

Charge Density of the Neutron

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[arXiv:0705.2409](https://arxiv.org/abs/0705.2409)

What do form factors really measure?

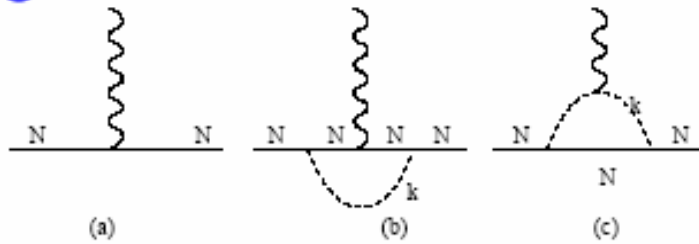
What is charge density at the center of the neutron?

- Neutron has no charge, but charge density need not vanish
- Is central density positive or negative?

Neutron: Need π cloud effect at low Q^2

Cloudy Bag Model 1980

TTM



Relativistic treatment needed Feynman graphs,

Light front cloudy bag model LFCBM 2002

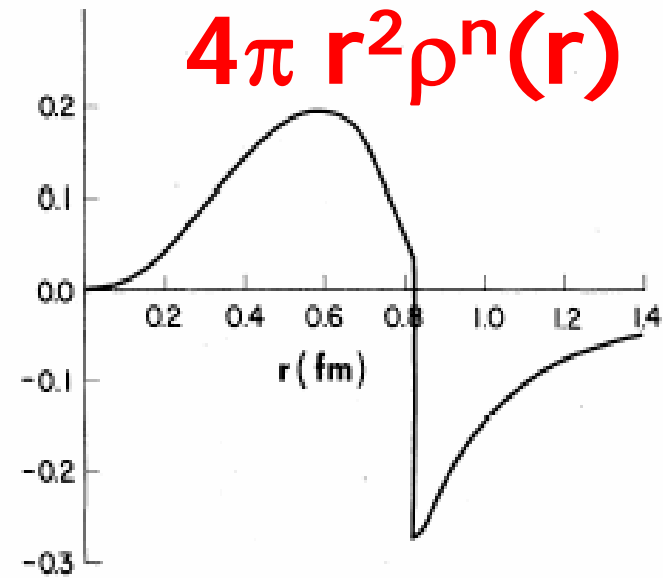


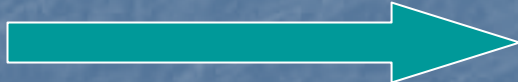
FIG. 11. Neutron charge density.

One gluon exchange also gives positive central charge density

Enough models- Today

model **ind**ependent information

Outline

- Electromagnetic form factors
- Light cone coordinates, kinematic subgroup
- GPDs + Bit of math 
- Two dimensional Fourier transf. of F_1 gives $\rho(b)$, Soper '77
- Data analysis, Interpretation

Definitions

$$\langle p', \lambda' | J^\mu(0) | p, \lambda \rangle = \bar{u}(p', \lambda') \left(\gamma^\mu F_1(Q^2) + i \frac{\sigma^{\mu\alpha} q_\alpha}{2M} F_2(Q^2) \right) u(p, \lambda)$$

$$G_E(Q^2) \equiv F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \quad G_M(Q^2) \equiv F_1(Q^2) + F_2(Q^2)$$

Old Interpretation- Breit frame $\vec{p}' = -\vec{p}$

G_E is helicity flip matrix element of J^0

Interpretation of Sachs - $G_E(Q^2)$ is Fourier transform of charge density

Correct non-relativistically

Non-relativistic two particle :

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, t) = e^{i\mathbf{P}\cdot\mathbf{R} - i(\frac{P^2}{2M} - \epsilon)t} \phi(\mathbf{r})$$

ϕ invariant under Galilean transformation

Relativity: $(\mathbf{r}_1, t_1), (\mathbf{r}_2, t_2) t_1 \neq t_2$

$e^{i H(t_1 - t_2)}$ Interactions!

