Single-Spin Asymmetries in Inclusive and Exclusive Hadronic Processes

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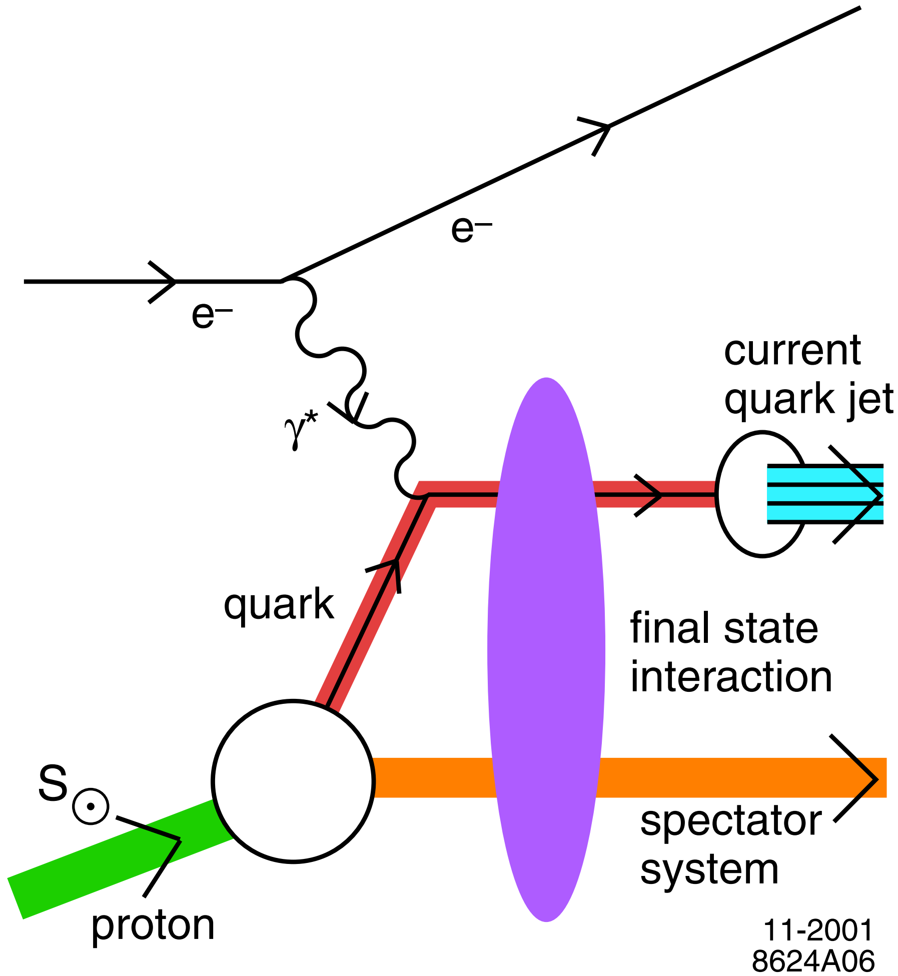
Single-Spin Asymmetry



Symmetric Spin-dependent Tensor

Existence of Symmetric Spin-dependent Hadronic Tensor Provides Sigle-Spin Asymmetries

Interpretation of Sigle-Spin Asymmetries in Inclusive and Exclusive Electron-proton Scattering and Electron-positron Annihilation in a Unified Manner

We Propose to Measure the Structure Function of this Symmetric Spin-dependent Hadronic Tensor at BABAR and BELLE, which will lead to the First Measurement of this Structure Function

