Light-Cone Sum Rules for Form Factors of the $N\gamma\Delta$ transition

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Schedule

Why examine $p\gamma \to \Delta^+$?

it is a possibility to study the proton

- a selection rule (1965) predicts only magnetic dipole transitions $M1$ for $p\gamma \to \Delta^+$
- the selection rule can be violated if the proton contains d-state contributions (S.L. Glashow, Physika 96A 1979)

interesting quantities at $Q^2 = 0$

$$R_{EM} = \frac{E2}{M1} \text{ and } G_M$$

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Experimental and theoretical results for R_{EM}

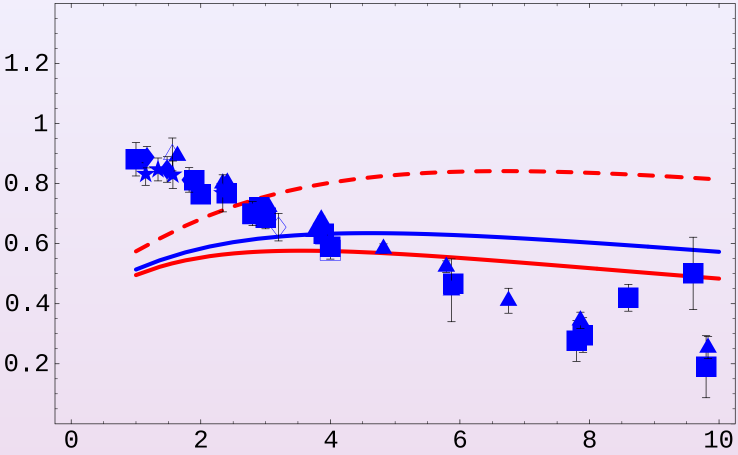
Experimental Results

Experiment	year	R_{EM}
MAMI	1997	$-2.5 \pm 0.4\%$
LEGS	1997	$-3.0 \pm 0.5\%$
MIT-Bates OOPS	2003	$-2.2 \pm 0.9\%$
MAMI	2004	$-2.73 \pm 0.03\%$

Theoretical Results

Approach	year	R_{EM}
MIT bag models	before 1990	$-2\%..0$
Skyrme model	1987	$\backsim -5\%$
Lattice	2004	$-2.0 \pm 1.0\%$
LCSR	2004	-6.8%
QCDSR	1984	???

LCSR and the form factor G_M



(Braun, Lenz, Peters & Radyushkin Phys.Rev. D73 (2006))

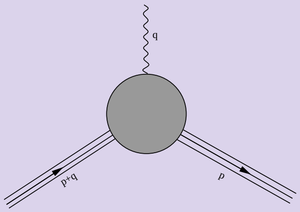
A Classical Problem

- consider the classical problem: the magnetic moments of nucleons
- the problem is well known and thus provides an excellent testing-ground for the technique
- classical QCD sum rules are known to work well

(Ioffe & Smilga NPB 232 (1984) 109-142 Balitsky & Yung PLB 129 (1983) 328)

- there are no unexpected subtleties
- LCSR based on nucleon distribution amplitudes work well

photon emission by a proton



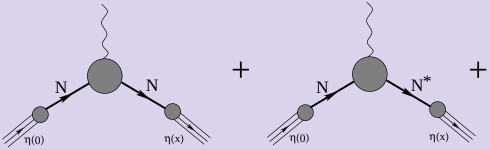
The Process on the Hadron Level

- the starting point for Sum Rule approaches is a correlation function:

$$\Pi^{\mu\nu}(p,q)e_\nu = i^2 \int d^4x \int d^4y \; e^{ipx+iqy} \langle 0| \mathcal{T} \eta(x) j^\nu_{\rm em}(y) \overline{\eta}(0) |0\rangle e_\nu$$

$\eta(x) = \varepsilon^{abc} \left(u(x)^a \mathcal{C}\gamma_\nu u(x)^b\right) \gamma_5 \gamma^\nu d^c(x)$ Ioffe Nucl. Phys. B188, 317 (1981)

