Hard-exclusive processes and Transition Distribution Amplitudes

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JLab, USA

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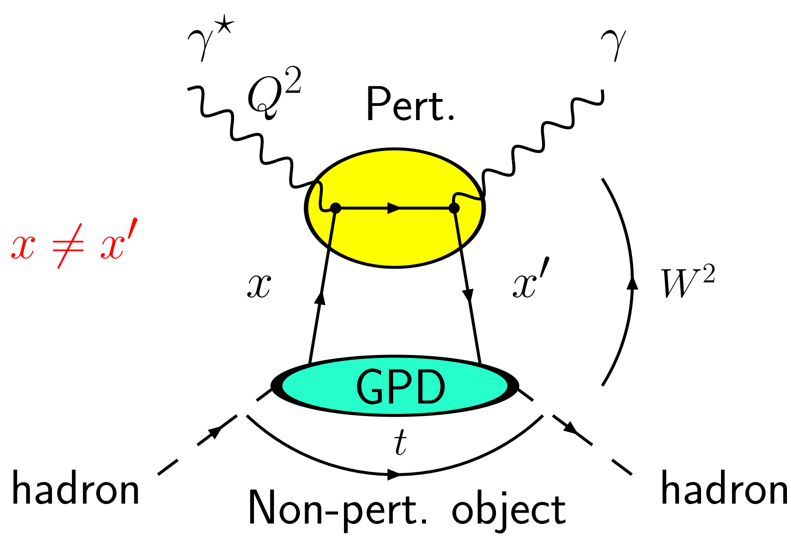
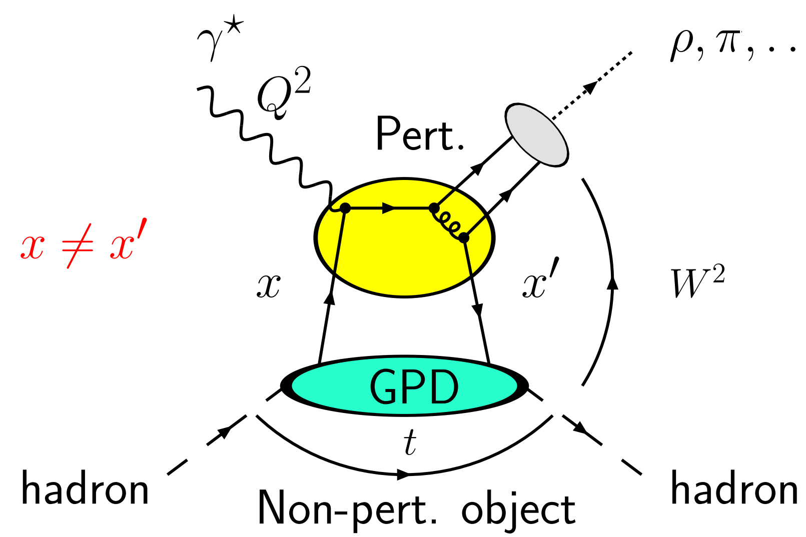
in collaboration with B. Pire and L. Szymanowski

Reminder on Generalised Parton Distributions

DVCS



Mesons Production



➩ Factorisation between the hard part (perturbatively calculable) and the soft part (non-perturbative) demonstrated for

$$Q^2 \to \infty,\ x_B = \frac{Q^2}{Q^2+W^2} \text{ fixed and } t \ll \text{ fixed}$$

TDAs : transition distribution amplitudes

B. Pire, L. Szymanowski, PRD 71 :111501,2005 ; PLB 622 :83,2005.

➯ For $u \ll$ DVCS, the non-perturbative part does not describe anymore a $H \to H$ transition, but rather

a hadron-photon or baryon-meson transition.

