

Implications of $G_E^p(Q^2)/G_M^p(Q^2)$.

*S. Dubnička*¹,

*A. Z. Dubničková*²,

OUTLINE:

1. JLab proton polarization data puzzle
2. Existence of two different $G_E^p(t)$ behaviors in space-like region
3. Consequences on the charge distribution within proton
4. Possible DIS insight by 'new sum rule'
5. Strange nucleon FF's from two different behaviors of $G_E^p(t)$
6. Compatibility of new $G_E^p(t)$ with Deuteron EM structure functions $A(t), B(t)$ data
7. Conclusions

¹Inst. of Physics, Slovak Acad. of Sciences, Bratislava, Slovak Republic

²Dept. of Theor. Physics, Comenius Univ., Bratislava, Slovak Republic

1. JLAB PROTON POLARIZATION DATA PUZZLE

Experimental information on nucleon EM form factors

I. Between the discovery of proton EM structure in the middle of the 1950's and 2000, abundant **proton EM FF data** in the space-like region ($t = -Q^2 < 0$) appeared (see Fig.1).

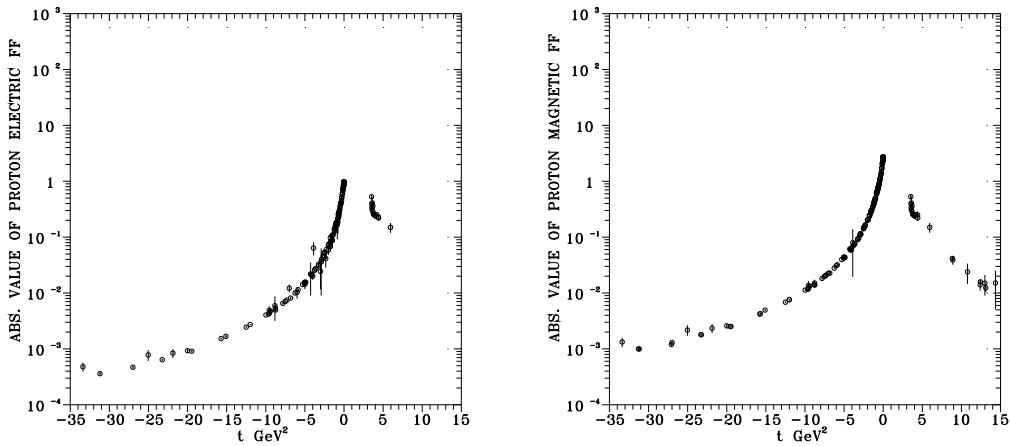


Figure 1: Experimental data on proton electric and magnetic form factors.

They have been obtained from the measured **cross section** of the **elastic scattering** of **unpolarized electrons** on **unpolarized protons** in the laboratory reference frame

$$\frac{d\sigma^{lab}(e^-p \rightarrow e^-p)}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \frac{1}{1 + (\frac{2E}{m_p}) \sin^2(\theta/2)} \cdot [A(t) + B(t) \tan^2(\theta/2)] \quad (1)$$

$\alpha = 1/137$, E -the incident electron energy

$$A(t) = \frac{G_{Ep}^2(t) - \frac{t}{4m_p^2} G_{Mp}^2(t)}{1 - \frac{t}{4m_p^2}},$$

$$B(t) = -2 \frac{t}{4m_p^2} G_{Mp}^2(t) \quad (2)$$

by **Rosenbluth technique**.

By a slightly more complicated method the **neutron electric and magnetic FF's data** have been obtained as presented in Fig.2

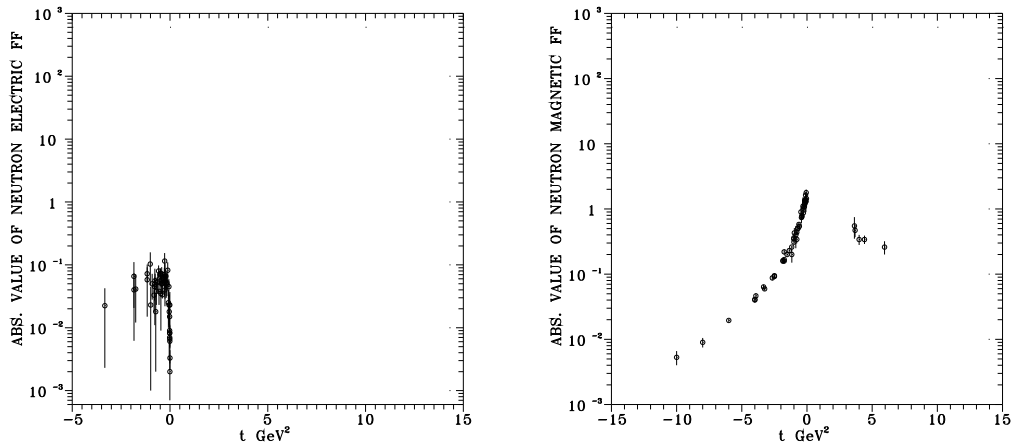


Figure 2: Experimental data on neutron electric and magnetic form factors.

