Spin-Orbit Correlations and SSAs

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Outline

- GPDs: probabilistic interpretation as Fourier transforms of impact parameter dependent PDFs
  - $H(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
  - $\tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\mathbf{\Delta}_\perp^2)$
    $\hookrightarrow \perp$ deformation of unpol. PDFs in $\perp$ pol. target
    - physics: orbital motion of the quarks

$\hookrightarrow$ intuitive explanation for SSAs (Sivers)

- intuitive explanation for Miller-effect
- $\bar{E}_T = 2\tilde{H}_T + E_T$
  $\longrightarrow \perp$ deformation of $\perp$ pol. PDFs in unpol. target
  - correlation between quark angular momentum and quark transversity

  $\hookrightarrow$ Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$
  - Are all Boer-Mulders functions alike?
- Summary

Impact parameter dependent PDFs

- define $\perp$ localized state

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has $\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
(cf.: working in CM frame in nonrel. physics)

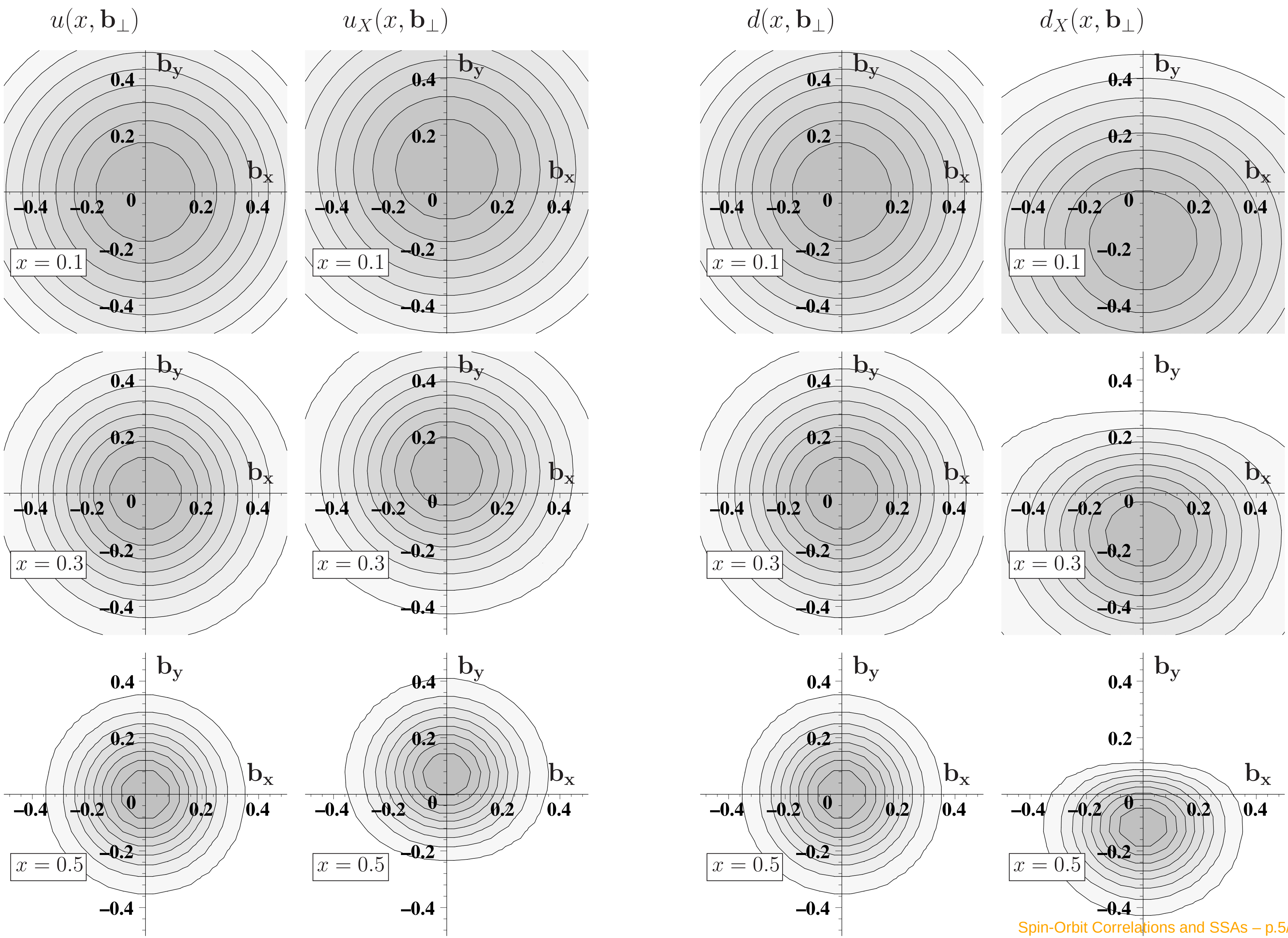
- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp) \gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$$\hookrightarrow \quad \begin{aligned} q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} H(x, 0, -\mathbf{\Delta}_\perp^2), \\ \Delta q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) \end{aligned}$$

Impact parameter dependent PDFs

- corollary (G.Miller's talk): Interpretation of two-dimensional Fourier transform of F_1 as j^+ charge distribution in impact parameter space;
  equivalent interpretation: FT of usual j^0 charge distribution accross the pizza (after nucleon has been boosted to ∞ momentum)
- analogously, impact parameter dependent distribution of quarks with $\pm$ helicity in longitudinally polarized nucleons obtained from 2d FT of $\frac{1}{2}\left(F_1 \pm G_A\right)$


$u(x, \mathbf{b}_\perp)$
$u_X(x, \mathbf{b}_\perp)$
$d(x, \mathbf{b}_\perp)$
$d_X(x, \mathbf{b}_\perp)$
$x = 0.1$
$x = 0.3$
$x = 0.5$
$\mathbf{b_x}$
$\mathbf{b_y}$

Transversely Deformed Distributions and $E(x,0,-\mathbf{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow | \bar{q}(0)\, \gamma^+ q(x^-) |\, P,\uparrow\rangle \quad = H(x,0,-\mathbf{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow | \bar{q}(0)\, \gamma^+ q(x^-) |\, P,\downarrow\rangle \quad = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle$.

$\hookrightarrow$ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

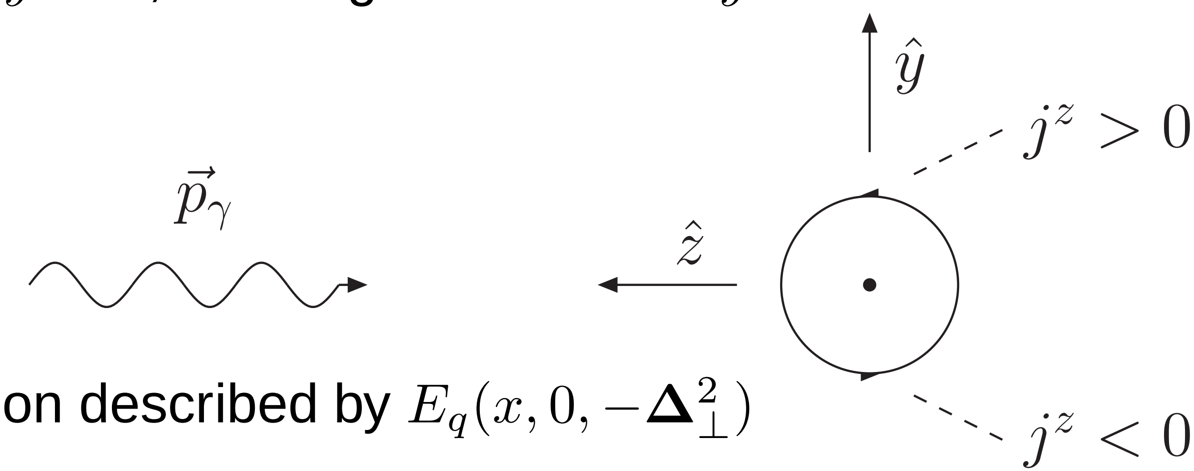
- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 !
  [X.Ji, PRL **91**, 062001 (2003)]

Intuitive connection with $\vec{L}_q$

- DIS probes quark momentum density in the infinite momentum frame (IMF). Quark density in IMF corresponds to $j^+ = j^0 + j^3$ component in rest frame ($\vec{p}_{\gamma^*}$ in $-\hat{z}$ direction)

↪ j^+ larger than j^0 when quarks move towards the γ^*; suppressed when they move away from γ^*

↪ For quarks with positive orbital angular momentum in $\hat{x}$-direction, j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



- Details of $\perp$ deformation described by $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$

↪ not surprising that $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$ enters Ji relation!

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x,0,0) + E_q(x,0,0) \right] x.$$

Transversely Deformed PDFs and $E(x,0,-\mathbf{\Delta}_\perp^2)$

- $q(x,\mathbf{b}_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons !
- mean $\perp$ deformation of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x,\mathbf{b}_\perp) b_y = \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q^p}{2M}$$