- in the region $p^2 = p_1^2 = m_P^2$ und $(p+q)^2 = p_2^2 = m_P^2$ the process $p \to p\gamma$ dominates



$$\propto \frac{\not{p} + m_N}{m_N^2 - p^2} \mathcal{V} \frac{\not{p} + \not{q} + m_N}{m_N^2 - (p+q)^2} + \int_{s_{01}}^{\infty} ds_1 \int_{s_{02}}^{\infty} ds_2 \; \frac{\rho(s_1, s_2)}{(s_1 - p^2)(s_1 - (p+q)^2)}$$

On the QCD side

- if $p^2 \ll 0$ and $(p+q)^2 \ll 0$ one can calculate the correlation function using QCD
- a *matching* of the hadronic representation and the QCD calculation allows the extraction of the form factors
- the only problem is how to treat the QCD side $\to$ OPE

in our case there is, however, one additional subtlety:

- as $q^2 = 0$, a large value of $|p^2|$ does not guarantee a small value of $|y|$

we will therefore have to make use of the so called background field method

LCSR vs. SVZ SR

- in classical SVZ sum rules the correlator is expanded in a power series of $\left(\frac{\Lambda_{QCD}^2}{-p^2}\right)^n$ and local condensates of increasing dimension

$$\langle\overline{q}q\rangle \quad \langle 0|\,\frac{\alpha_s}{\pi}G^a_{\mu\nu}G_a^{\mu\nu}\,|0\rangle \quad \langle 0|\,\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q\,|0\rangle$$

- as $q^2 = 0$ does not imply $q = 0$, terms like

$$\left(\frac{pq}{-p^2}\right)^m \left(\frac{\Lambda_{QCD}^2}{-p^2}\right)^n$$

  have to be taken into account
- one possible way: use $q \to 0$ and so $p = p + q$
- this will be a source for new problems

LCSR vs. SVZ SR II

- as one has to keep $q = 0$ in the SVZ approach. The ground state contribution can be distinguished from the contributions due to excited states by the double pole

$$\frac{\not p + m_N}{m_N^2 - p^2}\mathcal{V}\frac{\not p + m_N}{m_N^2 - p^2} + \frac{\not p + m_N}{m_N^2 - p^2}\mathcal{V}'\frac{\not p + m_{N'}}{m_{N'}^2 - p^2}$$

- however if one wants to study transition between hadrons of different mass, the double pole vanishes

$$\frac{1}{(m^2 - p^2)(m^{*2} - p^2)} = \frac{1}{m^{*2} - m^2}\left(\frac{1}{m^2 - p^2} - \frac{1}{m^{*2} - p^2}\right)$$

and the separation of ground state and continuum becomes difficult at best

LCSR vs. SVZ SR III

- the other option $q^2 = 0$, but $q \neq 0$ and $\left(\frac{2qp}{p^2}\right) \sim 1$
- the expansion then contains terms like

$$\left(\frac{\Lambda_{QCD}^2}{-p^2}\right)^k \left(\frac{2qp}{-p^2}\right)^n$$

  which have to be resummed
- this is possible by changing the expansion parameter from
  operator dimension $\rightarrow$ operator twist
- the price, one has to pay, is the introduction of new non-local operators – the distribution amplitudes

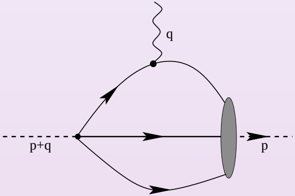
Distribution amplitudes

- consider again a three-point correlator

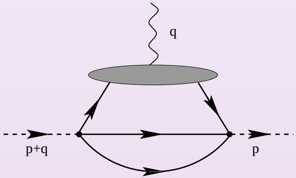
$$\langle 0 | \mathcal{T} \{ \eta_1(x) J_{em}(y) \eta_2(0) \} | 0 \rangle$$

- this can be represented in two ways

$$\langle 0 | \mathcal{T} \; \eta_1(x) J_{em}(y) \, | B(p) \rangle \qquad \langle 0 | \mathcal{T} \; \eta_1(x) \eta_2(0) \, | \gamma(q) \rangle$$



