentum transfers**. They **require fixed targets**, and are extremely difficult, if not impossible, to do for measuring space-like form factors of mesons at large momentum transfers; meson targets just do not exist!

Form Factors for Time-like Momentum Transfers

The measurements I am going to talk about are done with e^+e^- or $p\bar{p}$ annihilation, and therefore involve **time-like momentum transfers**. The e^+e^- measurements are done at e^+e^- colliders, and they can, in principle, be used to measure form factors for **any mesons or baryons**. The $p\bar{p}$ annihilations have so far been only done with $\bar{p}$ beams incident on fixed proton targets. These experiments of course only lead to proton form factors.

Important Note

The form factors are analytic functions of Q^2. The Cauchy theorem alone guarantees that

$$F(Q^2, \text{timelike}) \xrightarrow{Q^2 \to \infty} F(Q^2, \text{spacelike})$$

PROTONS

Cross Sections for Time-like Momentum Transfers

For protons, there are two form factors, Pauli and Dirac Form Factors, or more familiarly, the magnetic $G_M(s)$ and the electric $G_E(s)$ form factors, and the cross section $e^+e^- \to p\bar{p}$ is

$$\sigma_0(s) = \frac{4\pi\alpha^2}{3s}\beta_p\left[|G_M^p(s)|^2 + \frac{\tau}{2}|G_E^p(s)|^2\right]$$

(For $\sigma(p\bar{p} \to e^+e^-)$, replace β_p by $1/\beta_p$.)

At large momentum transfers separation between $G_M(s)$ and $G_E(s)$ is very difficult, and the results which are generally reported assume $G_E(s) = 0$, or $G_E(s) = G_M(s)$. (see BaBar result for G_E/G_M)

For pions and kaon, both of which have spin 0, there is no magnetic contribution, and only the electric form factor $F(s)$ exists. In this case the cross section for $e^+e^- \to m^+m^-$ is

$$\sigma_0(s) = \frac{\pi\alpha^2}{3s}\beta_m^3|F_m(s)|^2$$

Expectations with Large Momentum Transfers

Jumping the gun a little, let me point out that pQCD counting rules predict that the baryon form factors are proportional to Q^{-4} (or s^{-2}) and the meson form factors are proportional to Q^{-2} (or s^{-1}), so that

$$(d\sigma/d\Omega)_{\rm proton} \propto s^{-5}, \qquad (d\sigma/d\Omega)_{\rm meson} \propto s^{-3}$$

This tells you how rapidly the cross sections fall, and **how difficult** it becomes to measure any form factors at large momentum transfers.

For example, $\sigma(e^+e^- \to p\bar{p}) \approx 1$ pb at $s = Q^2 = 13.5$ GeV^2. At $s = 25$ GeV^2 one expects to drop down by a factor ~ 20, to ~ 50 fb.

This should prepare you for the large error bars you will see later!

Spacelike Form Factors of the Proton - 1

The spacelike magnetic form factors $G_M(Q^2)$ of the proton were measured with precision in the ep scattering experiments at SLAC, all the way up to $Q^2 = 31\ \mathrm{GeV}^2$ [2]. For $Q^2 \geq 15\ \mathrm{GeV}^2$, their **variation** follows the pQCD counting rule prediction that $Q^4 G_M(Q^2)/\mu_p$ is essentially constant and varies only as $\alpha^2(\mathrm{strong})$.

In the pQCD **factorization formalism** of Brodsky and Lepage [3]

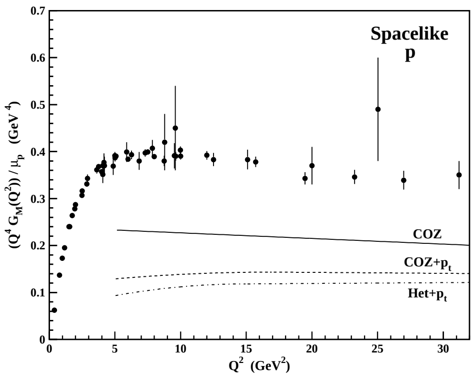
$$F(Q^2) = \int_0^1 \int_0^1 [dx][dy]\ \phi^*(y_i, Q^2)\ T_H(x_i, y_i, Q^2, \mu_R^2)\ \phi(x_i, Q^2),$$

- $T_H(x_i, y_i, Q^2, \mu_R^2)$ is the hard scattering amplitude.
- $\phi(x_i, Q^2)$ and $\phi^*(y_i, Q^2)$ are hadron distribution amplitudes (DA).
- μ_R^2 is the renormalization scale for the strong coupling constant, $\alpha_s(\mu_R^2)$.
- $i = 2$ and $i = 3$ refer to the constituent quarks in mesons and baryons, respectively.
- $[dx]$ and $[dy]$ are integration variables of the quark momentum fractions.

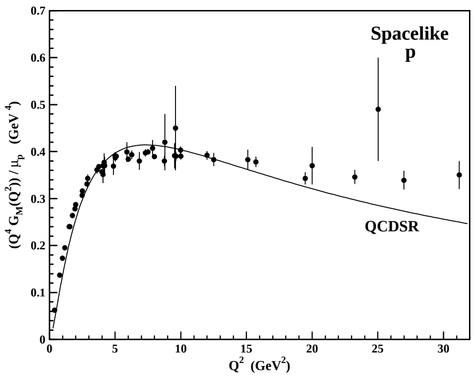
The non-perturbative part is entirely contained in the DA's, ϕ.

Spacelike Form Factors of the Proton - 2

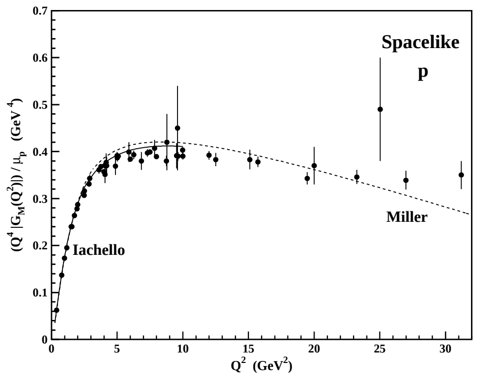
The asymptotic distribution amplitudes (DA) $\phi(y)$ and $\phi(z)$ lead to $G_M^P(Q^2) = 0$ for all Q^2. So, many different variations of asymmetric DA's have been considered, with and without **Sudakov corrections**, and with and without **transverse momenta**. QCD sum–rule predictions, and predictions based on GPD and meson–cloud pictures have also been made. With an appropriate choice of the parameters, the spacelike form factors of the proton can be **fitted**. Here is a montage of a few.



