Transverse Hadron Structure from Lattice QCD

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Deutsches Elektronen-Synchrotron DESY

– QCDSF Collaboration –

Special mention:

M. Göckeler, P. Hägler, T. Hemmert, R. Horsley, Y. Nakamura, D. Pleiter,

P.E.L. Rakow, W. Schroers, H. Stüben and J. Zanotti

Status Report

(selective)

Outline

Lattice Simulation

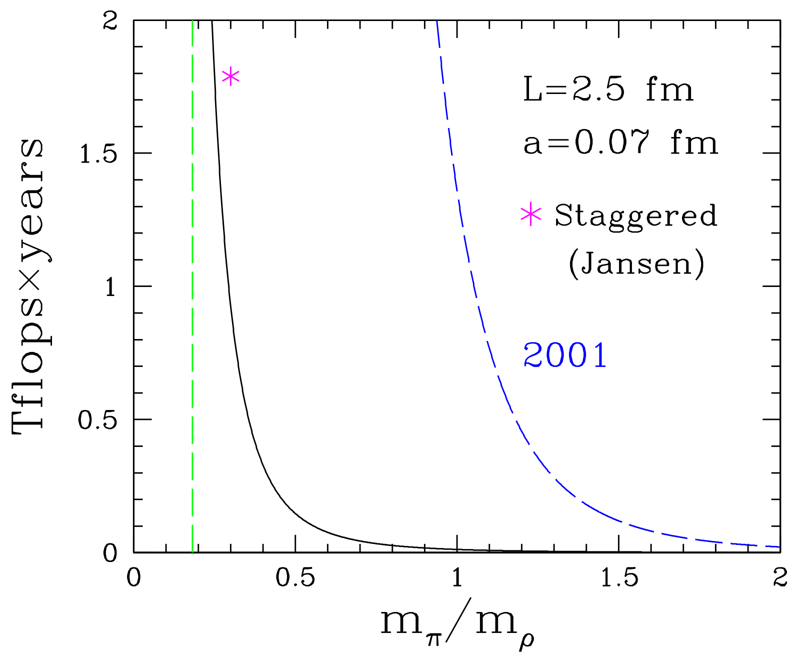
Choice of Fermion Actions

$$S_{QCD} = S_G + S_F$$

S_F	Chiral Symmetry	Flavor Symmetry	Comments
Overlap	Exact	Yes	Very Expensive
Domain Wall	$L_5 \longrightarrow \infty$	Yes	Expensive
Wilson/Clover	To $O(a^2)$	Yes	Fast
Twisted Mass	To $O(a^2)$	No	$m_{\pi^0}/m_{\pi^+} \approx 0.7$
Staggered	Mingles spin and flavor	No	Nonlocal

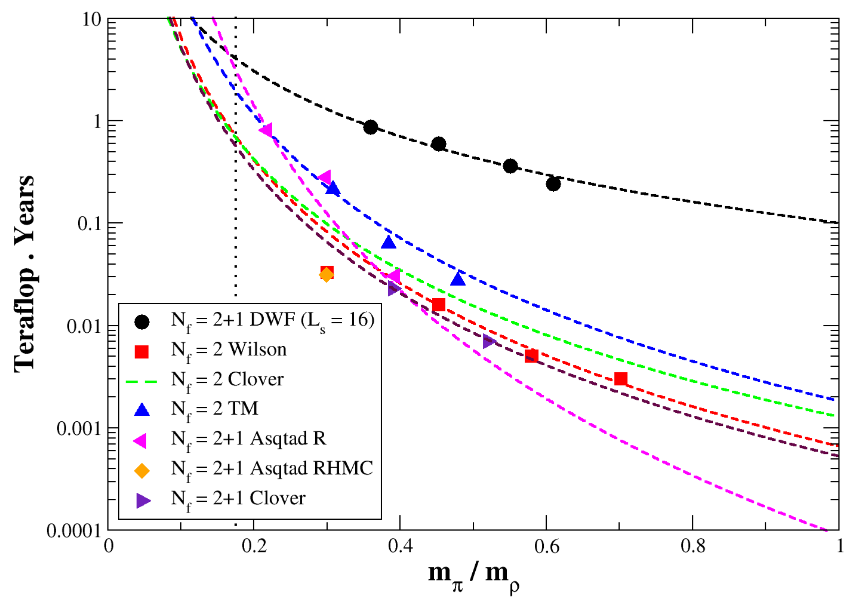
Cost of Simulation

1000 Configurations



Wilson

'Events'

