A Review of the Few-Body Form Factors

Workshop on Exclusive Reactions at High Momentum Transfer

Jefferson Lab
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Makis Petratos
Kent State University

Elastic Electron-Deuteron Scattering

- Cross Section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E'}{4E^3 \sin^4(\frac{\theta}{2})} \left[A(Q^2)\cos^2(\tfrac{\theta}{2}) + B(Q^2)\sin^2(\tfrac{\theta}{2})\right]$$

$$\begin{aligned} A(Q^2) &= F_C^2(Q^2) + \frac{8}{9}\tau^2 F_Q^2(Q^2) + \frac{2}{3}\tau F_M^2(Q^2) \\ B(Q^2) &= \frac{4}{3}\tau(\tau+1) F_M^2(Q^2) \end{aligned}$$

$$Q^2 = 4EE'\sin^2(\tfrac{\theta}{2}) \qquad \tau = Q^2/4M^2$$

- F_C, F_Q, F_M: Charge, Quadrupole, Magnetic form factors

- Forward angle scattering $\longrightarrow A(Q^2)$

- Backward angle scattering $\longrightarrow B(Q^2)$

Asymmetry Measurements

- Tensor polarization observables:

$$t_{20} = -\frac{1}{\sqrt{2}S}\left[\frac{8}{3}\tau F_C F_Q + \frac{8}{9}\tau^2 F_Q^2 + \frac{1}{3}\tau f(\theta) F_M^2\right]$$

$$t_{21} = \frac{2}{\sqrt{6}S}\tau\sqrt{\tau(1+f(\theta))}F_M F_Q \sec\frac{\theta}{2}$$

$$t_{22} = -\frac{1}{2\sqrt{3}S}\tau F_M^2$$

$$S = A + B\tan^2\tfrac{\theta}{2} \qquad f(\theta) = 1 + 2(1+\tau)\tan^2\tfrac{\theta}{2}$$

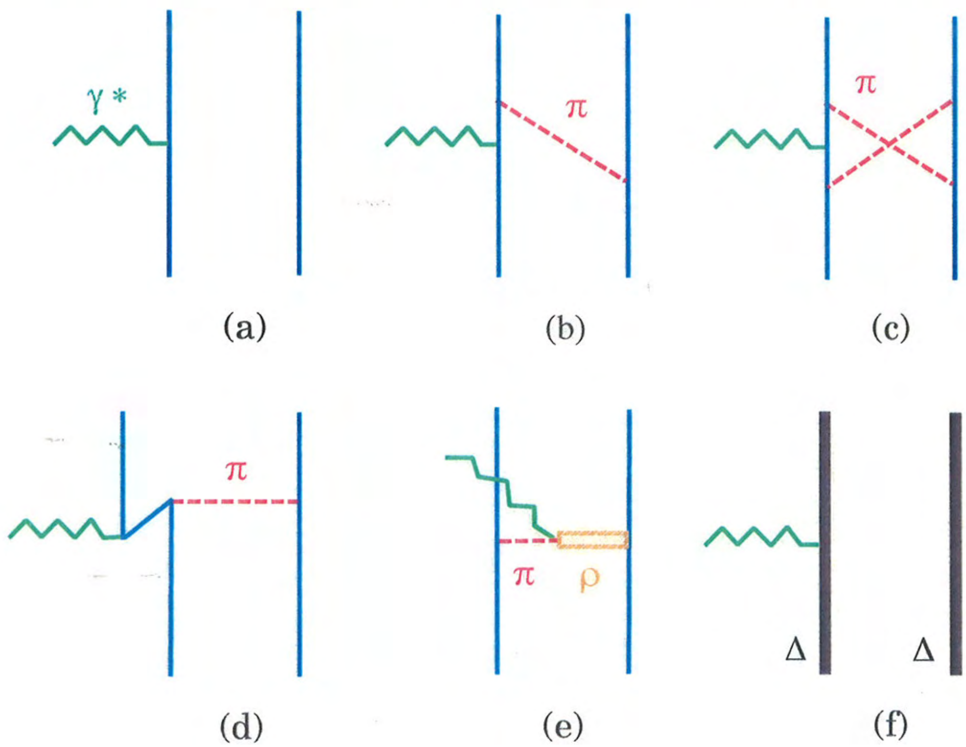
- Observable of choice :t_{20}; Neglect F_M contributions:

$$\tilde{t}_{20} = \sqrt{2}\frac{y(2+y)}{1+2y^2}$$

$$y = 2\tau F_Q/3F_C$$

- $\tilde{t}_{20}$ does not depend on nucleon form factors.

Deuteron Non-Relativistic Approach

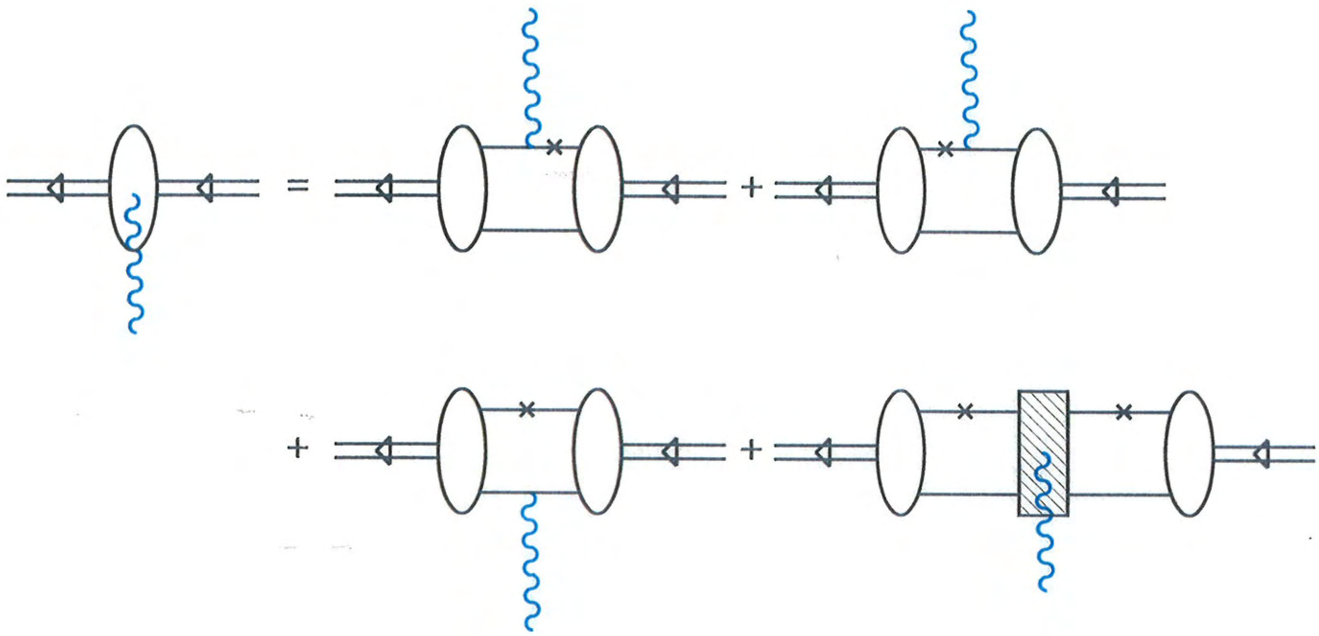


- Impulse Approximation
- Meson-Exchange Currents
- Isobar Configurations
- Six-Quark Admixtures
- Mainstream calculations (IA+MEC) fail to describe both $A(Q^2)$ and $B(Q^2)$ data

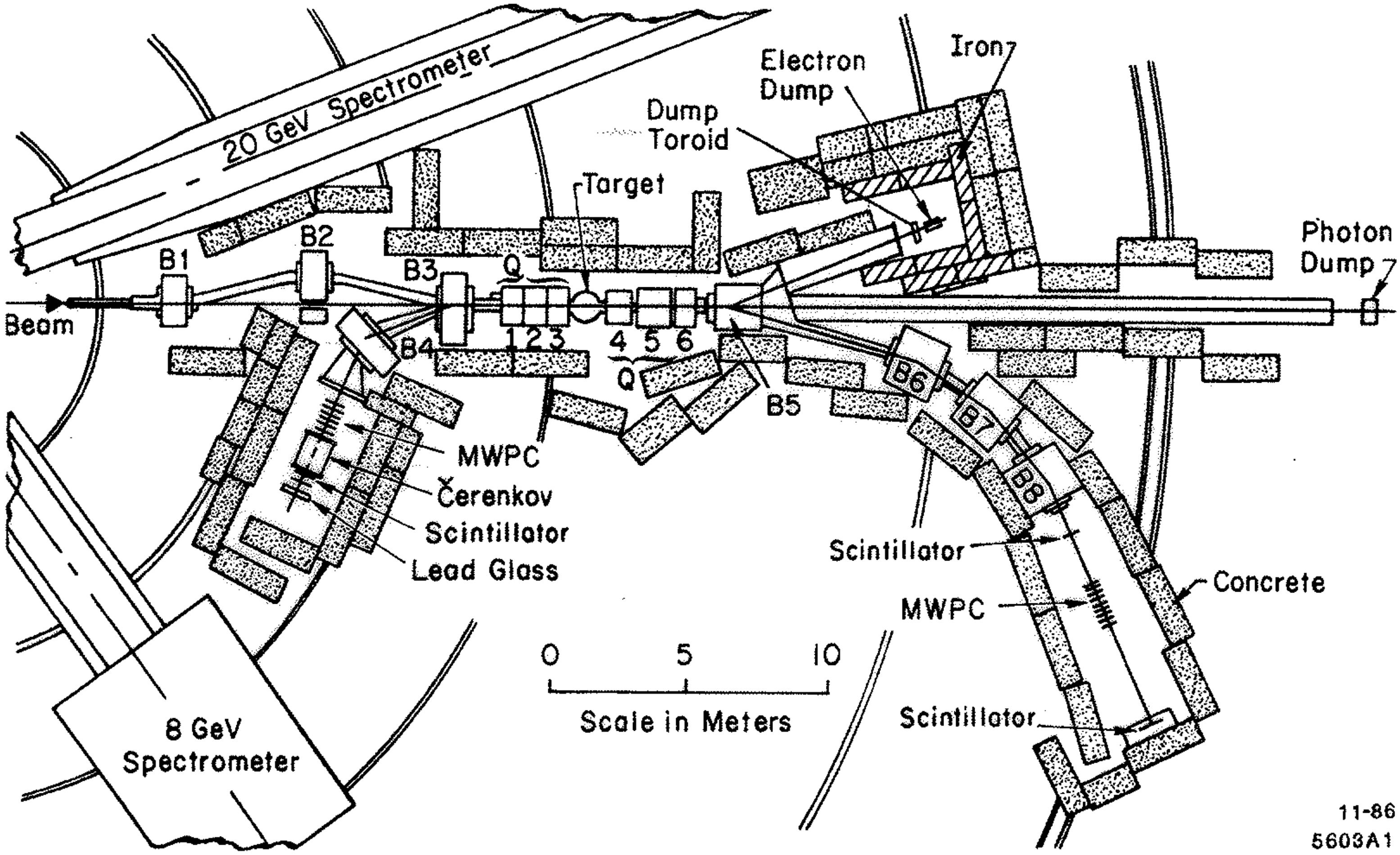
Deuteron Relativistic Approaches

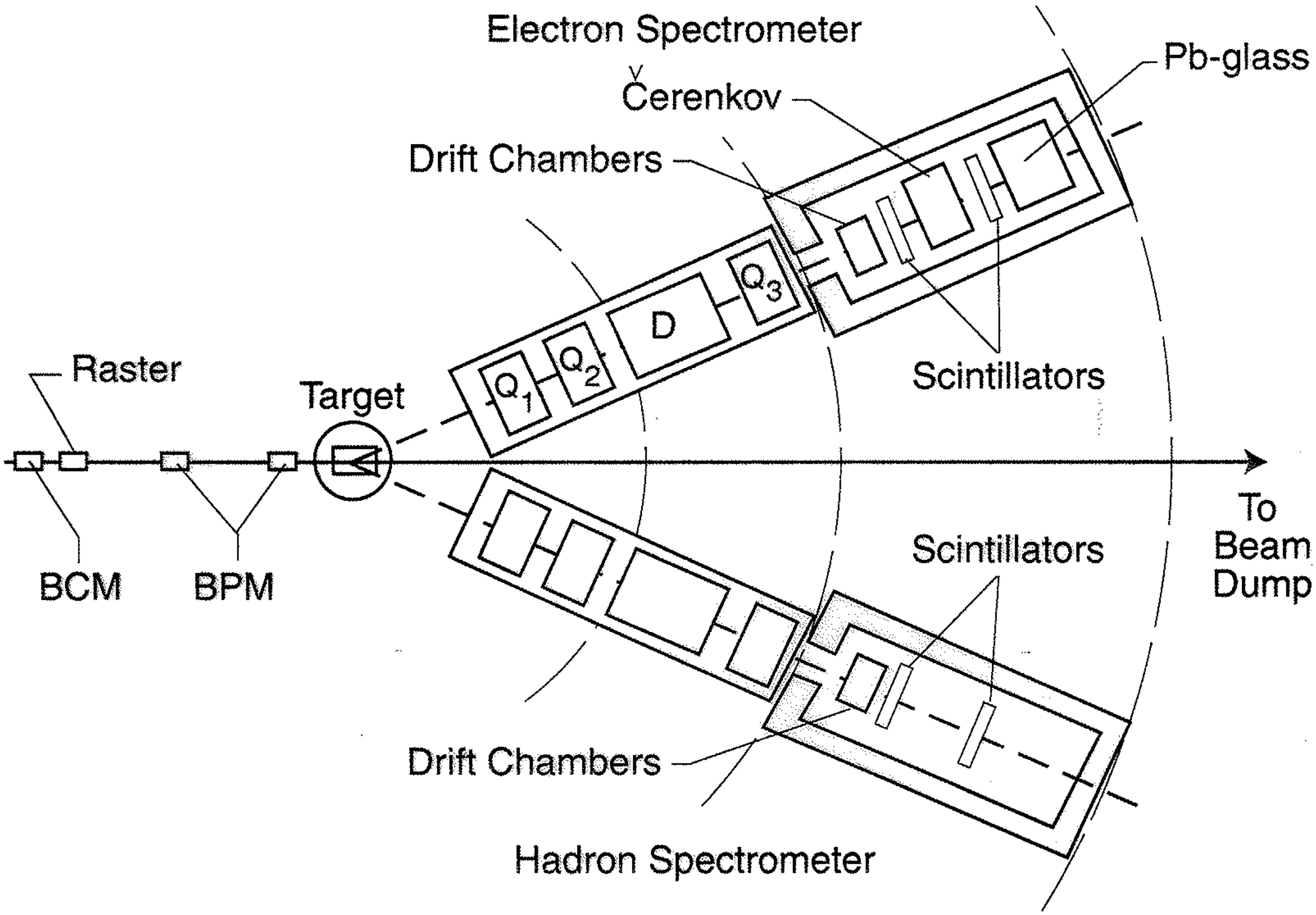
- Manifestly Covariant Dynamics using 3-D reductions of the Bethe-Salpeter (B-S) equation:
  - Lorentz transformations are kinematic
  - Inclusion of negative energy states is dynamic
  - Quasi-potential approximation
    - Gross equation (Van Orden, Devine, Gross)
    - Blankenbecler-Sugar equation (Hummel, Tjon)
  - Equal-time approximation
    - Wallace-Mandelzweig equation (Phillips, Wallace)
  - VDG Model describes all data fairly well
- Light Front Dynamics using field theories quantized on the light cone:
  - Conventional quantum mechanics
  - Some Lorentz transformations include the interaction
  - Models not as advanced yet as the B-S models