Hyer



Radushkin

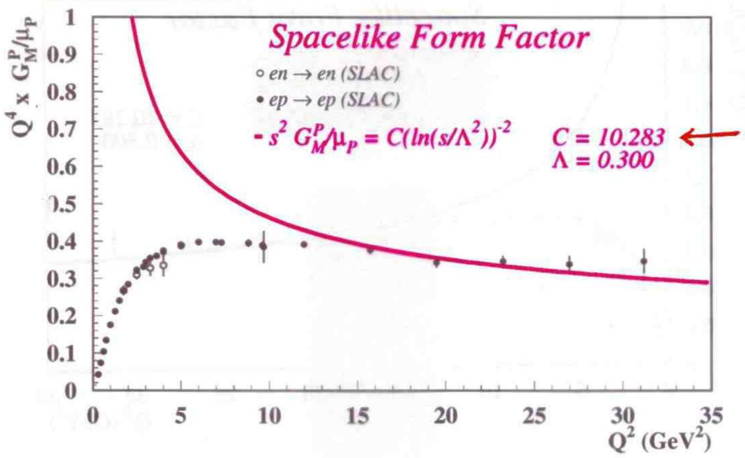


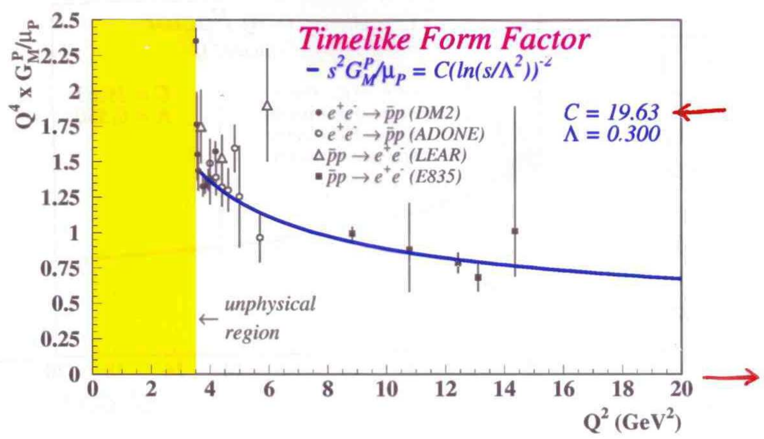
Iachello/Miller

Timelike Form Factors of the Proton – 1

Prior to the Fermilab (E760/E835) measurements in 1993/2003 [8,9,10] of the timelike form factors of the proton by the reaction $p\bar{p} \to e^+e^-$, the data were sparse, had large errors, and were confined to $|Q^2| < 5, 7\ \mathrm{GeV}^2$. The Fermilab measurements obtained $G_M(|Q^2|)$ for four $|Q^2|$ between 8.9 and 13.11 GeV^2. As the figure shows, while $Q^4 G_M(|Q^2|)$ was found to vary as $\alpha^2(\mathrm{strong})$, the value of the timelike form factor was **found to be twice as large** as the spacelike form factor, i.e.,

$$R \equiv G_M(\mathrm{timelike})/G_M(\mathrm{spacelike}) \approx 19.6/10.3 \approx 2$$

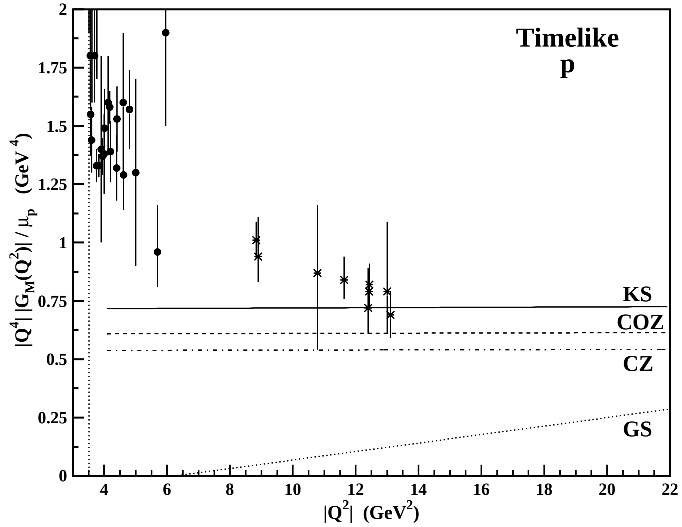




Timelike Form Factors of the Proton – 2

Prior to the Fermilab measurements there were few theoretical predictions of the timelike form factor of the proton. I quote two of them. Dubnicka [11] has been working on form factors within the framework of VDM in several iterations. Her 1990 version, for example, predicted the ratio $R \approx 0.8$ at $|Q^2| = 10\ \mathrm{GeV}^2$. Magnea and Sterman [13] calculated the ratio for the Sudakov form factor in a two–loop calculation to be ≈ 1.5 for $\alpha(strong) = 0.3$, but did not attempt to predict absolute values.

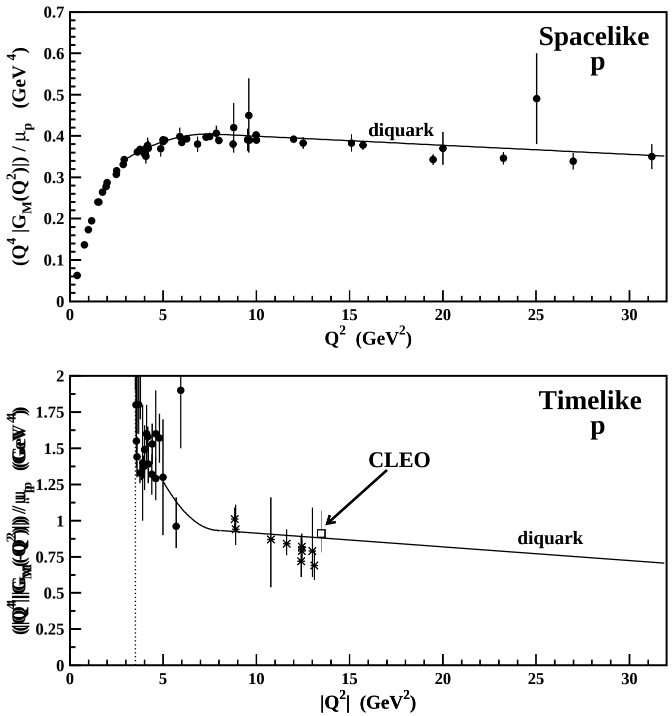
Following the Fermilab measurements, Hyer [13] reported predictions for timelike form factors within the pQCD formalism including Sudakov suppression. Hyer's predictions, shown in the figure, indicate the sensitivity to the distribution amplitudes, but do not address the question of the experimental ratio $R \approx 2$.