We can also Study this Structure Function at the JLAB, HERMES, COMPASS, GSI

Space-like Process $e^- p \to e^- X$

$$
\begin{aligned}
W_{\mu\nu} &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2,\nu) \\
&+ \frac{1}{M^2}\left(P_\mu - \frac{P\cdot q}{q^2} q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2} q_\nu\right) W_2(q^2,\nu) \\
&+ \frac{1}{M^3}\left[\left(P_\mu - \frac{P\cdot q}{q^2} q_\mu\right)\epsilon_{\nu abc} s^a P^b q^c \right. \\
&\qquad \left. + \left(P_\nu - \frac{P\cdot q}{q^2} q_\nu\right)\epsilon_{\mu abc} s^a P^b q^c\right] G_3(q^2,\nu) \, ,
\end{aligned}
$$

in which M is the proton mass and $q = k - k'$ where k and k' are the momenta of the initial and final electrons, respectively. $M\nu = P\cdot q$ and we use $\epsilon_{0123} = 1$. $-q^2 > 0$ and $1 < \omega \equiv \frac{2M\nu}{|q^2|} < \infty$ for the space-like processes.

$$\frac{d\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{|q^2|^2} \sin^2\frac{\theta}{2}\Big[\, 2W_1(q^2,\nu) \;+\; \cot^2\frac{\theta}{2} W_2(q^2,\nu)$$
$$+ \;\; (\vec{s}\cdot\hat{y})\, \frac{4(E+E')EE'}{M\,|q^2|}\, |\sin\theta|\; G_3(q^2,\nu)\,\Big]\,,$$

where E ($\hat{k}$) and E' ($\hat{k}'$) are the initial and final energies (momentum directions) and θ the scattering angle of the electron, and $\hat{y} = (\hat{k}\times\hat{k}')/|\hat{k}\times\hat{k}'|$.

From the previous equation the single-spin asymmetry is given by

$$\mathcal{P}_y = \frac{d\sigma(\vec{s}=\hat{y}) - d\sigma(\vec{s}=-\hat{y})}{d\sigma(\vec{s}=\hat{y}) + d\sigma(\vec{s}=-\hat{y})} = \frac{\frac{4(E+E')EE'}{M|q^2|}\ |\sin\theta|\ G_3(q^2,\nu)}{2W_1(q^2,\nu)\ +\ \cot^2\frac{\theta}{2}W_2(q^2,\nu)}\ .$$

Checking experimentally whether the above $\mathcal{P}_y$ is really zero corresponds to the study by Christ and Lee (Phys. Rev. 1966) on the T-violation.

This single-spin asymmetry can be investigated at the JLAB, HERMES, and COMPASS.

$e^- p \longrightarrow e^- p$

$$\langle P' | J^\mu(0) | P \rangle = \overline{u}(P') \left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2M}\sigma^{\mu\alpha}q_\alpha \right] u(P) \ .$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4M^2}F_2(q^2) \ , \qquad G_M(q^2) = F_1(q^2) + F_2(q^2) \ ,$$

where $G_E(0) = 1$ and the proton magnetic moment is defined by $\mu = \frac{e}{2M}G_M(0)$.

We have the relations

$$W_1^{\rm ex}(q^2,\nu) = -\frac{q^2}{4m^2}|G_M(q^2)|^2 \ \delta(2M\nu + q^2) \ ,$$

$$W_2^{\rm ex}(q^2,\nu) = \frac{|G_E(q^2)|^2 - \frac{q^2}{4M^2}|G_M(q^2)|^2}{1-\frac{q^2}{4M^2}} \ \delta(2M\nu + q^2) \ ,$$

$$G_3^{\rm ex}(q^2,\nu) = -\frac{{\rm Im}[G_M^* G_E]}{2\left(1-\frac{q^2}{4M^2}\right)} \ \delta(2M\nu + q^2) \ .$$

