Pion-Nucleon Light-Cone Distribution Amplitudes

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Outline

Why πN DAs?

1. Deep Inelastic Pion-Electroproduction:

   See recent articles:

   - V. M. Braun, D. Y. Ivanov, A. Lenz and A. Peters, "Deep inelastic pion electroproduction at threshold," Phys. Rev. D **75** (2007) 014021 [arXiv:hep-ph/0611386].
   - J. P. Lansberg, B. Pire and L. Szymanowski, "Hard exclusive electroproduction of a pion in the backward region," Phys. Rev. D **75** (2007) 074004 [arXiv:hep-ph/0701125].

2. Handle to learn more about nucleon DAs themselves

Motivation

- Polyakov, Pobylitsa, Strikman 2006

$$|p\uparrow\rangle = \frac{\phi_s(x)}{\sqrt{6}}|2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle + \frac{\phi_a(x)}{\sqrt{2}}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|p\uparrow\pi^0\rangle = \frac{\phi_s(x)}{2\sqrt{6}f_\pi}|6u_\uparrow d_\downarrow u_\uparrow + u_\uparrow u_\downarrow d_\uparrow + d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2\sqrt{2}f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|n\uparrow\pi^+\rangle = \frac{\phi_s(x)}{\sqrt{12}f_\pi}|2u_\uparrow d_\downarrow u_\uparrow - 3u_\uparrow u_\downarrow d_\uparrow - 3d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

- Rewrite in operator language
- Extend to higher twists

Leading-Twist Nucleon DAs

- Three-quark matrix element:

$$4\,\langle 0|\,\varepsilon^{ijk} u^i_\alpha(a_1 z) u^j_\beta(a_2 z) d^k_\gamma(a_3 z)\,|p(P,\lambda)\rangle_{\mathrm{twist}-3} = V_1^p\,(v_1)_{\alpha\beta,\gamma} + A_1^p\,(a_1)_{\alpha\beta,\gamma} + T_1^p\,(t_1)_{\alpha\beta,\gamma}$$

with

$$(v_1)_{\alpha\beta,\gamma} = (\not{p} C)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma$$
$$(a_1)_{\alpha\beta,\gamma} = (\not{p}\gamma_5 C)_{\alpha\beta}\, N^+_\gamma$$
$$(t_1)_{\alpha\beta,\gamma} = (i\sigma_{\perp p} C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^+\right)_\gamma$$

Leading-Twist Nucleon DAs

- Symmetry between the two up quarks:

$$V_1(1,2,3) = V_1(2,1,3), \; A_1(1,2,3) = -A_1(2,1,3)$$

- Isospin:

$$2T_1(1,2,3) = [V_1 - A_1](1,3,2) + [V_1 - A_1](2,3,1)$$

- In Brodsky notation $V_1 = \phi_s(x)$ and $A_1 = \phi_a(x)$
- V_1, A_1 can be expanded in contributions of operators with increasing conformal spin
- $\phi_N = [V_1 - A_1](1,2,3) =$Nucleon DA
- T_1 is not independent

Leading-Twist Pion-Nucleon DAs

- Aim: Find a representation of $V_1^{\pi N}$ etc. in terms of nucleon DAs
- Basic Ansatz:

$$4\,\langle 0|\,\varepsilon^{ijk} u^i_\alpha(a_1 z) u^j_\beta(a_2 z) d^k_\gamma(a_3 z)\,|\pi(k) N(P,\lambda)\rangle_{\mathrm{twist-3}} =$$

$$\text{-i/f}_\pi (\gamma_5)_{\delta\gamma} \{ V_1^{\pi N}\,(v_1)_{\alpha\beta,\delta} + A_1^{\pi N}\,(a_1)_{\alpha\beta,\delta} + T_1^{\pi N}\,(t_1)_{\alpha\beta,\delta} \}$$

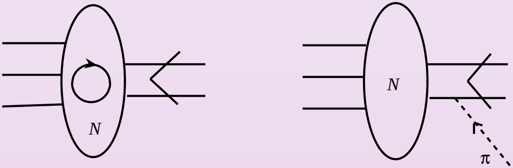
Soft-Pion theorem: Basic concepts

Take the Soft-Pion Theorem (SPT):

$$\langle 0|O|\pi^a(k)N_i(P,h)\rangle = -\frac{i}{f_\pi}\langle 0|[Q_5^a, O]|N_i(P,h)\rangle +$$