Timelike Form Factors of the Proton – 3

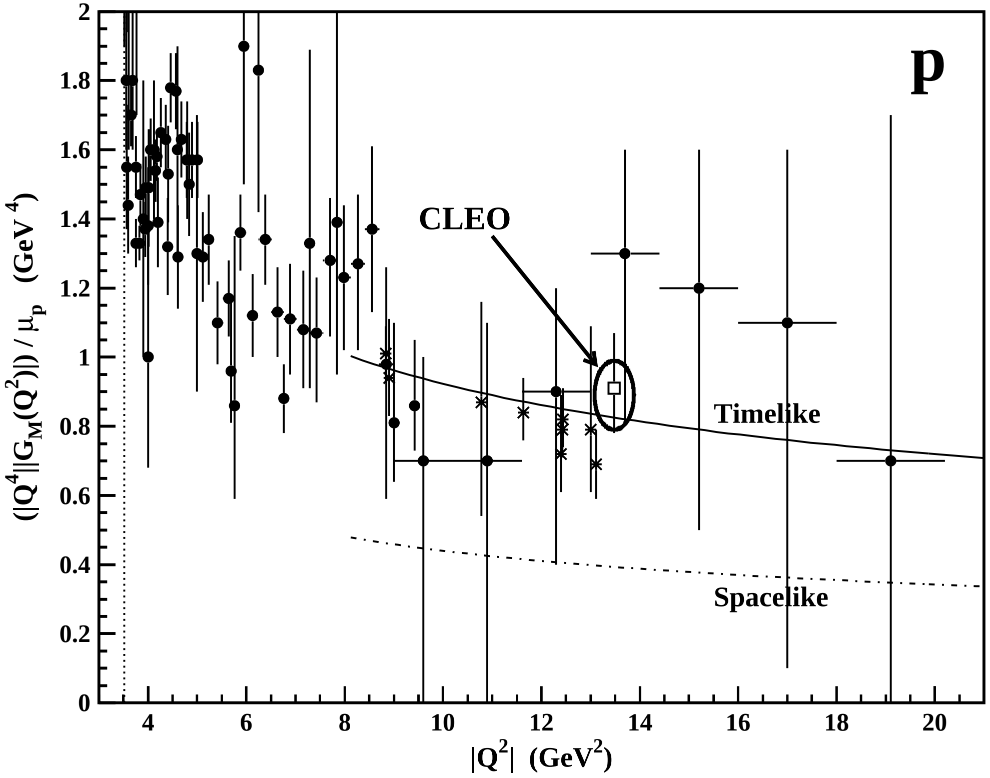
The attempt to explain the ratio $R \approx 2$ led Kroll and collaborators to propose the diquark–quark model of the nucleon. While this model has at least two extra parameters, it explains both spacelike and timelike G_M, and $R \approx 2$ very nicely.

On the experimental side, the new developments are that in e^+e^- annihilation at Cornell [14] we have made a measurement of $G_M(p)$ at $|Q^2| = 13.5\ \mathrm{GeV}^2$, BES [15] has made direct measurements at ten values of $|Q^2| = 4 - 9.4\ \mathrm{GeV}^2$, and BaBar [16] has made measurements using ISR from $\Upsilon(4S)$ at $|Q^2| = 3.6 - 20.3\ \mathrm{GeV}^2$. All these measurements, shown in the following figure, are consistent with each other, and confirm $R \approx 2$. BaBar has gone a step beyond, and has attempted to derive G_E/G_M from their data.



- I will let Prof. Baldini tell you about the BaBar measurements in detail.

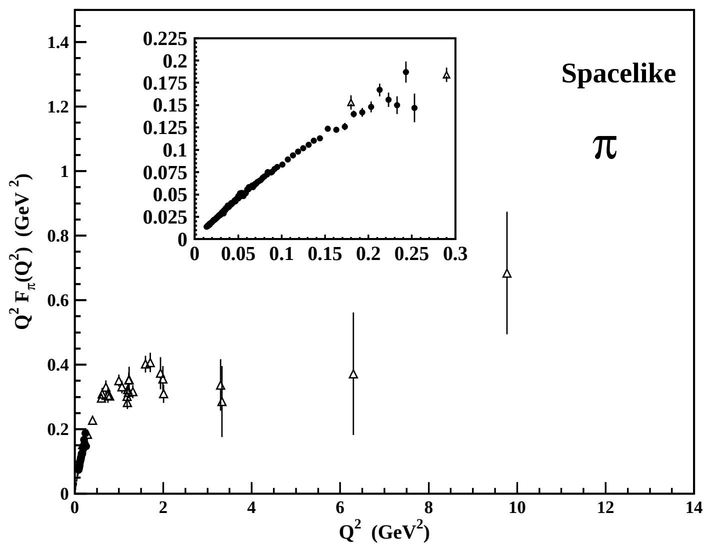
Timelike Form Factors of the Proton – 4

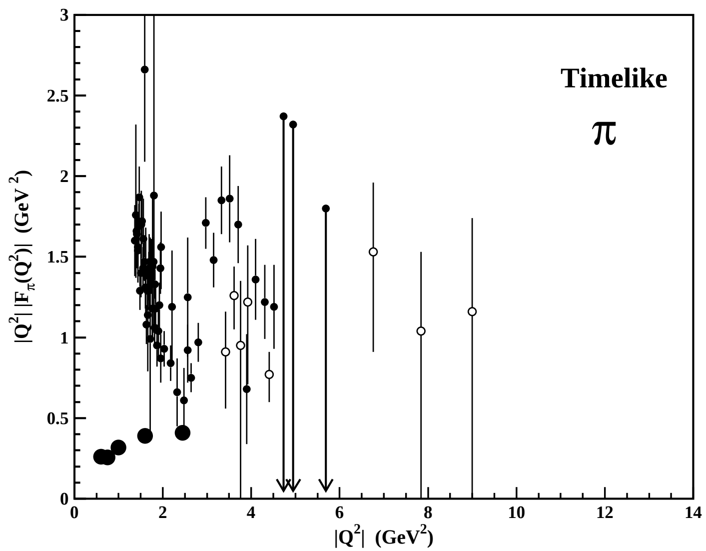


PIONS AND KAONS

Form Factors of Pions and Kaons

Mesons represent much simpler systems than **baryons**; two quark systems are expected to be easier to understand than three quark systems. It is because of this that the now-classic debate about when $|Q^2|$ is large enough for the **validity of pQCD** took place in the 1980s between Brodsky and collaborators on one side and Isgur and Llwellyn Smith on the other side. Unfortunately, the then existing experimental data on **pion form factors** was extremely poor, especially in the large $|Q^2|$ region which was the subject of the entire debate.





Spacelike Form Factors of Pions

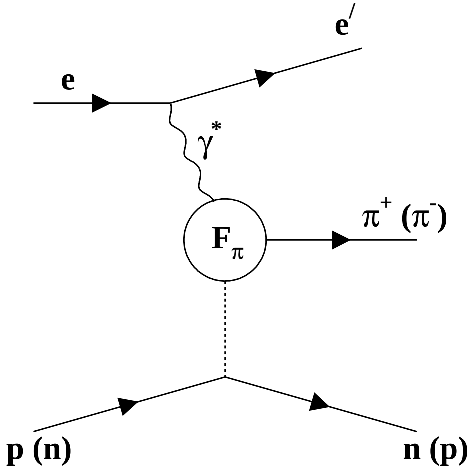
The problems with meson form factors for spacelike momentum transfers arises from the fact that **meson targets** do not exist. Two methods have been used to get around this problem.

