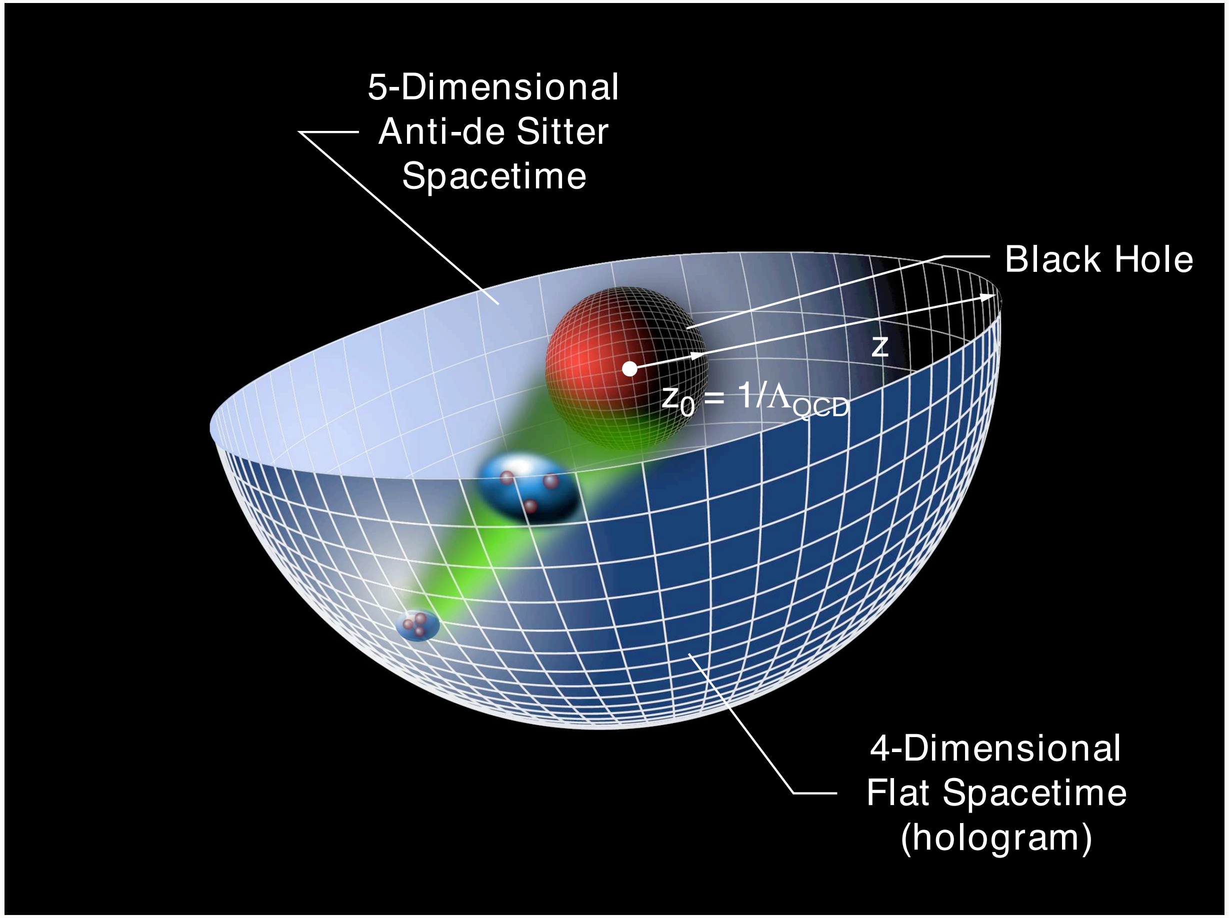
Exclusive Processes and AdS/QCD



Stan Brodsky, SLAC

Jlab Exclusive Processes

May 22, 2007

QCD Lagrangian

gluon dynamics — quark kinetic energy + quark-gluon dynamics — mass term

$$L_{QCD} = -\frac{1}{4g^2} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{nf} i \overline{\psi_f} D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{nf} m_f \overline{\psi_f} \psi_f$$

QCD color charge — field strength tensor — covariant derivative — quark field

Yang-Mills Gauge Principle: Invariance under Color Rotation and Phase Change at Every Point of Space and Time