II. More recently at Jefferson Lab,

M.K.Jones et al, Phys. Rev. Lett. 84 (2000) 1398

O.Gayou et al, Phys. Rev. Lett. 88 (2002) 092301

V.Punjabi et al, Phys. Rev. C71 (2005) 055202

measuring simultaneously **transverse**

$$P_t = \frac{h}{I_0}(-2)\sqrt{\tau(1 + \tau)}G_{Mp}G_{Ep} \tan(\theta/2) \quad (3)$$

and **longitudinal**

$$P_l = \frac{h(E + E')}{I_0m_p}\sqrt{\tau(1 + \tau)}G_{Mp}^2 \tan^2(\theta/2) \quad (4)$$

components of the **recoil proton's polarization** in the electron scattering plane of the polarization transfer process

(h is the electron beam helicity, I_0 is the unpolarized cross-section excluding σ_{Mott} and $\tau = Q^2/4m_p^2$)

one **obtained the data** on

$$G_{Ep}/G_{Mp} = -\frac{P_t(E + E')}{P_l 2m_p} \tan(\theta/2). \quad (5)$$

They are in strong disagreement with data obtained by Rosenbluth technique as shown in Fig.3

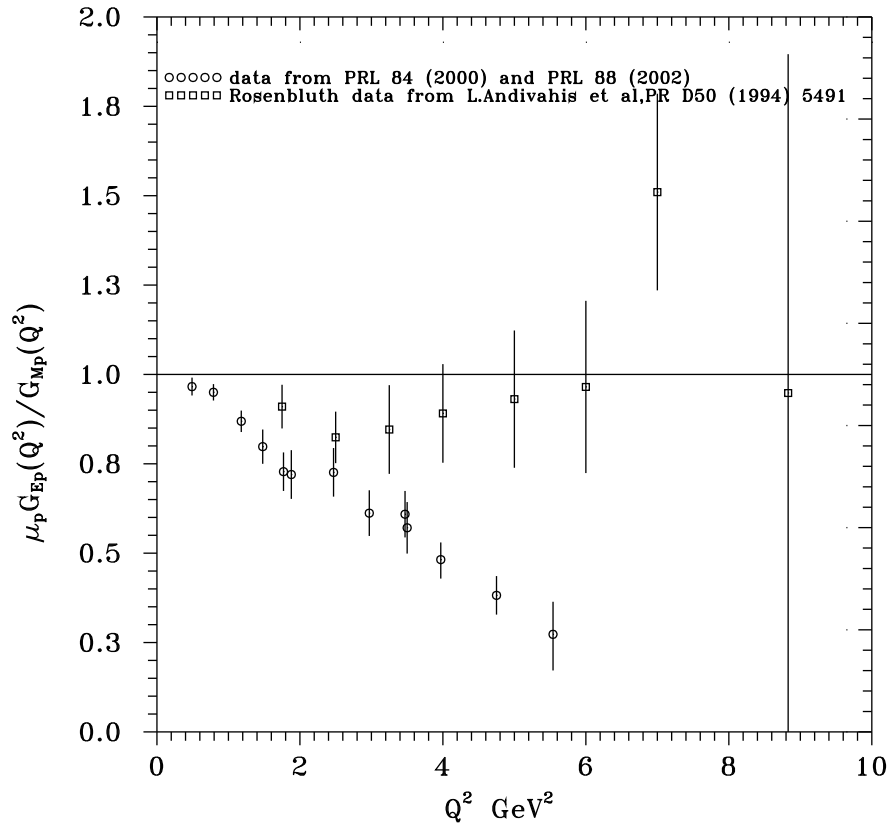


Figure 3: JLab proton polarization data puzzle

Important note:

The expressions for $\frac{d\sigma^{lab}(e^-p \rightarrow e^-p)}{d\Omega}$ and P_t, P_l are calculated

in the **one photon exchange approximation**

- to be justified **theoretically**.

