POLARIZATION PHENOMENA IN $e^+e^- \to p\bar{p}$ *REVISITED*

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1. SUCCESS OF POLARIZATION EXPERIMENT

More recently at Jefferson Lab *by means of double polarization experiment* a great success was achieved

M.K.Jones et al, Phys. Rev. Lett. 84 (2000) 1398

O.Gayou et al, Phys. Rev. Lett. 88 (2002) 092301

V.Punjabi et al, Phys. Rev C71 (2005) 055202

measuring simultaneously **transverse**

then P_t ; P_l

$$P_t = \frac{h}{I_0}(-2)\sqrt{\tau(1+\tau)}G_{Mp}G_{Ep}\tan(\theta/2) \quad (1)$$

and **longitudinal**

$$P_l = \frac{h(E+E')}{I_0 m_p}\sqrt{\tau(1+\tau)}G_{Mp}^2\tan^2(\theta/2) \quad (2)$$

components of the **recoil proton's polarization** in the electron scattering plane of the polarization transfer process (h is the electron beam helicity, I_0 is the unpolarized cross-section excluding σ_{Mott} and $\tau = Q^2/4m_p^2$)

$$G_{Ep}/G_{Mp} = -\frac{P_t}{P_l}\frac{(E+E')}{2m_p}\tan(\theta/2).$$

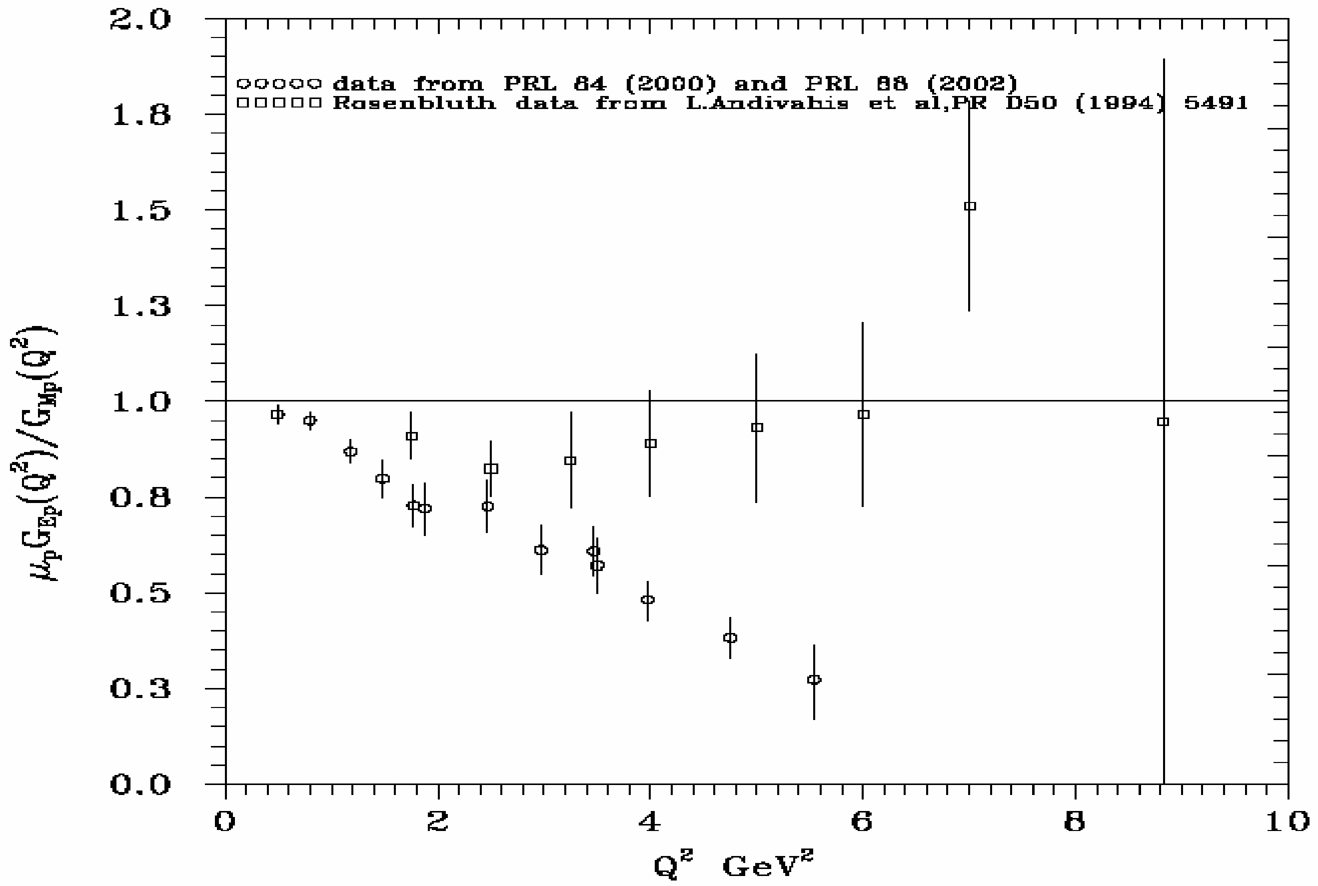


Figure 1: JLab polarization data on the ratio $\mu_p G_{Ep}(t)/G_{Mp}(t)$

A STRONG DISAGREEMENT with data obtained by Rosenbluth technique was revealed

A.Z. Dubničková, S. Dubnička, M.P. Rekalo, Nuovo Cim. **A109** *(1996) 241.*

2. SPIN POLARIZATION EFFECTS IN $e^+e^- \rightarrow p\bar{p}$

In this contribution **an important role of polarization observables** in time-like region of nucleon form factors **is demonstrated**.

By consideration of the $e^+e^- \rightarrow p\bar{p}$ process **one can in a flash eliminate unrealistic nucleon electromagnetic structure models with 'imaginary parts' of form factors to be zero**.

It is interesting to investigate:

first **single spin polarization observables** (either for proton or for antiproton) for the case of

- **unpolarized** incoming leptons
- incoming electron to be **longitudinally polarized**.

then **double spin polarization observables** (for proton and antiproton simultaneously) also for the case of

- **unpolarized** incoming leptons
- incoming electron to be **longitudinally polarized**.

The matrix element of the process $e^+e^- \longrightarrow p\bar{p}$ in the framework of the one-photon approximation is defined by the formulae

$$\begin{aligned} \mathcal{M} &= \frac{e^2}{k^2} j_\mu J_\mu, \\ j_\mu &= \bar{u}(-k_2)\gamma_\mu u(k_1), \\ J_\mu &= \bar{u}(p_1)[F_{1p}(s)\gamma_\mu - F_{2p}(s)\frac{\sigma_{\mu\nu}k_\nu}{2m_p}]u(-p_2), \end{aligned} \tag{4}$$

where $s = k^2 \geq 4m_p^2$.

