$\pi N \to$ Multi-πN Scattering in the $1/N_c$ Expansion

(HJK, Richard Lebed, **Phys.Rev.D75:016002,2007**)

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Outline

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3. Theoretical Results
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Baryon resonances in QCD

Baryon resonances pose a peculiar conundrum:

- continuum contribution affect Argand diagram for meson-baryon scattering or photoproduction processes
- states with well-defined masses and quantum numbers, occur with a regularity → a spectrum from some symmetry structure

One approach: construct scattering amplitudes that relate channels of different I, J, using other quantum numbers that emerge for QCD in the large N_c limit

Motivation: Skyrme/Chiral Soliton Model (1980's)

Scattering Amplitude Characteristics

1. The leading-order amplitudes in the $1/N_c$ expansion, expressed in terms of t-channel quantum numbers, have $I_t = J_t$ (holds for 3-flavor as well as 2-flavor processes)

   (see M.P. Mattis and M. Mukerjee, Phys. Rev. Lett. **61**, 1344 (1988))

2. For finite-N_c processes: Amplitudes with $|I_t - J_t| = n$ are suppressed by at least $1/N_c^n$ compared to the leading order → systematic expansion in $1/N_c$: lowest order + corrections from higher-order effects

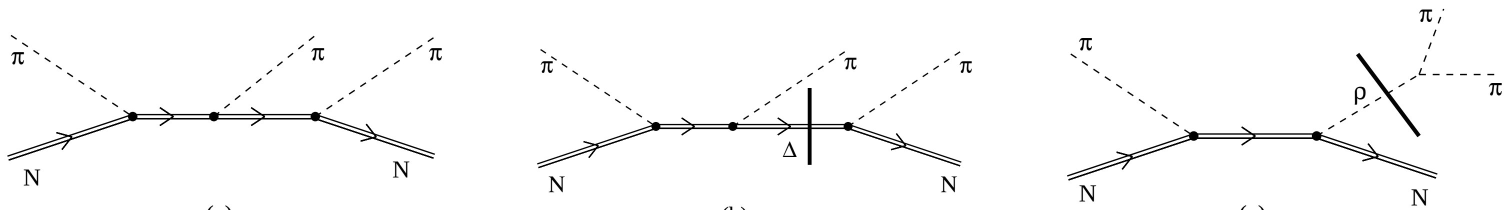
   (see for eq. T.D. Cohen, D.C. Dakin, A. Nellore, and R.F. Lebed, Phys. Rev. D **70**, 056004 (2004))

3. There exist linear relations among the scattering amplitudes in different channels → degeneracies among poles (resonance masses and widths)

HJK/Lebed Approach

- Up to recently: only Baryon+Meson $\longrightarrow$ Baryon+Meson $1/N_c$ amplitude analysis
- Want to extend to baryon resonances with multipion final state (2-flavor only)
- non-strange 3-flavor processes are cumbersome but tractable
- but our scattering amplitude formula only accommodate BM $\longrightarrow$ BM processes: must find a way to include multipion processes
- main goal: identifying the underlying pole structure by the presence/absence of certain decay channels (most incisive are η and mixed partial-wave $\pi\Delta$ final state)
- not good enough to predict numerical results for B.R.'s, need $1/N_c$ correction

N_c Power Counting



(a) (b) (c)

- $\pi N \to \pi N$ is $O(N_c^0)$
- $\pi N \to \pi\pi N$ is $O(N_c^{-1/2})$
- but can be $O(N_c^0)$ if it proceeds through:
  1. $\pi N \to \pi\Delta \to \pi(\pi N)$: Δ is stable for large N_c ($\Gamma \approx 1/N_c^2$)
     Real world: $N_c = 3$ and $\Gamma \approx 100$MeV, small compared to its mass
  2. $\pi N \to \rho N \to (\pi\pi)N$: ρ too has $\Gamma \approx 1/N_c^2$
- confident experiment can separate $\pi\Delta$ and ρN from $\pi\pi N$ background

$m + B \to m' + B'$ Scattering Amplitudes

$$
\begin{aligned}
S_{LL'SS'IJ} &= \sum_{K,\tilde{K},\tilde{K}'} [K]([R][R'][S][S'][\tilde{K}][\tilde{K}'])^{1/2} \\
&\times \begin{Bmatrix} L & i & \tilde{K} \\ S & R & s \\ J & I & K \end{Bmatrix} \begin{Bmatrix} L' & i' & \tilde{K}' \\ S' & R' & s' \\ J & I & K \end{Bmatrix} \\
&\times \tau_{K\tilde{K}\tilde{K}'LL'}
\end{aligned}
$$