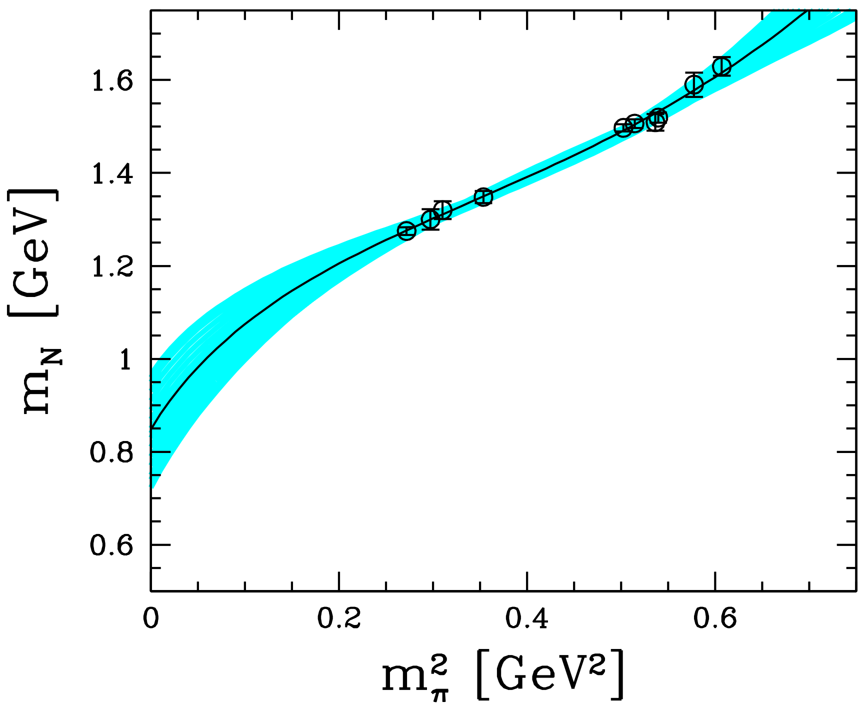


QCDSF, Lüscher, Urbach et al., · · ·

Clark

Recent Advances

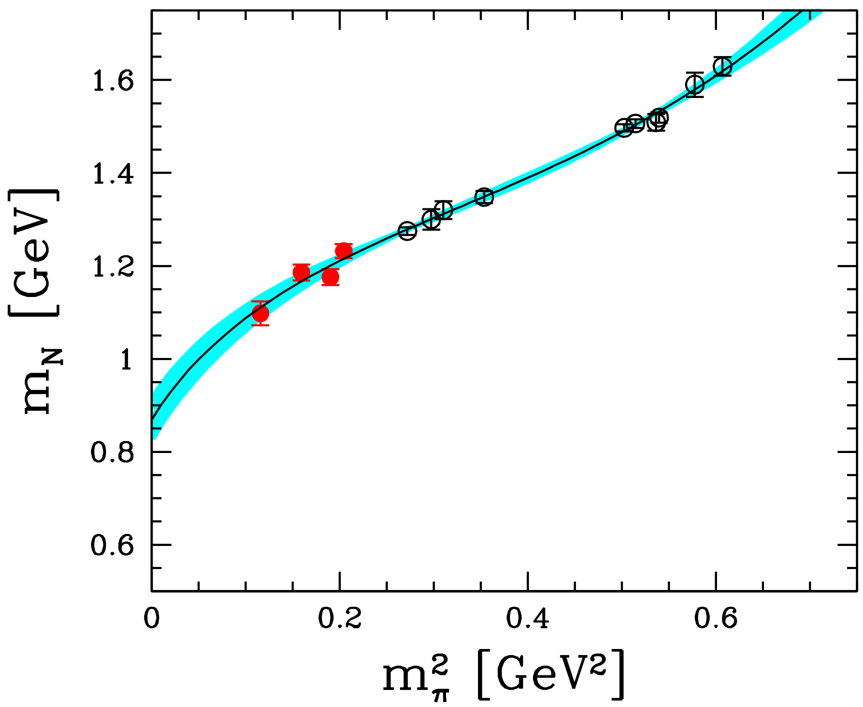
< 2006



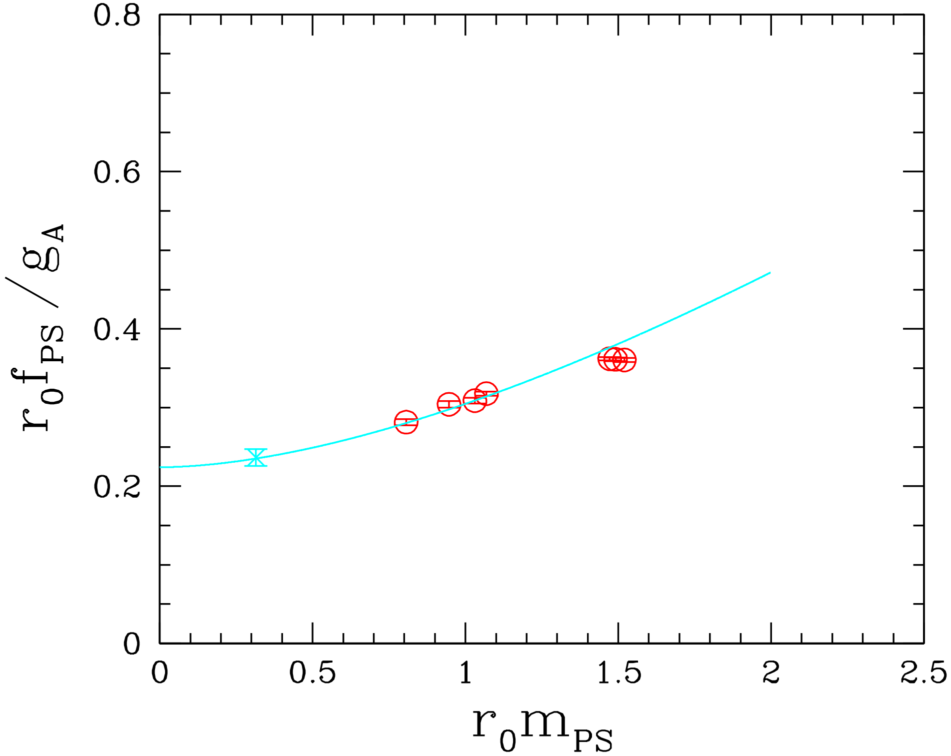
ChPT $O(p^4)$

68.3% CL

2007



Determination of Scale

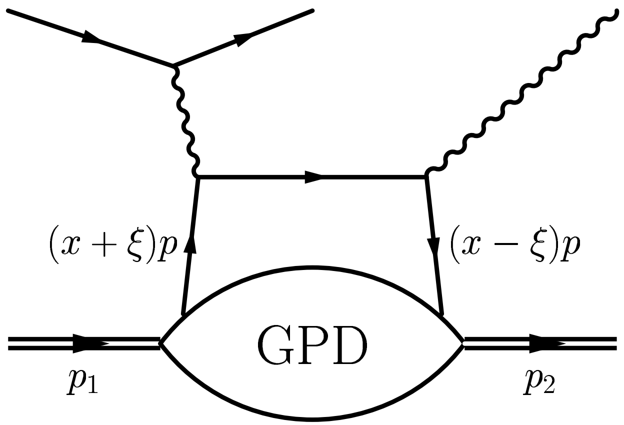


Z_A cancels , FS effects & leading log's largely cancel

$r_0 = 0.45(1)$ fm

Basics

OPE



$p = \frac{1}{2}(p_1+p_2)$, $\Delta = p_2-p_1$, $q = \frac{1}{2}(q_1+q_2)$

$\xi = 0$: Momentum transfer of the struck parton purely transverse, i.e. $\Delta = \mathbf{\Delta}_\perp$

$\Uparrow$

Of interest to us here only

$\Updownarrow$

Matrix elements of local operators

$$\mathcal{O}^q_{\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{q}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n}q$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n} \bar{q}\gamma_\sigma\gamma_5\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n}q$$

$$\mathcal{O}^{Tq}_{\mu\nu\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n} \bar{q}\sigma_{\mu\nu}\gamma_5\,\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n}q$$

Nucleon

$$\langle p_1, s|\, \mathcal{O}^q_{\{\mu_1\cdots\mu_n\}}\, |p_2, s\rangle = \bar{u}(p_1, s)\Big[A^q_n(\Delta^2)\ \gamma_{\{\mu_1}$$

$$+ B^q_n(\Delta^2)\ \frac{\mathrm{i}\Delta^\alpha}{2m_N}\sigma_{\alpha\{\mu_1}\Big] p_{\mu_2}\cdots p_{\mu_n\}}\, u(p_2, s)\ +\ \cdots$$

$$\langle p_1, s|\mathcal{O}^{5q}_{\{\mu\mu_1\cdots\mu_n\}}|p_2, s\rangle = \bar{u}(p_1, s)\Big[\tilde{A}^q_{n+1}(\Delta^2)\ \gamma_{\{\mu}\gamma_5 p_{\mu_1}\cdots p_{\mu_n\}}\Big]u(p_2, s)\ +\ \cdots$$

$$\langle p_1, s|\mathcal{O}^{Tq}_{\mu\{\nu\mu_1\cdots\mu_n\}}|p_2, s\rangle = \bar{u}(p_1, s)\Big[A^{Tq}_{n+1}(\Delta^2)\ \sigma_{\mu\{\nu}\gamma_5 - \tilde{A}^{Tq}_{n+1}(\Delta^2)\big(\frac{\Delta^2}{2m_N^2}\sigma_{\mu\{\nu} - \frac{\Delta_\mu\Delta_\alpha}{2m_N^2}\sigma_{\alpha\{\nu}\big)\gamma_5$$

$$+ \bar{B}^{Tq}_{n+1}(\Delta^2)\ \epsilon_{\alpha\beta\mu\{\nu}\frac{\Delta_\alpha\gamma_\beta}{2m_N}\Big]\ p_{\mu_1}\cdot\cdot p_{\mu_n\}}u(p_2, s)\ +\ \cdots$$

$$A_n^q(\Delta^2) = \int_0^1 dx\, x^{n-1} H^q(x, \Delta^2)$$

$$H^q(x,0) = q(x)$$

$$B_n^q(\Delta^2) = \int_0^1 dx\, x^{n-1} E^q(x, \Delta^2)$$

$$\tilde{A}_n^q(\Delta^2) = \int_0^1 dx\, x^{n-1} \tilde{H}^q(x, \Delta^2)$$

$$\tilde{H}^q(x,0) = \Delta q(x)$$

$$A_n^{Tq}(\Delta^2) = \int_0^1 dx\, x^{n-1} H^{Tq}(x, \Delta^2)$$

$$H^{Tq}(x,0) = \delta q(x)$$

↑ GFFs

↑ GPDs

$$\boxed{\frac{1}{2}\left(A_2^q(0) + B_2^q(0)\right) = J^q}$$

Ji

$$A_1^q\ (\Delta^2) = F_1^q(\Delta^2)$$

$$B_1^q\ (\Delta^2) = F_2^q(\Delta^2)$$

$$\tilde{A}_1^q\ (\Delta^2) = g_A^q\ (\Delta^2)$$

$$A_1^{Tq}(\Delta^2) = g_T^q\ (\Delta^2)$$

$$\Delta^2 = t = -Q^2$$

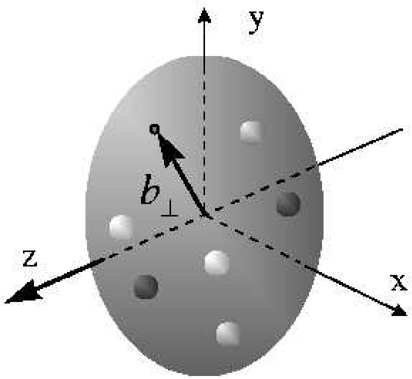
Impact Parameter Space

Generically

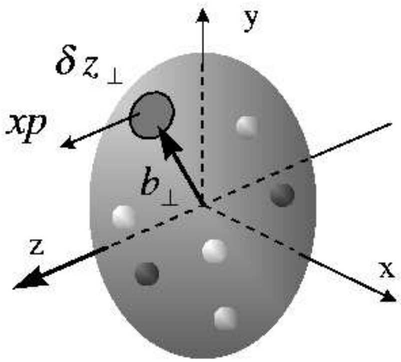
$$A_n^q(\mathbf{b}_\perp^2) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}\, \mathrm{e}^{\,\mathrm{i}\,\mathbf{b}_\perp \mathbf{\Delta}_\perp} A_n^q(\mathbf{\Delta}_\perp^2) \qquad \Longleftrightarrow \qquad \boxed{\langle p_+, s|\bar{q}(\mathbf{b}_\perp)\cdots q(\mathbf{b}_\perp)|p_+, s\rangle}$$

$$|p_+, s\rangle = \mathcal{N}\int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2}|p_+, \mathbf{p}_\perp, s\rangle$$

$$H^q(x, \mathbf{b}_\perp^2) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}\, \mathrm{e}^{\,\mathrm{i}\,\mathbf{b}_\perp \mathbf{\Delta}_\perp} H^q(x, \mathbf{\Delta}_\perp^2)$$



$F_1(\mathbf{b}_\perp^2) \equiv A_1(\mathbf{b}_\perp^2)$



$H(x, \mathbf{b}_\perp^2)$

Probability interpretation

Burkardt

$$H^q(x,\Delta^2) = \int_x^1 \frac{dy}{y}\, C\left(\frac{x}{y},\Delta^2\right) q(y)$$

Similarly for $\tilde{H}^q$ and $H^{T\,q}$

$$\int_0^1 dx\, x^n \underbrace{C(x,\Delta^2)} = \frac{A_{n+1}(\Delta^2)}{A_{n+1}(0)} = \frac{1}{(1-\Delta^2/M_n^2)^2}$$

By inverse Mellin transform

$$H^q(x,\mathbf{b}_\perp^2) = \int_x^1 \frac{dy}{y}\, C\left(\frac{x}{y},\mathbf{b}_\perp^2\right) q(y)$$

$$C(x,\mathbf{b}_\perp^2) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}\, e^{i\,\mathbf{b}_\perp \mathbf{\Delta}_\perp}\, C(x,\mathbf{\Delta}_\perp^2)$$