The c.m. system of the reaction $e^+e^- \longrightarrow p\bar{p}$ is **the most suitable** for the analysis of polarization effects.

Electromagnetic currents j_μ and J_μ are **conserved** $k \cdot j = k \cdot J = 0$, therefore in the c.m. system of $e^+e^- \longrightarrow p\bar{p}$ reaction $\ell_0 = J_0 = 0$

$\Rightarrow$ the matrix element $\mathcal{M}$ is **determined** only **by product of spatial components** of currents

$$\begin{aligned} \mathcal{M} &= -\frac{e^2}{s}\vec{j}\cdot\vec{J} \\ |\mathcal{M}|^2 &= \frac{e^4}{s^2} j_{ik} W_{ik}, \end{aligned} \quad (5)$$

where

$$j_{ik} = j_i j_k^* \qquad W_{ik} = J_i J_k^*.$$

The electromagnetic current $\vec{J}$ can be expressed through two-component spinors φ_1 and φ_2

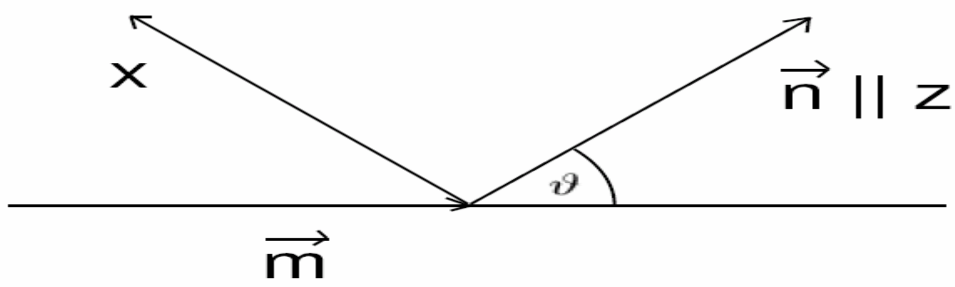
$$\vec{J} = \sqrt{s}\varphi_1^+ \left[G_{Mp}(s)(\vec{\sigma} - \vec{n}\vec{\sigma}\cdot\vec{n}) + \frac{2m_p}{\sqrt{s}} G_{Ep}(s)\vec{n}\vec{\sigma}\cdot\vec{n} \right] \varphi_2 \quad (6)$$

where we denote

$$\vec{F} = \sqrt{s}\,[\,G_M(s)(\vec{\sigma} - \vec{n}\,\vec{\sigma}.\vec{n}) + \frac{2M}{\sqrt{s}}\,G_E(s)\vec{n}\,\vec{\sigma}.\vec{n}\,]$$

$\vec{\sigma}$ are Pauli matrices.

$\vec{n}$ is the unit vector along the three momentum $\vec{q}$ of the proton, $\vec{m}$ is the unit vector of incoming electron, as is presented in following Figure



$$\vec{m} = (-\sin\vartheta, 0, \cos\vartheta)$$

$$\vec{n} = (0, 0, 1)$$

3. SINGLE SPIN POLARIZATION OBSERVABLES

(OF CREATED PROTONS REVISITED)

a) In the case of unpolarized initial leptons the corresponding **lepton tensor** takes the form

$$j_{ij} = 2s(\delta_{ij} - m_i m_j) \qquad (8)$$

The vector polarization in this case is

$$\vec{P} = \frac{j_{ij} Tr[F_i F_j^\dagger \vec{\sigma}]}{j_{ij} Tr[F_i F_j^\dagger]}. \qquad (9)$$

Calculating the corresponding trace in the numerator

$$\begin{aligned} Tr[F_i F_j^\dagger \sigma_x] &= 2it|G_M(t)|^2[\varepsilon_{ijx} - n_j n_k \varepsilon_{ikx} - n_i n_k \varepsilon_{kjx}] + \\ &+ i\frac{4M}{\sqrt{t}} G_M(t) G_E^*(t)[n_j n_k \varepsilon_{ikx}] + \\ &+ i\frac{4M}{\sqrt{t}} G_M^*(t) G_E(t)[n_i n_k \varepsilon_{kjx}]. \end{aligned} \tag{10}$$

and similarly in the denominator one obtaines the vector polarization

$$\vec{P} = \frac{\frac{4M}{\sqrt{t}} Im(G_M^*(t) G_E(t))(\vec{m}.\vec{n})}{\frac{4M^2}{t}|G_E|^2 \sin^2\vartheta + |G_M|^2(1+\cos^2\vartheta)}(\vec{n}\times\vec{m})$$

with only nonzero component P_y

Single spin polarization (unpolarized initial leptons)

$$\begin{aligned} P_x &= 0 \\ P_y &= -\frac{\frac{1}{\sqrt{\tau}} Im(G_M^*(t) G_E(t)) \sin 2\vartheta}{\frac{1}{\tau}|G_E|^2 \sin^2\vartheta + |G_M|^2(1+\cos^2\vartheta)} \\ P_z &= 0 \end{aligned} \qquad (12)$$

The y-axis is orthogonal to the scattering plane defined by the unit vectors $\vec{m}$ and $\vec{n}$ along the three-momentum of the electron and the three-momentum of the created proton, respectively.

b) The contributions of P_x and P_z in proton polarization are different from zero only **if the electron is longitudinally polarized**.

In this case the lepton tensor takes the form

$$j^{\mu\nu} = Tr[\, p^{\kappa}\gamma_{\kappa}\gamma^{\mu}p'^{\kappa}\gamma_{\kappa}\gamma^{\nu}(\frac{1+\gamma_5}{2})\,], \qquad (13)$$

which results in

$$j_{ij} = 2s(\,\delta_{ij} - m_i m_j + \lambda i \varepsilon_{ijl} m_l\,).$$

Then the vector polarization $\vec{P}$ of the reaction $e^+e^- \longrightarrow p\bar{p}$ has the form

$$
\begin{aligned}
\vec{P} \;=\; & 1/D\;\left\{\frac{4m_N}{\sqrt{t}} Im(G_M^*(t)G_E(t))(\vec{m}.\vec{n})(\vec{n}\times\vec{m})\right.\\
+\; & |G_M|^2(t)[\vec{m}-2(\vec{m}.\vec{n})\vec{n}] +\\
+\; & \left.\frac{4m_N}{\sqrt{t}} Re[G_M^*(t)G_E(t)](\vec{n}(\vec{m}.\vec{n})-\vec{m})\right\},
\end{aligned}
$$

where

$$
D = \frac{1}{\tau}|G_E|^2(t)\sin^2\vartheta + |G_M|^2(t)(1+\cos^2\vartheta),
$$

and

Results of spin polarization observables (polarized leptons)

$$P_x = -\frac{2\sin\theta \cdot Re[G_{Ep}(s)G^*_{Mp}(s)]\tau}{|G_{Ep}(s)|^2\sin^2\theta/\tau + |G_{Mp}(s)|^2(1+\cos^2\theta)} \quad (16)$$

$$P_y = -\frac{\frac{1}{\sqrt{\tau}} Im(G^*_M(t)G_E(t))\sin 2\vartheta}{\frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta + |G_M(t)|^2(1+\cos^2\vartheta)}$$

$$P_z = \frac{2\cos\theta|G_{Mp}(s)|^2}{|G_{Ep}(s)|^2\sin^2\theta/\tau + |G_{Mp}(s)|^2(1+\cos^2\theta)}, \quad (17)$$

assuming 100% i.e. $\lambda = 1$ longitudinal polarization of leptons.