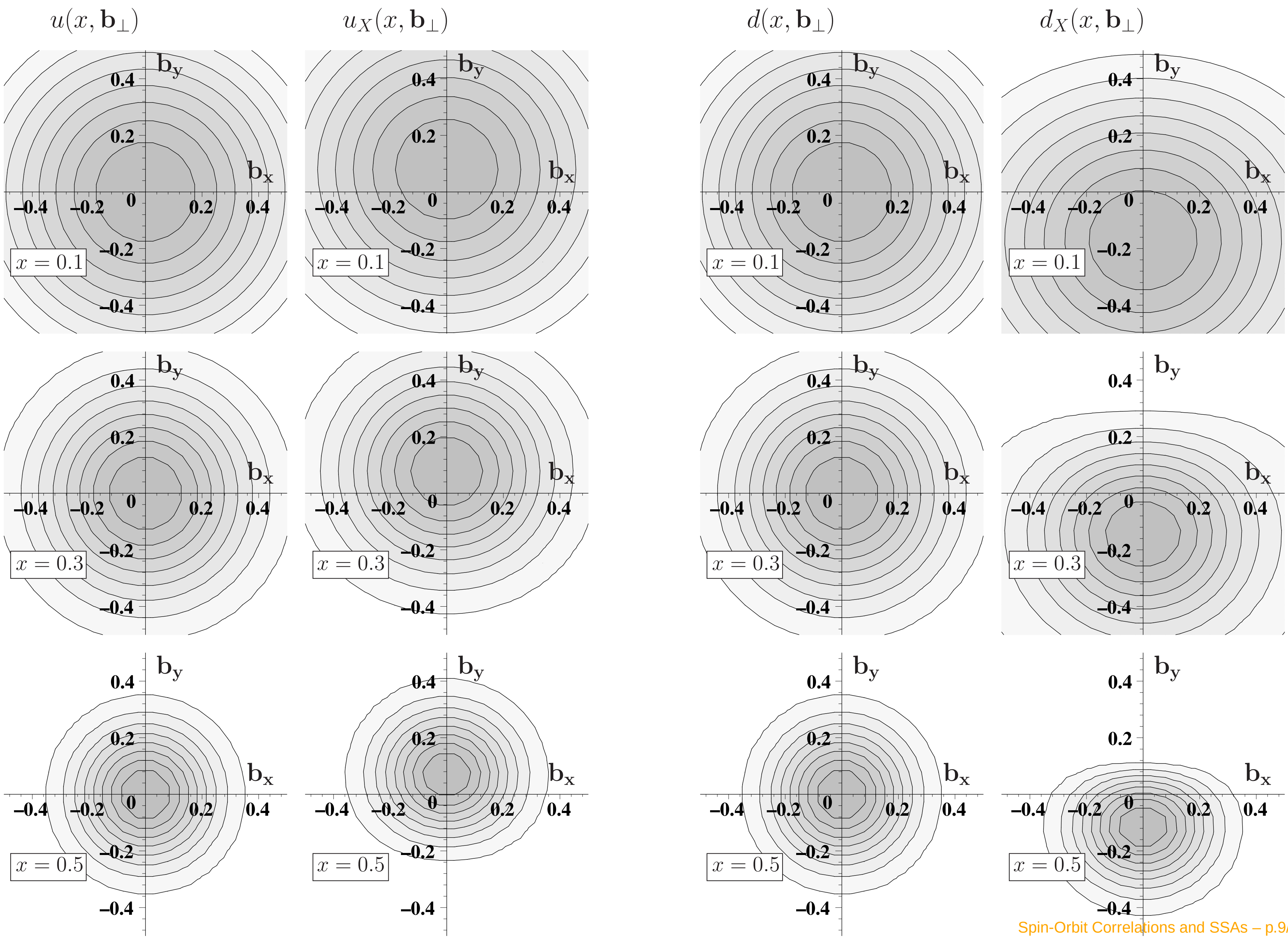
with $\kappa^p_{u/d} \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2 fm)$

- simple model: for simplicity, make ansatz where $E_q \propto H_q$

$$\begin{aligned} E_u(x,0,-\mathbf{\Delta}_\perp^2) &= \frac{\kappa_u^p}{2} H_u(x,0,-\mathbf{\Delta}_\perp^2) \\ E_d(x,0,-\mathbf{\Delta}_\perp^2) &= \kappa_d^p H_d(x,0,-\mathbf{\Delta}_\perp^2) \end{aligned}$$

with $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.$

- Model too simple but illustrates that anticipated deformation is very significant since κ_u and κ_d known to be large!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$x = 0.1$
$x = 0.3$
$x = 0.5$
$\mathbf{b_x}$
$\mathbf{b_y}$
-0.4
-0.2
0
0.2
0.4

$\perp$ flavor dipole moments $\leftrightarrow$ Ji-relation

[M.B., PRD**72**, 094020 (2005)]

- $J_\perp^q \propto \perp$ center of momentum (COM)

$$J_y^q = \frac{M}{4}\sum_i x_i b_i^y$$

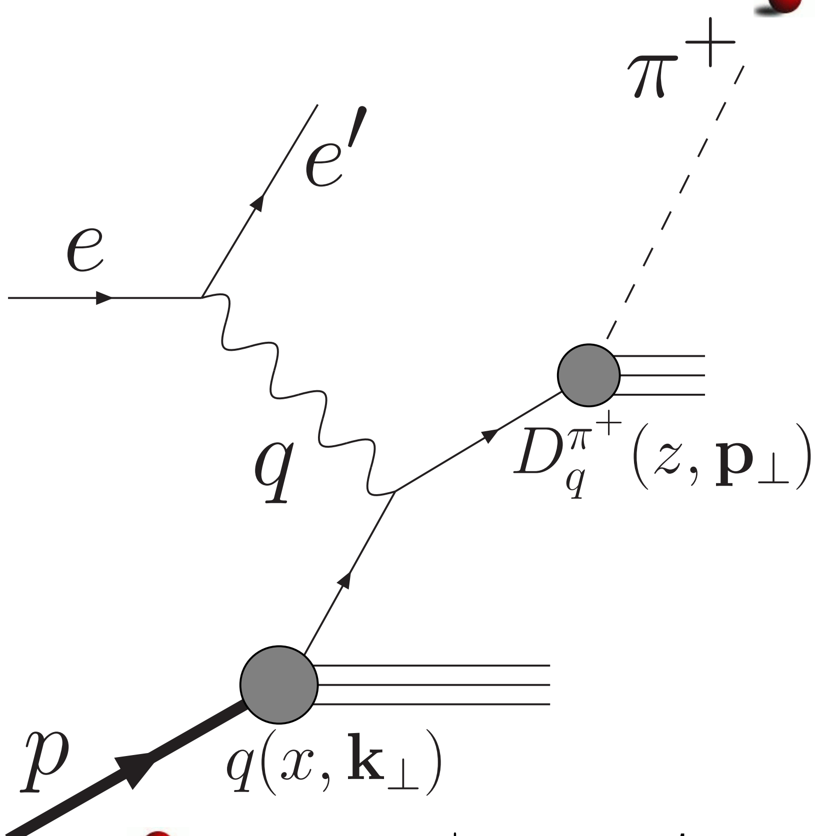
Note: two terms in $J_x^q \sim \int d^3r T^{tz}b^y - T^{ty}b^z$ equal by rot. inv.!

- $\perp$ COM for quark flavor q at $y = \frac{1}{2M}\int dx\, xE^q(x,0,0)$ (nucleon with COM at $\mathbf{R}_\perp = \mathbf{0}_\perp$ and polarized in $\hat{x}$ direction)
- additional $\perp$ displacement of the whole nucleon by $\frac{1}{2M}$ from boosting $\perp$ polarized nucleon wave packet from rest frame to ∞ momentum frame (Melosh ...)

$\hookrightarrow$ when $\perp$ polarized nucleon is boosted from rest to ∞ momentum, $\perp$ flavor dipole moment for quarks with flavor q is

$$\frac{1}{2M}\int dx\, xE^q(x,0,0) + \frac{1}{2M}\int dx\, xq(x) \qquad (\rightsquigarrow \text{Ji relation})$$

SSAs in SIDIS ($\gamma + p\uparrow \longrightarrow \pi^+ + X$)



- use factorization (high energies) to express momentum distribution of outgoing π^+ as convolution of
  - momentum distribution of quarks in nucleon

  $\hookrightarrow$ unintegrated parton density $f_{q/p}(x, \mathbf{k}_\perp)$
  - momentum distribution of π^+ in jet created by leading quark q

  $\hookrightarrow$ fragmentation function $D_q^{\pi^+}(z, \mathbf{p}_\perp)$
- average $\perp$ momentum of pions obtained as sum of
  - average $\mathbf{k}_\perp$ of quarks in nucleon (Sivers effect)
  - average $\mathbf{p}_\perp$ of pions in quark-jet (Collins effect)

GPD $\longleftrightarrow$ SSA (Sivers)

- Sivers: distribution of **unpol.** quarks in $\perp$ pol. proton

$$f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot S}{M}$$

- without FSI, $\langle\mathbf{k}_\perp\rangle = 0$, i.e. $f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2) = 0$
- with FSI, $\langle\mathbf{k}_\perp\rangle \neq 0$ (Brodsky, Hwang, Schmidt)
- Why interesting?
  - $\perp$ asymmetry involves nucleon helicity flip
  - quark density chirally even (no quark helicity flip)
  - $\hookrightarrow$ ‘helicity mismatch’ requires orbital angular momentum (OAM)
  - $\hookrightarrow$ (like κ), Sivers requires matrix elements between **wave function components that differ by one unit of OAM** (Brodsky, Diehl, ..)
  - Sivers requires nontrivial final state interaction phases
  - $\hookrightarrow$ **sensitive to space-time structure of hadrons**

$\perp$ Single-Spin Asymmetry (Sivers)

- treat FSI to lowest order in g

$\hookrightarrow$
$$\left\langle k_q^i \right\rangle = -\frac{g^2}{4p^+} \int \frac{d^2\mathbf{b}_\perp}{2\pi} \frac{b^i}{|\mathbf{b}_\perp|^2} \left\langle p, s \left| \bar{q}(0)\gamma^+ \frac{\lambda_a}{2} q(0)\rho_a(\mathbf{b}_\perp) \right| p, s \right\rangle$$

with $\rho_a(\mathbf{b}_\perp) = \int dr^- \rho_a(r^-, \mathbf{b}_\perp)$ summed over all quarks and gluons

$\hookrightarrow$ SSA related to dipole moment of density-density correlations

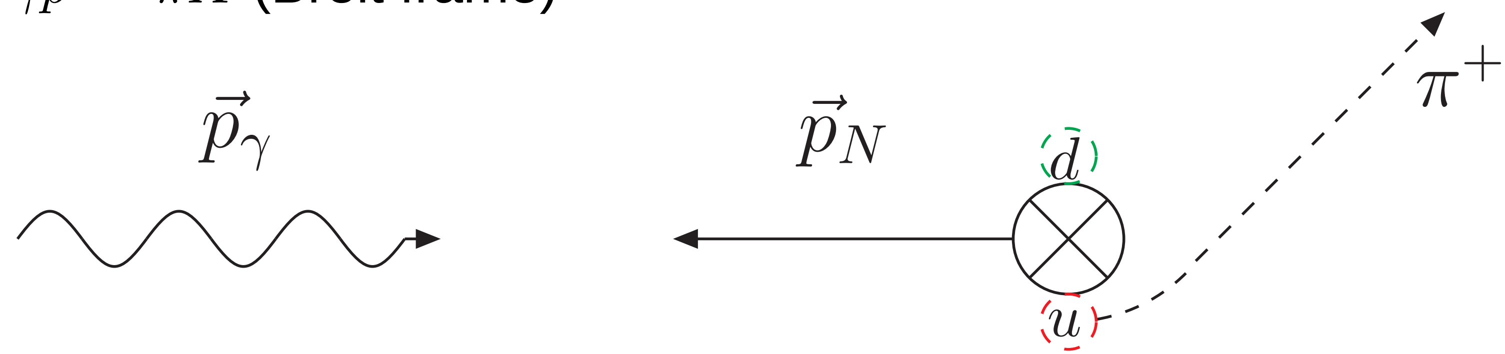
- GPDs (N polarized in $+\hat{x}$ direction): $u \longrightarrow +\hat{y}$ and $d \longrightarrow -\hat{y}$

