Obtaining Generalized Parton Distributions from hadronic observables and lattice QCD

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Outline

- Zero Skewness: GPDs from form factors and PDFs
- Non-Zero Skewness: using lattice results
- Nuclei:

  Deuteron: Angular Momentum Sum Rule

  ^{4}He: In Medium Form Factor through GPDs
- Conclusions

1. The Zero Skewness case

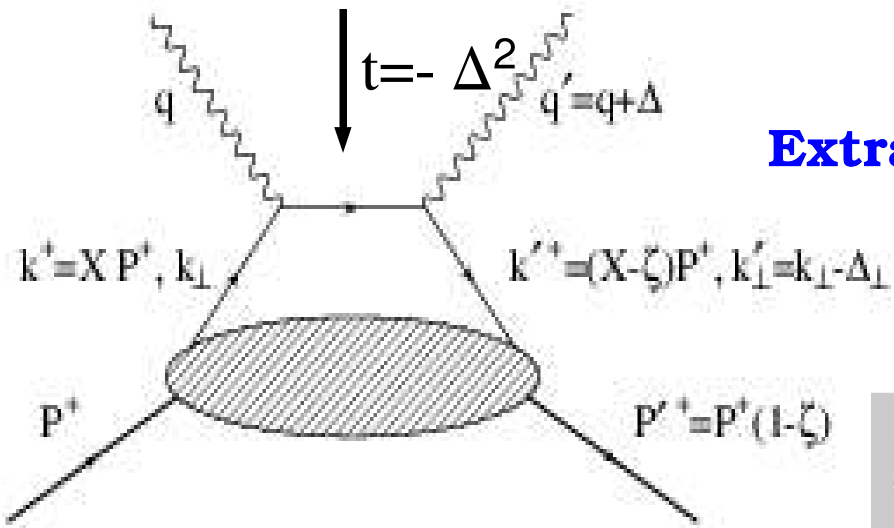
Motivation

- DVCS and other types of "exclusive" processes add a whole *new dimension* to studies of hadronic structure
- Experiments are hard and lenghty
- Theoretical tools :

  DD → "purely theoretical constructions"

  Mellin-Barnes moments → problem of "reconstructing" GPDs
- Need to devise new/alternative approaches to extract and interpret information from present and future (Jlab @12 GeV) experiments
- What are the prospects for obtaining spatial configurations from experiment?

DVCS and Generalized Parton Distributions



Extract "Generalized Parton Distributions"

$$\bar{p}^+ \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P', S' | \psi\left(-\frac{\xi^-}{2}\right) \gamma^+ \psi\left(\frac{\xi^-}{2}\right) | P, S\rangle =$$

$$\bar{u}(P', S')\left[\gamma^+ H(x,\xi,-\Delta^2) + \frac{i\sigma^{+\nu}q_\nu}{2M} E(x,\xi,-\Delta^2)\right] u(P,S)$$

- **GPDs are hybrids of PDFs and FFs: describe simultaneously x and t-dependences !**
- **GPDs give access to spatial d.o.f. of partons !**
- **GPDs give access to orbital angular momentum of partons!**

$$\int dx x \left[H_q(x,\zeta,t=0) + E_q(x,\zeta,t=0)\right] = 2J_q$$

X. Ji

Proposed Strategy

- Similarly to the inception of PDFs analyses:

Construct theoretically motivated parametrizations at a given *low* initial scale

- Merge data/information from:
  - Form factors → $\zeta=0$
  - PDFs → $\zeta=0$
  - Higher GPD moments (lattice calculations) → $\zeta\neq0$
  - DVCS data → $\zeta\neq0$

- Apply PQCD evolution to connect different sets of data

What goes into a theoretically motivated parametrization...?

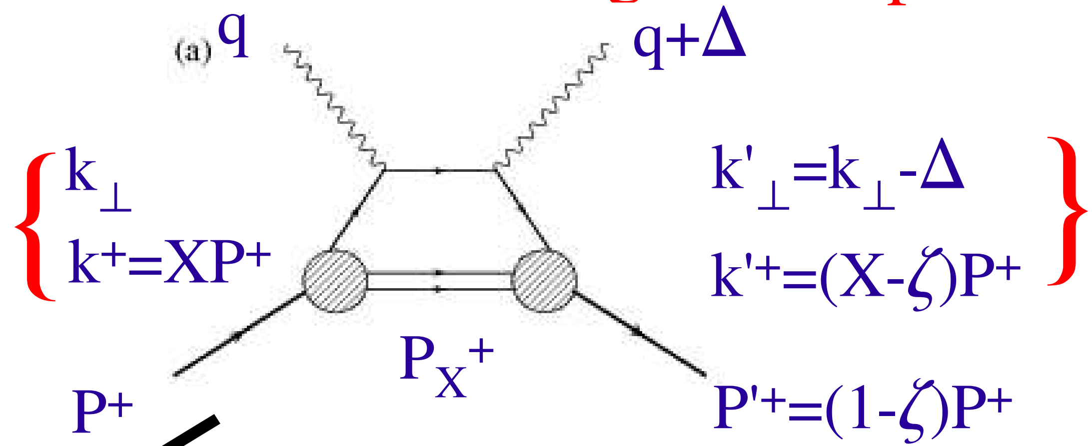
The name of the game: Devise a form combining essential dynamical elements with a flexible model that allows for a fully quantitative analysis constrained by the data

$$H_q(X,t) = R(X,t)\, G(X,t)$$

"Regge"

Quark-Diquark

For $\zeta = 0$ and in the DGLAP region $\Rightarrow$ partonic picture



$$R^I_{1(2)} = X^{-\alpha^I - \beta^I_{1(2)}(1-X)^{p^I_{1(2)}} t}$$

$$G^{\lambda}_{M_X}(X,t) = \mathcal{N}\frac{X}{1-X}\int d^2\mathbf{k}_\perp \frac{\phi(k^2,\lambda)}{D(X,\mathbf{k}_\perp)}\frac{\phi(k'^2,\lambda)}{D(X,\mathbf{k}_\perp+(1-X)\mathbf{\Delta}_\perp)}$$

Summary of Constraints

Constraints from Form Factors

$$\int_0^1 dX\, H^q(X,t) = F_1^q(t)$$ Dirac

$$\int_0^1 dX\, E^q(X,t) = F_2^q(t),$$ Pauli

$$F_{1(2)}^p(t) = \frac{2}{3}F_{1(2)}^u(t) - \frac{1}{3}F_{1(2)}^d(t) + \frac{1}{3}F_{1(2)}^s(t)$$ Dirac(Pauli) proton

$$F_{1(2)}^n(t) = -\frac{1}{3}F_{1(2)}^u(t) + \frac{2}{3}F_{1(2)}^d(t) + \frac{1}{3}F_{1(2)}^s(t),$$ Dirac(Pauli) neutron