Then, we get the differential cross section given by

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2 E'\cos^2\frac{\theta}{2}}{2E^3\sin^4\frac{\theta}{2}}\left(\frac{1}{1+\tau}\right)\Big[\,|G_E(q^2)|^2 + \frac{\tau}{\epsilon}|G_M(q^2)|^2$$

$$- \ (\vec{s}\cdot\hat{y})\ \frac{E+E'}{M}|\tan\frac{\theta}{2}|\ \mathrm{Im}[G_M^* G_E]\ \Big]\ ,$$

where $\hat{y} = (\hat{k}\times\hat{k}')/|\hat{k}\times\hat{k}'|$, $\tau = \frac{|q^2|}{4M^2}$, and $1/\epsilon = 1 + 2(1+\tau)\tan^2\frac{\theta}{2}$.

$$\mathcal{P}_y = \frac{-\ \frac{E+E'}{M}\ |\tan\frac{\theta}{2}|\ \mathrm{Im}[G_M^* G_E]}{|G_E(q^2)|^2 + \frac{\tau}{\epsilon}|G_M(q^2)|^2}\ .$$

In reality, the above $\mathcal{P}_y$ is expected to be zero since the Hermiticity of the electromagnetic current renders the space-like form factors real. However, it is useful to check experimentally whether it really vanishes.

This single-spin asymmetry corresponds to the $\mathcal{P}_y$ which will be measured at the up-graded proton form factor experiment at the JLAB.

Time-like Process $e^+e^- \to pX$

$$
\begin{aligned}
\overline{W}_{\mu\nu} &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\overline{W}_1(q^2,\nu) \\
&+ \frac{1}{M^2}\left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right)\overline{W}_2(q^2,\nu) \\
&+ \frac{1}{M^3}\left[\left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\epsilon_{\nu abc}s^a P^b q^c \right. \\
&\qquad \left. + \left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right)\epsilon_{\mu abc}s^a P^b q^c\right] G_3(q^2,\nu) \ ,
\end{aligned}
$$

in which M is the proton mass and $q = k + k'$ where k and k' are the momenta of the initial electron and positron, respectively. $M\nu = P\cdot q$ and we use $\epsilon_{0123} = 1$. $q^2 > 0$ and $0 < \omega \equiv \frac{2M\nu}{q^2} < 1$ for the time-like processes.

$\overline{W}_1$ and $\overline{W}_2$ are usual spin-independent structure functions (Drell, Levy, Yan, Phys. Rev. 1969), and G_3 is a symmetric spin-dependent structure function which has its origin in the strong-interaction phases, and was introduced by Gourdin (Nucl. Phys. B, 1972).

Here, we study the single-spin asymmetry which would be induced by G_3 in the time-like deep inelatic scattering.

In the center of mass frame of the initial electron and positron

$$
\begin{aligned}
&\frac{d^2\sigma}{dE d\cos\theta} \\
= \;& \frac{2\pi\alpha^2 M}{(q^2)^{\frac{3}{2}}}\Big(\frac{2M\nu}{q^2}\Big)\Big(1-\frac{q^2}{\nu^2}\Big)^{\frac{1}{2}} \Bigg(\Big[2\overline{W}_1(q^2,\nu) \\
&\qquad + \Big(\frac{2M\nu}{q^2}\Big)\Big(1-\frac{q^2}{\nu^2}\Big)\sin^2\theta\,\frac{\nu\overline{W}_2(q^2,\nu)}{2M}\Big] \\
+ \;& (\vec{s}\cdot\hat{y})\,\frac{(q^2)^{\frac{3}{2}}}{4M^3}\Big(\frac{2M\nu}{q^2}\Big)^2\Big(1-\frac{q^2}{\nu^2}\Big)\sin 2\theta\; G_3(q^2,\nu)\Bigg),
\end{aligned}
$$

where E is the energy of the produced proton, $\hat{k}$ and $\hat{p}$ the initial electron and produced proton momentum directions, and θ the angle between $\hat{k}$ and $\hat{p}$, and $\hat{y} = (\hat{k}\times\hat{p})/|\hat{k}\times\hat{p}|$.