Double spin polarization observables

4. DOUBLE SPIN POLARIZATION OBSERVABLES

(POLARIZATION COMPONENTS OF CREATED p AND $\bar{p}$)

A creation of simultaneously polarized p and $\bar{p}$ in $e^+e^- \to p\bar{p}$ process is characterized by tensor

$$P_{kl} = \frac{j_{ij} Tr[F_i \sigma_k F_j^\dagger \sigma_l]}{j_{ij} Tr[F_i F_j^\dagger]}. \qquad (17)$$

In (17) $(k, l = x, y, z)$

Calculating the trace in the numerator

Calculation of double spin polarization observables

$$\frac{1}{t}Tr[F_i\sigma_x F_j^\dagger\sigma_y] = 2|G_M|^2[\delta_{ix}\delta_{jy}$$

$$-\delta_{ij}\delta_{xy} + \delta_{iy}\delta_{xj} - \delta_{ix}n_jn_y + \delta_{xy}n_jn_i - \delta_{iy}n_jn_x - \delta_{jy}n_in_x$$

$$+\delta_{xy}n_in_j - \delta_{xj}n_in_y - \delta_{xy}n_in_j + 2n_in_jn_xn_y]$$

$$+\frac{4M}{\sqrt{t}}G_M(t)G_E^*(t)[\delta_{ix}n_jn_y - \delta_{xy}n_in_j + \delta_{iy}n_jn_x + \delta_{xy}n_in_j - 2n_in_jn_xn_y]$$

$$+\frac{4M}{\sqrt{t}}G_M^*(t)G_E(t)[\delta_{jy}n_in_x - \delta_{xy}n_in_j + \delta_{xj}n_in_y + \delta_{xy}n_in_j - 2n_in_jn_xn_y]$$

$$+\frac{8M^2}{t}|G_E(t)|^2(2n_in_jn_xn_y - \delta_{xy}n_in_j).$$

and similarly the trace in denominator, considering unpolarized

incoming leptons one finds

Double spin polarization observables – unpolarized initial leptons

$$
\begin{aligned}
P_{xx} &= \frac{|G_M(t)|^2\cos^2\vartheta - \frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta}{\frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta + |G_M(t)|^2(1+\cos^2\vartheta)};\\
P_{yy} &= \frac{|G_M(t)|^2(1+\sin^2\vartheta) - \frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta}{\frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta + |G_M(t)|^2(1+\cos^2\vartheta)};\\
P_{zz} &= \frac{|G_M(t)|^2\sin^2\vartheta - \frac{1}{\tau}|G_E(t)|^2\cos^2\vartheta}{\frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta + |G_M(t)|^2(1+\cos^2\vartheta)};\\
P_{xy} &= P_{yx} = 0;\\
P_{xz} &= P_{zx} = \frac{\frac{1}{\sqrt{\tau}}Re[G_M^*(t)G_E(t)]\sin 2\vartheta}{\frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta + |G_M(t)|^2(1+\cos^2\vartheta)};\\
P_{yz} &= P_{zy} = 0.
\end{aligned}
\tag{18}
$$

Now, for contraction using the expression for lepton tensor with longitudinally polarized electron

one obtains for **components of the polarization correlation tensor** P_{kl} of proton p and antiproton $\bar{p}$ the following formulas

Double spin polarization observables –polarized leptons

$$P_{xx} = \frac{|G_M(t)|^2\cos^2\vartheta - \frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta}{\frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta + |G_M(t)|^2(1+\cos^2\vartheta)}$$

$$P_{yy} = \frac{|G_M(t)|^2(1+\sin^2\vartheta) - \frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta}{\frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta + |G_M(t)|^2(1+\cos^2\vartheta)}$$

$$P_{zz} = \frac{|G_M(t)|^2\sin^2\vartheta - \frac{1}{\tau}|G_E(t)|^2\cos^2\vartheta}{\frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta + |G_M(t)|^2(1+\cos^2\vartheta)}$$

$$P_{xy} = P_{yx} = 0$$

$$P_{xz} = P_{zx} = \frac{\frac{1}{\sqrt{\tau}}Re[G_M^*(t)G_E(t)]\sin 2\vartheta}{\frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta + |G_M(t)|^2(1+\cos^2\vartheta)}$$

$$P_{yz} = P_{zy} = -2\frac{\frac{1}{\sqrt{\tau}}Im[G_M^*(t)G_E(t)]\sin\vartheta}{\frac{1}{\tau}|G_E(t)|^2\sin^2\vartheta + |G_M(t)|^2(1+\cos^2\vartheta)}$$

Every of components P_{kl} characterize a **polarization of the proton** p **at the direction of** k**, if antiproton** $\bar{p}$ **is polarized at the direction of** l and quantities P_{kl} are calculated for 100% polarization of one of the collided leptons.

Experimental data on proton electric and magnetic FFs

Experimental information on nucleon EM FFs

Between the discovery of proton EM structure in the middle of the 1950's and 2000, abundant **proton EM FF data** appeared (see Figs.).

