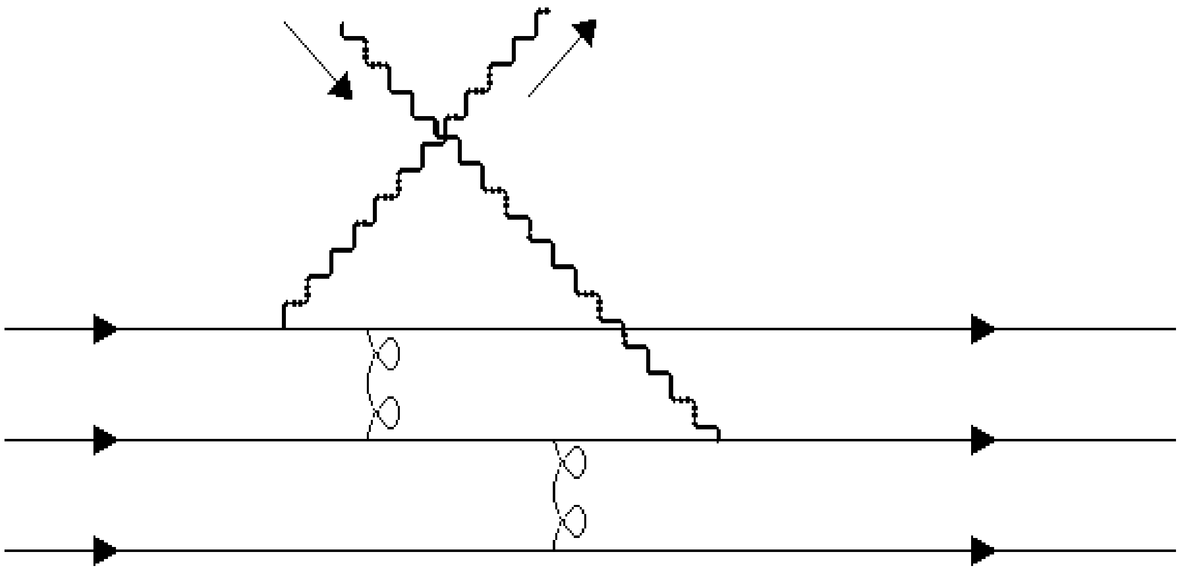
Real and Virtual Compton Scattering in Perturbative QCD



JLab, May 23, 2007

Motivation

- Forthcoming 12 GeV Upgrade and New JLab Data
  - -Hall A Collab, PRL98, 152001(2007): $s=5\text{-}11\,GeV^2, -t=2\text{-}7\,GeV^2$
  - -Cornell Data, PRD19, 1921(1979): $s=4.6\text{-}12.1\,GeV^2, -t=0.7\text{-}4.3\,GeV^2$
- Several Analyses of RCS but Different Results in the Past
  - -E.Maina and G.R.Farra, PLB206, 120(1988)
  - -G.R.Farrar and H.Zhang, PRD41, 3348(1990);42, 2413(E)(1990)
  - -A.S.Kronfeld and B.Nizic, PRD44, 3445(1991)
  - -M.Vanderhaeghen, P.Guichon and J.Van de Wiele, NPA622, 144(1997)
  - -T.Brooks and L.Dixon, PRD62, 114021(2000)

Recent Agreement with Brooks and Dixon's Result

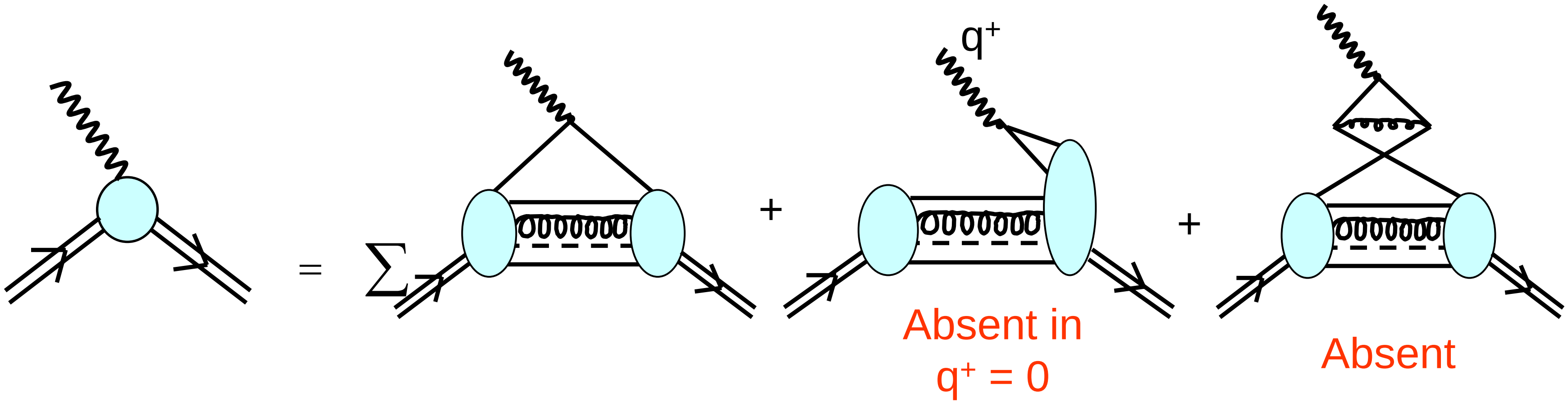
R.Thomson, A.Pang and C.Ji, PRD73,054023(2006)

- Extension to Virtual Compton Scattering
  - -Currently much interest in DVCS where GPD predictions are applicable
  - -Only one previous PQCD calculation (Farrar and Zhang) and their RCS result is in disagreement with Brooks and Dixon's and ours.
  - -GPD predictions can be compared with the PQCD results for DVCS. This can shed light on the applicability of both GPD and PQCD methods. $(Q^2 \gg -t \gg \Lambda^2)$