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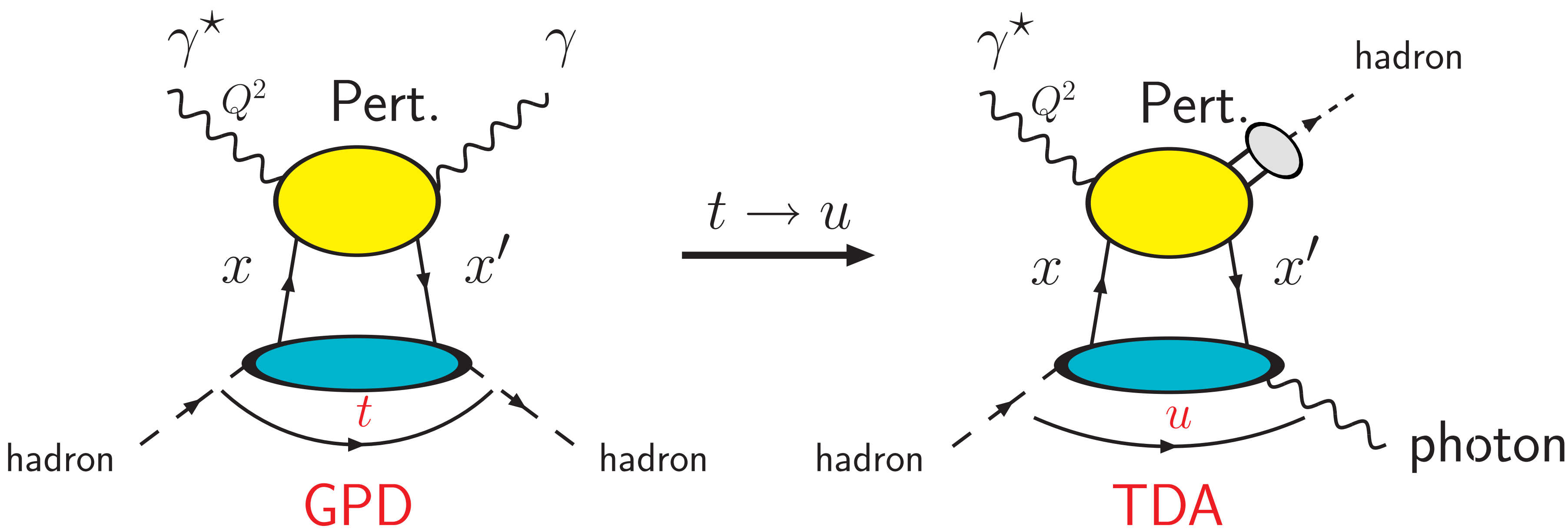


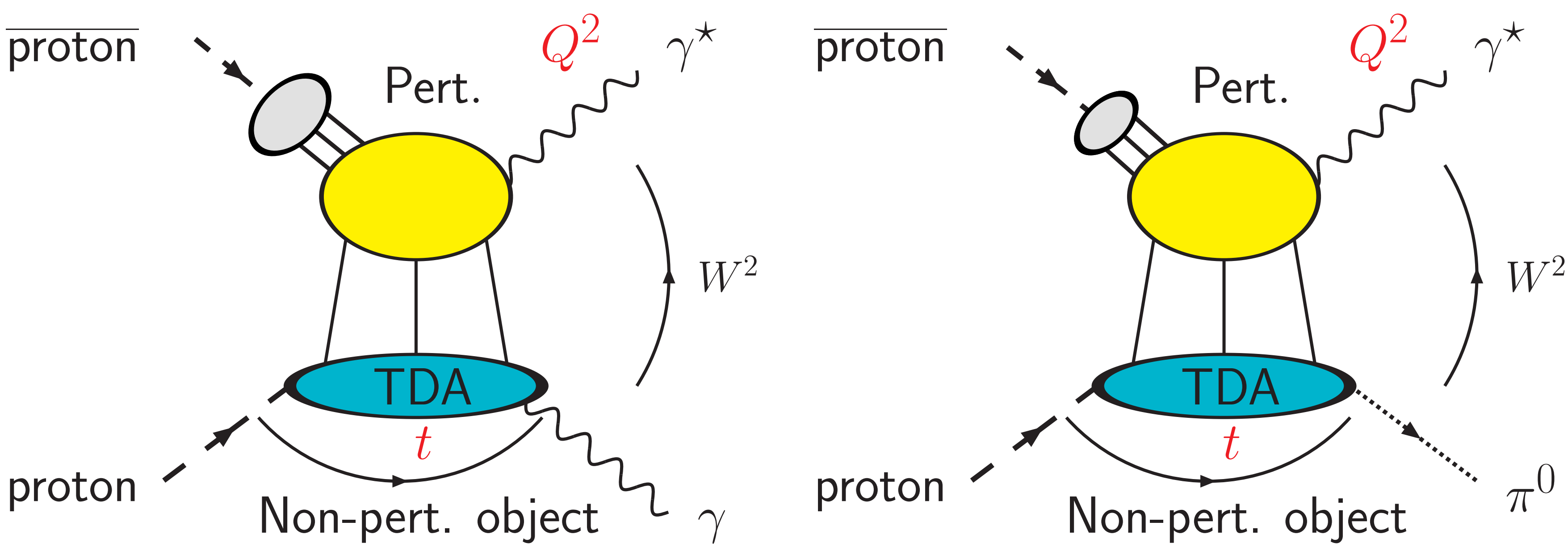
TDAs : transition distribution amplitudes