$\hookrightarrow$ expect density density correlation to show same asymmetry $\langle b^y \bar{u}(0)\gamma^+ \frac{\lambda_a}{2} u(0)\rho_a(\mathbf{b}_\perp)\rangle > 0$

$\hookrightarrow$ sign of SSA opposite to sign of distortion in position space

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \rightarrow \pi X$ (Breit frame)



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\hookrightarrow$ correlation between sign of κ_q^p and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q^p$

- $f_{1T}^{\perp q} \sim -\kappa_q^p$ confirmed by HERMES results (also consistent with COMPASS $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$)

GPD $\longleftrightarrow$ SSA (Sivers)

- $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$ also consistent with sum rule [M.B., PRD **69**, 091501 (2004)]

$$\int dx \sum_{i \in q,g} f_{1T}^{\perp q}(x, \mathbf{k}_\perp)\mathbf{k}_\perp^2 = 0.$$

- non-trivial sum rule, not a trivial consequence of momentum conservation (cf. Schäfer Teryaev sum rule for fragmentation) as it does not involve a summation over the whole final state, but only over active partons

$f_{1T}^{\perp}(x, \mathbf{k}_\perp)_{DY} = -f_{1T}^{\perp}(x, \mathbf{k}_\perp)_{SIDIS}$

- time reversal: FSI $\leftrightarrow$ ISI

$\hookrightarrow$ $f_{1T}^{\perp}(x, \mathbf{k}- \perp)_{DY} = -f_{1T}^{\perp}(x, \mathbf{k}- \perp)_{SIDIS}$ (Collins)

- Intuitive explanation (for simplicity first in QED)
  - compare FSI for bound e^- that is being knocked out with ISI for e^+ that is about to annihilate that bound e^-
  - FSI for knocked out e^- is attractive
  - ISI for the to-be-annihilated e^+ due to the spectators is repulsive.
  - annihilation local in $\mathbf{b}_\perp$

  $\hookrightarrow$ $\perp$ impulse opposite to $\perp$ impuls on e^-, since both are at same $\perp$ position

  - no $\perp$ impulse due to force from to-be-annihilated e^- as it is approached head-on

  $\hookrightarrow$ (after averaging over longitudinal positions of bound e^-) $\perp$ impulse in SIDIS must be equal and opposite to $\perp$ impulse in DY

$f_{1T}^{\perp}(x, \mathbf{k}_\perp)_{DY} = -f_{1T}^{\perp}(x, \mathbf{k}_\perp)_{SIDIS}$

- time reversal: FSI $\leftrightarrow$ ISI

$\hookrightarrow$ $f_{1T}^{\perp}(x, \mathbf{k}- \perp)_{DY} = -f_{1T}^{\perp}(x, \mathbf{k}- \perp)_{SIDIS}$ (Collins)

- Intuitive explanation (QCD)
  - compare FSI for 'red' q that is being knocked out with ISI for an anti-red $\bar{q}$ that is about to annihilate that bound q

  $\hookrightarrow$ FSI for knocked out q is attractive
  - nucleon is color singlet $\rightarrow$ when to-be-annihilated q is 'red', the spectators must be anti-red

  $\hookrightarrow$ anti-red spectators and anti-red approaching $\bar{q}$ repel each other

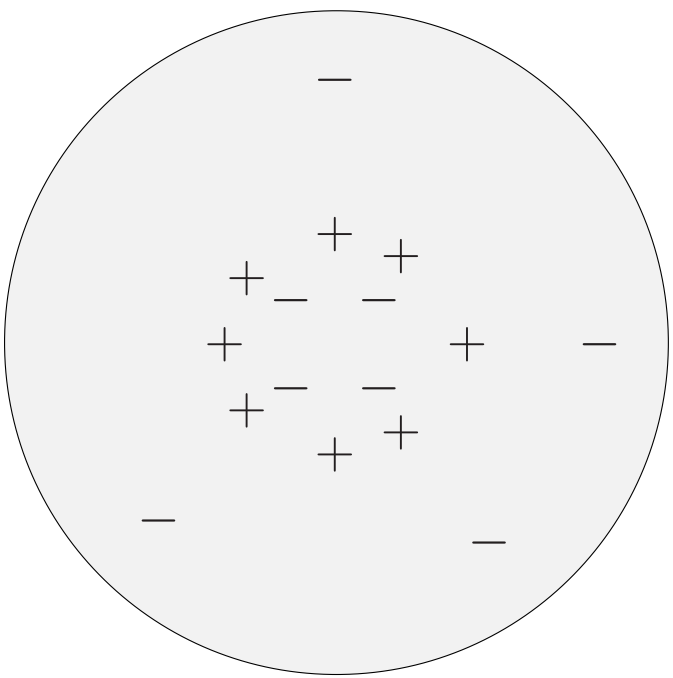
  $\hookrightarrow$ ISI is repulsive
  - no $\perp$ impulse due to force from to-be-annihilated q as it is approached head-on

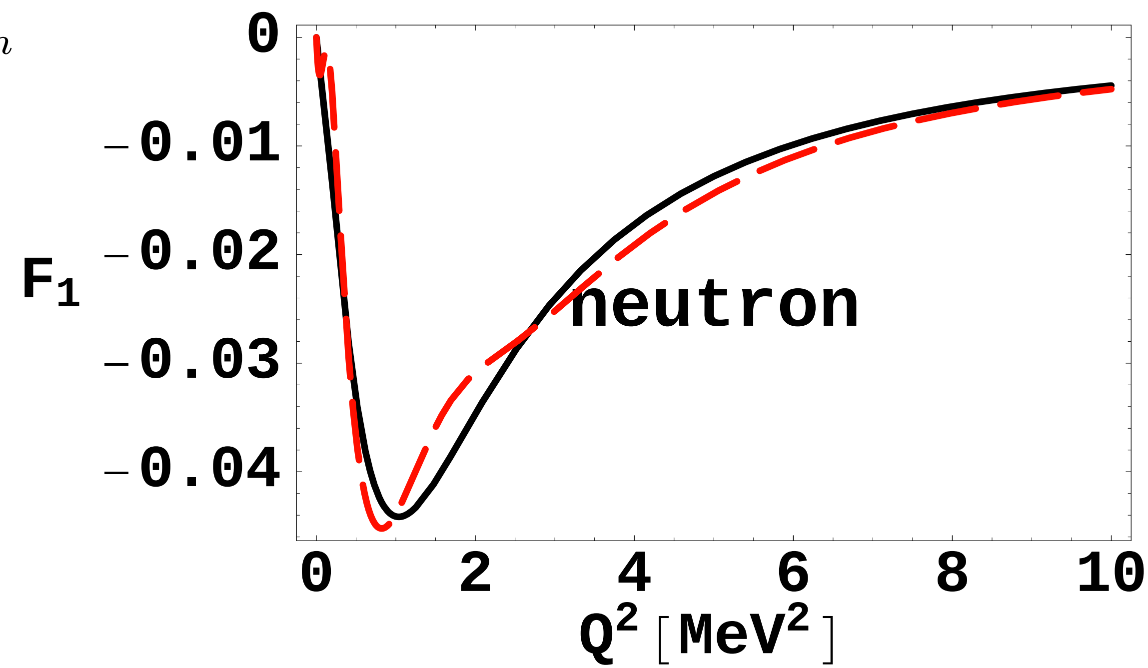
  $\hookrightarrow$ (after averaging over longitudinal positions of bound q) $\perp$ impulse in SIDIS must be equal and opposite to $\perp$ impulse in DY

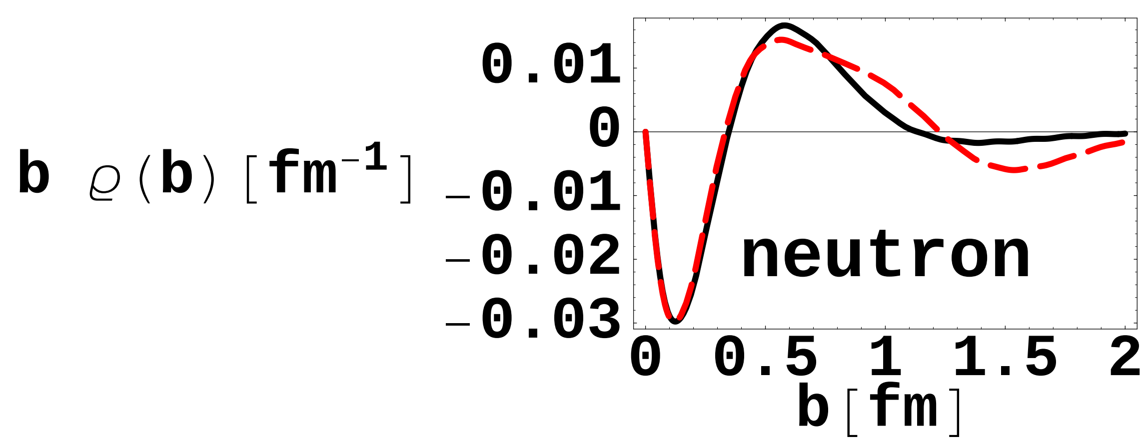
Intuitive Explanation for the 'Miller-Effect'

- Miller-effect: 2d FT of F_1^n

The neutron pizza







Intuitive Explanation for the 'Miller-Effect'