LCSR using baryon distribution amplitudes

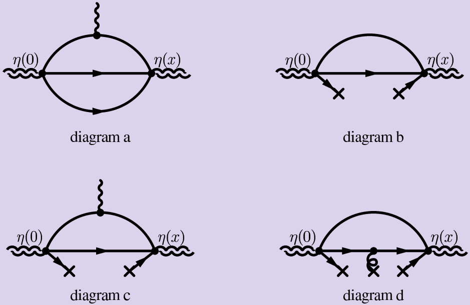


LCSR using photon distribution amplitudes

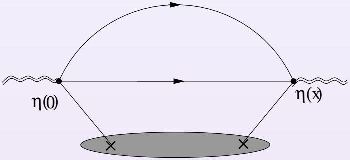
$P \to P\gamma$ on quark level

- in the kinematic region $-p^2 \ll 0$ and $-(p+q)^2 \ll 0$ we can calculate the correlation function using QCD

relevant diagrams up to twist-4



The leading-twist term

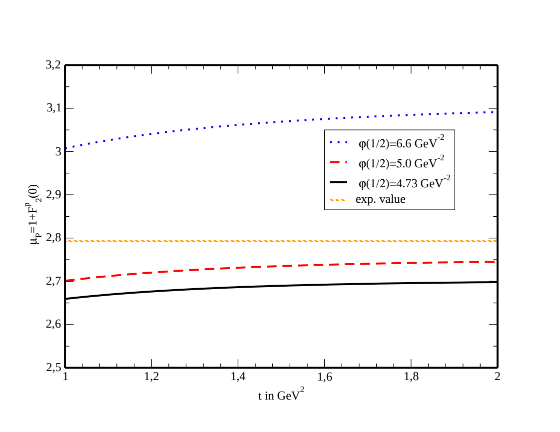


- equate the result for the diagram and the hadronic expression, containing the Pauli and Dirac form factors
- perform a Borel transformation to suppress contributions of the unknown continuum
- remove the continuum from both sides

$$F_2(0) = \frac{8\pi^2 m_p \langle \bar{q} q \rangle}{|\lambda_P|^2} e^{m_p^2/t} \left[\frac{e_d}{3} \varphi(1/2) t^2 \left(1 - e^{-S_0/t} \left(1 + \frac{S_0}{t} \right) \right) \right]$$

where t is the Borel parameter, S_0 is the continuum threshold and $\varphi(1/2) = 3/2\chi$

Results for the magnetic moment of the proton



J.R. Phys. Rev. D 75, 074025 (2007)

exp.value :

$$\mu_p = 2.793$$

LCSR result :

$$\mu_p = 2.68 \pm 0.3$$

Schedule

$N\gamma \to \Delta$

- the Δ has a spin of $3/2$ and is described by a *Rarita-Schwinger-Spinor* Δ_j^μ
- the correlator fullfils the Rarita-Schwinger-condition: $\gamma_\mu \Pi^{\mu\nu} = 0$

Transition matrix element

$$\langle \Delta^\mu(p) | j^\nu | P(p+q) \rangle = \Delta^\mu(p) \Big[G_2\left(q^2\right) \left(g_{\beta\nu} q \cdot \left(p + \frac{q}{2}\right) - q_\beta \left(p + \frac{q}{2}\right)_\nu \right) + G_1\left(q^2\right) \left(g_{\beta\nu} \not{q} - q_\beta \gamma_\nu \right) + G_3\left(q^2\right) \left(q_\beta q_\nu - q^2 g_{\beta\nu} \right) \gamma_5 \Big] P(p+q)$$

