Flavor Dependence of Transverse Momentum Distributions

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k_T-dependent Parton Distributions

The parton distributions $f_1(x)$, $g_1(x)$, $h_1(x)$ are given by integrating the unintegrated parton distributions over $\vec{k}_\perp$:

(1)
$$\begin{aligned} f_1(x) &= \int d^2\vec{k}_\perp \; f_1(x,\vec{k}_\perp) \;, \\ g_1(x) &= \int d^2\vec{k}_\perp \; g_1(x,\vec{k}_\perp) \;, \\ h_1(x) &= \int d^2\vec{k}_\perp \; h_1(x,\vec{k}_\perp) \;. \end{aligned}$$

k_T-dependent Parton Distributions

Defined through the vector, axial vector and tensor currents:

$$\int \frac{dy^- d^2\vec{y}_\perp}{16(\pi)^3}\, e^{ixP^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\, \langle P,\lambda'|\overline{\psi}(0)\,\gamma^+\,\psi(y)\,|P,\lambda\rangle\Big|_{y^+=0}$$

$$= \frac{1}{2P^+}\,\overline{U}(P,\lambda')\; f_1(x,\vec{k}_\perp)\;\gamma^+\;U(P,\lambda)\;,$$

$$\int \frac{dy^- d^2\vec{y}_\perp}{16(\pi)^3}\, e^{ixP^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\, \langle P,\lambda'|\overline{\psi}(0)\,\gamma^+\gamma_5\,\psi(y)\,|P,\lambda\rangle\Big|_{y^+=0}$$

$$= \frac{1}{2P^+}\,\overline{U}(P,\lambda')\; f_1(x,\vec{k}_\perp)\;\gamma^+\gamma_5\;U(P,\lambda)\;,$$

$$\int \frac{dy^- d^2\vec{y}_\perp}{16(\pi)^3}\, e^{ixP^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\, \langle P,\lambda'|\overline{\psi}(0)\,\sigma^{+i}\,\psi(y)\,|P,\lambda\rangle\Big|_{y^+=0}$$

$$= \frac{1}{2P^+}\,\overline{U}(P,\lambda')\; h_1(x,\vec{k}_\perp)\;\sigma^{+i}\;U(P,\lambda)\;.$$

LF Wavefunction Representation

The state of proton is represented by the light-cone Fock expansion:

$$
\begin{aligned}
&\left|\psi_p(P^+, \vec{P}_\perp)\right\rangle \\
&= \sum_n \prod_{i=1}^{n} \frac{\mathrm{d}x_i \, \mathrm{d}^2\vec{k}_{\perp i}}{\sqrt{x_i}\, 16\pi^3} \; 16\pi^3 \delta\left(1 - \sum_{i=1}^{n} x_i\right) \delta^{(2)}\left(\sum_{i=1}^{n} \vec{k}_{\perp i}\right) \\
&\qquad \times \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \left| n;\, x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i \right\rangle .
\end{aligned}
$$

The light-cone momentum fractions $x_i = k_i^+/P^+$ and $\vec{k}_{\perp i}$ represent the relative momentum of the constituents.

LF Wavefunction Representation

$$
\begin{aligned}
f_1(x, \vec{k}_\perp) &= \mathcal{A}\, \psi^{\uparrow\,*}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i)\, \psi^{\uparrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i)\ , \\
&= \mathcal{A}\, \psi^{\downarrow\,*}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i)\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i)\ , \\
g_1(x, \vec{k}_\perp) &= \mathcal{A}\, \lambda_1\, \psi^{\uparrow\,*}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i)\, \psi^{\uparrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i)\ , \\
&= \mathcal{A}\, (-\lambda_1)\, \psi^{\downarrow\,*}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i)\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i)\ , \\
h_1(x, \vec{k}_\perp) &= \mathcal{A}\, \psi^{\downarrow\,*}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda'_1 = \downarrow, \lambda_{i\neq 1})\, \psi^{\uparrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_1 = \uparrow, \lambda_{i\neq 1})\ , \\
&= \mathcal{A}\, \psi^{\uparrow\,*}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda'_1 = \uparrow, \lambda_{i\neq 1})\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_1 = \downarrow, \lambda_{i\neq 1})\ .
\end{aligned}
$$

$$
\mathcal{A} = \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\ 16\pi^3 \delta\left(1 - \sum_{j=1}^{n} x_j\right)\ \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)\ \delta(x - x_1)\, \delta^{(2)}(\vec{k}_\perp - \vec{k}_{\perp 1})\, .
$$