- Miller-effect: 2d FT of F_1^n

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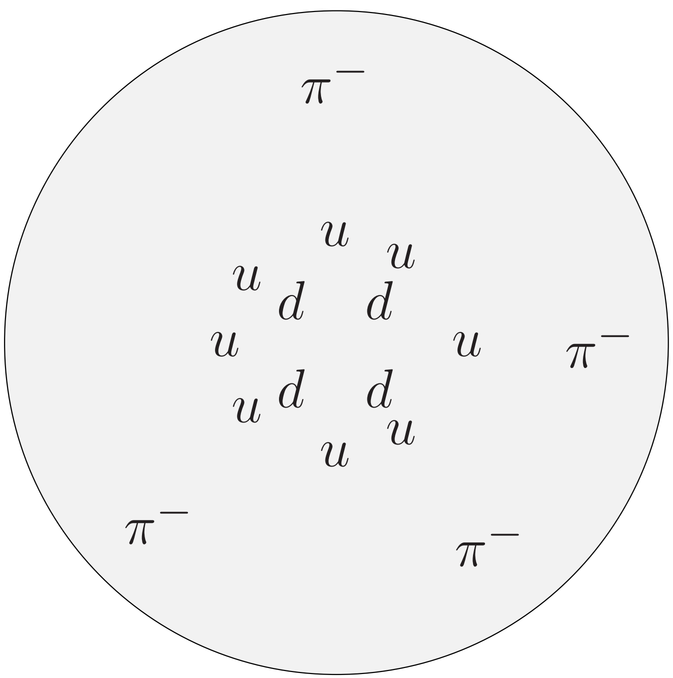
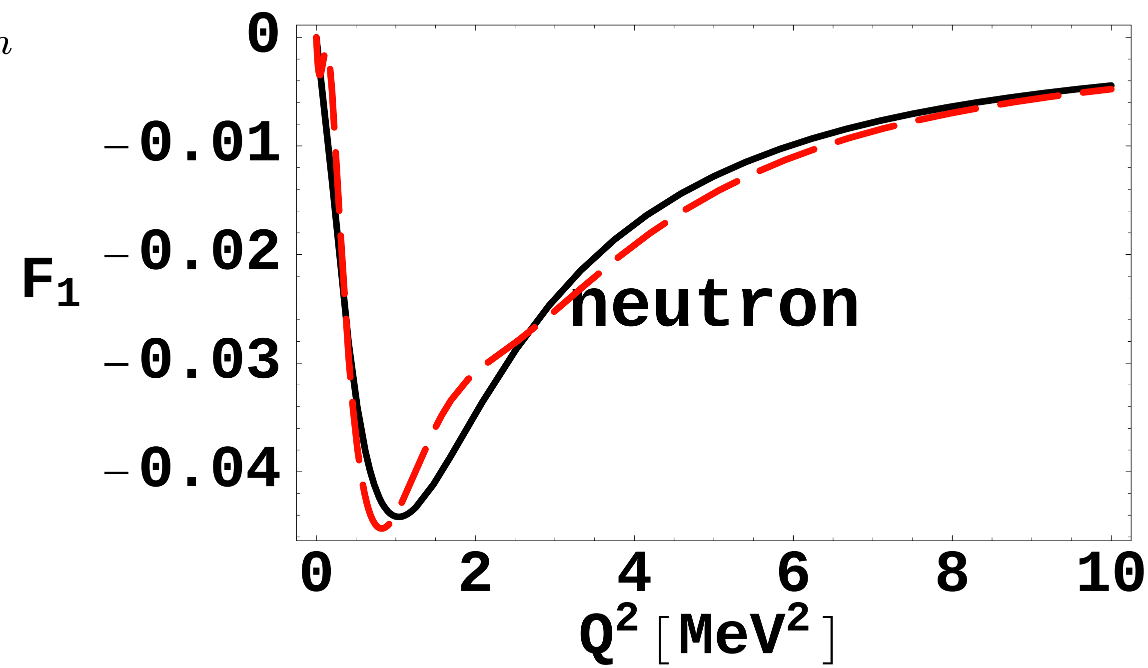
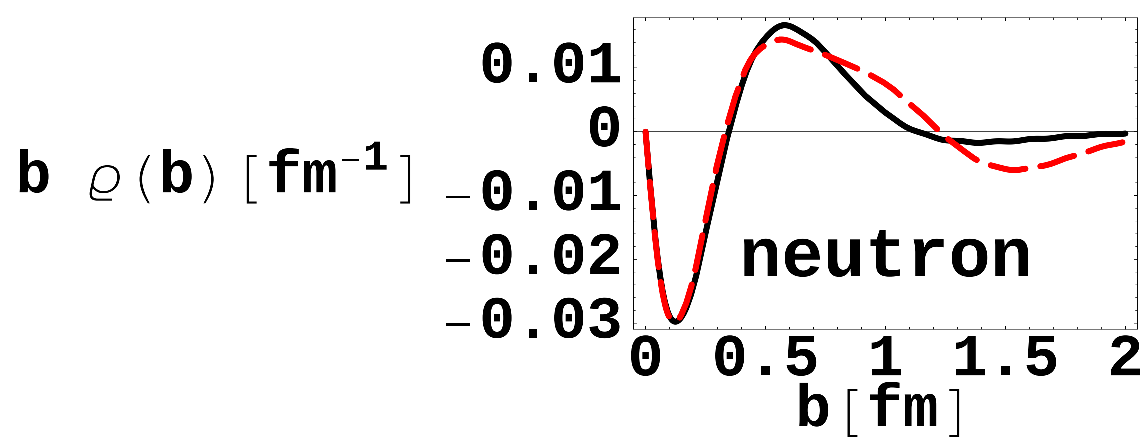






Intuitive Explanation for the ‘Miller-Effect’

- Miller-effect: 2d FT of F_1^n

↪ suppression of u quarks/enhancement of d quarks in center of neutron-pizza (in IMF)

- Explanation: several indications that, in proton, d-quarks in proton have larger p-wave component than u-quarks
  - after charge factors taken out, contribution from d quarks to anomalous magnetic moment of proton larger than from u quarks ($\kappa_u^p = 1.673$, $\kappa_d^p = -2.033$) — despite the fact that proton contains more u quarks .
  - HERMES: Sivers function for d quarks (in proton) at least as large as for u quarks — despite the fact that proton contains more u quarks .

↪ (in neutron), u quarks should have larger p-wave component than d quarks

- p wave suppressed at origin!

↪ **suppression of u quarks at center of neutron due to larger p-wave component**

Chirally Odd GPDs

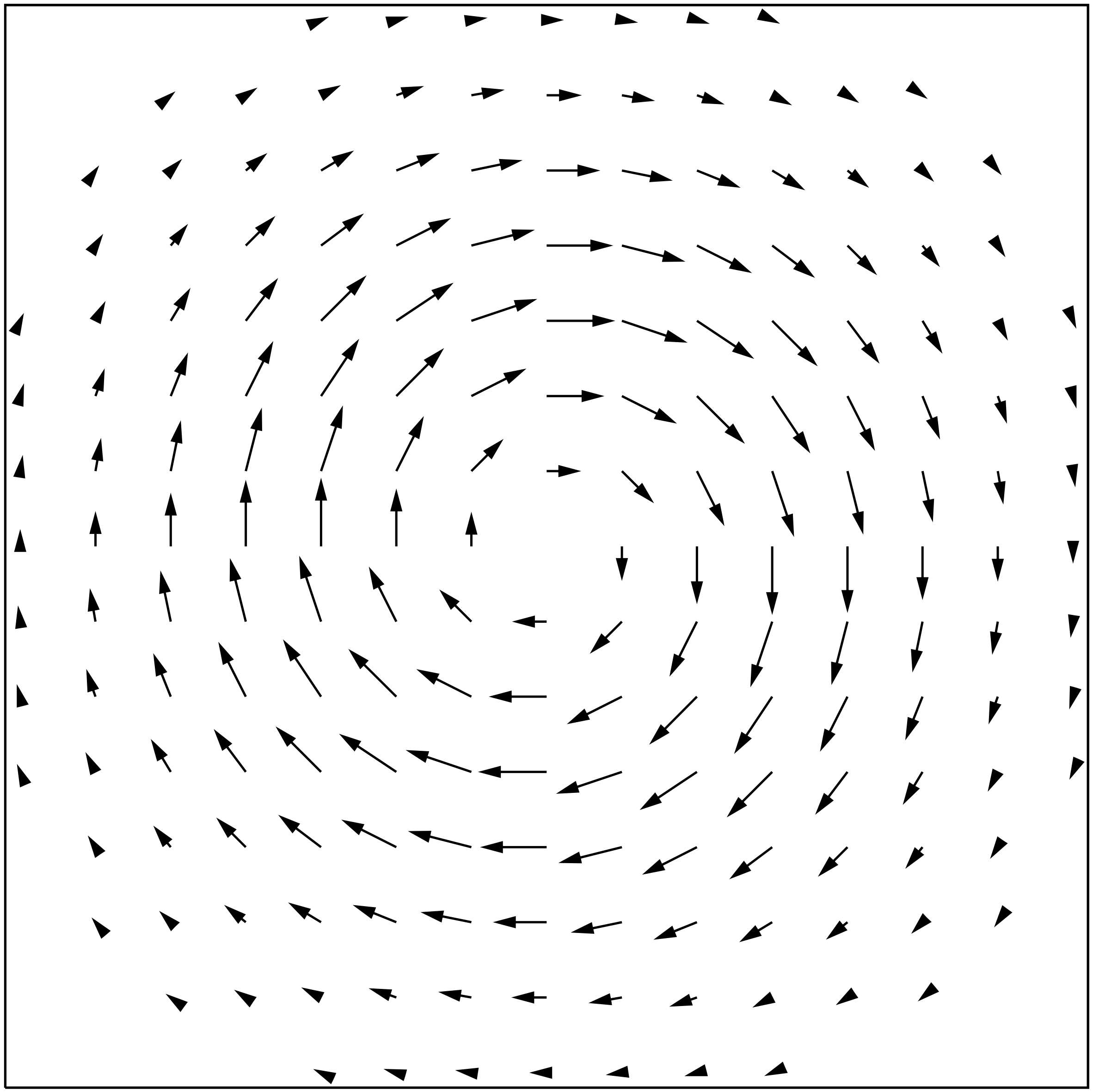
$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \sigma^{+j}\gamma_5 q\left(\frac{x^-}{2}\right) \right| p \right\rangle = \quad H_T \bar{u}\sigma^{+j}\gamma_5 u + \tilde{H}_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha P_\beta}{M^2} u$$
$$+ E_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha\gamma_\beta}{2M} u + \tilde{E}_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}P_\alpha\gamma_\beta}{M} u$$

- See also M.Diehl+P.Hägler, hep-ph/0504175.
- Fourier trafo of $\bar{E}_T^q \equiv 2\tilde{H}_T^q + E_T^q$ for $\xi = 0$ describes distribution of transversity for <u>un</u>polarized target in $\perp$ plane

$$q^i(x, \mathbf{b}_\perp) = \frac{\varepsilon^{ij}}{2M} \frac{\partial}{\partial b_j} \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} \bar{E}_T^q(x, 0, -\mathbf{\Delta}_\perp^2)$$

- origin: correlation between quark spin (i.e. transversity) and angular momentum

Transversity Distribution in Unpolarized Target



Boer-Mulders Function

- SIDIS: attractive FSI expected to convert position space asymmetry into momentum space asymmetry

↪ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction

↪ (qualitative) connection between Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$ and the chirally odd GPD $\bar{E}_T$ that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x, \mathbf{k}_\perp)$ and the GPD E.