From the above equation the single-spin asymmetry is given by

$$\mathcal{P}_y = \frac{\frac{(q^2)^{\frac{3}{2}}}{4M^3} \left(\frac{2M\nu}{q^2}\right)^2 \left(1 - \frac{q^2}{\nu^2}\right) \sin 2\theta \; G_3(q^2,\nu)}{2\overline{W}_1(q^2,\nu) \; + \; \left(\frac{2M\nu}{q^2}\right)\left(1 - \frac{q^2}{\nu^2}\right) \sin^2\theta \; \frac{\nu \overline{W}_2(q^2,\nu)}{2M}} \; .$$

In the Bjorken limit of the deep inelastic process (Drell, Levy, Yan, Phys. Rev. 1969),

$$M\overline{W}_1 = -F_1(\omega)\,, \qquad \nu\overline{W}_2 = F_2(\omega)\,, \qquad \text{and} \;\; F_1(\omega) = \frac{\omega}{2}F_2(\omega)\,.$$

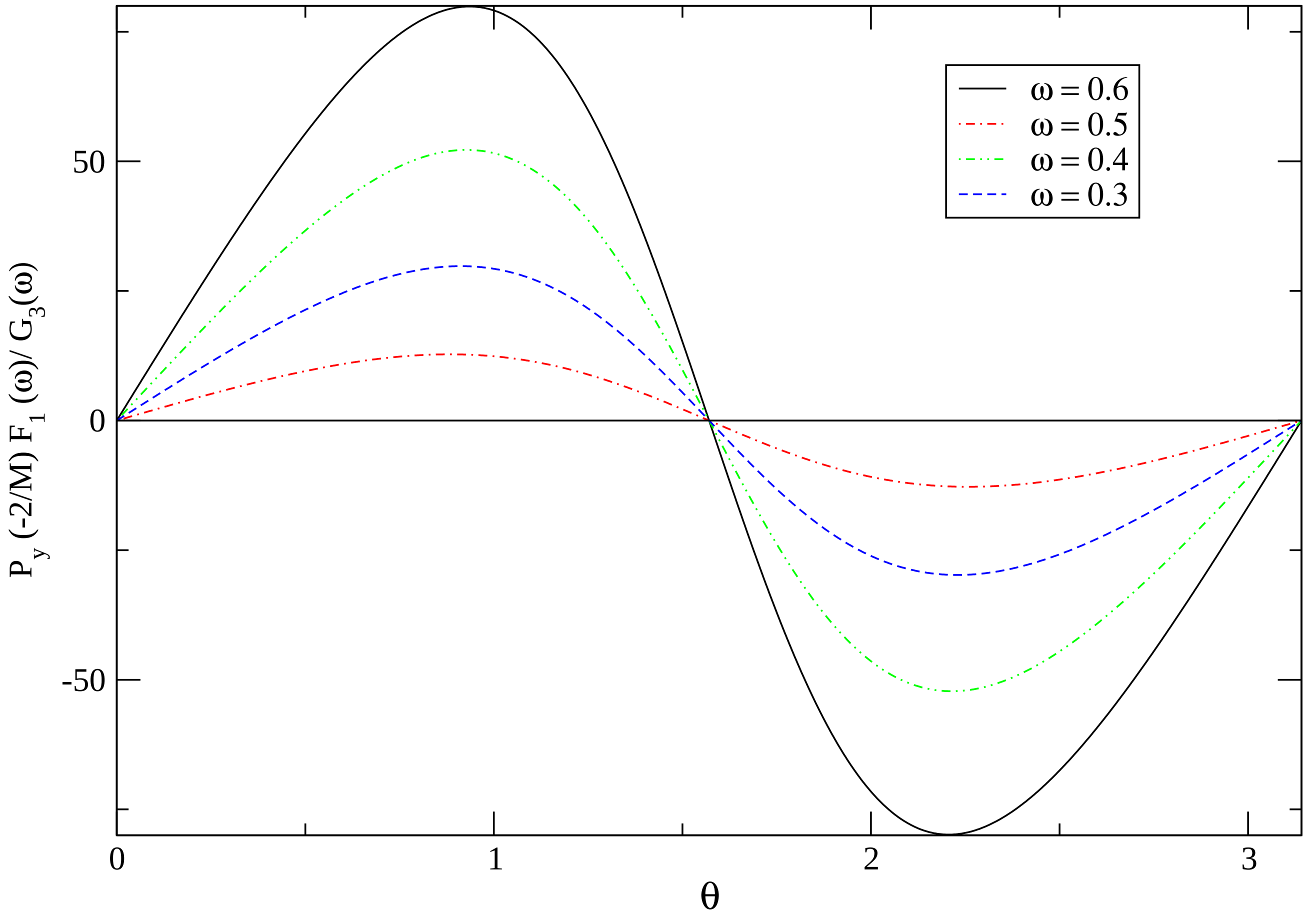
Then, the differential cross section is given by

