TMD parton distributions in semi-inclusive DIS

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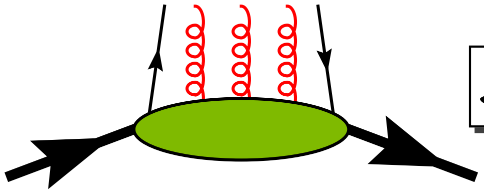
TMD-Parton Distributions

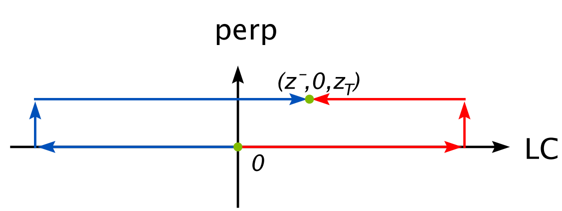


$$\boxed{\mathcal{FT}(z^-,\vec{z}_T)\langle P,S|\bar{\psi}_j(0)\,\mathcal{W}[0|(z^-,0^+,\vec{z}_T)]\psi_i(z^-,0^+,\vec{z}_T)|P,S\rangle}$$

Choice of the Wilson line: process dependent:

$$\boxed{\mathcal{W}[0,x|\text{path}] = \mathscr{P}\exp\left\{-ig\int_0^x ds_\mu\, A^\mu(s)\right\}}$$

SIDIS, DY: p_T-dependence



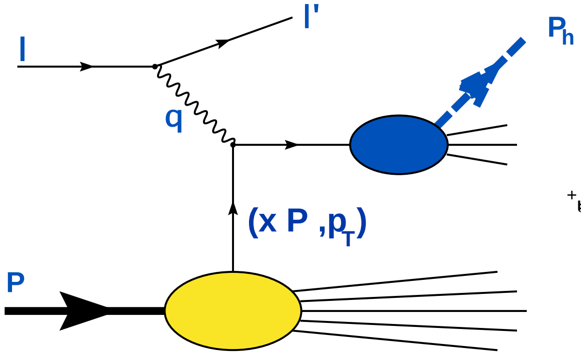
Twist-2 TMD parton distributions, parameterization, $f = f(x,\vec{p}_T^{\,2})$

$$\mathcal{FT}\left[\langle P,S|\bar{\psi}\gamma^+\mathcal{W}\psi|P,S\rangle\right] = f_1 - \frac{\epsilon_T^{ij}p_T^i S_T^j}{M}\underbrace{f_{1T}^\perp}_{\text{Sivers}}$$

$$\mathcal{FT}\left[\langle P,S|\bar{\psi}\gamma^+\gamma_5\mathcal{W}\psi|P,S\rangle\right] = \lambda g_{1L} + \frac{\vec{p}_T\cdot\vec{S}_T}{M}g_{1T}$$

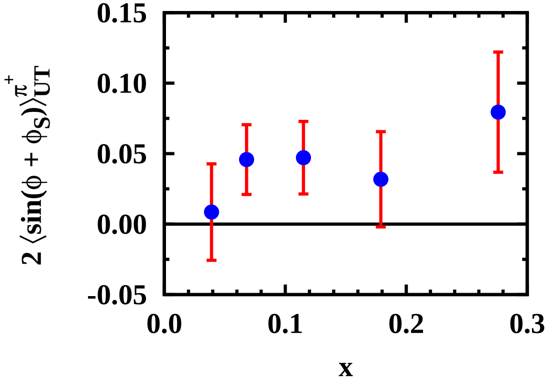
$$\mathcal{FT}\left[\langle P,S|\bar{\psi}i\sigma^{i+}\gamma_5\mathcal{W}\psi|P,S\rangle\right] = S_T^j\underbrace{\left(\delta^{ij}h_{1T} + \frac{p_T^i p_T^j}{M^2}h_{1T}^\perp\right)}_{\text{transversity } h_1(x,\vec{p}_T^{\,2})} + \lambda\frac{p_T^i}{M}h_{1L}^\perp + \frac{\epsilon_T^{ij}p_T^j}{M}\underbrace{h_1^\perp}_{\text{Boer-Mulders}}$$

Experiments & Model calculations



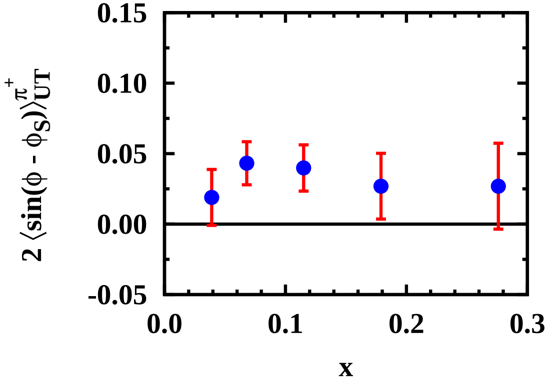
Collins-Effect:

$F_{UT}^{\sin(\phi_h+\phi_S)} = h_1 \otimes H_1^\perp$



Sivers-Effect:

$F_{UT}^{\sin(\phi_h-\phi_S)} = f_{1T}^\perp \otimes D_1$



- Boer-Mulders-Effect: (unpolarized processes)

$$\boxed{F_{UU}^{\cos(2\phi_h)} = \int d^2p_T d^2k_T\, \delta^{(2)}\left(\vec{p}_T - \vec{k}_T - \frac{\vec{P}_{h\perp}}{z_h}\right) \frac{\vec{k}_T \cdot \vec{p}_T - 2(\vec{h}\cdot\vec{k}_T)(\vec{h}\cdot\vec{p}_T)}{M m_\pi} h_1^\perp H_1^\perp}$$

in DY: $\cos(2\phi_h)$-distribution $\propto h_1^\perp \otimes \bar{h}_1^\perp$

- Estimates & Model-calculations of the Boer-Mulders function:

1. MIT-Bag model (F. Yuan, 2003): $h_1^{\perp(u)}$ and $h_1^{\perp(d)}$ both *negative*!

2. large-N_c arguments (P. Pobylitsa, 2003): $h_1^{\perp(u)}$ and $h_1^{\perp(d)}$ both *negative*!

3. Impact parameter space (M. Burkardt, 2005):
   Connection between Sivers-function $f_{1T}^{\perp}(x, \vec{b}_\perp)$ and GPD $E(x, 0, \vec{b}_\perp)$
   $\longrightarrow$ relation to anormalous magnetic moment $\kappa^{(q)}$
   Also: Connection between BM $h_1^{\perp}(x, \vec{b}_\perp)$ and GPDs $\tilde{H}_T$ and E_T
   $\Longrightarrow$ $h_1^{\perp(u)}$ and $h_1^{\perp(d)}$ both *negative*!

4. Lattice-QCD (P. Hägler et al., 2006): $h_1^{\perp(u)}$ and $h_1^{\perp(d)}$ both *negative*!

5. Diquark-spectator model (A. Bacchetta, A. Schäfer, J. Yang, 2003): $h_1^{\perp(u)}$ and $h_1^{\perp(d)}$ have opposite signs!

BM-function in the diquark-model

- *Quark-Quark Correlator*

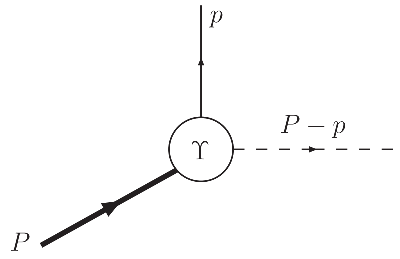
$$\boxed{\Phi_{ij}(x,\vec{p}_T) = \sum_X \int \frac{dz^- d^2 z_T}{(2\pi)^3}\, \mathrm{e}^{ixP^+z^- - i\vec{p}_T\cdot\vec{z}_T} \langle P,S|\,\bar{\psi}_j(0)\,\mathcal{W}[0,\infty^-]\,|X\rangle\,\langle X|\,\mathcal{W}[\infty^-,z]\,\psi_i(z)\,|P,S\rangle}$$

- Diquark-model: $|X\rangle \longrightarrow |dq;\, q,\, \lambda\rangle$ one particle-state!

- Two kinds of diquarks: Scalar (spin 0) and Axial-vector (spin 1).

  Specification of Nucleon-Diquark-Quark vertex:

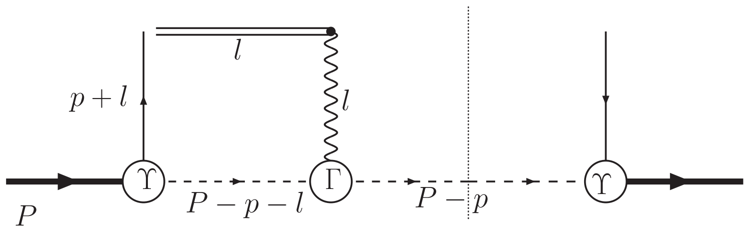
$$\langle dq; P-p, \lambda|\,\psi_i(0)\,|P,S\rangle =$$