- Boer-Mulders: distribution of $\perp$ **pol.** quarks in **unpol.** proton

$$f_{q^\uparrow/p}(x, \mathbf{k}_\perp) = \frac{1}{2}\left[f_1^q(x, \mathbf{k}_\perp^2) - h_1^{\perp q}(x, \mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp) \cdot S_q}{M}\right]$$

- $h_1^{\perp q}(x, \mathbf{k}_\perp^2)$ can be probed in DY (RHIC, J-PARC, GSI) and tagged SIDIS (JLab, eRHIC), using Collins-fragmentation

probing BM function in tagged SIDIS

- consider semi-inclusive pion production off unpolarized target
- spin-orbit correlations in target wave function provide correlation between (primordial) quark transversity and impact parameter

$\hookrightarrow$ (attractive) FSI provides correlation between quark spin and $\perp$ quark momentum $\Rightarrow$ BM function

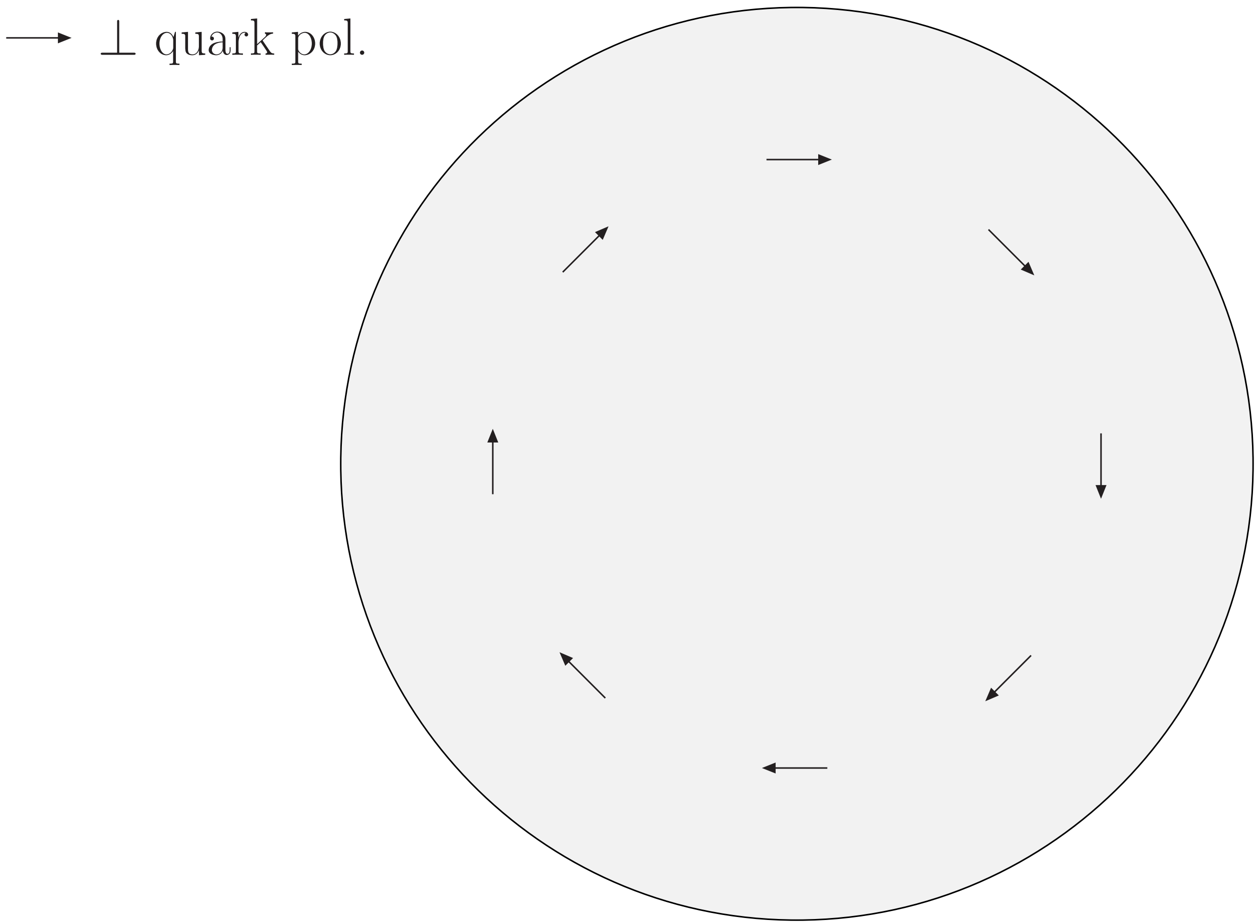
- Collins effect: left-right asymmetry of π distribution in fragmentation of $\perp$ polarized quark $\Rightarrow$ 'tag' quark spin

$\hookrightarrow$ $\cos(2\phi)$ modulation of π distribution relative to lepton scattering plane

$\hookrightarrow$ $\cos(2\phi)$ asymmetry proportional to: Collins $\times$ BM

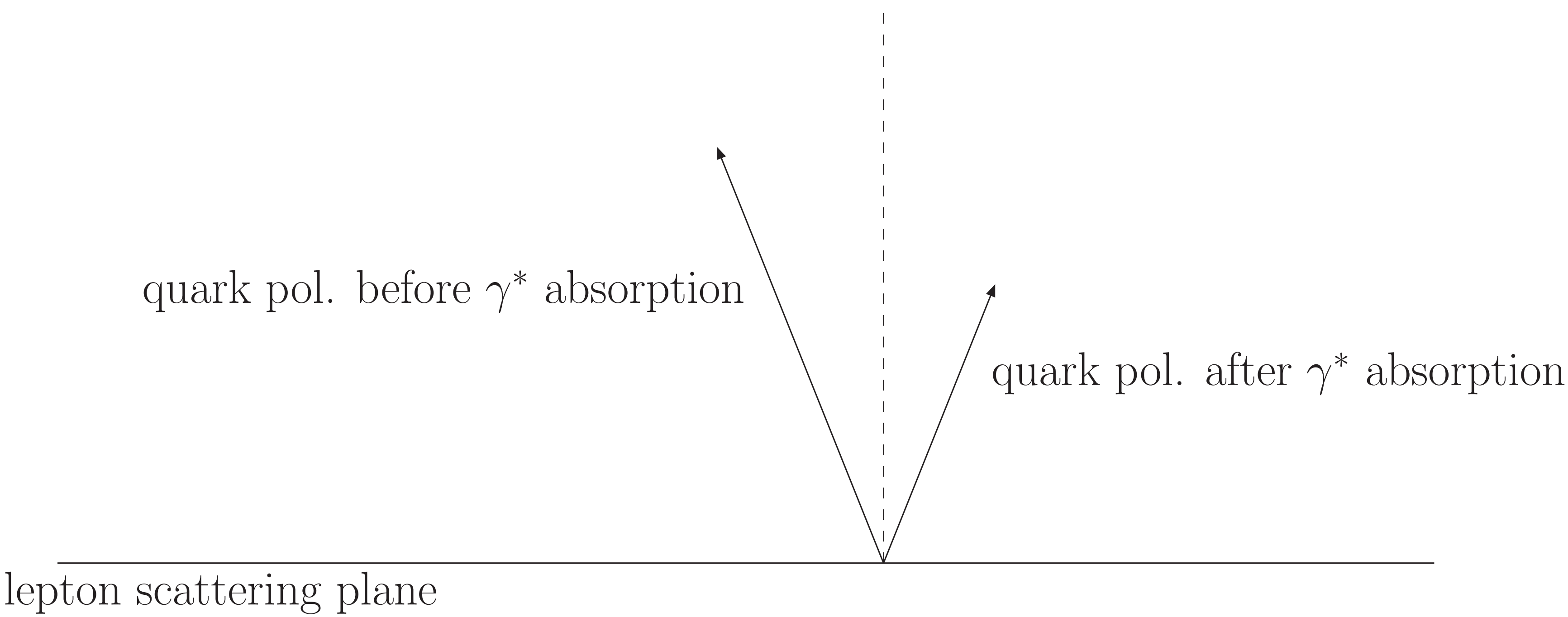
probing BM function in tagged SIDIS

Primordial Quark Transversity Distribution



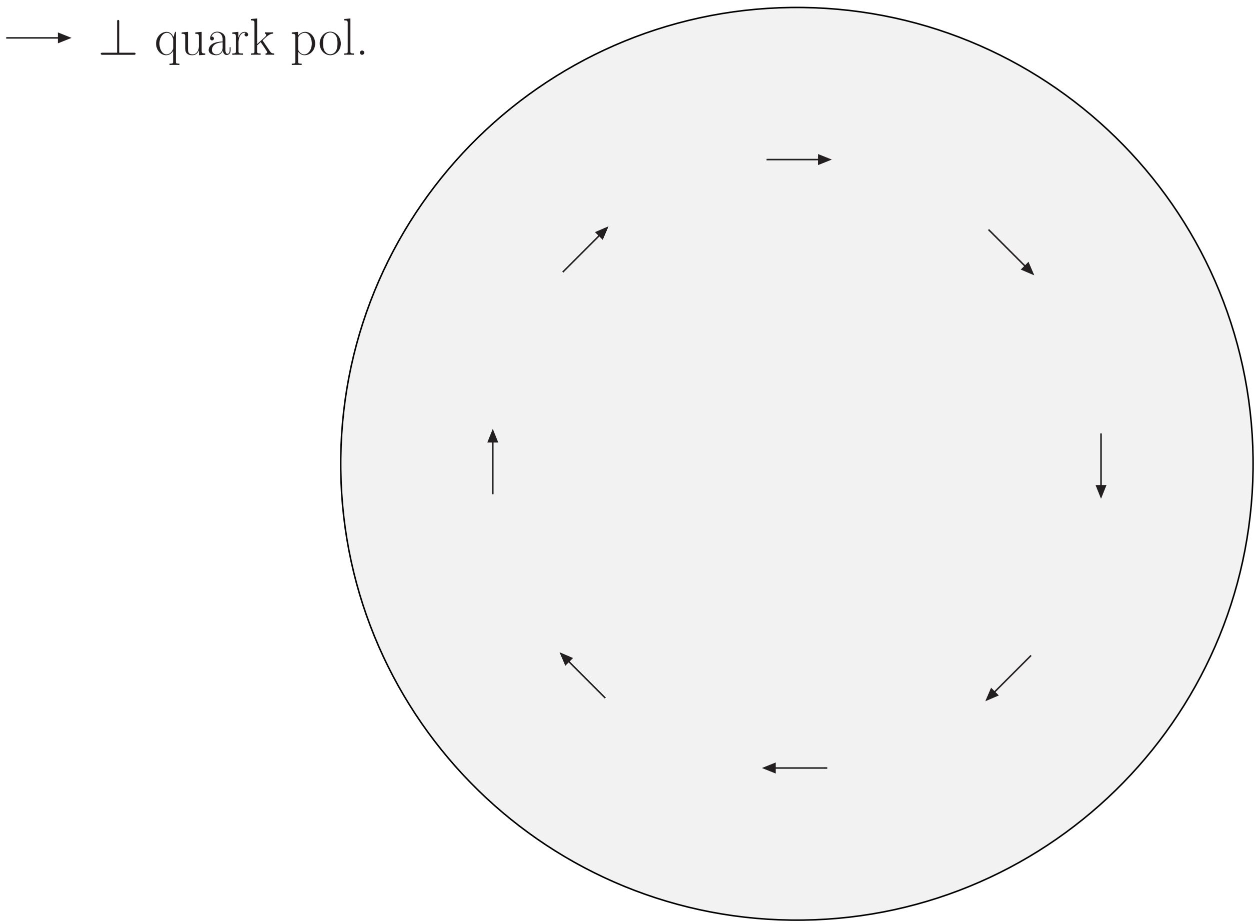
$\perp$ polarization and γ^* absorption

- QED: when the γ^* scatters off $\perp$ polarized quark, the $\perp$ polarization gets modified
  - gets reduced in size
  - gets tilted symmetrically w.r.t. normal of the scattering plane