Attempts to solve the problem

Despite the fact, that the **one photon exchange approximation is justified theoretically**, there were attempts to solve the problem **by inclusion of additional radiative correction terms, related to two-photon exchange approximations**

P.A.M.Guichon, M.Vanderhaeghen, Phys. Rev. Lett. 91 (2003) 142303-1

P.G.Blunder, W.Melnitchouk, J.A.Tjon, Phys. Rev. Lett. 91 (2003) 142304-1

Y.-C.Chen et al, Phys. Rev. Lett. 93 (2004) 122301-1

M.P.Rekalo, E.Tomasi-Gustafsson, Eur. Phys. J. A22 (2004) 331

A.V.Afanasiev et al, Phys. Rev. D72 (2005) 013008

S.Dubnicka et al, hep-ph/0507242

The analysis revealed:

- the two-photon exchange has a much **smaller effect on the polarization transfer** than on the Rosenbluth extractions
 \Rightarrow **JLab polarization data are more reliable**
- the size of the two-photon exchange correction is **larger** in the case of $G_E^p(Q^2)$ than of $G_M^p(Q^2)$ in the Rosenbluth process

The **latter conclusion**, together with **dominating $G_M^p(Q^2)$ factor** in $\frac{d\sigma^{lab}(e^-p \rightarrow e^-p)}{d\Omega}$ indicate - **$G_E^p(Q^2)$ is responsible for discrepancy** of $\mu_p G_{Ep}/G_{Mp}$ with Rosenbluth data for large Q^2 .

Now **we confirm this hypothesis** in the analysis of all nucleon FF data by 10-resonance model of nucleon EM structure.

2. EXISTENCE OF TWO DIFFERENT $G_E^p(t)$ BEHAVIORS IN SPACE-LIKE REGION

We have achieved **simultaneous description** of all existing proton and neutron form factor data in space-like and time-like regions by **10-resonance U&A model of nucleon EM structure**

*S.Dubnicka, A.Z.Dubnickova, P.Weisenpacher,
J. Phys. G29 (2003) 405*

formulated in the language of **isoscalar** $F_{1,2}^s(t)$ and **isovector** $F_{1,2}^v(t)$ Dirac and Pauli FF's, saturating them by $\omega, \phi, \omega', \omega'', \phi'$ and $\rho, \rho', \rho'', \rho''', \rho''''$, respectively.

The model **comprises all known nucleon FF properties**

- **experimental fact** of a **creation of unstable vector meson resonances** in electron-positron annihilation processes into hadrons
- **analytic** properties of FF's
- **reality** conditions
- **unitarity** conditions
- **normalizations**
- **asymptotic behaviours** as predicted by the quark model of hadrons.

First - the **analysis of all proton and neutron data obtained by Rosenbluth technique**, together with all proton and neutron data in time-like region, were **carried out**.

Then - the **new JLab proton polarization data** on $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$, $0.49\text{GeV}^2 \leq Q^2 \leq 5.54\text{GeV}^2$ were **substituted** for all $G_{Ep}(t)$ space-like data obtained by Rosenbluth technique and **analysed** together with all electric proton time-like data and all space-like and time-like magnetic proton, as well as electric and magnetic neutron data.

S.Dubnicka and A.Z.Dubnickova, Fizika B13 (2004)

287

*C.Adamuscin, S.Dubnicka, A.Z.Dubnickova, P.Weisenpacher,
Prog. Part. Nucl. Physics 55 (2005) 228*

The results are presented in Figs.4 and 5.

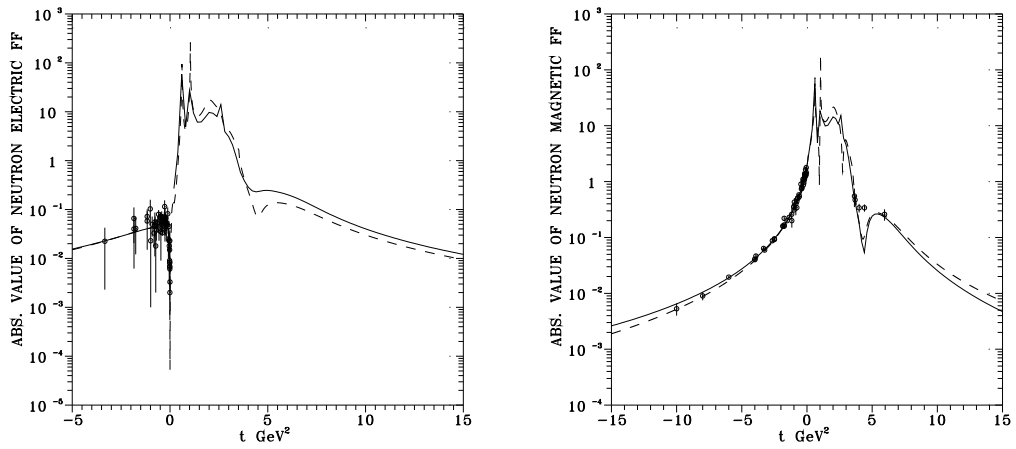


Figure 4: Theoretical behavior of neutron electric and magnetic form factors.