Soffer's Inequality

$$\left[\left(f_1(x,\vec{k}_\perp)+g_1(x,\vec{k}_\perp)\right)\ \pm\ 2\,h_1(x,\vec{k}_\perp)\right]\ =\ \mathcal{A}$$
$$\times\left[\ \psi^{\uparrow\ *}_{(n)}(x_i,\vec{k}_{\perp i},\lambda'_1=\uparrow,\lambda_{i\neq 1})\ \pm\ \psi^{\downarrow\ *}_{(n)}(x_i,\vec{k}_{\perp i}\lambda'_1=\downarrow,\lambda_{i\neq 1})\ \right]$$
$$\times\left[\ \psi^{\uparrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_1=\uparrow,\lambda_{i\neq 1})\ \pm\ \psi^{\downarrow}_{(n)}(x_i,\vec{k}_{\perp i}\lambda_1=\downarrow,\lambda_{i\neq 1})\ \right].$$

Therefore, we get the Soffer's inequality:

$$\left(f_1(x,\vec{k}_\perp)+g_1(x,\vec{k}_\perp)\right)\ \pm\ 2\,h_1(x,\vec{k}_\perp)\ \geq\ 0\ .$$

The equality holds when

$$\psi^{\uparrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_1=\uparrow,\lambda_{i\neq 1})\ \pm\ \psi^{\downarrow}_{(n)}(x_i,\vec{k}_{\perp i}\lambda_1=\downarrow,\lambda_{i\neq 1})\ =\ 0\ .$$

Yukawa Model

$$\begin{cases} \psi^{\uparrow}_{+\frac{1}{2}}(x,\vec{k}_\perp) = (M + \frac{m}{x})\,\varphi\ , \\ \psi^{\uparrow}_{-\frac{1}{2}}(x,\vec{k}_\perp) = -\frac{(k^1+\mathrm{i}k^2)}{x}\,\varphi\ , \end{cases}$$

where $\varphi(x,\vec{k}_\perp) = \frac{e}{\sqrt{1-x}}\ \frac{1}{M^2 - \frac{\vec{k}_\perp^2+m^2}{x} - \frac{\vec{k}_\perp^2+\lambda^2}{1-x}}$.

$$\begin{aligned} f_1(x,\vec{k}_\perp) &= \int \frac{\mathrm{d}^2\vec{k}_\perp \mathrm{d}x}{16\pi^3}\left[(M+\frac{m}{x})^2 + \frac{\vec{k}_\perp^2}{x^2}\right]|\varphi|^2\ , \\ g_1(x,\vec{k}_\perp) &= \int \frac{\mathrm{d}^2\vec{k}_\perp \mathrm{d}x}{16\pi^3}\left[(M+\frac{m}{x})^2 - \frac{\vec{k}_\perp^2}{x^2}\right]|\varphi|^2\ , \\ h_1(x,\vec{k}_\perp) &= \int \frac{\mathrm{d}^2\vec{k}_\perp \mathrm{d}x}{16\pi^3}\left[(M+\frac{m}{x})^2\right]|\varphi|^2\ . \end{aligned}$$

$$\Big(f_1(x,\vec{k}_\perp) + g_1(x,\vec{k}_\perp)\Big) - 2\,h_1(x,\vec{k}_\perp) = 0\ .$$

