

Exclusive Reactions at High Momentum Transfer  
Jefferson Lab, Newport News, VA  
May 21-24, 2007

# Imaging the Proton via Hard Exclusive Production in Diffractive pp Scattering

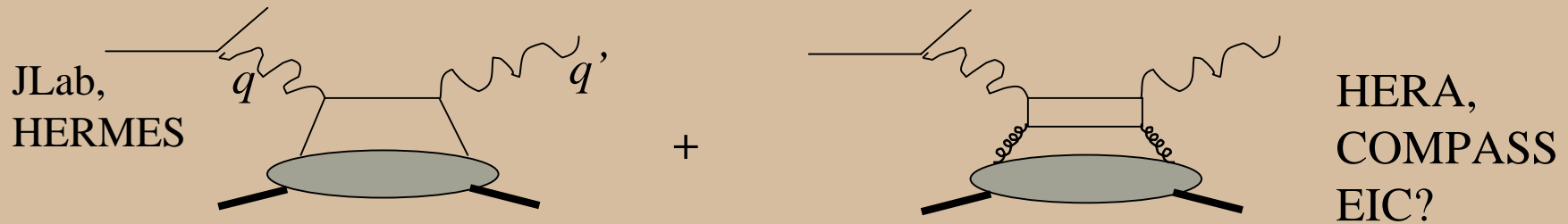
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L.Frankfurt, Ch.E.H., M. Strikman, Ch.Weiss, *Phys.Rev.* **D75** 054009 (2007).  
Ch.E.H., L.Frankfurt, M. Strikman, Ch.Weiss, DIS2006: hep-ph/0608312

# Quark & Gluon Imaging

- Example of Deeply Virtual Compton Scattering (DVCS)  $ep \rightarrow ep\gamma$



- Large Virtuality  $-q^2=Q^2: \Rightarrow$  Factorization of perturbative sub-process.
- Longitudinal momentum transfer:
  - $2\xi = 2x_{Bj}/(2-x_{Bj}) =$  skewness = difference of initial and final momentum fractions
  - $\xi = 0$ : Densities;  $\xi > 0$ : Wigner functions
- $\Delta = (q-q')$ ,  $\Delta^2 = t$   $\Delta_{\perp}^2 \approx t_{\min} - t$
- $\Delta_{\perp}$  Fourier conjugate to *impact parameter*,  $\mathbf{b}$ .
- *Parton Tomography*:
  - *Correlation of parton longitudinal momentum and transverse position*

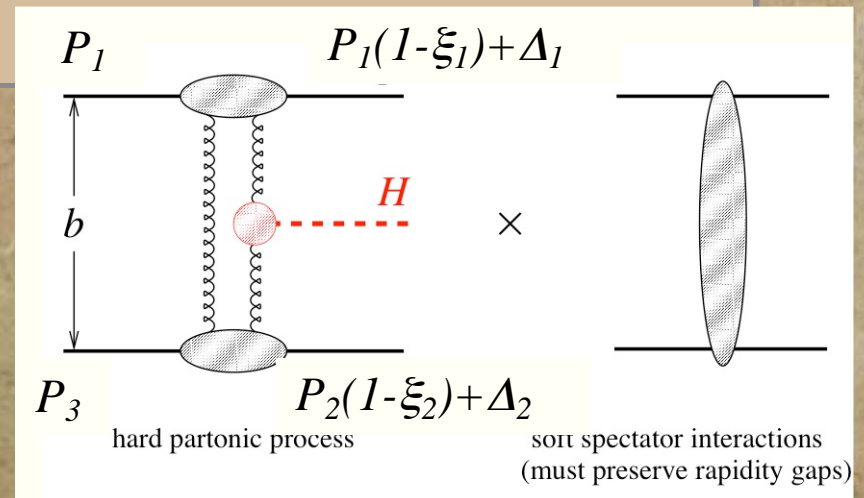


# Idea of pp Diffractive Production

- $p_1 + p_2 \rightarrow p_3 + \text{gap} + H + \text{gap} + p_4$ 
  - $H = \text{Higgs, di-jet, Upsilon, } J/\Psi, \text{ di-hadron, di-lepton, di-photon...}$
- $M_H^2 = \xi_1 \xi_2 s \ll s$ : Phase space for rapidity gaps
- $M_H^2 \gg \Lambda_{\text{QCD}}^2$ : Perturbative mechanism
- $t_1 = (p_3 - p_1)^2 \approx -(\Delta_1)^2$        $t_2 = (p_4 - p_2)^2 = -(\Delta_2)^2$
- $\Delta_{1,2}$ : Fourier Conjugate to impact parameters  $\mathbf{b}_{1,2}$  of hard scattering process
  - pp impact parameter  $\mathbf{b} = \mathbf{b}_2 - \mathbf{b}_1$

## Irreducible soft interactions:

- Convolute hard and soft contributions to  $\Delta_{1,2}$
- Measure/compute soft interaction effects?
- Does hard scattering measure properties of ground state or of soft collisional excited state?



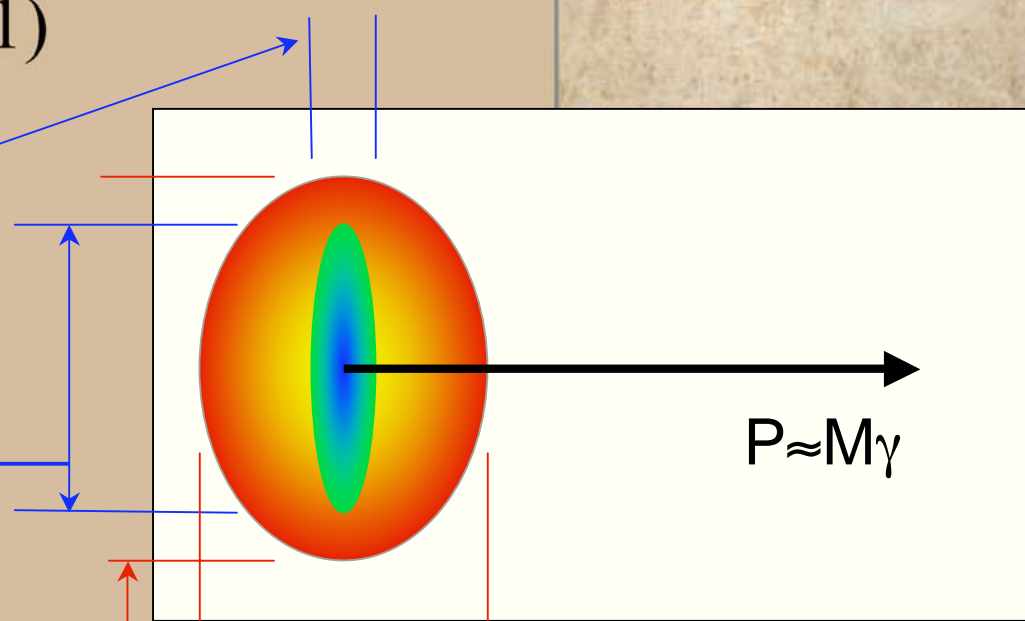
# Gribov Picture of Fast Projectiles

hep-ph/0006158

- Large- $x$  partons ( $x > 0.01$ )

- confined to a volume:

$$\langle R^2 \rangle_{\perp} \otimes \left[ \frac{\sqrt{\langle R^2 \rangle}}{\gamma} \right]_{\parallel}$$



- Low- $x$  partons ( $x \geq 1/\gamma$ ).