Nucleon Structure

$N_f = 2$ ‘Valence’ quark distributions

Form Factors

Sachs form factors

$$F_1(\Delta^2) = A_1(\Delta^2)$$

$$F_2(\Delta^2) = B_1(\Delta^2)$$

$$G_e(\Delta^2) \ = F_1(\Delta^2) + \frac{\Delta^2}{4m_N^2} F_2(\Delta^2)$$

$$G_m(\Delta^2) = F_1(\Delta^2) + F_2(\Delta^2)$$

$$F_1(0) = e^N$$

$$F_2(0) = \mu^N - e^N = \kappa^N$$

$$G_e(0) \ = e^N$$

$$G_m(0) = \mu^N = 1 + \kappa^N$$

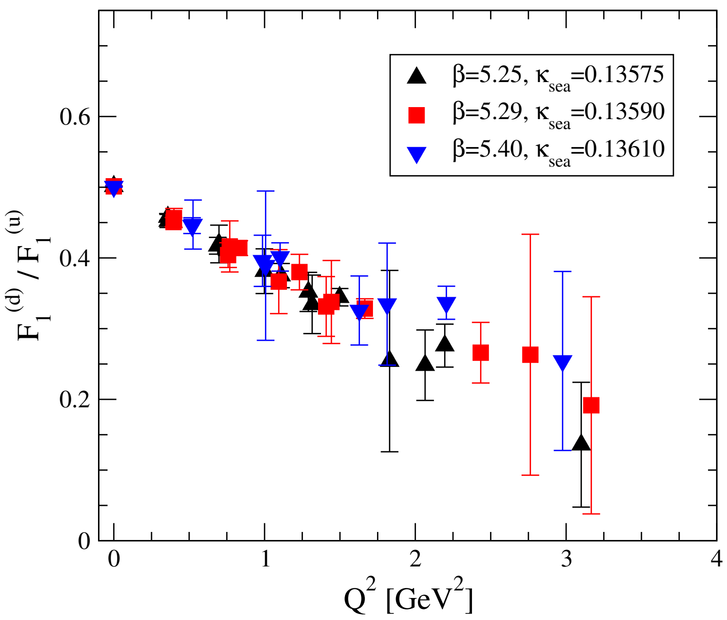
Benchmark calculation

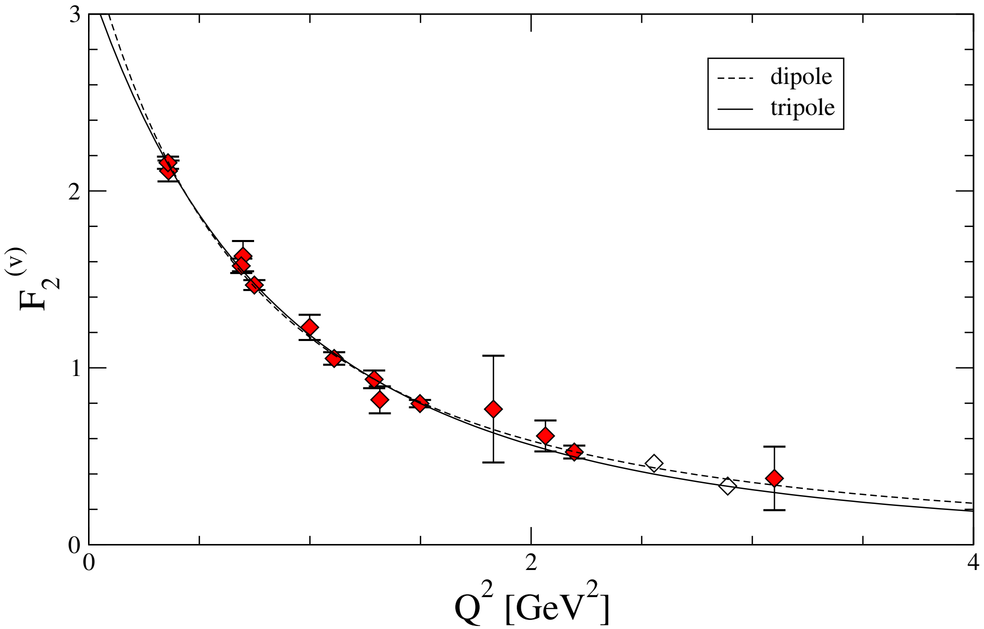
Expect (dimensional counting)

$$F_1(Q^2) \propto \frac{1}{(Q^2)^2}$$

$$F_2(Q^2) \propto \frac{1}{(Q^2)^3}$$

$$Q^2 = -\Delta^2$$





$$F^p = \frac{2}{3}F^u - \frac{1}{3}F^d$$

$$F^n = -\frac{1}{3}F^u + \frac{2}{3}F^d$$

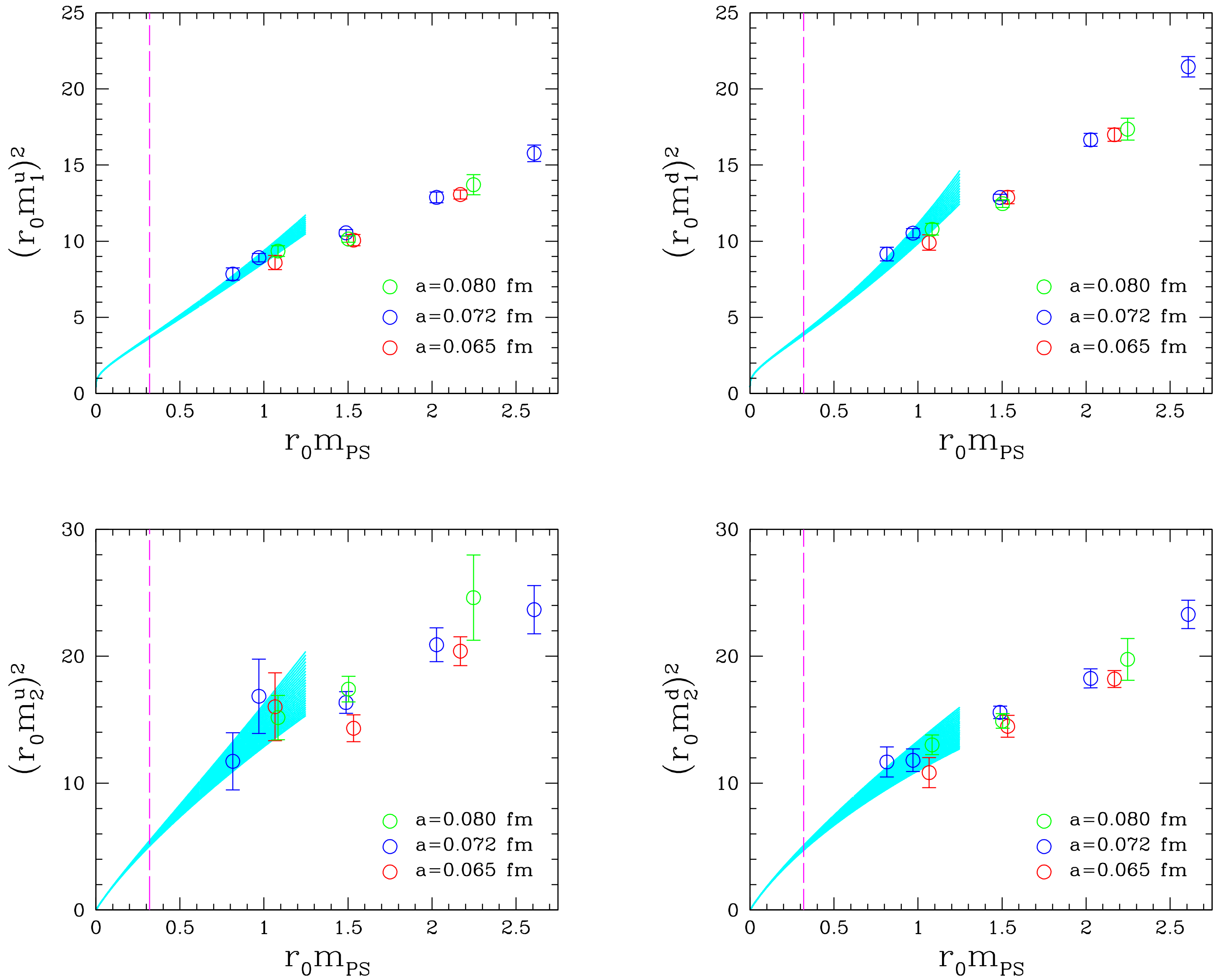
$$F^v = F^u - F^d$$

$$\boxed{F(Q^2) = F(0)\left(1 + Q^2/m^2\right)^{-n}}$$

$F_1^u \qquad n = 2$

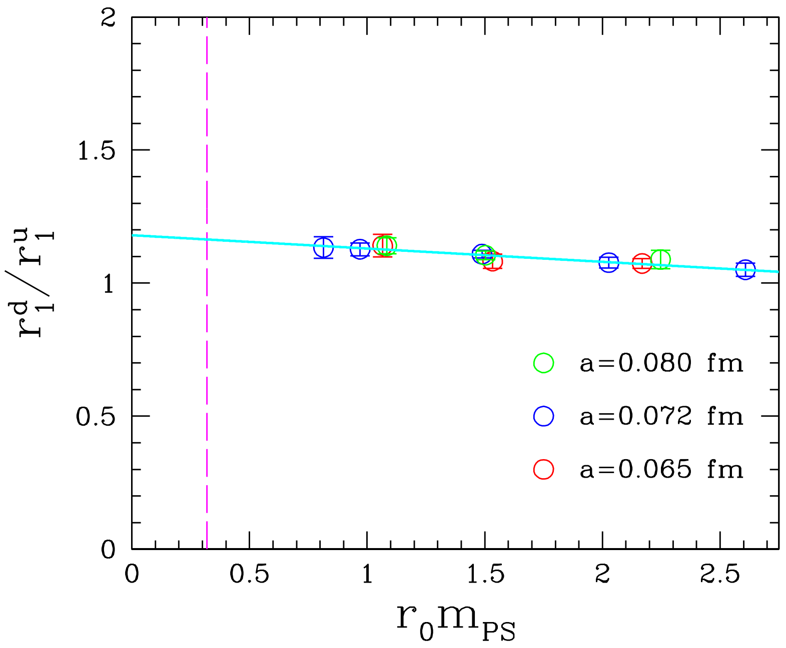
$\left.\begin{array}{l} F_1^d \\ F_2^{u,d} \end{array}\right\} \quad n = 3$