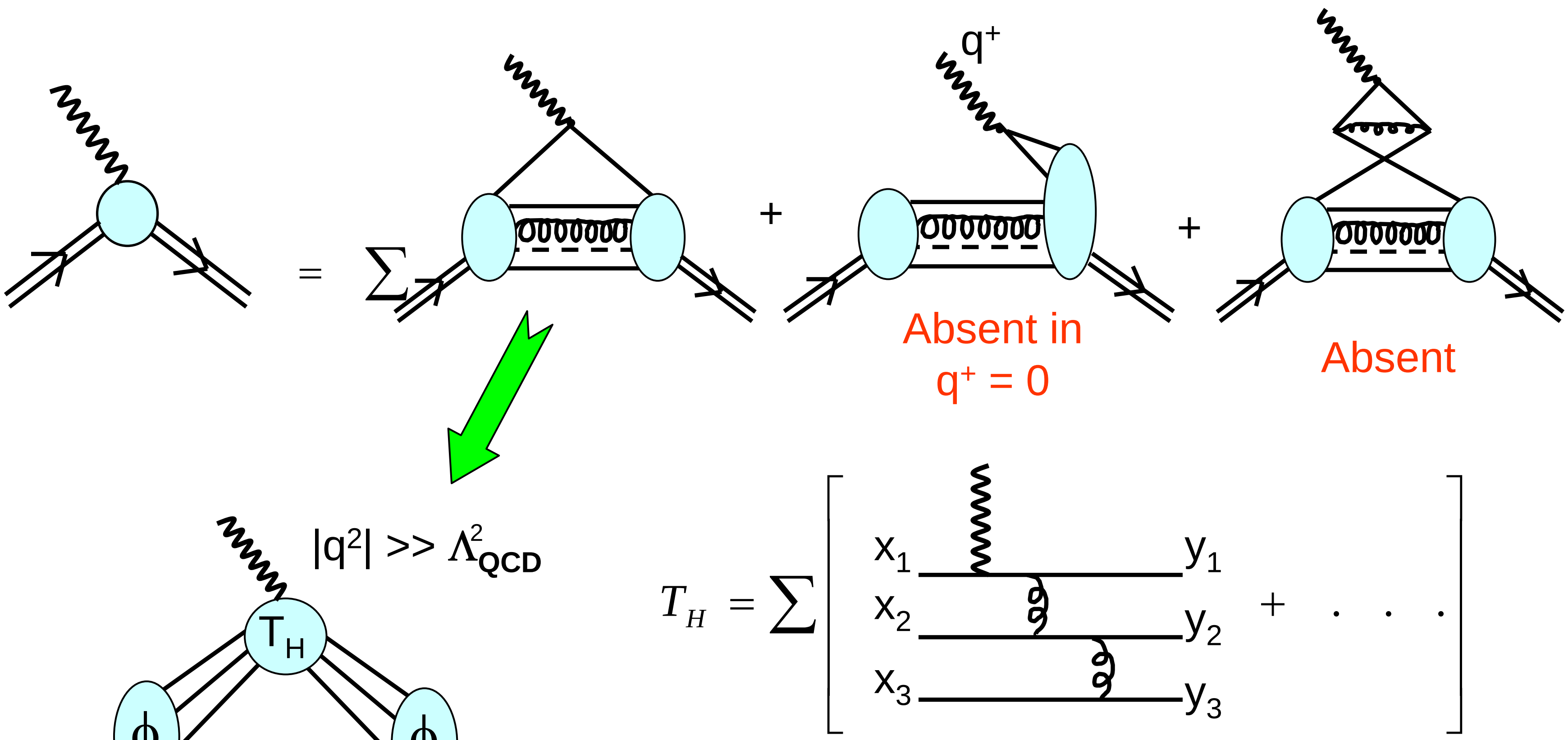
Outline

- PQCD in Light-Front Dynamics(LFD)
  - Hard Scattering Amplitude
  - Distribution Amplitude
  - Remarks on Computational Methods
- Real Compton Scattering Results
  - Comparison with Previous Computations
  - Comparison with JLab Data
- Extension to Virtual Compton Scattering
  - Comparison with Previous Computations
  - Link to GPD and Handbag Dominance
- Conclusions

LFD in Exclusive Processes



LFD in Exclusive Processes



$$= \frac{\alpha_s^2}{Q^4} f(x_i, y_i)$$

Classification of Diagrams

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

A, C, E $\longleftrightarrow$ B,D,F under 1 $\longleftrightarrow$ 3;

Color factor for all type: $C^{(d)} = 4/9$;

Triple gluon contribution is absent due to the color factor.

For RCS,

$$M_{hh'}^{\lambda\lambda'}(s,t) = M_{\bar{h}\bar{h}'}^{\bar{\lambda}\bar{\lambda}'}(s,t) \quad due\,to \quad Parity\ Inv.$$

$$M_{hh'}^{\lambda\lambda'}(s,t) = M_{h'h}^{\lambda'\lambda}(s,t) \quad Time-reversal\ Inv.$$

Number of Contributing Diagrams

Single Photon (e.g. F_1^P)

Attachment of a photon: (6/2)x7 = 21

Nonzero Diagrams: A1,A4,A7,C2,C7,E1,E5

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Two Photons: (6/2)x7x8 = 168

RCS needs only A and C types due to T-inv.

52 Nonzero Diagrams:A11,$\underline{\text{A11}}$,…C12,…,$\underline{\text{C77}}$.

-A.S.Kronfeld and B.Nizic, PRD44, 3445(1991)

Use h=h'=1 for proton helicities and

$|\gamma_{in}>=\alpha|\uparrow>+\beta|\downarrow>$, $|\gamma_{out}>=\gamma|\uparrow>+\delta|\downarrow>$.

Virtual Compton Scattering can't take advantage of T-inv.

96 Nonzero Diagrams:A11,…,$\underline{\text{C77}}$; A77,$\underline{\text{A77}}$,…,E12,…,$\underline{\text{E55}}$.

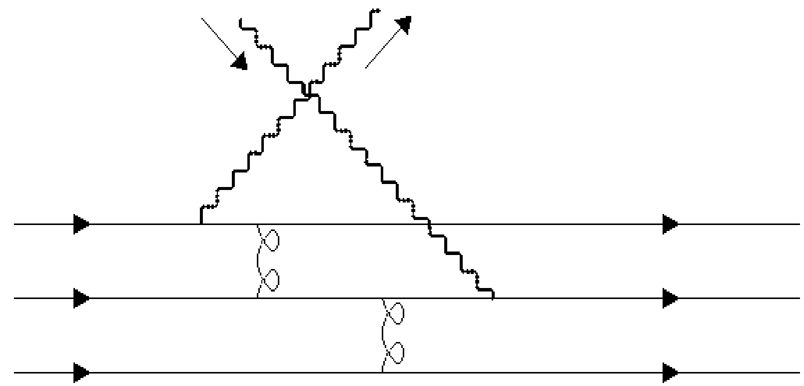
-Alex (Chiu-Yan) Pang's Thesis, NCSU (1995)

Virtual photon has also the longitudinal polarization.

A51

```
   A          B
   A         BB
    AA       BB
     A      BB
      AA    BB
       A   BB
        AABB
         **
        BBAA
       BB  AA
      BB    A
     BB      AA
    BB        A
   BB          AA
AAAAAAA*AAAAA*AAAAAAAAAAAAAAAAAAAAAAAAAAAA
         C     AA
         C      AA
         C       AA
         C        AA
         C         AA
         C          AA
         C           AA
         C            AA
         C             AA
         C              AA
         C               AA
         C                AA
         C                 AA
FFFFFFFFFFFF*FFFFFFFFFFFF*FFFFFF*FFFFFFF
                   D
                   D
                   D
                   D
                   D
                   D
                   D
                   D
                   D
                   D
                   D
                   D
EEEEEEEEEEEEEEEEEEEEEEEEEEE*EEEEEEEEEEEEEE
```

tdiag:=[[1,p1],[p3,2],[p1,1,2,p2],[p2,p3,3,3]];
plotsett:=DrawDiag(3,2,tdiag);



A.Pang and C.Ji, J.Comp.Phys.115,267(94)
COMPUTE;FeynComp;FeynGen;FeynDraw
-Alex (Chiu-Yan) Pang’s Thesis, NCSU (1995)