$$\begin{aligned}\frac{d^2\sigma}{dE\,d\cos\theta} \;=\; & \frac{2\pi\alpha^2 M}{(q^2)^{\frac{3}{2}}}\,\omega\left(1-\frac{4M^2}{q^2}\frac{1}{\omega^2}\right)^{\frac{1}{2}} \\ \times\; & \left(\frac{2}{M}\left(-F_1(\omega)\right)\left[1-\frac{1}{2}\left(1-\frac{4M^2}{q^2}\frac{1}{\omega^2}\right)\sin^2\theta\right]\right. \\ & \left. +\,(\vec{s}\cdot\hat{y})\,\frac{(q^2)^{\frac{3}{2}}}{4M^3}\,\omega^2\left(1-\frac{4M^2}{q^2}\frac{1}{\omega^2}\right)\sin 2\theta\; G_3(\omega)\right)\,,\end{aligned}$$

and the single-spin asymmetry is given by

$$\mathcal{P}_y = \frac{G_3(\omega)\,\frac{(q^2)^{\frac{3}{2}}}{4M^3}\,\omega^2\left(1-\frac{4M^2}{q^2}\frac{1}{\omega^2}\right)}{\left(-F_1(\omega)\right)\frac{2}{M}}\;\frac{\sin 2\theta}{\left[1-\frac{1}{2}\left(1-\frac{4M^2}{q^2}\frac{1}{\omega^2}\right)\sin^2\theta\right]}\,.$$

We propose that BABAR and BELLE perform this measurement.



Here, we used the proton mass for M, and $q^2 = (10.6\ \mathrm{GeV})^2$ which is the value of BABAR and BELLE.

$e^-e^+ \to p\overline{p}$

$$\langle pp'|J^\mu(0)|0\rangle = \overline{u}(P)\left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2M}\sigma^{\mu\alpha}q_\alpha\right]v(P')\ .$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m^2}F_2(q^2)\ , \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)\ .$$

We have the relations

$$\overline{W}_1^{\text{ex}}(q^2,\nu) = \frac{q^2}{4M^2}|G_M(q^2)|^2\ \delta(2M\nu - q^2)\ ,$$

$$\overline{W}_2^{\text{ex}}(q^2,\nu) = -\frac{|G_E(q^2)|^2 - \frac{q^2}{4M^2}|G_M(q^2)|^2}{1 - \frac{q^2}{4M^2}}\ \delta(2M\nu - q^2)\ ,$$

$$G_3^{\text{ex}}(q^2,\nu) = \frac{\text{Im}[G_M^* G_E]}{2\left(1 - \frac{q^2}{4M^2}\right)}\ \delta(2M\nu - q^2)\ .$$