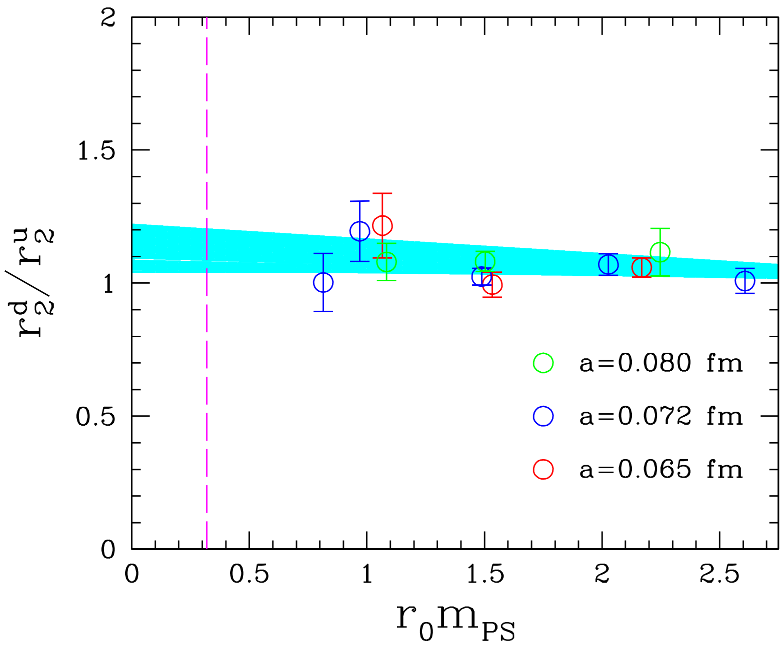
Chiral extrapolation



$$F_i(Q^2) = F_i(0)\left(1 - \frac{1}{6} r_i^2 Q^2 + O(Q^4)\right)$$

$$r_i^2 = 6\, n/m_i^2$$





$$r^d_{1,2} > r^u_{1,2}$$

ChPT

$$r_1^2 = -\frac{1}{(4\pi F_\pi)^2}\left\{1 + 7g_A^2 + (10g_A^2 + 2)\log\left[\frac{m_\pi}{\lambda}\right]\right\} - \frac{12B_{10}^{(r)}(\lambda)}{(4\pi F_\pi)^2}$$
$$+ \frac{c_A^2}{54\pi^2 F_\pi^2}\left\{26 + 30\log\left[\frac{m_\pi}{\lambda}\right] + 30\frac{\Delta}{\sqrt{\Delta^2 - m_\pi^2}}\log R(m_\pi)\right\}$$

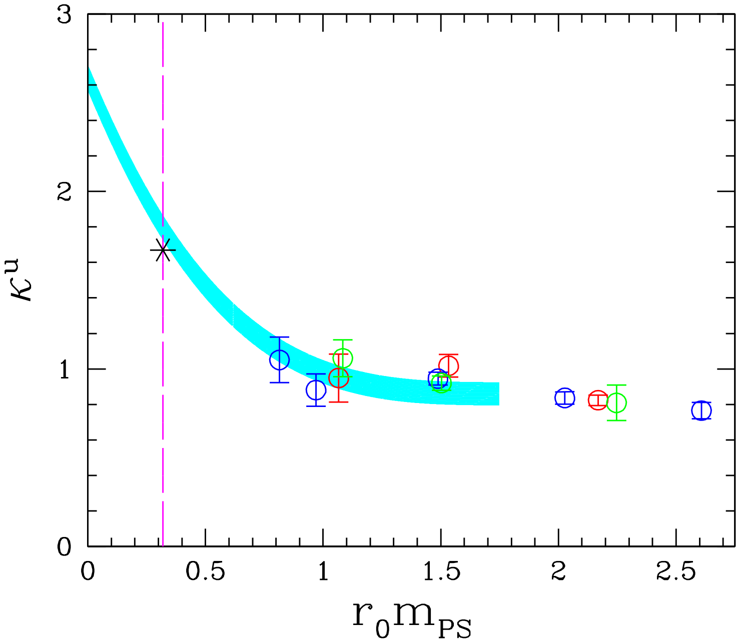
$$r_2^2 = \frac{g_A^2 m_N}{8F_\pi^2 \kappa \pi m_\pi} + \frac{c_A^2 m_N}{9F_\pi^2 \kappa \pi^2 \sqrt{\Delta^2 - m_\pi^2}}\log R(m_\pi) + \frac{24 m_N}{\kappa} B_{c2}$$

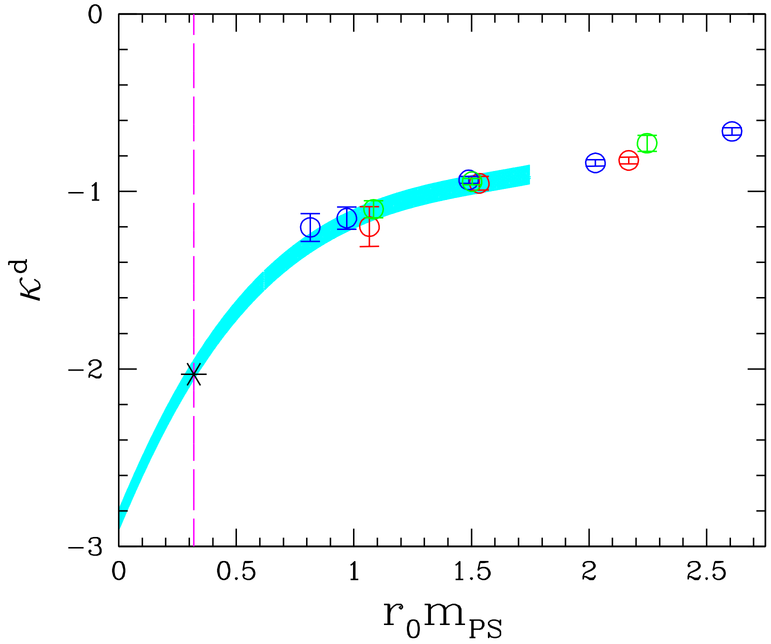
$$R(m) = \frac{\Delta}{m} + \sqrt{\frac{\Delta^2}{m^2} - 1}$$

Radii

[fm^2]	Lattice	Experiment	ChPT
$(r_1^u)^2$	0.67(3)		
$(r_1^d)^2$	0.93(4)		
$(r_1^v)^2$	0.41(5)	0.58	0.71
$(r_2^u)^2$	0.69(3)		
$(r_2^d)^2$	0.74(5)		
$(r_2^v)^2$	0.72(6)	0.80	0.60

Chiral extrapolation (ctd.)





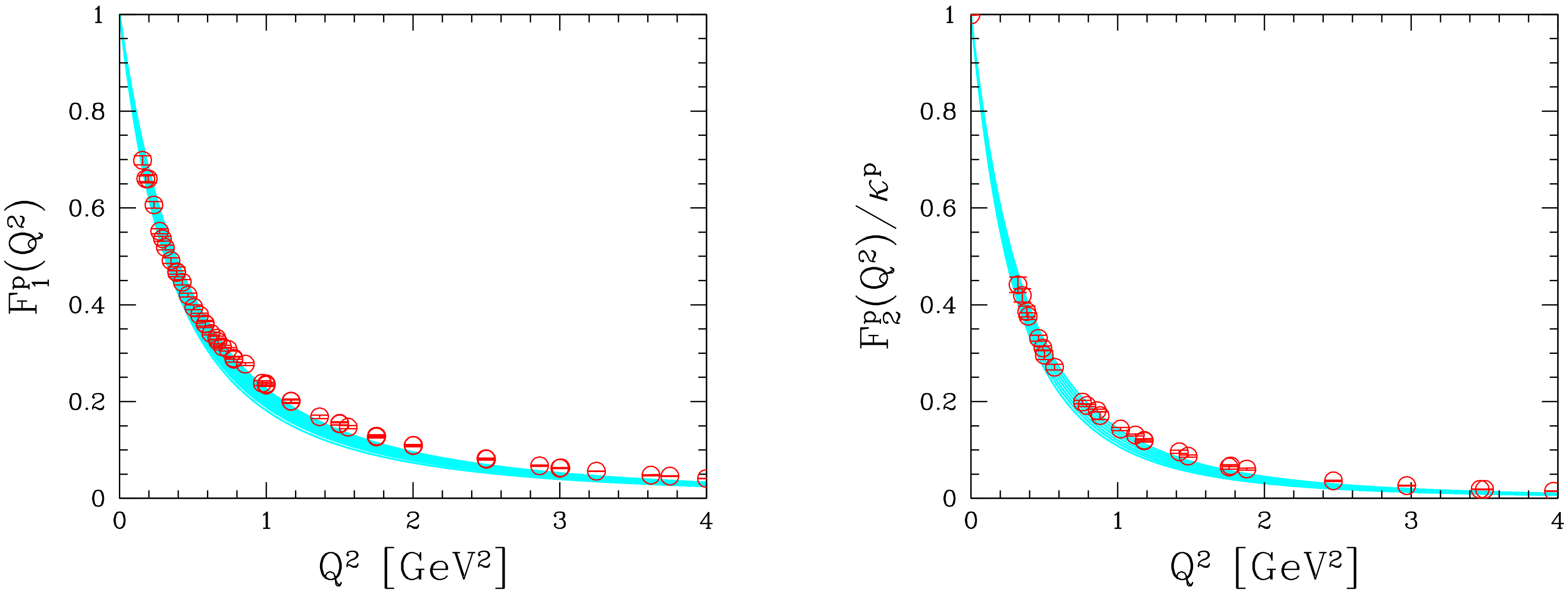
$$\kappa^p = \frac{2}{3}\kappa^u - \frac{1}{3}\kappa^d$$