1. Elastic scattering of pions off atomic electrons: $\pi^- e^- \to \pi^- e^-$

   Limitation: Only very low Q^2 possible. For example, with 300 GeV pion beams, $Q^2(\max) = 0.12\ \mathrm{GeV}^2$ was realized at CERN.

2. Electroproduction of pions: $e^- p \to e^- \pi^- p,\ e^- \pi^+ n$

   Limitation: Also limited to low Q^2. Many theoretical objections.



Electroproduction of Pions

The longitudinal part of the cross-section $\sigma(L)$ is related to the spacelike form factor, F_π,

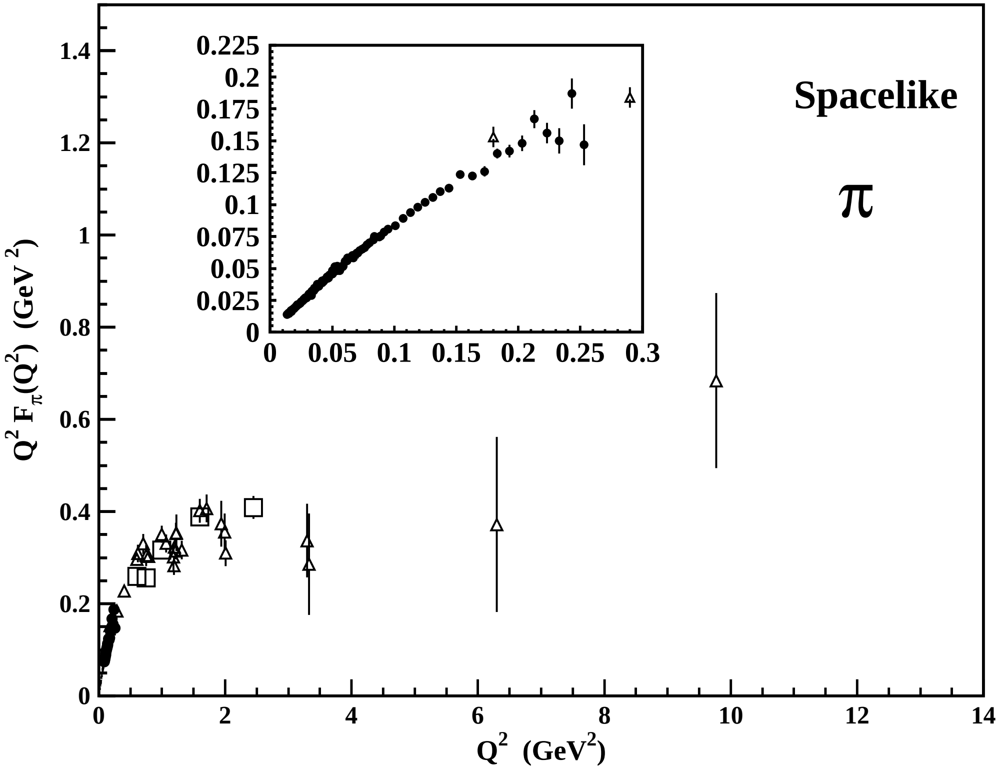
$$\sigma(L) \approx -\frac{t}{(t-m_\pi)^2} g^2_{\pi NN}(t) F^2_\pi(Q^2)$$

The latest measurements are from JLab for $Q^2 = 0.6 - 2.45\ \mathrm{GeV}^2$ in which longitudinal/transverse separation was done [17], unlike the earlier Cornell measurements [18]. The three larger $Q^2 = 3.3$, 6.3 and $9.8\ \mathrm{GeV}^2$ results from Cornell have $\pm 38\%$, $\pm 51\%$ and $\pm 27\%$ quoted errors, respectively, and have additional uncertainties due to the ad-hoc way of subtracting transverse cross-sections .

Objections

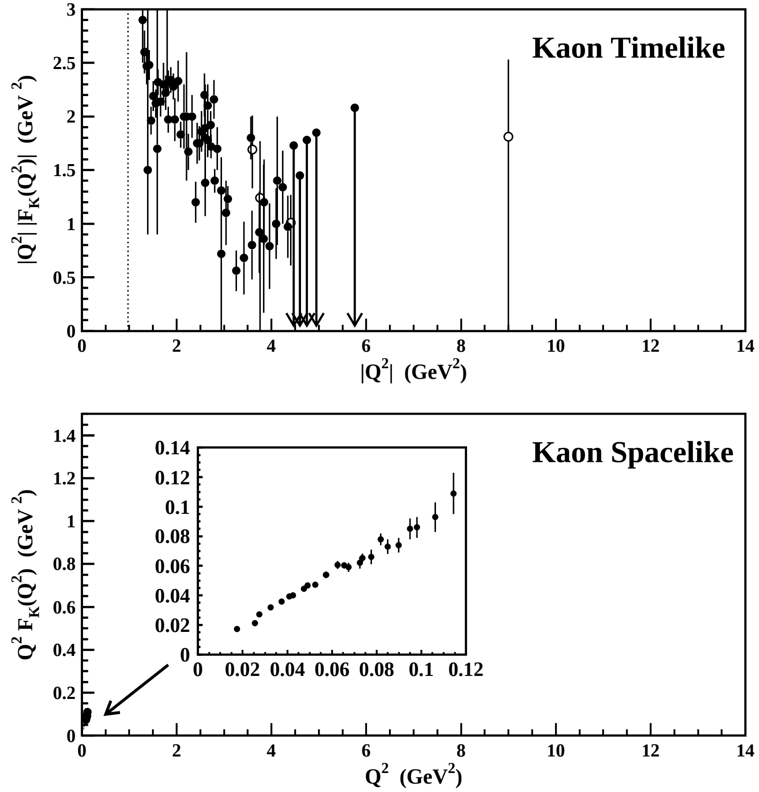
1. The struck pion is off-shell and one must extrapolate to the physical pion pole at $t = m^2_\pi$.
2. The t dependence of $g_{\pi NN}$, the pion–nucleon coupling, is uncertain.
3. Other hard processes which can not be separated compete with the t–channel process [19].