Dimensionless Coupling
Renormalizable
Asymptotic Freedom
Color Confinement

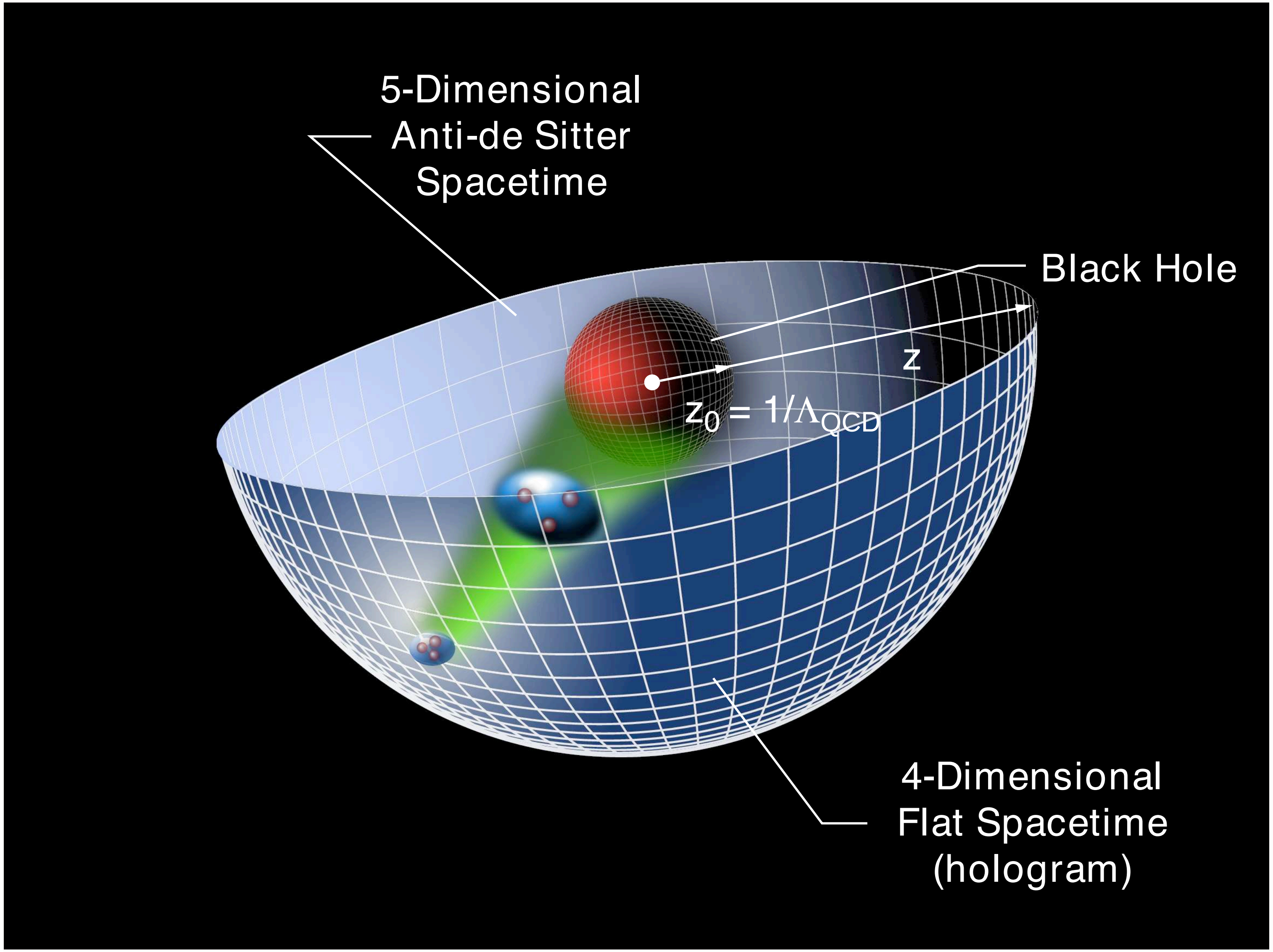
Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**

- **Analogous to the Schrodinger Equation for Atomic Physics**

- *AdS/QCD Holographic Model*

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

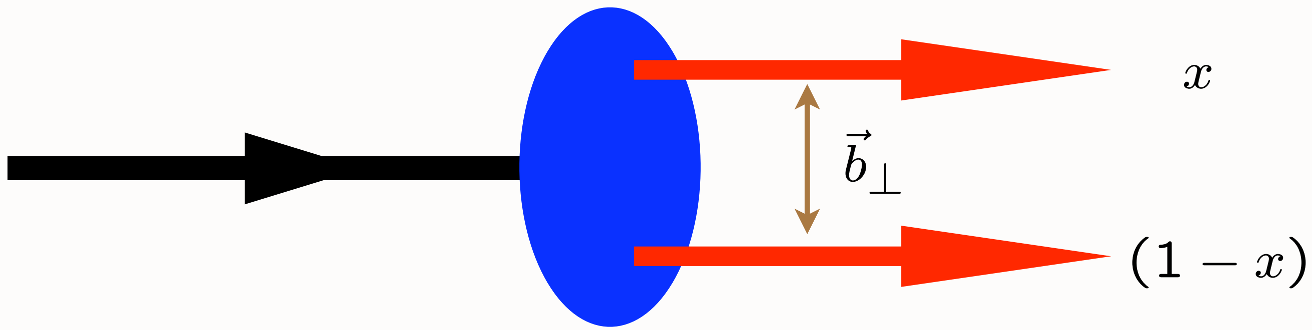
Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic radial equation: *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

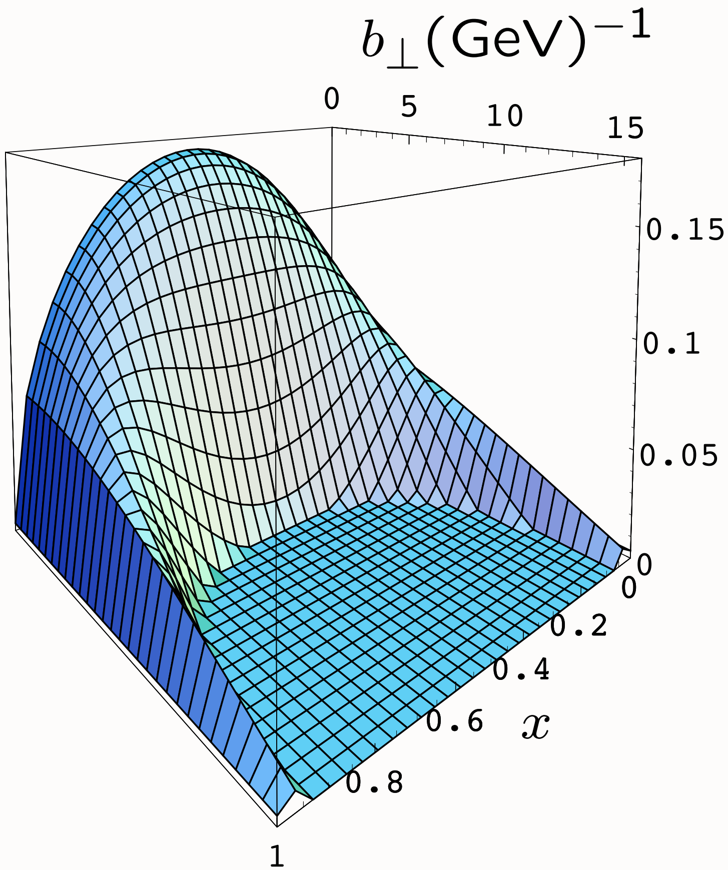
$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