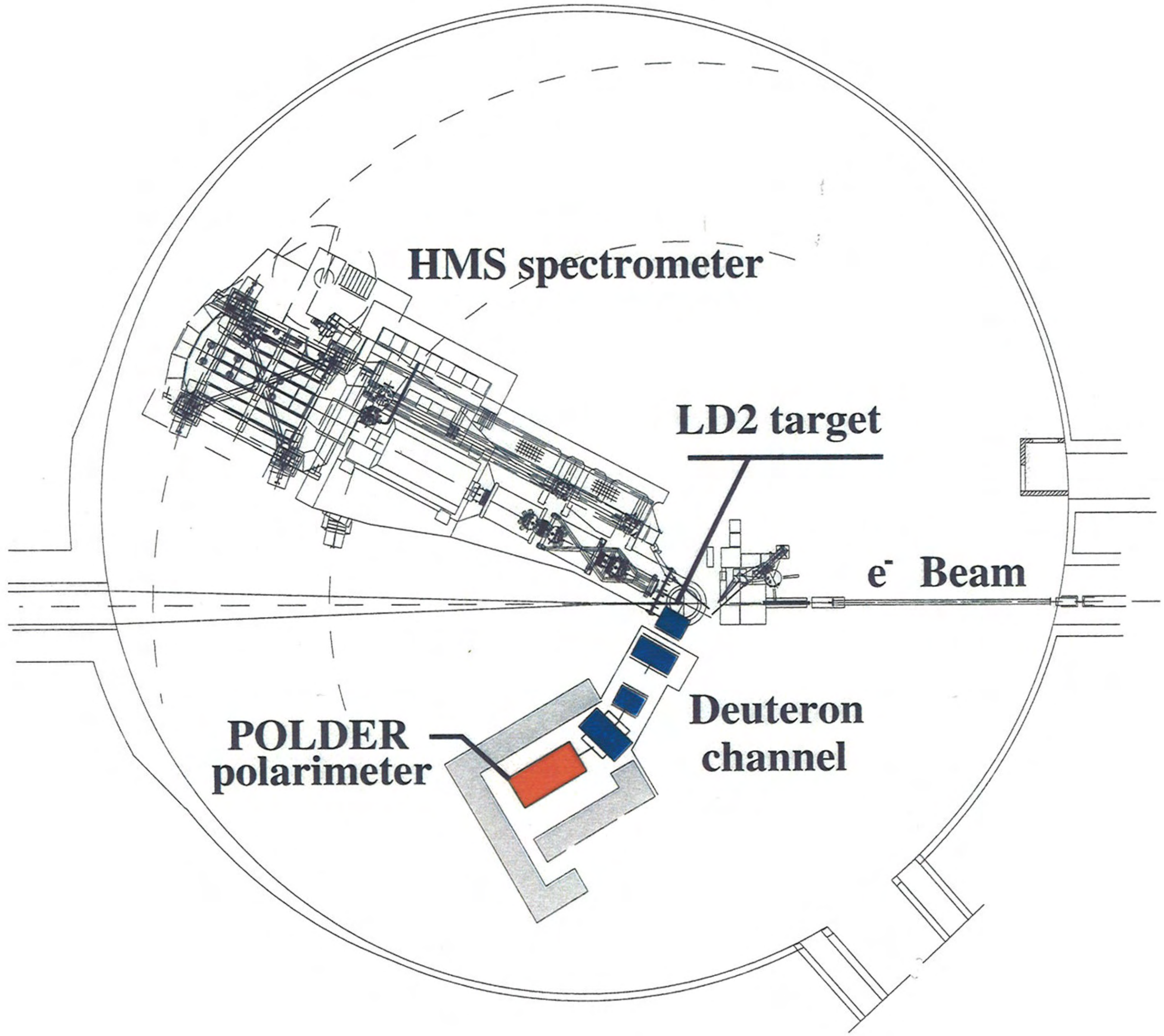
Spectator Model with Gross Equation

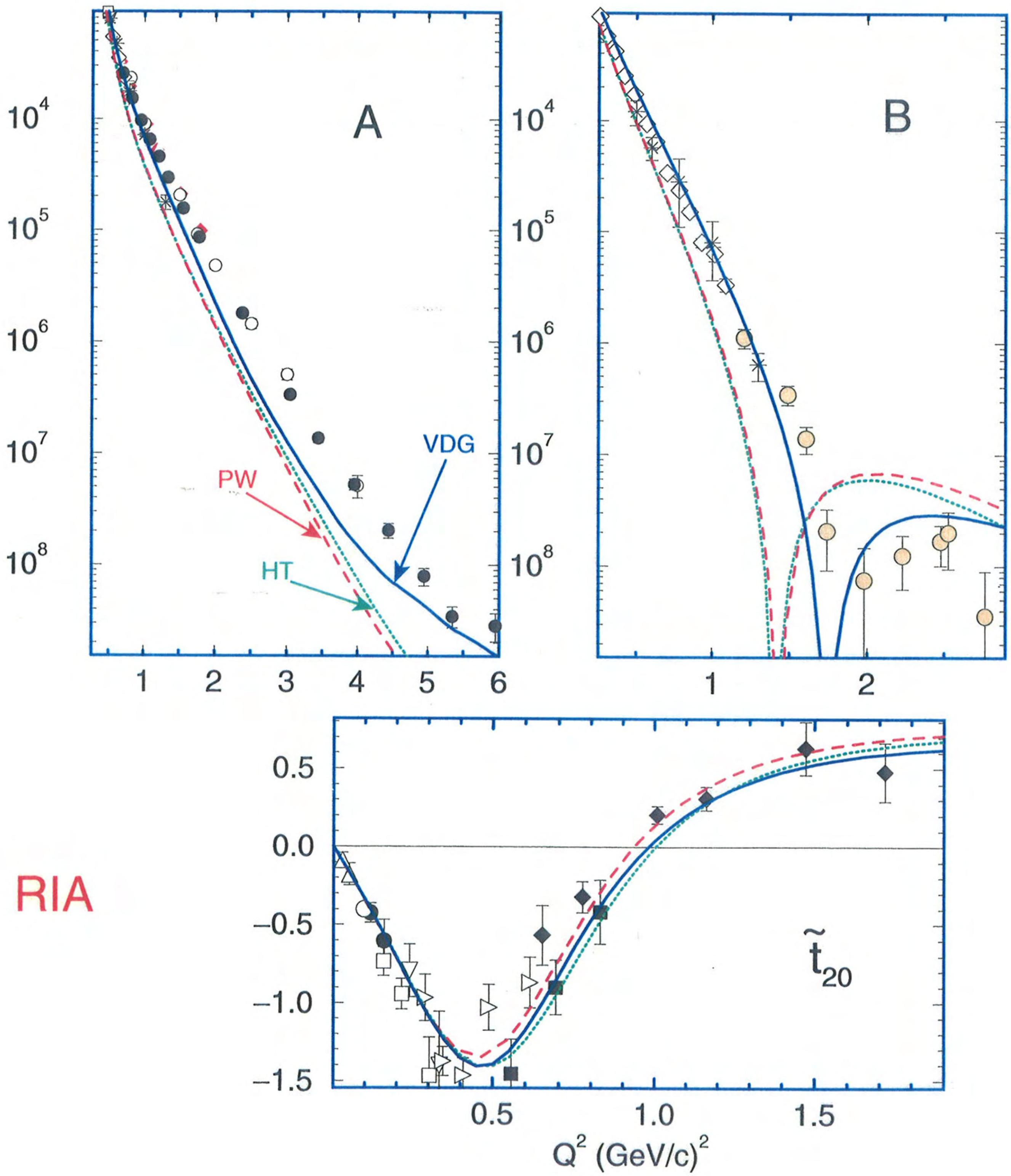


- One-boson-exchange model
- Model sensitive to:
  - Magnitude of relativistic small P states of deuteron wave function
  - $F_{\rho\pi\gamma}$ form factor
  - Nucleon off-shell mass treatment (F_3 form factor)

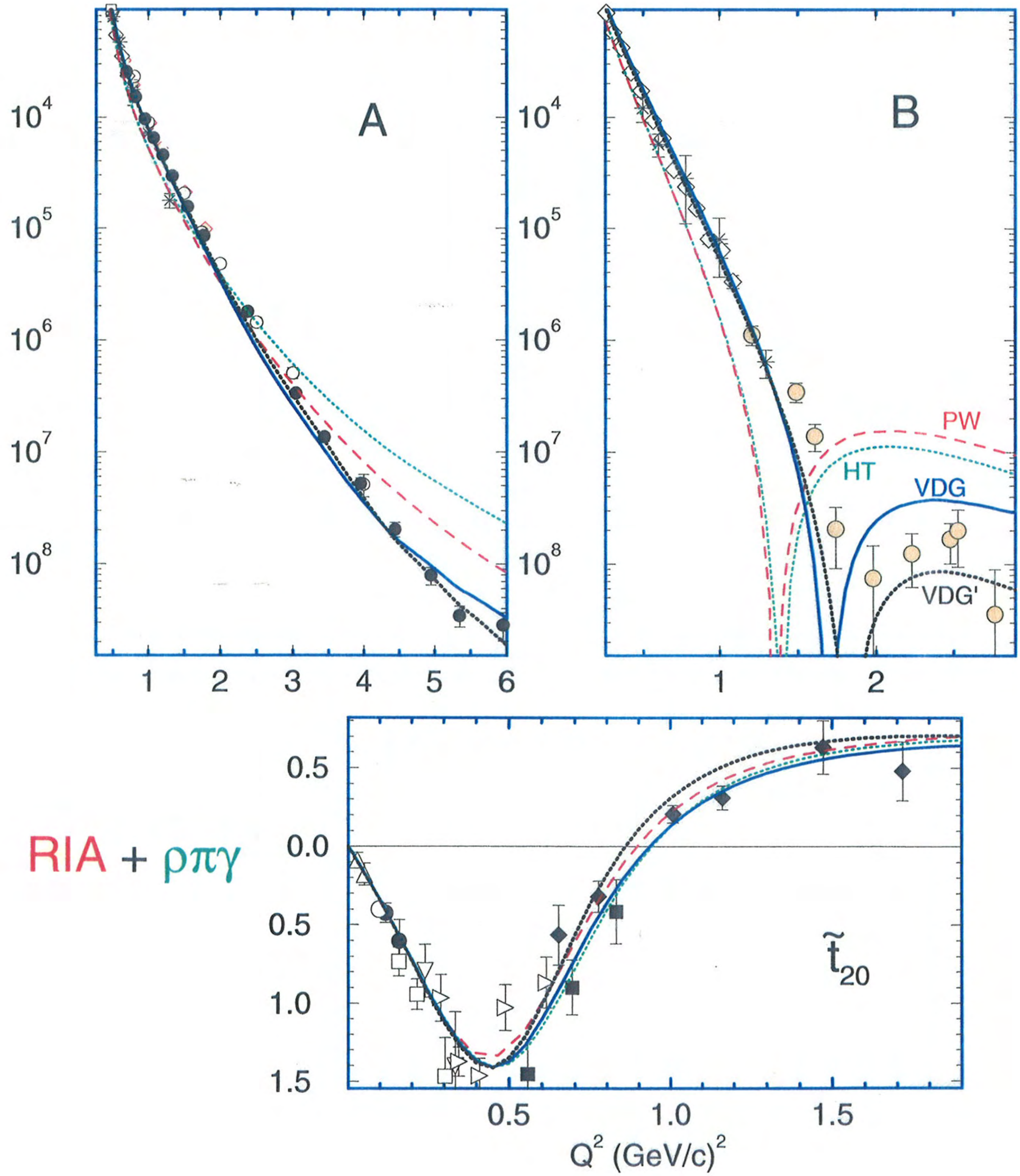

20 GeV Spectrometer
Electron Dump
Iron
Dump Toroid
Target
Photon Dump
B1
B2
B3
Q
Beam
1 2 3
4 5 6
B4
Q
B5
B6
B7
B8
MWPC
Čerenkov
Scintillator
Lead Glass
Scintillator
Concrete
MWPC
0 5 10
Scale in Meters
Scintillator
8 GeV Spectrometer
11-86
5603A1


Electron Spectrometer
Čerenkov
Pb-glass
Drift Chambers
Q1
Q2
D
Q3
Scintillators
Raster
Target
BCM
BPM
To
Beam
Dump
Scintillators
Drift Chambers
Hadron Spectrometer


HMS spectrometer
LD2 target
e- Beam
Deuteron
channel
POLDER
polarimeter



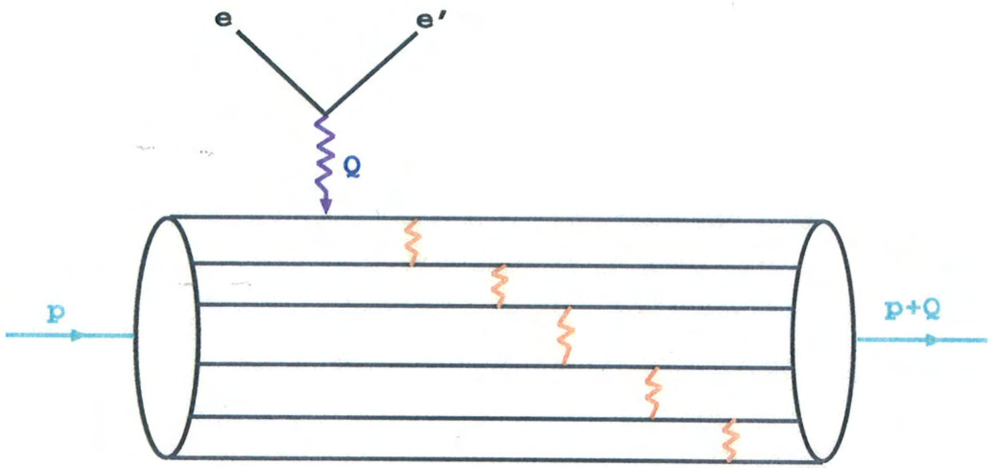
A
B
VDG
PW
HT
10^4
10^5
10^6
10^7
10^8
1 2 3 4 5 6
1 2
0.5
0.0
−0.5
−1.0
−1.5
0.5 1.0 1.5
$\tilde{t}_{20}$
Q^2 $(GeV/c)^2$
RIA