B. Pire, L. Szymanowski, PLB 622 :83,2005.
J.P. Lansberg, B. Pire, L. Szymanowski, in preparation.

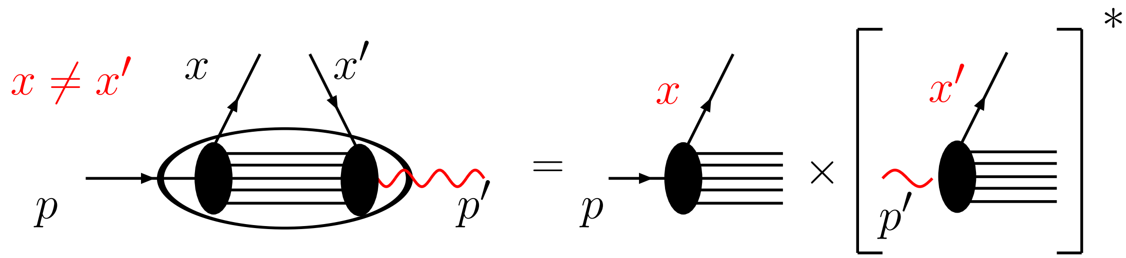
Also appear in exclusive $p\bar{p} \to \gamma^\star\gamma$ and $p\bar{p} \to \gamma^\star\pi^0$ reactions at $t \ll$ (GSI)

➯ Large Q^2 would provide us with a hard scale :

$\to$ perturbative expansion

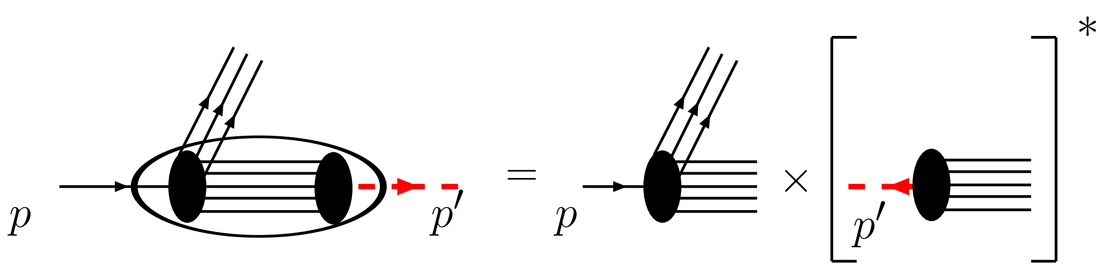


Interpretation of the TDAs



➯ The mesonic TDAs possess an interpretation at the amplitude level and provide with information on correlations between a meson DA and a photon DA

whereas



➯ The baryonic TDAs rather provide information on how one can find a meson or a photon in the baryon

TDAs vs GPDs : meson case

	GPDs	TDAs
Matrix elements	$\langle M(p')\|\Phi^\dagger(z)\Phi(0)\|M(p)\rangle$	$\langle \gamma(p',\varepsilon)\|\Phi^\dagger(z)\Phi(0)\|M(p)\rangle$
Diagonal limit $\xi \to 0,\ t \to 0$	GPDs $\to$ PDFs $H^q(x,0,0) = q(x)$	N/A
Sum rules : $\int dx$ $\to$ local operator	$\int dx H(x,\xi,t) = F(t)$	$\int dx T(x,\xi,t) = F_{A\to B}(t)$

➪ In view of the sum rules, both GPDs and TDAs are such that their integral on x is independent of ξ !

➟ possible modelling of the TDAs through double distributions (cf. Radyushkin)

Models for the mesonic TDAs

➠ **Double distributions :**

JPL, B. Pire, L. Szymanowski, PRD 73 :074014,2006.

B. Tiburzi PRD 72 :094001,2005.

➠ **Spectral Quark Model :**

W. Broniowski, E. Ruiz Arriola, PLB 649 :49,2007

➠ **NJL : S. Noguera *et al.*, on-going work**

➠ **BSE and DSE : used for PDFs ; previous studies could be extended**

e.g. M.B. Hecht, Craig D. Roberts, S.M. Schmidt, PRC 63 :025213,2001

➠ **Lattice : as for GPDs, TDA moments are certainly calculable**

e.g. QCDSF/UKQCD Collab, D. Brömmel *et al.*, PoS LAT2005 :360,2006.

TDAs : baryonic case

B. Pire, L. Szymanowski, PLB 622 :83,2005.
JPL, B. Pire, L. Szymanowski,PRD 75 :074004,2007.

➫ Both for Baryon $\rightarrow$ Meson and Baryon $\rightarrow$ photon,

3 quarks should be exchanged in the t-channel

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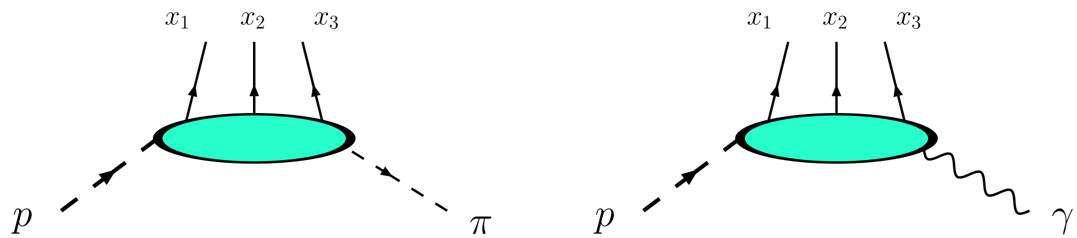