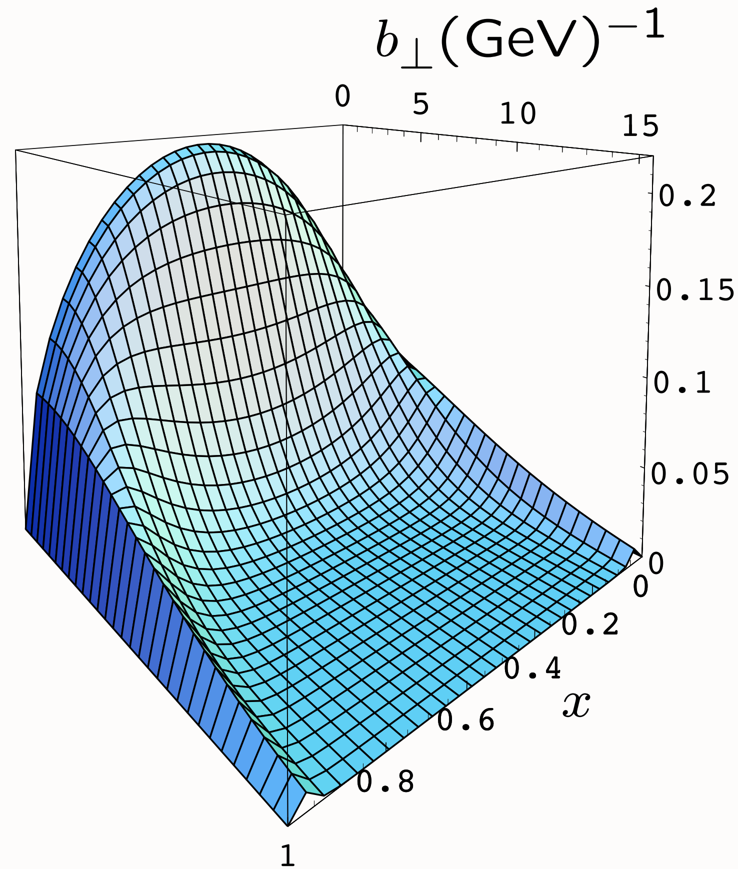
$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$



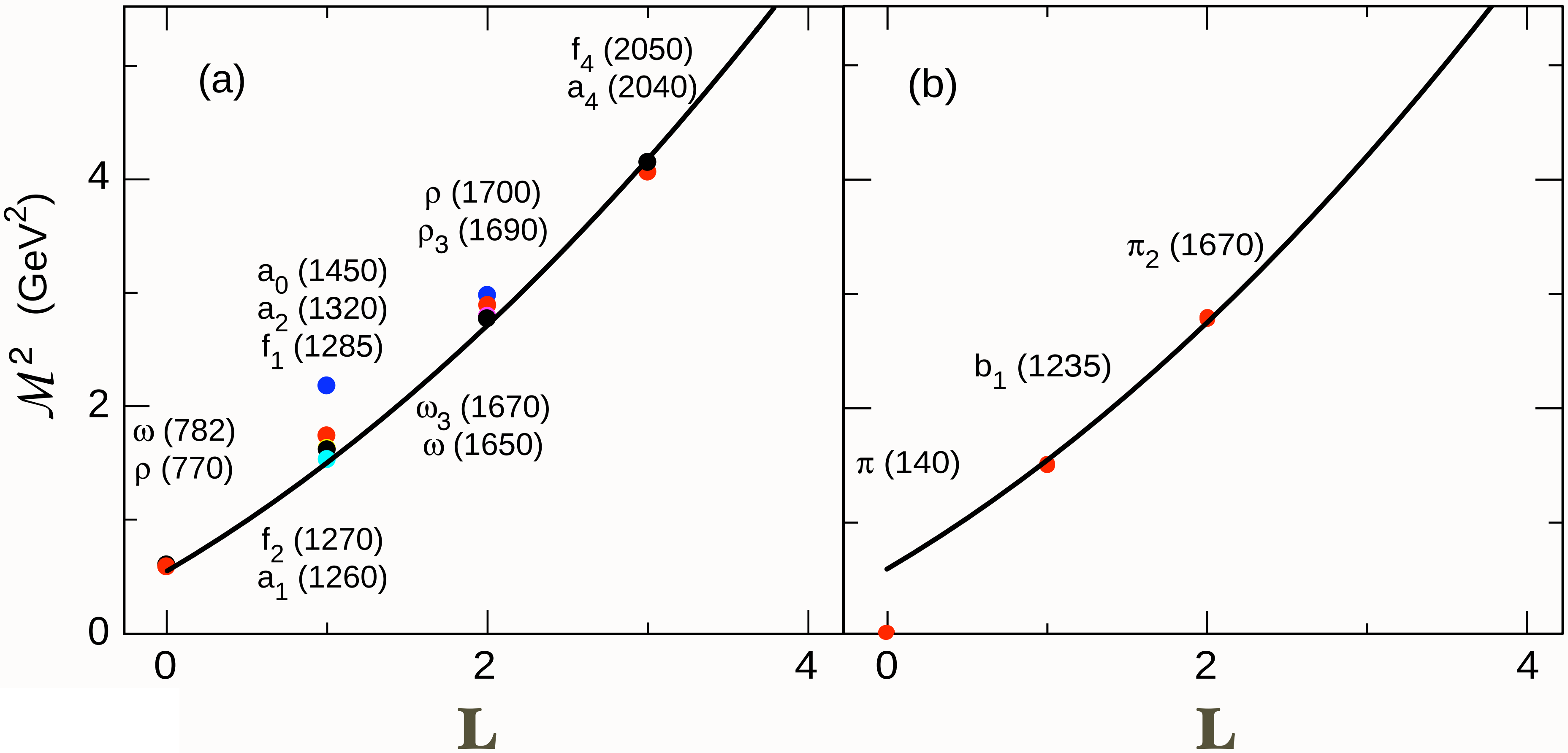
$$\Lambda_{\rm QCD} = 0.32 \text{ GeV}$$