- Transverse diffusion

- ...  $\sqrt{\langle R^2 \rangle} \ln(x_0/x)$

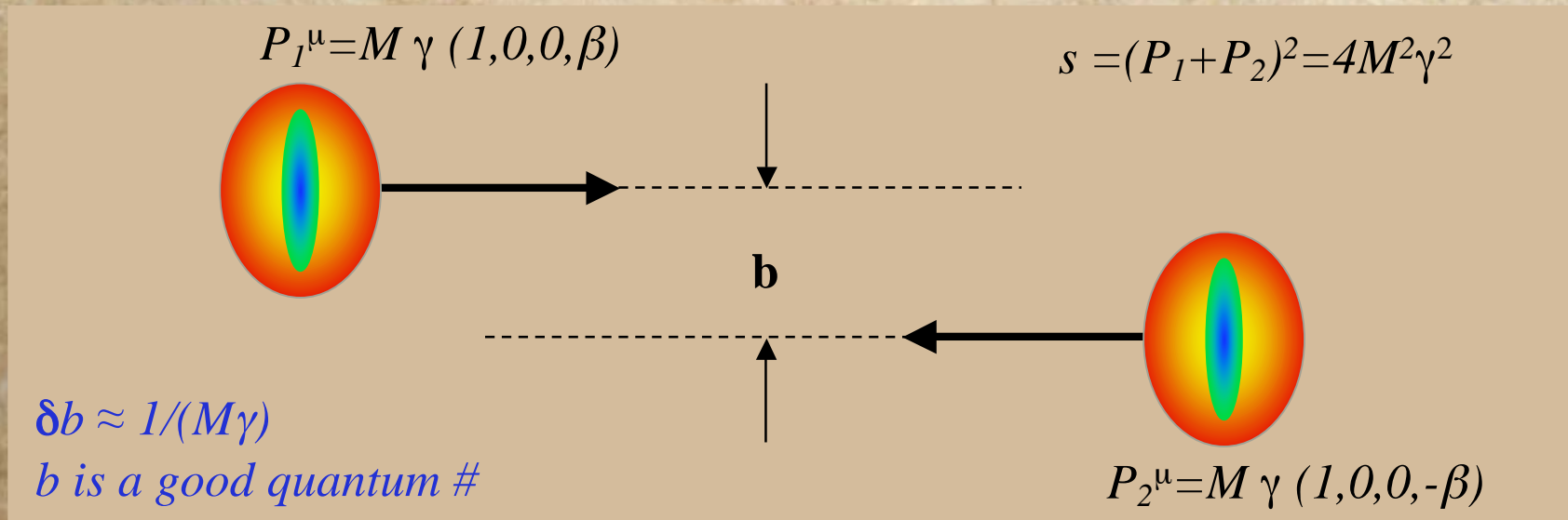
- Longitudinal spread ...

- Invariant longitudinal size.

$$\frac{\langle x \rangle}{x} \frac{\sqrt{\langle R^2 \rangle}}{\gamma} \approx \sqrt{\langle R^2 \rangle}$$

# Elastic Scattering & Black Disk Limit

## Impact Parameter ( $\mathbf{b}$ ) Representation



Elastic Scattering Amplitude  $T_{EL}$ , for  $\perp$ -Momentum Transfer  $\Delta$ :

Elastic scattering = diffraction pattern of  $\mathbf{b}$ -dependent absorption  $i\Gamma(\mathbf{b}, s)$ .

$$T_{EL}(\Delta, s) = [is/(4\pi)] \int d^2\mathbf{b} e^{i\Delta \cdot \mathbf{b}} \Gamma(\mathbf{b}, s)$$

Center of Proton is Black: Black Disk Limit (BDL) = Pure Absorption

$$\Gamma(\mathbf{0}, s) = 1 \quad (s \geq \text{TeV}) \quad \rightarrow \text{Stabilizes numerical estimates}$$



# Elastic, Inelastic, and Total Scattering Cross Sections

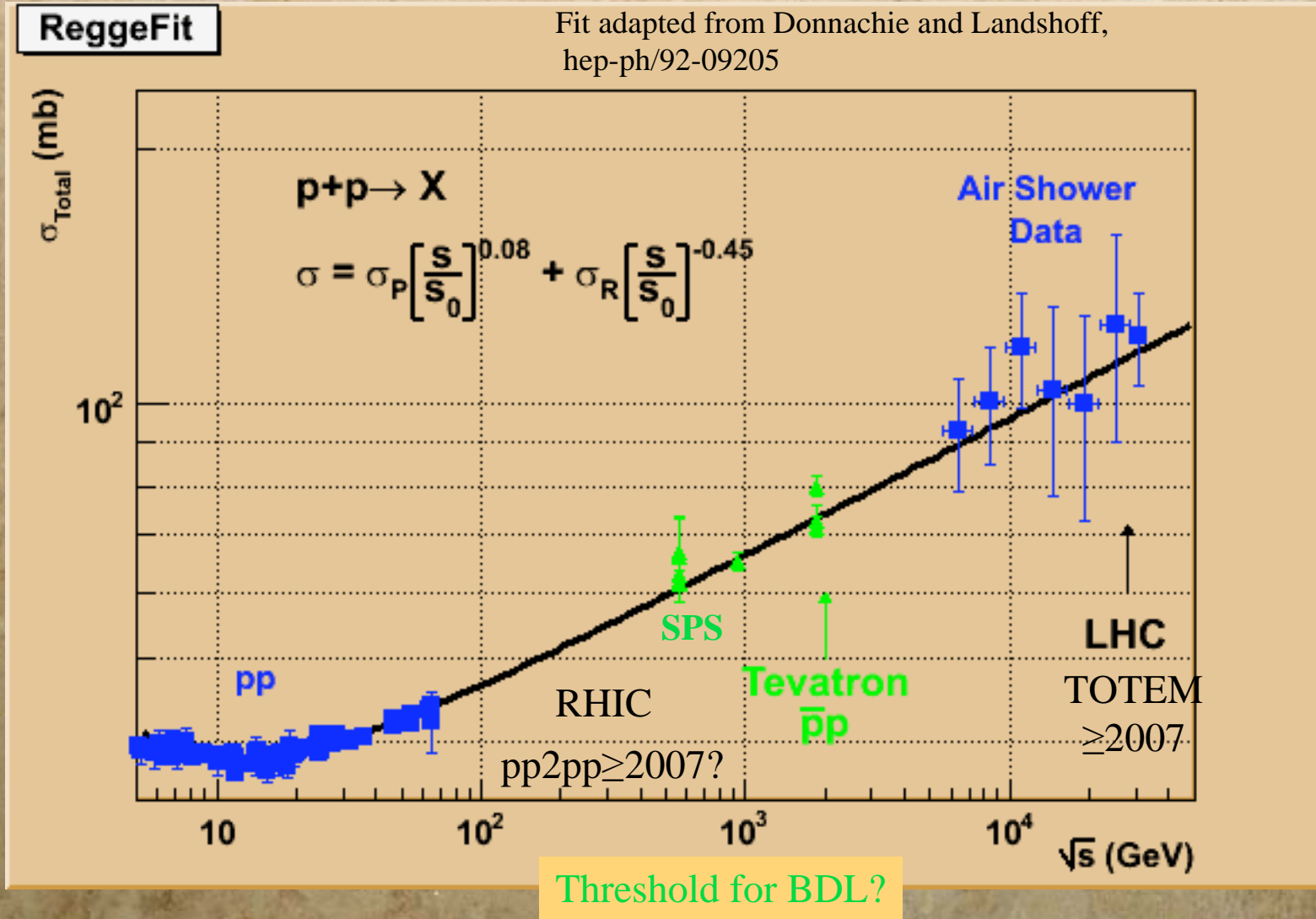
- Impact Parameter Representation:

$$d^2\sigma_{\text{El}}(s) = \frac{d^2\Delta}{(2\pi)^2} \left| \int d^2\mathbf{b} e^{-i\mathbf{b}\cdot\Delta} \Gamma(\mathbf{b}; s) \right|^2$$

$$\left. \begin{array}{l} \sigma_{\text{El}}(s) \\ \sigma_{\text{Tot}}(s) \\ \sigma_{\text{Inel}}(s) \end{array} \right\} = \int d^2\mathbf{b} \times \left\{ \begin{array}{l} |\Gamma(\mathbf{b}; s)|^2 \\ 2\Re[\Gamma(\mathbf{b}; s)] \\ [1 - |1 - \Gamma(\mathbf{b}; s)|^2] \end{array} \right.$$