Spacelike Form Factors of Pions



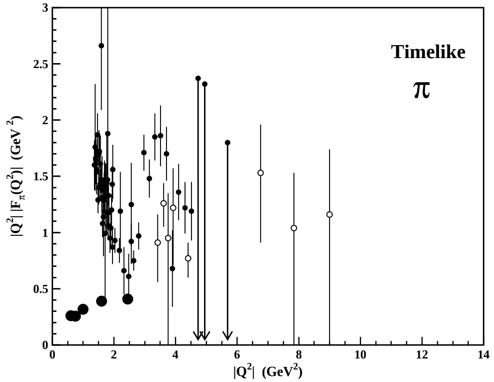
Spacelike Form Factors of Kaons

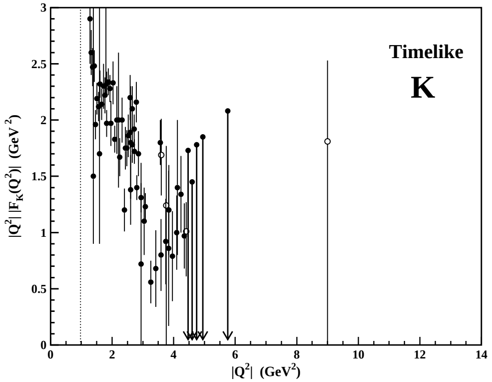
For kaons things are in worse shape, and only form factor measurements by elastic scattering of 250 GeV kaons from atomic electrons are available. As for pions, these are confined to very low momentum transfers, $Q^2 < 0.3\ \mathrm{GeV}^2$ and have only been used to measure the kaon radius [20].

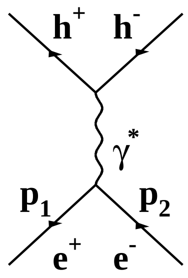


Timelike Form Factors of Pions and Kaons

As already mentioned, these can be determined by e^+e^- annihilation, $e^+e^- \to \pi^+\pi^-, K^+K^-$. Prior to the present measurements at CLEO [14], the only existing data were the ones shown below.





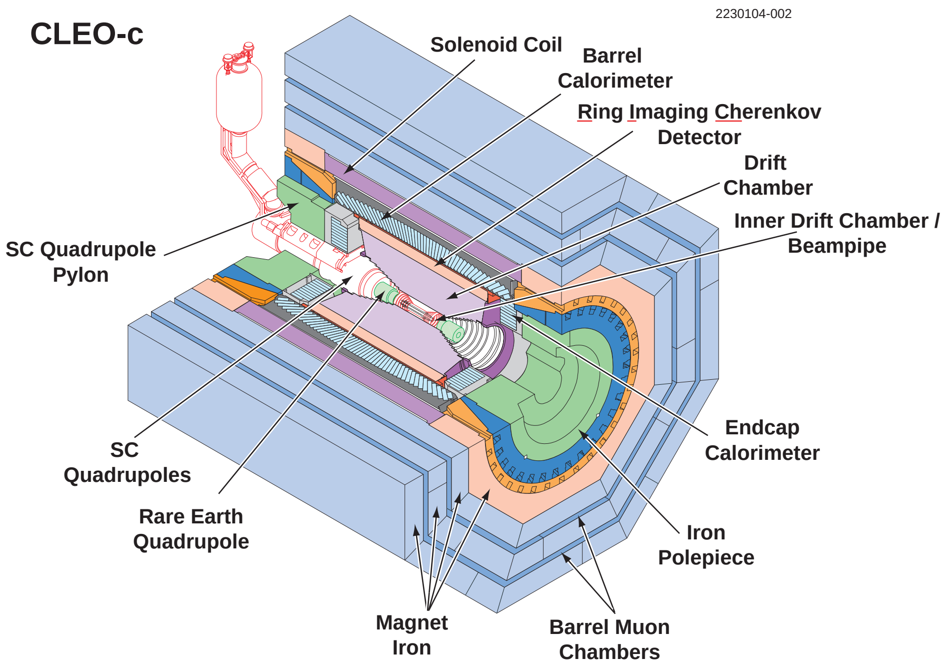


All measurements have large errors, approaching +100%–200% at the highest $|Q^2|$. This is mainly due to the great difficulty in the identification of $\pi^\pm$ and $K^\pm$ in the presence of **monstrous backgrounds** from the QED production of e^+e^- and $\mu^+\mu^-$.

- For example, in the ADONE measurements at $|Q^2| > 3.5$ GeV^2, the few h^+h^- pairs observed could not be assigned to $\pi^+\pi^-$ or K^+K^-. At $|Q^2| = 9$ GeV^2, $\sigma(\pi^+\pi^-) = 60 \pm 60$ pb and $\sigma(K^+K^-) = 80 \pm 80$ pb were reported
- Later DM2 measurements at Orsay failed to find any kaon pairs for $|Q^2| > 4.4$ GeV^2.

Timelike Form Factors of Pions and Kaons

I want to describe a bit in detail how the **formidable problem** of $\pi^\pm$ and $K^\pm$ identification was overcome in our new measurements at CLEO. The summary of the answer is that you need a state-of-the-art detector and a very good graduate student (in this case, Peter Zweber).



CLEO Measurements of Pion and Kaon Form Factors – 1

The CLEO measurements were made with the CLEO-c detector using 20.7 pb^{-1} of e^+e^- data taken at $\sqrt{s} = 3.671$ GeV, i.e., 15 MeV below the ψ' resonance. The data were originally taken for background studies for the ψ' decays which were being studied. It is ironic that these background studies have provided the world's best measurements of pion and kaon form factors.

- To illustrate the formidable problem of backgrounds, let me jump a bit ahead to tell you that the CLEO measured form factor cross-sections at 3.67 GeV turn out to be

$$\sigma(e^+e^- \to \pi^+\pi^-) \approx \mathbf{9\ pb}, \quad \sigma(e^+e^- \to K^+K^-) \approx \mathbf{6\ pb}$$

  The background cross-sections for $|\cos\theta| < 0.8$ are

$$\begin{aligned} \sigma(e^+e^- \to e^+e^-) &\approx \mathbf{130\ nb} \\ \sigma(e^+e^- \to \mu^+\mu^-) &\approx \mathbf{5\ nb} \\ \sigma(e^+e^- \to h\bar{h}) &\approx \mathbf{10\ nb} \end{aligned}$$

- In other words, the background cross-sections are 10^3 to 10^5 times larger than the form factor cross-sections to be measured.

To reject backgrounds at this level you must use everything at your disposal.