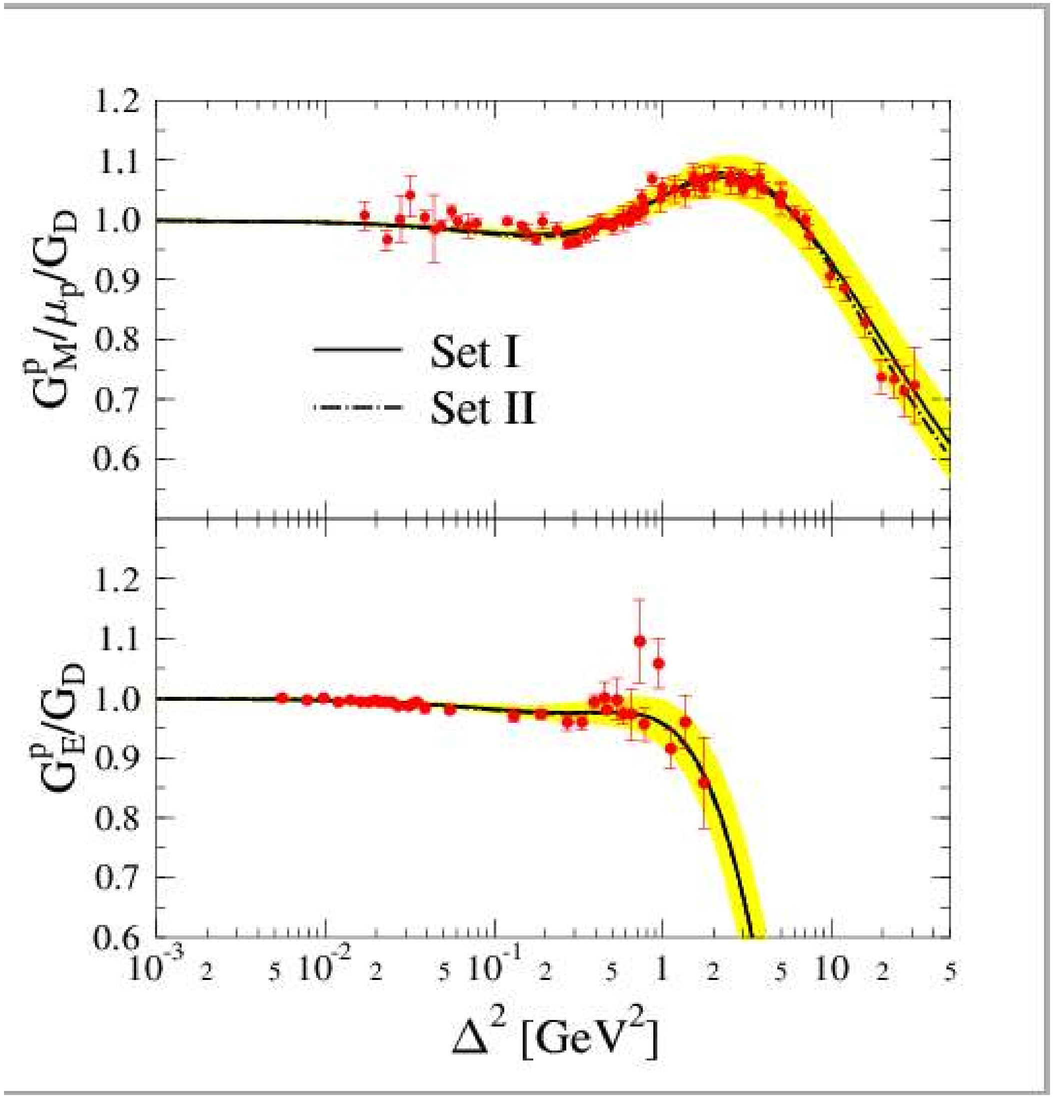
Constraints from PDFs

$$q(x) = H_q(x,0,0)$$

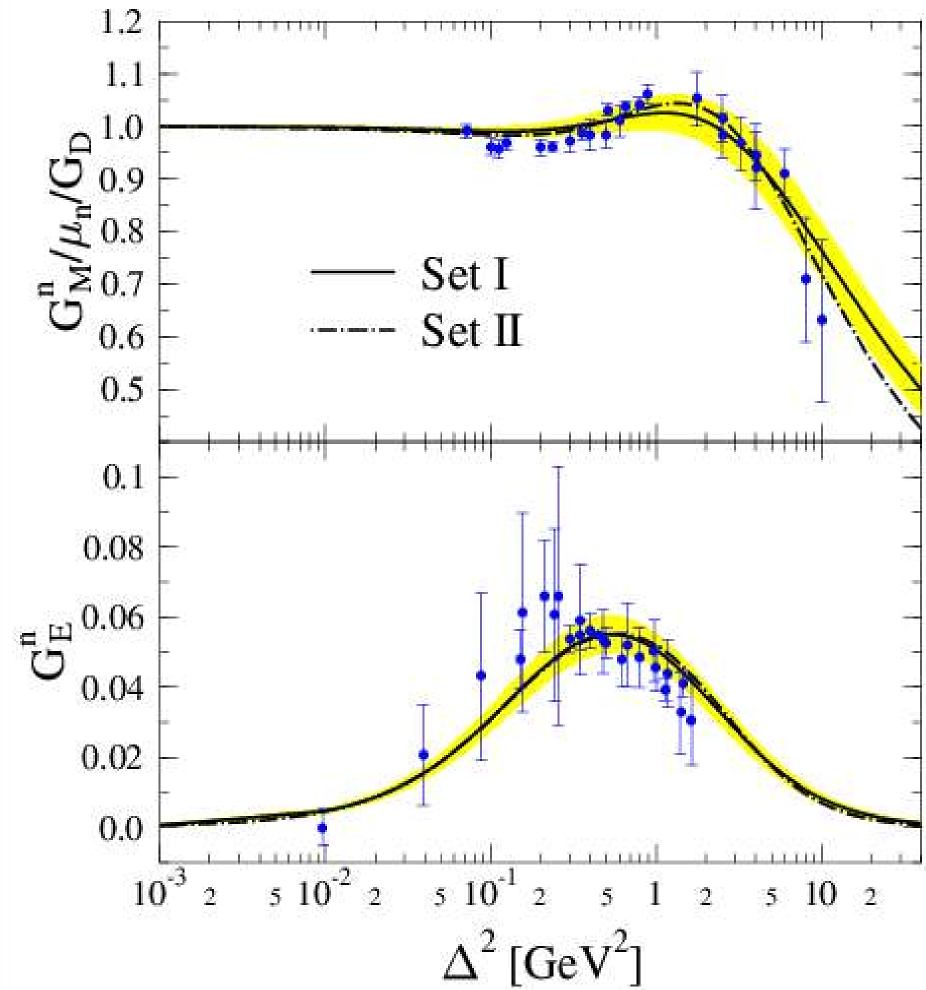
$\zeta = 0$

GPDs from available data 1

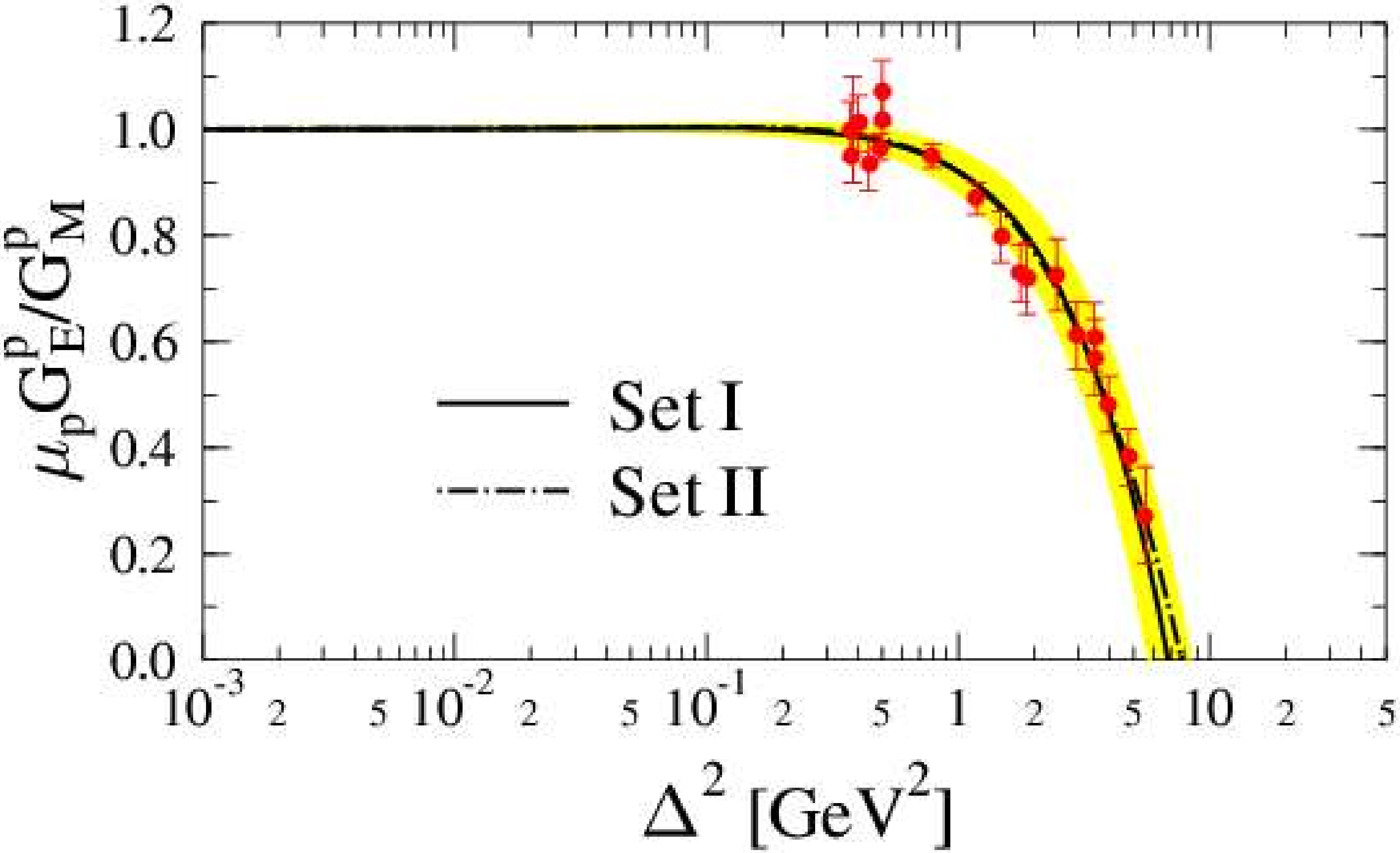
Nucleon Form Factors



S. Ahmad, H. Honkanen, S. L., S.K. Taneja, hep-ph/0611046 to be published in PRD



1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
$G_M^n/\mu_n/G_D$
Set I
Set II
0.1
0.08
0.06
0.04
0.02
0.0
G_E^n
10^{-3} 2 5 10^{-2} 2 5 10^{-1} 2 5 1 2 5 10 2
Δ^2 [GeV^2]



$\mu_p G_E^p / G_M^p$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Set I
Set II
10^{-3} 2 5 10^{-2} 2 5 10^{-1} 2 5 1 2 5 10 2 5
Δ^2 [GeV^2]

GPDs from available data 2

Parton Distribution Functions

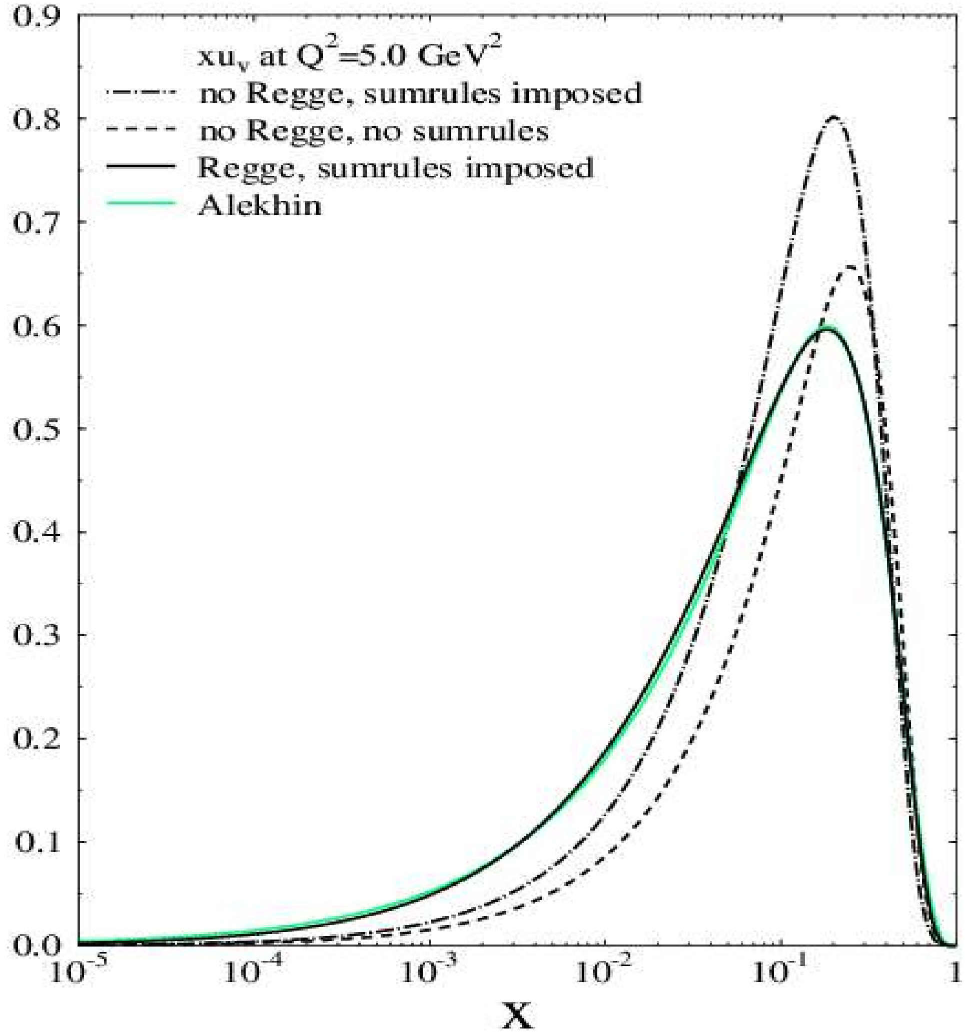
Notice! GPD parametric form is given at $Q^2=\mu^2$ and evolved to Q^2 of data.