From these Figures two consequences follow:

- The fact, that **almost nothing is changed** in a description of $G_{En}(t)$, $G_{Mn}(t)$ and $G_{Mp}(t)$ in both, the space-like and time-like regions, and also $|G_{Ep}(t)|$ in the time-like region - **supports the hypothesis** - the **discrepancy between the old and new ratios** $G_{Ep}(t)/G_{Mp}(t)$ is really **created by different behaviors** of $G_{Ep}(t)$.

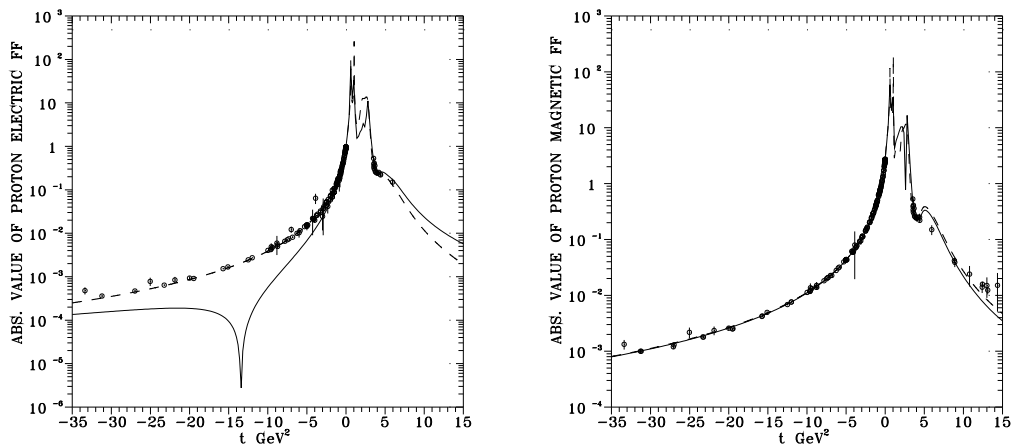


Figure 5: Theoretical behavior of proton electric and magnetic form factors.

- The new behavior of $G_{Ep}(t)$ (the **full line in Fig.5**) extracted from the JLab polarization data on $G_{Ep}(t)/G_{Mp}(t)$ is **consistent with all known FF properties**, including also the asymptotic behavior - but **strongly requires an existence of FF zero around $t = -13\text{GeV}^2$** .

As a result of our analyses one gets **two different behaviors** of $G_{Ep}(t)$ in $t = -Q^2 < 0$ region.

3. CONSEQUENCES ON CHARGE DISTRIBUTION WITHIN PROTON

The **proton charge distribution** (assuming to be spherically symmetric) is an **inverse Fourier transform of the proton electric FF**

$$\rho_p(r) = \frac{1}{(2\pi)^3} \int e^{-iQr} G_{Ep}(Q^2) d^3Q \quad (6)$$

from where

$$\rho_p(r) = \frac{4\pi}{(2\pi)^3} \int_0^\infty G_{Ep}(Q^2) \frac{\sin(Qr)}{Qr} Q^2 dQ. \quad (7)$$

Substituting for the $G_{Ep}(Q^2)$ under the integral:

- either the **Rosenbluth behavior**
- or the **JLab polarization behavior** with the zero

one gets **different charge distributions within the proton** given in Fig.6 by dashed and full lines, respectively.

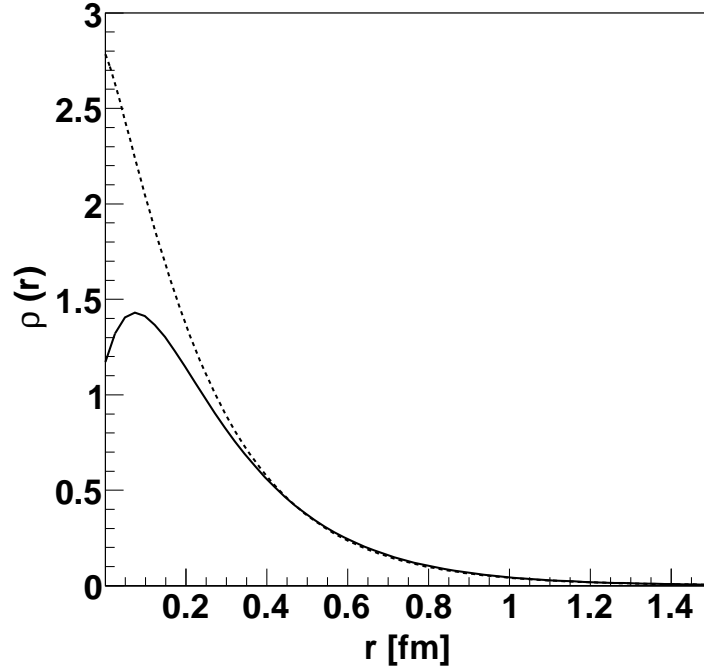


Figure 6: Charge distribution behavior within the proton

That all leads also to **different mean square proton charge radii**. The **old proton charge radius** takes the value $\langle r_p^2 \rangle = 0.68 \text{fm}^2$. If JLab proton polarization data are correct \Rightarrow the **new charge radius** $\langle r_p^2 \rangle = 0.72 \text{fm}^2$ is **larger**.