Then, we get the differential cross section given by

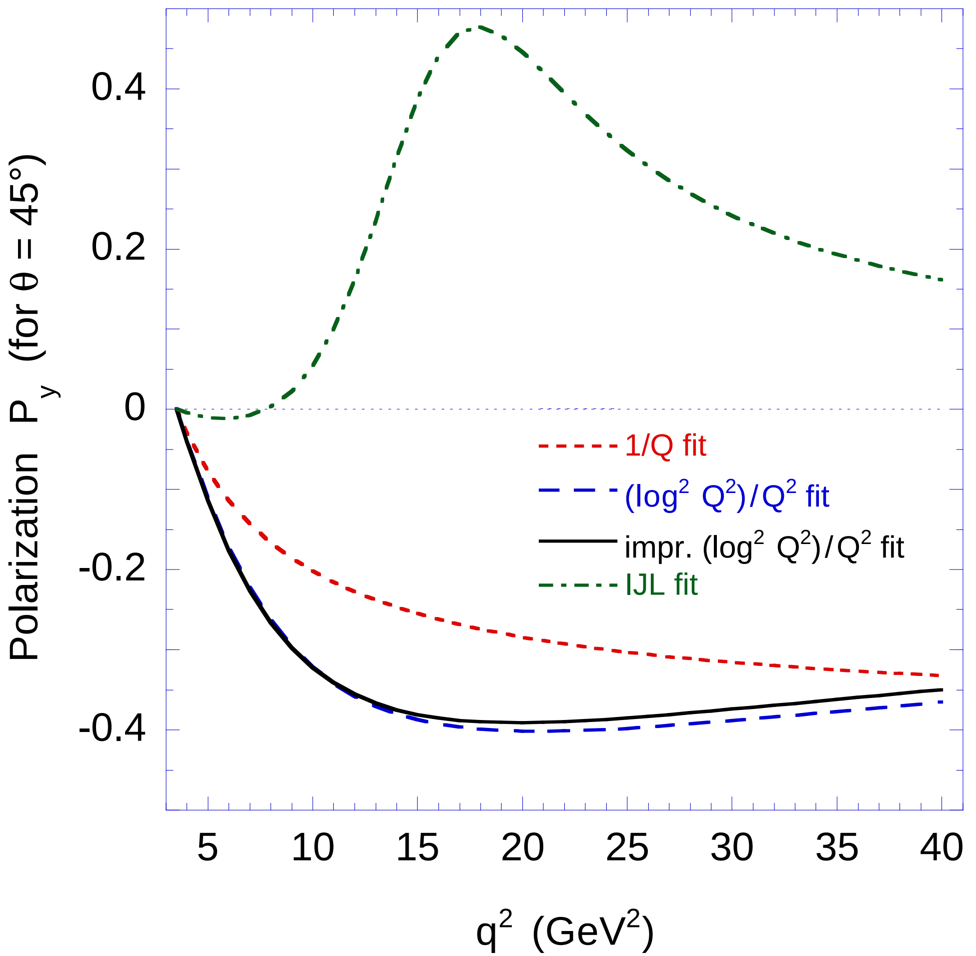
$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\sqrt{1-\frac{4M^2}{q^2}}}{2q^2}$$
$$\times \Big[\,|G_M(q^2)|^2(1+\cos^2\theta) \;+\; \frac{4M^2}{q^2}|G_E(q^2)|^2\sin^2\theta$$
$$-\,(\vec{s}\cdot\hat{y})\,\frac{2M}{\sqrt{q^2}}\,\sin2\theta\;\mathrm{Im}[G_M^* G_E]\,\Big]\,,$$

where $\hat{y} = (\hat{k}\times\hat{p})/|\hat{k}\times\hat{p}|$.