- Ingredients, sufficient for T-even PDFs, e.g. $f_1^{(u)}$ and $f_1^{(d)}$

$$\Upsilon^\mu_{ax} = \frac{g(p^2)}{\sqrt{3}}\gamma_5\left(\gamma^\mu - R_g\frac{P^\mu}{M}\right),\ \Upsilon^\mu_{sc} = g(p^2)\mathbb{1}$$

- T-odd PDFs: consequence of Gauge link $\longrightarrow$ 1 Gluon exchange approximation



- Further ingredients:

gauge boson-axial vector diquark coupling

$$\Gamma_{ax}^{\mu\nu_1\nu_2} = \mathrm{WW}\gamma - \text{vertex } (\textit{incl. anom. magn. moment } \kappa) \quad ; \quad \Gamma_{sc}^{\mu} = \text{scalar QCD} - \text{vertex}$$

axial-vector diquark and scalar diquark propagator:

$$\mathcal{D}_{ax}^{\mu\nu}(q) = \frac{-i(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_s^2})}{q^2 - m_s^2 + i0} \quad ; \quad \mathcal{D}_{sc}(q) = \frac{-i}{q^2 - m_s^2 + i0}$$

$\longrightarrow$ Loop-Integral (axial-vector diquark):

$$\int \frac{d^4 l}{(2\pi)^4}\, g((l+p)^2) g(p^2) \mathcal{D}_{\rho\eta}(P-p-l) \left(\sum_\lambda \epsilon_\sigma^* \epsilon_\mu\right) \Gamma_{ax}^{\lambda\rho\sigma} \frac{n_{-\lambda}}{[l^+ + i0]} \times$$

$$\frac{\mathrm{Tr}\left[(\not{P}+M)\left(\gamma^\mu - R_g \frac{P^\mu}{M}\right)(\not{p}-m_q)\gamma^+\gamma^i(\not{l}+\not{p}+m_q)\left(\gamma^\eta + R_g\frac{P^\eta}{M}\right)\gamma_5\right]}{\left[l^2-\lambda^2+i0\right]\left[(l+p)^2-m_q^2+i0\right]}$$

Specifying the form factor

- Simplification of the numerator $\longrightarrow$ sort by powers of loop-momentum l

$$\longrightarrow \boxed{J^{(i)\alpha_1\ldots\alpha_i} = \int \frac{d^4l}{(2\pi)^4} \frac{g((l+p)^2)g(p^2)l^{\alpha_1}\ldots l^{\alpha_i}}{[(v\cdot l)+i0]\,[l^2+i0]\,[(l+p-P)^2-m_s^2+i0]\,\left[(l+p)^2-m_q^2+i0\right]}}$$

- $v = \left[1^-, 0^+, \vec{0}_T\right]$, $l^+ \rightarrow 0$, $l^- \rightarrow \infty$, $\alpha_k = - \Rightarrow$ *Light cone divergence*!

- Regularization procedure:

1) Clean procedure: (Gamberg, Hwang, Metz, MS, 2006) Introduction of Wilson lines *off* the light cone, $v = \left[1^-, \lambda^+, \vec{0}_T\right]$,

$$\longrightarrow h_1^{\perp,ax}(x, \vec{p}_T^{\,2}, v) \propto \ln\left(\frac{v^2}{v\cdot P}\right)$$

2) Phenomenological procedure: Introduction of add. poles via the form factor $g(p^2)$

$$\boxed{g(p^2) = N^{2n} \frac{\left[p^2 - m_q^2\right] F(p^2)}{\left[p^2 - \Lambda^2 + i0\right]^n}}$$

$\longrightarrow$additional pole produces additional factor $[l^+]^n$ in numerator $\longrightarrow$ Regularization.

- Transverse Integral:

$$h_1^{\perp,ax} \propto \int d^2 l_T \frac{F((l+p)^2)\left[A\vec{l}_T^4 + B\vec{l}_T^2(\vec{p}_T \cdot \vec{l}_T) + C\vec{l}_T^2 + D(\vec{p}_T \cdot \vec{l}_T) + E\right]}{\left[\vec{l}_T^2\right]\left[(\vec{l}_T + \vec{p}_T)^2 + \tilde{m}_\Lambda^2\right]^3}$$

- $E = 0$: *no IR-divergence*!