A
B
PW
HT
VDG
VDG'
RIA + ρπγ
$\tilde{t}_{20}$
Q^2 (GeV/c)^2

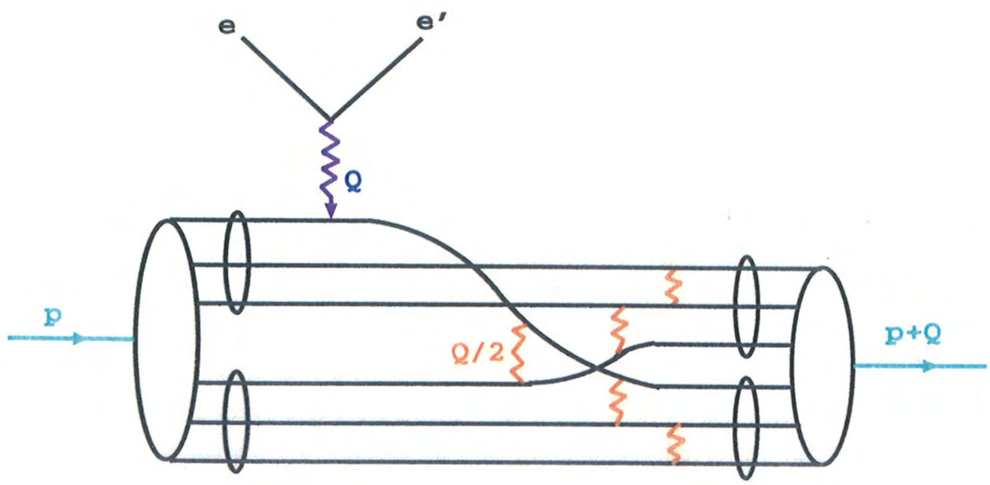
Quark Dimensional Scaling

- Dimensional Scaling Quark Model (DSQM)

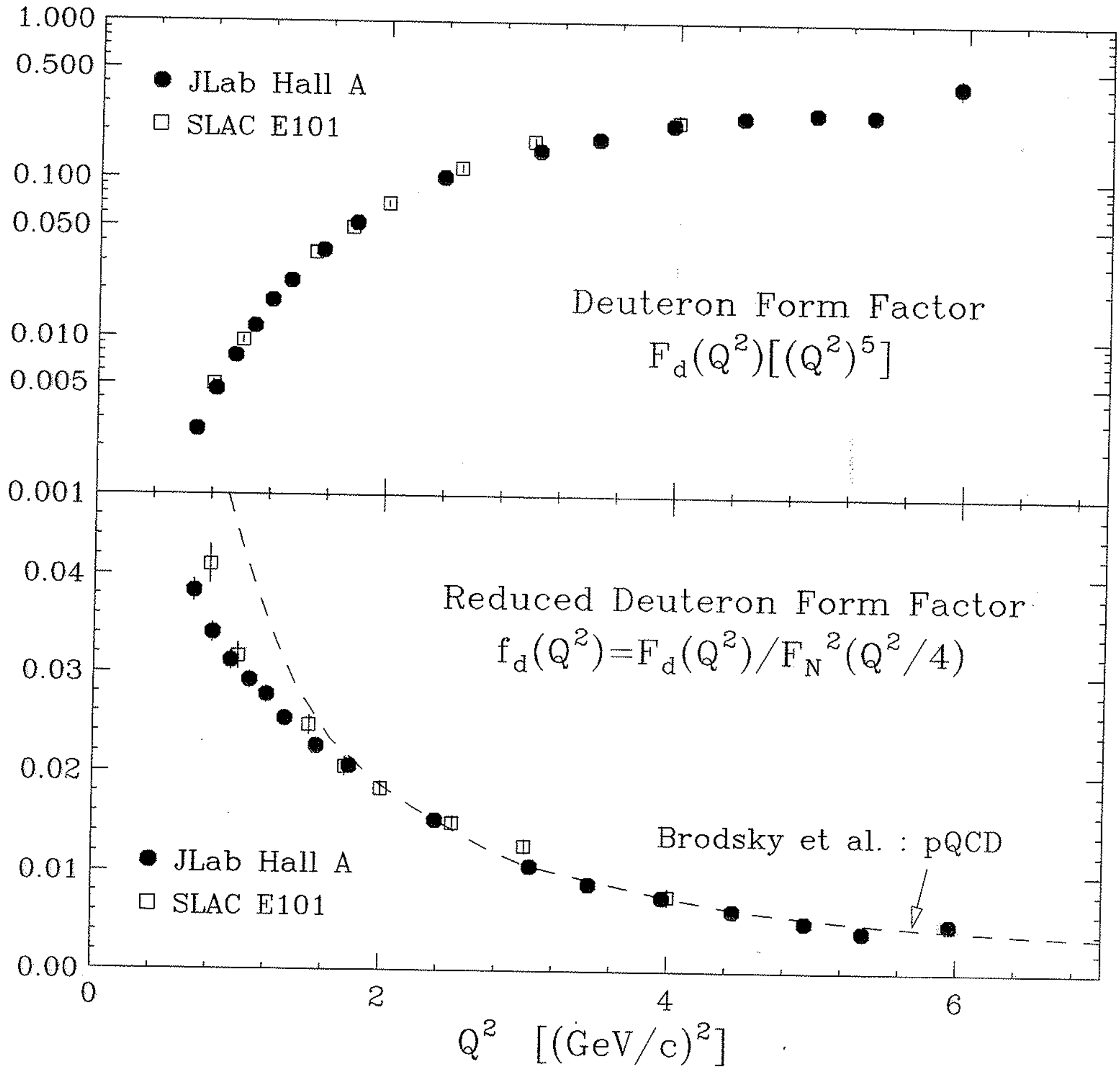
$$F_d(Q^2) \equiv \sqrt{A(Q^2)} \sim (Q^2)^{-5}$$
$$\sqrt{B(Q^2)} \sim (Q^2)^{-6}$$



(a)

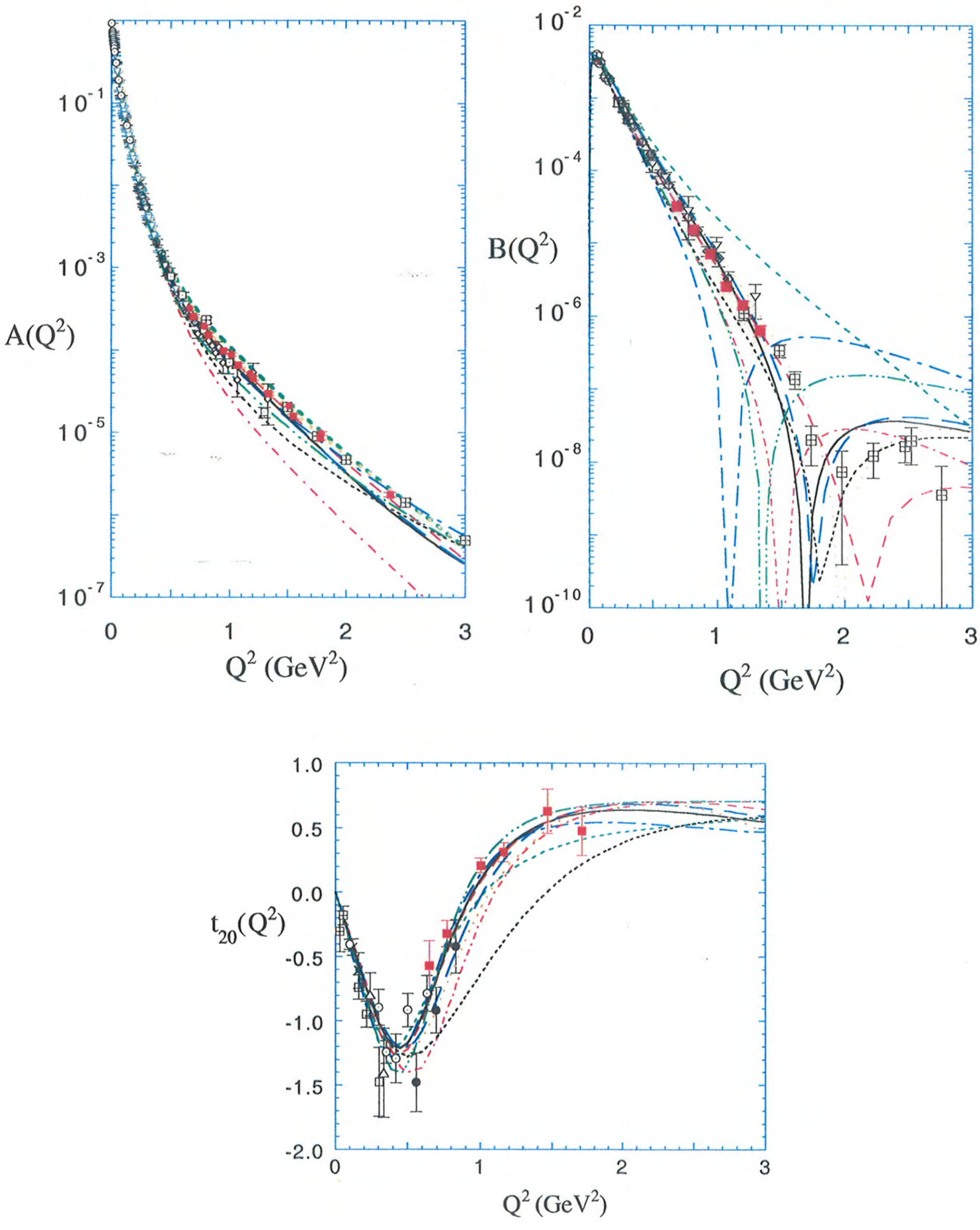


(b)

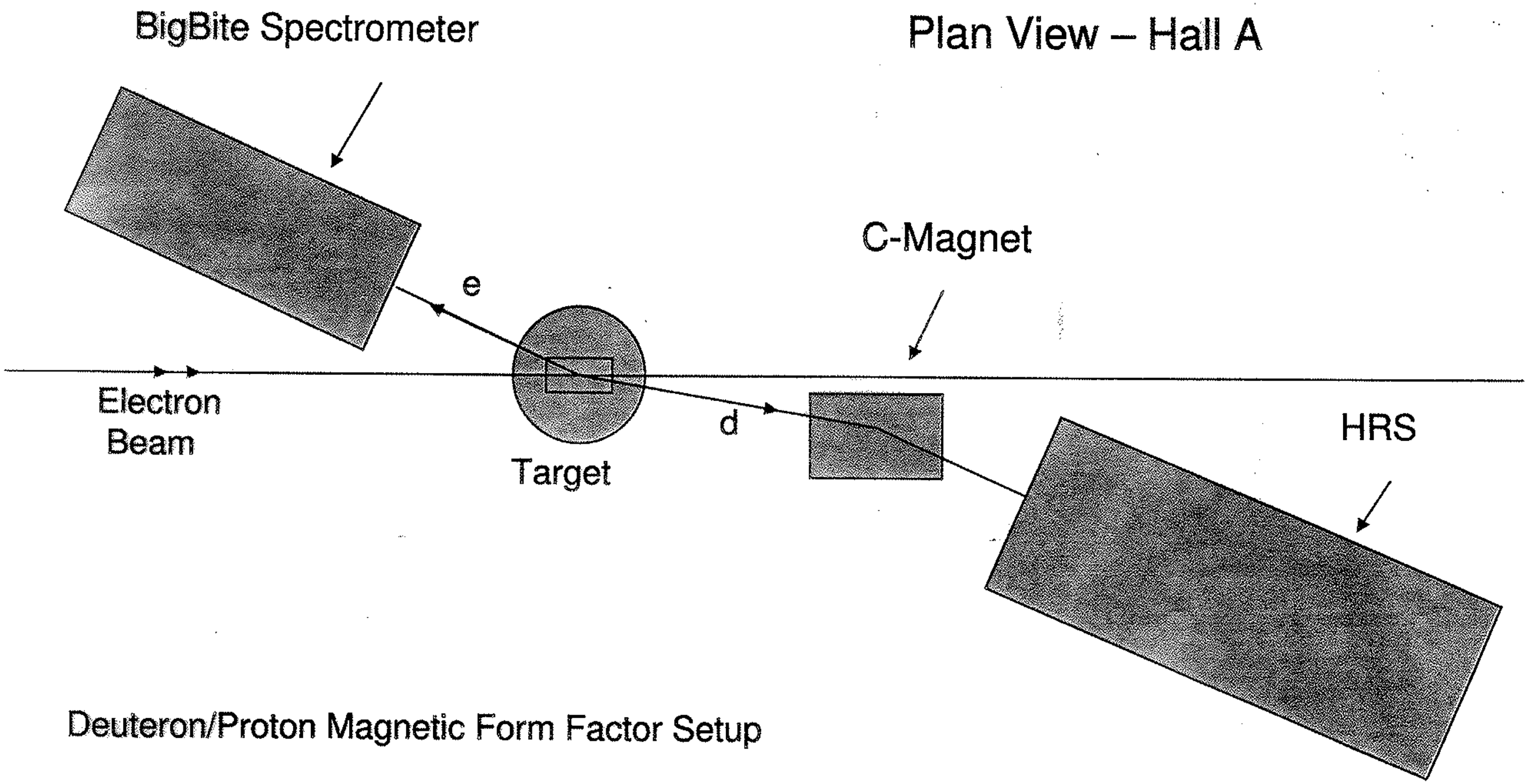


1.000
0.500
0.100
0.050
0.010
0.005
0.001
JLab Hall A
SLAC E101
Deuteron Form Factor
$F_d(Q^2)[(Q^2)^5]$
0.04
0.03
0.02
0.01
0.00
Reduced Deuteron Form Factor
$f_d(Q^2)=F_d(Q^2)/F_N{}^2(Q^2/4)$
Brodsky et al. : pQCD
JLab Hall A
SLAC E101
0
2
4
6
Q^2 $[(GeV/c)^2]$