ϕ is frame dependent,
interpretation of Sachs wrong

Why relativity if $Q^2 \ll M^2$

QCD- photon hits \approx massless quarks

No matter how small Q^2 is, there is a boost correction that is $\propto Q^2$

$$F_1 \sim Q^2 R_N^2 (|\psi|^2 + C / (m_q R_N)^2)$$

Light cone coordinates

“Time” $x^+ = (ct + z)/\sqrt{2} = (x^0 + x^3)/\sqrt{2}$

“Evolution” $p^- = (p^0 - p^3)/\sqrt{2}$

“Space” $x^- = (ct - z)/\sqrt{2} = (x^0 - x^3)/\sqrt{2}$, If $x^+ = 0$, $x^- = -\sqrt{2}z$

“Momentum” $p^+ = (p^0 + p^3)/\sqrt{2}$

Transverse : “Position” b “Momentum” p

Relativistic formalism- kinematic subgroup of Poincare

- Lorentz transformation –transverse velocity v

$$k^+ \longrightarrow k^+, \quad \mathbf{k} \longrightarrow \mathbf{k} - k^+ \mathbf{v}$$

k^- such that k^2 not changed

Just like non-relativistic

Generalized Parton Distribution

$$H_q(x, t) = \int \frac{dx^-}{4\pi} \langle p^+, p', \lambda | \bar{q}(-\frac{x^-}{2}, 0) \gamma^+ q(\frac{x^-}{2}, 0) | p^+, p, \lambda \rangle e^{ixp^+ x^-}$$

$$H_q(x, \xi = 0, t) \equiv H_q(x, t)$$

$$A^+ = 0, t = (p - p')^2 = -Q^2 = - (\mathbf{p}' - \mathbf{p})^2$$

$$H_q(x, t) = \int \frac{dx^-}{4\pi} \langle p^+, \mathbf{p}', \lambda | \bar{q}\left(-\frac{x^-}{2}, 0\right) \gamma^+ q\left(\frac{x^-}{2}, 0\right) | p^+, \mathbf{p}, \lambda \rangle e^{ixp^+ x^-}$$

$$H_q(x, 0) = q(x) \quad F_1(t) = \sum_q e_q \int dx H_q(x, t)$$

transverse center of mass \mathbf{R}

$$|p^+, \mathbf{R} = \mathbf{0}, \lambda\rangle \equiv \mathcal{N} \int \frac{d^2 \mathbf{p}}{(2\pi)^2} |p^+, \mathbf{p}, \lambda\rangle$$

$$\hat{O}_q(x, \mathbf{b}) \equiv \int \frac{dx^-}{4\pi} q_+^\dagger\left(-\frac{x^-}{2}, \mathbf{b}\right) q_+\left(\frac{x^-}{2}, \mathbf{b}\right) e^{ixp^+ x^-}$$

$$H_q(x, t) = \int \frac{dx^-}{4\pi} \langle p^+, \mathbf{p}', \lambda | \bar{q}\left(-\frac{x^-}{2}, 0\right) \gamma^+ q\left(\frac{x^-}{2}, 0\right) | p^+, \mathbf{p}, \lambda \rangle e^{ixp^+ x^-}$$

$$\hat{O}_q(x, \mathbf{b}) \equiv \int \frac{dx^-}{4\pi} q_+^\dagger\left(-\frac{x^-}{2}, \mathbf{b}\right) q_+\left(\frac{x^-}{2}, \mathbf{b}\right) e^{ixp^+ x^-}$$

$$q(x, \mathbf{b}) \equiv \langle p^+, \mathbf{R} = \mathbf{0}, \lambda | \hat{O}_q(x, \mathbf{b}) | p^+, \mathbf{R} = \mathbf{0}, \lambda \rangle.$$

Burkardt

$$q(x, \mathbf{b}) = \int \frac{d^2 q}{(2\pi)^2} e^{i \mathbf{q} \cdot \mathbf{b}} H_q(x, t = -\mathbf{q}^2),$$

**Integrate on x, Left: sets $x^- = 0 \rightarrow q_+^\dagger(0, \mathbf{b}) q_+(0, \mathbf{b})$
DENSITY; right 2 Dim. Fourier T. of F_1**

RESULT

$$\rho(b) \equiv \sum_q e_q \int dx \underset{\text{Density}}{q(x, \mathbf{b})} = \int \frac{d^2 q}{(2\pi)^2} F_1(Q^2 = \mathbf{q}^2) e^{i \mathbf{q} \cdot \mathbf{b}}.$$