CLEO Measurements of Pion and Kaon Form Factors – 2

For the CLEO-c measurements, it means that in addition to the standard requirements of charged particle track quality and event vertex, we use the back-to-back nature of m^+ and m^-. We use measurements of:

- momenta and dE/dx from the main **drift chamber**,
- energy loss E_{CC} in the **central calorimeter**,
- and the particle identification capabilities of the **RICH detector**,

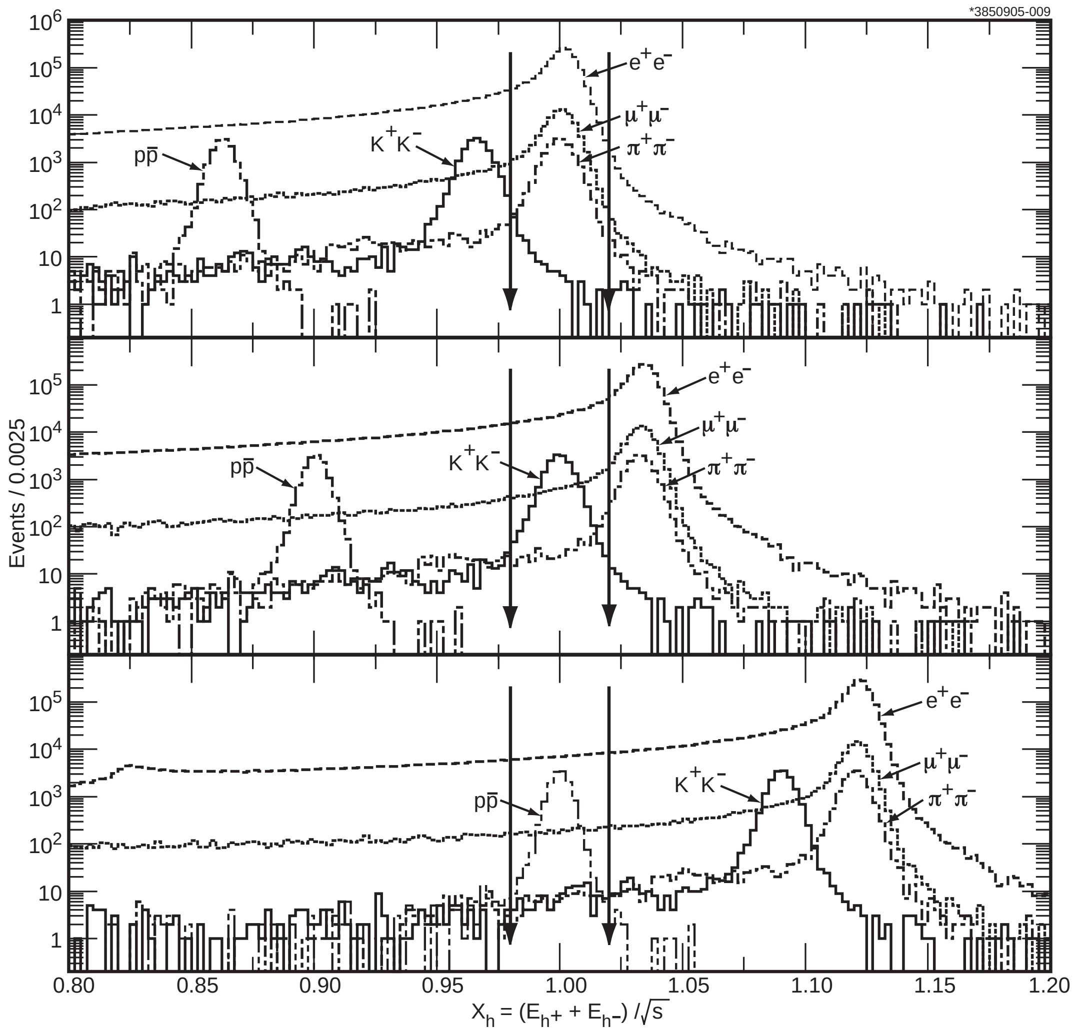
to achieve background reduction at the required level.

The challenge of particle identification is best illustrated by the following figure which shows Monte Carlo distributions (on arbitrary scale) as functions of the normalized energy variable, $X_h \equiv (E(h^+) + E(h^-))/\sqrt{s}$.

It is clear that we have minimal problems with protons, manageable problems with kaons, and formidable problems with pions. However, all problems were solved with a very fancy likelihood cut defined as

$$L(p,K) - L(l) = (L_{RICH}(p,K) + \sigma^2_{dE/dx}(p,K)) - (L_{RICH}(l) + \sigma^2_{dE/dx}(l))$$

for separating $p\bar{p}$ and K^+K^- from their leptonic background, and making specialized likelihood and E_{CC} cuts for separating $\pi^+\pi^-$ from leptonic and kaonic backgrounds.



π

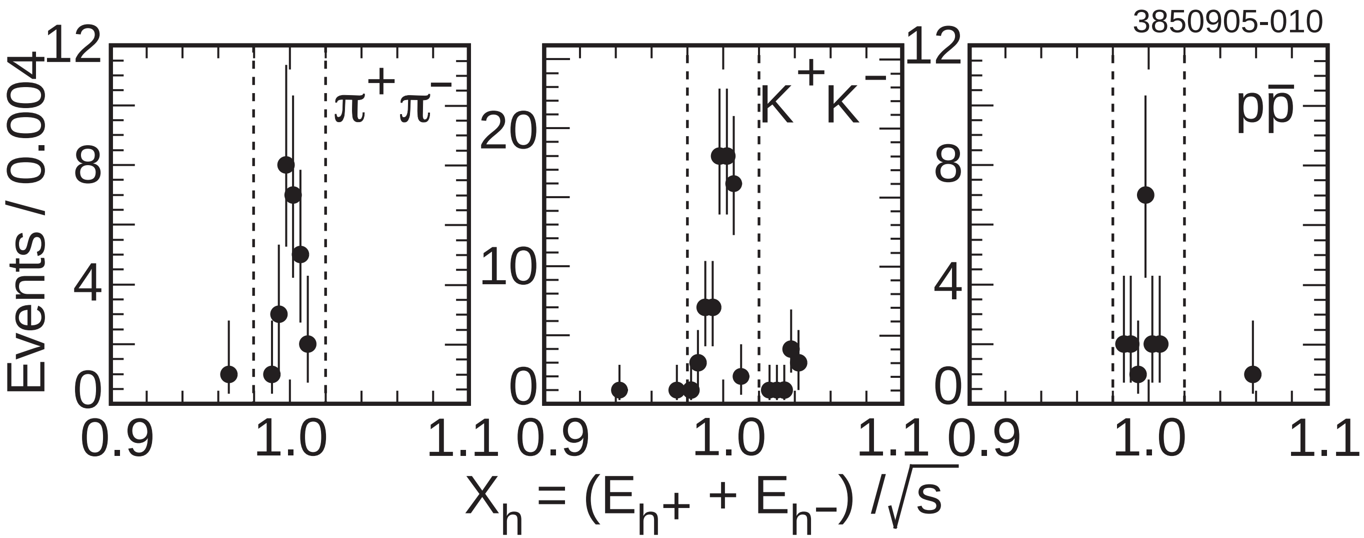
K

p

$$X_h = (E_{h^+} + E_{h^-})/\sqrt{s}$$

CLEO Measurements of Pion and Kaon Form Factors – 3

The resulting form factor events, though small in number, are almost completely free of backgrounds and are shown below.