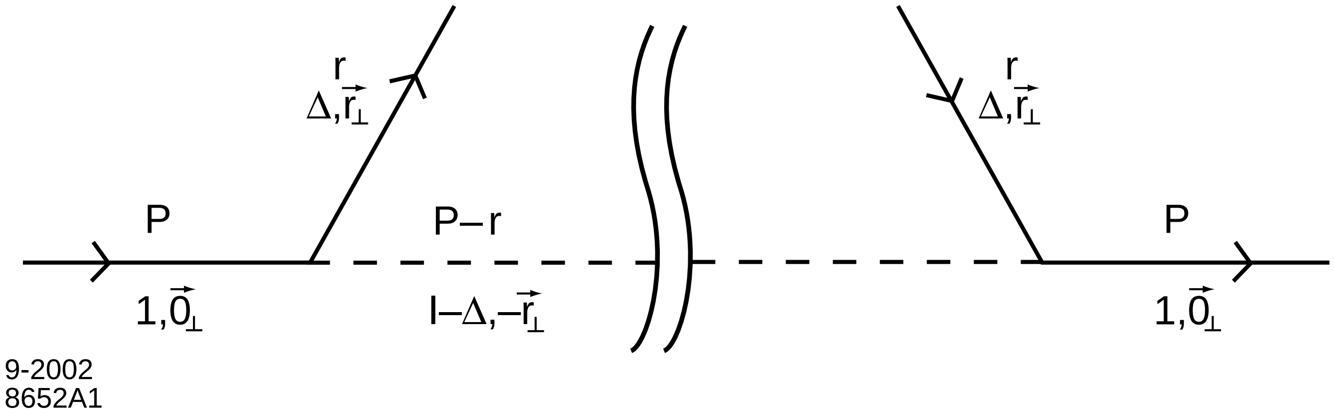
QED Model

$$
\begin{cases}
\psi^{\uparrow}_{+\frac{1}{2}\,+1}(x,\vec{k}_\perp) = -\sqrt{2}\frac{(-k^1+\mathrm{i}k^2)}{x(1-x)}\,\varphi\ , \\
\psi^{\uparrow}_{+\frac{1}{2}\,-1}(x,\vec{k}_\perp) = -\sqrt{2}\frac{(+k^1+\mathrm{i}k^2)}{1-x}\,\varphi\ , \\
\psi^{\uparrow}_{-\frac{1}{2}\,+1}(x,\vec{k}_\perp) = -\sqrt{2}(M-\frac{m}{x})\,\varphi\ , \\
\psi^{\uparrow}_{-\frac{1}{2}\,-1}(x,\vec{k}_\perp) = 0\ .
\end{cases}
$$

$$
\begin{aligned}
f_1(x,\vec{k}_\perp) &= \int \frac{\mathrm{d}^2\vec{k}_\perp \mathrm{d}x}{16\pi^3}\, 2\left[\frac{\vec{k}_\perp^2}{x^2(1-x)^2} + \frac{\vec{k}_\perp^2}{(1-x)^2} + (M-\frac{m}{x})^2\right]|\varphi|^2\ , \\
g_1(x,\vec{k}_\perp) &= \int \frac{\mathrm{d}^2\vec{k}_\perp \mathrm{d}x}{16\pi^3}\, 2\left[\frac{\vec{k}_\perp^2}{x^2(1-x)^2} + \frac{\vec{k}_\perp^2}{(1-x)^2} - (M-\frac{m}{x})^2\right]|\varphi|^2\ , \\
h_1(x,\vec{k}_\perp) &= \int \frac{\mathrm{d}^2\vec{k}_\perp \mathrm{d}x}{16\pi^3}\, 4\left[\frac{\vec{k}_\perp^2}{x(1-x)^2}\right]|\varphi|^2\ .
\end{aligned}
$$

$$
\begin{aligned}
&\Big(f_1(x,\vec{k}_\perp) + g_1(x,\vec{k}_\perp)\Big) - 2\,h_1(x,\vec{k}_\perp) \\
&= \int \frac{\mathrm{d}^2\vec{k}_\perp \mathrm{d}x}{16\pi^3}\, 2\left[\frac{\vec{k}_\perp^2}{x^2(1-x)^2}\right](x-1)^2\,|\varphi|^2 \ \geq\ 0.
\end{aligned}
$$

Diquark Model



(Jakob, Mulders, Rodrigues, Nucl. Phys. A 1997)

Scalar and Axial-vector Diquarks:

$$\Upsilon^s(N) = \mathbf{1} g_s(r^2) \, ,$$

$$\Upsilon^{a\mu}(N) = \frac{g_a(r^2)}{\sqrt{3}} \gamma_\nu \gamma_5 \frac{\not{P} + M}{2M} \Big(- g^{\mu\nu} + \frac{P^\mu P^\nu}{M^2} \Big) \, .$$

$$g(\tau) = g_s(\tau) = g_a(\tau) = N \frac{\tau - m^2}{|\tau - \Lambda^2|^\alpha} \; ,$$

$$\frac{\tau - \Lambda^2}{x} = \frac{r^2(x, \mathbf{p}_T^2) - \Lambda^2}{x} = -\frac{\mathbf{p}_T^2}{x(1-x)} - \frac{M_R^2}{1-x} - \frac{\Lambda^2}{x} + M^2 \; .$$

$\Lambda = 0.5$, M=0.94, m=0.3, $\alpha = 2$.