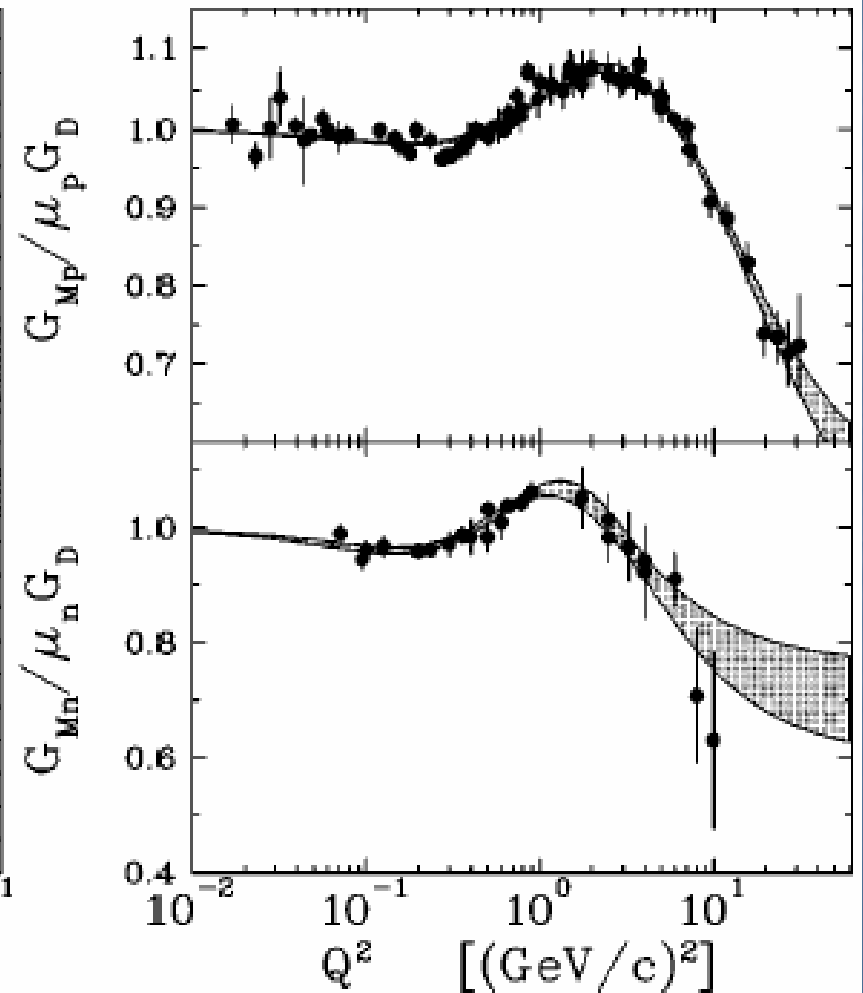
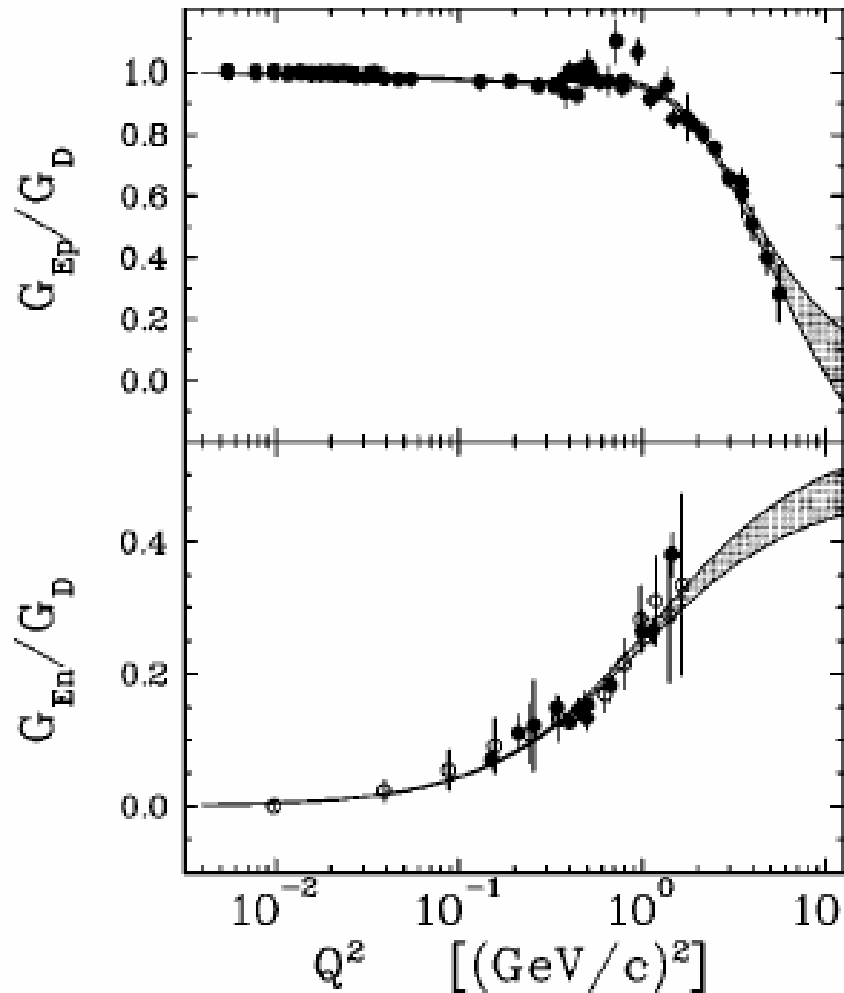
Soper '77