Truncated Space



$$\kappa = 0.76 \text{ GeV}$$

Harmonic Oscillator



Light meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Guy de Teramond
SJB

AdS/CFT: Anti-de Sitter Space ↔ Conformal Field Theory

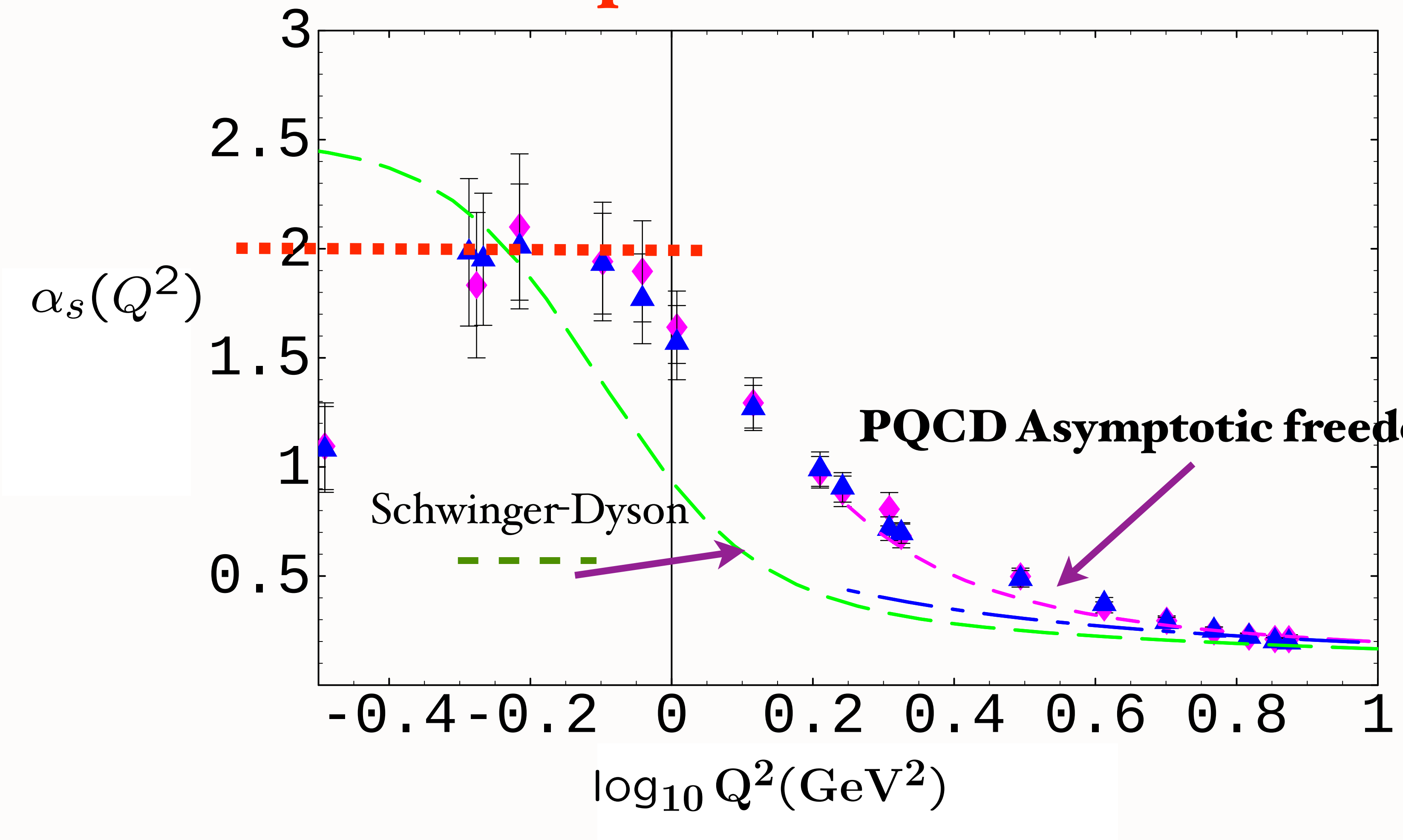
Maldacena:

Map $AdS_5 \times S_5$ to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window:** $\alpha_s(Q^2) \simeq \mathrm{const\ at\ small}\ Q^2$
- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**
- **Evidence for IR Fixed Point**

Conformal window
Infrared fixed-point

$$\beta(Q^2) = \frac{d\alpha_s(Q^2)}{d\log Q^2} \to 0$$



Shirkov
Gribov
Dokshitser
Siminov
Maxwell
Cornwall

lattice: Furui, Nakajima (MILC)

DSE: Alkofer, Fischer, von Smekal et al.