Notice! We provide a parametrization for GPDs that simultaneously fits the PDFs:

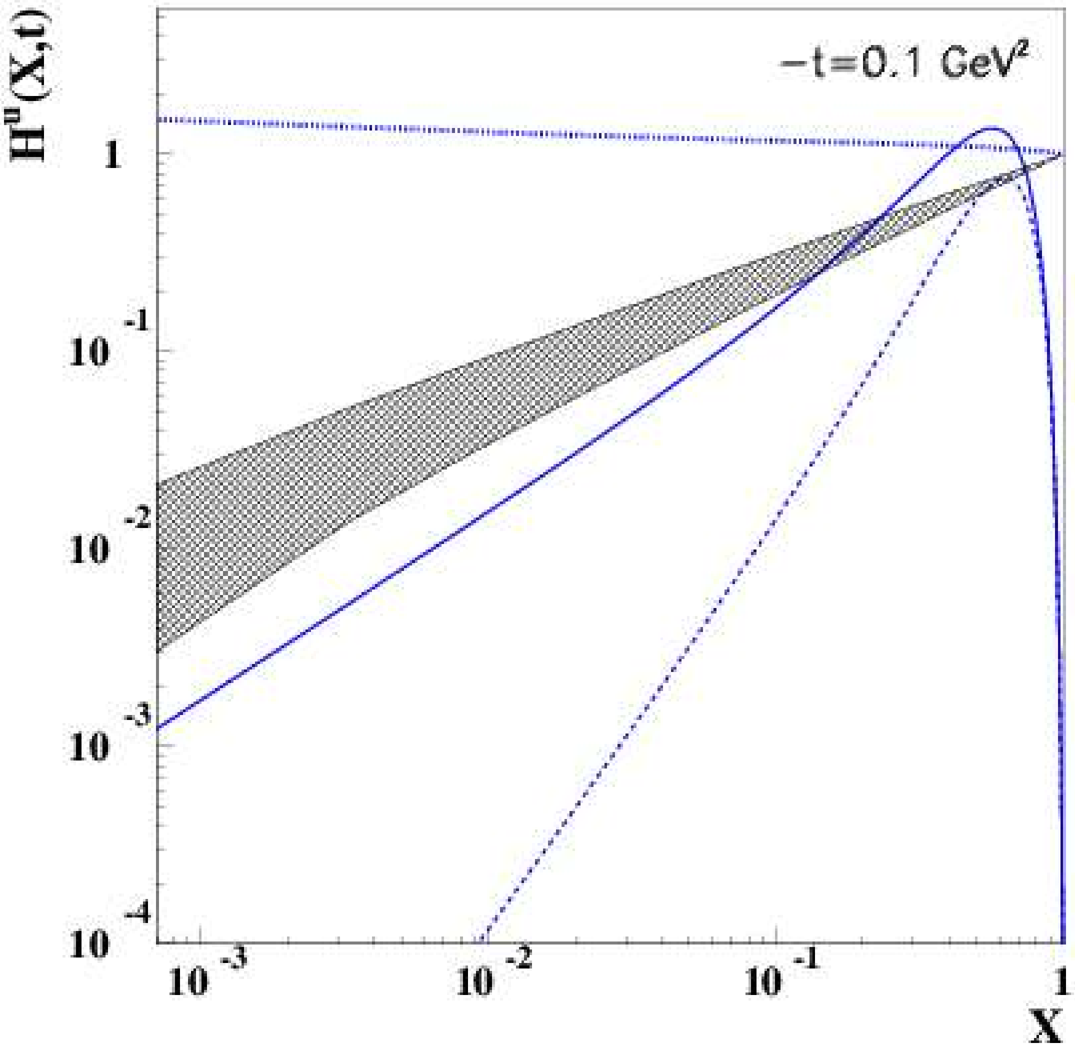
$H_q(X,t) = R(X,t)\, G(X,t)$

Regge

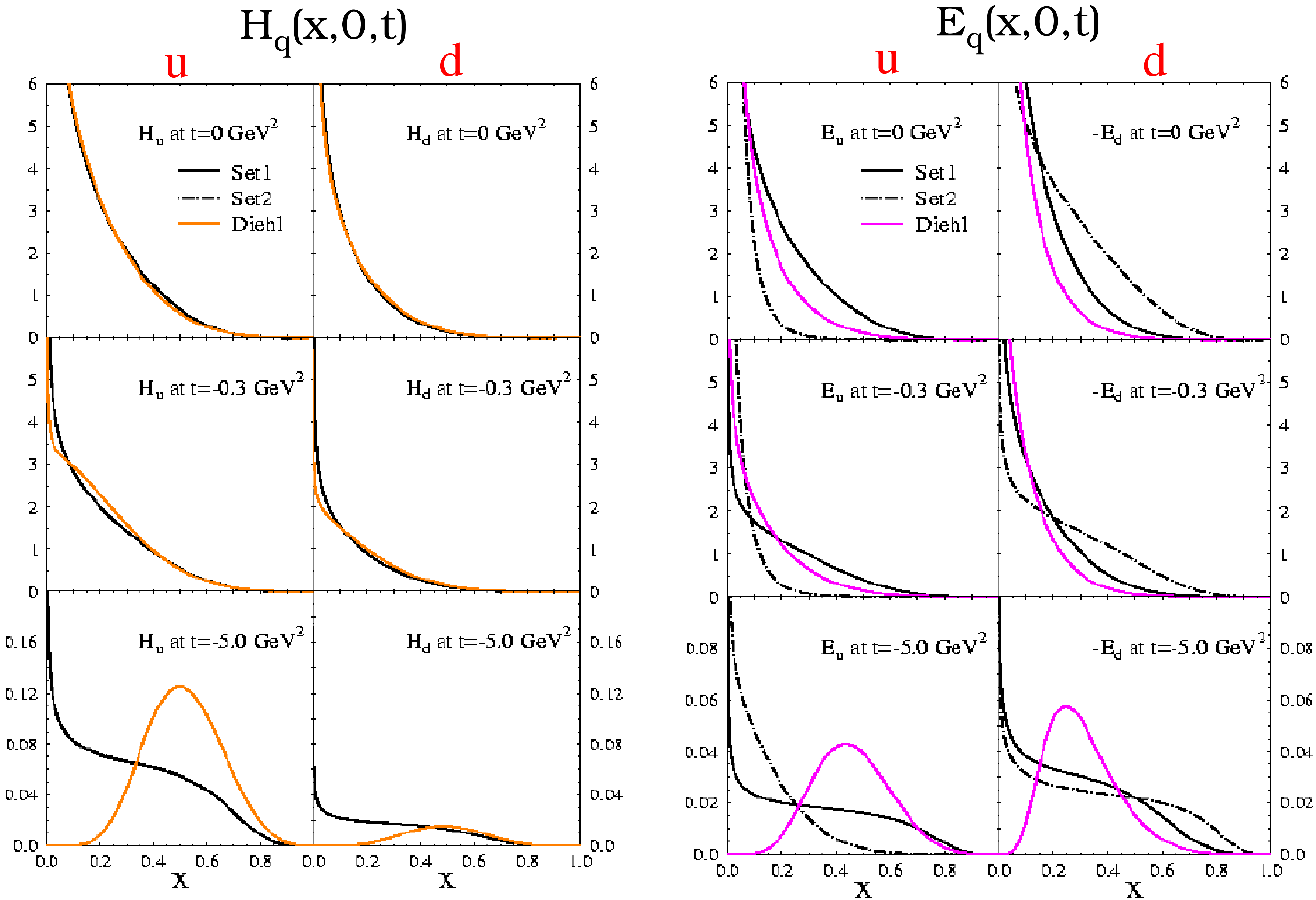
Quark-Diquark



Role of "Regge" term



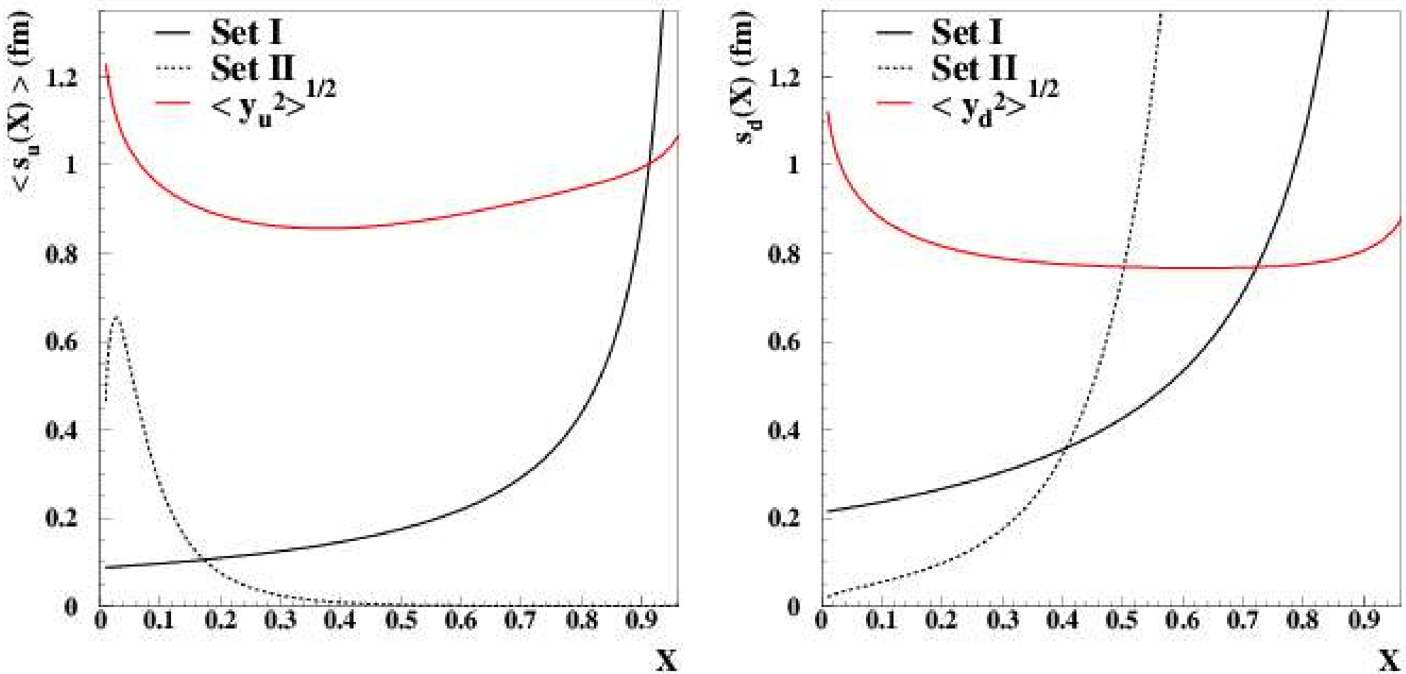
Comparison with similar parametrizations at ζ=0



Evaluation of interparton distances

y is the average distance of quark **q** from the spectator quarks

s is the average shift of quark along the y-axis when proton is (transversely) polarized along the x-axis



Sensitive to E!

2. Non-zero Skewness

Higher moments (n=2,3,...)

n=2

$$H_2^q = A_{20}^q(t) + \left(-\frac{2\zeta}{2-\zeta}\right)^2 C_2^q(t)$$

$$E_2^q = B_{20}^q(t) - \left(-\frac{2\zeta}{2-\zeta}\right)^2 C_2^q(t)$$

n=3

$$H_3^q = A_{30}^q(t) + \left(-\frac{2\zeta}{2-\zeta}\right)^2 A_{32}^q(t)$$

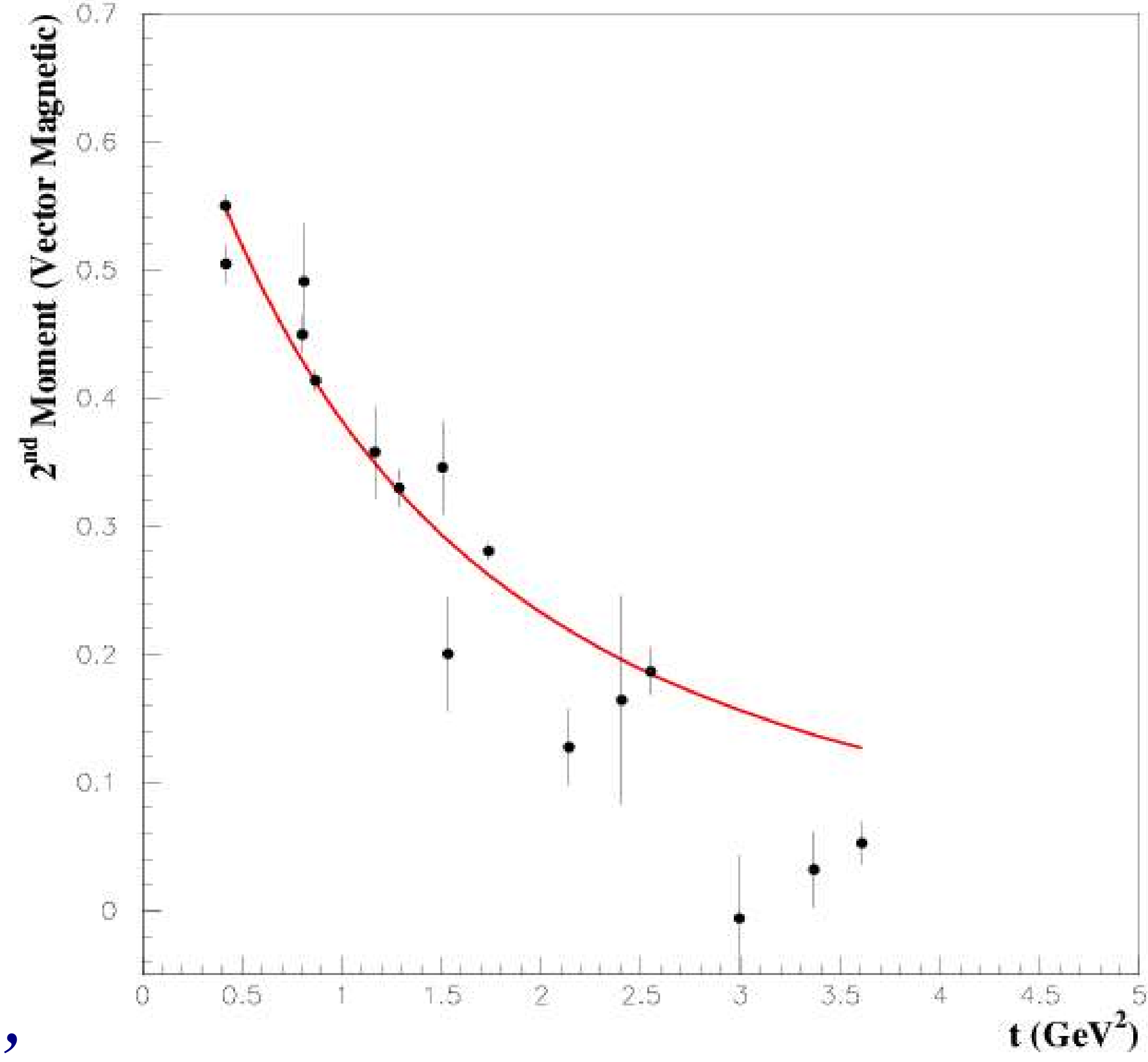
$$E_3^q = B_{30}^q(t) + \left(-\frac{2\zeta}{2-\zeta}\right)^2 B_{32}^q(t)$$

$\zeta \neq 0$