$$\rho(b) = \int_0^\infty \frac{dQ Q}{2\pi} J_0(Qb) \frac{G_E(Q^2) + \tau G_M(Q^2)}{1 + \tau}$$

$$\tau = Q^2 / 4M^2$$

Simple parametrization of nucleon form factors

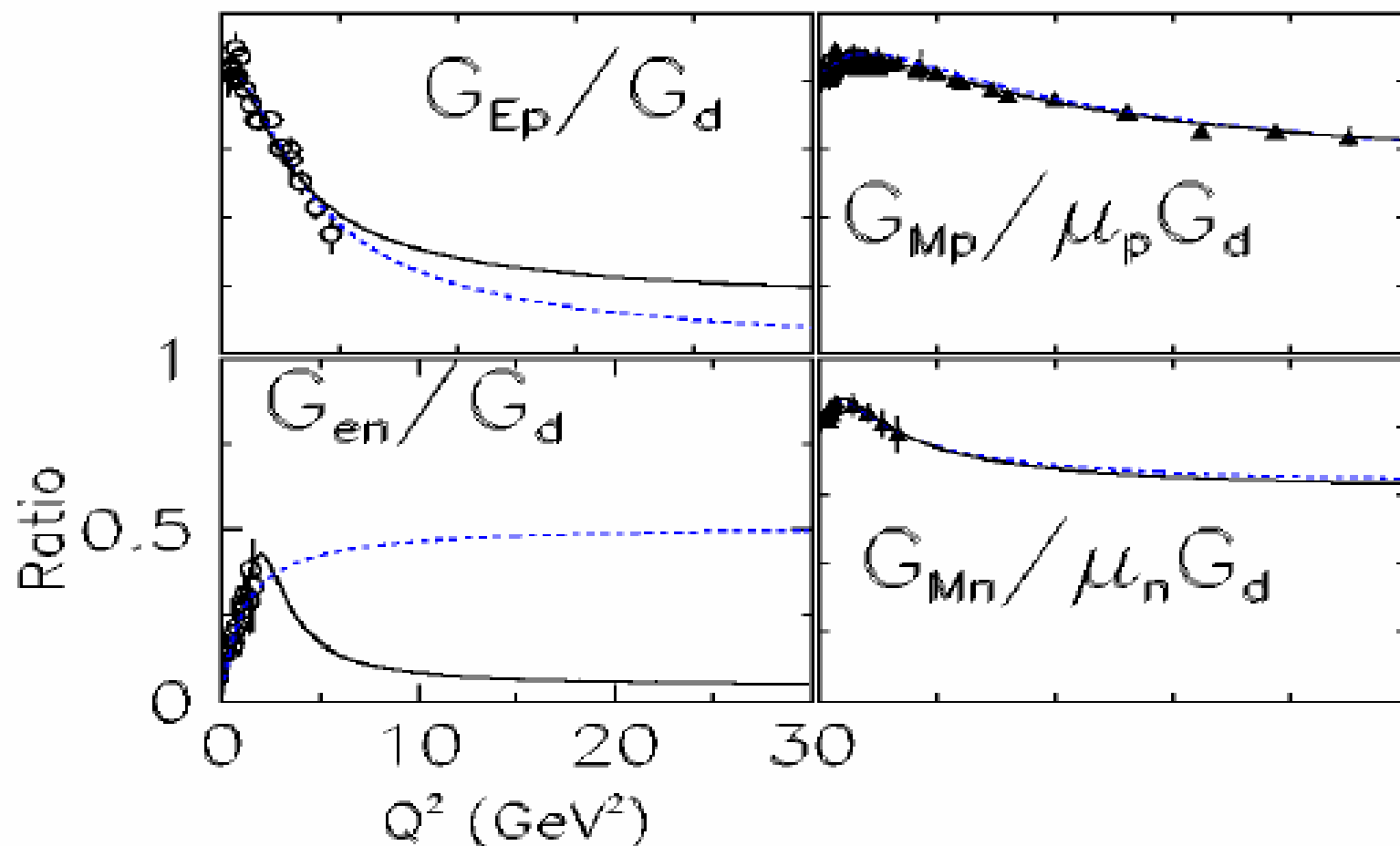
J. J. Kelly



A New Parameterization of the Nucleon Elastic Form Factors

R. Bradford,^a A. Bodek,^a H. Budd,^a and J. Arrington^b

hep-ex/0602017



— BBBA — May 05

..... J. Kelly — December 04

Results

$\varrho(\mathbf{b})$ [fm⁻²]

1.5

1

0.5

0

proton

0 0.5 1 1.5 2

b [fm]



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Kelly

$\varrho(\mathbf{b})$ [fm⁻²]