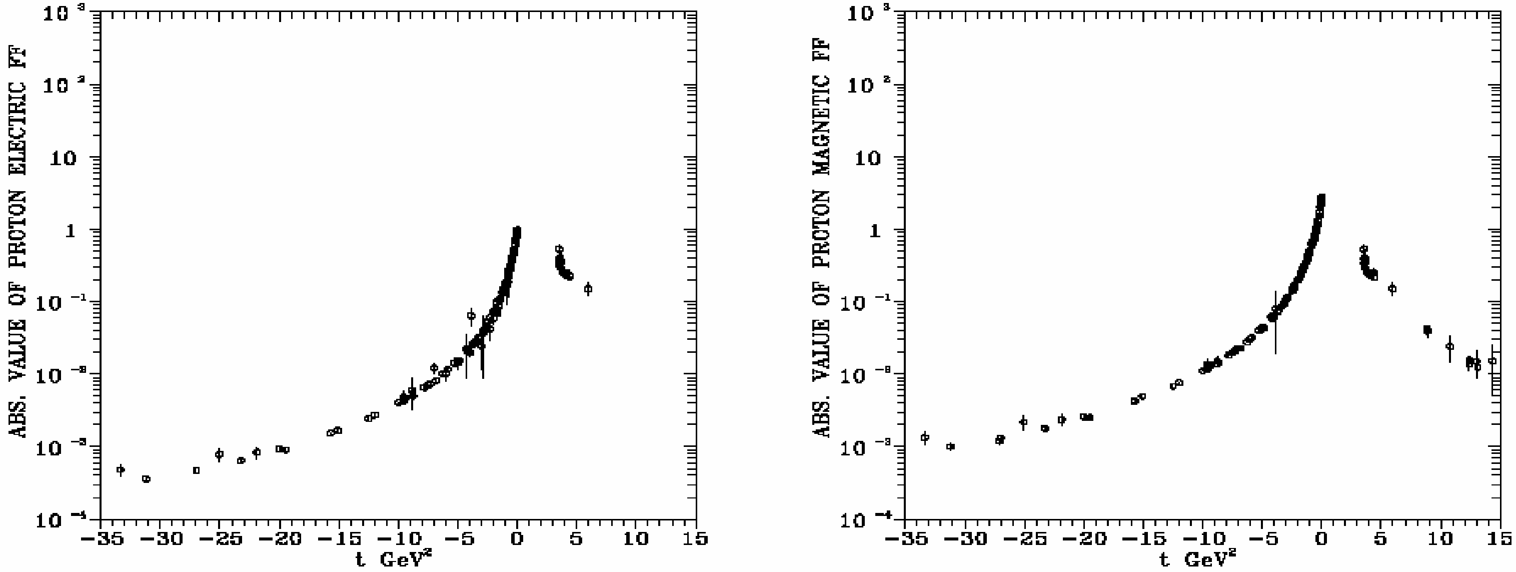


Figure 2: Experimental data on proton electric and magnetic form factors.

Experimental data on neutron electric and magnetic FFs

By a slightly more complicated method the **neutron electric and magnetic FF's data** have been obtained as presented in Fig.

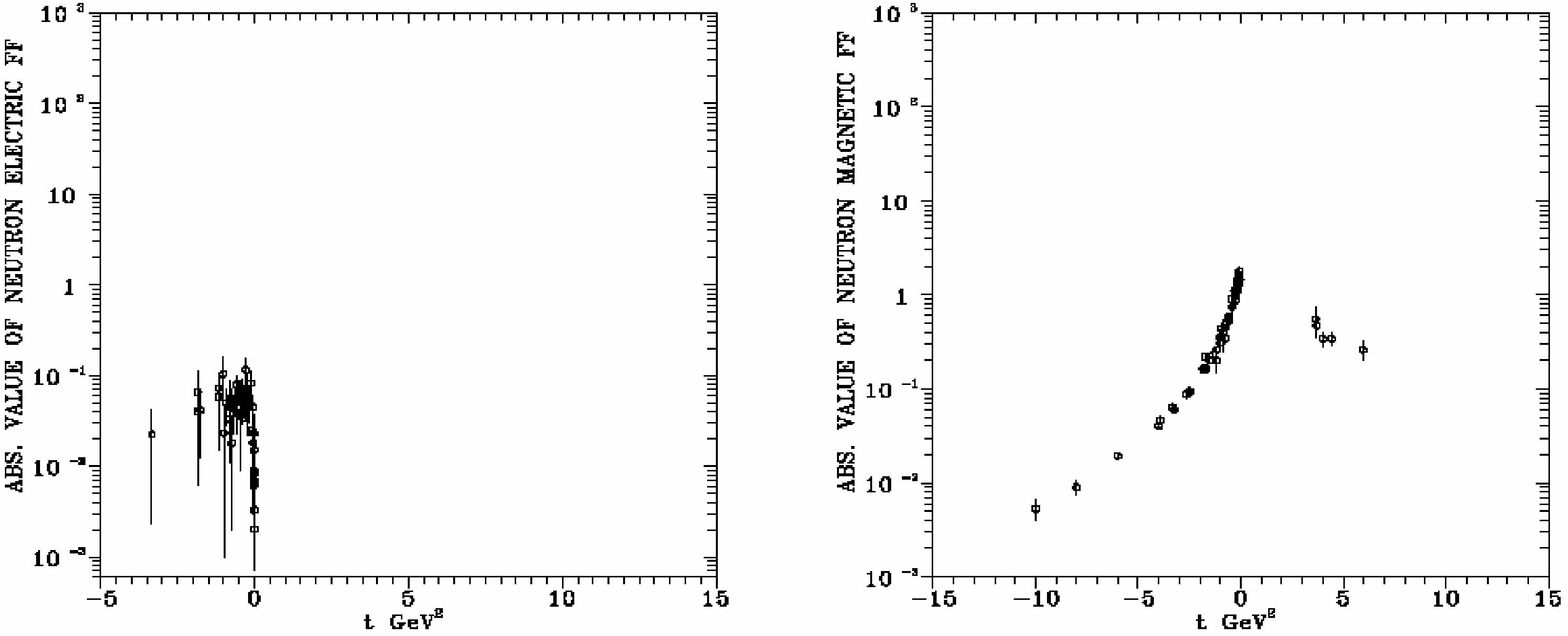


Figure 3: Experimental data on neutron electric and magnetic form factors.

They have been obtained by measuring the

$$\frac{d\sigma^{lab}(e^-p \longrightarrow e^-p)}{d\Omega} = \frac{\alpha^2}{4E^2}\frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\frac{1}{1+(\frac{2E}{m_p})\sin^2(\theta/2)}.$$

$$\left[A(t) + B(t)\tan^2(\theta/2)\right] \qquad (20)$$

$\alpha = 1/137$, E-the incident electron energy

$$A(t) = \frac{G^2_{Ep}(t) - \frac{t}{4m_p^2}G^2_{Mp}(t)}{1-\frac{t}{4m_p^2}}, \qquad (21)$$

$$B(t) = -2\frac{t}{4m_p^2}G^2_{Mp}(t) \qquad (22)$$

$\alpha = 1/137$, E-the incident electron energy

exploiting so-called Rosenbluth technique, i.e. linear dependence of

$$\frac{d\sigma^{lab}(e^-p \longrightarrow e^-p)}{d\Omega} \sim [A(Q^2) + B(Q^2)\tan^2(\theta/2)] \qquad (23)$$

on $\tan^2(\theta/2)$ to be **valid for target particle with arbitrary nonzero spin** in one-photon exchange approximation

And measurements of the **total cross-sections**

$$\sigma_{tot}^{c.m.}(e^+e^- \rightarrow p\bar{p}) = \frac{4\pi\alpha^2\beta_N}{3s}[\mid G_{Mp}(s) \mid^2 + \frac{2m_p^2}{s} \mid G_{Ep}(s) \mid^2]$$

with

$$\beta_N = \sqrt{1 - \frac{4m_p^2}{s}}$$

and

$$\sigma_{tot}^{c.m.}(\bar{p}p \rightarrow e^+e^-) = \frac{2\pi\alpha^2}{3p_{c.m.}\sqrt{s}}[\mid G_{Mp}(s) \mid^2 + \frac{2m_p^2}{s} \mid G_{Ep}(s) \mid^2]$$

providing experimental information on $|G_{Ep}(s)|$, $|G_{Mp}(s)|$ in time-like region for $s > 4m_p^2$, where FFs are **complex functions**.