Use information from Lattice QCD:

(1) lattice results follow dipole behavior for n=1,2,3

$$G(Q^2) = \frac{G(0)}{(1 + Q^2/\Lambda^2)^2}$$



Lattice results from:

M. Gockeler et al. (2006);

J. Zanotti: hep-ph/0501029;

and "*44th Winter School in Schladming, Austria, March 2006.'*

$\zeta \neq 0$ **(2)** chiral extrapolate dipole masses

$$\langle r^2 \rangle_M^v \sim \frac{\chi_1}{m_\pi}\frac{2}{\pi}\arctan(\mu/m_\pi) + \frac{\chi_2}{2}\ln\left(\frac{m_\pi^2}{m_\pi^2+\mu^2}\right)$$

Modified to suppress chiral loops at large m_π

Ashley, Leinweber, Thomas, Young (2003)

$$\chi_1 = \frac{g_A^2 m_N}{8\pi f_\pi^2 \kappa_v},$$

$$\chi_2 = -\frac{5g_A^2+1}{8\pi^2 f_\pi^2},$$

$$\langle r^2 \rangle_M^v \sim \frac{\chi_1}{m_\pi} + \chi_2 \ln\left(\frac{m_\pi}{\mu}\right).$$

V. Bernard et al. (1995)

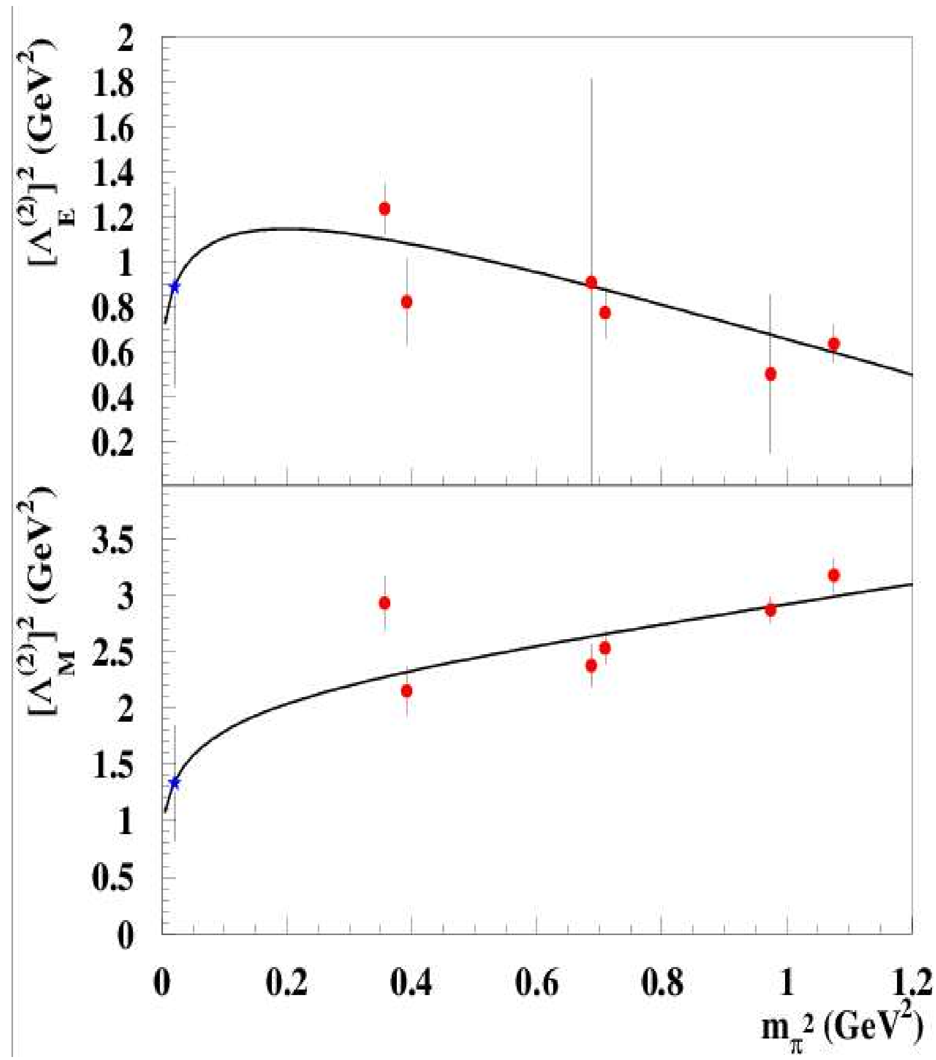
We extend method to n>1

$$[(\Lambda^V_{M(E)})^2]_n = \frac{12(1+\alpha_n^{M(E)} m_\pi^2)}{\beta_n^{M(E)} + \gamma_n \ln\left(\frac{m_\pi^2}{m_\pi^2+\mu^2}\right)}$$