0.1

0

-0.1

-0.2

-0.3

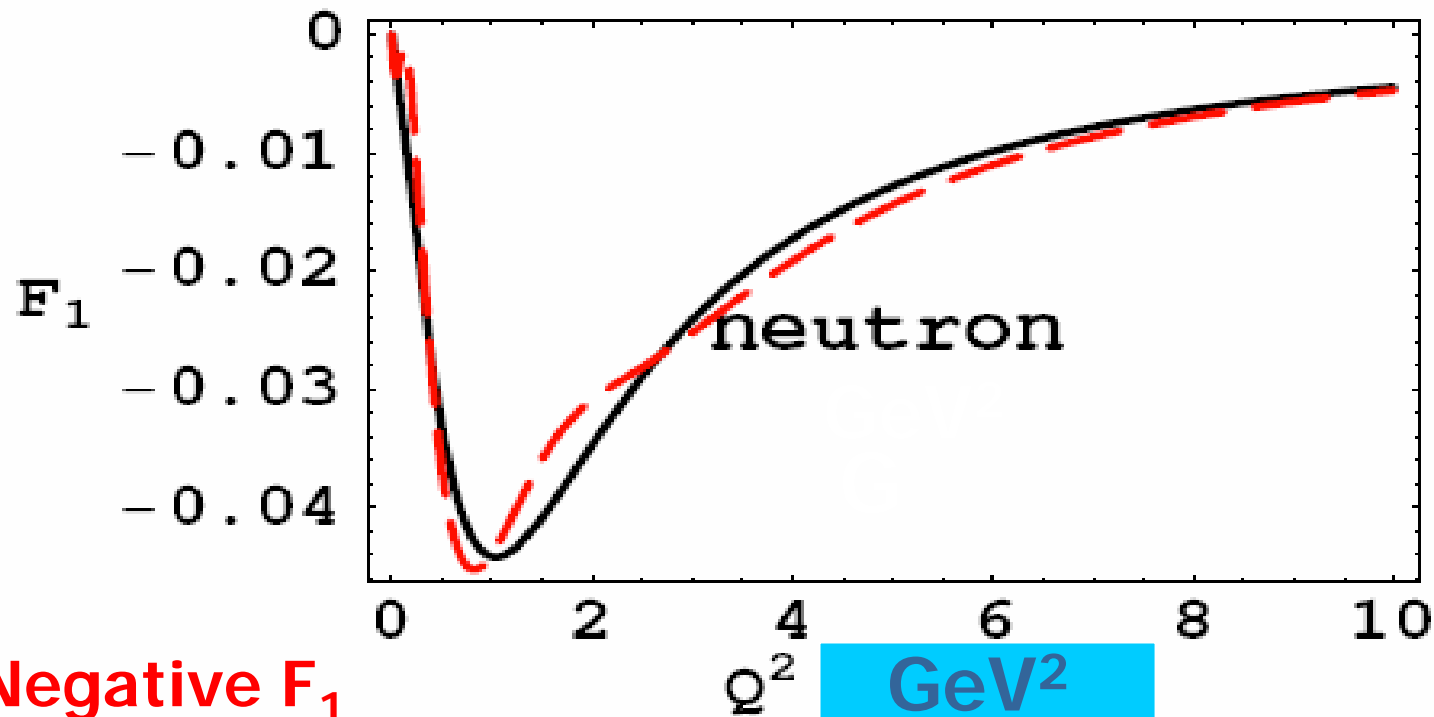
-0.4

neutron

0 0.5 1 1.5 2

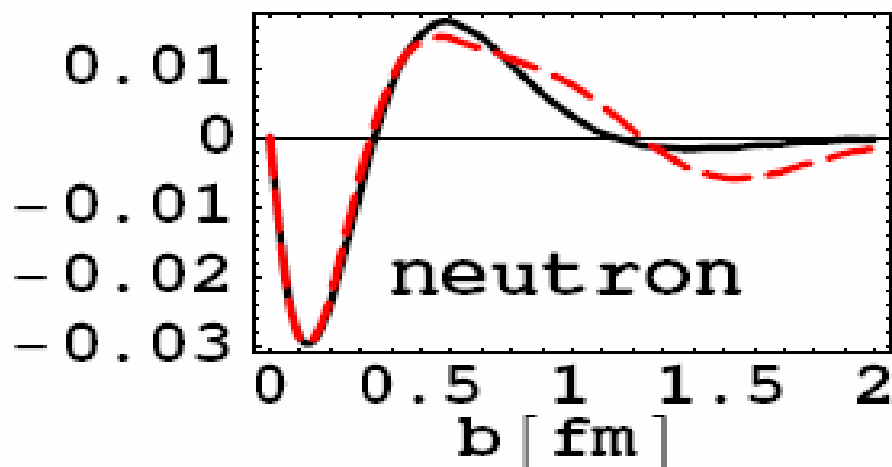