4. POSSIBLE DIS INSIGHT BY 'NEW SUM RULE'

Recently in the paper

*E.Bartos, S.Dubnicka, E.A.Kuraev, Phys. Rev. D70
(2004) 117901-1*

considering very high energy process in **one photon approximation**

$$e^-(p_1) + N(p) \rightarrow e^-(p'_1) + X \quad (8)$$

with peripheral production of a hadronic state X **moving closely** to a direction of initial nucleon

one comes to **Weizsaecker-Williams** like relation.

On the other hand, **investigating analytic properties** of parts of forward virtual Compton scattering amplitude on nucleons $\tilde{A}^{(N)}(s_1, \mathbf{q})$ which by a construction is only a part of the total forward virtual Compton

scattering amplitude $A^{(N)}(s_1, \mathbf{q})$ on nucleon one comes to

new sum rule

$$\begin{aligned}
& F_{1p}^2(-\mathbf{q}^2) + \frac{\mathbf{q}^2}{4m_p^2} F_{2p}^2(-\mathbf{q}^2) \\
& - F_{1n}^2(-\mathbf{q}^2) - \frac{\mathbf{q}^2}{4m_n^2} F_{2n}^2(-\mathbf{q}^2) = \quad (9) \\
& 1 - 2 \frac{(\mathbf{q}^2)^2}{\pi\alpha^2} \left(\frac{d\sigma^{e^-p \rightarrow e^-X}}{d\mathbf{q}^2} - \frac{d\sigma^{e^-n \rightarrow e^-X}}{d\mathbf{q}^2} \right),
\end{aligned}$$

giving into a **relation**:

- **nucleon electromagnetic form factors**
- with Q^2 -**dependent differential cross-sections** of deep inelastic electron-proton and electron-neutron differential cross-sections.

Evaluating FFs on the left-hand side **by using two different $G_E^p(Q^2)$** behaviors in $t = -Q^2 < 0$ region one gets **two different curves in Fig.7**

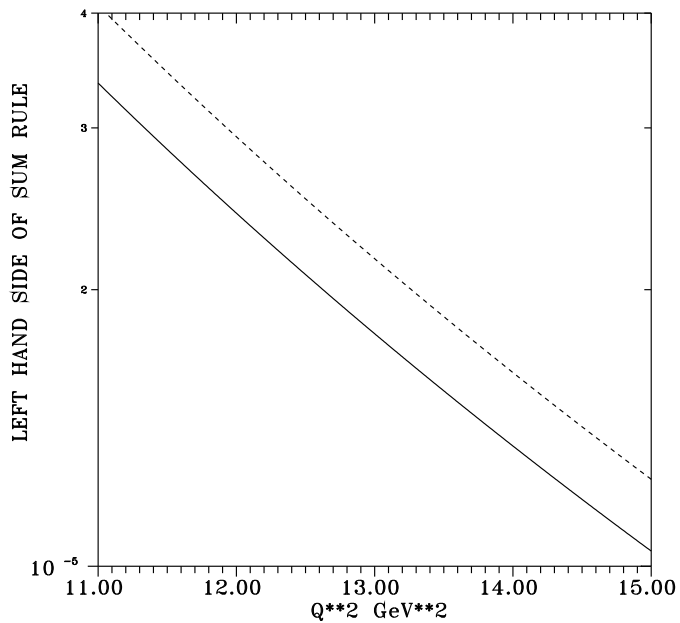


Figure 7: Sum rule interpretation in s_1 plane.

By accurate measurements of the right hand side of the sum rule the **true behavior of $G_E^p(Q^2)$ in space-like region** could be distinguished (see Fig. 7).

5. STRANGE NUCLEON FF'S FROM TWO DIFFERENT BEHAVIORS OF $G_E^p(Q^2)$ IN $t = -Q^2 < 0$ REGION

The **momentum dependence** of the nucleon matrix element of the strange-quark vector current $J_\mu^s = \bar{s}\gamma_\mu s$ is **contained** in the **Dirac** $F_1^s(t)$ and **Pauli** $F_2^s(t)$ **strange FF's**

$$\langle p' | \bar{s}\gamma_\mu s | p \rangle = \bar{u}(p') \left[\gamma_\mu F_1^s(t) + i \frac{\sigma_{\mu\nu} q^\nu}{2m_N} F_2^s(t) \right] u(p) \quad (10)$$

or in the **strange electric** and **strange magnetic FF's**

$$G_E^s(t) = F_1^s(t) + \frac{t}{4m_N^2} F_2^s(t), \quad G_M^s(t) = F_1^s(t) + F_2^s(t). \quad (11)$$

As a consequence of the **isospin zero value** of the strange quark, **the strange FF's contribute only to the behaviour of the isoscalar nucleon FF's** and never to isovector ones.