Bremsstrahlungs contributions

$$Q_5^a = \int d^3x\, \bar{q}(x)\gamma_0\gamma_5\frac{\tau^a}{2}q(x)$$



Leading-Twist Pion-Nucleon DAs

- For $n\pi^+$ specify:

$$\langle 0|O|\pi^+(k)n(P,h)\rangle = -\frac{i}{f_\pi}\langle 0|[Q_5^+,O]|n(P,h)\rangle$$

- Define trilocal operator as:

$$O^{uud}_{\alpha\beta\gamma} = \epsilon^{ijk} u^i_\alpha(a_1 z) u^j_\beta(a_2 z) d^k_\gamma(a_3 z)$$

together with

$$[Q_5^1, O^{uud}_{\alpha\beta\gamma}] = -\frac{1}{2}\{(\gamma_5)_{\alpha\lambda} O^{ddu}_{\lambda\gamma\beta} + (\gamma_5)_{\beta\lambda} O^{ddu}_{\lambda\gamma\alpha} + (\gamma_5)_{\gamma\lambda} O^{uuu}_{\alpha\beta\lambda}\}$$

$$[Q_5^2, O^{uud}_{\alpha\beta\gamma}] = \frac{i}{2}\{(\gamma_5)_{\alpha\lambda} O^{ddu}_{\lambda\gamma\beta} + (\gamma_5)_{\beta\alpha} O^{ddu}_{\lambda\gamma\alpha} - (\gamma_5)_{\gamma\lambda} O^{uuu}_{\alpha\beta\lambda}\}$$

Results for $n\pi^+$

The final comparison with the Ansatz leads us to:

$$V_1^{n\pi^+}(1,2,3) = \frac{1}{\sqrt{2}}\Big\{V_1^n(1,3,2) + V_1^n(1,2,3) + V_1^n(2,3,1) + A_1^n(1,3,2) + A_1^n(2,3,1)\Big\},$$

$$A_1^{n\pi^+}(1,2,3) = -\frac{1}{\sqrt{2}}\Big\{V_1^n(3,2,1) - V_1^n(1,3,2) + A_1^n(2,1,3) + A_1^n(2,3,1) + A_1^n(3,1,2)\Big\},$$

$$T_1^{n\pi^+}(1,2,3) = \frac{1}{2\sqrt{2}}\Big\{A_1^n(2,3,1) + A_1^n(1,3,2) - V_1^n(2,3,1) - V_1^n(1,3,2)\Big\}.$$

Results for $n\pi^+$

- The amplitudes have the natural symmetry property

$$V_1^{n\pi^+}(1,2,3) = V_1^{n\pi^+}(2,1,3),\ A_1^{n\pi^+}(1,2,3) = -A_1^{n\pi^+}(2,1,3)$$

- They are always built nucleon DAs of the same twist
- But they do not fulfill the Isospin relation anymore as they contain both $I = 1/2$ and $I = 3/2$ contributions

Results for $p\pi^0$

- The calculation for $p\pi^0$ DAs is much easier as we do not need any Fierz transform

$$[Q_5^3, O^{uud}_{\alpha\beta\gamma}] = -\frac{1}{2}\{(\gamma_5)_{\alpha\lambda}O^{uud}_{\lambda\beta\gamma} + (\gamma_5)_{\beta\lambda}O^{uud}_{\alpha\lambda\gamma} - (\gamma_5)_{\gamma\lambda}O^{uud}_{\alpha\beta\lambda}\}$$

$$V_1^{p\pi^0}(1,2,3) = \frac{1}{2}V_1^p(1,2,3)\,,$$

$$A_1^{p\pi^0}(1,2,3) = \frac{1}{2}A_1^p(1,2,3)\,,$$

$$T_1^{p\pi^0}(1,2,3) = \frac{3}{2}T_1^p(1,2,3)\,.$$

Results for $p\pi^+$

- $p\pi^+$ DAs are built of $n\pi^+$ DAs and nucleon DAs

$$O^{uuu}_{\alpha\gamma\beta} = \epsilon^{ijk} u^i_\alpha(a_1 z) u^j_\beta(a_2 z) u^k_\gamma(a_3 z)$$