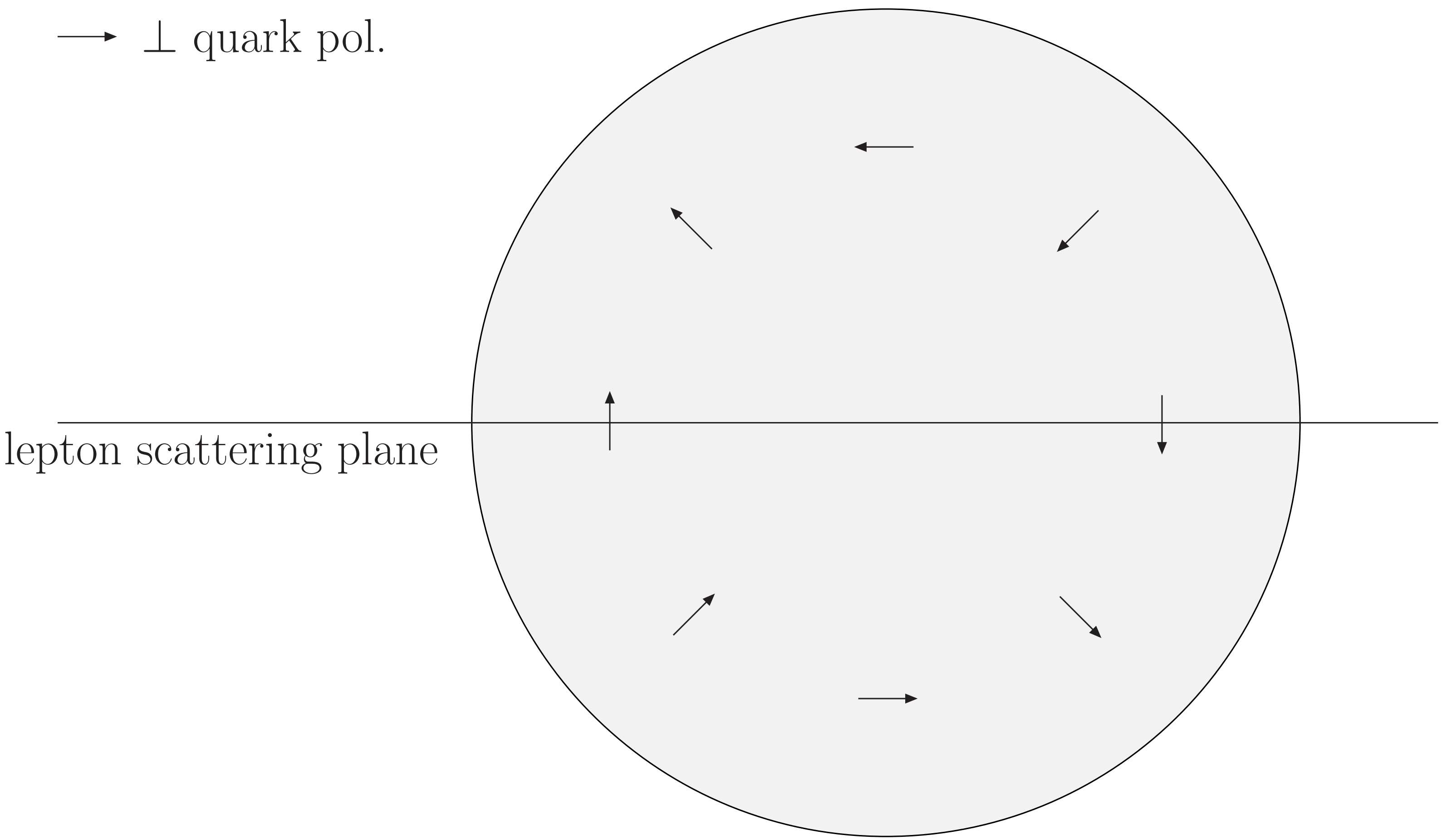
probing BM function in tagged SIDIS

Primordial Quark Transversity Distribution



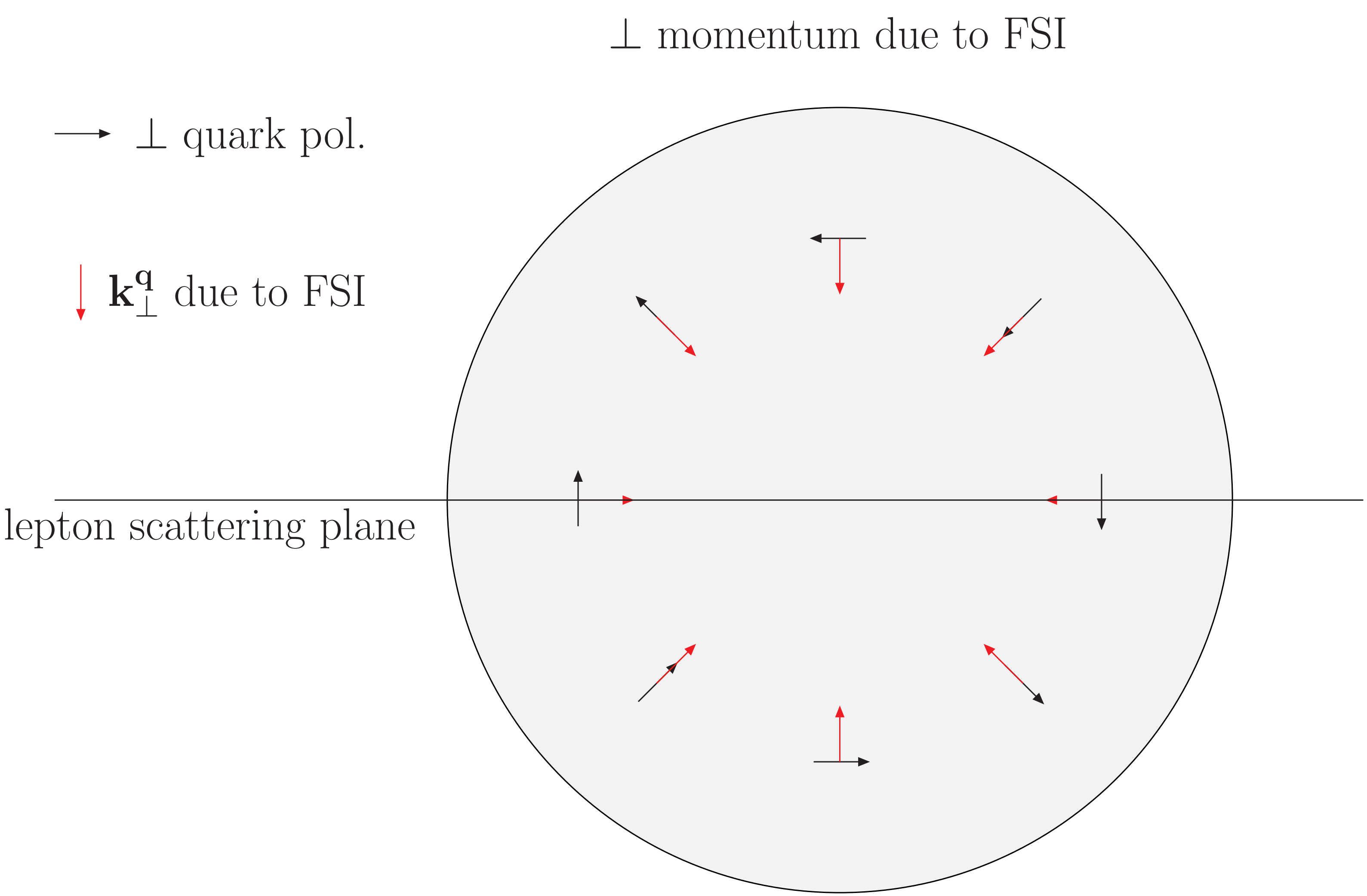
probing BM function in tagged SIDIS

Quark Transversity Distribution after γ^* absorption



quark transversity component in lepton scattering plane flips

probing BM function in tagged SIDIS



on average, FSI deflects quarks towards the center

Collins-Effect

- When a $\perp$ polarized struck quark fragments, the strucure of jet is sensitive to polarization of quark
- distribution of hadrons relative to $\perp$ polarization direction may be left-right asymmetric
- asymmetry parameterized by **Collins fragmentation function**
- Artru model:
  - struck quark forms pion with $\bar{q}$ from $q\bar{q}$ pair with ${}^3P_0$ 'vacuum' quantum numbers

  $\hookrightarrow$ pion 'inherits' OAM in direction of $\perp$ spin of struck quark

  $\hookrightarrow$ produced pion preferentially moves to left when looking into direction of motion of fragmenting quark with spin up
- Artru model confirmed by HERMES experiment
- more precise determination of Collins function under way (BELLE)