- $|1 - \Gamma(\mathbf{b}; s)|^2 =$  Probability of no inelastic scattering at impact parameter  $\mathbf{b}$ .
- Elastic S-matrix:  $S_{\text{El}}(\Delta, s) = \int e^{i\Delta\cdot\mathbf{b}} d^2\mathbf{b} [1 - \Gamma(\mathbf{b}, s)]$

# pp Total Cross Sections

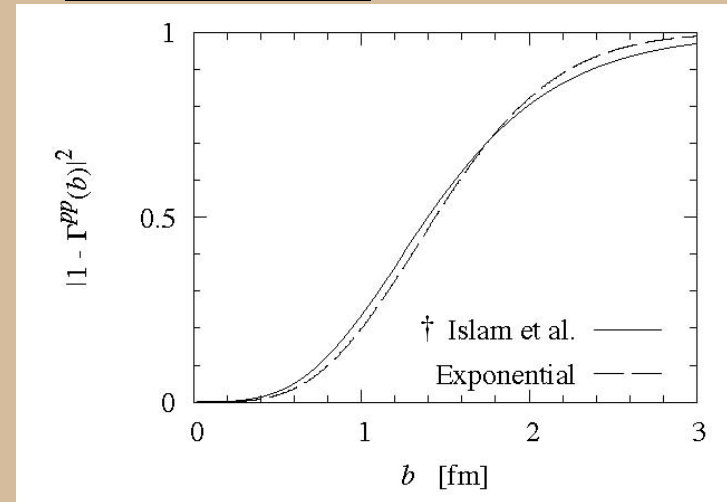


# Behavior of $\sigma_{\text{Total}}$ & $\Gamma(\mathbf{b};s)$

BDL:  
 $1 - \Gamma(0,s) = 0$

Ultrapерipheral:  
 $\Gamma(\infty,s) = 0$

- Gaussian model:
  - $\Gamma(\mathbf{b},s) = \exp[-\mathbf{b}^2/B(s)].$
  - $\sigma_{\text{Total}} = 4\pi B(s)$
- Regge Fit
  - $\sigma_{\text{Total}} \rightarrow \sigma_0 (s/s_0)^{0.08} + \dots$ 
    - $B(s) = B_0(s/s_0)^{0.08}.$
    - $B = 21.8 \text{ GeV}^{-2}$



$s = (14 \text{ TeV})^2$ : LHC.

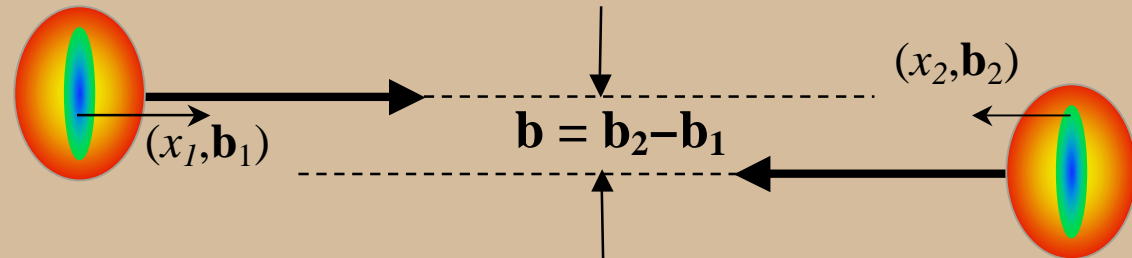
- $\Gamma(\mathbf{b},s)$  is a slowly varying function of  $s$  at high energy.

† M. Islam, *et al*, *PLA***18**, 743, 2003.  
 Model of Kaidalov, Khoze, Martin, and Ryskin gives similar results