R. Thomson’s hard work!

Summary of Theory

Leading twist PQCD approximation for proton Compton scattering gives helicity amplitude

$$M_{hh'}^{\lambda\lambda'} = \sum_{i,d} \int dx_1 dx_2 dx_3 dy_1 dy_2 dy_3 \phi_i(x) T_i^{(d)} \phi_i^*(y)$$

subject to constraints $x_1 + x_2 + x_3 = 1$ and $y_1 + y_2 + y_3 = 1$

$h, \lambda (h', \lambda')$ helicities of i/c (o/g) proton, photon

$x(y)$ longitudinal momentum fractions of i/c (o/g) quarks

i labels independent Fock states of the proton with distribution amplitudes $\phi_i(x)$

d labels the Feynman diagrams that contribute to hard scattering amplitude $T_i^{(d)}$

Distribution Amplitude

$$| p_{\uparrow} \rangle = \frac{f_N}{8\sqrt{6}} \int [dx] \sum_i \phi_i(x_1, x_2, x_3) \, | i, x_1, x_2, x_3 \rangle;$$

$$| 1; x_1, x_2, x_3 \rangle = | u_{\uparrow}(x_1) u_{\downarrow}(x_2) d_{\uparrow}(x_3) \rangle,$$

$$| 2; x_1, x_2, x_3 \rangle = | u_{\uparrow}(x_1) d_{\downarrow}(x_2) u_{\uparrow}(x_3) \rangle,$$

$$| 3; x_1, x_2, x_3 \rangle = | d_{\uparrow}(x_1) u_{\downarrow}(x_2) u_{\uparrow}(x_3) \rangle;$$

$$\phi_2(x_1, x_2, x_3) = -[\phi_1(x_1, x_2, x_3) + \phi_1(x_3, x_2, x_1)],$$

$$\phi_3(x_1, x_2, x_3) = \phi_1(x_3, x_2, x_1);$$

$$[dx] = dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3);$$

$$f_N = (5.2 \pm 0.3) \times 10^{-3} \, (GeV/c)^2.$$

V.Chernyak and I.Zhitnitsky, Nucl.Phys.B246,52(84)

$$\phi_{CZ} = 120 x_1 x_2 x_3 (1.69 - 9.26 x_1 - 10.94 x_3 + 22.70 x_1^2 + 13.45 x_3^2 + 9.26 x_1 x_3)$$

V.Chernyak,A.Ogloblin and I.Zhitnitsky,Sov.J.Nucl.Phys.48,536(88)

$$\phi_{COZ} = 120 x_1 x_2 x_3 (5.880 - 25.956 x_1 - 20.076 x_3 + 36.792 x_1^2 + 19.152 x_3^2 + 25.956 x_1 x_3)$$

I.King and C.Sachrajda,Nucl.Phys.B279,785(87)

$$\phi_{KS} = 120 x_1 x_2 x_3 (8.40 - 26.88 x_1 - 35.28 x_3 + 35.28 x_1^2 + 37.80 x_3^2 + 30.24 x_1 x_3)$$

M.Gari and N.G.Stefanis,Phys.Lett.B175,462(86);PRD35,1074(87)

$$\phi_{GS} = 120 x_1 x_2 x_3 (6.040 - 16.775 x_1 - 34.985 x_3 - 1.027 x_1^2 + 12.307 x_3^2 + 111.320 x_1 x_3)$$

Asymptotic DA

$$\phi_{ASY} = 120 x_1 x_2 x_3$$

AdS/CFT?

It is possible to write the helicity amplitude in the form

$$M_{hh'}^{\lambda\lambda'} = \frac{4}{9}(4\pi\alpha_{em})(4\pi\alpha_s)^2\left(\frac{120 f_N}{8\sqrt{6}}\right)^2 \sum_d \sum_{m,n} C^{(d)}(m_1, m_3, n_1, n_3) I^{(d)}(m_1, m_3, n_1, n_3)$$

where

$$I^{(d)}(m_1, m_3, n_1, n_3) = \int_0^1 dx_1 dx_2 dx_3 dy_1 dy_2 dy_3 \tilde{T}^{(d)} x_1^{m_1+1} x_2 x_3^{m_3+1} y_1^{n_1+1} y_2 y_3^{n_3+1}$$

m_1, m_3, n_1, n_3 are powers of momentum fractions (from ϕ_i)

$C^{(d)}$ is a coefficient that sums contributions for 3 Fock states.

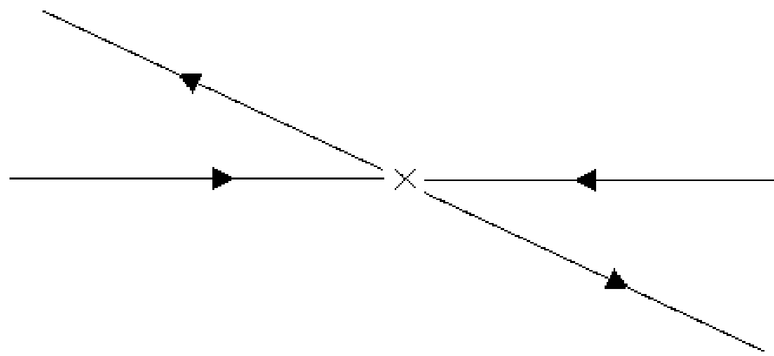
It is dependent on $Z_i^{(d)}$ (product of charges of struck quarks)

and the coefficients appearing in ϕ_i

f_N is a normalization constant (taken as 0.0052 GeV^2)

$\tilde{T}^{(d)}$ is the color/flavor independent part of $T_i^{(d)}$

Kinematics



$$p = P(1,0,0,1) \qquad k = P(\sqrt{1 - Q^2/P^2}, 0, 0, -1)$$

$$p' = E(1, \sin\theta, 0, \cos\theta) \qquad k' = E(1, -\sin\theta, 0, -\cos\theta)$$

- Define $S = (2E)^2$ (Mandelstam invariant)

  $R = 1 + Q^2/S$ (Virtuality parameter)

- For real photon: R=1, for DVCS: R=2
- $\tilde{T}^{(d)}$ calculated as a function of S and R: *e.g.*

$$\tilde{T}_{A16}^{\uparrow\uparrow} = \frac{8}{S^2} \frac{R^{3/2} s^4}{c} \frac{1 - R x_3}{<\bar{y}_1, x_3 >< \bar{y}_1, \bar{x}_1 >< y_3, x_3 > (1 - R\bar{x} + i\varepsilon)}$$

$$< y, x >= y(1 - Rs^2 x) - Rc^2 x + i\varepsilon$$

$$s = \sin(\theta/2), c = \cos(\theta/2), \bar{y} = 1 - y$$

- See details of calculation for A51 in Appendix A of
  Thomson,Pang and Ji, PRD73, 054023 (2006).

- The pole expressions have been expanded into real and imaginary parts using

$$\frac{1}{<y,x>+i\varepsilon} = P\left(\frac{1}{<y,x>}\right) - i\pi\delta(<y,x>)$$

where *P* means principal value.

- Delta function integrals have been evaluated explicitly.
- The remaining principal value integrals have been transformed using the ‘folding method’ of Kronfeld & Nizic which renders the integrand finite over the range of integration.
- Brooks and Dixon used a different method to do the integrations

  (contour deformation)
- The integration has then been completed numerically using a Monte Carlo algorithm in Fortran.