$N(\pi^+\pi^-) = 26 \pm 5 \qquad N(K^+K^-) = 72 \pm 9 \qquad N(p\bar{p}) = 16 \pm 5$

CLEO Measurements of Pion and Kaon Form Factors – 4

The signal counts N are related to the Born cross-section as

$$\sigma_0(e^=e^- \to m^+m^-) = N/[\epsilon\mathcal{L}(1+\delta)]$$

where ϵ is the efficiency, $\mathcal{L}$ is the integrated luminosity and $(1+\delta)$ is the radiative correction factor $(\sim 0,8)$. Finally,

$$\sigma_0(s) = \frac{\pi\alpha^2}{3s}\beta_m^3|F_m(s)|^2$$

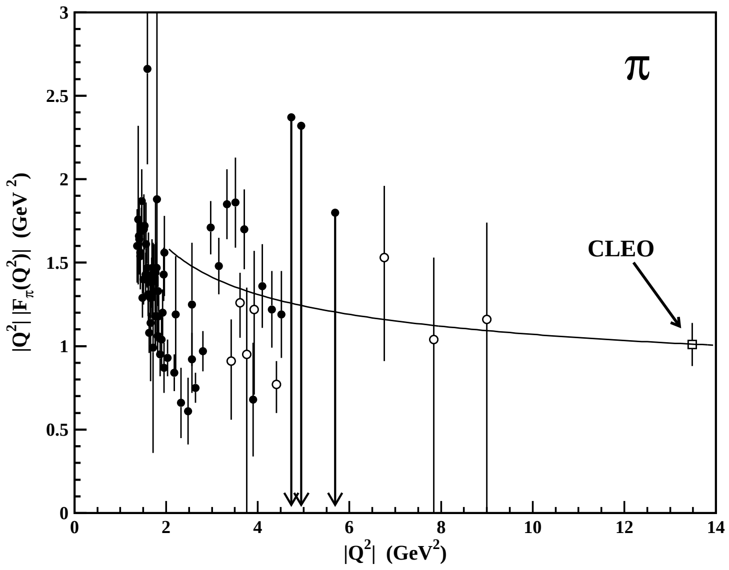
The results for $|Q^2| = 13.48\ \mathrm{GeV}^2$ are:

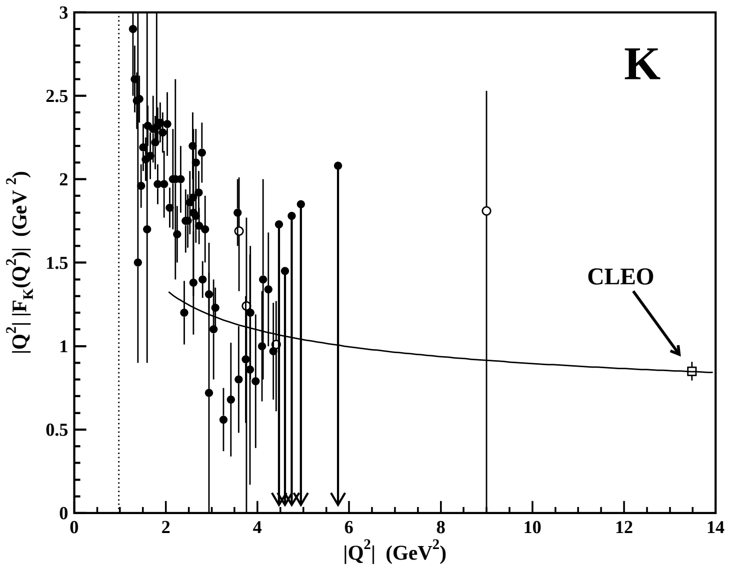
PION: $|Q^2|F_\pi(|Q^2|) = \mathbf{1.01 \pm 0.11 \pm 0.07\ GeV^2}$
KAON: $|Q^2|F_K(|Q^2|) = \mathbf{0.85 \pm 0.05 \pm 0.02\ GeV^2}$

$$F_\pi(13.48\ \mathrm{GeV}^2)/F_K(13.48\ \mathrm{GeV}^2) = \mathbf{1.19 \pm 0.07}$$

These are the **world's first** measurements of the form factors of any mesons at this large a momentum transfer, and with precision of this level, $\pm 13\%$ for pions and $\pm 6\%$ for kaons. They are shown in the figure along with the old world data, and arbitrarily normalized curves showing the pQCD predicted **variation** of $|Q^2|F_\pi$ and $|Q^2|F_K$ with α_S.

Timelike Form Factors of Pions and Kaons





A Theoretical Measurement

Before our experimental measurement of F_π, Milana, Nussinov and Olsson [21] made a very bold conjecture to obtain F_π at $|Q^2| = M^2(J/\psi)$. They considered the three different intermediaries, a photon, three gluons, and two gluons plus a photon, by which J/ψ could decay into a meson pair, i.e.,

$$\mathcal{B}(J/\psi \to M\overline{M}) = K|A_\gamma + A_{ggg} + A_{\gamma gg}|^2$$

They argued that for $\pi^+\pi^-$ decay, A_{ggg} and $A_{\gamma gg}$ were negligably small, and therefore,

$$\frac{\mathcal{B}(J/\psi \to \pi^+\pi^-)}{\mathcal{B}(J/\psi \to e^+e^-)} = 2F_\pi^2(M_{J/\psi}^2) \times \left(\frac{p_\pi}{M_{J/\psi}}\right)^3$$

They thus obtained $|Q^2|F_\pi(9.6\ \mathrm{GeV}^2) = \mathbf{0.94 \pm 0.06\ GeV^2}$

As the figure shows, this estimation is in excellent agreement with our result $|Q^2|F_\pi(|Q^2|) = \mathbf{1.01 \pm 0.13\ GeV^2}$.

This agreement appears to justify the arguments of Milana et al.

It occured to me that if Milana et al. are right, a similar argument can be used to determine $F_K(M^2_{J/\psi})$. It could be easily argued that $A^K_{\gamma gg}$ was negligable in this case as well. On the other hand, A^K_{ggg} is not negligable. Fortunately, its contribution can be reliable subtracted. It has been shown that A^K_γ and A^K_{ggg} are nearly 90° out of phase, and A^K_{ggg} is given by the K_SK_L decay of J/ψ. Therefore,

$$\mathcal{B}_\gamma(J/\psi \to K^+K^-) = \mathcal{B}(J/\psi \to K^+K^-) - \mathcal{B}(J/\psi \to K_SK_L)$$

and one obtains the relation analgous to that for $\pi^+\pi^-$ decay. Using the known branching ratios, we obtain [22]