The **main idea of a prediction of strange nucleon FF's behaviour** is based on two assumptions:

- on **the $\omega - \phi$ mixing** - to be valid also for coupling constants between EM (quark) current and vector meson

$$\begin{aligned} \frac{1}{f_\omega} &= \frac{1}{f_{\omega_0}} \cos \epsilon - \frac{1}{f_{\phi_0}} \sin \epsilon \\ \frac{1}{f_\phi} &= \frac{1}{f_{\omega_0}} \sin \epsilon + \frac{1}{f_{\phi_0}} \cos \epsilon \end{aligned} \quad (12)$$

- on **the assumption** that the **quark current of some flavour couples** with universal strength κ **exclusively to the vector-meson wave function component of the same flavour**

$$\langle 0 | \bar{q}_r \gamma_\mu q_r | (\bar{q}_t q_t)_V \rangle = \kappa m_V^2 \delta_{rt} \varepsilon_\mu \quad (13)$$

which **result in the relations**

$$\begin{aligned} (f_{\omega NN}^{(i)} / f_\omega^s) &= -\sqrt{6} \frac{\sin \varepsilon}{\sin(\varepsilon + \theta_0)} (f_{\omega NN}^{(i)} / f_\omega^e) \\ (f_{\phi NN}^{(i)} / f_\phi^s) &= -\sqrt{6} \frac{\cos \varepsilon}{\cos(\varepsilon + \theta_0)} (f_{\phi NN}^{(i)} / f_\phi^e) \quad (14) \\ (i &= 1, 2) \end{aligned}$$

where f_ω^s, f_ϕ^s are **strange-current** $\leftrightarrow V = \omega, \phi$ **coupling constants** and $\varepsilon = 3.7^\circ$ is a deviation from the ideally mixing angle $\theta_0 = 35.3^\circ$.

So, **if one knows** from the fit of nucleon FF data **free parameters** $(f_{\omega NN}^{(i)}/f_{\omega}^e), (f_{\phi NN}^{(i)}/f_{\phi}^e)$ ($i=1,2$) of the **suitable model** of $F_1^{I=0}(t), F_2^{I=0}(t)$

$$F_i^{I=0}(t) = f \left[t; (f_{\omega NN}^{(i)}/f_{\omega}^e), (f_{\phi NN}^{(i)}/f_{\phi}^e) \right] \quad (i = 1, 2) \quad (15)$$

where $f_{\omega NN}^{(i)}, f_{\phi NN}^{(i)}$ are coupling constants of ω and ϕ to nucleons and f_{ω}^e, f_{ϕ}^e are **virtual photon** \leftrightarrow **V** $=\omega, \phi$

coupling constants

given by **leptonic decay widths** $\Gamma(V \rightarrow e^+e^-)$,

\Rightarrow the **unknown free parameters** $(f_{\omega NN}^{(i)}/f_{\omega}^s), (f_{\phi NN}^{(i)}/f_{\phi}^s)$

of a **strange nucleon FF's model**

$$F_i^s(t) = \bar{f} \left[t; (f_{\omega NN}^{(i)}/f_{\omega}^s), (f_{\phi NN}^{(i)}/f_{\phi}^s) \right] \quad (i = 1, 2) \quad (16)$$

of **the same analytic structure** are calculated by the previous relations (14).

By using the U&A model of N EM structure with two different $G_E^p(Q^2)$ in space-like region strange nucleon FFs were predicted as given in Fig.8.