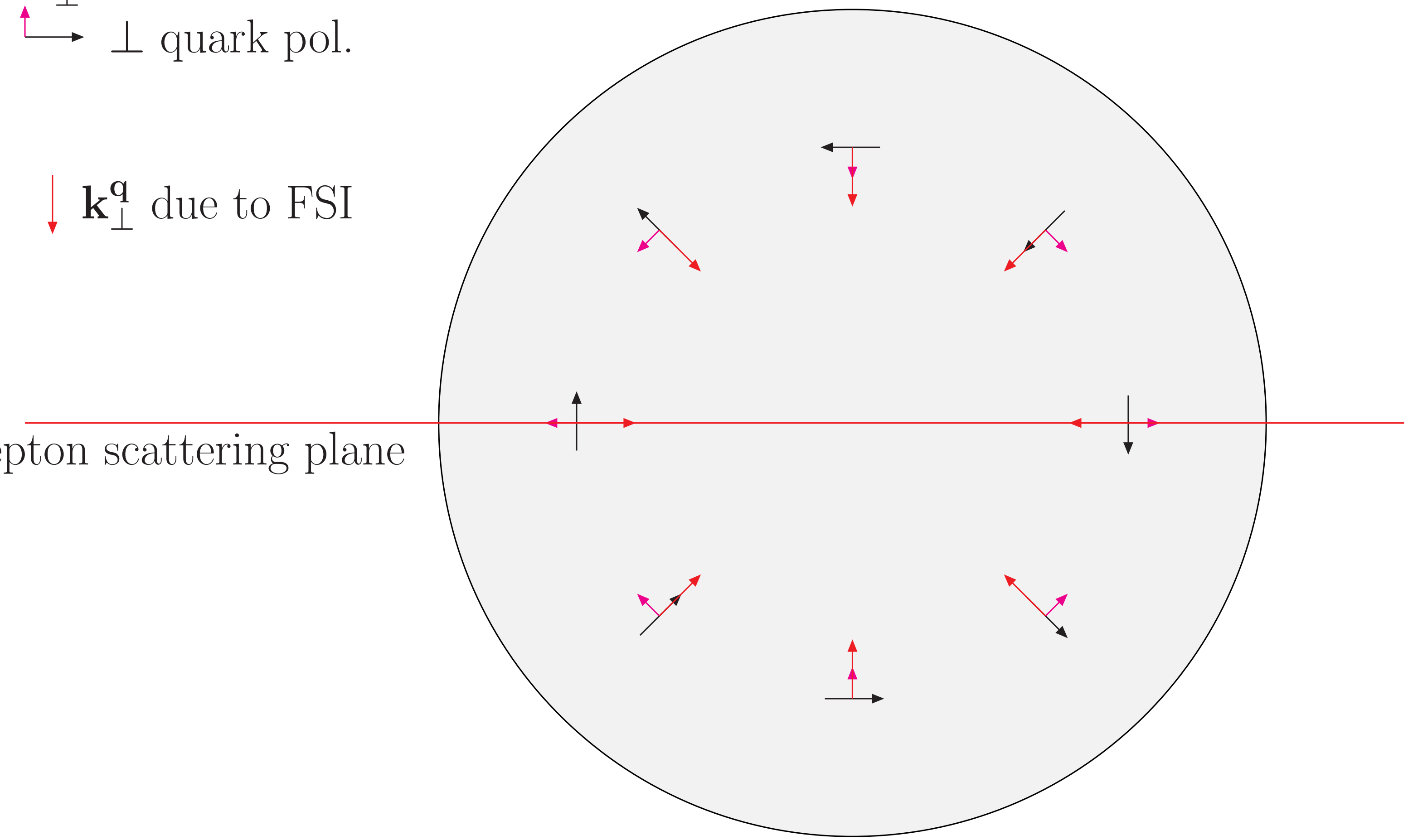
probing BM function in tagged SIDIS

⊥ momentum due to Collins

$\mathbf{k}_\perp$ due to Collins

⊥ quark pol.

$\mathbf{k}_\perp^{\mathbf{q}}$ due to FSI

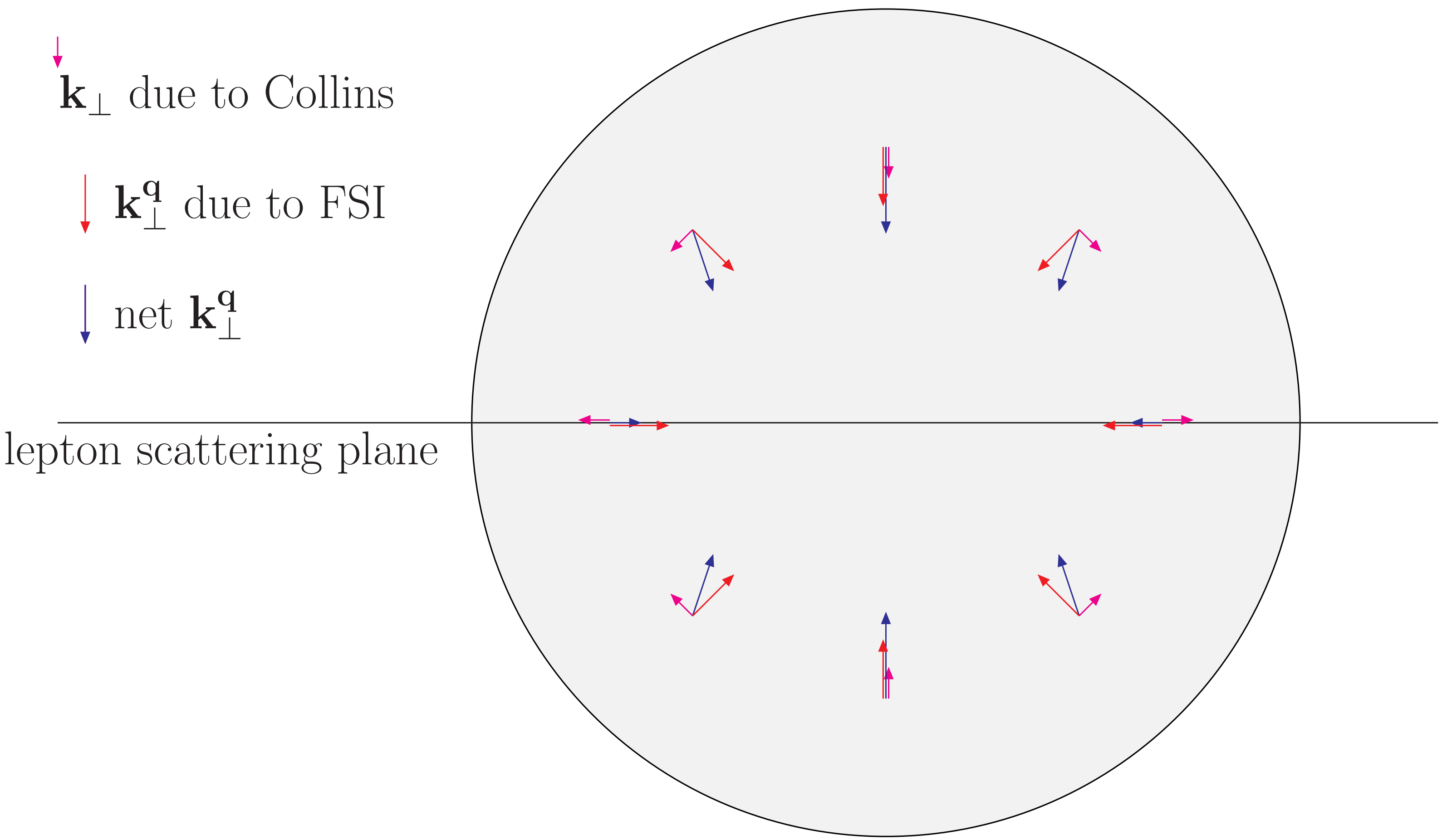


lepton scattering plane

SSA of π in jet emanating from ⊥ pol. q

probing BM function in tagged SIDIS

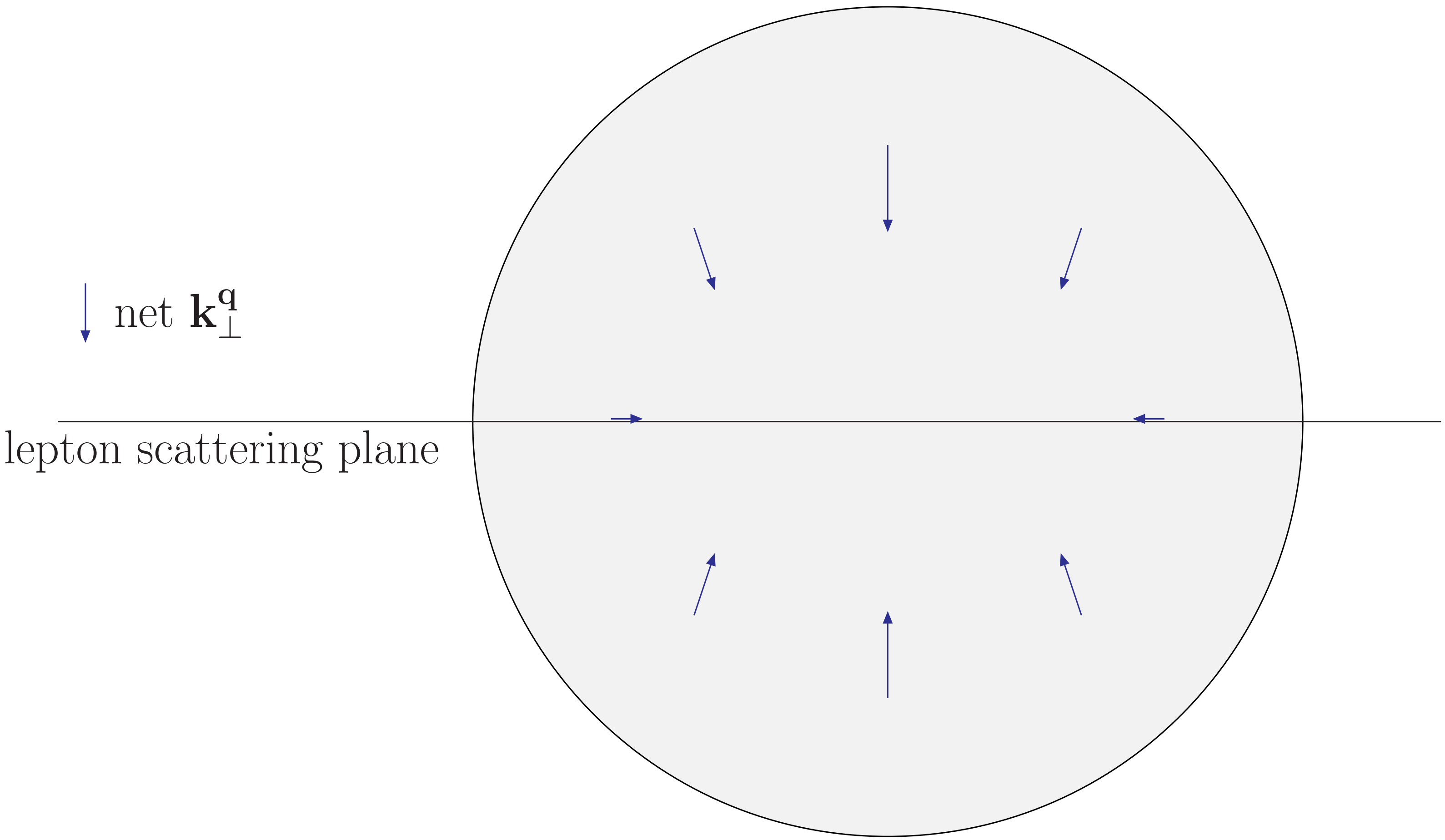
net ⊥ momentum (FSI+Collins)