proton $\to \pi$ proton $\to \gamma$

- More than the two regions ERBL and DGLAP

- Sum rules

$\to \xi$-independence of the moments of the TDA

  - QUADRUPLE distributions : being worked out
  - Diquark picture and double distribution ?

would suit some regions only ?

- Closest object : Baryon Distribution Amplitude : $\to$ SOFT LIMIT ?

$p \to \pi$: parametrisation

➫ $p \to \pi$ **(at Leading twist accuracy)**

➟ $\Delta_T = 0$ **: 3 TDAs** ($3 \times p(\uparrow) \to uud(\uparrow\uparrow\downarrow) + \pi$)

TDA

$$4\langle \pi^0 |\, \epsilon^{ijk} u^i_\alpha(z_1 n) u^j_\beta(z_2 n) d^k_\gamma(z_3 n)\, | p\rangle \propto$$

$$\Big[V_1^{\pi^0}(x_i,\xi,\Delta^2)(\not p\, C)_{\alpha\beta}(N)_\gamma + A_1^{\pi^0}(x_i,\xi,\Delta^2)(\not p\,\gamma^5 C)_{\alpha\beta}(\gamma^5 N)_\gamma + T_1^{\pi^0}(x_i,\xi,\Delta^2)(\sigma_{\rho p} C)_{\alpha\beta}(\gamma^\rho N)_\gamma \Big]$$

DA (Chernyak-Zhitnitsky)

$$4\langle 0 |\epsilon^{ijk} u^i_\alpha(z_1 n) u^j_\beta(z_2 n) d^k_\gamma(z_3 n) | p\rangle \propto$$

$$\Big[V(x_i)(\not p C)_{\alpha\beta}(\gamma^5 N)_\gamma + A(x_i)(\not p \gamma^5 C)_{\alpha\beta} N_\gamma + T(x_i)(i\sigma_{\rho p}\, C)_{\alpha\beta}(\gamma^\rho \gamma^5 N)_\gamma \Big]$$

$p \longrightarrow \pi$: parametrisation

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➟ **$\Delta_T = 0$: 3 TDAs** ($3 \times p(\uparrow) \rightarrow uud(\uparrow\uparrow\downarrow) + \pi$)

TDA

$$4\langle \pi^0 | \, \epsilon^{ijk} u^i_\alpha(z_1 n) u^j_\beta(z_2 n) d^k_\gamma(z_3 n) \, |p\rangle \propto \Big[V_1^{\pi^0}(x_i,\xi,\Delta^2)(\not p \, C)_{\alpha\beta}(N)_\gamma + A_1^{\pi^0}(x_i,\xi,\Delta^2)(\not p \, \gamma^5 C)_{\alpha\beta}(\gamma^5 N)_\gamma + T_1^{\pi^0}(x_i,\xi,\Delta^2)(\sigma_{\rho p} C)_{\alpha\beta}(\gamma^\rho N)_\gamma \Big]$$

DA (Chernyak-Zhitnitsky)

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➟ **$\Delta_T \neq 0$: 8 TDAs** ($\frac{1}{2} \times 2 \times (2 \times 2 \times 2) \times 1$)

$$4\langle \pi^0(p_\pi) | \, \epsilon^{ijk} u^i_\alpha(z_1 n) u^j_\beta(z_2 n) d^k_\gamma(z_3 n) \, |p(p_1,s)\rangle = \frac{i f_N}{f_\pi} \times$$
$$\Big[V_1^{\pi^0}(x_i,\xi,\Delta^2)(\not p \, C)_{\alpha\beta}(N^+)_\gamma + V_2^{\pi^0}(x_i,\xi,\Delta^2)(\not p \, C)_{\alpha\beta}(\not\Delta_T N^+)_\gamma$$
$$+ A_1^{\pi^0}(x_i,\xi,\Delta^2)(\not p \, \gamma^5 C)_{\alpha\beta}(\gamma^5 N^+)_\gamma + A_2^{\pi^0}(x_i,\xi,\Delta^2)(\not p \, \gamma^5 C)_{\alpha\beta}(\gamma^5 \not\Delta_T N^+)_\gamma$$
$$+ T_1^{\pi^0}(x_i,\xi,\Delta^2)(\sigma_{p\mu} C)_{\alpha\beta}(\gamma^\mu N^+)_\gamma + T_2^{\pi^0}(x_i,\xi,\Delta^2)(\sigma_{p\Delta_T} C)_{\alpha\beta}(N^+)_\gamma$$
$$+ T_3^{\pi^0}(x_i,\xi,\Delta^2)(\sigma_{p\mu} C)_{\alpha\beta}(\sigma^{\mu\Delta_T} N^+)_\gamma + T_4^{\pi^0}(x_i,\xi,\Delta^2)(\sigma_{p\Delta_T} C)_{\alpha\beta}(\not\Delta_T N^+)_\gamma \Big]$$