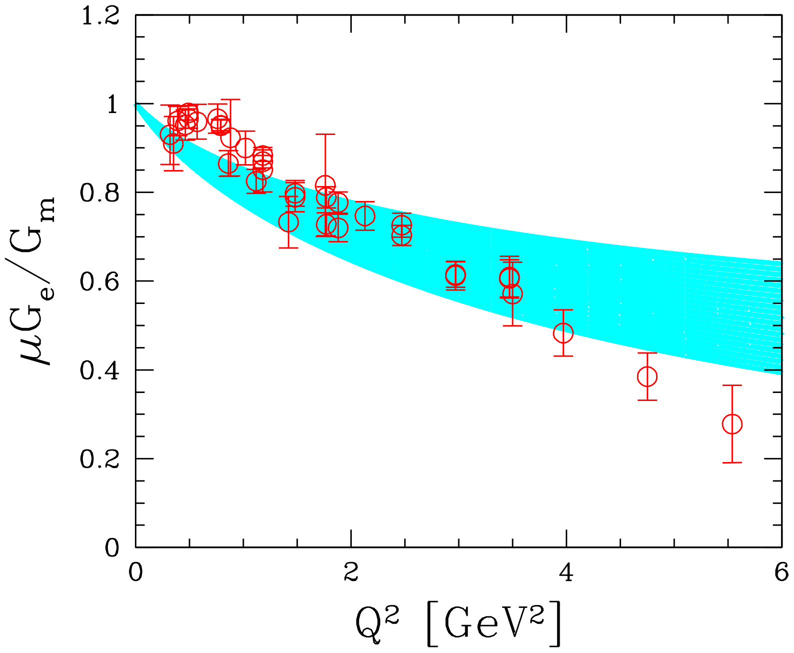
$$\kappa^n = -\frac{1}{3}\kappa^u + \frac{2}{3}\kappa^d$$

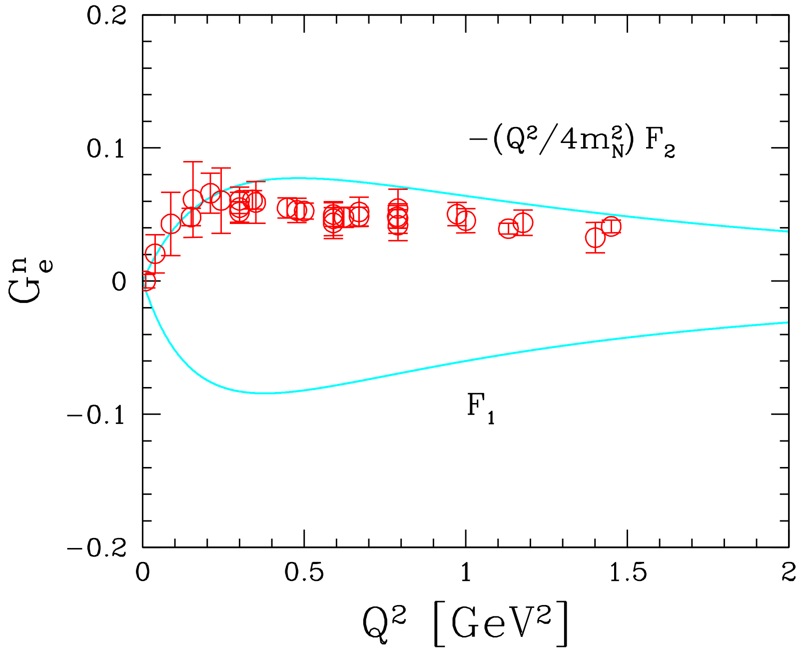
ChPT

$$
\begin{aligned}
\kappa(m_\pi) = \kappa_v^0 &- \frac{g_A^2 m_\pi M_N}{4\pi F_\pi^2} + \frac{2c_A^2 \Delta M_N}{9\pi^2 F_\pi^2}\left\{\sqrt{1-\frac{m_\pi^2}{\Delta^2}}\log R(m_\pi) + \log\left[\frac{m_\pi}{2\Delta}\right]\right\} \\
&- 8E_1^{(r)}(\lambda) M_N m_\pi^2 + \frac{4c_A c_V g_A M_N m_\pi^2}{9\pi^2 F_\pi^2}\log\left[\frac{2\Delta}{\lambda}\right] + \frac{4c_A c_V g_A M_N m_\pi^3}{27\pi F_\pi^2 \Delta} \\
&- \frac{8c_A c_V g_A \Delta^2 M_N}{27\pi^2 F_\pi^2}\left\{\left(1-\frac{m_\pi^2}{\Delta^2}\right)^{3/2}\log R(m_\pi) + \left(1-\frac{3m_\pi^2}{2\Delta^2}\right)\log\left[\frac{m_\pi}{2\Delta}\right]\right\}
\end{aligned}
$$

Finally







Very preliminary

Spin Asymmetries

Transverse spin density

$\Downarrow$

$\lambda_\perp$ quark spin
$s_\perp$ nucleon spin

$$\langle p_+, s_\perp | \bar{q}(\mathbf{b}_\perp) \left[\gamma_+ - \lambda_{\perp i}\, \sigma_{+j}\, \gamma_5\right] q(\mathbf{b}_\perp) | p_+, s_\perp \rangle = \Big\{ A_1^q(\mathbf{b}_\perp^2) + \lambda_{\perp i}\, s_{\perp i} \Big[A_1^{Tq}(\mathbf{b}_\perp^2) - \frac{1}{4m_N^2} \Delta_{b_\perp} \tilde{A}_1^{Tq}(\mathbf{b}_\perp^2) \Big] - \frac{1}{m_N} \epsilon_{ij} b_{\perp j} \Big[s_{\perp i} B_1^q(\mathbf{b}_\perp^2)' + \lambda_{\perp i}\, \bar{B}_1^{Tq}(\mathbf{b}_\perp^2)' \Big] + \frac{1}{m_N^2} \lambda_{\perp i} \left(2 b_{\perp i}\, b_{\perp j} - \mathbf{b}_\perp^2 \delta_{ij}\right) s_{\perp j}\, \tilde{A}_1^{Tq}(\mathbf{b}_\perp^2)'' \Big\}$$

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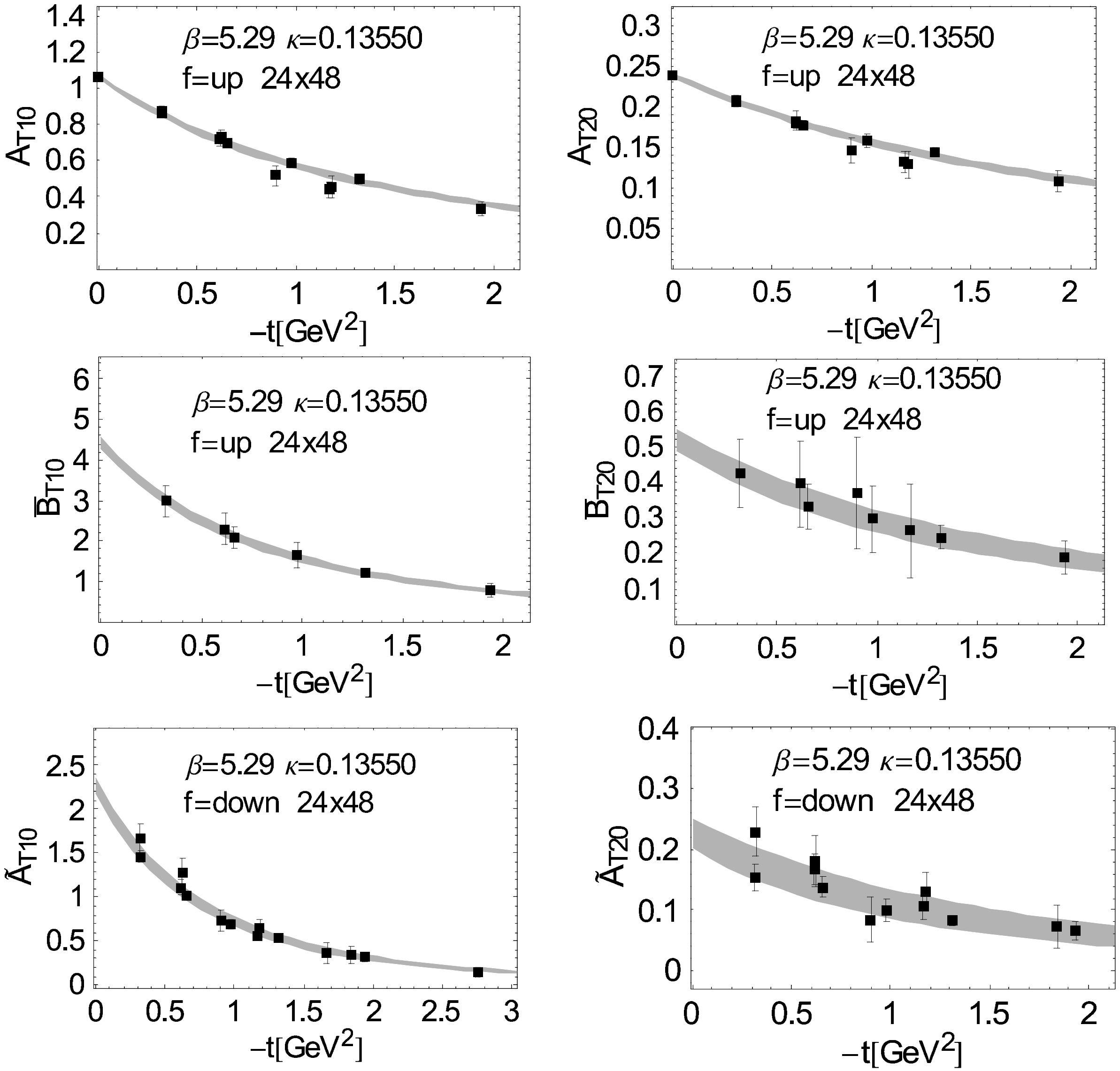
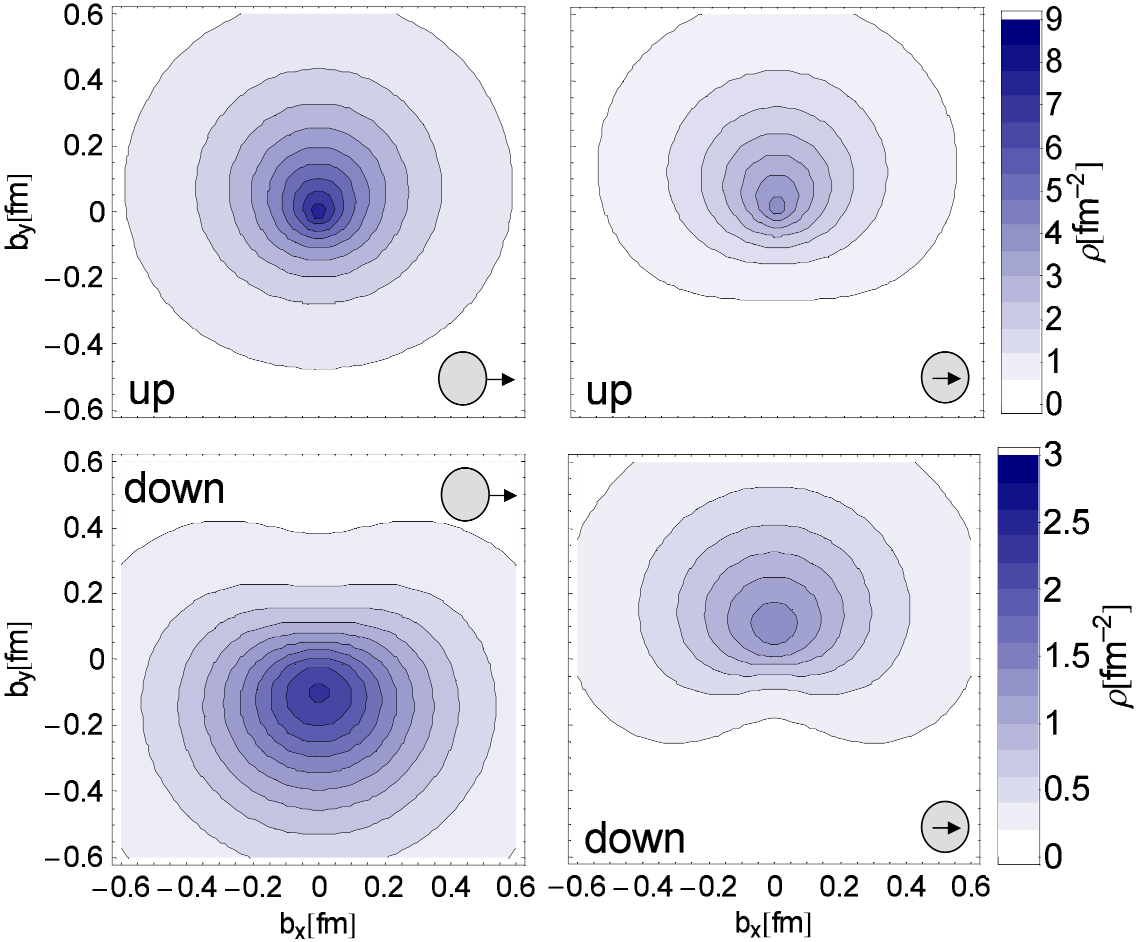
Quadrupole