Extra parameter

Results

n	$(\Lambda^V_E)^2$ (GeV^2)	$(\Lambda^V_M)^2$ (GeV^2)
1	0.457 ± 0.048	0.576 ± 0.060
2	0.704 ± 0.163	1.531 ± 0.129
3	1.814 ± 0.158	1.429 ± 0.144



H and E from isovector moments

n=1

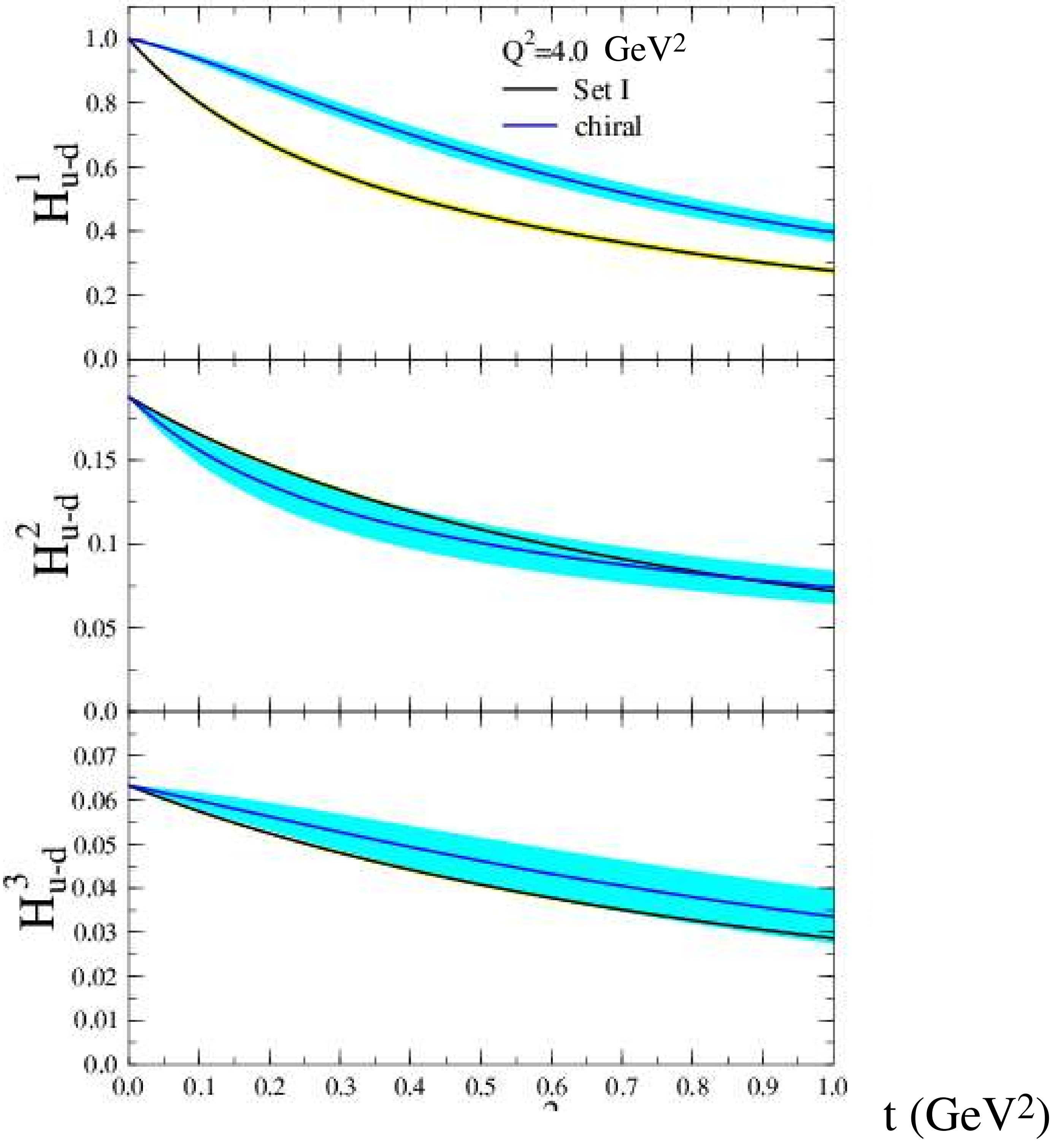
$$H_1^{u-d} \equiv \int dX (H^u - H^d) = \frac{\tau G_M^V + G_E^V}{1+\tau}$$

$$E_1^{u-d} \equiv \int dX (E^u - E^d) = \frac{G_M^V - G_E^V}{1+\tau}.$$

"any" n

$$H_n^{u-d} \equiv \int dX X^{n-1} (H^u - H^d) = \frac{\tau (H_M^V)_n + (H_E^V)_n}{1+\tau}$$

$$E_n^{u-d} \equiv \int dX X^{n-1} (E^u - E^d) = \frac{(E_M^V)_n - (E_E^V)_n}{1+\tau},$$



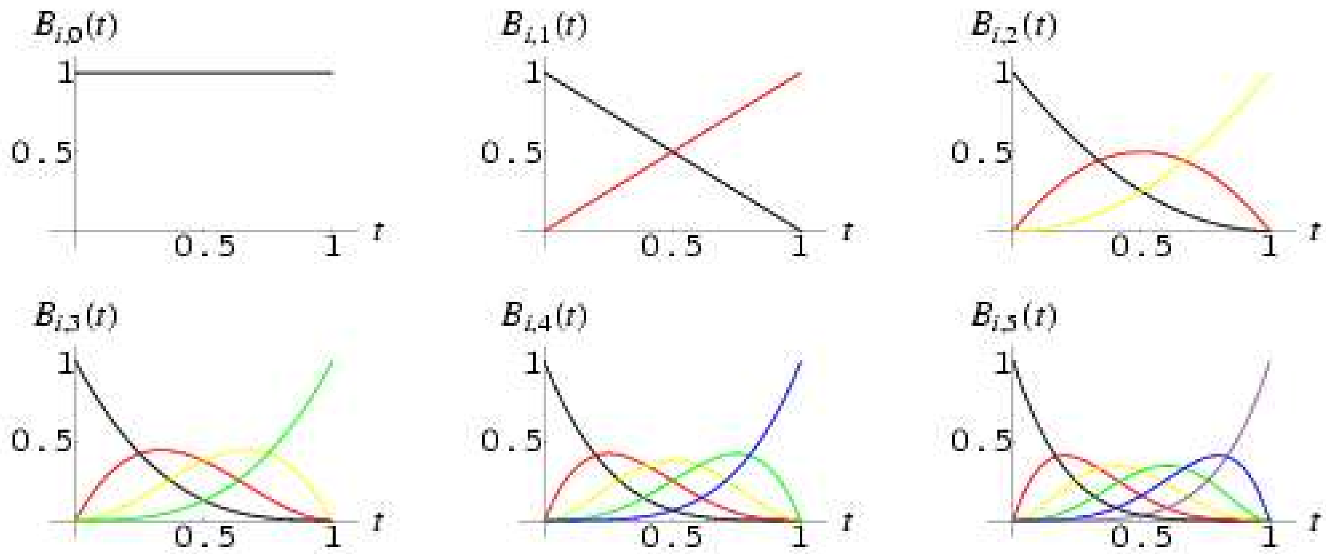
Q^2=4.0 GeV^2
Set I
chiral
H^1_{u-d}
H^2_{u-d}
H^3_{u-d}
t (GeV^2)

(3) reconstruct GPD from its moments: Bernstein polynomials

Weighted Average ⇒ $$\overline{H}(X,\zeta,t) = \int_0^1 H(X,\zeta,t) b_{k,n}(X) dX \quad k = 1,\ldots n,$$

X-bin ⇒ $$\overline{X}_{k,n} = \int_0^1 X b_{k,n}(X) dX = \frac{k+1}{n+1},$$

Dispersion ⇒ $$\Delta_{k,n} = \left(\overline{X^2}_{k,n} - \overline{X}^2_{k,n}\right)^{1/2}$$



$$\overline{H}(X,\zeta,t) = \frac{(n+1)!}{k!}\sum_{l=0}^{n-k}\frac{(-1)^l}{l!(n-k-l)!}M_{l+k}$$