Define QCD Coupling from Observable

Grunberg

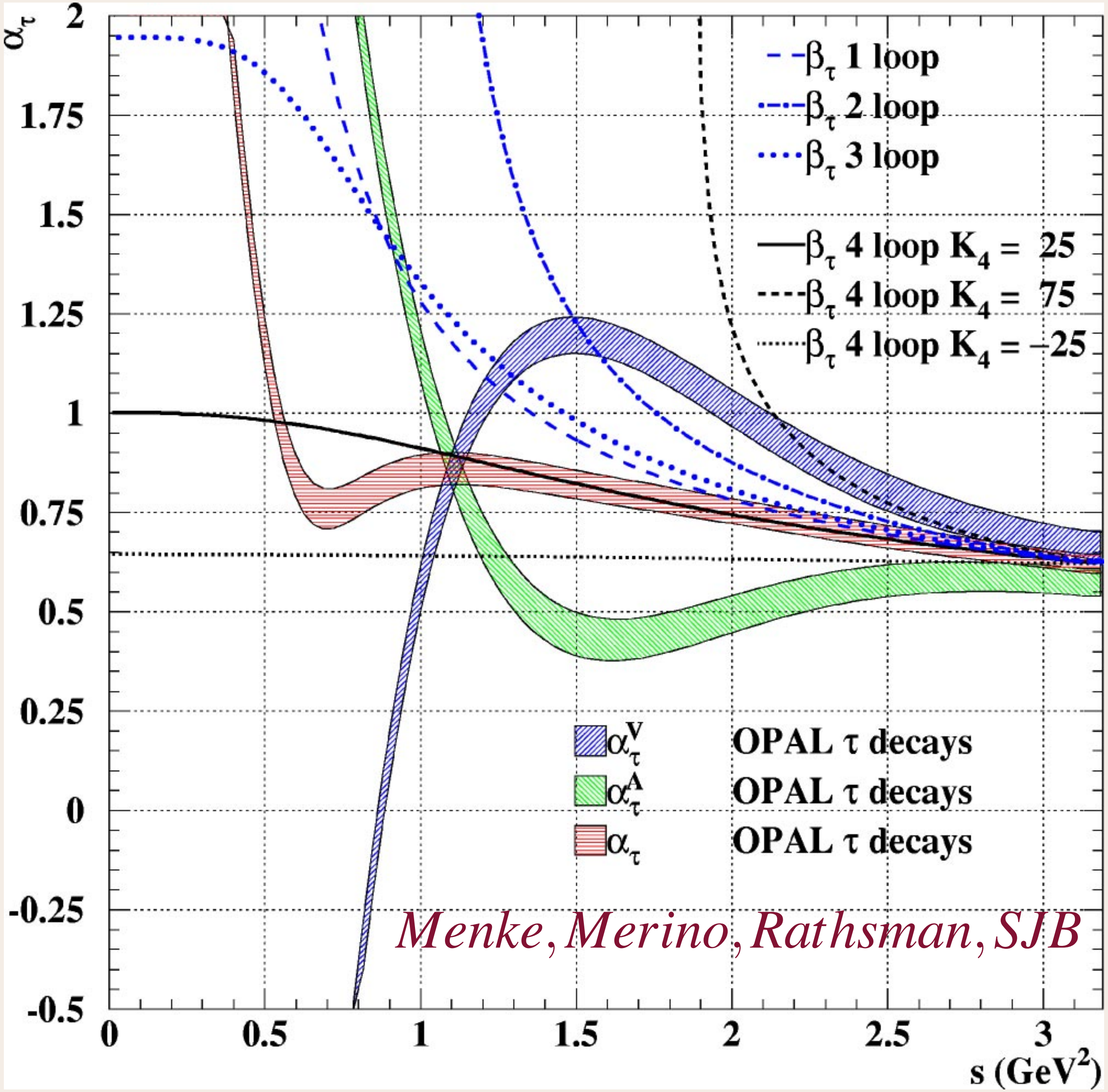
$$R_{e^+e^- \to X}(s) \equiv 3\Sigma_q e_q^2 \; [1 + \frac{\alpha_R(s)}{\pi}]$$

$$\Gamma(\tau \to X e \nu)(m_\tau^2) \equiv \Gamma_0(\tau \to u \bar{d} e \nu) \times [1 + \frac{\alpha_\tau(m_\tau^2)}{\pi}]$$