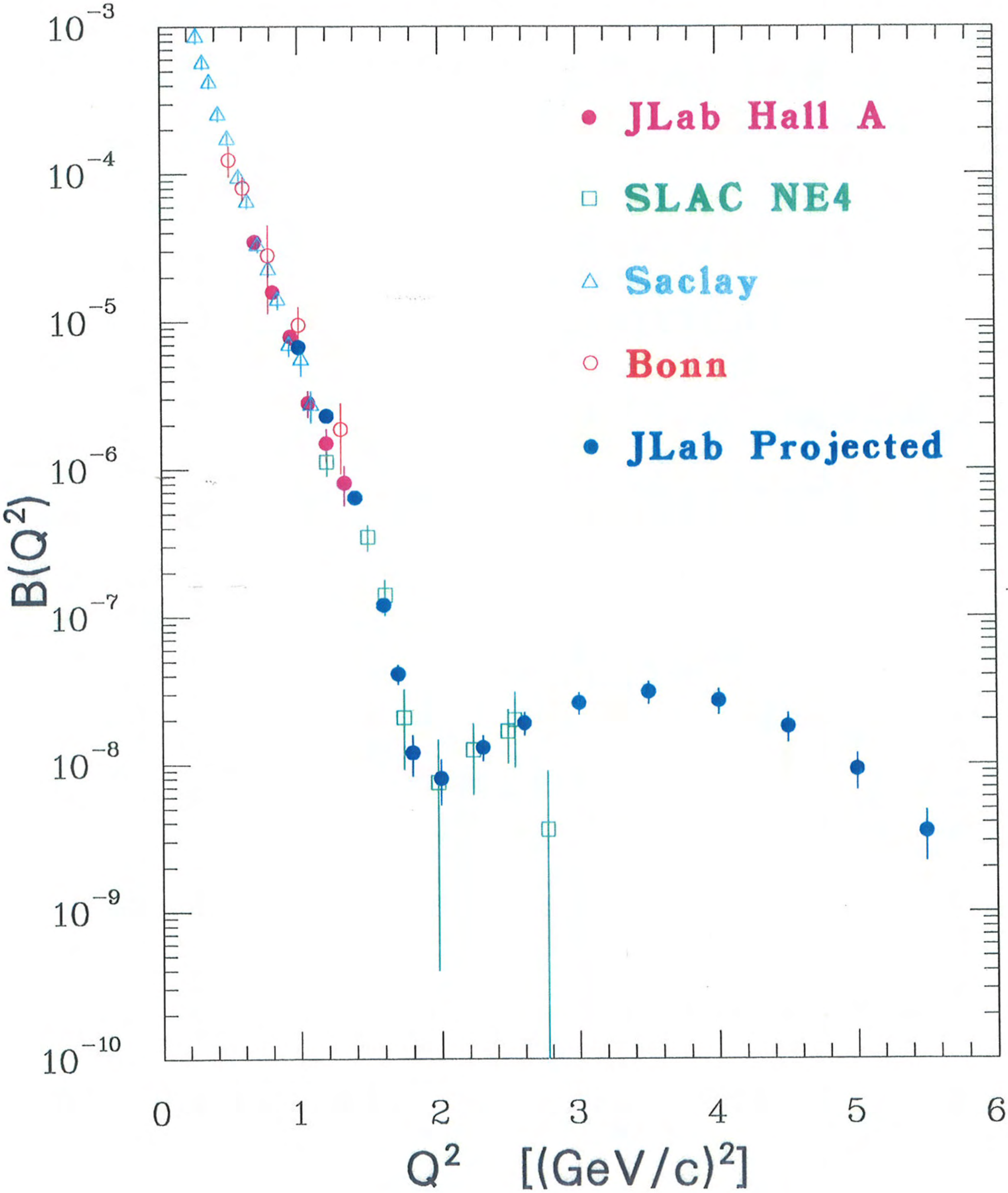
Sensitivity of $B(Q^2)$ to Model Details



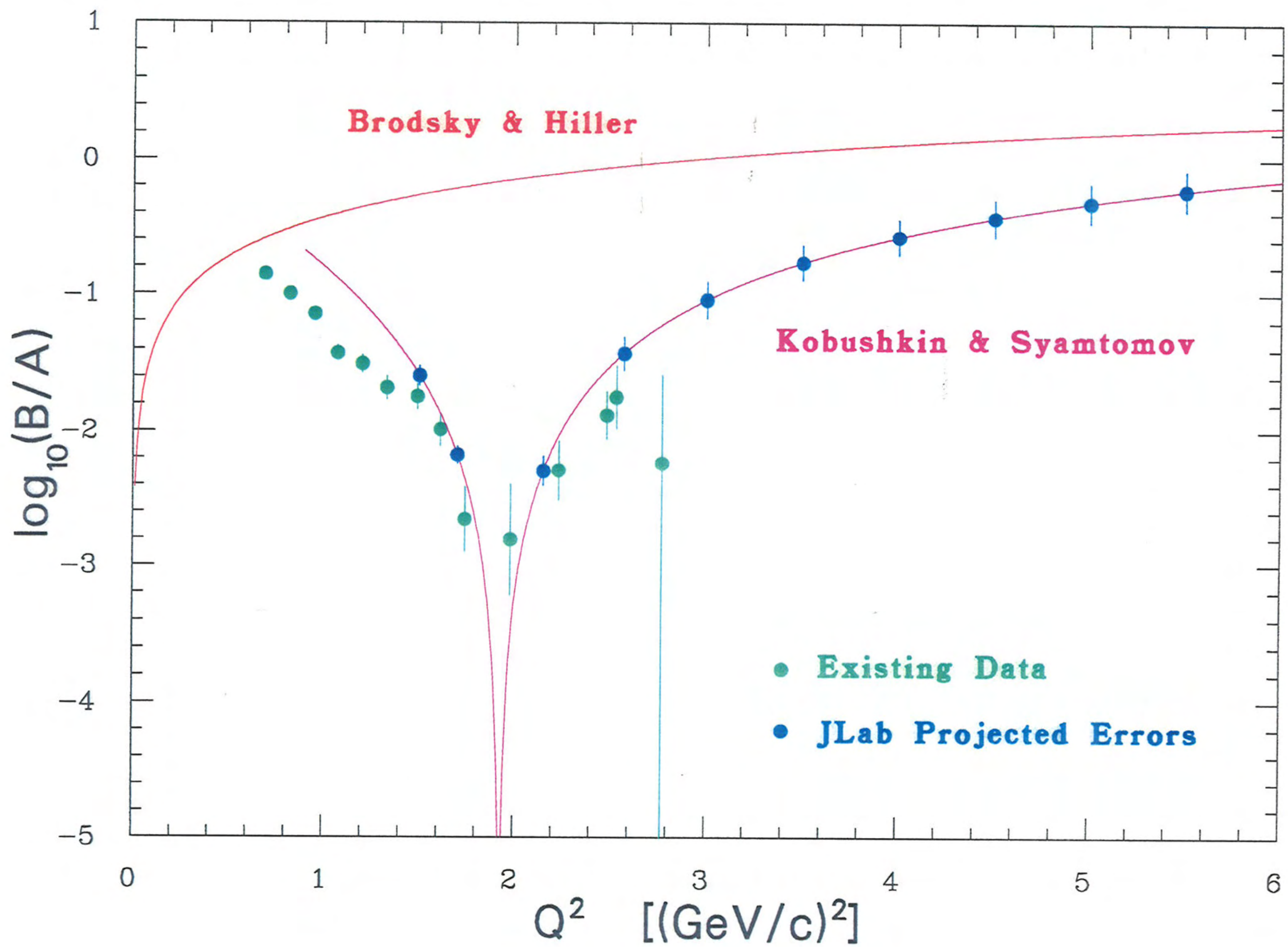
Plan View – Hall A



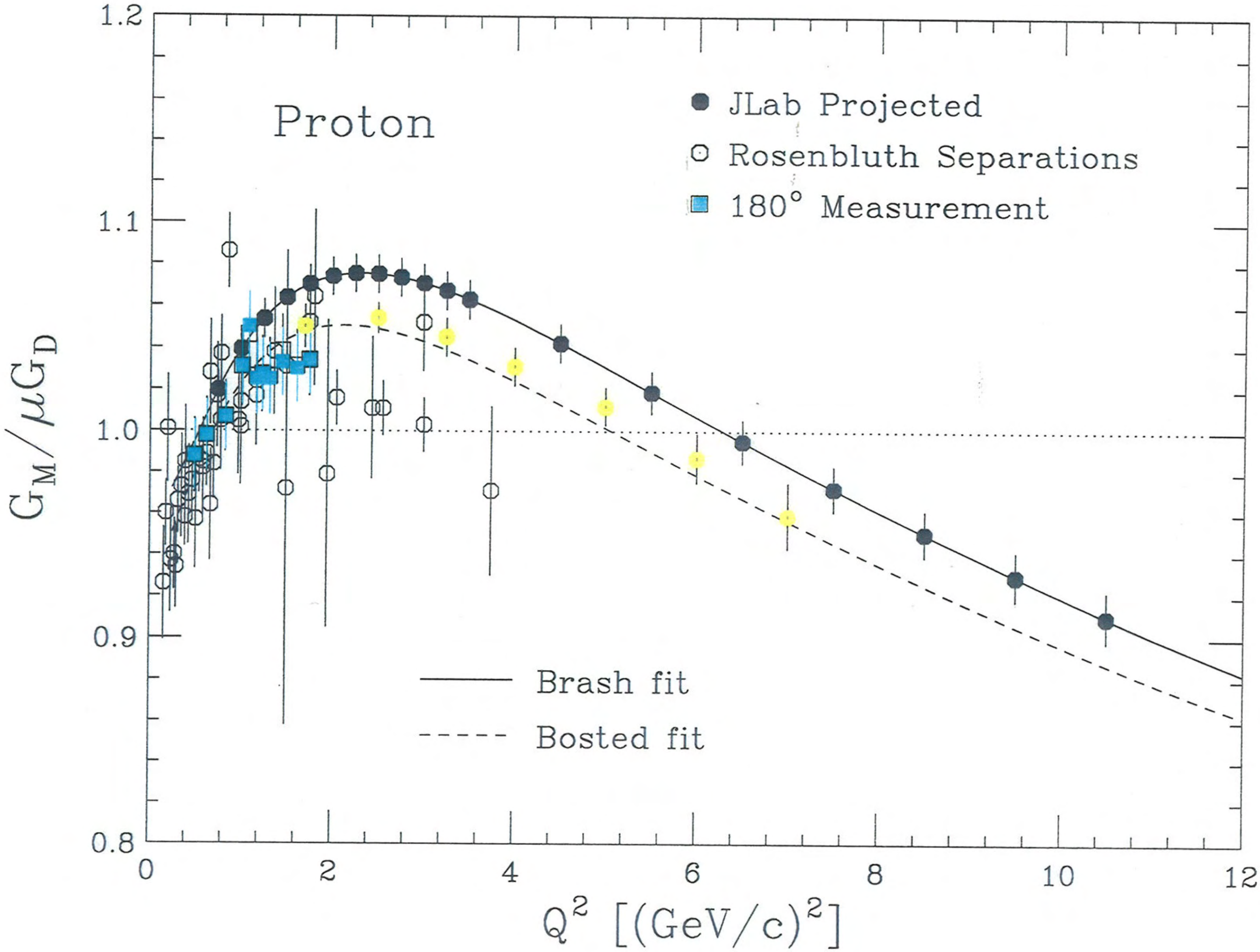
Deuteron/Proton Magnetic Form Factor Setup



JLab Hall A
SLAC NE4
Saclay
Bonn
JLab Projected
B(Q²)
Q² [(GeV/c)²]



1
0
−1
−2
−3
−4
−5
$\log_{10}(B/A)$
0
1
2
3
4
5
6
Q^2 [(GeV/c)^2]
Brodsky & Hiller
Kobushkin & Syamtomov
Existing Data
JLab Projected Errors



Proton
JLab Projected
Rosenbluth Separations
180° Measurement
Brash fit
Bosted fit
$G_M/\mu G_D$
Q^2 [(GeV/c)^2]
1.2
1.1
1.0
0.9
0.8
0
2
4
6
8
10
12

Elastic Electron-Helium/Tritium Scattering

- $^3He/^3H$ charge and magnetic form factors:

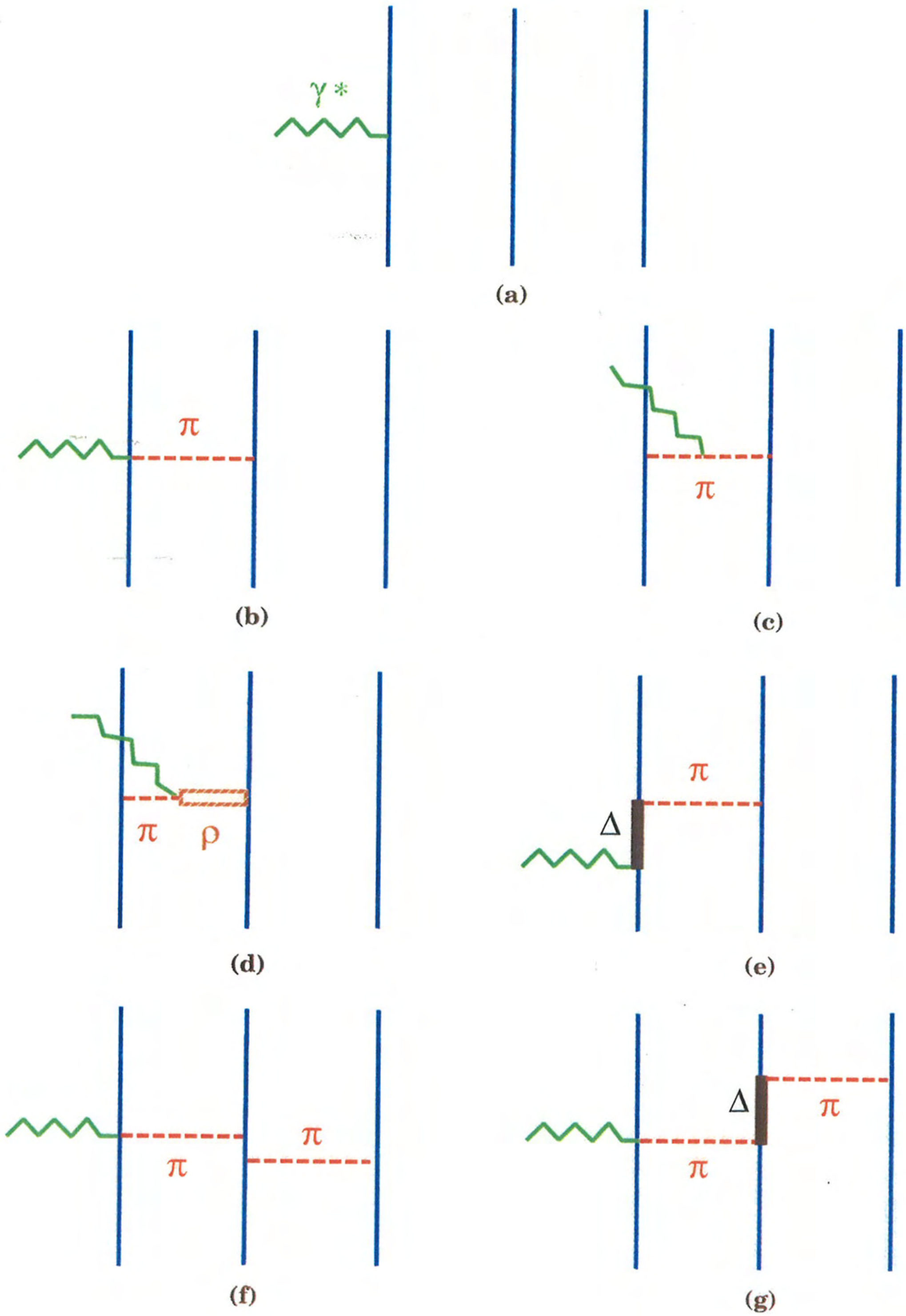
$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2 E'}{4E^3 \sin^4(\frac{\theta}{2})} \left[A(Q^2)\cos^2(\tfrac{\theta}{2}) + B(Q^2)\sin^2(\tfrac{\theta}{2})\right]$$

$$\begin{aligned} A(Q^2) &= \frac{F_C^2(Q^2) + \mu^2\tau F_M^2(Q^2)}{1+\tau} \\ B(Q^2) &= 2\tau\mu^2 F_M^2(Q^2) \end{aligned}$$

$$Q^2 = 4EE'\sin^2(\tfrac{\theta}{2}) \qquad \tau = Q^2/4M^2$$

- ^4He charge form factor

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2 E' \cos^2\left(\frac{\theta}{2}\right)}{4E^3 \sin^4\left(\frac{\theta}{2}\right)} F_C^2(Q^2)$$