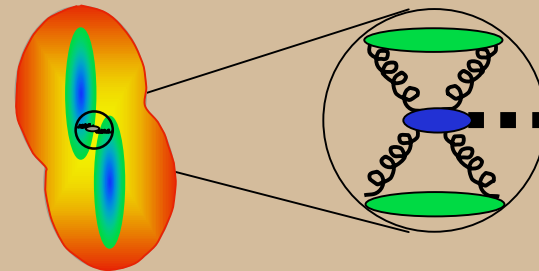
# Diffractive High Mass Production

$t \ll -R$

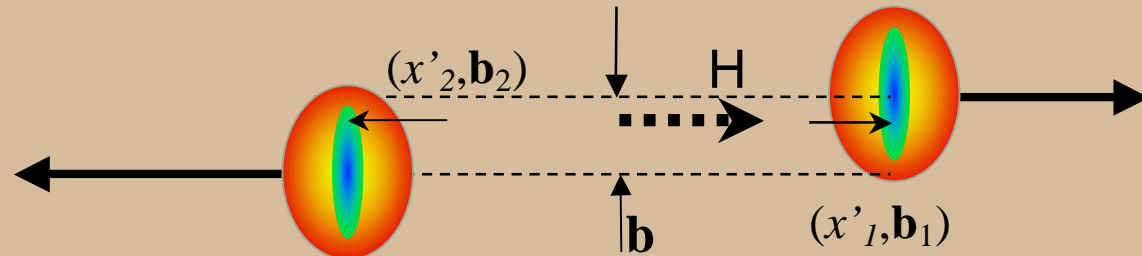


Soft interactions for  
 $-R \leq t \leq R$

*Hard interactions for*  
 $|t| \leq R/\gamma$

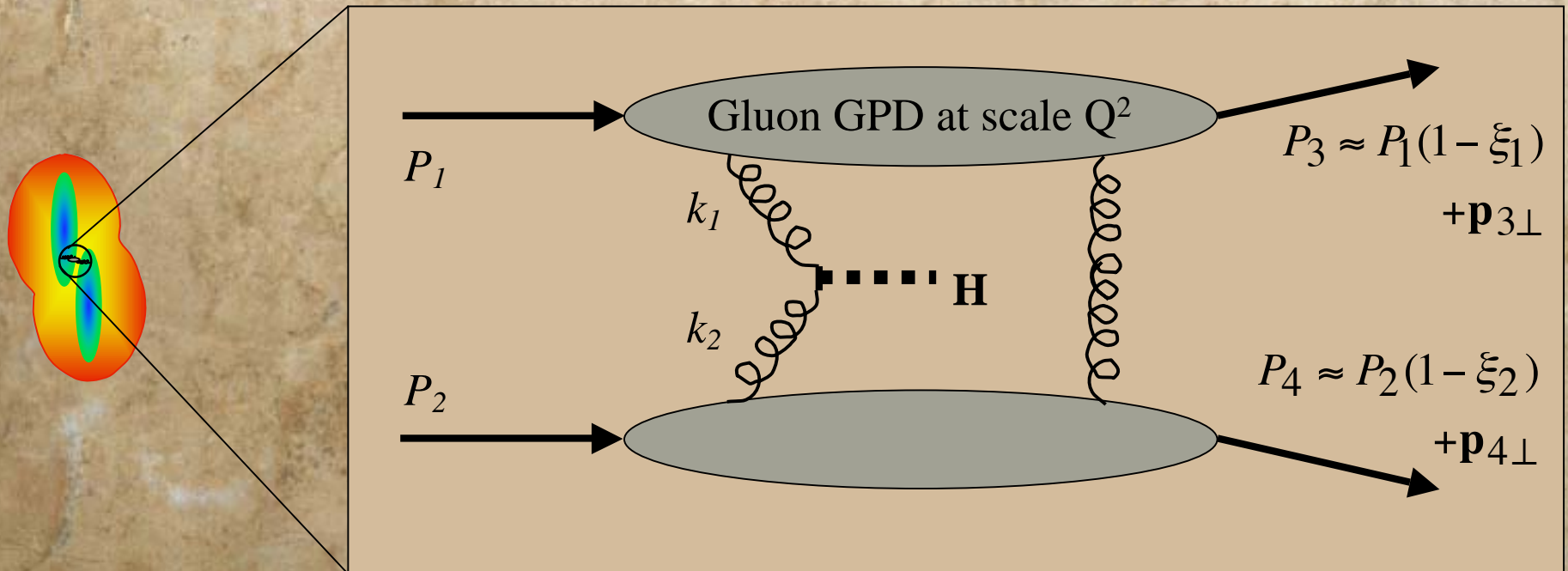


$t > +R$



Time scales of hard and soft interactions differ by factor  $\gamma$ .

# Hard Scattering Kinematics, $s=(P_1+P_2)^2$



- $k_1 \approx x_1 P_1 + \mathbf{k}_{1\perp}$        $k_2 \approx x_2 P_2 + \mathbf{k}_{2\perp}$
- $k = (x_1 - \xi_1)P_1 + \mathbf{k}_{\perp} - (x_2 - \xi_2)P_2 \approx \mathbf{k}_{\perp}$
- Post Selection of Rapidity Gap  $\Rightarrow$   
Suppression of gluon bremsstrahlung:  
Loop Virtuality:  $k_1^2 \sim k_2^2 \sim k^2 \sim Q^2$ .

**Kinematic Hierarchy:**

$$\Lambda_{QCD}^2 \ll \langle Q^2 \rangle \ll M_H^2 \ll s$$

$$x_1 x_2 s \approx M_H^2$$

$$(x_i - \xi_i) \ll x_i \approx \xi_i \ll 1$$

# Factorization of Hard & Soft Scattering

- $T_{\text{Diff}} = \langle p_3 p_4 | S_{\text{Soft}}(\infty, 0) V_{\text{Hard}}(H) S_{\text{Soft}}(0, -\infty) | p_1 p_2 \rangle$ 
  - $V_{\text{Hard}}$  :
    - Diagonal in impact parameter
    - Time scale =  $R/\gamma \ll R$  = Time scale of  $S_{\text{Soft}}$
    - Does not mix Fock sub-spaces in diffractive production.
    - Conserves parton helicity
  - $[V_{\text{Hard}}, S_{\text{Soft}}] \approx 0$ 
    - $T_{\text{Diff}} = \langle P_3 P_4 | S_{\text{Soft}}(\infty, -\infty) | X \rangle \langle X | V_{\text{Hard}}(H) | P_1 P_2 \rangle$
    - Broken by transverse correlations of hard and soft partons
  - $V_{\text{Hard}}$  and  $S_{\text{Soft}}$  populate orthogonal inelastic intermediate states. Excitation of low mass  $N^*$  by  $S_{\text{Soft}}$  suppressed at LHC energies (Goulianos hep-ph/0510035).
- $T_{\text{Diff}} \rightarrow \langle P_3 P_4 | S_{\text{Elastic}}(\infty, -\infty) | p' p'' \rangle \langle p' p'' | V_{\text{Hard}}(H) | P_1 P_2 \rangle$



# Scattering Amplitude in Impact Parameter Space

- GPD  $h_g$  in impact parameter space.

$$h(x, \xi, \rho) = \int \frac{d^2 \Delta}{(2\pi)^2} e^{i\Delta \cdot \rho} H(x, \xi, t = -\Delta^2)$$

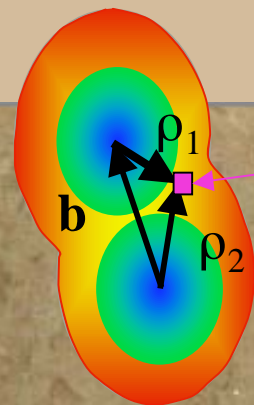
- Scattering amplitude

$$T_{Diff}(\xi_1, \mathbf{p}_3, \xi_2, \mathbf{p}_4) = \int d^2 \rho_1 \int d^2 \rho_2 e^{-i[\rho_1 \cdot \mathbf{p}_3 + \rho_2 \cdot \mathbf{p}_4]}$$

$$\kappa(\xi_1, \xi_2) h(x_1, \xi_1, \rho_1, Q^2) h(x_2, \xi_2, \rho_2, Q^2)$$

$$[1 - \Gamma(\rho_2 - \rho_1, s)]$$

← Soft Scattering



$$\mathbf{b} = \rho_2 - \rho_1$$

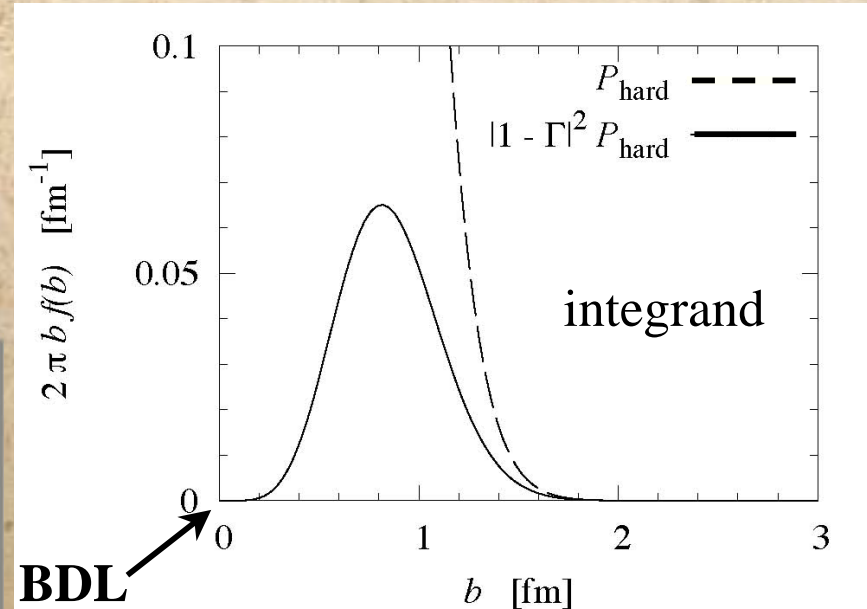
“Beam-pipe view”:

One projectile coming towards us, one moving away.