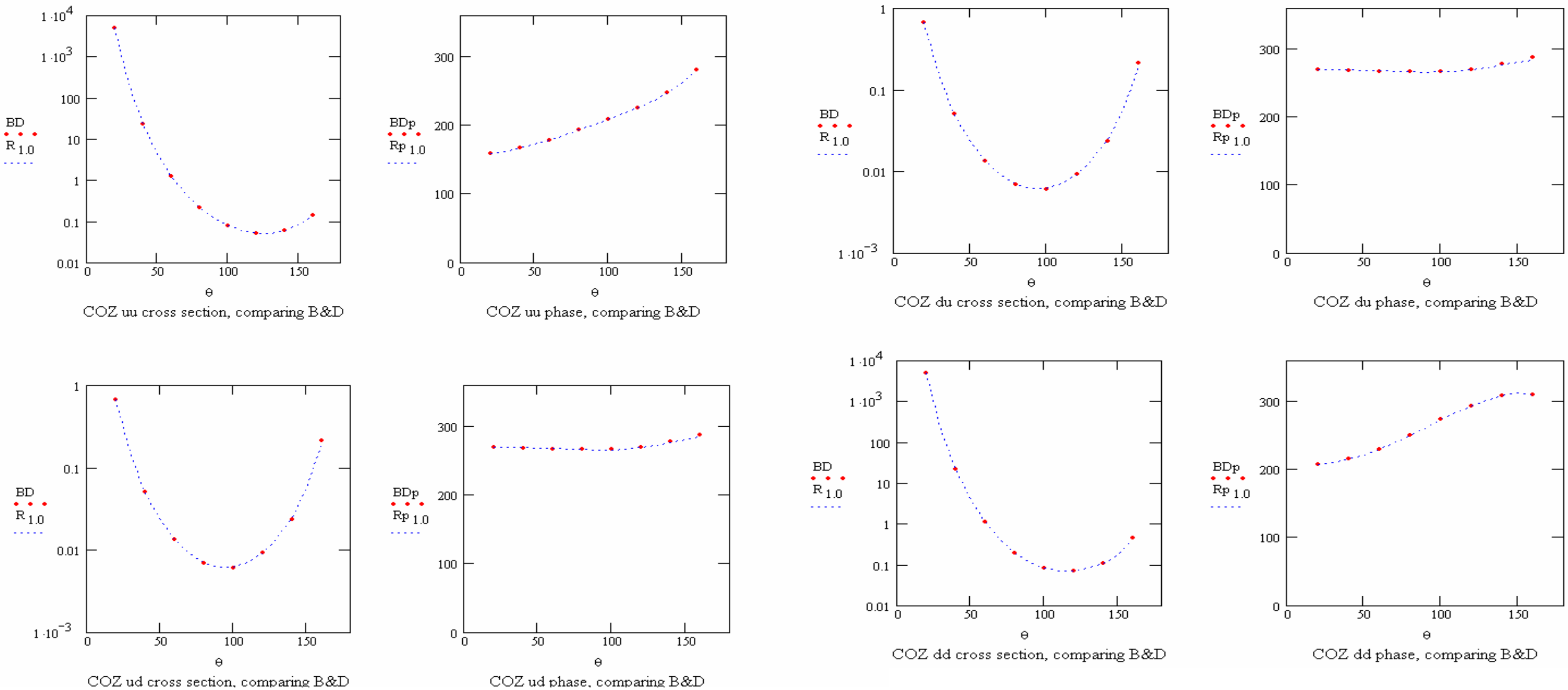
Real and imaginary parts of all helicity amplitudes were tabulated in the

basis of Appell polynomial expansion up to A_3:

Thomson,Pang and Ji, hep-ph/0602164v2

for the computation of cross sections and phase calculations preformed in the range of *R* values (1.0, 1.25, 1.5, 1.75, 2.0) and angles (20^0 to 160^0).

Comparison with previous work for real photon



- T. Brooks and L. Dixon, PRD62, 114021 (2000)

⋯ R. Thomson, A. Pang and C. Ji, PRD73, 054023 (2006)

Initial-State Helicity Correlation

$$A_{LL}(or\ K_{LL}) = \frac{\frac{d\sigma_+^+}{dt} - \frac{d\sigma_+^-}{dt}}{\frac{d\sigma_+^+}{dt} + \frac{d\sigma_+^-}{dt}} = \frac{|M_{\uparrow\uparrow}^{\uparrow\uparrow}|^2 + |M_{\uparrow\uparrow}^{\uparrow\downarrow}|^2 - |M_{\uparrow\uparrow}^{\downarrow\uparrow}|^2 - |M_{\uparrow\uparrow}^{\downarrow\downarrow}|^2}{|M_{\uparrow\uparrow}^{\uparrow\uparrow}|^2 + |M_{\uparrow\uparrow}^{\uparrow\downarrow}|^2 + |M_{\uparrow\uparrow}^{\downarrow\uparrow}|^2 + |M_{\uparrow\uparrow}^{\downarrow\downarrow}|^2}$$

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

—— T. Brooks and L. Dixon, PRD62, 114021 (2000)

--- M. Vanderhaeghen, P.Guichon and J. Van de Wiele, NPA622, 144 (1997)

– – A. S. Kronfeld and B. Nizic, PRD44, 3445 (1991)

····· G. R. Farrar and H. Zhang, PRD41, 3348 (1990);42, 2413(E) (1990)

JLab Data: $0.678 \pm 0.083 \pm 0.04$ at 120^0

$s=6.9$ GeV^2, $t=-4.0$ GeV^2, $u=-1.1$GeV^2

PRL94,242001(2005)

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Handbag: M.Diehl,T.Feldmann,R.Jakob and P.Kroll, PLB460,204(1999)

Diquark: P.Kroll, M.Schurmann and W.Schwiger, IJMPA6,4107(1991)

Handbag approach to wide-angle RCS using GPD

- It is argued, at currently accessible kinematics, the Compton scattering amplitude is dominated by soft overlap contributions which can be described in light front gauge via the handbag diagram.
- In this approach, the helicity amplitude is given by

(for example)