b [fm]

Negative

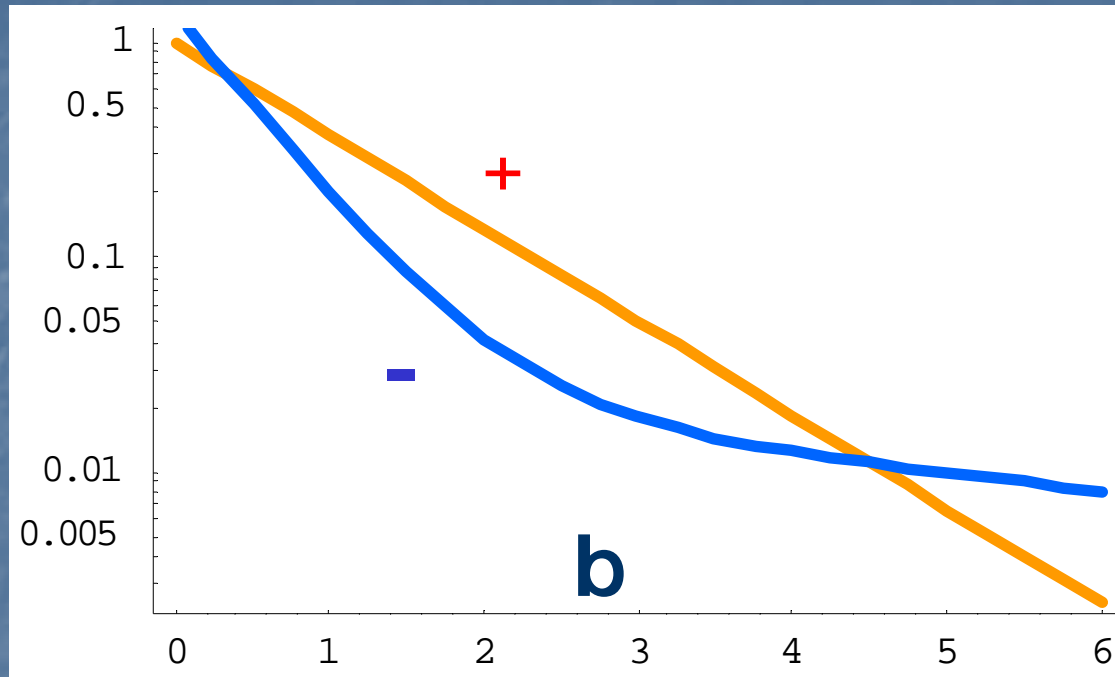


**Negative F_1
 means central
 density negative**

$\rho(\mathbf{b})$ [fm⁻¹]