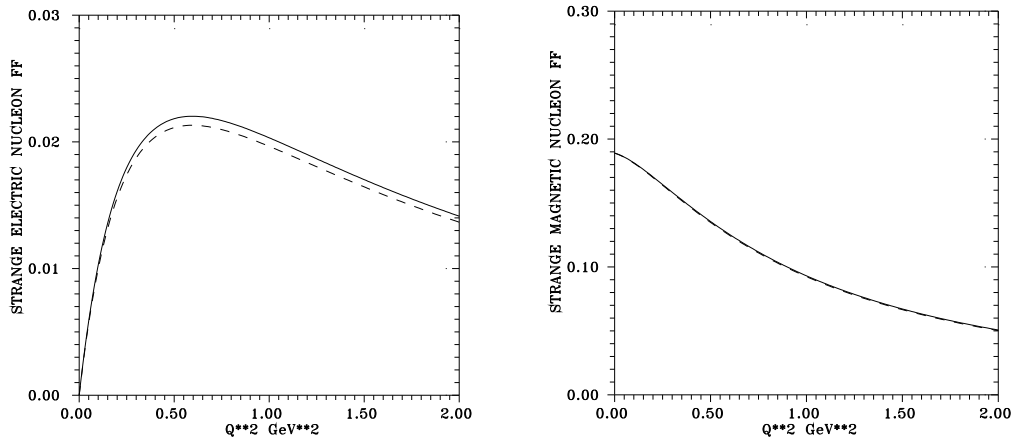


Figure 8: Theoretical prediction of strange electric and magnetic form factors.

As one can see from Fig.8b, a reasonable **value of the strangeness nucleon magnetic moment is predicted** $\mu_s = +0.19[\mu_N]$.

The **behavior of strange nucleon FFs doesn't feel too much the difference in contradicting behaviors of $G_{Ep}(t)$ in space-like region.**

A reasonable **description of the recent data on the combination** $G_E^s(Q^2) + \eta(Q^2)G_M^s(Q^2)$ for $0.12\text{GeV}^2 < Q^2 < 1.0\text{GeV}^2$ is achieved (see Fig.9)

D.S.Armstrong et al, Phys. Rev. Lett. 95 (2005)

092001

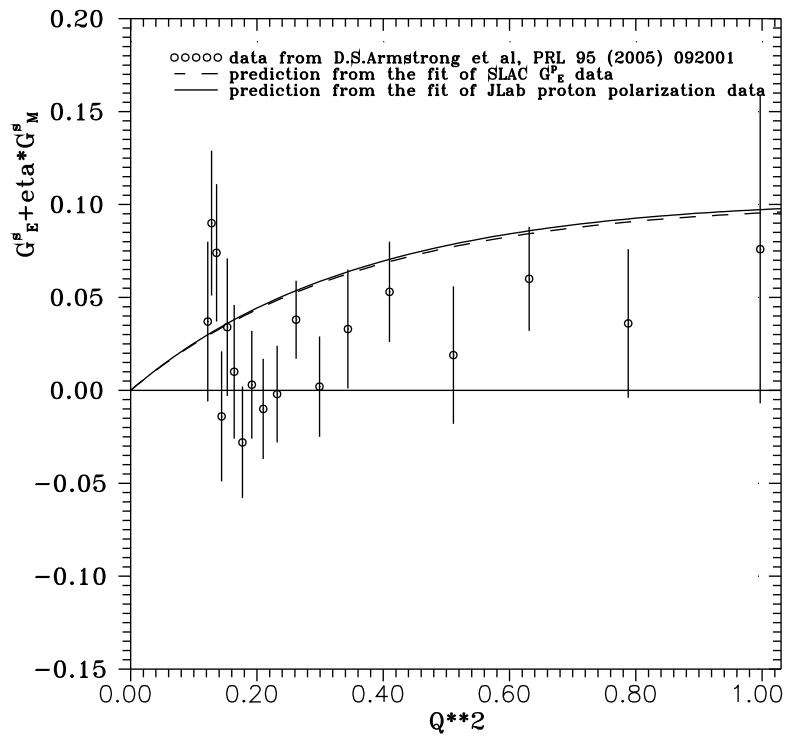


Figure 9: Prediction for the behaviour of the combination of strange nucleon form factors $G_E^s(Q^2) + \eta(Q^2)G_M^s(Q^2)$ by 8-resonance U&A model

**6. COMPATIBILITY
OF NEW $G_E^p(Q^2)$ WITH DEUTERON EM
STRUCTURE FUNCTIONS $A(Q^2), B(Q^2)$
DATA.**

The cross-section of **elastic electron scattering on deuteron** is

$$\frac{d\sigma^{lab}(e^- D \rightarrow e^- D)}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \frac{1}{1 + (\frac{2E}{m_D}) \sin^2(\theta/2)} \cdot [A(t) + B(t) \tan^2(\theta/2)] \quad (17)$$

Similarly **to the nucleons** one can draw out from previous formula the data on $A(Q^2)$ and $B(Q^2)$ as presented in Figs.10