- the current

Δ^+ current

$$\eta^\mu(x) = \left[\left(u^a(x) \mathcal{C} \gamma^\mu u^b(x)\right) d^c(x) + 2 \left(u^a(x) \mathcal{C} \gamma^\mu d^b(x)\right) u^c(x) \right] \varepsilon^{abc}$$

has non-vanishing overlap with $J^P = 1/2^-$ states

Lorentz Structures

$$
\begin{aligned}
\mathcal{R}_1 &= qp\,\gamma^\mu \not{e}\not{q}\gamma_5 - ep\,\gamma^\mu \not{q}\not{p}\gamma_5 + 4\left(qp\,p^\mu \not{e}\gamma_5 - ep\,p^\mu \not{q}\gamma_5\right)\\
\mathcal{R}_2 &= p^2\left(pq\,\gamma^\mu \not{e}\gamma_5 - pe\,\gamma^\mu \not{q}\gamma_5\right) - 4\left(pq\,p^\mu \not{e}\not{p}\gamma_5 - ep\,p^\mu \not{q}\not{p}\gamma_5\right)\\
\mathcal{R}_3 &= pq\,\gamma^\mu \not{e}\gamma_5 - pe\,\gamma^\mu \not{q}\gamma_5 + 2\left(p^\mu \not{e}\not{q}\gamma_5\right) - \tfrac{1}{2}\left(\gamma^\mu \not{e}\not{q}\not{p}\gamma_5\right)\\
\mathcal{R}_4 &= 4\left(p^\mu \not{e}\not{q}\not{p}\gamma_5\right) - p^2\left(\gamma^\mu \not{e}\not{q}\gamma_5\right) + 2\left(pq\,e^\mu \not{q}\gamma_5 - pe\,q^\mu \not{q}\gamma_5\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_5 &= qp\left(\gamma^\mu \not{e}\not{q}\gamma_5\right) - 4\left(pq\,e^\mu \not{q}\gamma_5 - pe\,q^\mu \not{q}\gamma_5\right)\\
\mathcal{R}_6 &= \gamma^\mu \not{e}\not{q}\gamma_5 - 2\left(e^\mu \not{q}\gamma_5 - q^\mu \not{e}\gamma_5\right)\\
\mathcal{R}_7 &= pq\,\gamma^\mu \not{e}\gamma_5 - pe\,\gamma^\mu \not{q}\gamma_5 - 4\left(pq\,e^\mu \gamma_5 - pe\,q^\mu \gamma_5\right)\\
\mathcal{R}_8 &= 4\left(pq\,e^\mu \not{q}\not{p}\gamma_5 - pe\,q^\mu \not{q}\not{p}\gamma_5\right) - qp\left(\gamma^\mu \not{e}\not{q}\not{p}\gamma_5\right)\\
\mathcal{R}_9 &= 2\left(q^\mu \not{e}\not{p}\gamma_5 - e^\mu \not{q}\not{p}\gamma_5\right) + \gamma^\mu \not{e}\not{q}\not{p}\gamma_5\\
\mathcal{R}_{10} &= 4\left(pq\,e^\mu \not{p}\gamma_5 - pe\,q^\mu \not{p}\gamma_5\right) - qp\,\gamma^\mu \not{e}\not{q}\gamma_5 - ep\,\gamma^\mu \not{q}\not{p}\gamma_5\\
\mathcal{R}_{11} &= q^\mu \not{e}\not{q}\not{p}\gamma_5\\
\mathcal{R}_{12} &= q^\mu \not{e}\not{q}\gamma_5
\end{aligned}
$$

Quantities known from experiments

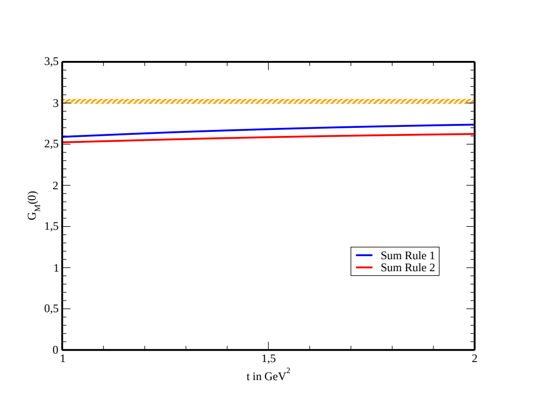
$$G_M(0) = \frac{m_p}{3(m_p + m_\Delta)}\left[(m_p + 3m_\Delta)(m_\Delta + m_p)\frac{G_1(0)}{m_\Delta} + (m_\Delta^2 - m_p^2)G_2(0)\right]$$

$$G_E(0) = \frac{m_p}{3(m_p + m_\Delta)}(m_\Delta^2 - m_p^2)\left[\frac{G_1(0)}{m_\Delta} + G_2(0)\right]$$

$$R_{EM} = -\frac{G_E(0)}{G_M(0)}$$

(Jones & Scadron Ann.Phys 82 (1973))