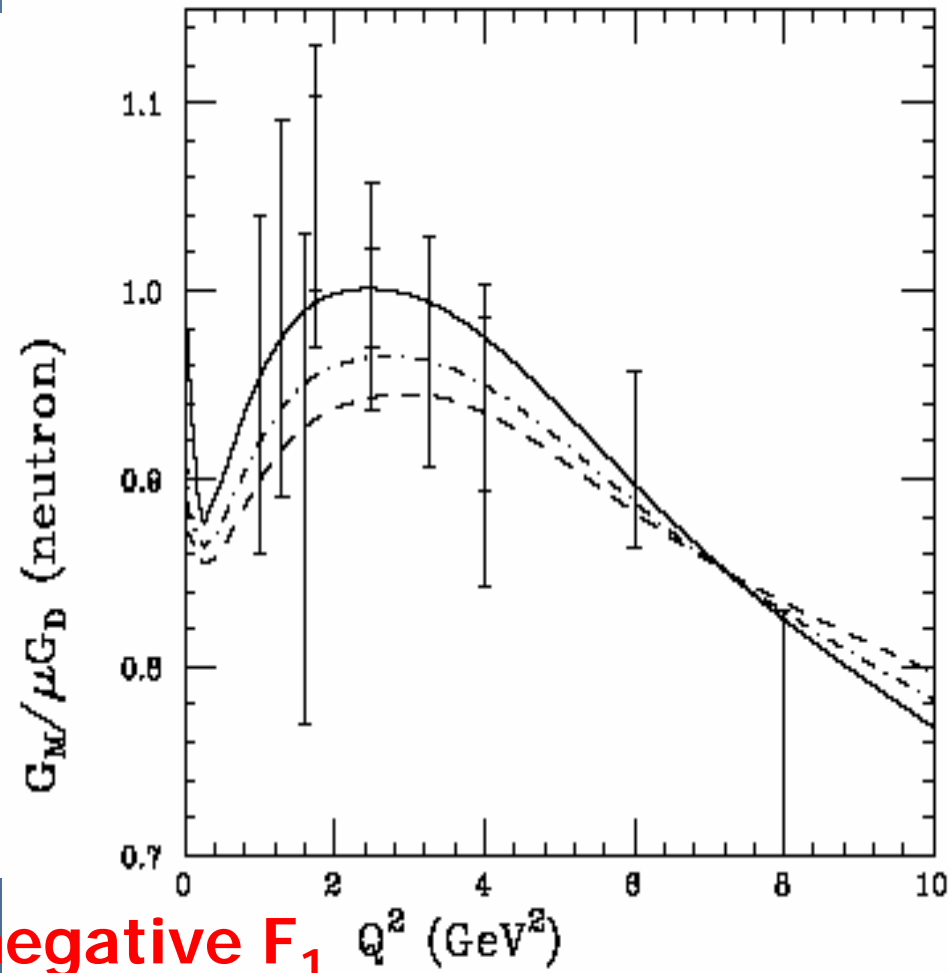
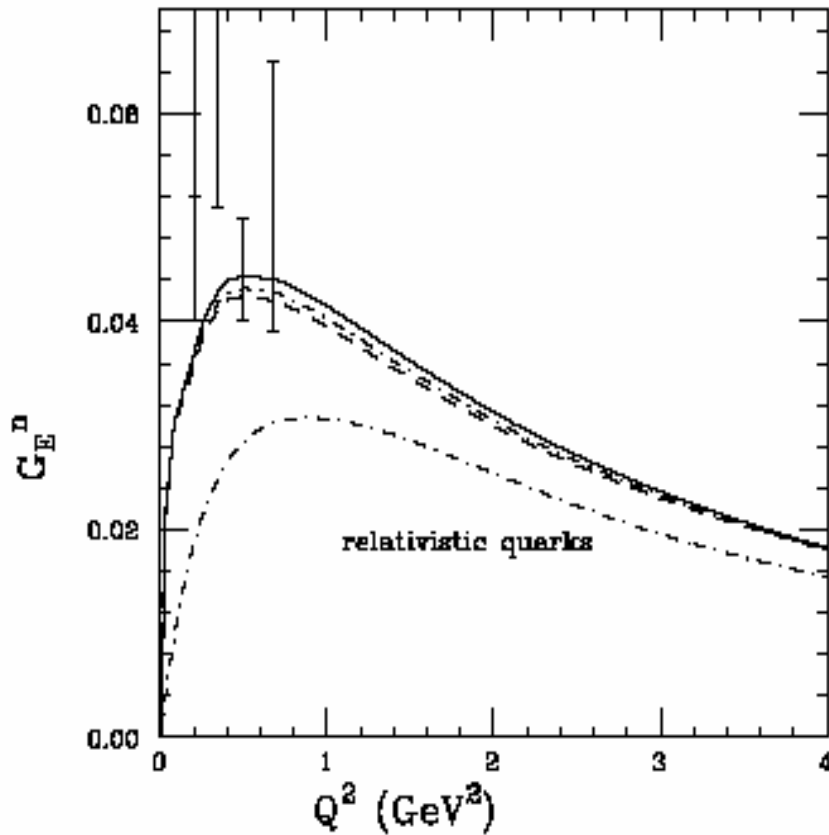
Neutron Interpretation



? π^- at short distance ?