Soft pion limit for proton to pion TDAs

➟ **soft pion limit :** $\xi \to 1$ & $\Delta_T \to 0 \Rightarrow P \to p$

$$\langle \pi^a(k)|\mathcal{O}|p(p,s)\rangle = -\frac{i}{f_\pi}\langle 0|[Q_5^a,\mathcal{O}]|p(p,s)\rangle + \frac{ig_A}{4f_\pi p\cdot k}\sum_{s'}\bar{u}(p,s')\not{k}\gamma_5\tau^a u(p,s)\langle 0|\mathcal{O}|p(p,s')\rangle$$

➟ Using $[Q_5^b,\psi] = -\frac{\tau^b}{2}\gamma^5\psi$, the baryonic DAs appear and we get the following limiting values :

$$V_1^{\pi^0}(2x_1,2x_2,2x_3,\xi\to 1) \to V(x_1,x_2,x_3)$$
$$A_1^{\pi^0}(2x_1,2x_2,2x_3,\xi\to 1) \to A(x_1,x_2,x_3)$$
$$T_1^{\pi^0}(2x_1,2x_2,2x_3,\xi\to 1) \to 3T(x_1,x_2,x_3)$$

➟ Same relations obtained for the proton-pion DAs $\langle 0|\mathcal{O}|\pi(k)p(p,s)\rangle$

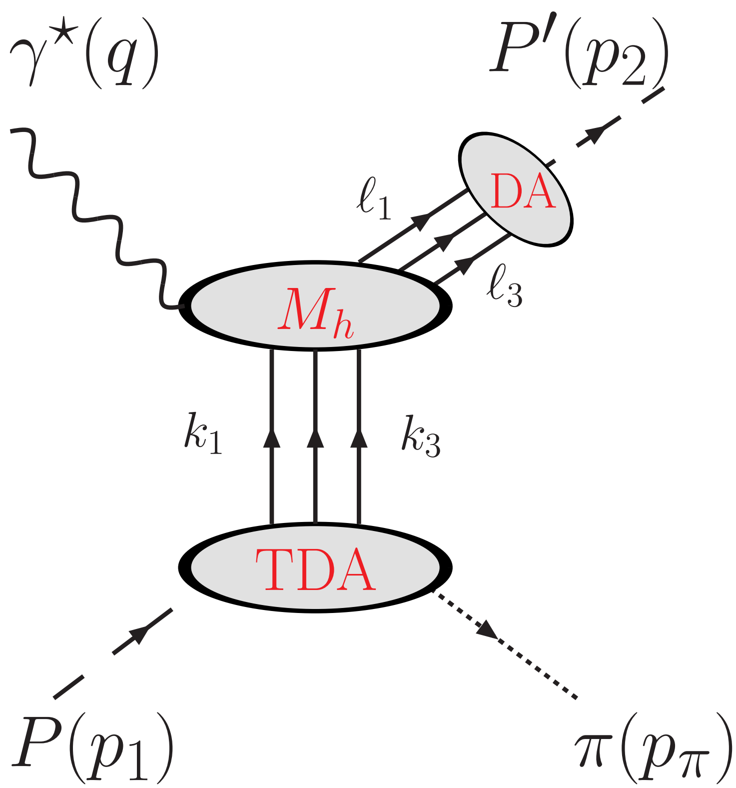
V.M Braun *et al.* **PRD75 :014021,2007**

Application to backward electroproduction of a pion

JPL, B. Pire, L. Szymanowski,PRD 75 :074004,2007.