$\hookrightarrow$ in this example, enhancement of pions with ⊥ momenta ⊥ to lepton plane

probing BM function in tagged SIDIS

net $k_\perp^\pi$ (FSI + Collins)



$\hookrightarrow$ expect enhancement of pions with $\perp$ momenta $\perp$ to lepton plane

Chirally Odd GPDs (sign)

- LC-wave function representation: matrix element for $\bar{E}_T$ involves quark helicity flip [M.B.+B.Hannafious, hep-ph/0705.1573]

$\hookrightarrow$ interference between wave function components that differ by one unit of OAM (e.g. s-p interference)

$\hookrightarrow$ sign of $\bar{E}_T$ depends on rel. sign between s & p components

- bag model: p-wave from lower component

$$\Psi_m = \begin{pmatrix} i f \chi_m \\ -g(\vec{\sigma}\cdot\hat{\vec{x}})\chi_m \end{pmatrix},$$

(relative sign from free Dirac equation $g = \frac{1}{E}\frac{d}{dr}f$)

- $\bar{E}_T \propto -f \cdot g$. Ground state wave function: f peaked at $r = 0 \Rightarrow$ $\bar{E}_T > 0$
- more general potential model: $\frac{1}{E} \rightarrow \frac{1}{E - V_0(r) + m + V_S(r)}$

$\hookrightarrow$ sign of $\bar{E}_T$ same as in Bag model!

Chirally Odd GPDs: sign (M.B. + Brian Hannafious)

- relativistic constituent model: spin structure from SU(6) wave functions plus “Melosh rotation”
  - $\hookrightarrow$ $\bar{E}_T > 0$ (B.Pasquini et al.)
  - origin of sign: “Melosh rotation” is free Lorentz boost
  - $\hookrightarrow$ relative sign between upper and lower component same as for free Dirac eq. (bag)
- diquark models: nucleon structure from perturbative splitting of spin $\frac{1}{2}$ ‘nucleon’ into quark & scalar/a-vector diquark: $\bar{E}_T > 0$
  - origin of sign: interaction between q and diquark is point-like
  - $\hookrightarrow$ except when q & diquark at same point, q is noninteracting
  - $\hookrightarrow$ relative sign between upper and lower component same as for free Dirac eq. (bag)
- NJL model (pion): $\bar{E}_T > 0$
  origin of sign: NJL model also has contact interaction!
- lattice QCD (u, d in nucleon; pion): $\bar{E}_T > 0$ (P.Hägler et al.)

Chirally Odd GPDs (magnitude)

- large N_C: $\bar{E}_T^u = \bar{E}_T^d$
- Bag model/potential models: correlation between quark orbit and quark spin same for all quark states (regardless whether $j_z = +\frac{1}{2}$ or $j_z = -\frac{1}{2}$)

$\hookrightarrow$ all quark orbits contribute coherently to $\bar{E}_T$

- compare E (anomalous magnetic moment), where quark orbits with $j_z = +\frac{1}{2}$ and $j_z = -\frac{1}{2}$ contribute with opposite sign

$\hookrightarrow$ E, which describes correlation between quark OAM and nucleon spin smaller than $\bar{E}_T$, which describes correlation between quark OAM and quark spin: $\bar{E}_T > |E|$

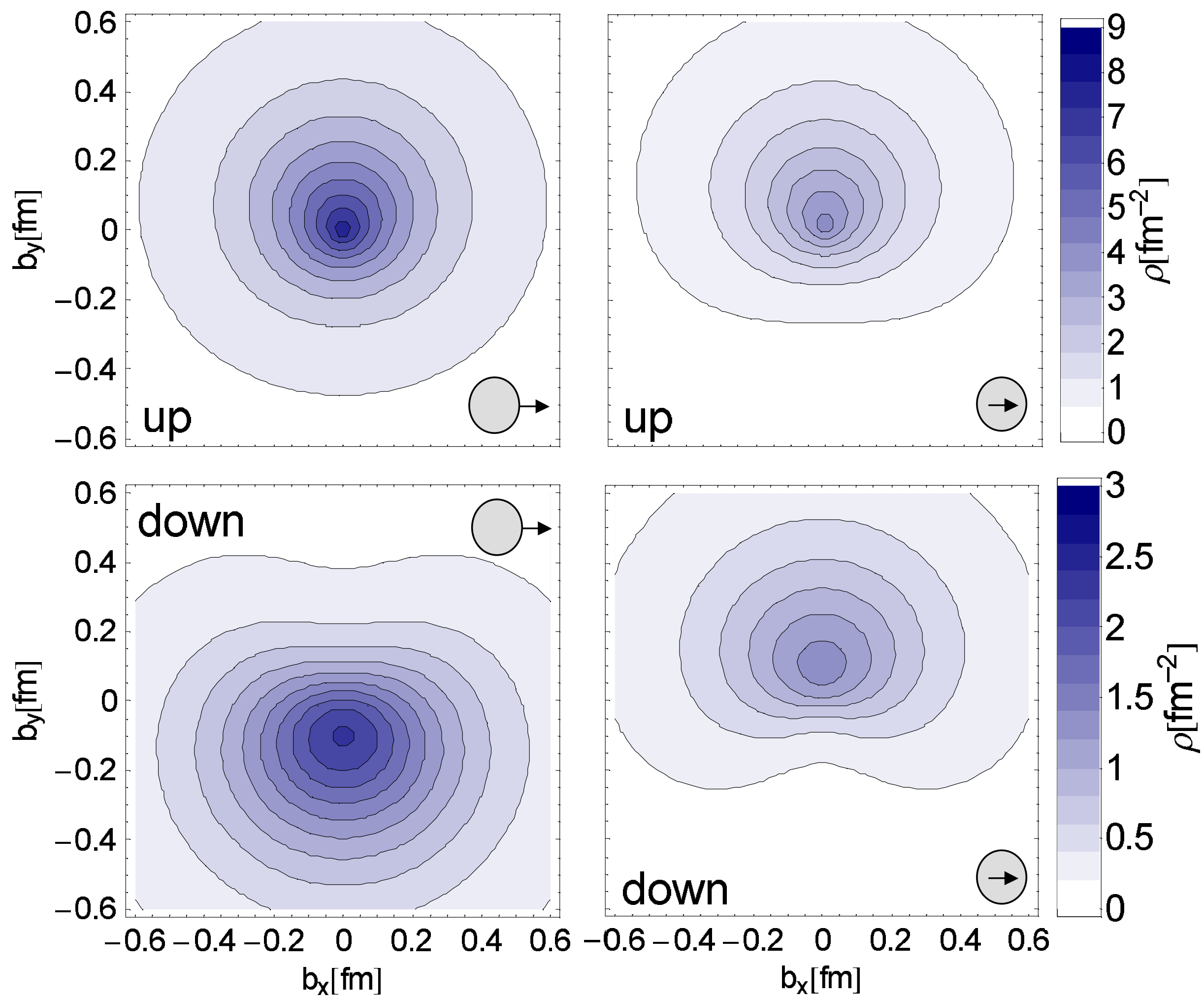
- potential models: $\bar{E}_T \propto$ # of q $\quad \Rightarrow \quad \bar{E}_T^u = 2\bar{E}_T^d$

$\hookrightarrow$ expect $2\bar{E}_T^d > \bar{E}_T^u > \bar{E}_T^d$

- all of the above confirmed in LGT calcs. (e.g. P.Hägler et al.)

IPDs on the lattice (Hägler et al.)

- lowest moment of distribution of unpol. quarks in ⊥ pol. proton (left) and of ⊥ pol. quarks in unpol. proton (right):



Transversity decomposition of J_q

- $J^i = \frac{1}{2}\varepsilon^{ijk}\int d^3x \left[T^{0j}x^k - T^{0k}x^j\right]$
- J_q^x diagonal in transversity, projected with $\frac{1}{2}(1 \pm \gamma^x\gamma_5)$, i.e. one can decompose

$$J_q^x = J_{q,+\hat{x}}^x + J_{q,-\hat{x}}^x$$

where $J_{q,\pm\hat{x}}^x$ is the contribution (to J_q^x) from quarks with positive (negative) transversity

$\hookrightarrow$ derive relation quantifying the correlation between $\perp$ quark spin and angular momentum [M.B., PRD72, 094020 (2006); PLB639, 462 (2006)]

$$\left\langle J_{q,+\hat{y}}^y \right\rangle = \frac{1}{4}\int dx \left[H_T^q(x,0,0) + \bar{E}_T^q(x,0,0)\right] x$$

(note: this relation is not a decomposition of J_q into transversity and orbital)

Summary

- GPDs $\stackrel{FT}{\longleftrightarrow}$ IPDs (impact parameter dependent PDFs)
- $E(x,0,-\boldsymbol{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target

$\hookrightarrow$ origin for deformation: orbital motion of the quarks

$\hookrightarrow$ simple mechanism (attractive FSI) to predict sign of f_{1T}^q

$$f_{1T}^u < 0 \qquad\qquad f_{1T}^d > 0$$

- intuitive explanation for 'Miller-effect': $|\vec{L}_{u/n}| > |\vec{L}_{d/n}|$
- distribution of $\perp$ polarized quarks in unpol. target described by chirally odd GPD $\bar{E}_T^q = 2\bar{H}_T^q + E_T^q$

$\hookrightarrow$ origin: correlation between orbital motion and spin of the quarks

$\hookrightarrow$ attractive FSI $\Rightarrow$ measurement of $h_1^\perp$ (DY,SIDIS) provides information on $\bar{E}_T^q$ and hence on spin-orbit correlations

- expect:

$$h_1^{\perp,q} < 0 \qquad\qquad |h_1^{\perp,q}| > |f_{1T}^q|$$