M_R=0.6 for scalar; M_R=0.8 for axial-vector.

$$f_1(x, \mathbf{p}_T^2) = A \frac{(xM+m)^2 + \mathbf{p}_T^2}{(\mathbf{p}_T^2 + \lambda_R^2)^{2\alpha}} ,$$

$$g_1(x, \mathbf{p}_T^2) = a_R \, A \frac{(xM+m)^2 - \mathbf{p}_T^2}{(\mathbf{p}_T^2 + \lambda_R^2)^{2\alpha}} ,$$

where

$$a_s = 1 , \qquad a_a = -\frac{1}{3} ,$$

$$\lambda_R^2(x) = (1-x)\Lambda^2 + xM_R^2 - x(1-x)M^2 ,$$

$$A = \frac{N^2(1-x)^{2\alpha-1}}{16\pi^3} .$$

Scalar Diquark

$$q_s^+(x,\mathbf{p}_T^2) \;=\; \frac{1}{2}\Big(f_{1s}+g_{1s}\Big) \;=\; A\,\frac{(xM+m)^2}{(\mathbf{p}_T^2+\lambda_R^2)^{2\alpha}}\,,$$

$$q_s^-(x,\mathbf{p}_T^2) \;=\; \frac{1}{2}\Big(f_{1s}-g_{1s}\Big) \;=\; A\,\frac{\mathbf{p}_T^2}{(\mathbf{p}_T^2+\lambda_R^2)^{2\alpha}}\,,$$

Axial-vector Diquark

$$q_a^+(x,\mathbf{p}_T^2) \;=\; \frac{1}{2}\Big(f_{1a}+g_{1a}\Big) \;=\; A\,\frac{\frac{1}{3}(xM+m)^2+\frac{2}{3}\mathbf{p}_T^2}{(\mathbf{p}_T^2+\lambda_R^2)^{2\alpha}}\,,$$

$$q_a^-(x,\mathbf{p}_T^2) \;=\; \frac{1}{2}\Big(f_{1a}-g_{1a}\Big) \;=\; A\,\frac{\frac{2}{3}(xM+m)^2+\frac{1}{3}\mathbf{p}_T^2}{(\mathbf{p}_T^2+\lambda_R^2)^{2\alpha}}\,.$$

Since spin-0 diquarks are in a flavor singlet state and spin-1 are in a flavor triplet state, in order to combine to a symmetric spin-flavor wavefunction as demanded by the Pauli principle, the proton wavefunction has the well-known $SU(4)$ structure (Jakob, Mulders, Rodrigues, Nucl. Phys. A 1997):

$$
\begin{aligned}
|p\uparrow> \;\; &= \;\; \frac{1}{\sqrt{2}}|u\uparrow S_0^0> + \frac{1}{\sqrt{18}}|u\uparrow A_0^0> - \frac{1}{3}|u\downarrow A_0^1> \\
&- \;\; \frac{1}{3}|d\uparrow A_1^0> + \sqrt{\frac{2}{9}}|d\downarrow A_1^1> ,
\end{aligned}
$$

where S (A) represents a scalar (axial-vector) diquark and upper (lower) indices represent the projections of the spin (isospin) along a definite direction.

Since the coupling of the spin has already been included in the vertices, we need the flavor coupling (Jakob, Mulders, Rodrigues, Nucl. Phys. A 1997):

$$|p> = \frac{1}{\sqrt{2}}|uS_0> + \frac{1}{\sqrt{6}}|uA_0> - \frac{1}{\sqrt{3}}|dA_1>,$$

to find that for the nucleon the flavor distributions are

$$\begin{aligned} f_1^u &= \frac{3}{2}f_1^s + \frac{1}{2}f_1^a, \\ f_1^d &= f_1^a. \end{aligned}$$

u^+, u^-, d^+, d^-

$$u^+(x, \mathbf{p}_T^2) = A\, \frac{\frac{5}{3}(xM+m)^2 + \frac{1}{3}\mathbf{p}_T^2}{(\mathbf{p}_T^2+\lambda_R^2)^{2\alpha}}\,,$$