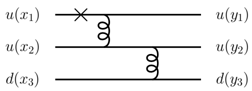
- First evaluation : valid at large ξ

  i.e. small pion energy
- TDAs extrapolated from their limiting value at $\xi = 1$ $(E_\pi \to 0)$
- DGLAP contribution neglected : safe for large ξ

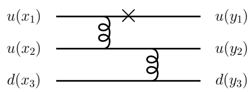


Perturbative part : M_h for $\gamma^\star p \to p\pi^0$ at $\Delta_T = 0$

JPL, B. Pire, L. Szymanowski,PRD 75 :074004,2007.



$$\frac{-Q_u(2\xi)^2[(V_1^{p\pi^0}-A_1^{p\pi^0})(V^p-A^p)+4T_1^{p\pi^0}T^p]}{(2\xi-x_1-i\epsilon)^2(x_3-i\epsilon)(1-y_1)^2y_3}$$



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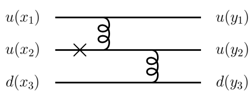
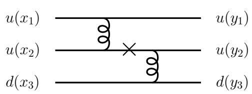
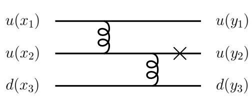


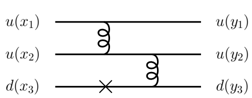
$$\frac{Q_u(2\xi)^2[4T_1^{p\pi^0}T^p]}{(x_1-i\epsilon)(2\xi-x_2-i\epsilon)(x_3-i\epsilon)y_1(1-y_1)y_3}$$



$$\frac{-Q_u(2\xi)^2[(V_1^{p\pi^0}-A_1^{p\pi^0})(V^p-A^p)]}{(x_1-i\epsilon)(2\xi-x_3-i\epsilon)(x_3-i\epsilon)y_1(1-y_1)y_3}$$



$$\frac{Q_u(2\xi)^2[(V_1^{p\pi^0}+A_1^{p\pi^0})(V^p+A^p)]}{(x_1-i\epsilon)(2\xi-x_3-i\epsilon)(x_3-i\epsilon)y_1(1-y_2)y_3}$$



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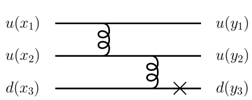
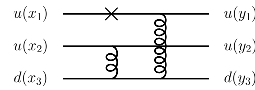


$$\frac{-Q_d(2\xi)^2[2(V_1^{p\pi^0}V^p+A_1^{p\pi^0}A^p)]}{(x_1-i\epsilon)(2\xi-x_3-i\epsilon)^2y_1(1-y_3)^2}$$



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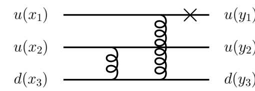
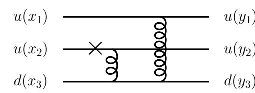


$$\frac{-Q_u(2\xi)^2[(V_1^{p\pi^0}-A_1^{p\pi^0})(V^p-A^p)+4T_1^{p\pi^0}T^p]}{(2\xi-x_1-i\epsilon)^2(x_2-i\epsilon)(1-y_1)^2y_2}$$



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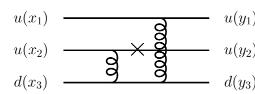

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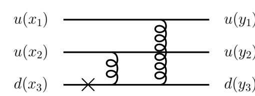
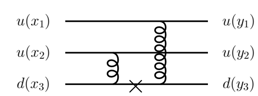


$$\frac{Q_d(2\xi)^2[(V_1^{p\pi^0}+A_1^{p\pi^0})(V^p+A^p)]}{(x_1-i\epsilon)(x_2-i\epsilon)(2\xi-x_3-i\epsilon)y_1(1-y_2)y_2}$$



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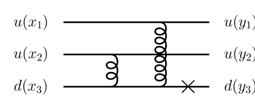


$$\frac{Q_d(2\xi)^2[(V_1^{p\pi^0}-A_1^{p\pi^0})(V^p-A^p)]}{(x_1-i\epsilon)(2\xi-x_1-i\epsilon)(x_2-i\epsilon)y_1y_2(1-y_3)}$$

Some Remarks about the scaling

dominance of perturbative mechanism over Feynman one ?