# $S^2$ : Rapidity Gap Survival Probability for Hard Exclusive Production

## ■ Surface Peaking:

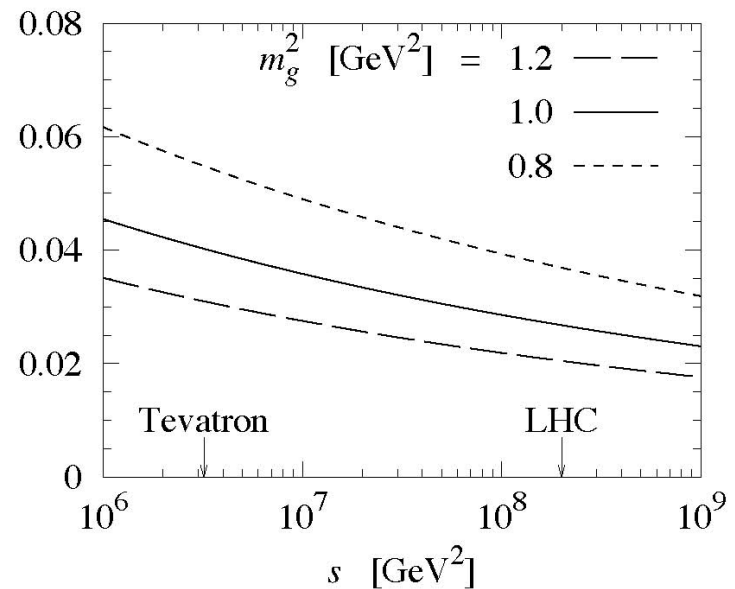
- Integrand = gluon density  $\times [1-\Gamma]$



$$S^2 \equiv \frac{\sigma_{DD}(\text{Full})}{\sigma_{DD}(\text{no\_soft} : \Gamma = 0)}$$

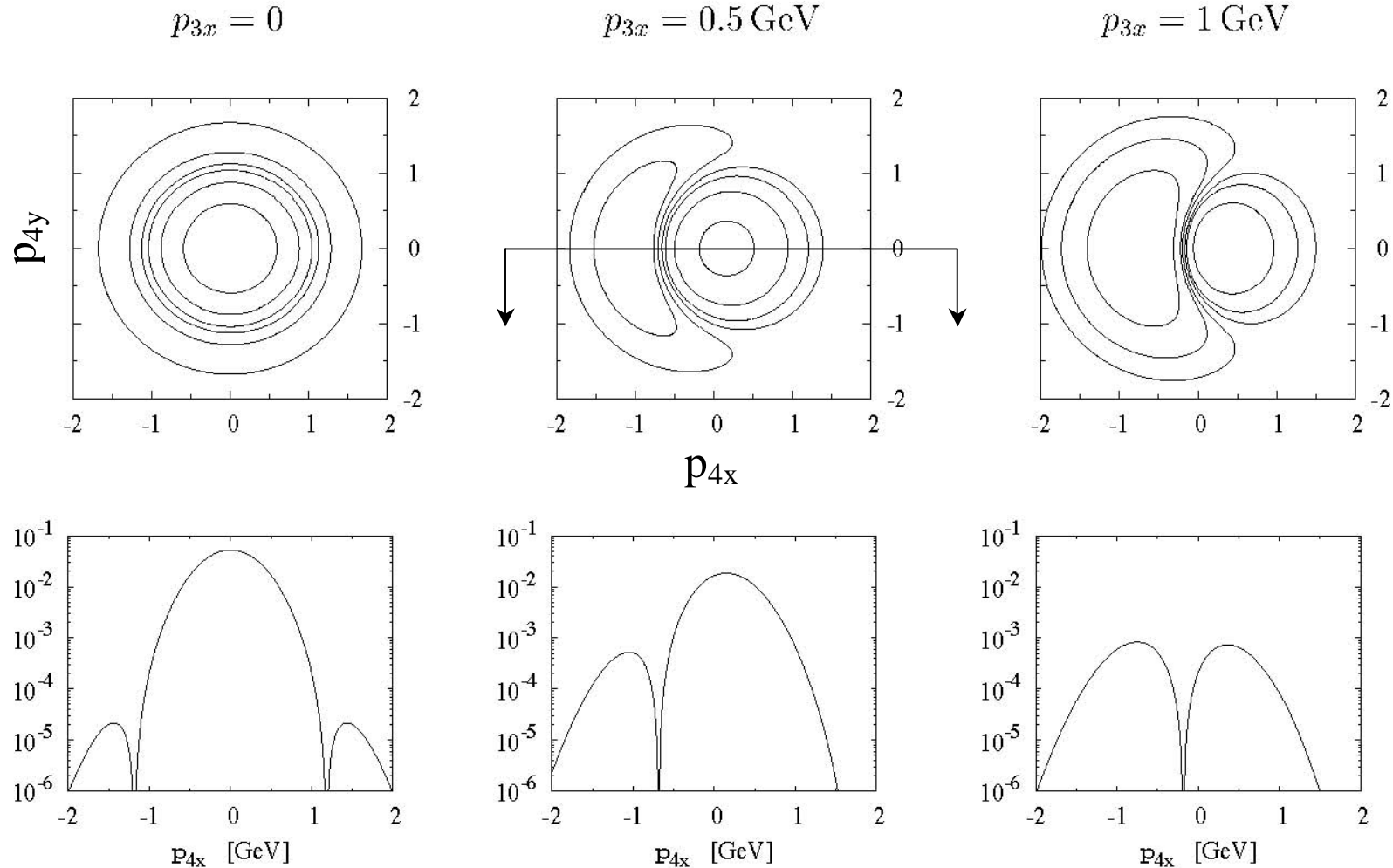
$$= \int d^2\mathbf{b} P_{hard}(\mathbf{b}) |1 - \Gamma(\mathbf{b}, s)|^2$$

Rapidity Gap Survival probability  $S^2$



# Diffractive images in transverse plane ( $p_{3y}=0$ )

Optical analog:  $\left\{ \begin{array}{l} H(\Delta) = \text{fourier transform of Diffraction Grating,} \\ [1 - \Gamma(\rho_2 - \rho_1)] = \text{single slit profile of each grating} \end{array} \right.$



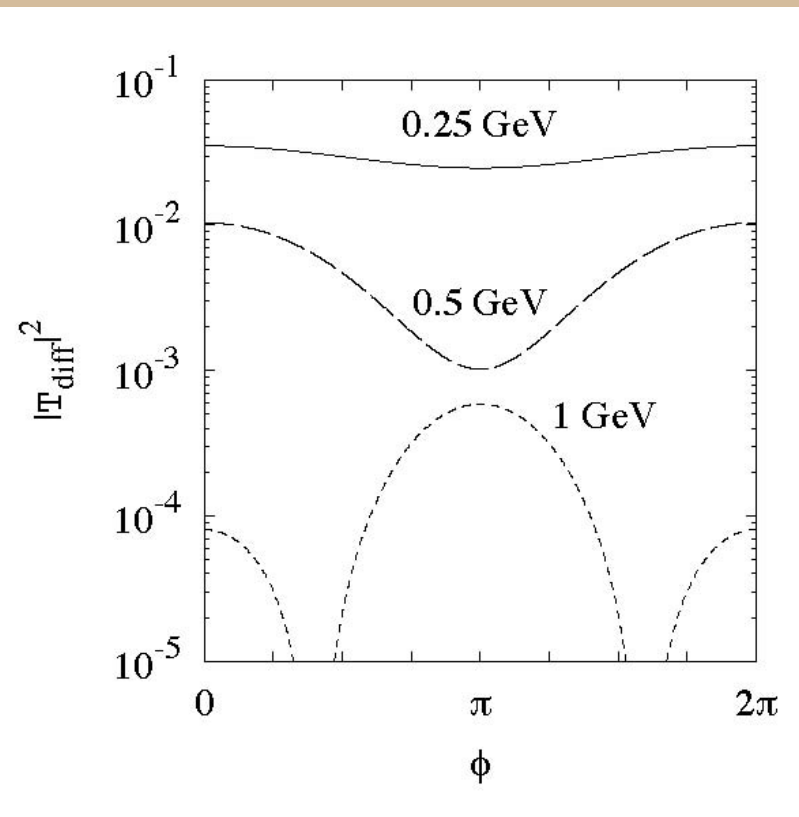


# Azimuthal Distribution of $\mathbf{p}_4$ vs $\mathbf{p}_3$

- $\phi = \phi_4 - \phi_3$ .
  - $0^+$  production ( $g^{\mu\nu}$ ) maximizes  $\mathbf{p}_3 \cdot \mathbf{p}_4$ 
    - $\phi = \pi$ : both projectiles recoil in same direction
  - $0^-$  production ( $\varepsilon^{\mu\nu\rho\sigma}$ ) maximizes  $E_1 \mathbf{p}_2 \cdot (\mathbf{p}_3 \times \mathbf{p}_4)$ 
    - $\phi = \pi/2$ : projectiles recoil at right angles.
- Sensitivity to parity of new particles