Diehl & Hägler



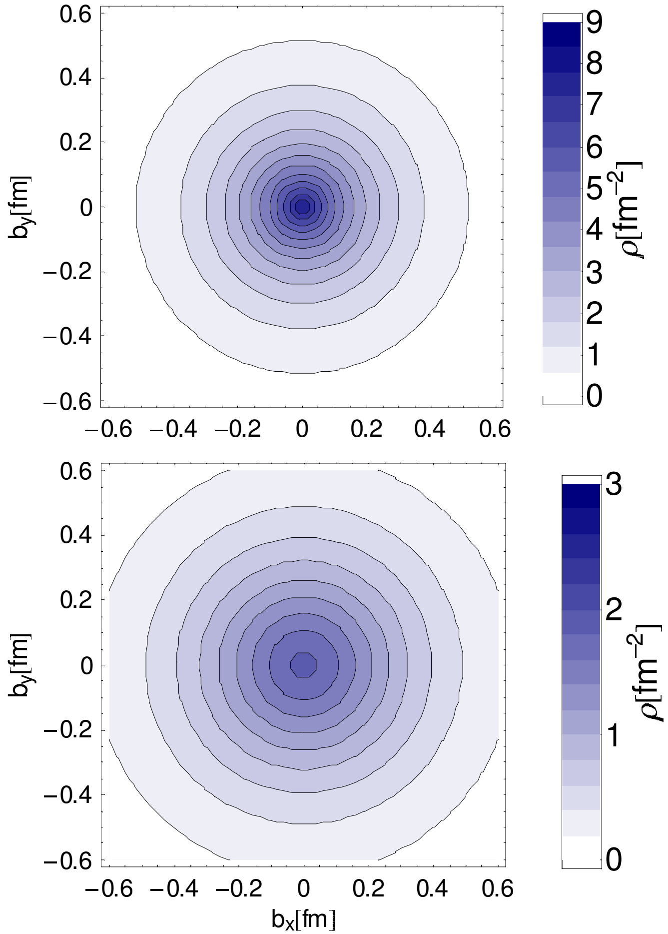
Dipole fit $\quad$ 1^{st} moment

To be extrapolated to chiral limit



Sivers effect

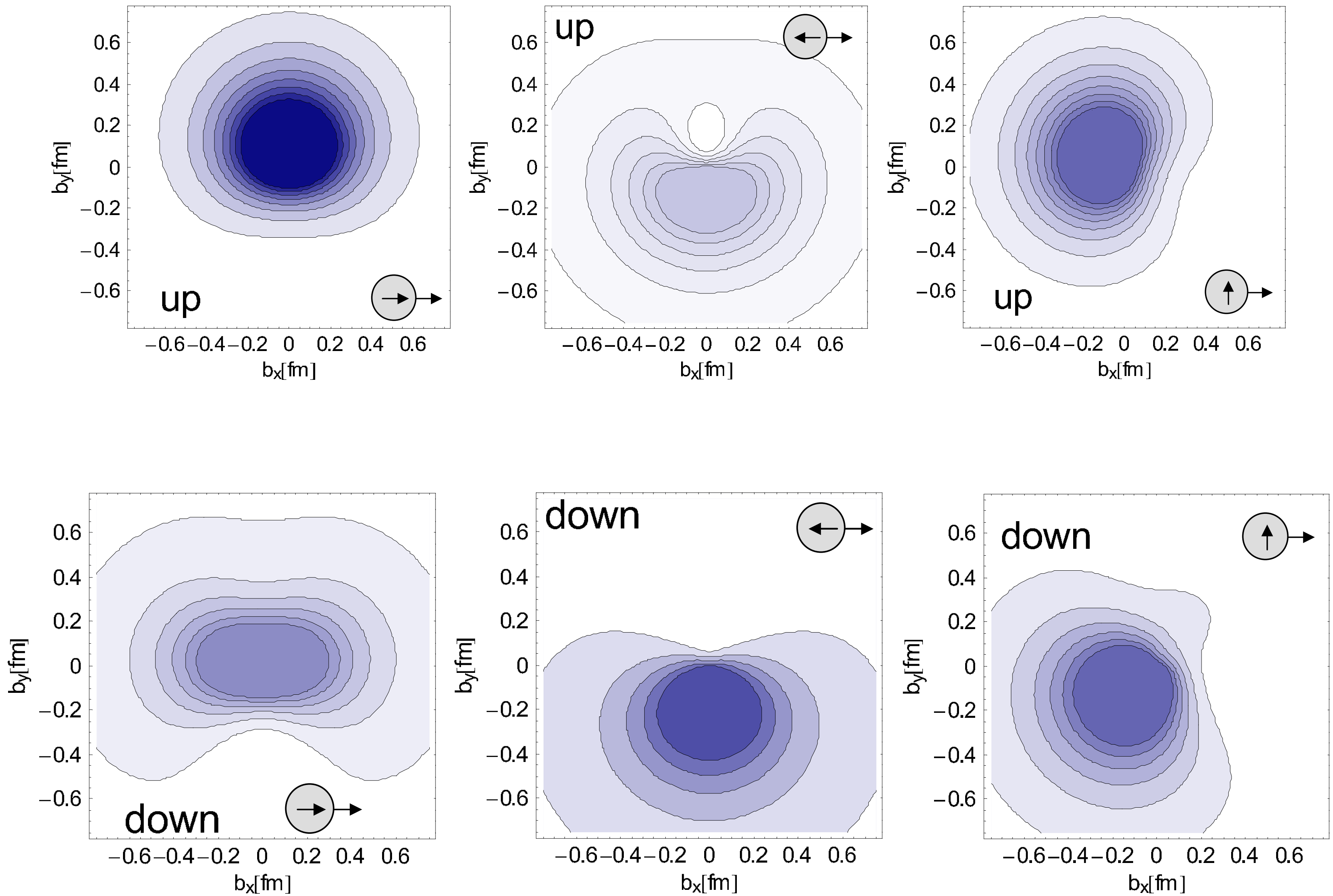
Boer-Mulders effect



Unpolarized

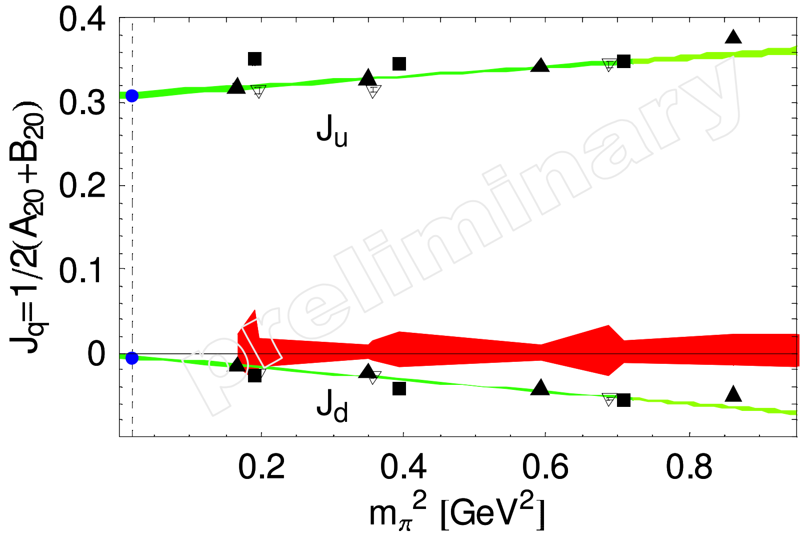
$r_T^d > r_T^u$

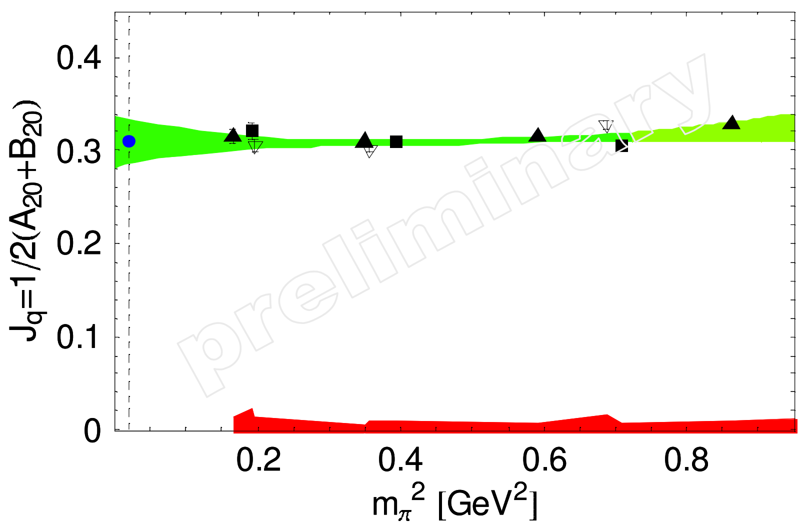
Nucleon and quarks both polarized



(Orbital) Angular Momentum

$$J^q = \frac{1}{2}\left(A_2^q(0) + B_2^q(0)\right) \equiv \frac{1}{2}\Delta\Sigma^q + L^q$$

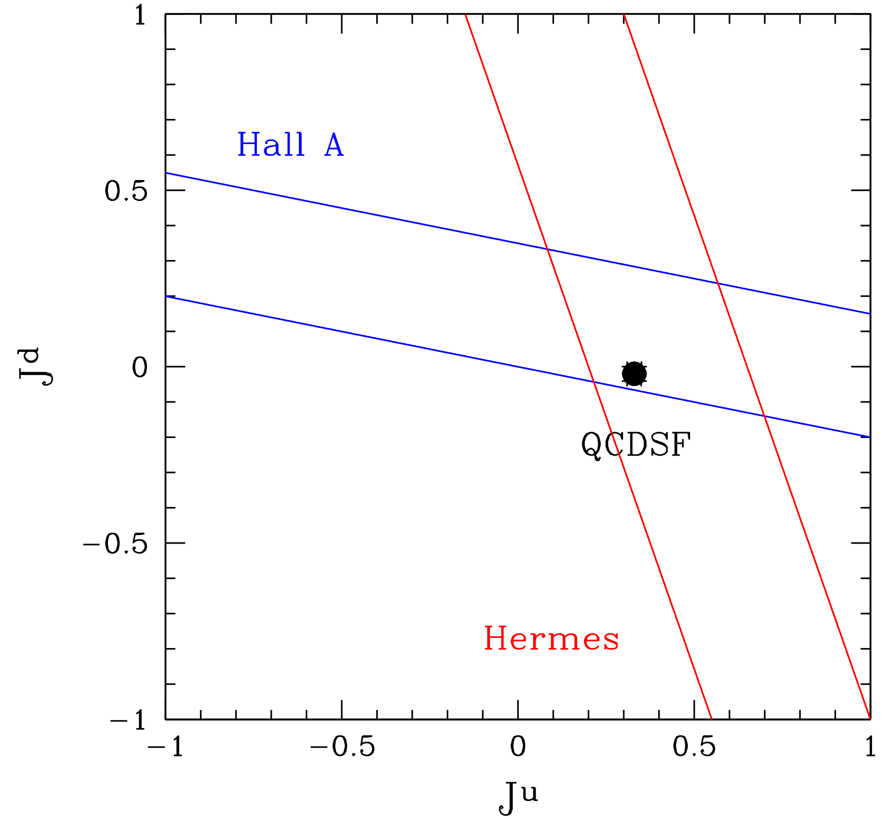




ChPT fit   Chen & Ji

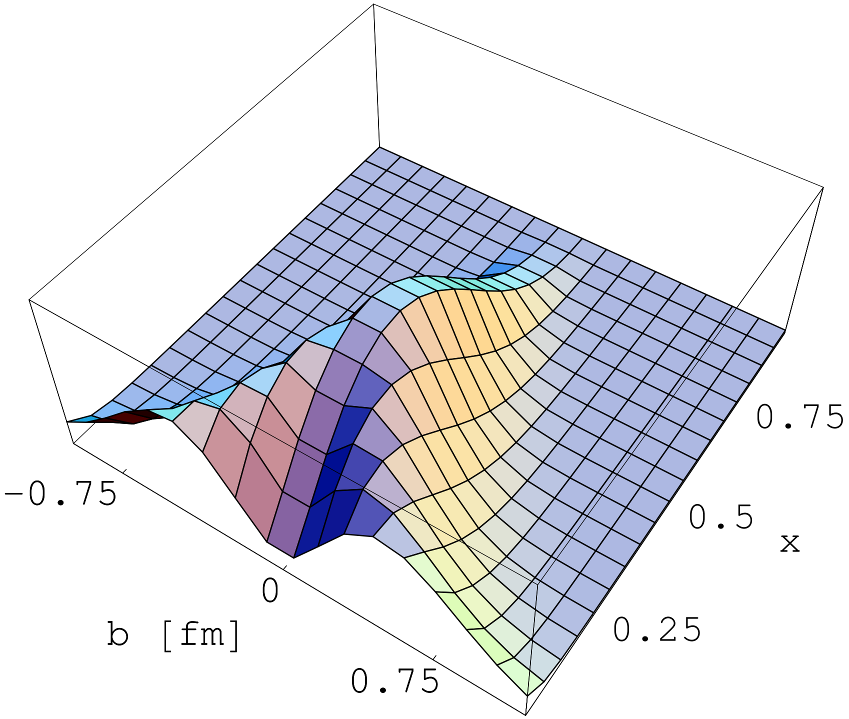