Effective Charges: analytic at quark mass thresholds, finite at small momenta

Deur et al: Effective Charge from Bjorken Sum Rule

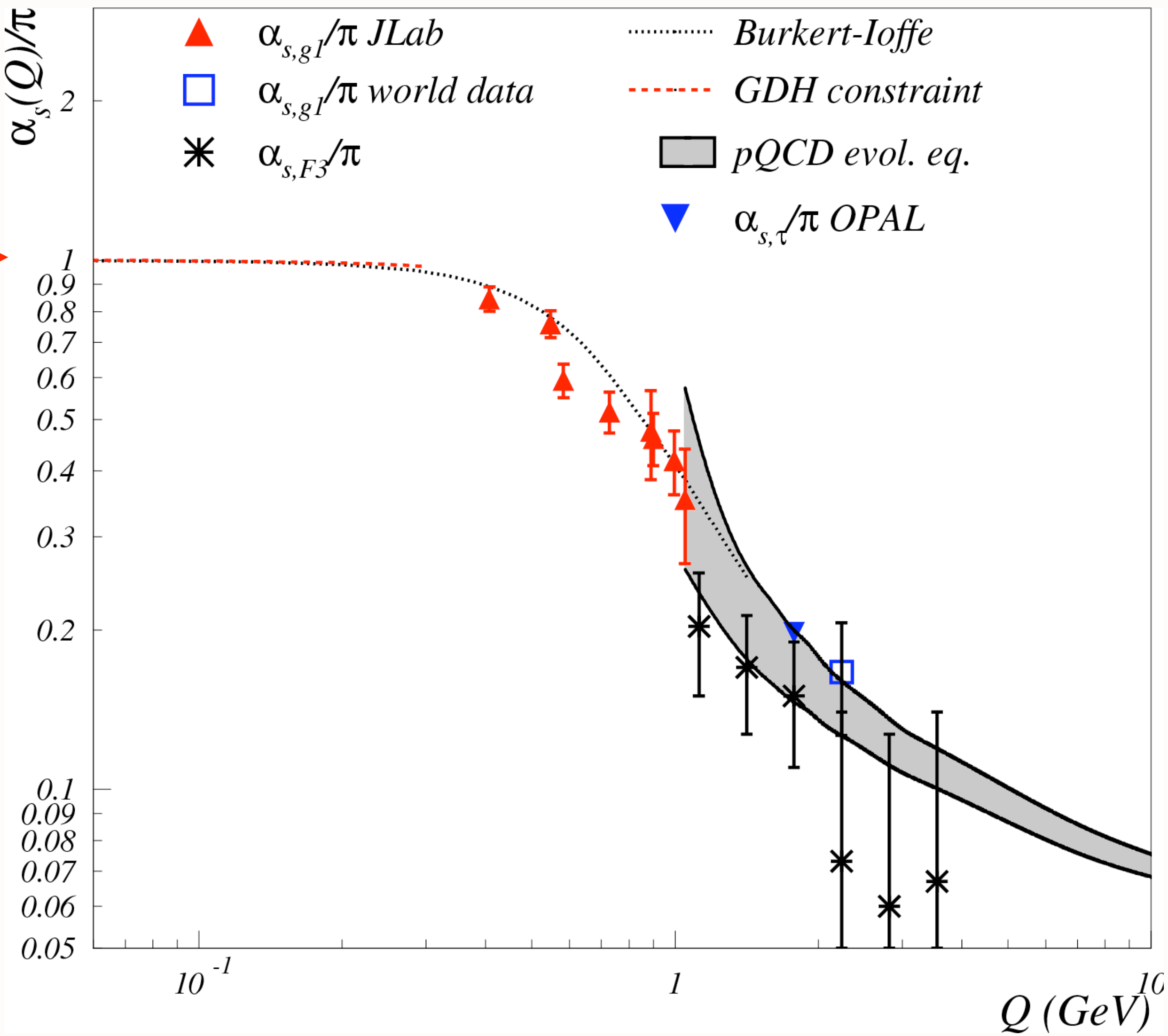
QCD Effective Coupling from *hadronic τ decay*



Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6}[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi}]$$

GDH constraint →



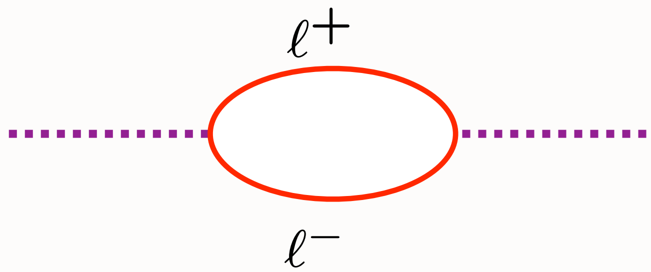
$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell Mann-Low Effective Charge for QED

IR Fixed-Point for QED!

QED vacuum polarization



$t = -Q^2 < 0$

(t spacelike)

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}\left[\frac{5}{3} - \frac{4m^2}{Q^2} - \left(1 - \frac{2m^2}{Q^2}\right)\sqrt{1 + \frac{4m^2}{Q^2}}\,\log\frac{1+\sqrt{1+\frac{4m^2}{Q^2}}}{\left|1-\sqrt{1+\frac{4m^2}{Q^2}}\right|}\right]$$

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}\frac{\log Q^2}{m^2} \qquad Q^2 >> 4M^2$$

$$\beta = \frac{d(\frac{\alpha}{4\pi})}{d\log Q^2} = \frac{4}{3}\left(\frac{\alpha}{4\pi}\right)^2 n_\ell > 0 \qquad \alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

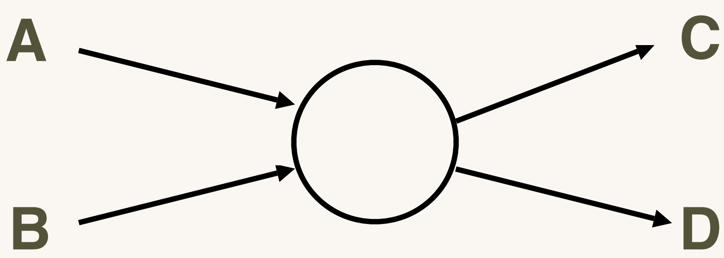
$$\Pi(Q^2) = \frac{\alpha(0)}{15\pi}\frac{Q^2}{m^2} \qquad Q^2 << 4M^2$$

Serber-Uehling

$$\beta \propto \frac{Q^2}{m^2}$$

vanishes at small momentum transfer

Constituent Counting Rules



$$\frac{d\sigma}{dt}(s,t) = \frac{F(\theta_{\text{cm}})}{s^{[n_{\text{tot}}-2]}} \qquad s = E^2_{\text{cm}}$$

$$F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H - 1}$$

$$n_{tot} = n_A + n_B + n_C + n_D$$

Fixed t/s or $\cos\theta_{cm}$

Farrar & sjb; Matveev, Muradyan, Tavkhelidze

Conformal symmetry and PQCD predict leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