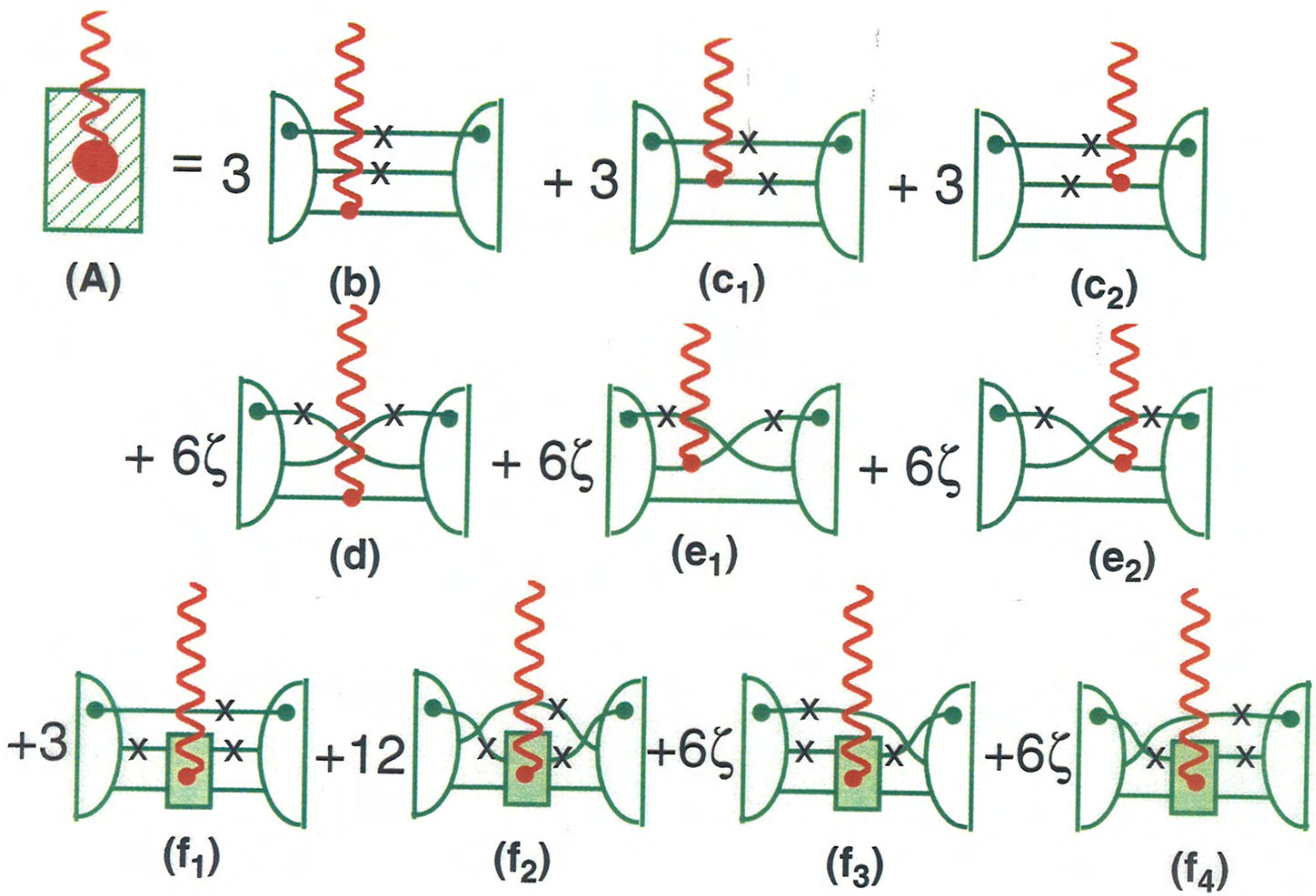

γ*
(a)
π
(b)
π
(c)
π
ρ
(d)
π
Δ
(e)
π
π
(f)
Δ
π
π
(g)

The Three- and Four-Body Standard Model

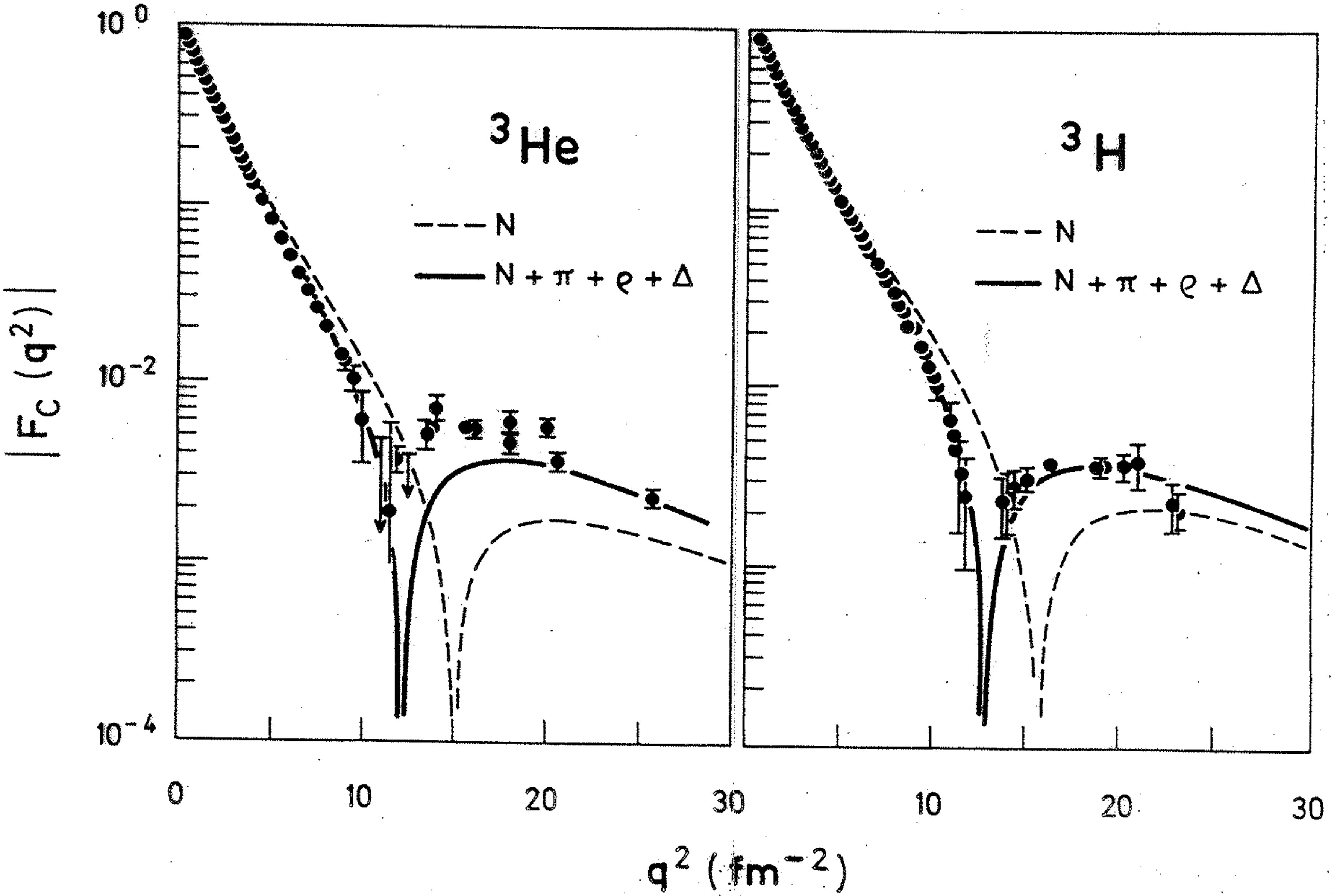
- Non-relativistic Impulse Approximation (IA)
- Solve the nuclear ground state using:
  - Numerical solutions of Faddeev equations
  - Correlated Hyperspherical Harmonics (CCH)
  - Monte Carlo methods
    - Variational Monte Carlo (VMC)
    - Green's Function Monte Carlo (GFMC)
- Relativistic Corrections
  - Darwin - Foldy
  - Spin - orbit
- "Standard Model" to be challenged soon by the three-body covariant Bethe-Salpeter relativistic model by Gross and collaborators!

Beyond the Impulse Approximation

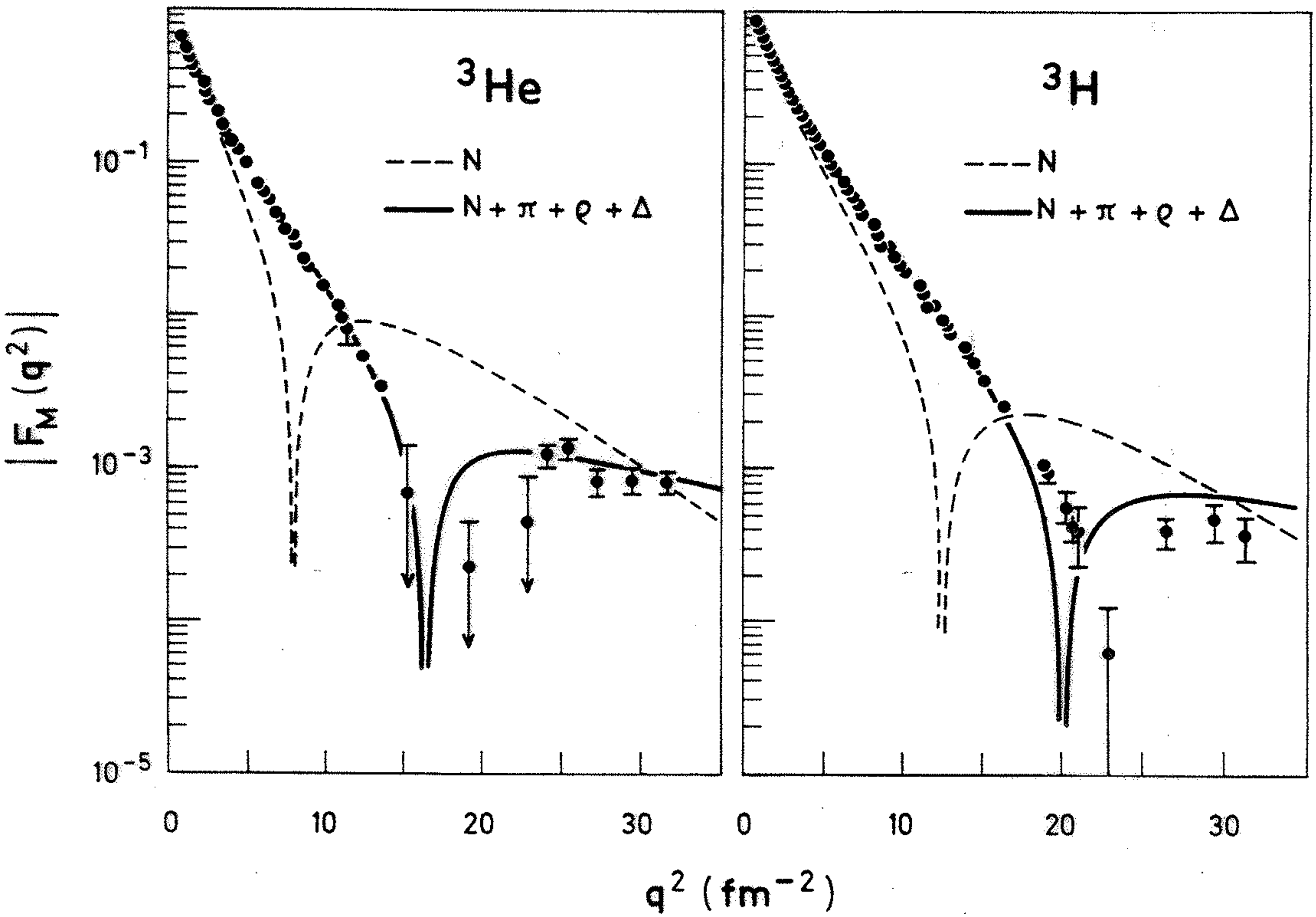
- IA alone cannot describe the few-body form factor data
- Position of first diffraction minimum and height of secondary maximum either overestimated or underestimated
- Inclusion of Meson-Exchange Currents (MEC) brings theory in better agreement with data
  - Model-independent (pion-like exchanges)
  - Model-dependent (transition currents)
- Isobar Configurations calculated to have small effects
- Three-Body Force effects have been shown to be small at low Q^2
- Multi-Quark Admixtures in nuclear wave function