Results for G_M



exp.value :

$$G_M(0) = 3.02 \pm 0.03$$

LCSR result :

$$G_M(0) = 2.70 \pm 0.27$$

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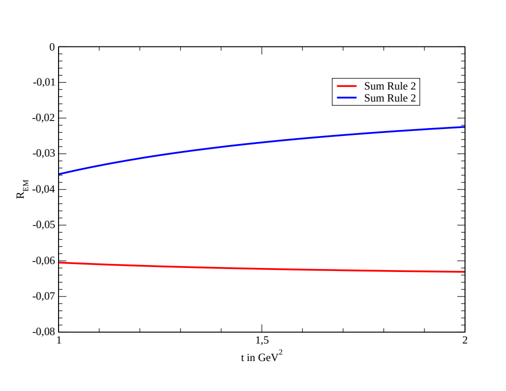
Results for R_{EM}



J.R. Phys. Rev. D 75, 074025 (2007)

exp.value :

$$R_{EM}(0) = -2.5 \pm 0.4\%$$

LCSR result :

$$R_{EM}(0) = -6.4 \pm 0.8\%$$

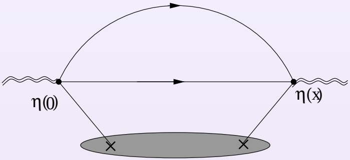
Schedule

Conclusions

- LCSR are able to handle radiative transition matrix elements
- the magnetic moment of the proton is reproduced surprisingly well, the same holds for G_M
- what can be done to improve this technique further?
  - determine the values of the non-perturbative parameters, especially χ and the twist-4 parameters more precisely
  - α_s-corrections
  - expand photon DAs from $Q^2 = 0$ to photon virtual photons

    (Yu, Liu & Zhu Phys.Rev. D73 (2006))

The leading-twist term



$$\langle 0 | \mathcal{T} \eta_p(x) \overline{\eta}_p(0) | 0 \rangle_F = -\frac{8}{\pi^4 x^8} \langle 0 | \gamma^5 \not{x} d^a(x) \overline{d}^a(0) \not{x} \gamma_5 | 0 \rangle_F + \ldots$$

- use Fierz identity to decompose the Dirac matrix $d^a(x)_i \overline{d}^a(0)_j$ to the Dirac basis
- upon insertion of the twist-2 photon DA we get

$$-ip^\alpha p_\mu \sigma^{\mu\nu} F_{\alpha\nu} \left[\frac{e_d \langle \overline{q} q \rangle}{12\pi^2} \chi \int_0^1 du\, \varphi(u) \ln\left(\frac{\mu^2}{-\overline{u} p_1^2 - u p_2^2} \right) \right]$$

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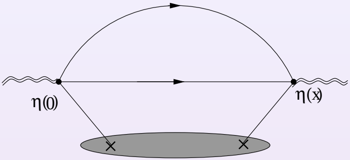

$$\langle 0|\,\mathcal{T}\eta_p\,(x)\,\overline{\eta}_p\,(0)\,|0\rangle_F \quad = \quad -\frac{8}{\pi^4 x^8}\,\langle 0|\,\gamma^5 \not{x}\, d^a(x)\overline{d}^a(0)\not{x}\gamma_5\,|0\rangle_F \;+\; \dots$$

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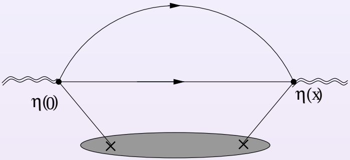
The leading-twist term



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