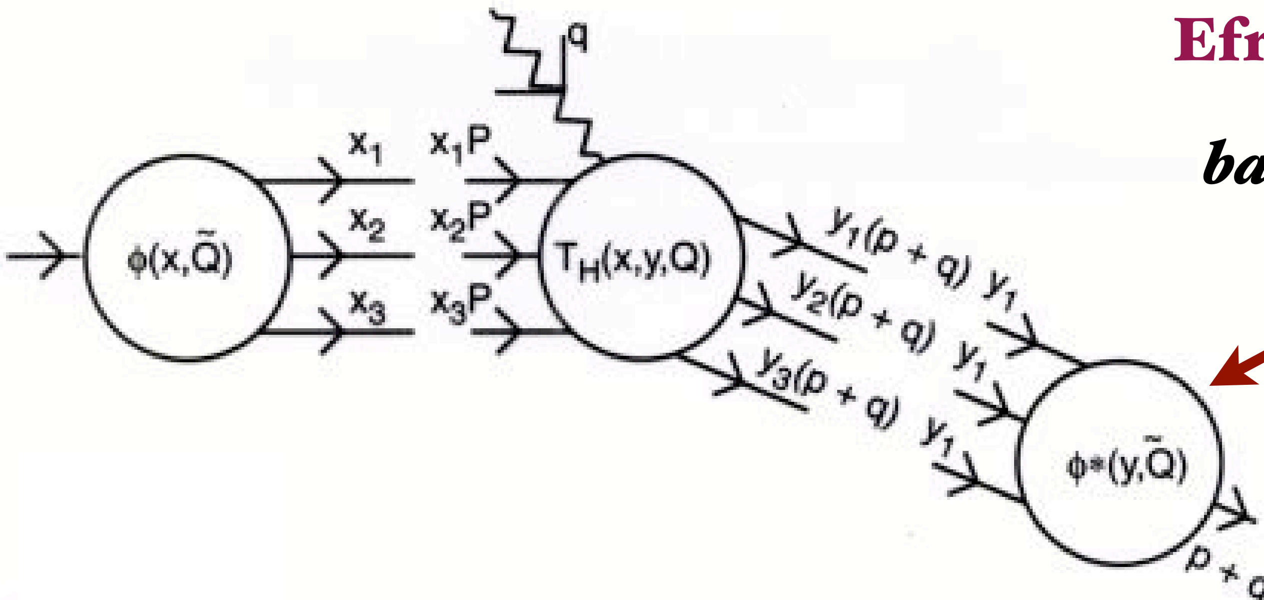
Nonperturbative derivation from AdS/CFT

Polchinski & Strassler, de Teramond and sjb; Grigorian and Radyushkin

Many new J-Lab (12), J-PARC, GSI, Belle, Babar tests

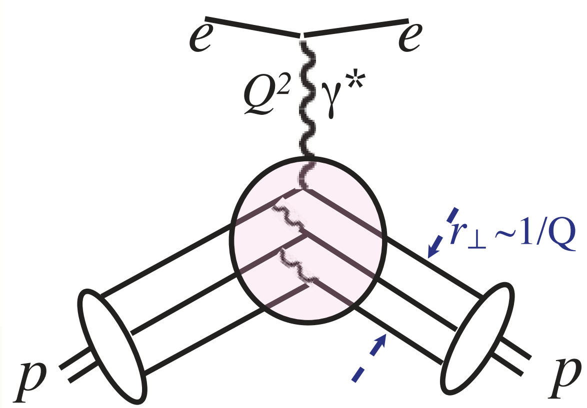
Leading-Twist PQCD Factorization for form factors, exclusive amplitudes

Lepage, sjb
Efremov, Radyushkin



baryon distribution amplitude

$$M = \int \Pi dx_i dy_i \phi_F(x_i, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \times \phi_I(y_i, \tilde{Q})$$