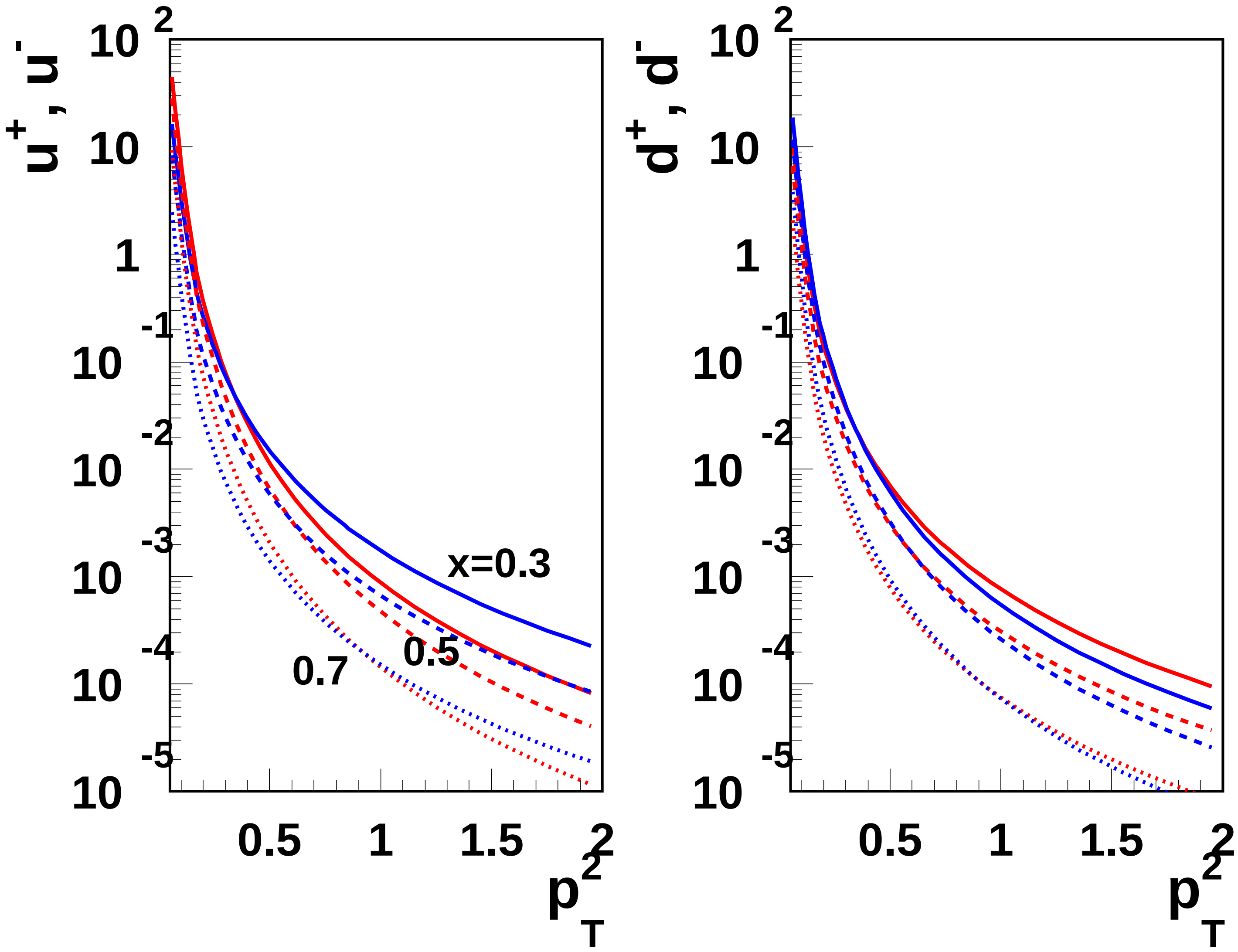
$$u^-(x, \mathbf{p}_T^2) = A\, \frac{\frac{1}{3}(xM+m)^2 + \frac{5}{3}\mathbf{p}_T^2}{(\mathbf{p}_T^2+\lambda_R^2)^{2\alpha}}\,,$$

$$d^+(x, \mathbf{p}_T^2) = A\, \frac{\frac{1}{3}(xM+m)^2 + \frac{2}{3}\mathbf{p}_T^2}{(\mathbf{p}_T^2+\lambda_R^2)^{2\alpha}}\,,$$