Mellin moments

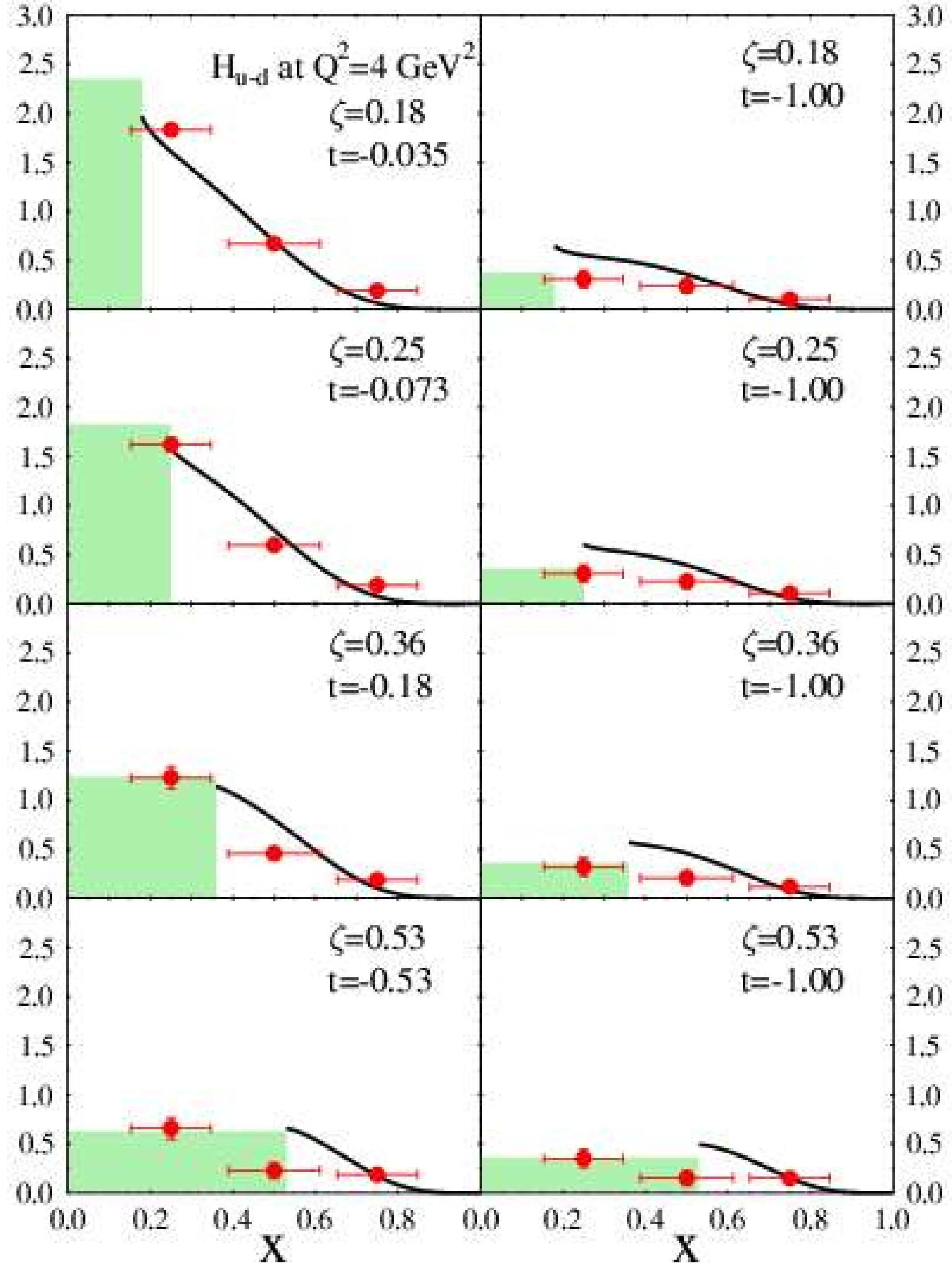
GPDs from Bernstein moments

$$\overline{H}_{02}(X_{02}) = 3A_{10} - 6A_{20} + 3\left[A_{30} + \left(\frac{2\zeta}{2-\zeta}\right)^2 A_{32}\right],$$

$$\overline{H}_{12}(X_{12}) = 6A_{20} - 6\left[A_{30} + \left(\frac{2\zeta}{2-\zeta}\right)^2 A_{32}\right],$$

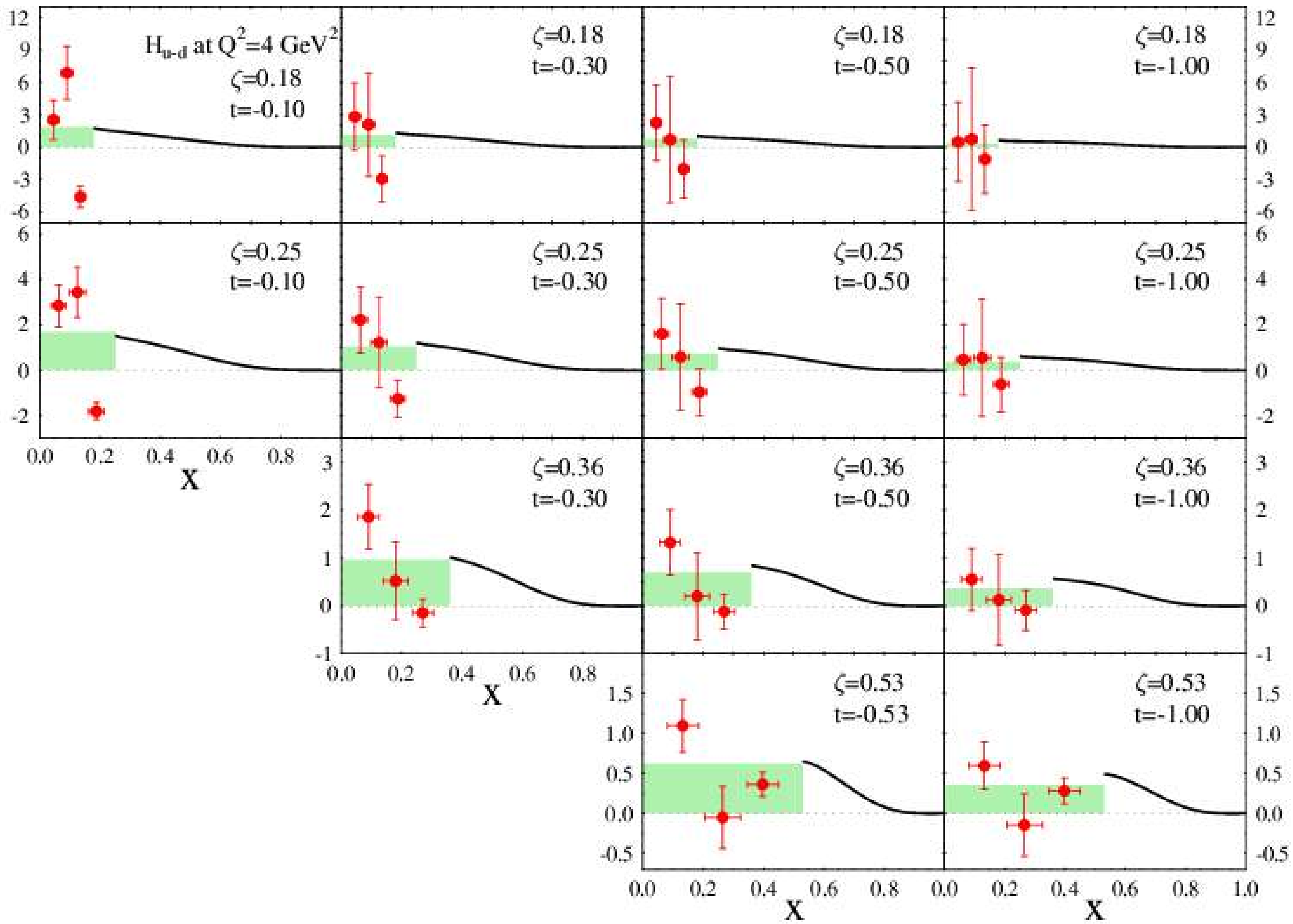
$$\overline{H}_{22}(X_{22}) = 3A_{30} + \left[\left(\frac{2\zeta}{2-\zeta}\right)^2 A_{32}\right].$$

Test in X=[0,1]



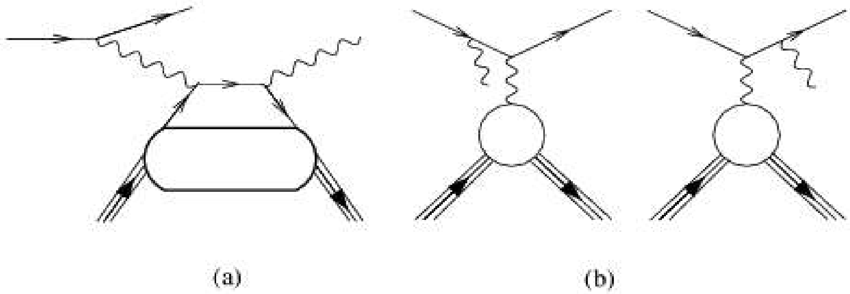
H_{u-d} at Q^2=4 GeV^2
ζ=0.18
t=-0.035
ζ=0.18
t=-1.00
ζ=0.25
t=-0.073
ζ=0.25
t=-1.00
ζ=0.36
t=-0.18
ζ=0.36
t=-1.00
ζ=0.53
t=-0.53
ζ=0.53
t=-1.00
X
X