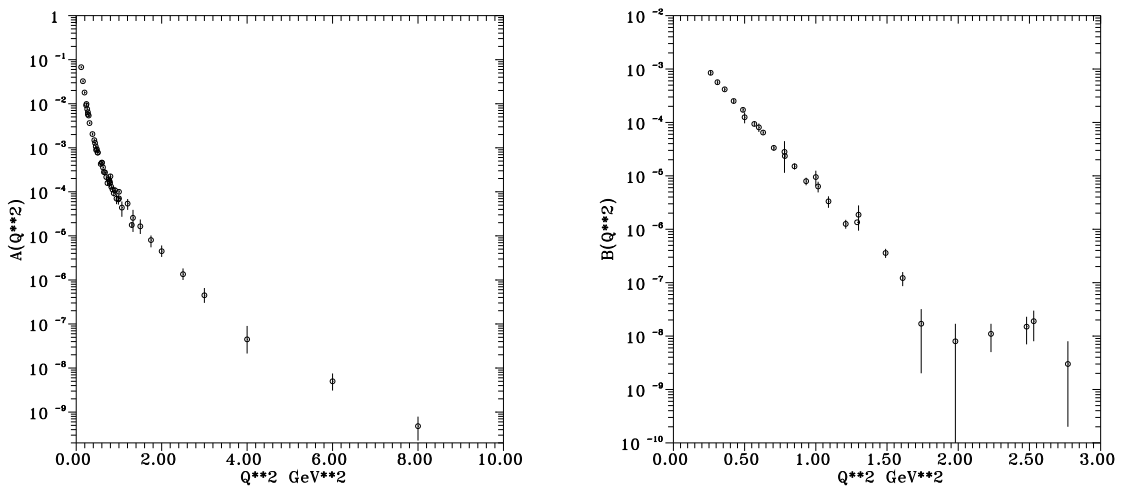


Figure 10: Experimental data on deuteron EM structure functions.

The dependence of $A(Q^2)$ and $B(Q^2)$ on deuteron EM FF's is given by the relations

$$\begin{aligned}
 A(Q^2) &= G_C^2(Q^2) + \frac{8}{9}\eta^2 G_Q^2(Q^2) + \frac{2}{3}\eta G_M^2(Q^2) \\
 B(Q^2) &= \frac{4}{3}\eta(1 + \eta)G_M^2(Q^2) \\
 \eta &= Q^2/(4m_D^2)
 \end{aligned} \tag{18}$$

In the non-relativistic impulse approximation deuteron EM FFs can be expressed simply through the **isoscalar parts of the nucleon electric and**

magnetic FFs $G_E^s = G_E^p + G_M^n$ and $G_M^s = G_M^p + G_M^n$
as follows

$$\begin{aligned} G_C &= G_E^s D_C; G_Q = G_E^s D_Q; \\ G_M &= \frac{m_D}{2m_p} (G_M^s D_M + G_E^s D_E) \end{aligned} \quad (19)$$

with

$$\begin{aligned} D_C(Q^2) &= \int_0^\infty dr [u^2(r) + w^2(r)] j_0(qr/2) \\ D_Q(Q^2) &= \frac{3}{\sqrt{2}\eta} \int_0^\infty dr w(r) [u(r) - w(r)/\sqrt{8}] j_2(qr/2) \\ D_M(Q^2) &= \int_0^\infty dr [2u^2(r) - w^2(r)] j_0(qr/2) \\ &\quad + [\sqrt{2}u(r)w(r) + w^2(r)] j_2(qr/2) \quad (20) \\ D_E(Q^2) &= \frac{3}{2} \int_0^\infty dr w^2(r) [j_0(qr/2) + j_2(qr/2)] \end{aligned}$$

and $u(r)$, $w(r)$ are the **reduced S- and D-state deuteron wave-functions**.

The **set of nucleon FF's with the new behaviour** of $G_E^p(Q^2)$ from the JLab polarization experiments **gives almost two-times lower value** of $\chi^2=1402$ in a description of $A(Q^2)$ and $B(Q^2)$ than the set of FF's obtained by Rosenbluth technique $\chi^2=2653$ what can be considered as a **support for the $G_E^p(Q^2)$ behavior with the zero** to be correct.

CONCLUSIONS

- **Rosenbluth and JLab proton polarization data** were analysed by using 10-resonance U&A model of nucleon EM structure and **two contradicting $G_E^p(Q^2)$ space-like behaviors are revealed**
- **implications** of these two contradicting space-like behaviors of $G_E^p(Q^2)$ in various physical phenomena **have been investigated**

- **New charge distribution within the proton** was found with maximum to be shifted from the centre by 1fm and **leading to the enlarged proton charge radius** $\langle r_p^2 \rangle = 0.72\text{fm}^2$
- **New sum rule** relating nucleon FFs and purely Q^2 dependent differential cross-sections of DIS on protons and neutrons **is suggested to give another support for the space-like $G_E^p(Q^2)$ behavior** with the zero
- It is demonstrated that predicted **Strange nucleon FFs do not feel contradiction** between two found $G_E^p(Q^2)$ behaviors in space-like region
- **Deuteron structure functions $A(Q^2), B(Q^2)$ data** through NIA **prefer the space-like $G_E^p(Q^2)$ behavior with the zero**