$$[Q^1_5, O^{uuu}_{\alpha\beta\gamma}(z)] = -\frac{1}{2}\left\{(\gamma_5)_{\alpha\lambda} O^{duu}_{\lambda\beta\gamma}(z) + (\gamma_5)_{\beta\lambda} O^{udu}_{\alpha\lambda\gamma}(z) + (\gamma_5)_{\gamma\lambda} O^{uud}_{\alpha\beta\lambda}(z)\right\}$$

$$[Q^2_5, O^{uuu}_{\alpha\beta\gamma}(z)] = \frac{i}{2}\left\{(\gamma_5)_{\alpha\lambda} O^{duu}_{\lambda\beta\gamma}(z) + (\gamma_5)_{\beta\lambda} O^{udu}_{\alpha\lambda\gamma}(z) + (\gamma_5)_{\gamma\lambda} O^{uud}_{\alpha\beta\lambda}(z)\right\}$$

$$V_i^{p\pi^+} = \pm V_i^{n\pi^+} + V_i^p \;, \quad A_1 \;\; \text{and} \;\; T_1 \;\; \text{similar}$$

Higher Twists

- General Definition of Higher Twists

$$4 \cdot \langle 0 | \, \varepsilon^{ijk} u^i_\alpha(a_1 z) u^j_\beta(a_2 z) d^k_\gamma(a_3 z) \, | N(P,\lambda)\pi(k) \rangle = -(\gamma_5)_{\gamma\delta} \frac{i}{f_\pi} \Big[$$
$$S_1^{\pi N}(s_1)_{\alpha\beta,\delta} + S_2^{\pi N}(s_2)_{\alpha\beta,\delta} + P_1^{\pi N}(p_1)_{\alpha\beta,\delta} + P_2^{\pi N}(p_2)_{\alpha\beta,\delta}$$
$$+V_1^{\pi N}(v_1)_{\alpha\beta,\delta} + V_2^{\pi N}(v_2)_{\alpha\beta,\delta} + \tfrac{1}{2} V_3^{\pi N}(v_3)_{\alpha\beta,\delta} + \tfrac{1}{2} V_4^{\pi N}(v_4)_{\alpha\beta,\delta}$$
$$+V_5^{\pi N}(v_5)_{\alpha\beta,\delta} + V_6^{\pi N}(v_6)_{\alpha\beta,\delta} + A_1^{\pi N}(a_1)_{\alpha\beta,\delta} + A_2^{\pi N}(a_2)_{\alpha\beta,\delta}$$
$$+A_3^{\pi N}(a_3)_{\alpha\beta,\delta} + \tfrac{1}{2} A_4^{\pi N}(a_4)_{\alpha\beta,\delta} + A_5^{\pi N}(a_5)_{\alpha\beta,\delta} + A_6^{\pi N}(a_6)_{\alpha\beta,\delta}$$
$$+T_1^{\pi N}(t_1)_{\alpha\beta,\delta} + T_2^{\pi N}(t_2)_{\alpha\beta,\delta} + T_3^{\pi N}(t_3)_{\alpha\beta,\delta} + T_4^{\pi N}(t_4)_{\alpha\beta,\delta}$$
$$+T_5^{\pi N}(t_5)_{\alpha\beta,\delta} + T_6^{\pi N}(t_6)_{\alpha\beta,\delta} + \tfrac{1}{2} T_7^{\pi N}(t_7)_{\alpha\beta,\delta} + \tfrac{1}{2} T_8^{\pi N}(t_8)_{\alpha\beta,\delta} \Big] .$$

- Higher twist calculation follows the same procedure but much more extensive
- x^2-corrections can also be calculated

Summary

- We defined a trilocal Operator O and made use of the SPT to calculate πN DAs
- We obtained πN DAs which have the natural symmetry property. They are expressed in terms of nucleon DAs having the same twist

Outlook

- Apply to deep inelastic Pion Electroproduction (see talk of V.Braun)
- etc. ...

References:

V. M. Braun, D. Y. Ivanov, A. Lenz and A. Peters, Phys. Rev. D **75** *(2007) 014021 [arXiv:hep-ph/0611386].*