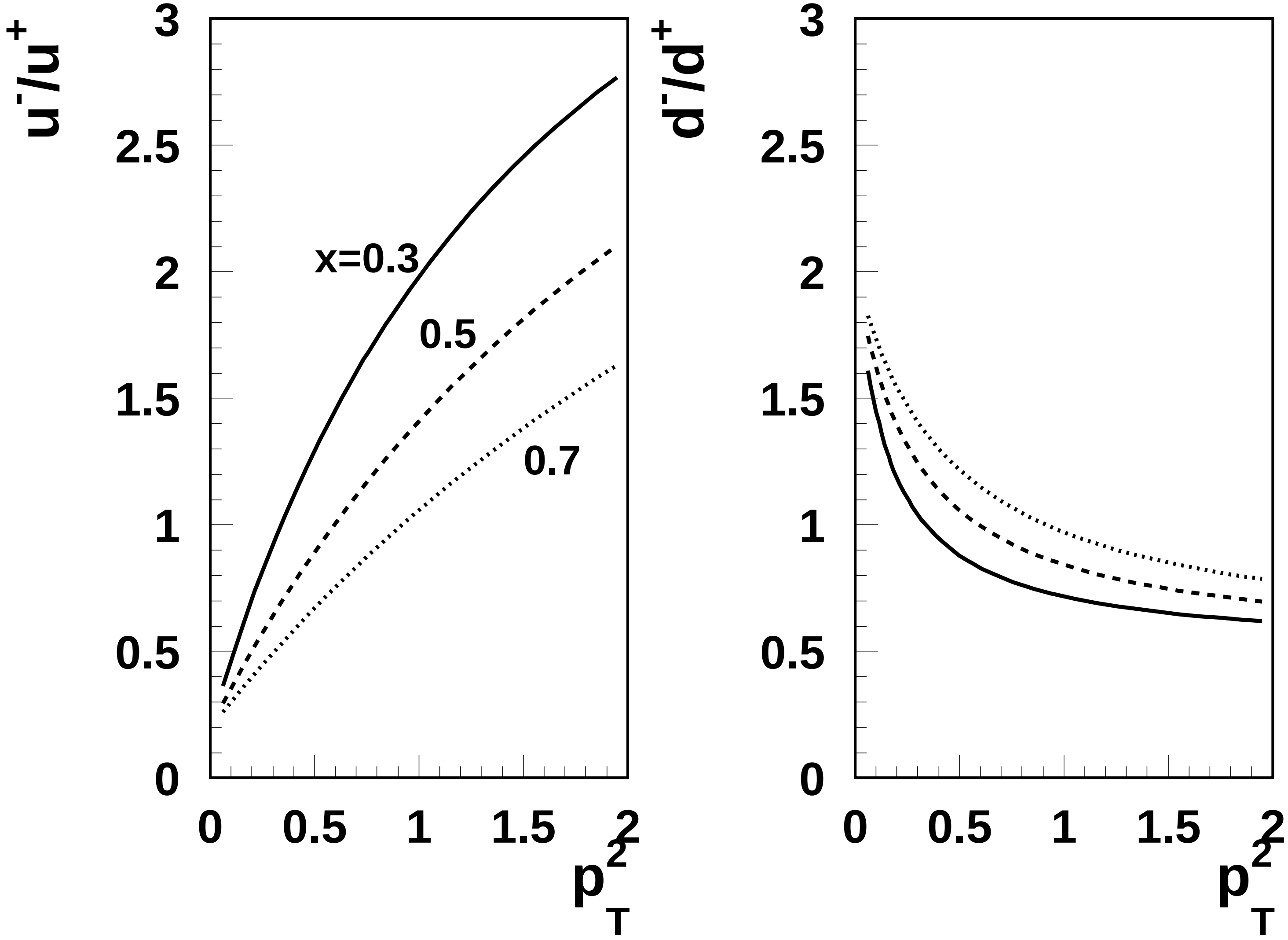
$$d^-(x, \mathbf{p}_T^2) = A\, \frac{\frac{2}{3}(xM+m)^2 + \frac{1}{3}\mathbf{p}_T^2}{(\mathbf{p}_T^2+\lambda_R^2)^{2\alpha}}\,.$$

$$A = \frac{N^2(1-x)^{2\alpha-1}}{16\pi^3}\,.$$

u^+, u^-, d^+, d^- $(x = 0.3, 0.5, 0.7)$



$$\frac{u^+}{u^-}, \frac{d^+}{d^-} \; (x = 0.3, 0.5, 0.7)$$



Conclusion

Scalar and axial-vector diquark models give different k_T-distributions of quarks. $\longrightarrow$
u and d quark in proton have different k_T-distributions.

k_T distributions make interesting physical phenomena possible:

Orital Angular Momentum

Pauli Form Factor, Sivers Function, and so on.

Single-spin Asymmetry and Double-spin Asymmetry in SIDIS.