- $s\,(s')$: spin of mesons $i\,(i')$: isospin of mesons
- $L\,(L')$: meson to baryon relative orbital angular momentum
- $R\,(R')$: baryon spin = isospin
- $S\,(S')$: total spin angular momentum (not including L/L') of the meson and baryon
- I, J: isospin, spin of the intermediate state
- $[X] = 2X + 1$
- $\{\quad\}$: 9j symbol $\to$ 6j (one zero) $\to$ 3j (two zeros)

More on Scattering Amplitudes Formula

- $\mathbf{K}$: Grand Spin $\equiv \mathbf{I}+\mathbf{J}$
- $\tilde{\mathbf{K}} \equiv \mathbf{i}+\mathbf{L}$, and $\tilde{\mathbf{K}}' \equiv \mathbf{i}'+\mathbf{L}'$ (so that $\mathbf{K}=\tilde{\mathbf{K}}+\mathbf{s}=\tilde{\mathbf{K}}'+\mathbf{s}'$)
- first derivation: chiral soliton model, but this amplitude is the result of Large N_c QCD limit, not depending on any model assumption (see appendix of Cohen and Lebed, Phys. Rev. D **67**, 096008 (2003))
- **The point**: more $S_{LL'SS'IJ}$ amplitudes than $\tau_{K\tilde{K}\tilde{K}'LL'}$ amplitudes
  1. linear relations among scattering amplitudes
  2. multiplets of baryon resonances with degenerate mass and width

  pole in $S_{LL'SS'IJ} \rightarrow$ pole in $\tau_{K\tilde{K}\tilde{K}'LL'} \rightarrow$ pole in other $S_{LL'SS'IJ}$'s
- poles/resonant poles depend only on K

Partial-wave amplitudes for positive-parity $N_{1/2}$ resonances in multipion processes (the πN final state is included for comparison). Expansions are given in terms of K amplitudes.

State	Poles	Partial Wave, K-Amplitudes		
$N^+_{1/2}$	$K = 0, 1$	$P_{11}^{(\pi N)(\eta N)}$	$=$	$-\frac{\sqrt{2}}{\sqrt{3}}\tau_{11111}$
		$P_{11}^{(\pi N)(\pi N)}$	$=$	$\frac{1}{3}\tau_{00011} + \frac{2}{3}\tau_{11111}$
		$P_{11}^{(\pi N)(\pi \Delta)}$	$=$	$\frac{\sqrt{2}}{3}\tau_{00011} - \frac{\sqrt{2}}{3}\tau_{11111}$
		$P_{11}^{(\pi N)(\omega N)_1}$	$=$	$\frac{1}{3}\tau_{00111} + \frac{2}{3}\tau_{11111}$
		$P_{11}^{(\pi N)(\omega N)_3}$	$=$	$\frac{\sqrt{2}}{3}\tau_{00111} - \frac{\sqrt{2}}{3}\tau_{11111}$
		$P_{11}^{(\pi N)(\rho N)_1}$	$=$	$\frac{\sqrt{2}}{3\sqrt{3}}\tau_{00111} - \frac{\sqrt{2}}{9}\tau_{11011}$ $+\frac{2\sqrt{10}}{9}\tau_{11211}$
		$P_{11}^{(\pi N)(\rho N)_3}$	$=$	$-\frac{1}{3\sqrt{3}}\tau_{00111} - \frac{4}{9}\tau_{11011}$ $+\frac{1}{\sqrt{3}}\tau_{11111} + \frac{\sqrt{5}}{9}\tau_{11211}$

$$L(L')_{2I2J}^{(initial)(final)_{2S'}}$$

Partial-wave amplitudes for negative-parity $N_{1/2}$ resonances in multipion processes (the πN final state is included for comparison). Expansions are given in terms of K amplitudes.