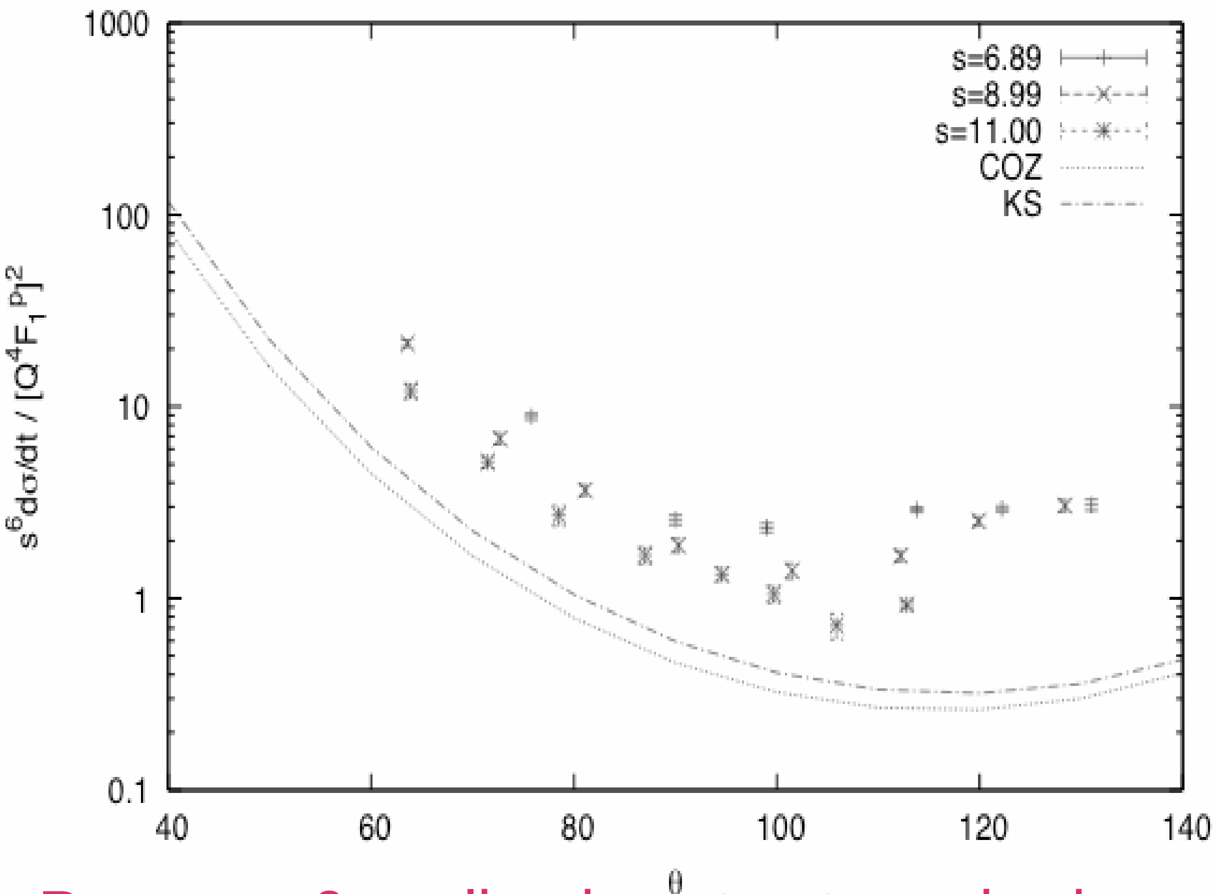
$$M_{++}^{\lambda\lambda'}(s,t) = 2\pi\alpha_{em}\Big(H_{++}^{\lambda\lambda'}(s,t)(R_V(t)+R_A(t)) + H_{--}^{\lambda\lambda'}(s,t)(R_V(t)-R_A(t))\Big)$$

where $H_{++}^{\lambda\lambda'}(s,t)$ denotes the amplitude for the sub-process $\gamma q \rightarrow \gamma q$ and $R_A(t),\ R_V(t)$ are soft form factors which can be expressed in terms of GPDs. For example, summing over flavors a

$$R_V(t) = \sum_a e_a^2 \int_{-1}^{1} \frac{d\bar{x}}{\bar{x}} H^a(\bar{x}, \xi = 0, t)$$

Scaled Unpolarized RCS Cross Section

Hall A Collab, PRL98, 152001(2007)

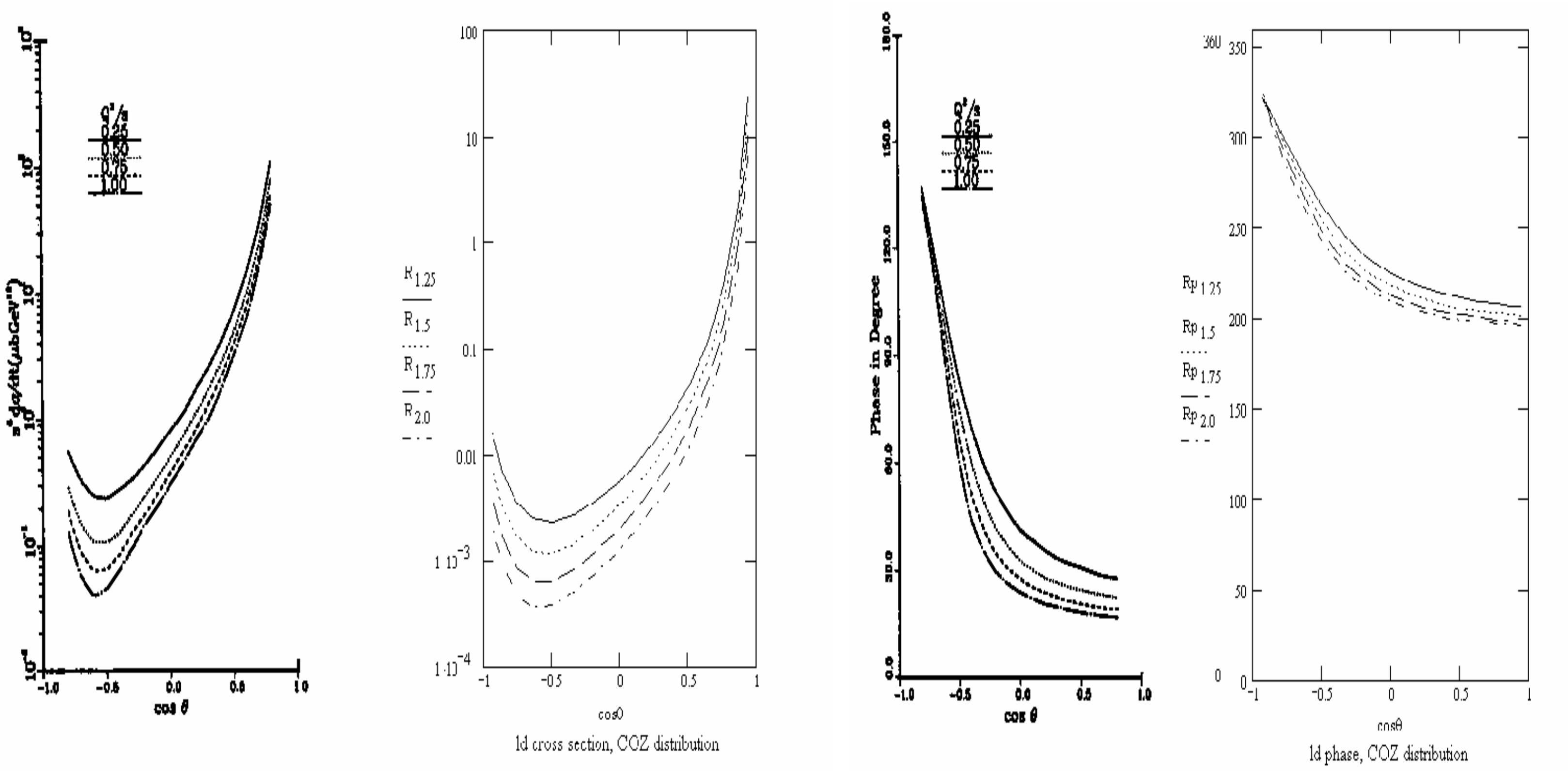


Suppress the Uncertainty in Normalization:

$0.19<\alpha_s<0.39$,
$0.75<\phi_{evol}<1.33$,
$f_N=(5.2\pm 0.3)\times 10^{-3}$ GeV^2.