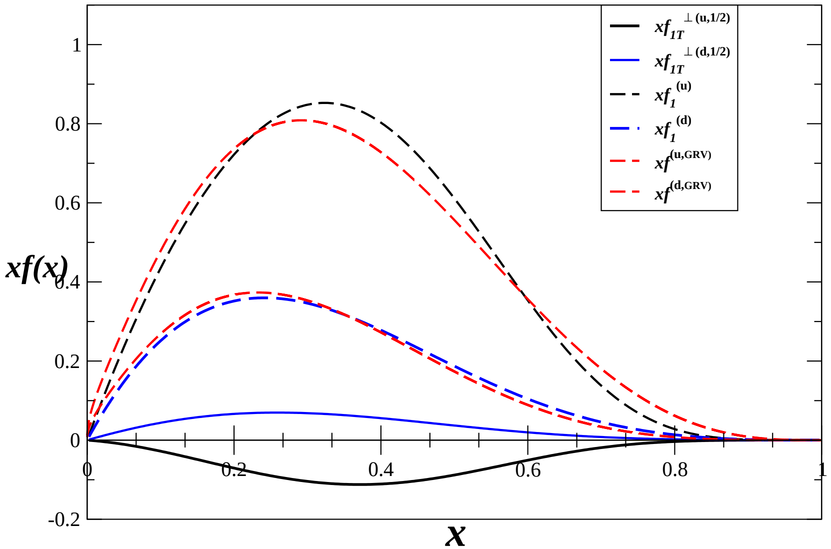
- $A \neq 0$: *UV-divergence* $\Longrightarrow$ Further specification of form factor:

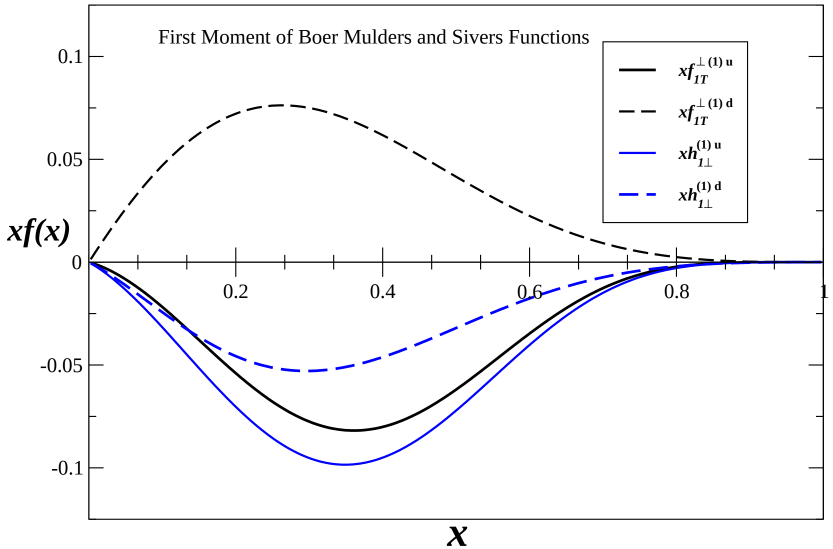
$$\boxed{g(p^2) = N^2 \frac{\left[p^2 - m_q^2\right] \mathrm{e}^{-b|p^2|}}{\left[p^2 - \Lambda^2 + i0\right]^3}}$$

- Integration leads to incomplete Gamma-functions $\Gamma(n,x) \equiv \int_x^\infty \mathrm{e}^{-t} t^{n-1} dt \,,\, n > 0$.

Results

Construction of flavor-dependent PDFs from diquark models: $u = \frac{3}{2}sc + \frac{1}{2}ax$, $d = ax$, moments: $f^{(n/2)}(x) = \int d^2\vec{p}_T \, \frac{|\vec{p}_T|^n}{2M^n} f(x, \vec{p}_T^2)$





- Comparison to parametrization of f_1 (Glück, Reya, Vogt) $\longrightarrow$ parameters of the model, e.g. diquark masses, normalization...
- Comparison to parameterization of Sivers function $f_{1T}^{\perp}$ $\longrightarrow$ size and sign of FSI.

Summary

- T-odd PDF $h_1^{\perp}(x, \vec{p}_T)$ (Boer-Mulders function) essential ingredient for azimuthal $\cos(2\phi)$-asymmetries in unpolarized SIDIS and DY.

- Various approaches predict negative signs for u and d Boer-Mulders functions
$\longrightarrow$ Model calculation for $h_1^{\perp}$ in an axial-vector and scalar diquark spectator model.

- Inclusion of full axial-vector diquark propagator leads to loop-integrals divergent on the light cone
$\longrightarrow$ Use of form factors as regulators.

- Results: u and d Boer-Mulders function *negative* also in the diquark spectator model
$\longrightarrow$ Input for phenomenology of azimuthal $\cos(2\phi)$-asymmetries in unpol. SIDIS and DY.