In this determinations of proton FFs one typically takes into account:

- $G_{Ep}(s)$ contribution plays a **minor role**
- and experimental results are usually given in terms of $|G_{Mp}(s)|$

under the assumptions:

- either $G_{Ep}(s) = 0$- the **arbitrary hypothesis**
- or $|G_{Ep}(s)| = |G_{Mp}(s)|$ for $s > 4m_p^2$ - but there is **no theoretical argument** which justifies its validity.

Moreover, one can't determine the **phases of FFs**.

A complete observation of EM structure of hadrons – POLARIZATION EXPERIMENTS

A complete observation of the electromagnetic structure of hadrons is possible only in **polarization experiments**.

The polarization effects in all three processes mentioned above are completely different.

This difference consists in the fact that **there are noticeable polarization effects** in the $e^+e^- \to p\bar{p}$ process if there are no polarized particles in the initial state.

The appearance of polarization effects is due to $G_{Ep}(s)$ and $G_{Mp}(s)$ **being complex with non-zero relative phase!**

U&A MODEL OF NUCLEON EM STRUCTURE

Now we will predict behaviours of all polarization observables by using our U&A model of nucleon elmag structure

which is formulated in the language of **isoscalar** $F^{s,v}_{1,2}(t)$ and **isovector** $F^{s,v}_{1,2}(t)$ parts of the Dirac and Pauli FF's

$$\begin{aligned}
G^p_E(t) &= [F^s_1(t)] + F^v_1(t)] + \frac{t}{4m_p^2}[F^s_2(t) + F^v_2(t)] \\
G^p_M(t) &= [F^s_1(t) + F^v_1(t)] + [F^s_2(t) + F^v_2(t)] \qquad (24) \\
G^n_E(t) &= [F^s_1(t) - F^v_1(t)] + \frac{t}{4m_n^2}[F^s_2(t) - F^v_2(t)]; \\
G^n_M(t) &= [F^s_1(t) - F^v_1(t)] + [F^s_2(t) - F^v_2(t)],
\end{aligned}$$

and **comprises all known nucleon FF properties** like

10 resonance U&A model fulfils

- experimental fact of a creation of unstable vector meson resonances in electron-positron annihilation processes into hadrons
- analytic properties of FF's
- reality conditions
- unitarity conditions
- normalizations
- asymptotic behaviours as predicted by the quark model of hadrons.

The results of the description of all existing nucleon form factor data by using 10 resonance U&A model are presented in following figures

S.Dubnicka, A.Z.Dubnickova, P.Weisenpacher: J. Phys. G29 (2003) 405

And in the paper C Adamuščín, S.Dubnička, A.Z.D.,P.Weisenpacher, Prog.Part.Nucl.Phys.55 (2005)228

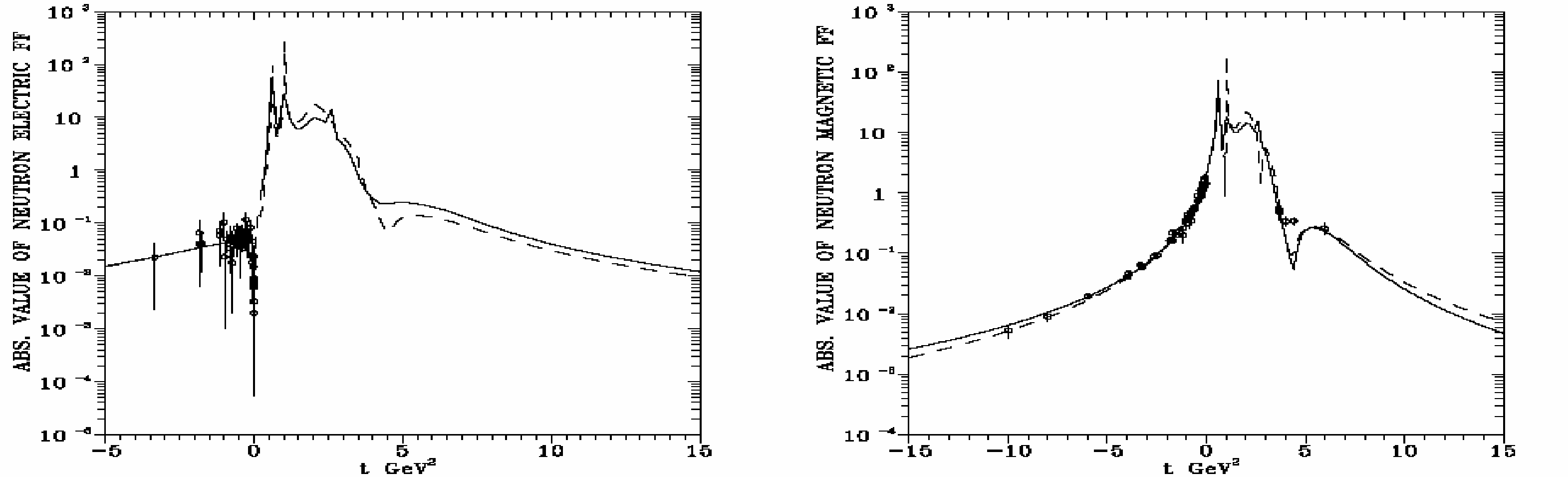


Figure 4: Theoretical prediction of the behaviour of neutron electric and magnetic form