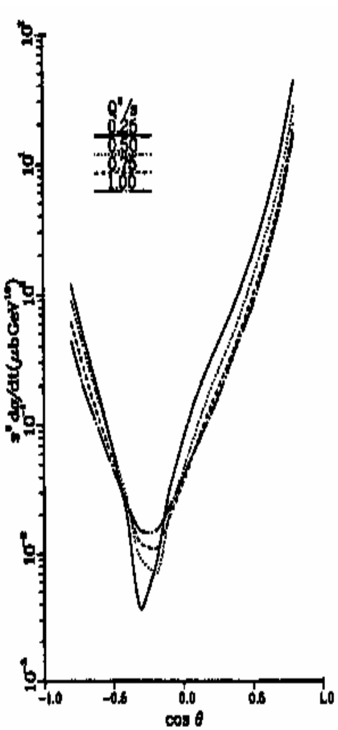
Power n=6 scaling is not yet reached.
12 GeV upgrade is anticipated.

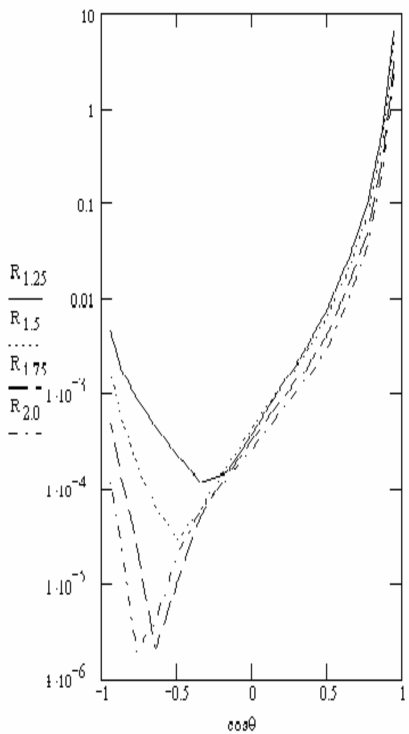
Virtual photon comparison: longitudinal polarization
(l to d)



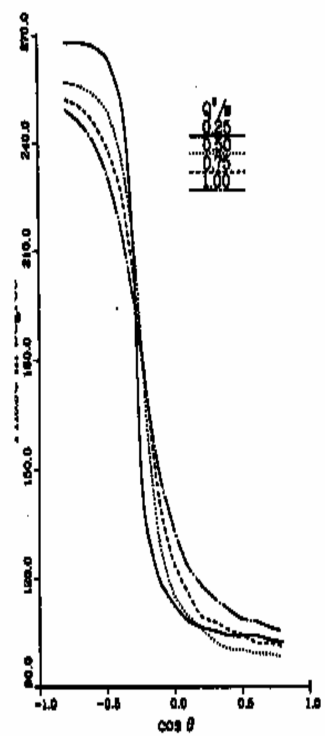
The results have been compared with Farrar and Zhang, PRD41,3348(1990).

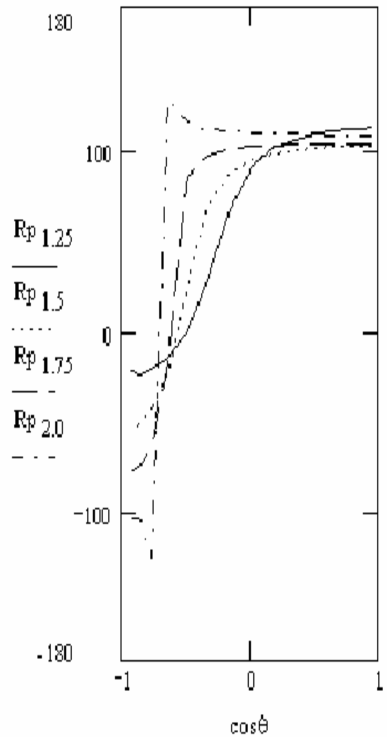
Virtual photon comparison: longitudinal polarization
(l to u)