➩ Late scaling for proton Form Factors

does not imply late scaling for other channels

➩ The Asymptotic DAs, $\phi_N = 120 x_1 x_2 x_3$, lead to

vanishing $G_M^p(Q^2)$ at the leading twist accuracy

This is not the case for other amplitudes

➩ The Feynman mechanism seems to be important for $G_M^p(Q^2)$

it does not imply that it is so for other amplitudes

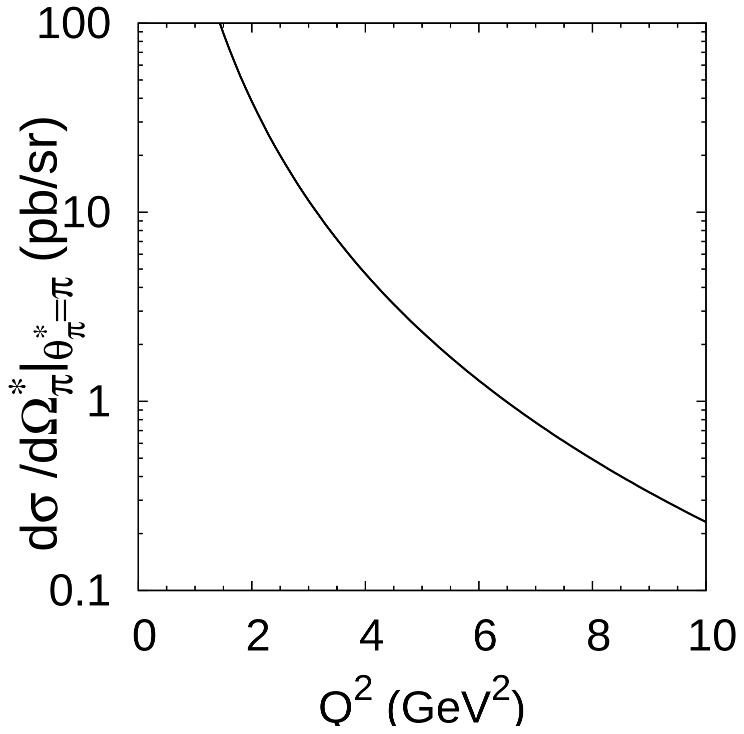
➩ Only experiments can tell

Application to backward electroproduction of a pion

JPL, B. Pire, L. Szymanowski, PRD 75 :074004, 2007.

➠ **The (leading-twist) amplitude reads :**

$$\mathcal{M}^{\lambda}_{s_1 s_2} = -i\frac{(4\pi\alpha_s)^2\sqrt{4\pi\alpha_{em}}f_N^2}{54 f_\pi Q^4}\bar{u}(p_2,s_2)\not{\epsilon}(\lambda)\gamma^5 u(p_1,s_1)\int\limits_{-1+\xi}^{1+\xi} d^3x \int\limits_{0}^{1} d^3y \left(2\sum_{\alpha=1}^{7} T_\alpha + \sum_{\alpha=8}^{14} T_\alpha\right)$$



➠ **Data exist (at least) from JLab**

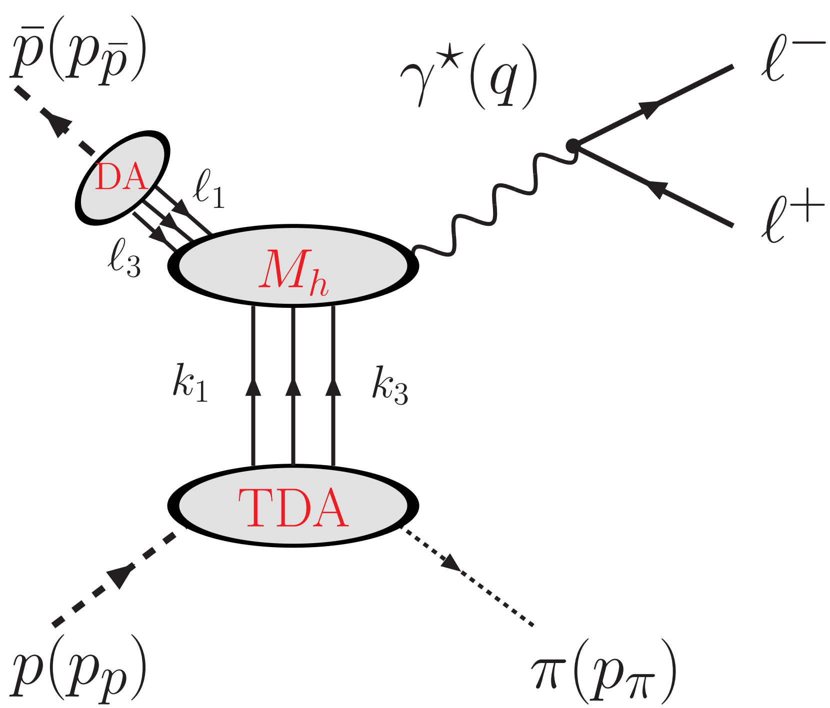
➠ **We need more information about the TDAs**

Application to proton-antiproton annihilations

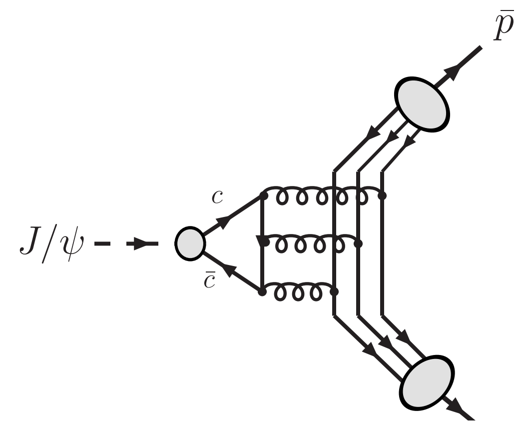
JPL, B. Pire, L. Szymanowski, in preparation

- $\bar{p}p \to \gamma^\star \pi^0$ **can be studied by PANDA**
- **Same TDAs as for backward electroproduction**