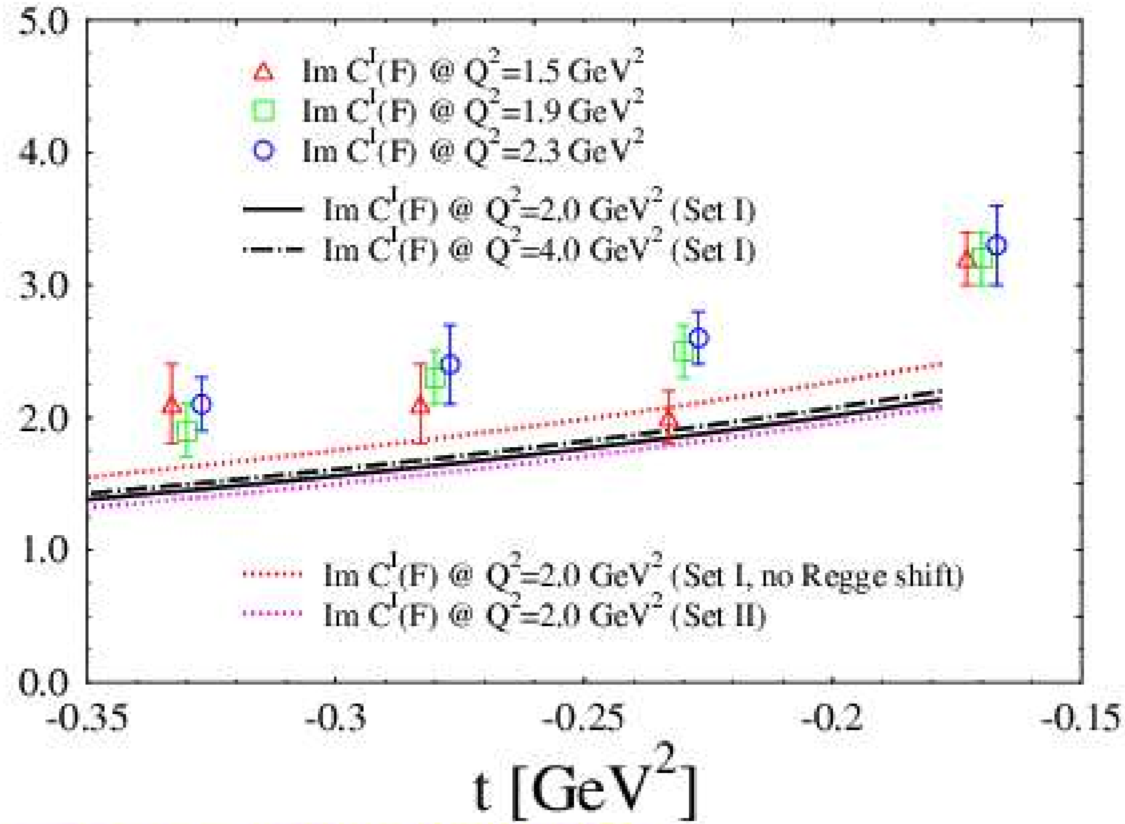
Assuming DGLAP region is constrained ⇒ fit ERBL region



First "model independent" extraction of GPDs!!!

Comparison with Jlab Hall A data





- Observable given by Interference Term between DVCS (a) and BH(b):

$$d\sigma^{\rightarrow} - d\sigma^{\leftarrow} \propto \sin\phi \left[F_1(\Delta^2)\mathcal{H} + \frac{x}{2-x}(F_1+F_2)\tilde{\mathcal{H}} + \frac{\Delta^2}{M^2}F_2(\Delta^2)\mathcal{E} \right]$$

$$\mathcal{H} = \sum_q e_q^2 (H(\xi,\xi,\Delta^2) - H(-\xi,\xi,\Delta^2))$$

3. Nuclei

Deuteron: New sum rules

S.L., S.K.Taneja

$$\frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ix(Pz)}\langle p'|\bar{q}(-\tfrac{1}{2}z)\not{n}_-\,q(\tfrac{1}{2}z)\,|p\rangle\Big|_{z=\lambda n_-} = -(\epsilon'^*\epsilon)\,H_1$$
$$+\frac{(\epsilon n_-)(\epsilon'^*P)+(\epsilon'^*n_-)(\epsilon P)}{Pn_-}H_2-\frac{2(\epsilon P)(\epsilon'^*P)}{m^2}H_3$$
$$+\frac{(\epsilon n_-)(\epsilon'^*P)-(\epsilon'^*n_-)(\epsilon P)}{Pn_-}H_4$$
$$+\left[m^2\frac{(\epsilon n_-)(\epsilon'^*n_-)}{(Pn_-)^2}+\frac{1}{3}(\epsilon'^*\epsilon)\right]H_5\,,$$

Cano and Pire (2001)

Form Factors
$$\int_{-1}^{1}dx\,H_i(x,\xi,t)=G_i(t)\qquad(i=1,2,3)$$

$$G_C=G_1+\frac{2}{3}\eta\,G_Q,$$
$$G_Q=G_1-G_2+(1+\eta)G_3,$$
$$G_M=G_2$$

Energy momentum tensor

$$\langle p'|\theta^{\mu\nu}|p\rangle = -\frac{1}{2}\left[P^\mu P^\nu - \frac{g^{\mu\nu}}{4}P^2\right](\epsilon'^*\epsilon)G_{1,2}(t) - \frac{1}{4}\left[P^\mu P^\nu - \frac{g^{\mu\nu}}{4}P^2\right]\frac{(\epsilon P)(\epsilon'^* P)}{M^2}G_{2,2}(t) \quad (2)$$

$$-\frac{1}{2}\left[\Delta^\mu\Delta^\nu - \frac{g^{\mu\nu}}{4}\Delta^2\right](\epsilon'^*\epsilon)G_{3,2}(t) - \frac{1}{4}\left[\Delta^\mu\Delta^\nu - \frac{g^{\mu\nu}}{4}\Delta^2\right]\frac{(\epsilon P)(\epsilon'^* P)}{M^2}G_{4,2}(t)$$

$$+\frac{1}{4}\left[\left(\epsilon'^{*\mu}(\epsilon P) + \epsilon^\mu(\epsilon'^* P)\right)P^\nu + \mu\leftrightarrow\nu - g^{\mu\nu}(\epsilon P)(\epsilon'^* P)\right]G_{5,2}(t)$$

$$+\left[\left(\epsilon'^{*\mu}(\epsilon P) - \epsilon^\mu(\epsilon'^* P)\right)\Delta^\nu + \mu\leftrightarrow\nu + g^{\mu\nu}(\epsilon P)(\epsilon'^* P) - \left(\epsilon'^{*\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu\right)\Delta^2 + \frac{g^{\mu\nu}}{2}(\epsilon'^*\epsilon)\Delta^2\right]G_{6,2}(t)$$

.. and relation with deuteron GPDs:

$$\int dx\, x H_1(x,\xi,t) - \frac{1}{3}\int dx\, x H_5(x,\xi,t) = G_{1,2}(t) + \xi^2 G_{3,2}(t)$$

$$\int dx\, x H_2(x,\xi,t) = G_{5,2}(t)$$

$$\int dx\, x H_3(x,\xi,t) = G_{2,2}(t) + \xi^2 G_{4,2}(t)$$

$$\frac{1}{4\xi}\int dx\, x H_4(x,\xi,t) = \frac{M^2}{t}\int dx\, x H_5(x,\xi,t) = G_{6,2}(t)$$

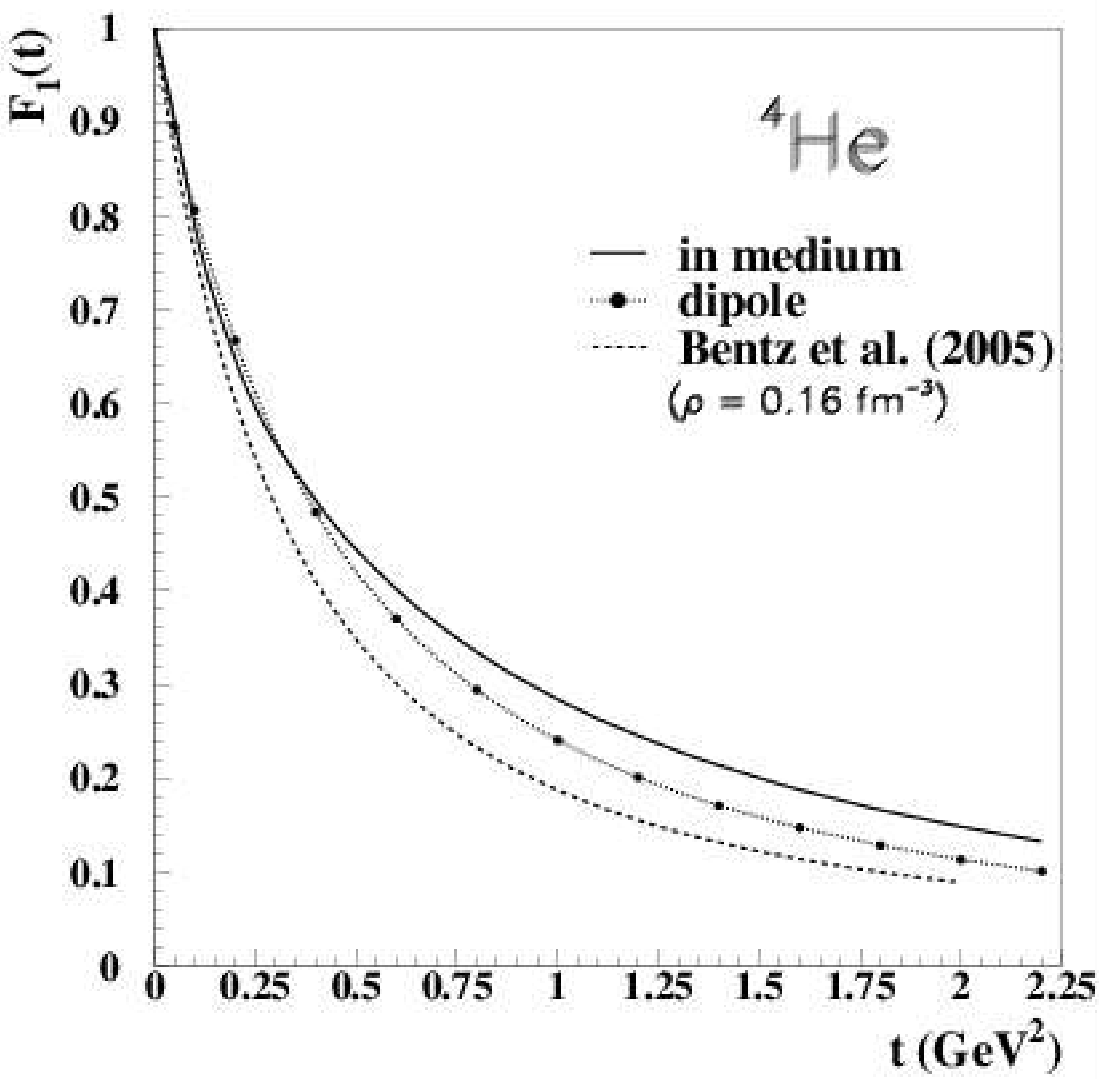
...inserting the energy momentum tensor in: $\langle p'| \int d^3x(\vec{x} \times \vec{T}^{0i}_{q,g})_z |p\rangle$