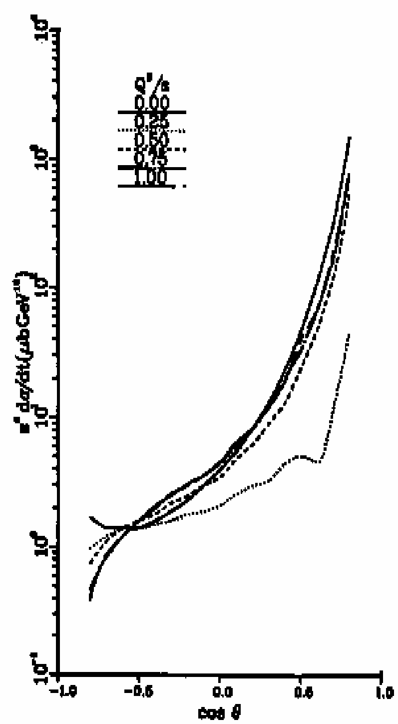
lu cross section, COZ distribution

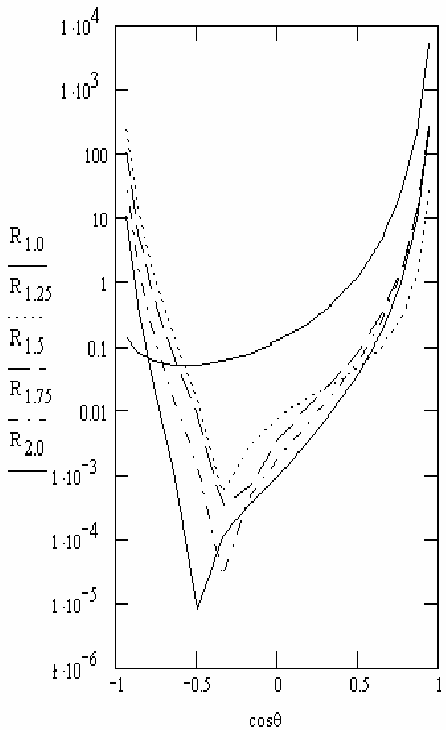




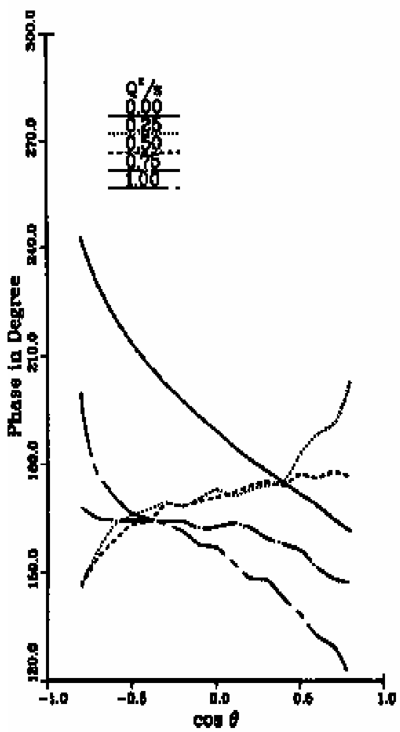
lu phase, COZ distribution

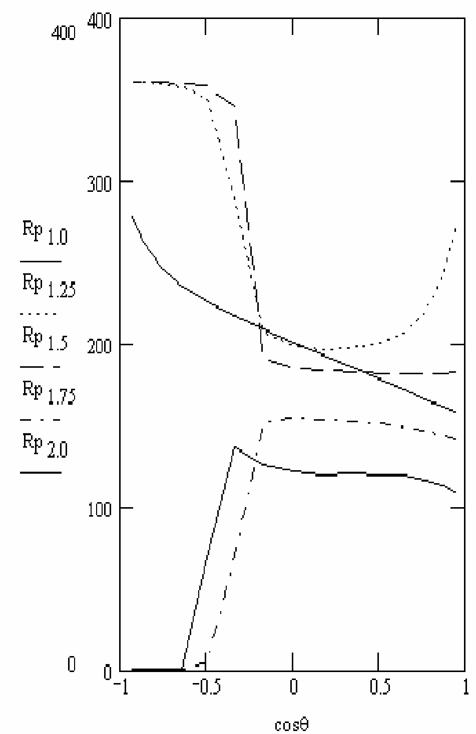
Virtual photon comparison: up to up





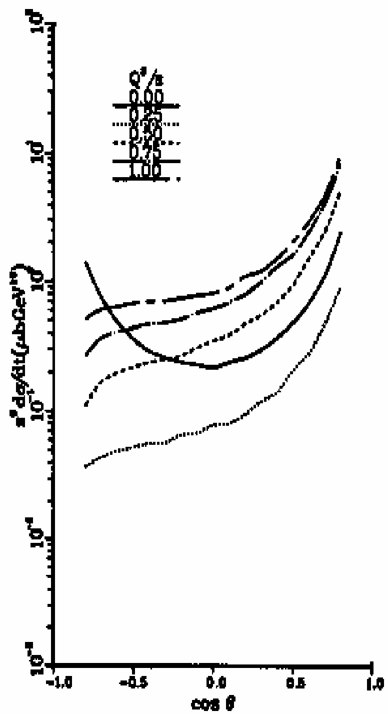
uu cross section, COZ distribution

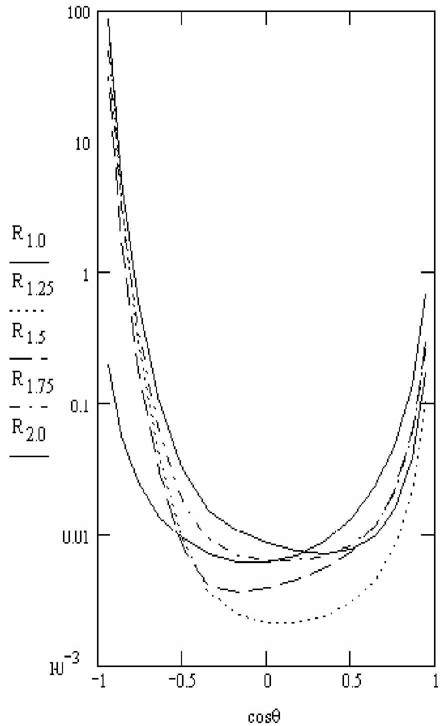




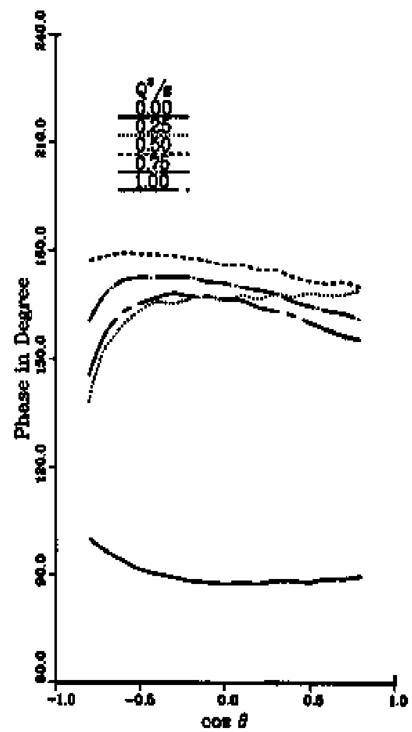
uu phase, COZ distribution

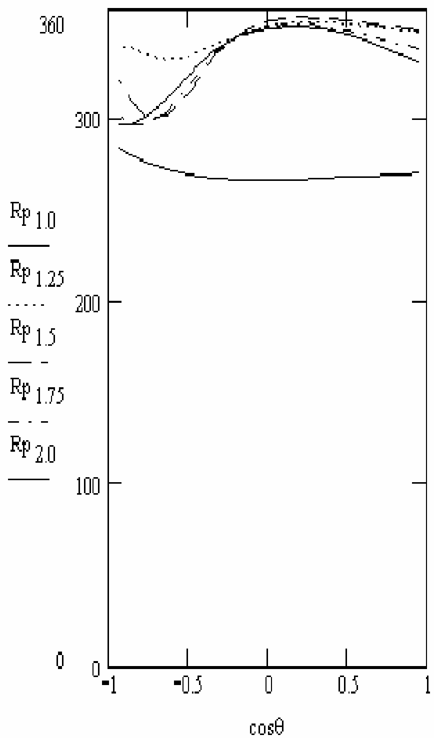
Virtual photon comparison: up to down





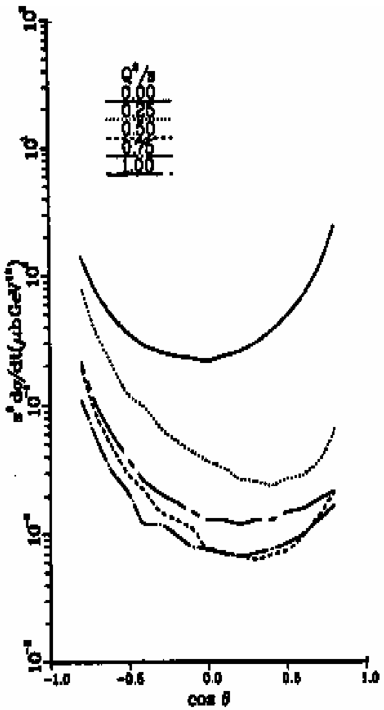
ud cross section, COZ distribution

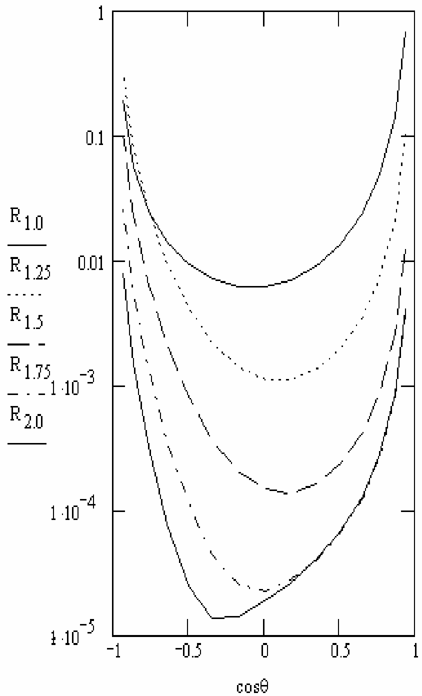




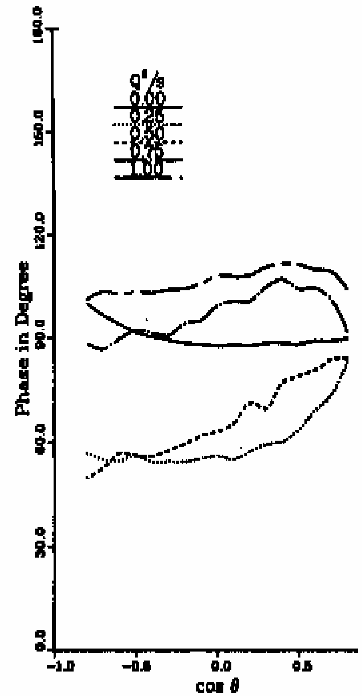
ud phase, COZ distribution

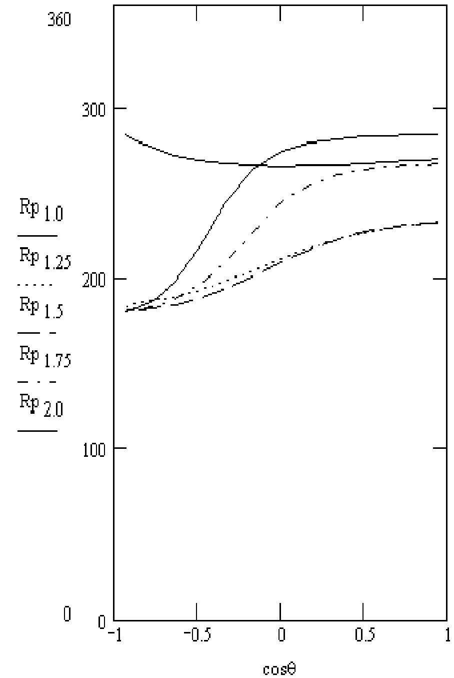
Virtual photon comparison: down to up





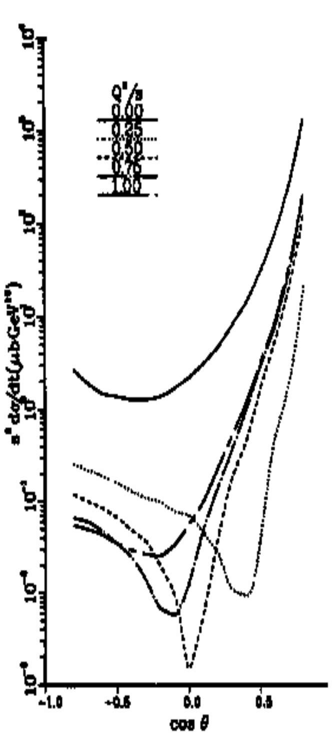
du cross section, COZ distribution

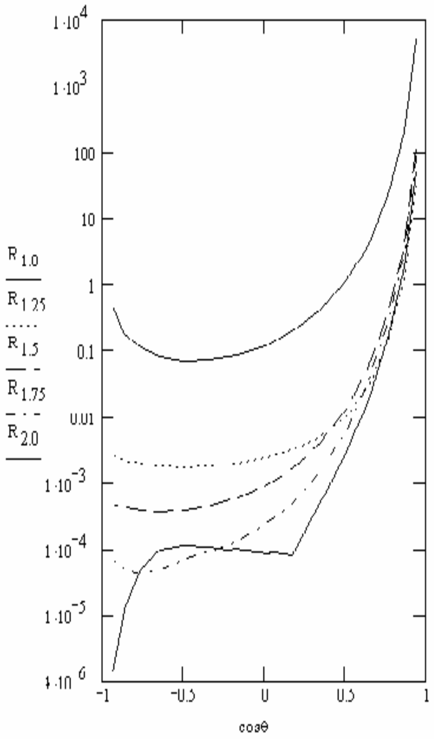




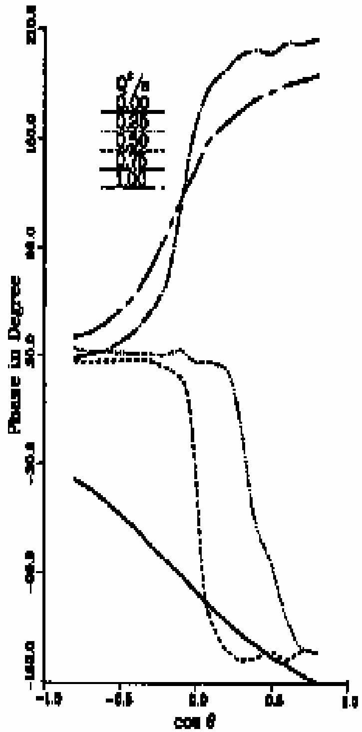
du phase, COZ distribution