In the GPD case, after crossing, we have to deal with GDAs



Future application to charmonium production

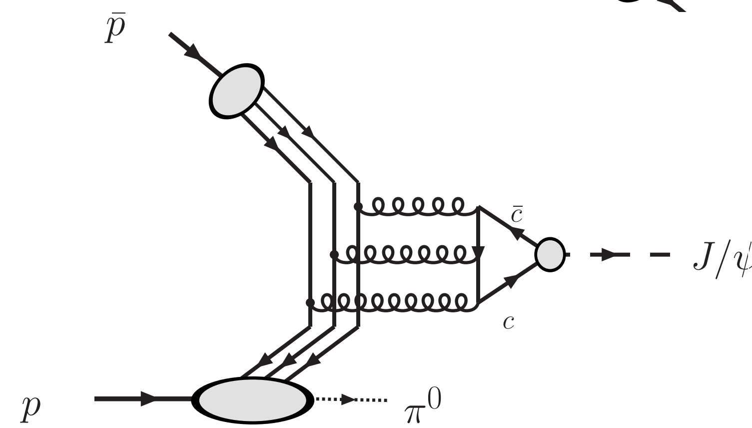


➩ J/ψ **decay in proton antiproton**

well accounted by the perturbative mechanism

➩ $p\bar{p} \to J/\psi\pi^0$ **at small** t

can be described likewise



➟ **this process is used to search for new charmonium states (**h_c**,. . .)**

➟ **will be extensively studied at GSI**

➟ **For now, comparisons are possible with previous calculations**

Soft pion limit **M.K. Gaillard,** *et al.***, PLB 110 :489,1982.**

T.Barnes, X.Li, PRD 75 :054018,2007

Lattice calculations

Gavela, King, Sachrajda, Martinelli,...

a lattice computation of proton decay amplitudes

Nucl.Phys.B312 :269,1989

➡ **Calculation of the matrix elements for the GUT decays**

$$p \to \pi^0 e^+ \qquad p \to \pi^+ \bar{\nu} \qquad p \to K^0 + lepton$$

➡ **Evaluation of the two matrix elements**

$$\epsilon^{ijk} \langle \pi^0 | (u^i C d^j) u^k_\gamma | P \rangle = A_1 N_\gamma \quad \epsilon^{ijk} \langle \pi^0 | (u^i C \gamma_5 d^j)(\gamma_5 u^k)_\gamma | P \rangle = A_2 N_\gamma$$

➡ **Update of this study would be very useful**
- ➡ **would fix the normalisation of the TDAs via Sum Rules**
- ➡ **would give information on their t-dependence**

Conclusions and outlooks

➡ Model-independent predictions : baryon case

- **Scaling law for the amplitude :**

$$\mathcal{M}(Q^2) \propto \frac{\alpha_s^2(Q^2)}{Q^4}$$

- **Approximate Q^2-independence of the ratios**

$$\frac{\mathcal{M}(\gamma^* p \to p\pi)}{\mathcal{M}(\gamma^* p \to p\gamma)}, \quad \frac{\mathcal{M}(\gamma^* p \to p\gamma)}{\mathcal{M}(\gamma^* p \to p)} \text{ and } \frac{\frac{d\sigma(p\bar{p}\to\ell^+\ell^-\pi^0)}{dQ^2}}{\frac{d\sigma(p\bar{p}\to\ell^+\ell^-)}{dQ^2}}$$

- **Dominance of γ_T^* emission in $p\bar{p} \to \gamma^*\pi^0 \to \ell^+\ell^-\pi^0$**

  Dilepton angular dependence : $1 + \cos^2\theta$

Conclusions and outlooks

➡ **Quantitative predictions require models**

➟ **Meson case :**

double distribution : ok ; models used for GPDs should be suitable

two already used

➟ **Baryon case :**

4-ple distribution : on-going work...

➡ **Lattice computations can play a key role**

➡ **Data needed to test the picture**

and to extract physical information

➪ ...expected from

➟ JLab : backward electroproduction of mesons and backward DVCS

➟ GSI : $p\bar{p} \to \gamma^\star \pi^0$, $p\bar{p} \to J/\psi \pi^0$, $p\bar{p} \to \gamma^\star \gamma$, . . .

➟ Hermes (DVCS on pion)

➟ B-factories ($\gamma^\star \gamma \to MM$) data extraction possible

DVCS on pion

D. Amrath, M. Diehl, JPL, in preparation

➾ **To be analysed by HERMES through :** $ep \to e'\gamma\pi^+n$

- **Non-trivial kinematics ; DVCS subset of** $2 \to 4$ **process**
- **Cannot trivially distinguish between the small** t **(GPD) and small** u **(TDA) regions in the LAB frame**