Neutron Form Factors in LFCBM

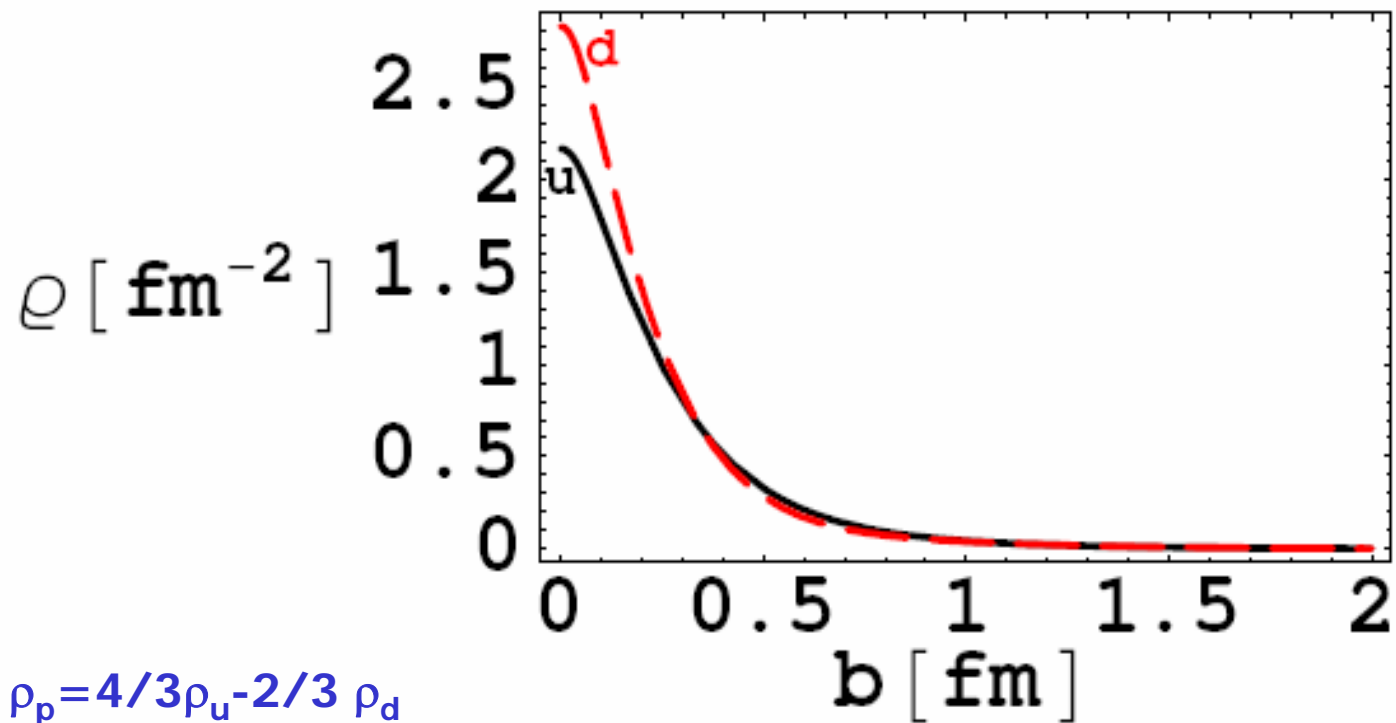
Miller 2002



These give negative F_1

Charge symmetry: u in proton is d in neutron, d in proton is u in neutron

$$\rho_u = \rho_p - \rho_n/2 \quad \rho_d = \rho_p - 2\rho_n$$



?Quark interpretation?

- $b=0$, high transverse momentum, low Bjorken x
- low x , sea
- $u \bar{u}$ is suppressed by Pauli principal, Signal & Thomas

Summary

- Model independent information on charge density

$$\rho(b) \equiv \sum_a e_q \int dx q(x, \mathbf{b}) = \int d^2q F_1(Q^2 = q^2) e^{i \mathbf{q} \cdot \mathbf{b}}$$

- Central charge density of neutron is negative
- Pion cloud at large b