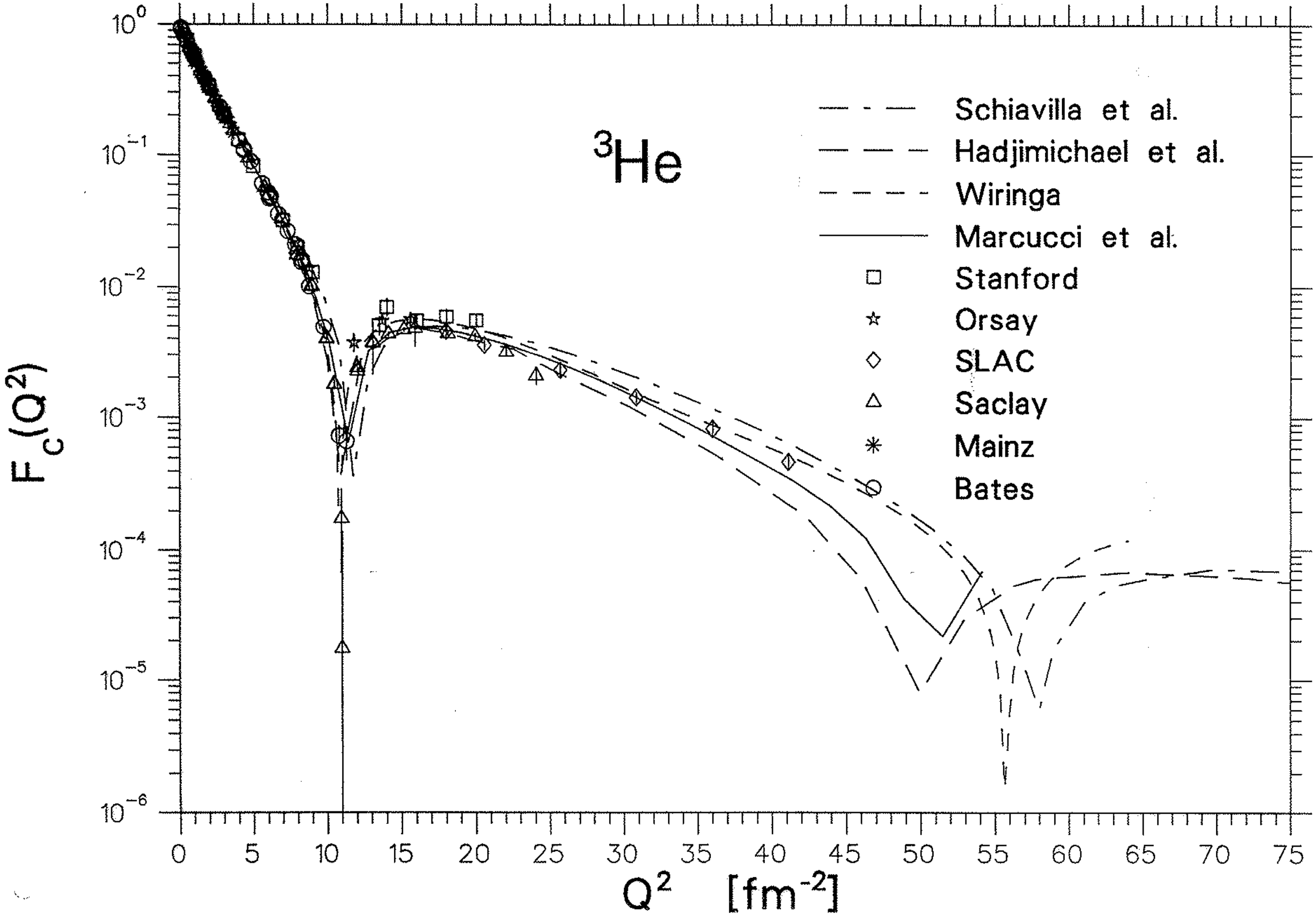
(A)
= 3
(b)
+ 3
(c1)
+ 3
(c2)
+ 6ζ
(d)
+ 6ζ
(e1)
+ 6ζ
(e2)
+3
(f1)
+12
(f2)
+6ζ
(f3)
+6ζ
(f4)



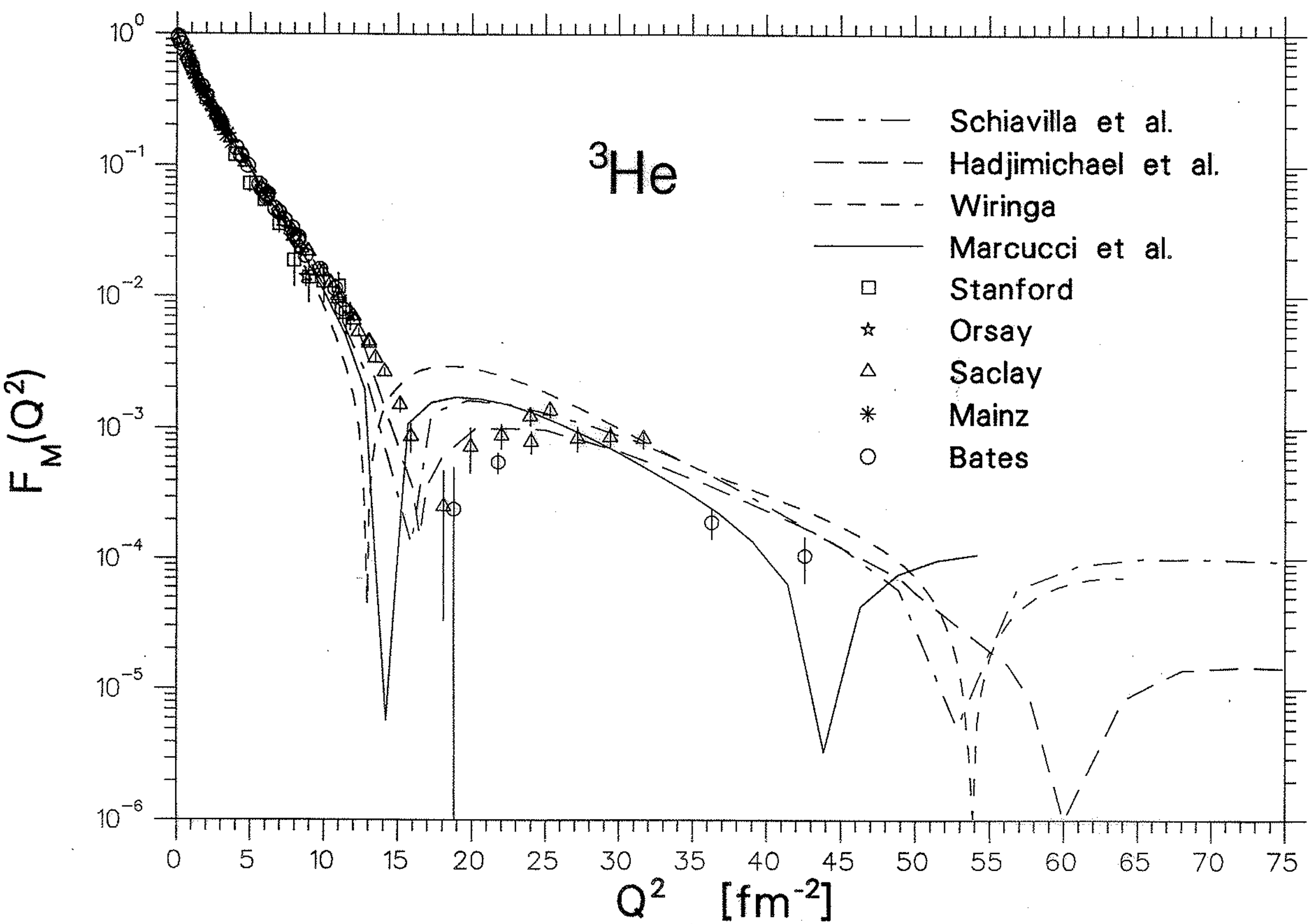
^{3}He
^{3}H
N
N + π + ρ + Δ
$|F_C(q^2)|$
10^0
10^{-2}
10^{-4}
0
10
20
30
q^2 (fm^{-2})



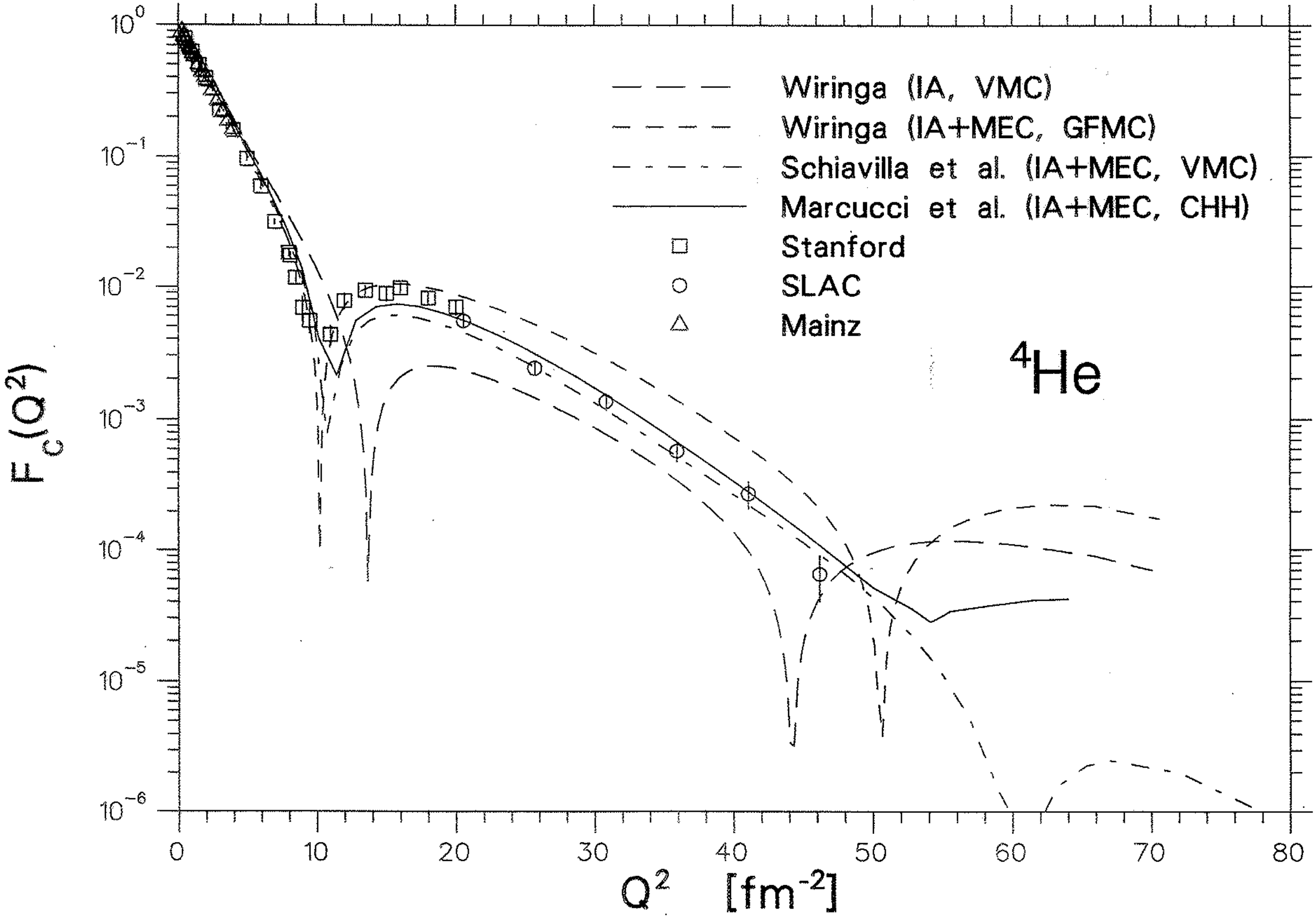
3He
3H
N
N + π + ρ + Δ
10⁻¹
10⁻³
10⁻⁵
|F_M (q²)|
0
10
20
30
q² (fm⁻²)



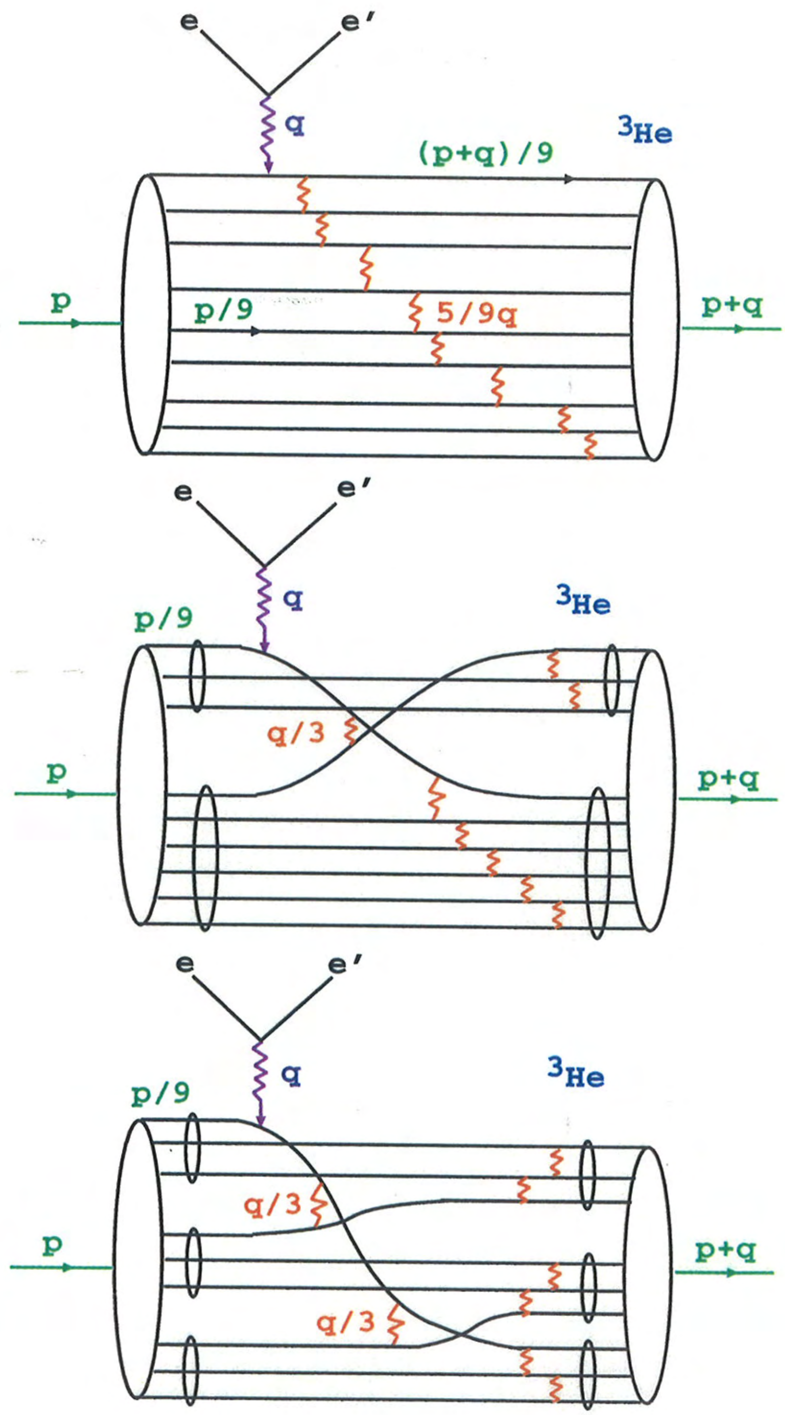
^{3}He
Schiavilla et al.
Hadjimichael et al.
Wiringa
Marcucci et al.
Stanford
Orsay
SLAC
Saclay
Mainz
Bates
$F_c(Q^2)$
Q^2 [fm^{-2}]