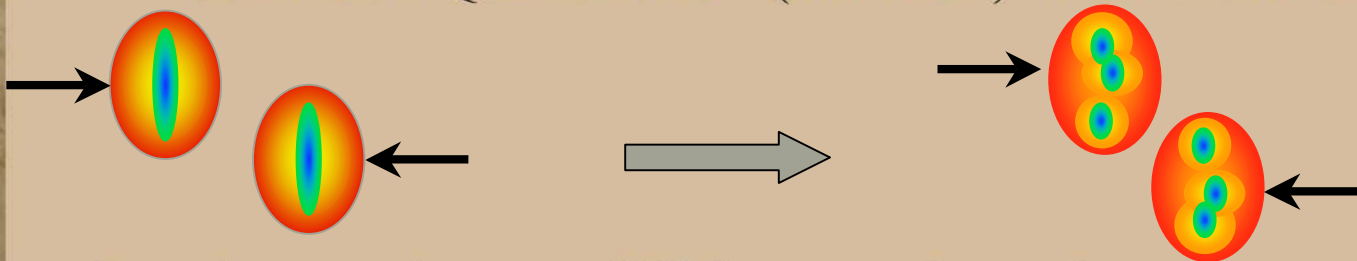
$0^+$  production:

140 GeV Higgs at 7 TeV



# Correlations of soft and hard partons in transverse plane

- Hard Scattering of parton at impact parameter  $\rho_1$  in projectile 1 on parton at impact parameter  $\rho_2$  in projectile 2.
- Local density of soft partons in each projectile near  $\rho_{1,2}$  is greater than average density in each projectile.
  - Partons are clustered in hadrons
- Sources of parton clustering:
  - Pion-cloud for  $\xi < m_\pi/M \rightarrow 6\%$  reduction of  $S^2$ .
  - Constituent Quarks: Size =  $(600 \text{ MeV})^{-1}$  from Instantons



- Previous estimate of  $S^2$  is upper bound

# Conceptual Conclusions:

## Rapidity Gap Survival in Central Hard Diffraction

- Generalized Parton Distributions:
  - Unifying Concept for Hard Exclusive Reactions:  $ep, pp, \dots$
- Impact Parameter representation gives physical picture
  - Quantum Numbers (parity) of new particles
  - [Approximate] factorization of hard & soft interactions
  - Transverse-spatial imaging of [quark &] gluon distributions
  - Model independent Parameterization of Soft Interactions
  - Black Disk Limit at  $b=0$ ,  $s \geq \text{Tevatron}$  highly constrains numerical estimates
  - $P_T$  distributions (spatial distribution of gluons) Depend on Rapidity
- Correlations of Hard and Soft partons in impact parameter
  - Complicates separation of hard and soft scattering
  - Examples: Pion-cloud, constituent-quarks, instantons:
  - Diffraction may provide new tool for observation of correlations.
  - Bound on Rapidity Gap Survival:  $S^2 \leq 0.03$ .



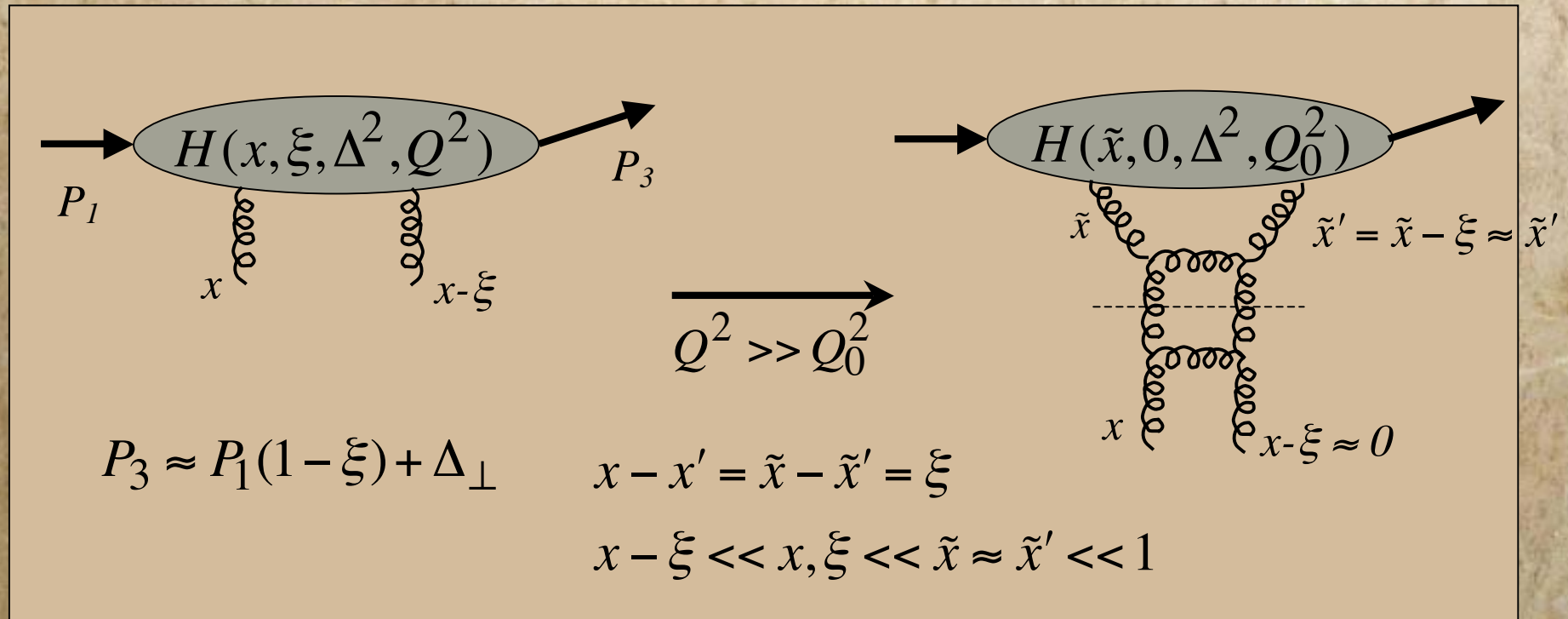
# Experimental Outlook

- RHIC: pp2pp @ STAR  $\sqrt{s}=200$  GeV
  - Si tracker at  $\approx 60$  m  $\geq 2007$ 
    - Elastic polarized pp scattering amplitude
    - Constrain onset of Black Disk Limit?
  - Si tracker at  $\approx 20$  m  $\geq 2008$ 
    - Rapidity gap events with central production in STAR
  - $\sqrt{s}=500$  GeV  $\leq 2010?$
- LHC
  - TOTEM (CMS  $\geq 2007$ ): Si tracker at 200 m
    - Elastic Scattering, total cross section, Re/Im ratio at  $t=0$ .
    - Rapidity gap tagging for  $0.01 < \xi < 0.1$
  - LHC420 ( $\geq 2008$ ): Si tracker at 420 m
    - Rapidity gap tagging for  $0.001 < \xi < 0.01$