State	Poles	Partial Wave, K-Amplitudes		
$N^-_{1/2}$	$K=1$	$S_{11}^{(\pi N)(\eta N)}$	$=$	0
		$S_{11}^{(\pi N)(\pi N)}$	$=$	τ_{11100}
		$SD_{11}^{(\pi N)(\pi \Delta)}$	$=$	$-\tau_{11102}$
		$S_{11}^{(\pi N)(\omega N)_1}$	$=$	τ_{11000}
		$SD_{11}^{(\pi N)(\omega N)_3}$	$=$	$-\tau_{11202}$
		$S_{11}^{(\pi N)(\rho N)_1}$	$=$	$\sqrt{\frac{2}{3}}\tau_{11100}$
		$SD_{11}^{(\pi N)(\rho N)_3}$	$=$	$\frac{1}{\sqrt{6}}\tau_{11102}+\frac{1}{\sqrt{2}}\tau_{11202}$

Phenomenological Results

1. Consider only 3- or 4-star resonances as classified by the PDG
2. Association of resonances to poles labeled by K (determined by decay channels that occur prominently versus those that are absent or weak) seem robust, eq.:
   (a) $\pi N \to \eta N$ contains a single K amplitude [with $K = L$]
   (b) mixed partial wave $\pi N(L) \to \pi\Delta(L')$ contains a single K amplitude [with $K = \frac{1}{2}(L + L')$]
3. Prediction of the ratio of BRs between two decay channels is not always in accord with experiment, eq.: the ratios of πN to $\pi\Delta$ BR's at leading $[O(N_c^0)]$ order
4. But can easily be explained by $1/N_c$ corrections
   (see Cohen, Dakin, Nellore and Lebed in PRD **69**, 056001 (2004) next-to-leading order amplitude relations for πN to $\pi\Delta$)

Difficulties

1. Overall analysis does not yet include $1/N_c$ corrections: impossible to draw any conclusion from such mountain of information

2. Mesons involved (π, η, ρ and ω) in these scatterings are widely different:

   - mass: 140 MeV $\rightarrow$ 783 MeV, compensated in phase space using simple two-body decay formula

   - π and η: Pseudo-Nambu-Goldstone boson of spontaneous χSB
     ρ and ω: vector mesons with masses set by QCD scale

3. No chiral symmetry analysis

4. Data (PDG) is filled with internal contradictions: another reason we did not include $1/N_c$ correction

The following is example analysis for one channel:

$N^{+}_{1/2}$ (P_{11}): $N(1440)$ (the Roper) and $N(1710)$

- Our calculation →two poles: $K=0$ and $K=1$
- $N(1440)$ has a very small, $(0\pm1)\%$, ηN BR
- $N(1710)$ has a small but nonnegligible ηN BR, $(6.2 \pm 1.0)\%$
- Comparing this to our tabulated result suggests that the Roper is a $K=0$ pole and the $N(1710)$ is a $K=1$ pole
- Agrees well with the Roper as a radial excitation of ground-state N, which is a (nonresonant) $K=0$ state
- But leading-order prediction of $\pi N \to \pi N$ to $\pi N \to \pi \Delta$ BR's does not agree well with experiment
- As mentioned above, this discrepancy can be cured by $1/N_c$-suppressed amplitudes

Summary

1. QCD symmetry in Large N_c limit: Baryon resonances multiplets emerge from scattering amplitudes
2. The scattering amplitude approach can be extended to include multipion final state processes by introducing stable intermediate stated in Large N_c limit
3. Pole determination from absence/presence of certain decay channels is robust
4. Prediction of ratio of B.R.'s of two channels is not always accurate but can be accommodated by including $1/N_c$ correction

$N^{-}_{1/2}$ (S_{11}): $N(1535)$ and $N(1650)$.

1. Both resonances have significant ηN BR, $(53\pm1)\%$ and 3–10%
2. our leading-order results predict them to be zero
3. The $\eta N \to \eta N$ amplitude is purely $K=0$ at leading order, strongly suggesting that $N(1535)$ is a $K=0$ pole that has a πN coupling through $O(1/N_c)$ mixing to $K=1$, while $N(1650)$ is a $K=1$ pole that has an ηN coupling through $O(1/N_c)$ mixing to $K=0$. Further analysis for $\pi\Delta$ and ρN channels supports this assignment. The $K=1$ $\pi\Delta$ mixed partial wave SD_{11} has a BR of $< 1\%$ for $N(1535)$ but 1–7% for $N(1650)$. Moreover, the ρ and ω couplings are purely $K=1$ at leading order, while the $N(1535)$ has a ρN BR of $< 4\%$, the $N(1650)$ has 4–12% (although available phase space may be an important factor for these cases)