$J^u = 0.33(2)$ $J^d = -0.02(2)$ $L^u = -0.13(4)$ $L^d = 0.15(4)$

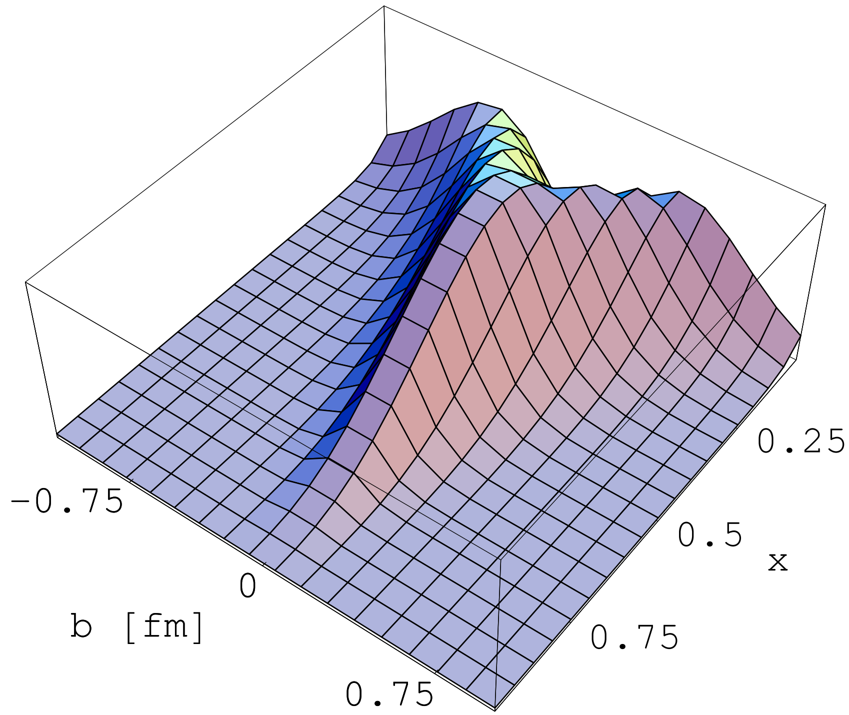
Comparison with experiment



GPDs

$H^u(x, \mathbf{b}_\perp^2)$ $Q^2 = 4\ \mathrm{GeV}^2$





$$\langle b^2 \rangle = \frac{7}{2}\,\alpha^{\vee\,2}\,(1-x)^2 + \mathcal{O}\left((1-x)^3\right)$$

$$\langle r^2 \rangle = \frac{7}{2}\alpha^{\vee\,2} + \mathcal{O}\,(1-x)$$

Pion Structure

$N_f = 2$ 'Valence' quark distributions

Form Factor

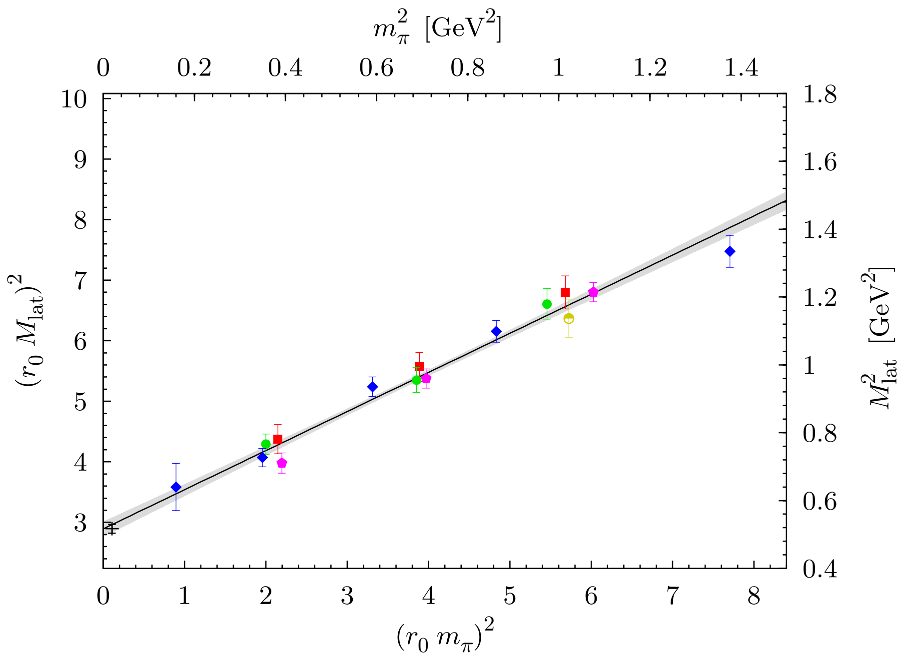
$$F^{\pi}(\Delta^2) = A_1(\Delta^2)$$

Expect (dimensional counting)