# Additional Transparencies



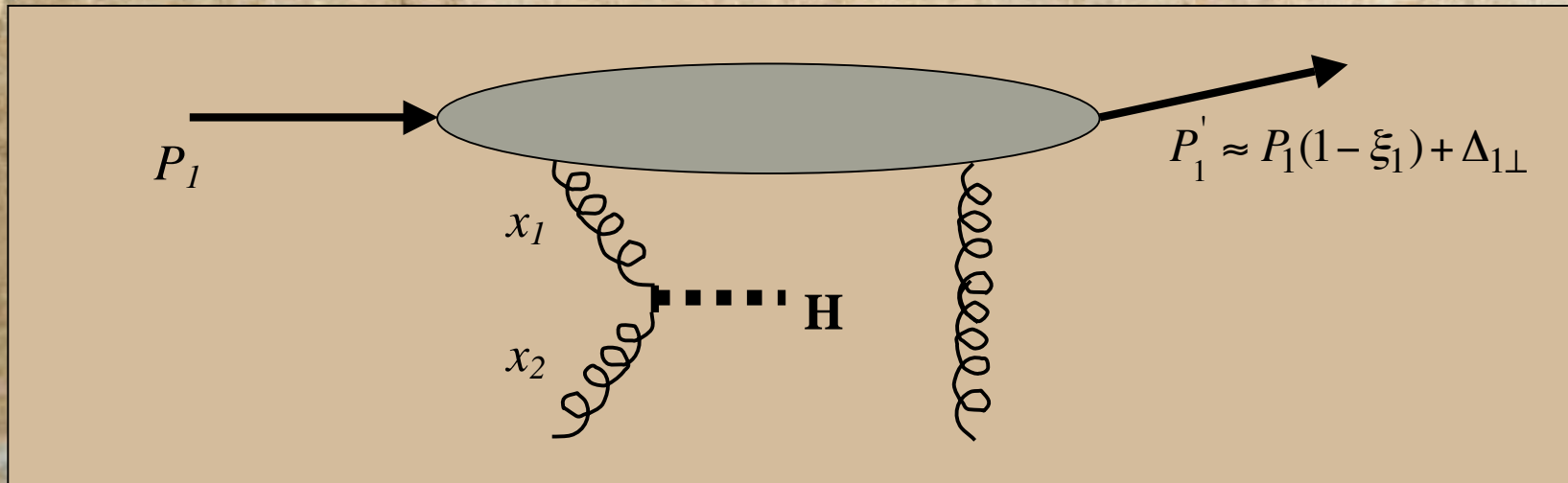
# Evolution and Parton model



- Example:  $p = 7 \text{ TeV}$        $M_H = 100 \text{ GeV}$   
 $x \approx 10^{-2} \ll \tilde{x} \ll 1$
- Virtuality  $Q^2$ : evolution local in impact parameter space,



# Hard Scattering Amplitude: Un-integrated Generalized Parton distributions



- Perturbative kernel for  $gluon(x_1) + gluon(x_2) \rightarrow H$
- Soft matrix elements of projectiles
  - *Gluon + Proton*  $\rightarrow$  *Gluon + Proton*
  - Compton amplitude for gluons of virtuality  $Q^2$ 
    - $\Lambda_{QCD}^2 \ll \langle Q^2 \rangle \ll M_H^2$ .
  - *Virtuality*  $Q^2$ : evolution is local in impact parameter space

# Scattering Amplitude in Momentum Transfer Space

pQCD kernel
Gluon GPDs of Projectiles

$$T_{Diff}(\xi_1, \mathbf{p}_3, \xi_2, \mathbf{p}_4) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} \kappa(\xi_1, \xi_2) H_g(x_1, \xi_1, t_3, Q^2) H_g(x_2, \xi_2, t_4, Q^2)$$

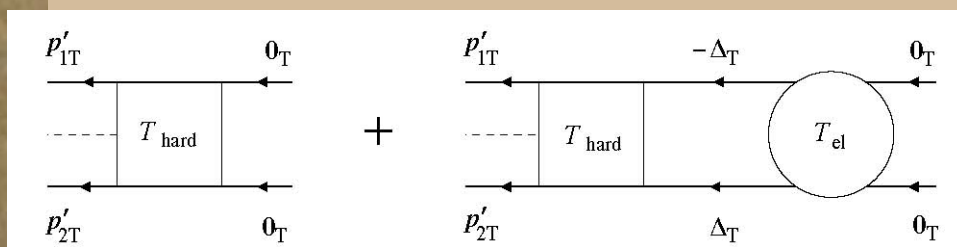
$$\left[ (2\pi)^2 \delta^{(2)}(\Delta_\perp) + \frac{4\pi i}{s} T_{El}(s, t) \right] dx_1 dx_2$$

$S_{Soft}$

$$t = -\Delta^2$$

$$t_3 = -(\mathbf{p}_3 - \Delta)^2$$

$$t_4 = -(\mathbf{p}_4 + \Delta)^2$$



# Gaussian Model

$$\left. \begin{aligned} H_g(x, \xi, \Delta^2) &= H_0(x, \xi) e^{-\Delta^2 B_g(\xi)} \\ T_{El} &= \int d^2 \mathbf{b} e^{-i \Delta \cdot \mathbf{b}} \Gamma(\mathbf{b}, s) \\ \Gamma(\mathbf{b}, s) &= e^{-\mathbf{b}^2 / B} \end{aligned} \right\}$$

*J/ψ* Photoproduction,

$$B_g \approx 3.24 \text{ GeV}^{-2} \ll B \approx 22 \text{ GeV}^{-2}$$

Transverse size  $\sqrt{B_g}$  of ‘hard’ gluons is smaller than hadronic radius  $\sqrt{B}$ .

*GPD*( $x \approx 0.05$ ,  $Q_0^2 \approx 3 \text{ GeV}^2$ )  
dominates determination of  
*GPD*( $x = 0.01$ ,  $Q^2 \approx 20 \text{ GeV}^2$ )

$M_H = 140 \text{ GeV}$  @ LHC

$$T_{\text{Diff}} = e^{-\left[ B_g(\xi_1) \mathbf{p}_3^2 + B_g(\xi_2) \mathbf{p}_4^2 \right] / 2} \left\{ 1 - \frac{B}{B_{\text{Tot}}} e^{\left[ B_g(\xi_1) \mathbf{p}_3 + B_g(\xi_2) \mathbf{p}_4 \right]^2 / [2 B_{\text{Tot}}]} \right\}$$

$$B_{\text{Tot}} = B_g(\xi_1) + B_g(\xi_2) + B$$



# Rapidity Dependence

## ■ Rapidity $y$ of Higgs

- $\xi_0 = [\xi_1 \xi_2]^{1/2} = M_H / \sqrt{s}$
- $\xi_{1,2} = \xi_0 e^{\pm y}$
- $B_g(\xi_{1,2}) \approx B_g(\xi_0) + \alpha_g' \ln[\xi_{1,2} / \xi_0]$
- $B_g(\xi_{1,2}) \approx B_g(\xi_0) \pm y \alpha_g'$