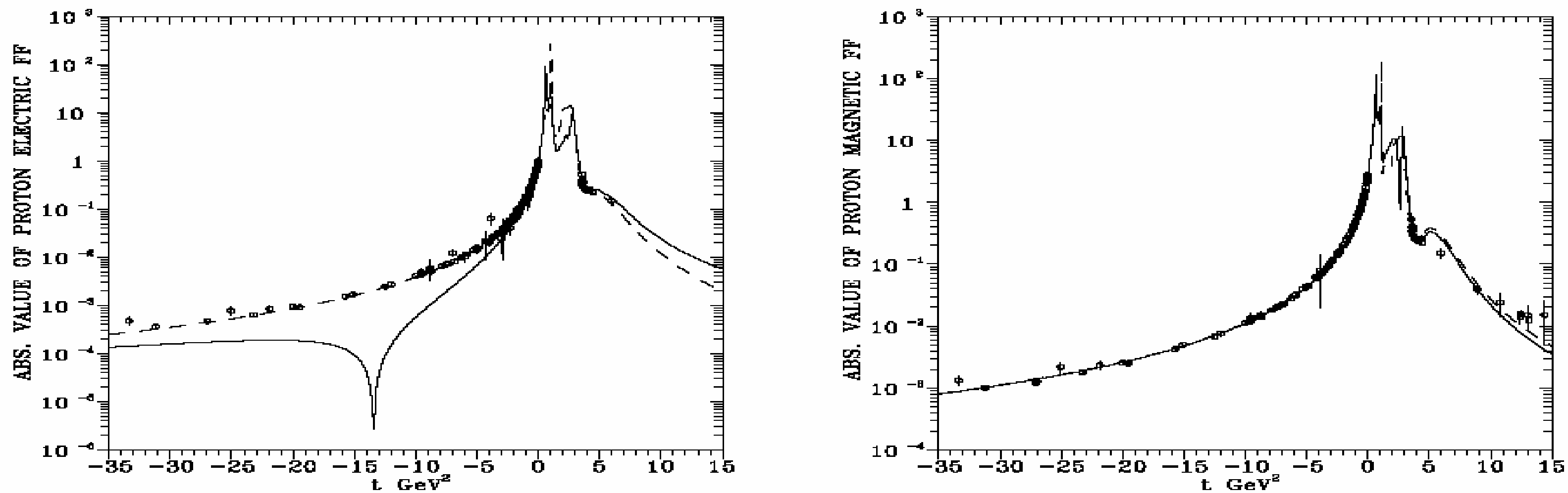


Figure 5: Teoretical prediction of the behaviour of proton electric and magnetic form factors

10 resonance U&A model fit

For numerical evaluation of the parameters of U&A model we have collected 512 experimental data. The best description of them was achieved with $\chi^2/ndf = 1.46$ and the following values of free parameters

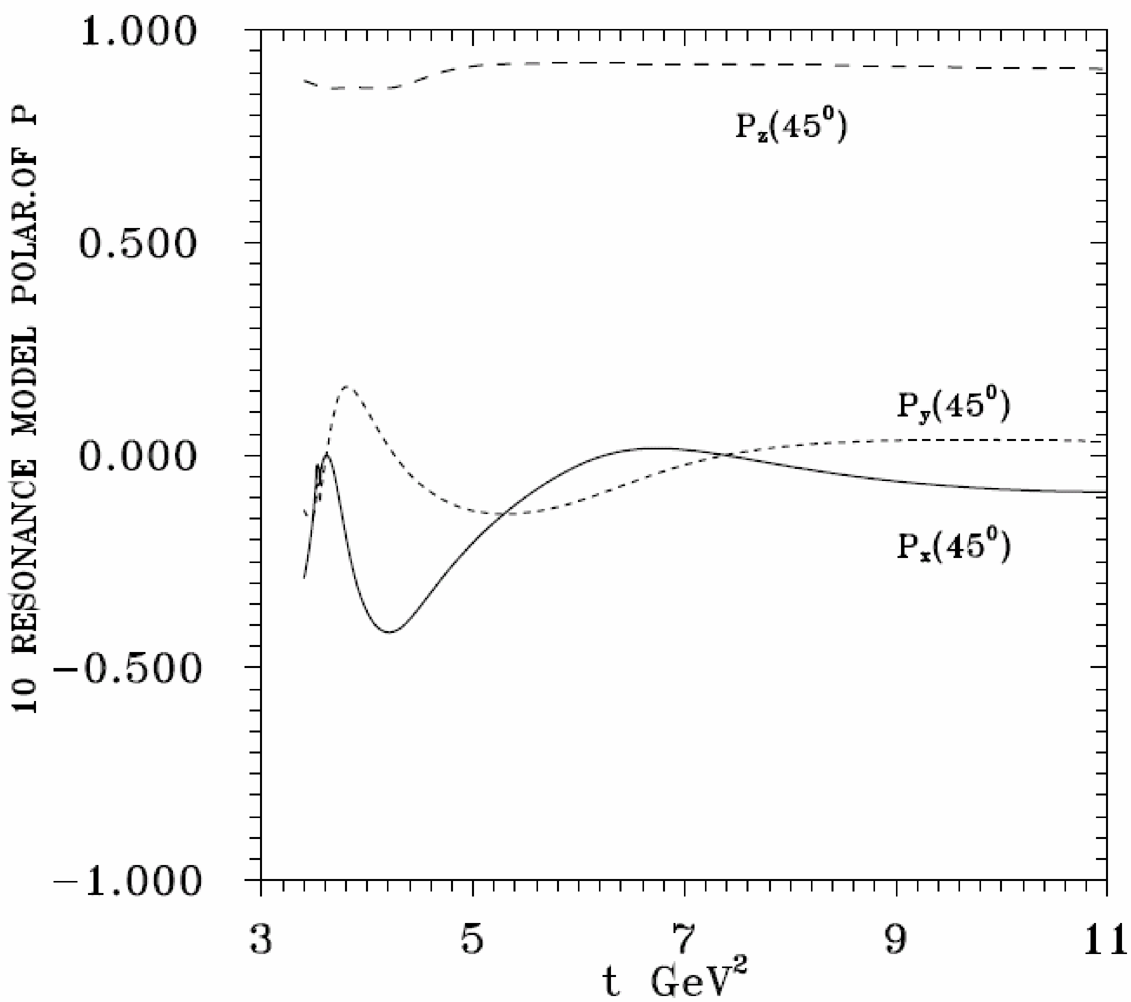
- **it does not matter, if with the analysis of old SLAC or new JLab data** - the behaviours of the spin polarization parameters are predicted in Figures

Figures of predicted single and double spin polarization observables in time-like region process e^+e^to pp by U&A model