$$\Downarrow$$

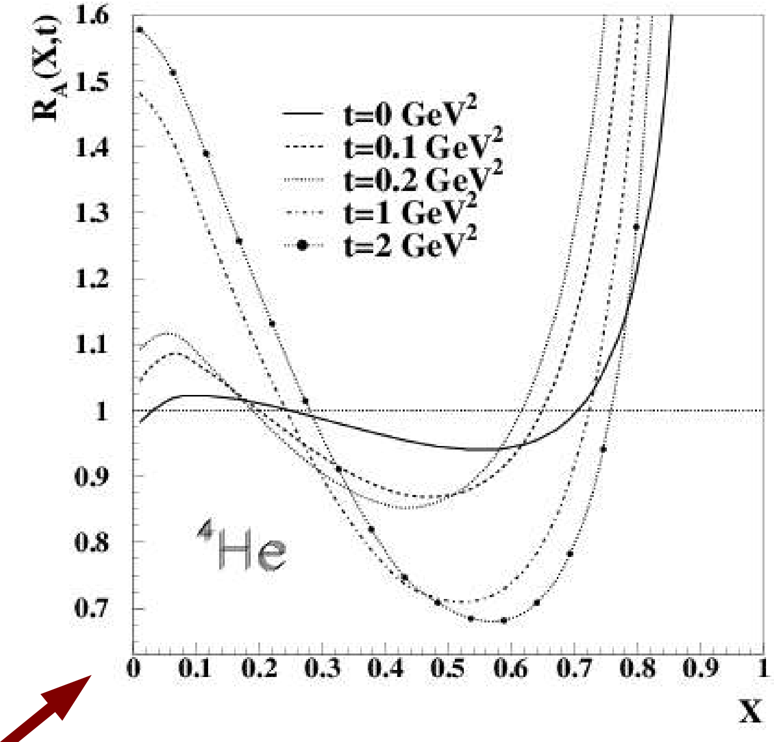
$$J_q = \frac{1}{2} \int dX\, X\, H_2^q(X,0,0) \equiv \frac{1}{2} G_{5,2}(0)$$

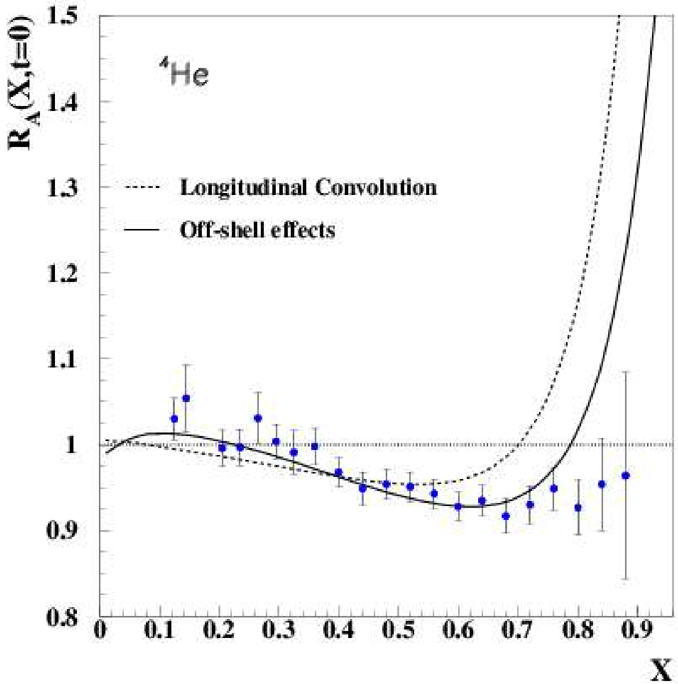
An essential piece of information for extracting quarks angular momentum!

4He: nuclear GPDs as tools to study in medium modifications

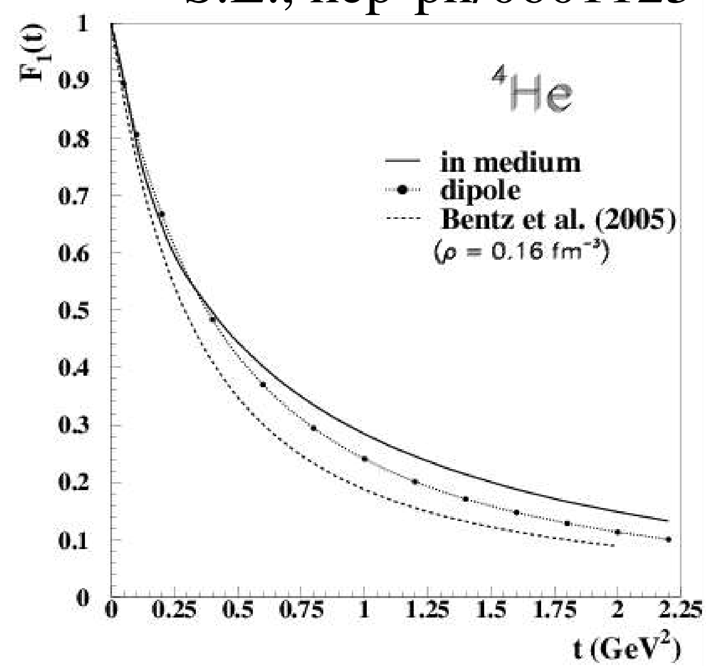


S.L., S.K. Taneja,
PRC72, 032201 (2005)

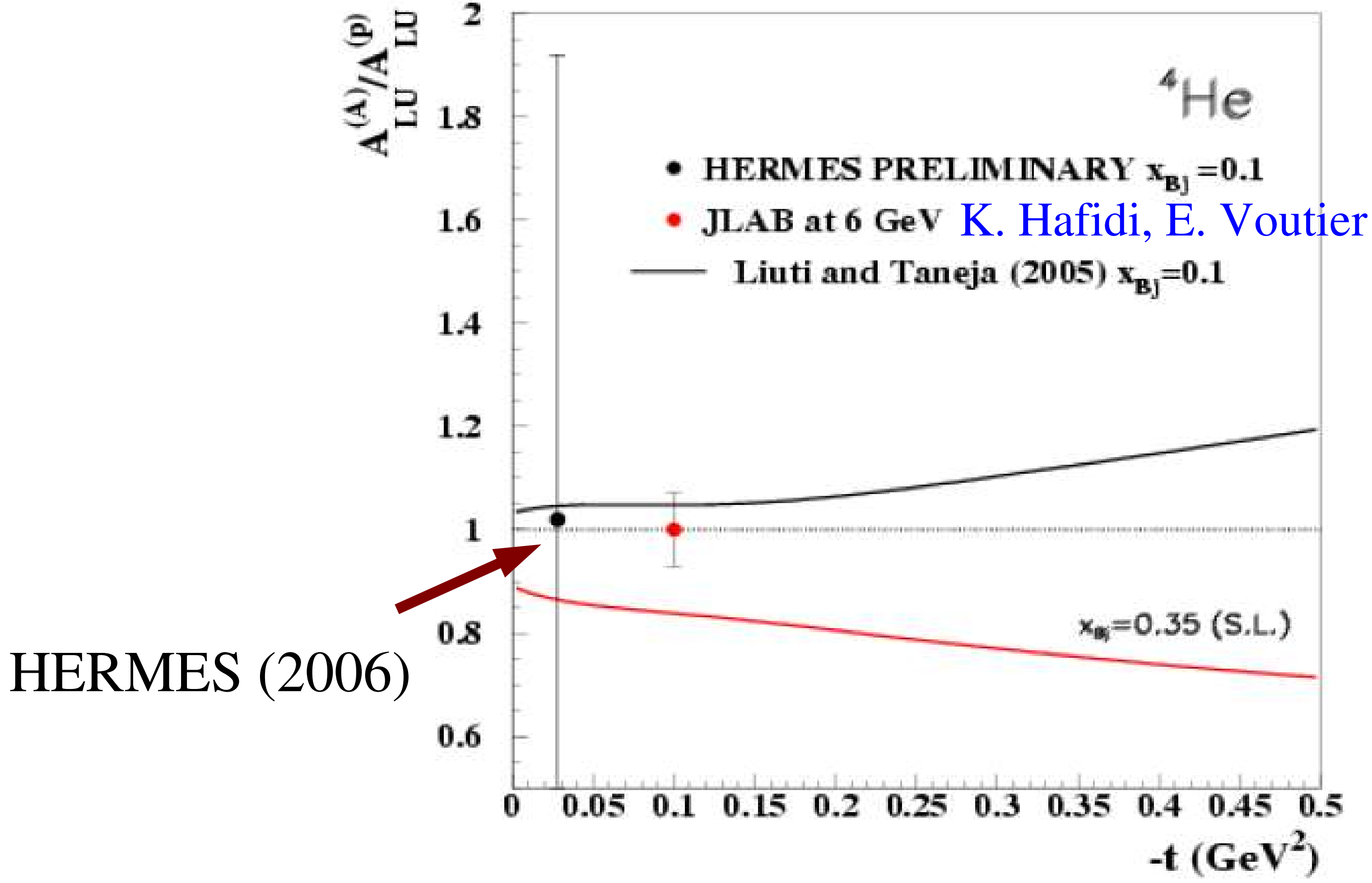




S.L., hep-ph/0601125



Experiments on ^{4}He are feasible at Jlab:



Conclusions

- We presented a method to extract GPDs from available experimental data on inclusive experiments, using constraints from lattice QCD.
- Our analysis is a first attempt to obtain a model independent view of the behavior of GPDs
- Higher "n" lattice moments along with a validation of the chiral extrapolation methods used so far, are crucial for future extractions of GPDs from experiment
- Nuclei are interesting! Deuteron Sum Rule, in medium form factors .....