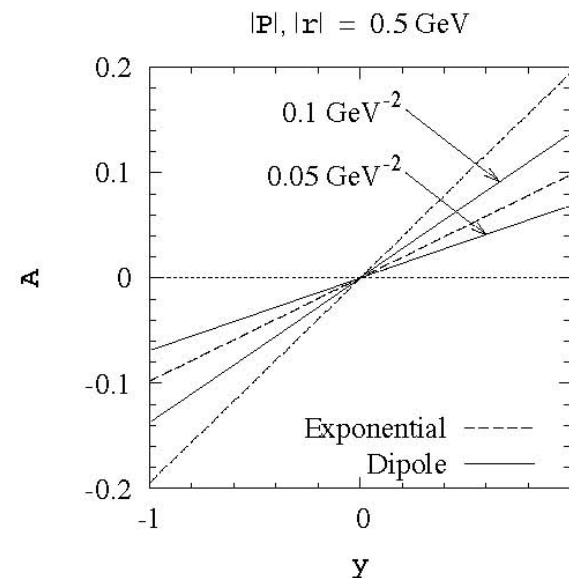
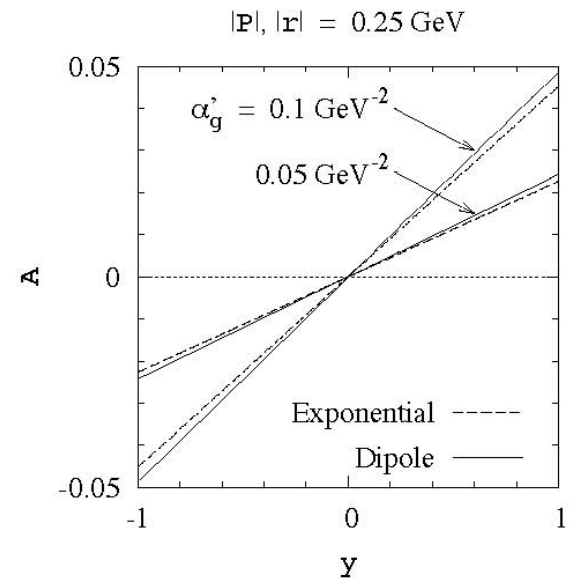
## ■ Forward backward asymmetry depends only on $\alpha_g'$

$$A = \frac{d\sigma(\xi_1, \xi_2) - d\sigma(\xi_2, \xi_1)}{d\sigma(\xi_1, \xi_2) + d\sigma(\xi_2, \xi_1)}$$

$$\propto y \alpha_g'$$

$$\mathbf{r} = (\mathbf{p}_3 - \mathbf{p}_4)$$

$$\mathbf{P} = (\mathbf{p}_3 + \mathbf{p}_4) / 2$$



# Rapidity Gap Survival Probability: $S^2$

- $S^2$  not an observable, but a useful statistic
  - $\sigma_{DD}$  = total Double Diffractive cross section.

$$\begin{aligned} \sigma_{DD}(\xi_1, \xi_2) &\propto \int \frac{d^2 \mathbf{p}_{3\perp}}{(2\pi)^2} \int \frac{d^2 \mathbf{p}_{4\perp}}{(2\pi)^2} |T_{\text{Diff}}(\xi_1, \mathbf{p}_{3\perp})|^2 \\ &\propto \int d^2 \rho_1 \int d^2 \rho_2 h_g^2(\xi_1, \rho_1) h_g^2(\xi_2, \rho_2) |1 - \Gamma(\rho_2 - \rho_1, s)|^2 \end{aligned}$$

$$\begin{aligned} S^2 &\equiv \frac{\sigma_{DD}(\text{Full})}{\sigma_{DD}(\text{no\_soft} : \Gamma = 0)} \\ &= \int d^2 \mathbf{b} P_{\text{hard}}(\mathbf{b}) |1 - \Gamma(\mathbf{b}, s)|^2 \end{aligned}$$

$S^2$  depends on hard sub-process

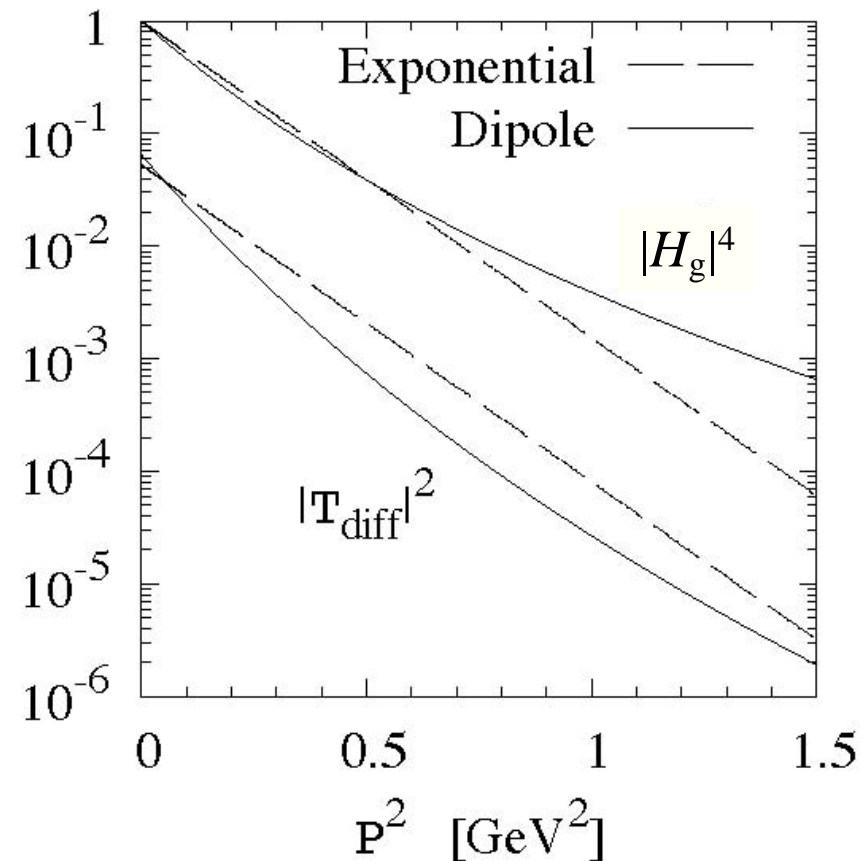
$$P_{\text{hard}}(\mathbf{b}) = \int d^2 \rho_1 \int d^2 \rho_2 \delta^{(2)}(\mathbf{b} + \rho_1 - \rho_2) \left[ \frac{h_g^2(\xi_1, \rho_1)}{\int d^2 \rho' h_g^2(\xi_1, \rho')} \right] \left[ \frac{h_g^2(\xi_2, \rho_2)}{\int d^2 \rho'' h_g^2(\xi_2, \rho'')} \right]$$

# Effect of Rapidity Gap Survival Probability

- $|T_{\text{diff}}|^2 = \text{Full Diffraction calculation}$
- $|H_g|^4 = |T_{\text{diff}}|^2$  with  $S_{\text{soft}} = 1$  ( $\Gamma=0$ , No soft absorption)

$$\mathbf{r} = (\mathbf{p}_3 - \mathbf{p}_4)$$

$$\mathbf{P} = (\mathbf{p}_3 + \mathbf{p}_4)/2$$





# Sensitivity to form of Gluon Distribution $H_g$

- Exponential and Dipole forms

$$H_g(x, \xi, -\Delta^2) = \begin{cases} e^{-\Delta^2 B_g(\xi)/2} \\ \frac{1}{\left[1 + \Delta^2 / m_g^2\right]^2} \end{cases}$$