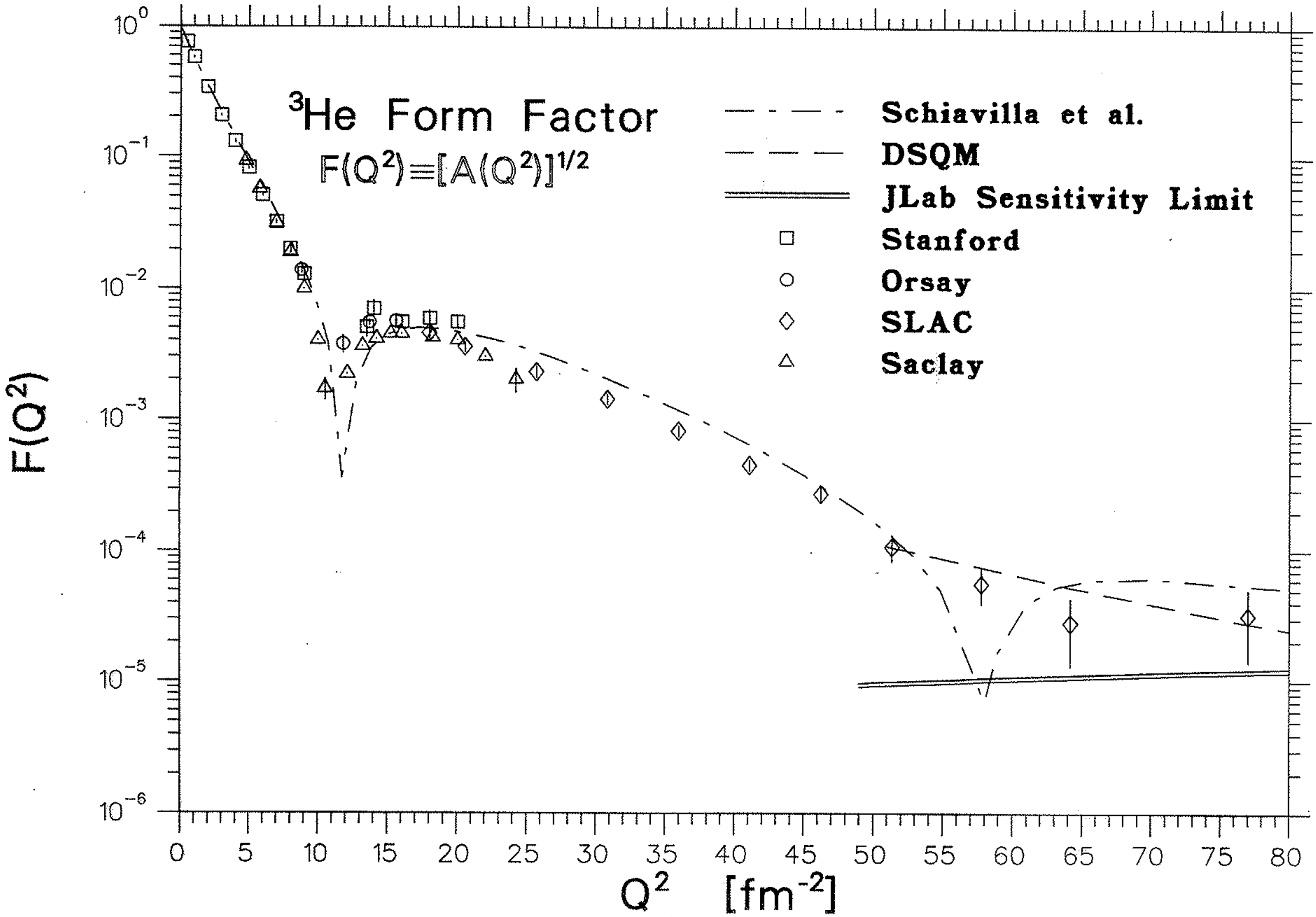
^{3}He
Schiavilla et al.
Hadjimichael et al.
Wiringa
Marcucci et al.
Stanford
Orsay
Saclay
Mainz
Bates
$F_M(Q^2)$
Q^2 [fm^{-2}]



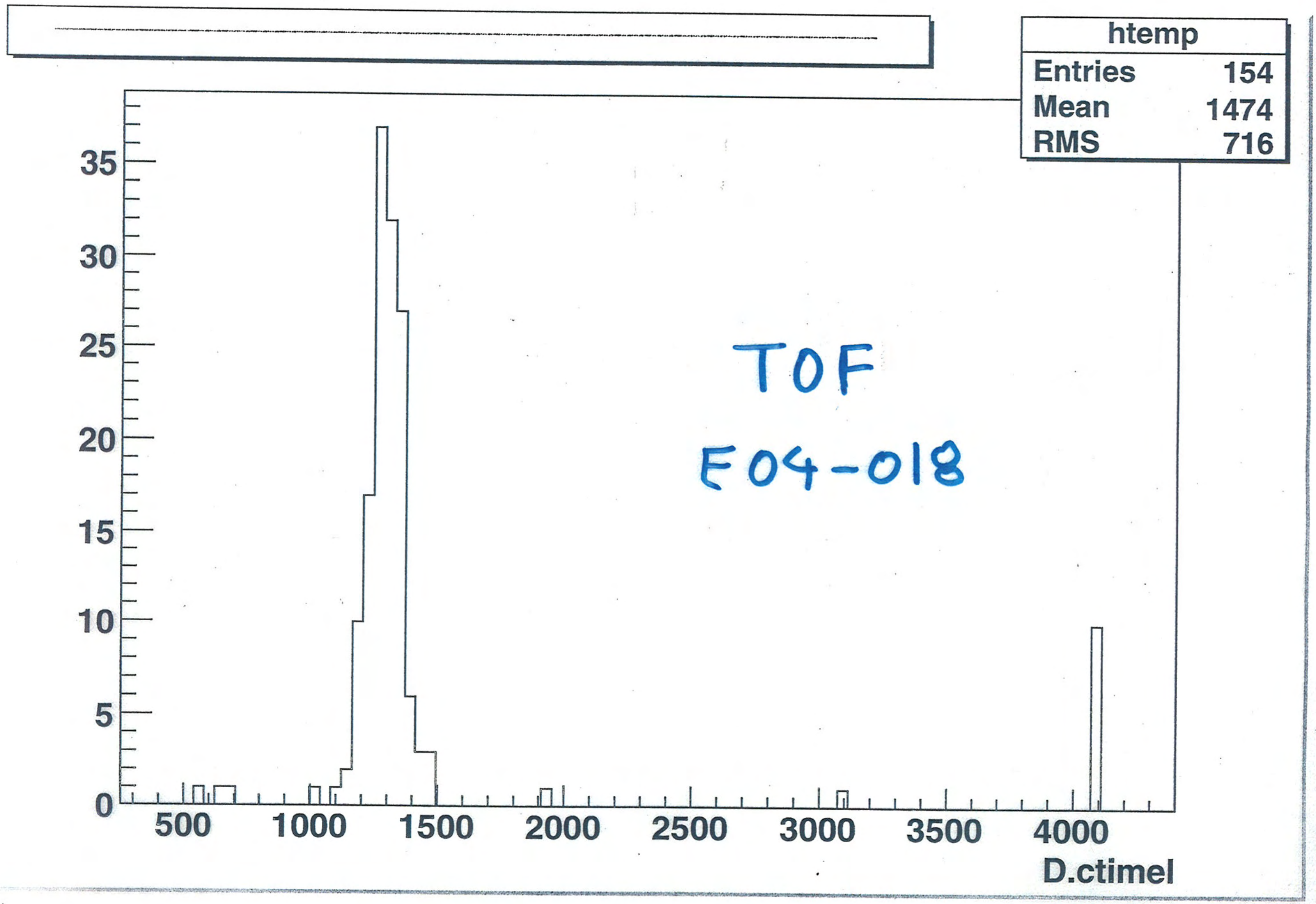
Wiringa (IA, VMC)
Wiringa (IA+MEC, GFMC)
Schiavilla et al. (IA+MEC, VMC)
Marcucci et al. (IA+MEC, CHH)
Stanford
SLAC
Mainz
^{4}He
$F_C(Q^2)$
Q^2 [fm^{-2}]
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
10
20
30
40
50
60
70
80



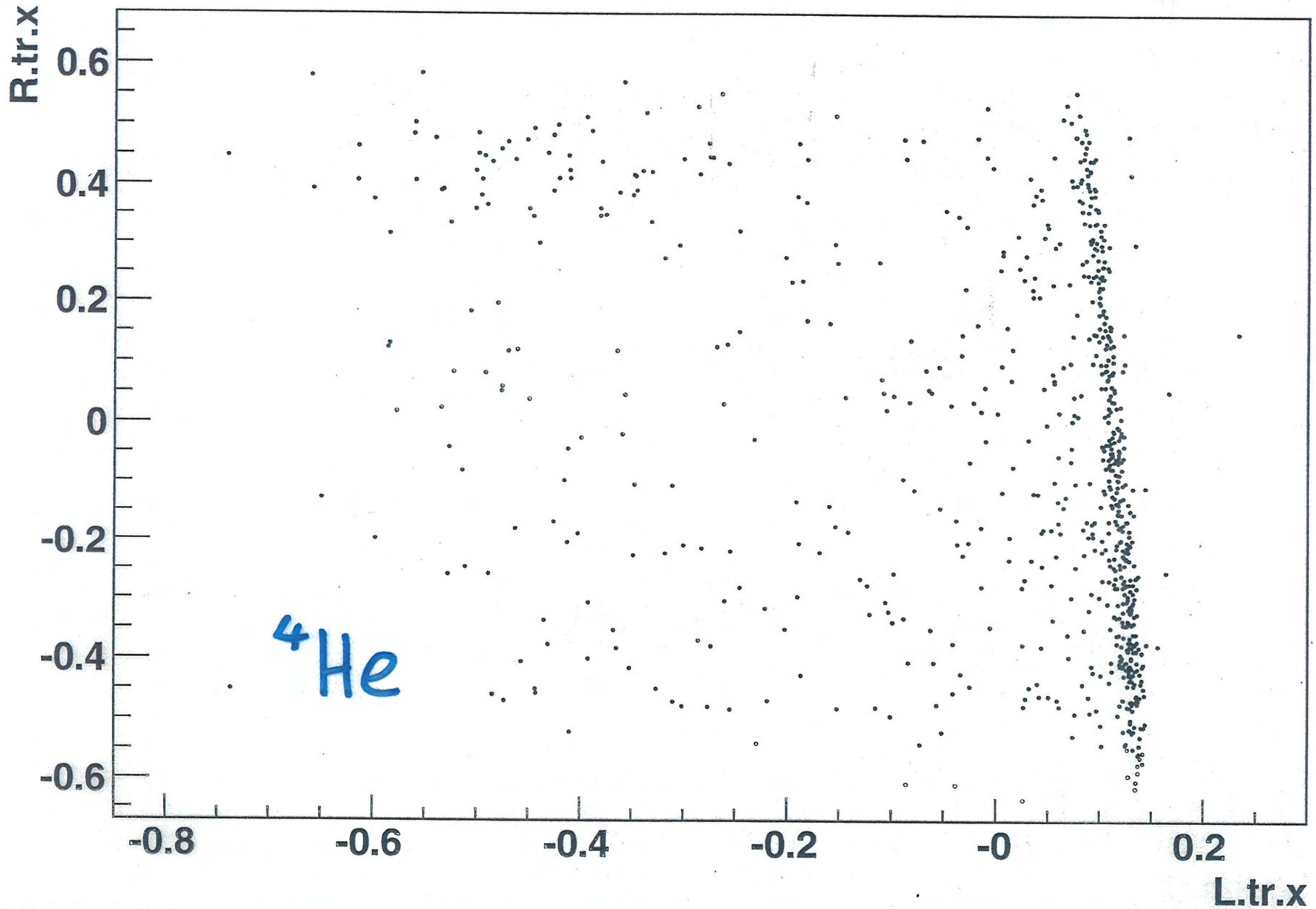
e
e′
q
³He
(p+q)/9
p
p/9
5/9q
p+q
e
e′
q
³He
p/9
q/3
p
p+q
e
e′
q
³He
p/9
q/3
p
p+q
q/3

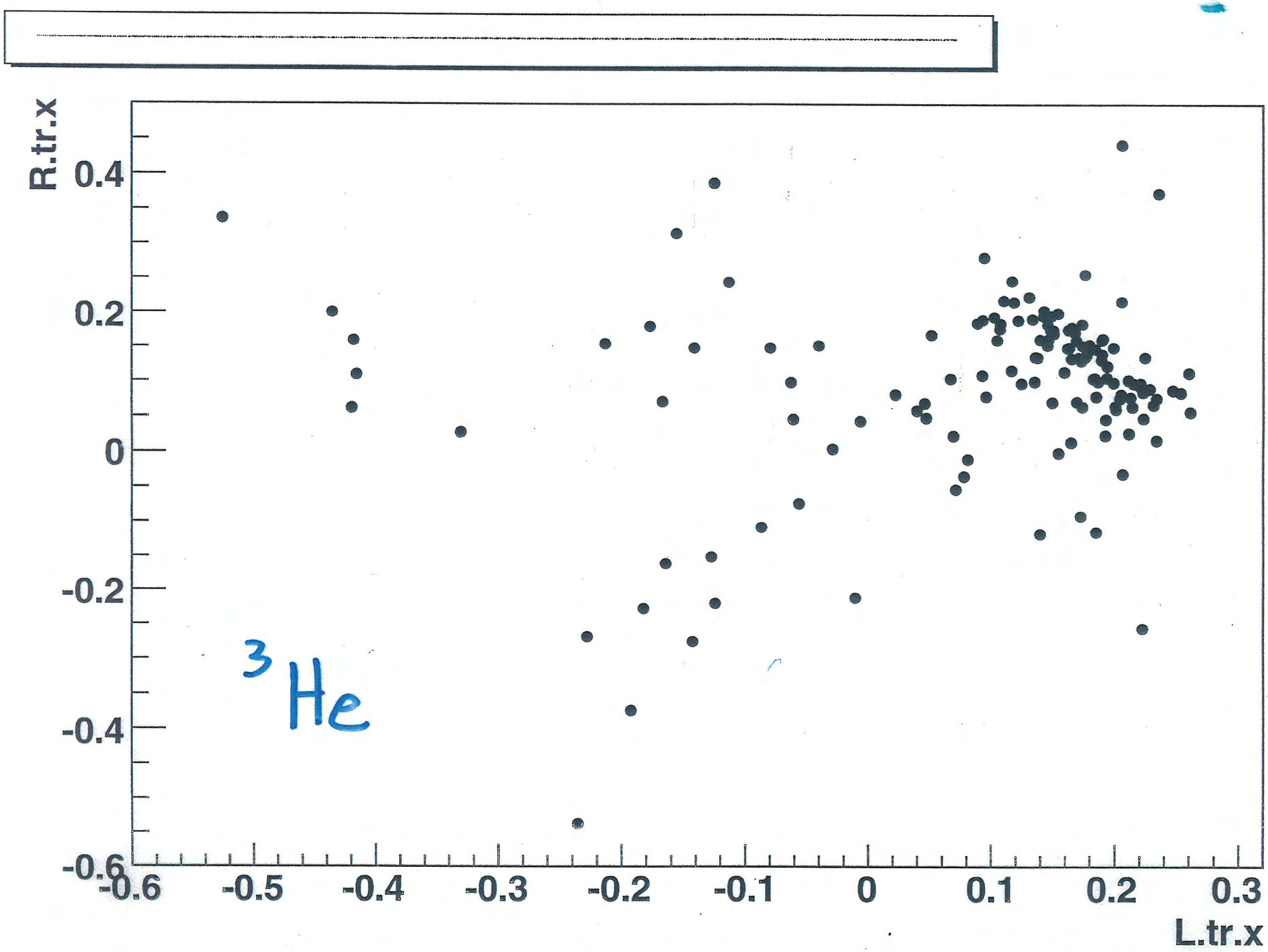


³He Form Factor
F(Q²)≡[A(Q²)]^1/2
Schiavilla et al.
DSQM
JLab Sensitivity Limit
Stanford
Orsay
SLAC
Saclay
F(Q²)
Q² [fm⁻²]
10⁰
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