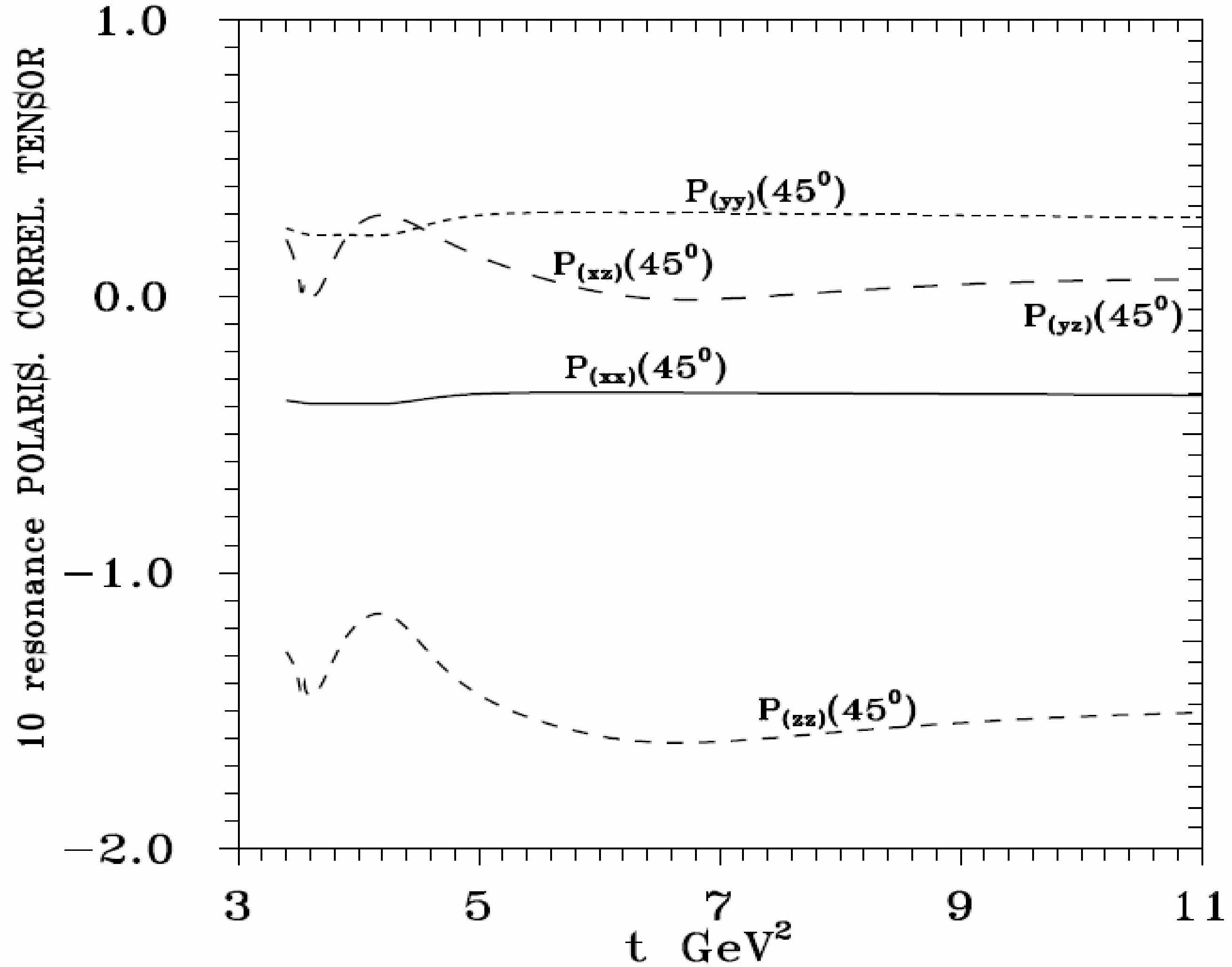
6. Prediction of single and double spin polarization observables

Now, exploiting behaviours of proton electric and magnetic FFs in time-like region, one predicts behaviour of single and double spin polarization observables of the $e^+e^- \to p\bar{p}$ processes as demonstrated in Figures