$$|Q^2|F_K(9.6\ \mathrm{GeV}^2) = \mathbf{0.81 \pm 0.06\ GeV^2},$$

which is once again in remarkably good agreement with our measured value

$$|Q^2|F_K(|Q^2|) = \mathbf{0.85 \pm 0.05\ GeV^2}$$

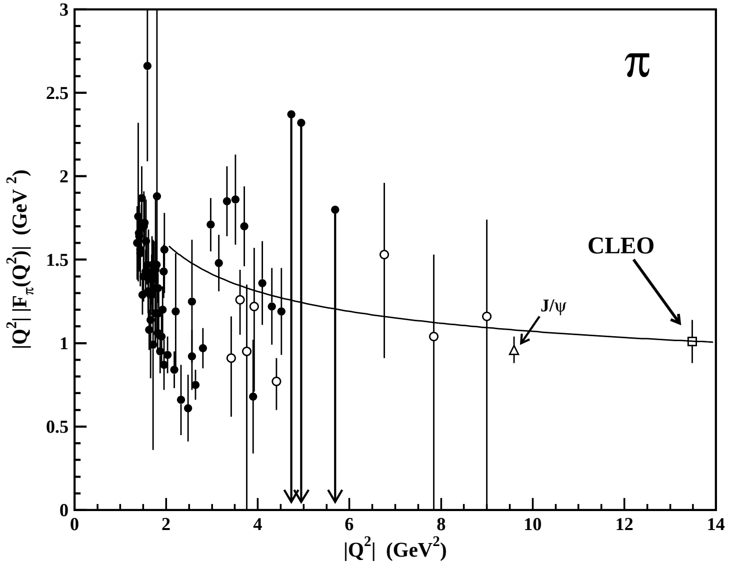
We also note that

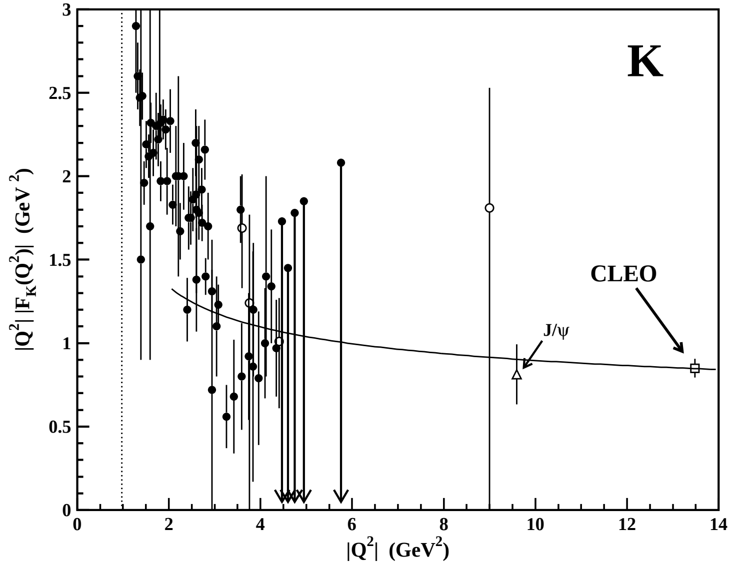
$$F_\pi(M^2_{J/\psi})/F_K(M^2_{J/\psi}) = \mathbf{1.16 \pm 0.27},$$

which is also in excellent agreement with the result of the CLEO measurement

$$F_\pi(13.48\ \mathrm{GeV}^2)/F_K(13.48\ \mathrm{GeV}^2) = \mathbf{1.19 \pm 0.07}$$

Timelike Form Factors of Pions and Kaons





Fun and Games With Theoretical Models

There are no independent theoretical predictions for kaon form factors apart from the fact that pQCD predicts that

$$F_\pi(|Q^2|)/F_K(|Q^2|) = (f_\pi^2/f_K^2) = \mathbf{0.67 \pm 0.01}$$

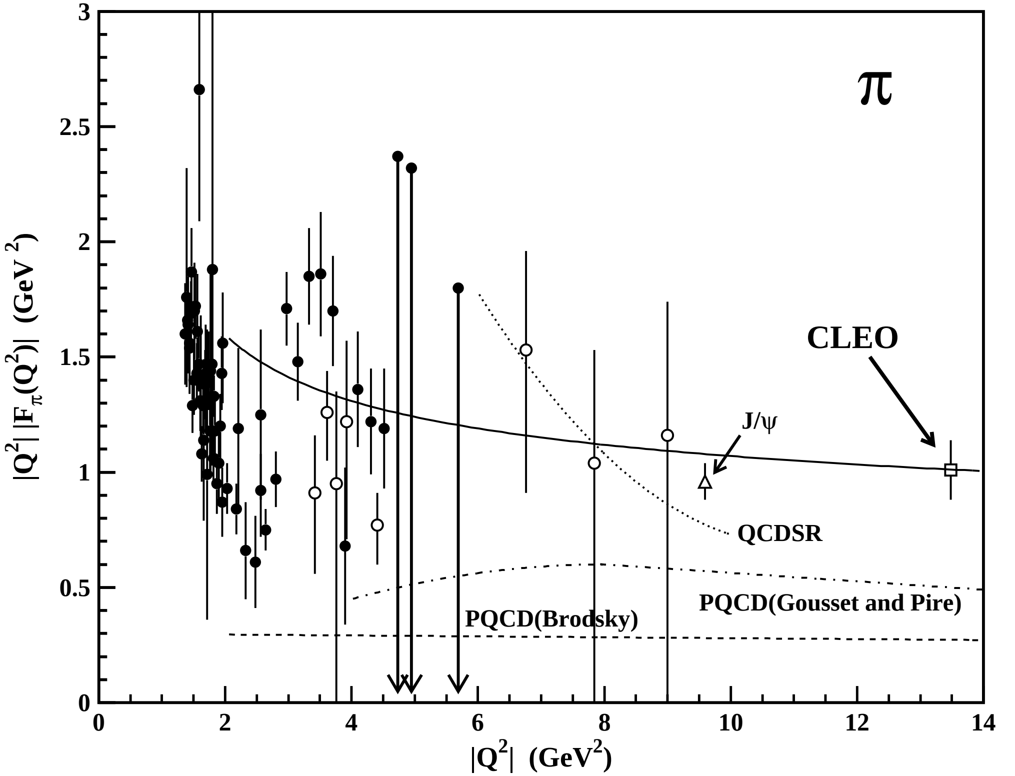
Obviously, this is in strong disagreement with our result,

$$\boldsymbol{F_\pi(13.48\ \mathrm{GeV}^2)/F_K(13.48\ \mathrm{GeV}^2) = 1.19 \pm 0.7}$$

- The fun and games relate to the theoretical prediction for F_π.

  I do not want to get in trouble with so many eminent theorists present here, but let me just show a sample of the theoretical predictions. The authors will recognize their predictions.

Fun and Games With Theoretical Models



- **My summary of the theoretical situation is that nothing seems to work.** This was perhaps excusable when there were no precision results for large $|Q^2|$. Now there is no excuse. The experts must go back to work.

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