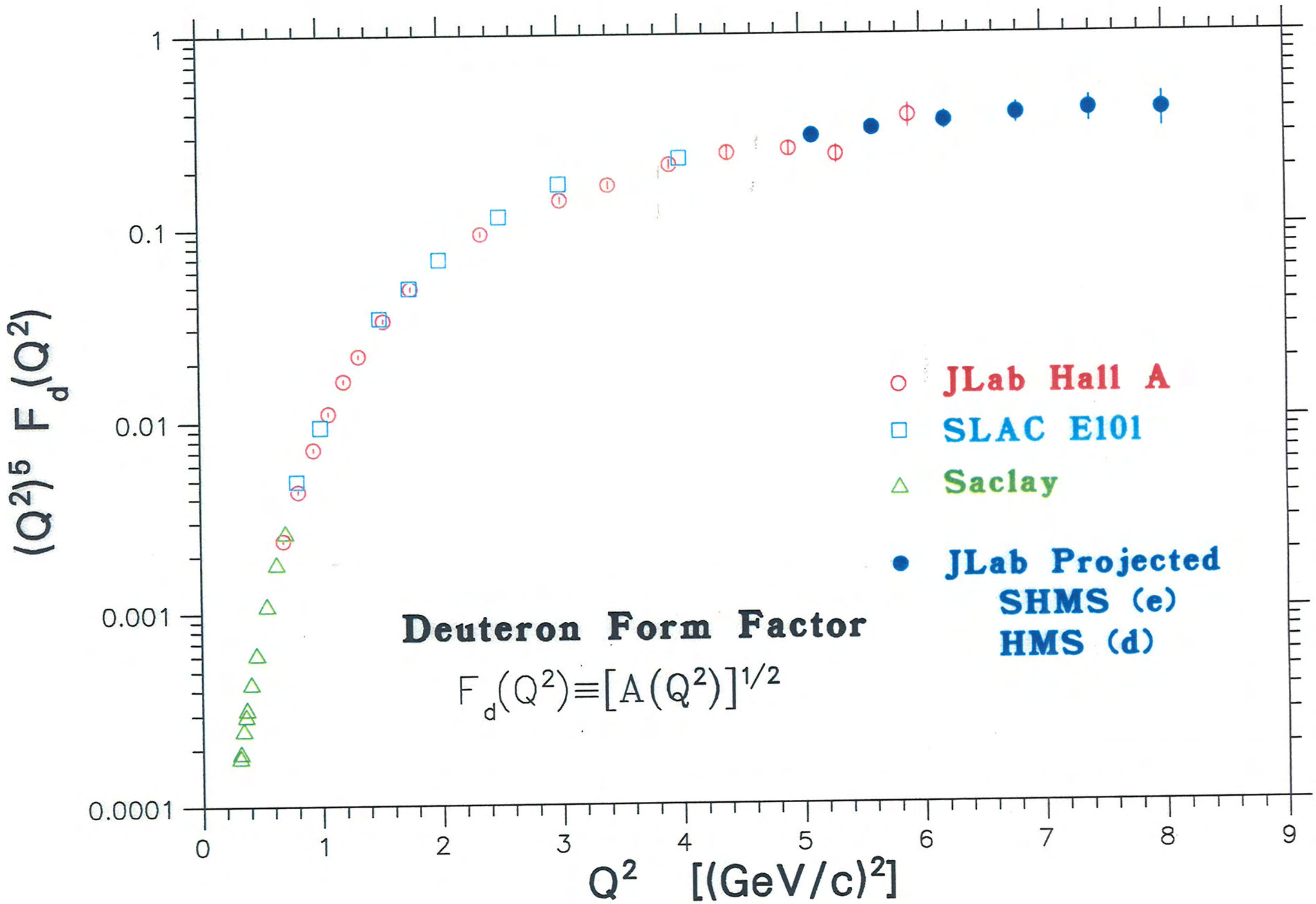

htemp
Entries 154
Mean 1474
RMS 716
0
5
10
15
20
25
30
35
500
1000
1500
2000
2500
3000
3500
4000
D.ctimel
TOF
E04-018

```
R.tr.x:L.tr.x {((((((((D.evtypebits&0x20)==0x20)||((D.evtypebits&0x40)==0x40&&(D.evtypebits&0x20)!=0x20))&&(D.edtpl==0))&&((L.prl1.asum_p+L.prl2.as um_p)>3000))&&(abs(R.tr.x)<1&&abs(L.tr.x)<1))&&(L.cer.asum_c>400))&&(D.ctimel>1300&&D.ctimel<2300))&&(((R.s1.la_p[0]+R.s1.ra_p[0]>1600)||(R.s1.la_p[1]+R.s1.ra_p[1]>1600)||(R.s1.la_p[2]+R.s1.ra_p[2]>1600)||(R.s1.la_p[3]+R.s1. ra_p[3]>1600)||(R.s1.la_p[4]+R.s1.ra_p[4]>1600)||(R.s1.la_p[5]+R.s1.ra_p[5]>1600)))}
```

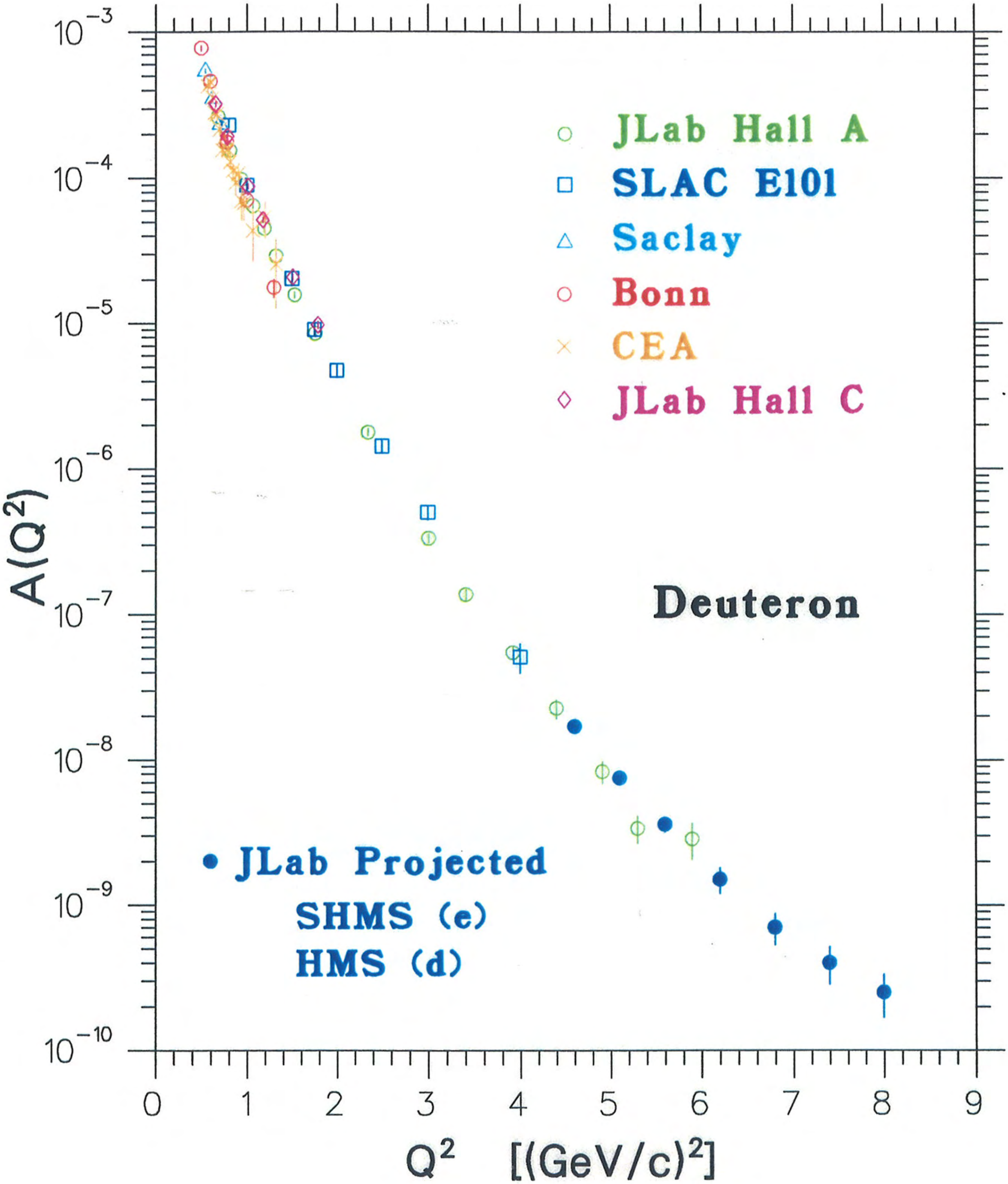




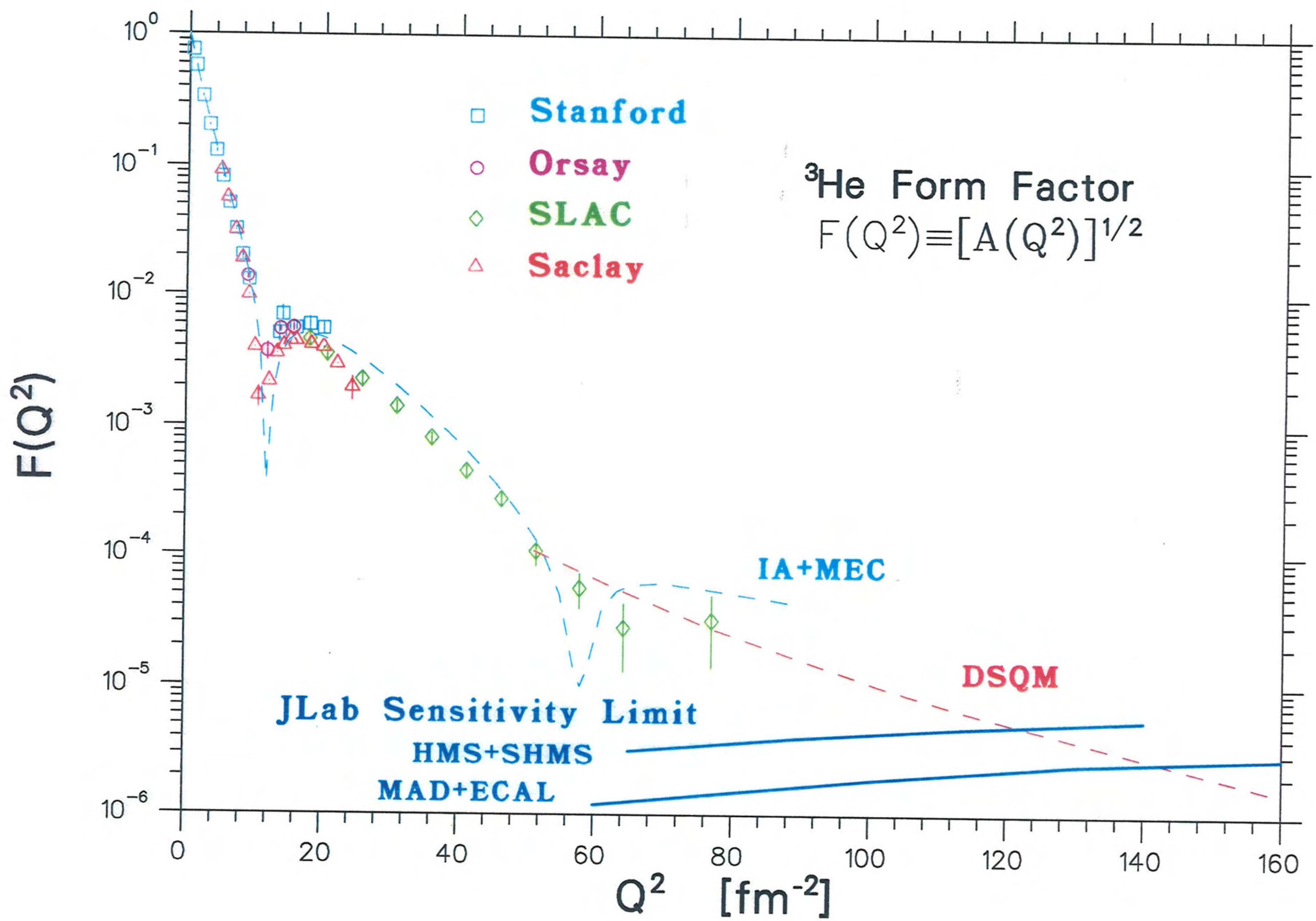
R.tr.x
0.4
0.2
0
-0.2
-0.4
-0.6
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
L.tr.x
^3He



1
0.1
0.01
0.001
0.0001
0
1
2
3
4
5
6
7
8
9
(Q²)⁵ F_d(Q²)
Q² [(GeV/c)²]
JLab Hall A
SLAC E101
Saclay
JLab Projected
SHMS (e)
HMS (d)
Deuteron Form Factor
F_d(Q²)≡[A(Q²)]^{1/2}



JLab Hall A
SLAC E101
Saclay
Bonn
CEA
JLab Hall C
Deuteron
JLab Projected
SHMS (e)
HMS (d)
A(Q²)
Q² [(GeV/c)²]
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
0
1
2
3
4
5
6
7
8
9



F(Q²)
Stanford
Orsay
SLAC
Saclay
³He Form Factor
F(Q²)≡[A(Q²)]^1/2
IA+MEC
DSQM
JLab Sensitivity Limit
HMS+SHMS
MAD+ECAL
Q² [fm⁻²]
10⁰
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
0
20
40
60
80
100
120
140
160

Summary

- JLab is the ideal place to improve the few-body form factor data taken over previous 40 years and extend them to the higher Q^2 possible
- JLab has provided deuteron $A(Q^2)$ and t_{20} data in qualitative agreement with relativistic calculations
- High Q^2 measurements of deuteron $B(Q^2)$ are essential for a complete theoretical description
- JLab is in progress of measuring the three- and four-body helium form factors
- Expected ^{3}He and ^{4}He data will be pivotal for the establishment of a canonical meson nucleon model
- JLab 12 GeV upgrade offers opportunity for even larger Q^2 measurements to test quark scaling and pQCD