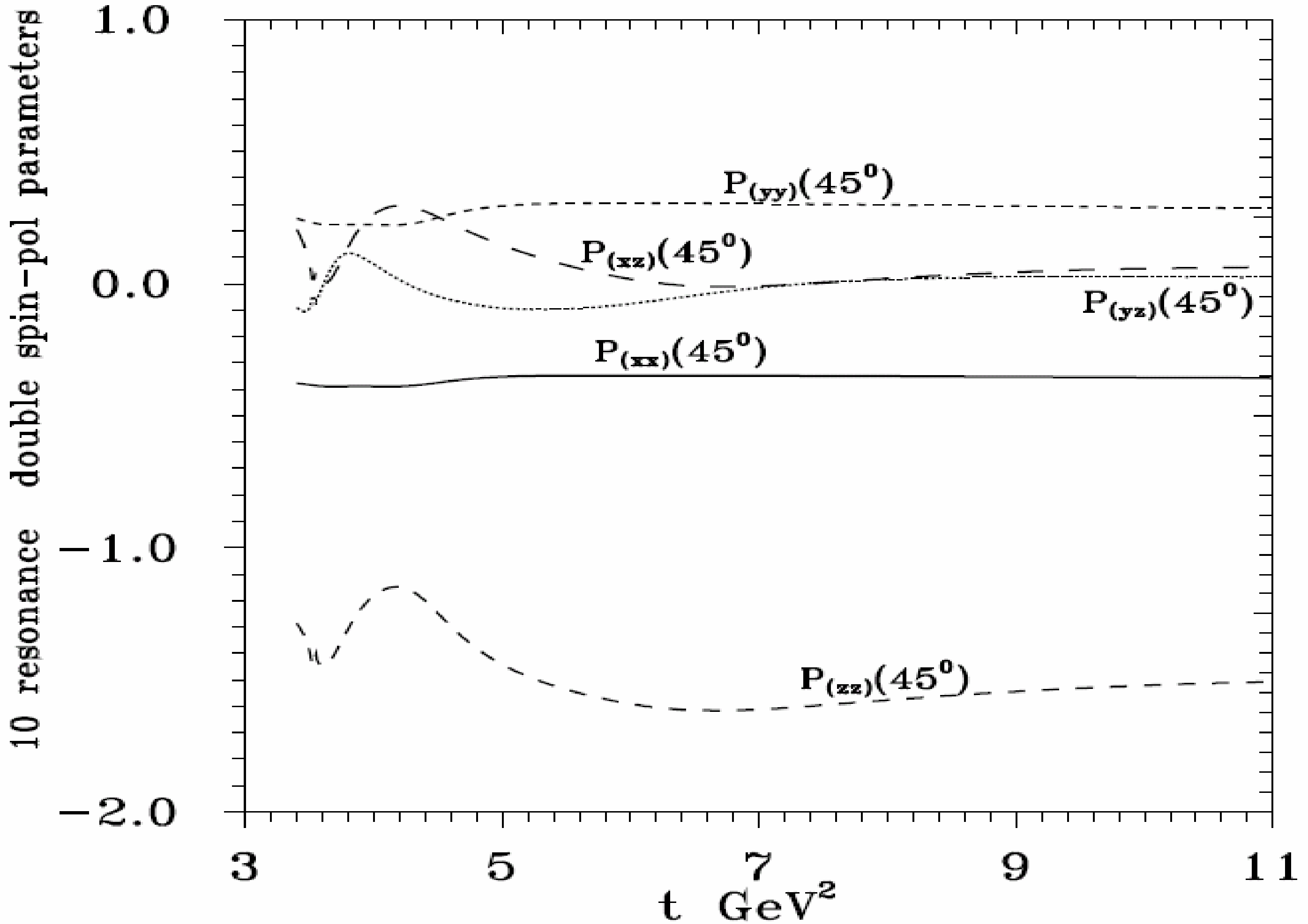
Single spin polarization observables



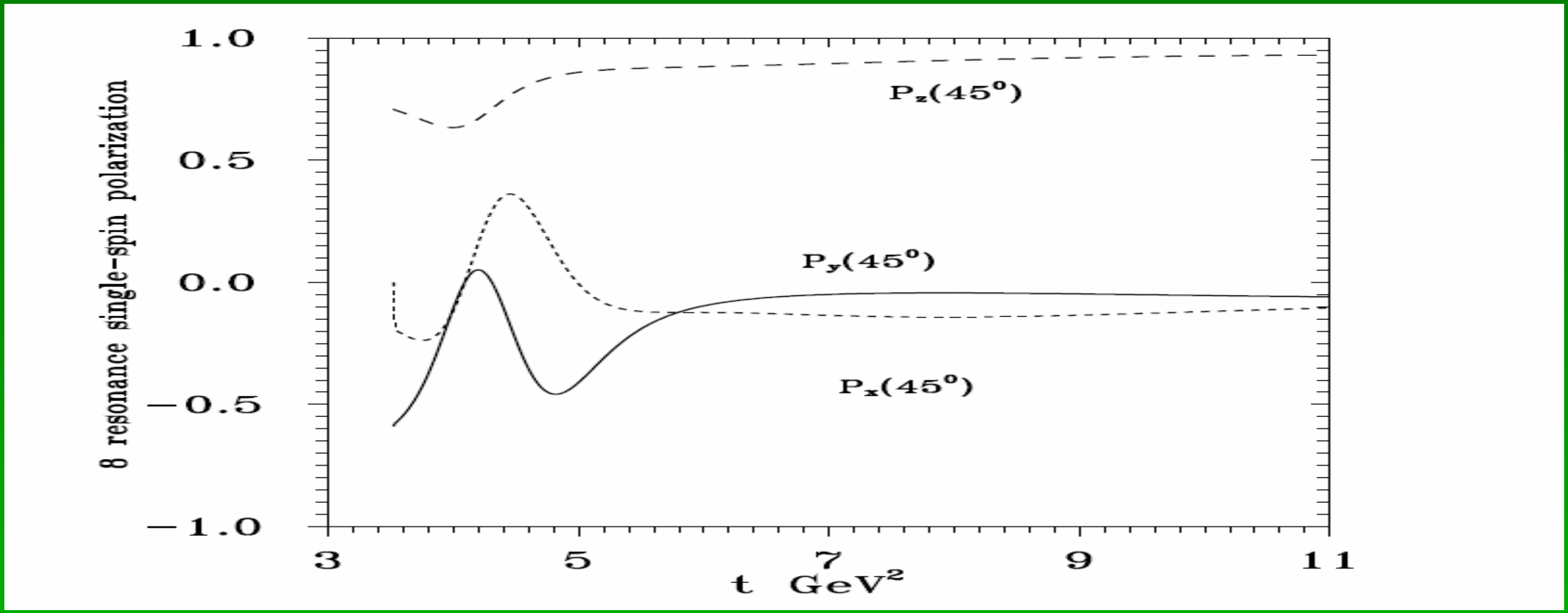
Double spin polarized observables (unpolarized leptons)

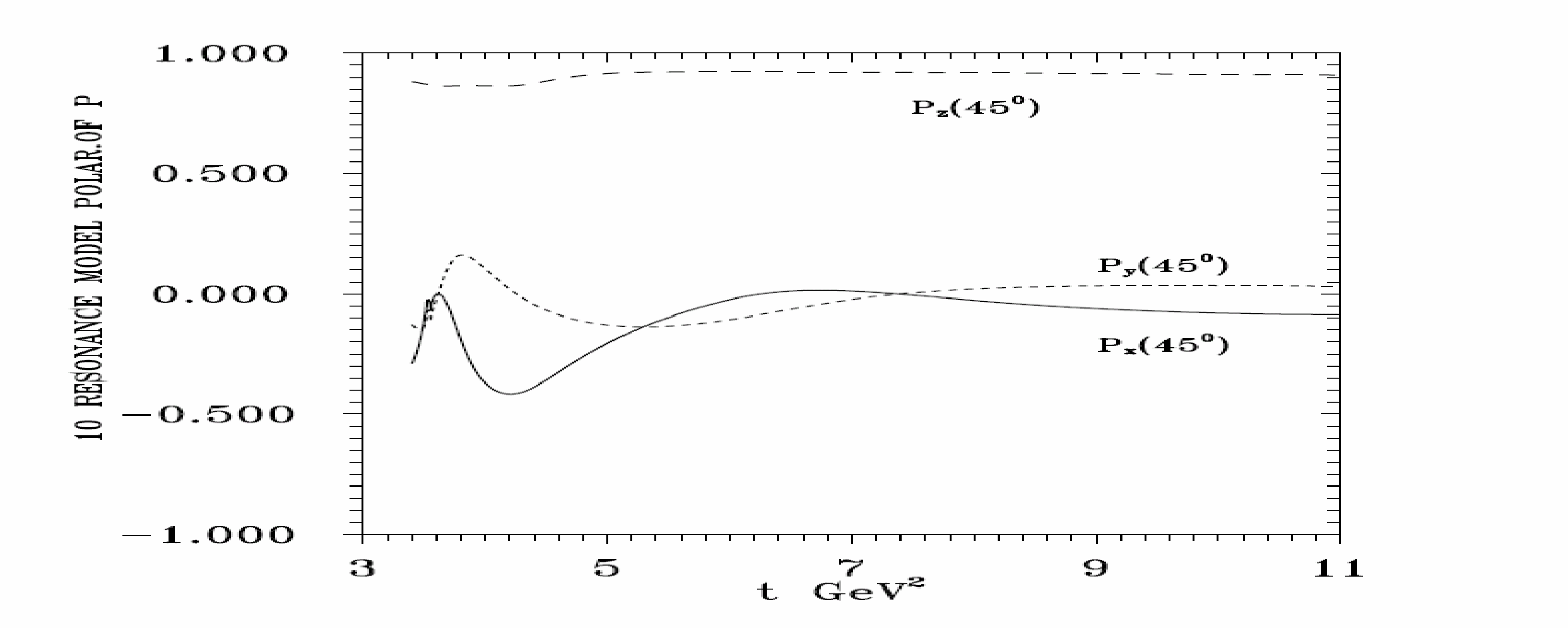


Duoble spin polarization observables (polarized leptons)



Comparison 8 resonances and 10 resonances U&A models





Conclusions

- **polarization effects** in $e^+e^- \longrightarrow p\bar{p}$ were **reinvestigated** in detail and single and double spin polarization observables were explicitly given
- **various predictions** for them were presented **by using our U&A model** of nucleon EM structure **with natural creation of the imaginary parts of nucleon form factors** and completed by **new experimental information**
- it was clearly **demonstrated** that even in the framework of the same U&A model, however, with different **number resonances (8 and 10)** the predictions of **observables are different**. Therefore **experimental measurements** of single and double spin polarization observables in the $e^+e^- \longrightarrow p\bar{p}$ are **highly desirable**