$$F^{\pi}(Q^2) \propto \frac{1}{Q^2} \qquad Q^2 = -\Delta^2$$

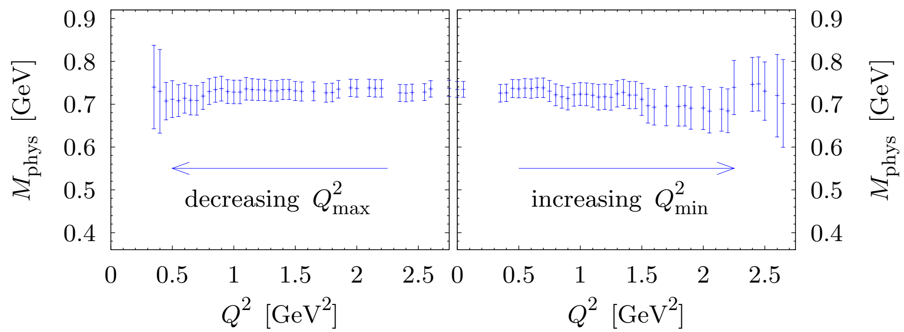
Ansatz

$$\boxed{F^{\pi}(Q^2) = 1/(1 + Q^2/m^2)^{-1}}$$



$$M_{\rm lat} = m$$

$$M_{\rm phys} = M_{\rm lat}\,(m_{\pi,{\rm phys}})$$

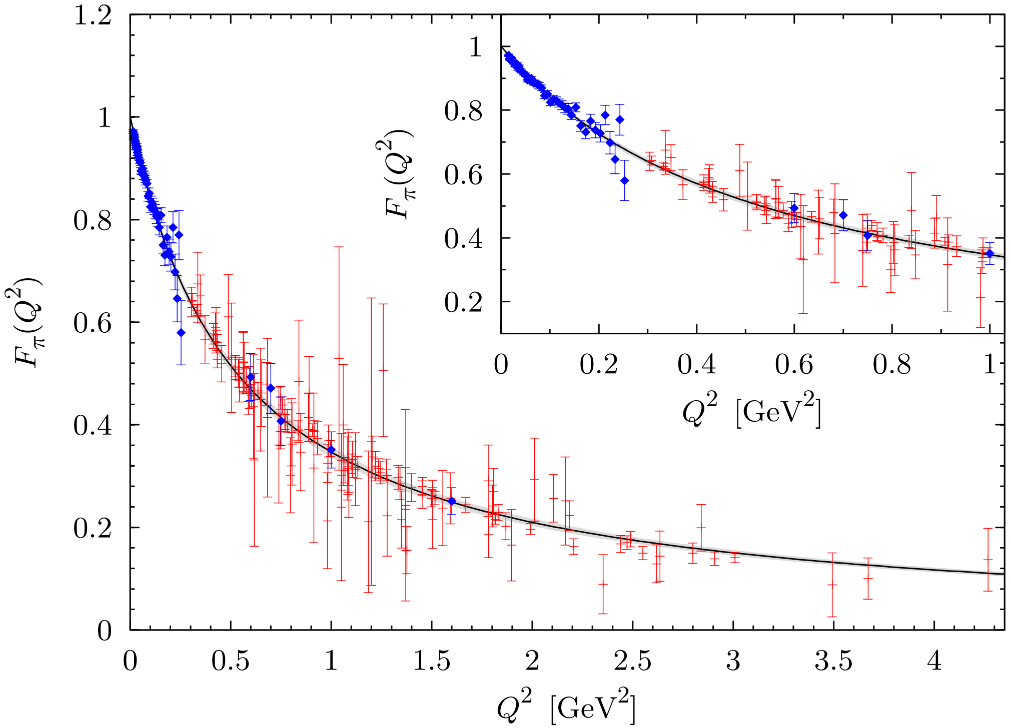


$\Leftarrow$ Stability of monopole fit

$M_{\rm phys} = 773(17)$ MeV $Q^2 < 1$ GeV^2

VDM

$M_{\rm phys} = 729(16)$ MeV all Q^2



◆ Experiment   + Lattice

$\Uparrow$

Shifted by $(1 + Q^2/M^2_{\rm phys})^{-1} - (1 + Q^2/M^2_{\rm lat})^{-1}$

Spin Asymmetries

Transverse spin density $\Downarrow$

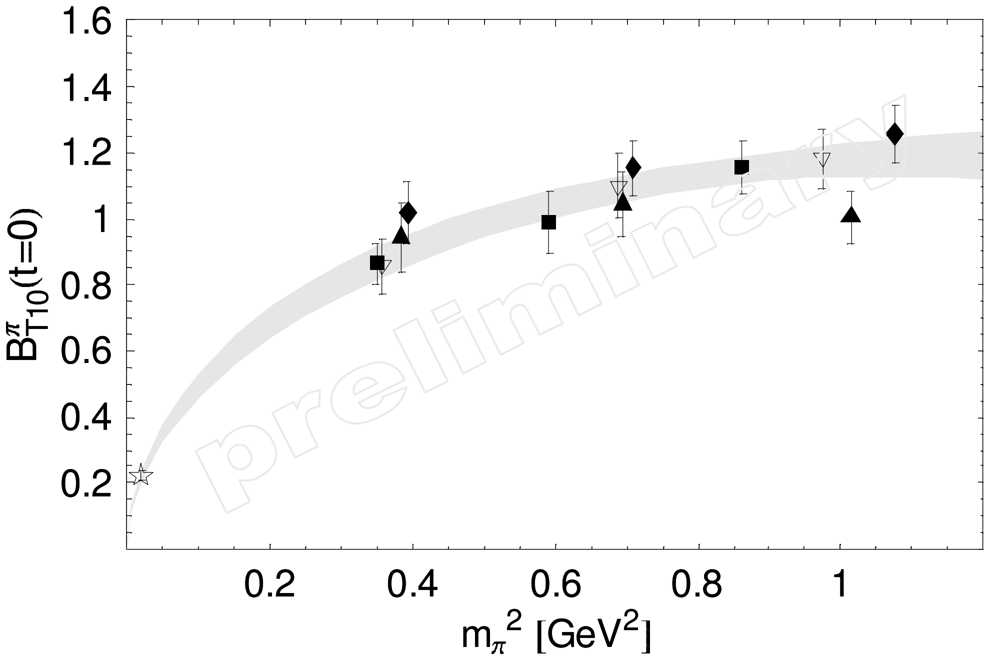
$\lambda_\perp$ quark spin

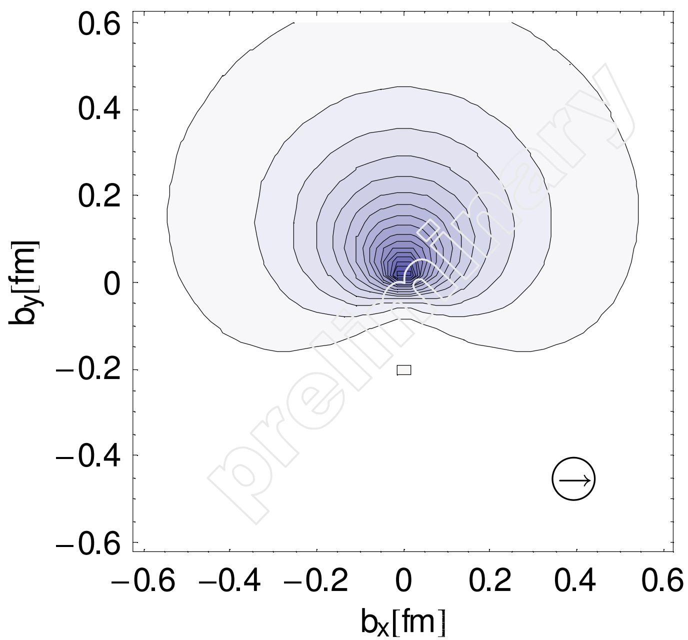
$$\langle p_+|\bar{q}(\mathbf{b}_\perp)\left[\gamma_+ - \boldsymbol{\lambda}_{\perp i}\,\boldsymbol{\sigma}_{+j}\,\gamma_5\right]q(\mathbf{b}_\perp)|p_+\rangle = \left\{A_1^q(\mathbf{b}_\perp^2) - \frac{1}{m_\pi}\epsilon_{ij}b_{\perp j}\boldsymbol{\lambda}_{\perp i}\,\bar{B}_1^{Tq}(\mathbf{b}_\perp^2)'\right\}$$

$\Uparrow$ Monopole

$\Uparrow$ Dipole

Hägler et al.





Boer-Mulders effect

Conclusions & Outlook

- Simulations at small pion masses m_π with Wilson-type fermions feasible now

- Improvement of algorithms
- Increase of computing power

- Extrapolation to chiral limit and infinite volume greatly improved

FS corrections surprisingly well described by ChPT

- Current simulations done at $m_\pi = O(300)$ MeV

- 2007/8: $m_\pi \longrightarrow 250$ MeV
- 2008/9: $m_\pi \longrightarrow 200$ MeV

On spatial volumes $\gtrsim (3\,\text{fm})^3$

$\Downarrow$

Resolution of pion cloud

- Challenge: Evaluation of disconnected diagrams