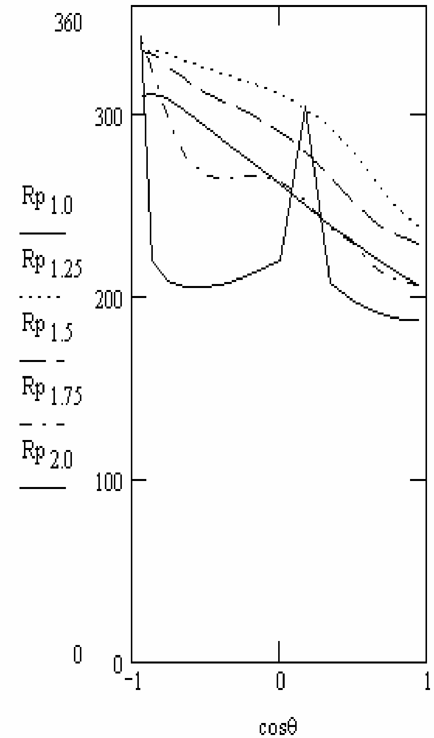
Virtual photon comparison: down to down





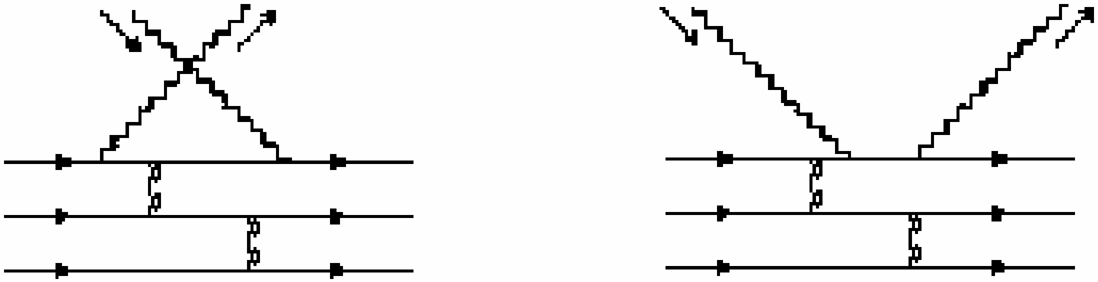
dd cross section, COZ distribution





dd phase, COZ distribution

Checking Handbag Dominance in PQCD



- FeynComp converted to work in Light Front gauge.
- Hand calculation of a few sample diagrams has been used to check the output of FeynComp.
- Explicit calculation using FeynComp shows the gauge invariance:

$$\sum_{LightFront} amplitudes = \sum_{Feynman} amplitudes$$

where the sum is over diagrams with photons attached to the same quarks.

Work in progress …

Conclusions

- Agreement with Brooks & Dixon in RCS results gives more confidence on our VCS calculation over the previous one by Farrar &Zhang.
- JLab 12 GeV upgrade is highly desirable to shed some light on the validity of PQCD in exclusive processes.
- GPDs can be expressed in terms of DAs using PQCD at large momentum transfer and the previous results are currently under investigation.
- Handbag dominace check in PQCD analysis is also in progress.