From the above equation the single-spin asymmetry is given by (Brodsky, Carlson, Hiller, Hwang, Phys. Rev. D, 2004)

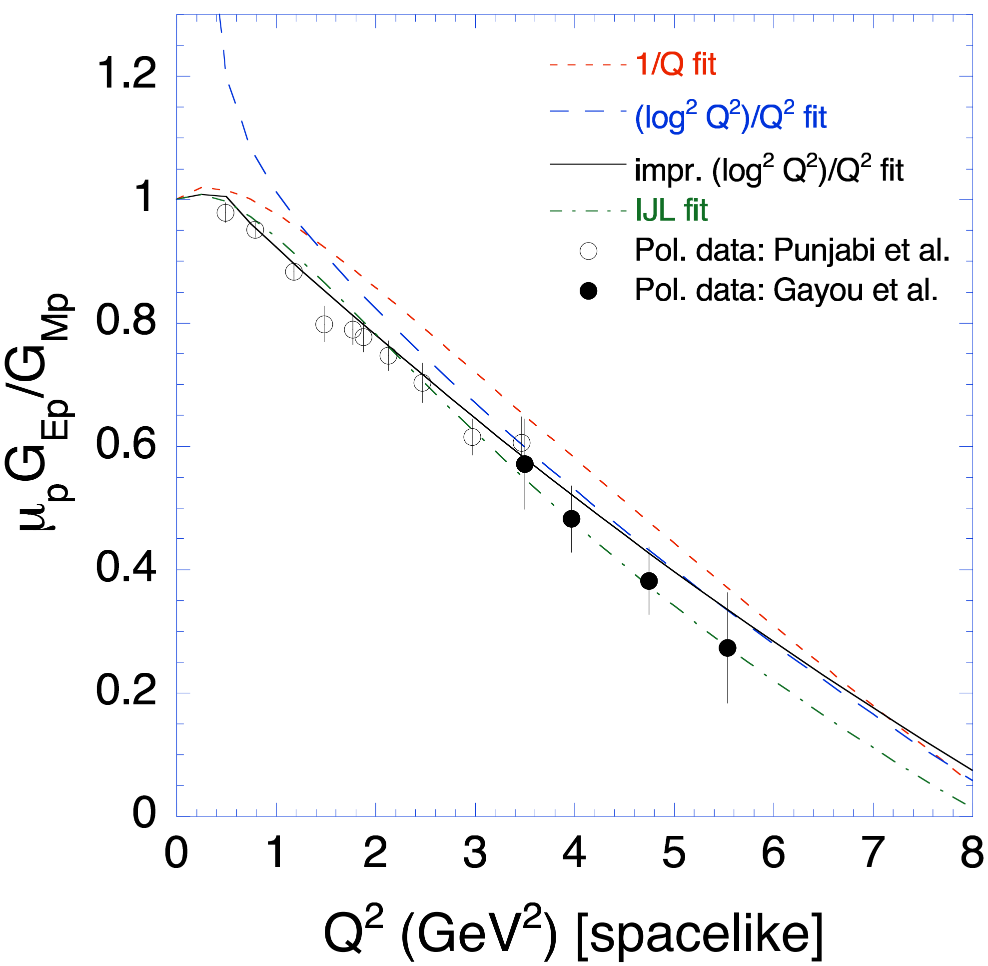
$$\mathcal{P}_y = \frac{-\frac{2M}{\sqrt{q^2}}\,\sin 2\theta\ \mathrm{Im}[G_M^* G_E]}{|G_M(q^2)|^2(1+\cos^2\theta) + \frac{4M^2}{q^2}|G_E(q^2)|^2 \sin^2\theta}\ .$$

This single-spin asymmetry may be measured at the future GSI $p\overline{p}$ experiment and the up-graded DAFNE experiment.

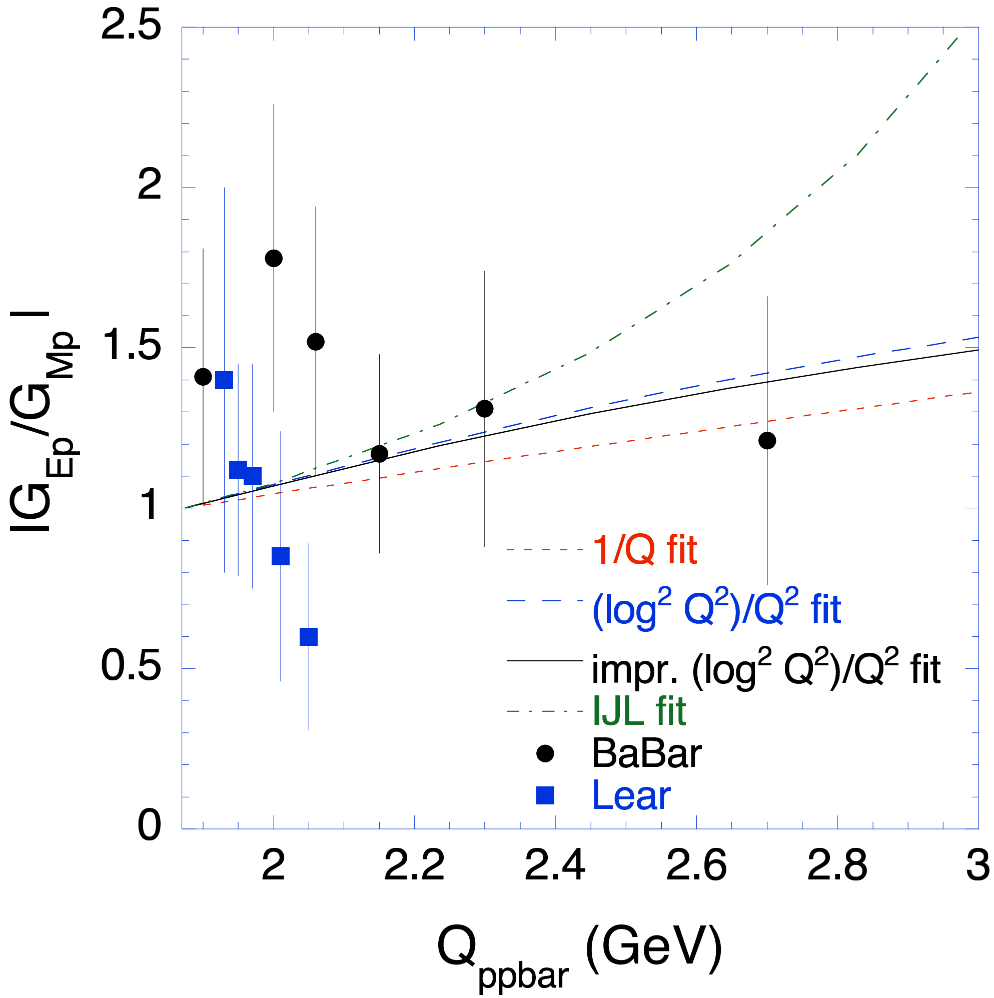


Predicted polarization $\mathcal{P}_y$ in the timelike region for selected form factor fi ts described in BCHH (Brodsky, Carlson, Hiller, Hwang, Phys. Rev. D, 2004). The plot is for $\theta = 45^\circ$. The four curves are for an $F_2/F_1 \propto 1/Q$ fi t, using BCHH 1; the $(\log^2 Q^2)/Q^2$ fi t of Belitsky et al.; an improved $(\log^2 Q^2)/Q^2$ fi t, BCHH 2; and a fi t from Iachello et al.

Space-like FF Ratios (Brodsky, Carlson, Hiller, Hwang)



Time-like FF Ratios (Brodsky, Carlson, Hiller, Hwang)



Conclusion

The symmetric spin-dependent structure function, $G_3(q^2, \nu)$, can exist in the time-like processes from the strong phases which are made by the final-state interactions, without violating the T-invariance.

It is important to measure the single-spin asymmetries in both inclusive and exclusive processes for the Λ production at the present B-factories. This will lead to the first measurement of $G_3(q^2, \nu)$.

The existence of $G_3(q^2, \nu)$ in the space-like processes corresponds to the T-violation.