If $\alpha_s(\tilde{Q}^2) \simeq$ constant

$Q^4 F_1(Q^2) \simeq$ constant

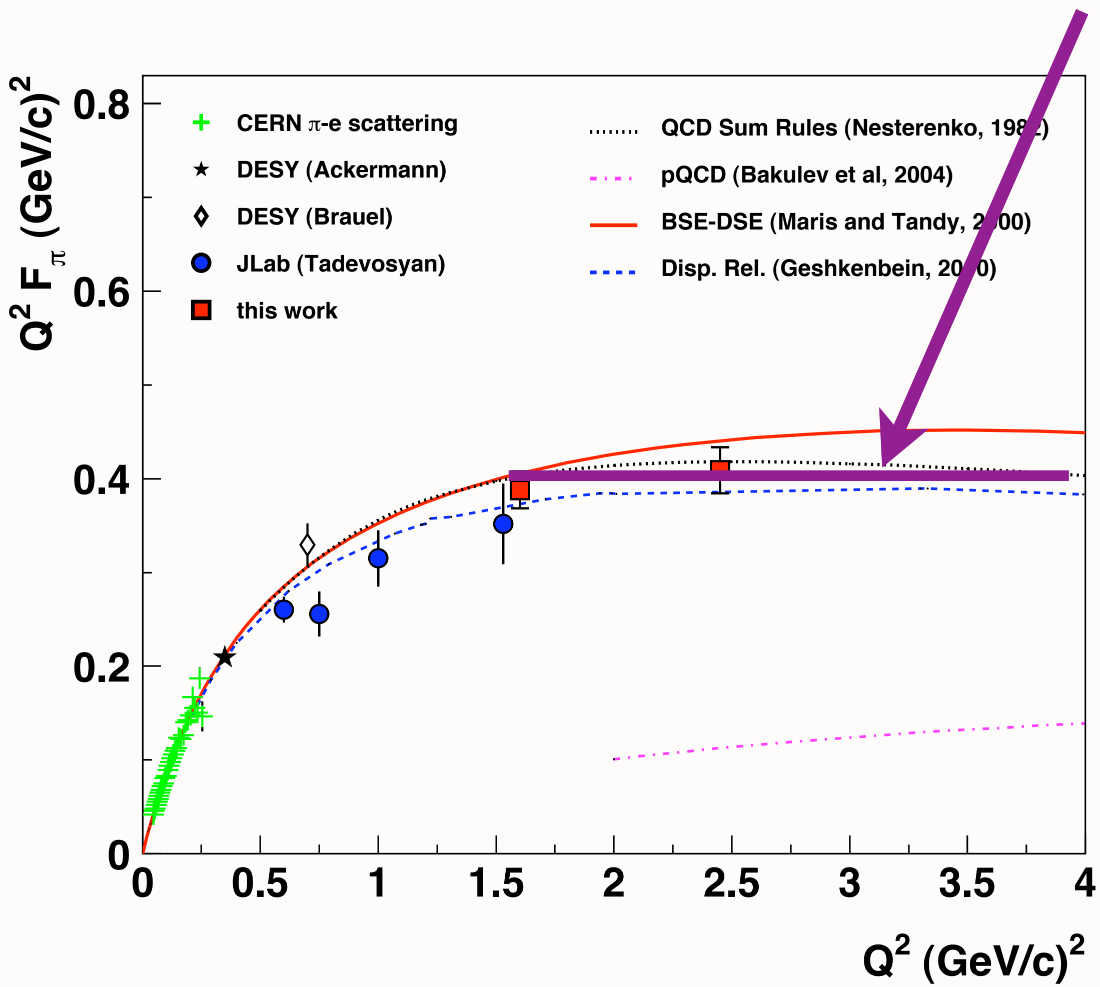
Hadron Distribution Amplitudes

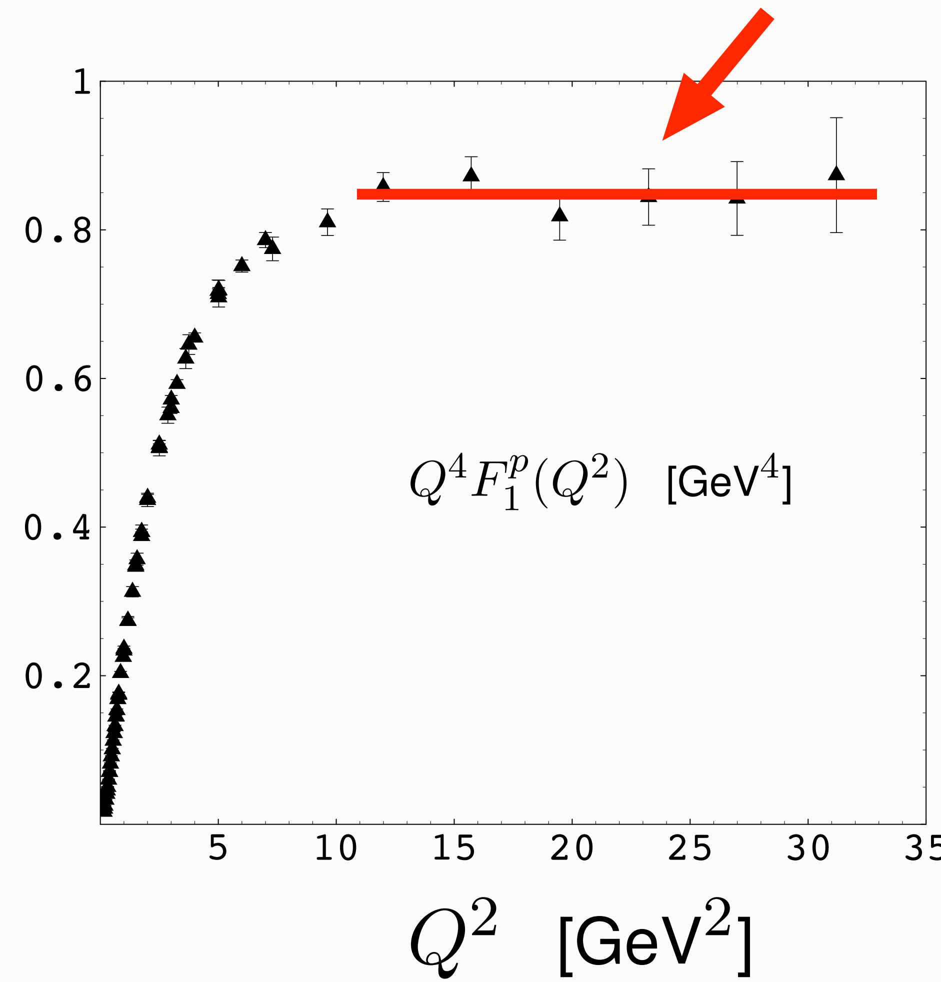
$$\phi(x_i, Q) \equiv \Pi_{i=1}^{n-1} \int^Q d^2\vec{k}_\perp \ \psi_n(x_i, \vec{k}_{\perp i})$$

- Fundamental measure of valence wavefunction
- Gauge Invariant (includes Wilson line)
- Evolution Equations, OPE — ERBL
- Conformal Expansion
- Hadronic Input in Factorization Theorems

Conformal behavior: $Q^2 F_\pi(Q^2) \to$ const

$Q^4 F_1(Q^2) \to$ const





Determination of the Charged Pion Form Factor at Q2=1.60 and 2.45 (GeV/c)2.
By Fpi2 Collaboration (T. Horn *et al.*). Jul 2006. 4pp.
e-Print Archive: **nucl-ex/0607005**

G. Huber

Generalized parton distributions from nucleon form-factor data.
M. Diehl (**DESY**) , **Th. Feldmann** (**CERN**) ,
R. Jakob, **P. Kroll** (**Wuppertal U.**) .
DESY-04-146, CERN-PH-04-154, WUB-04-08, Aug 2004. 68pp.
Published in Eur.Phys.J.C39:1-39,2005
e-Print Archive: **hep-ph/0408173**

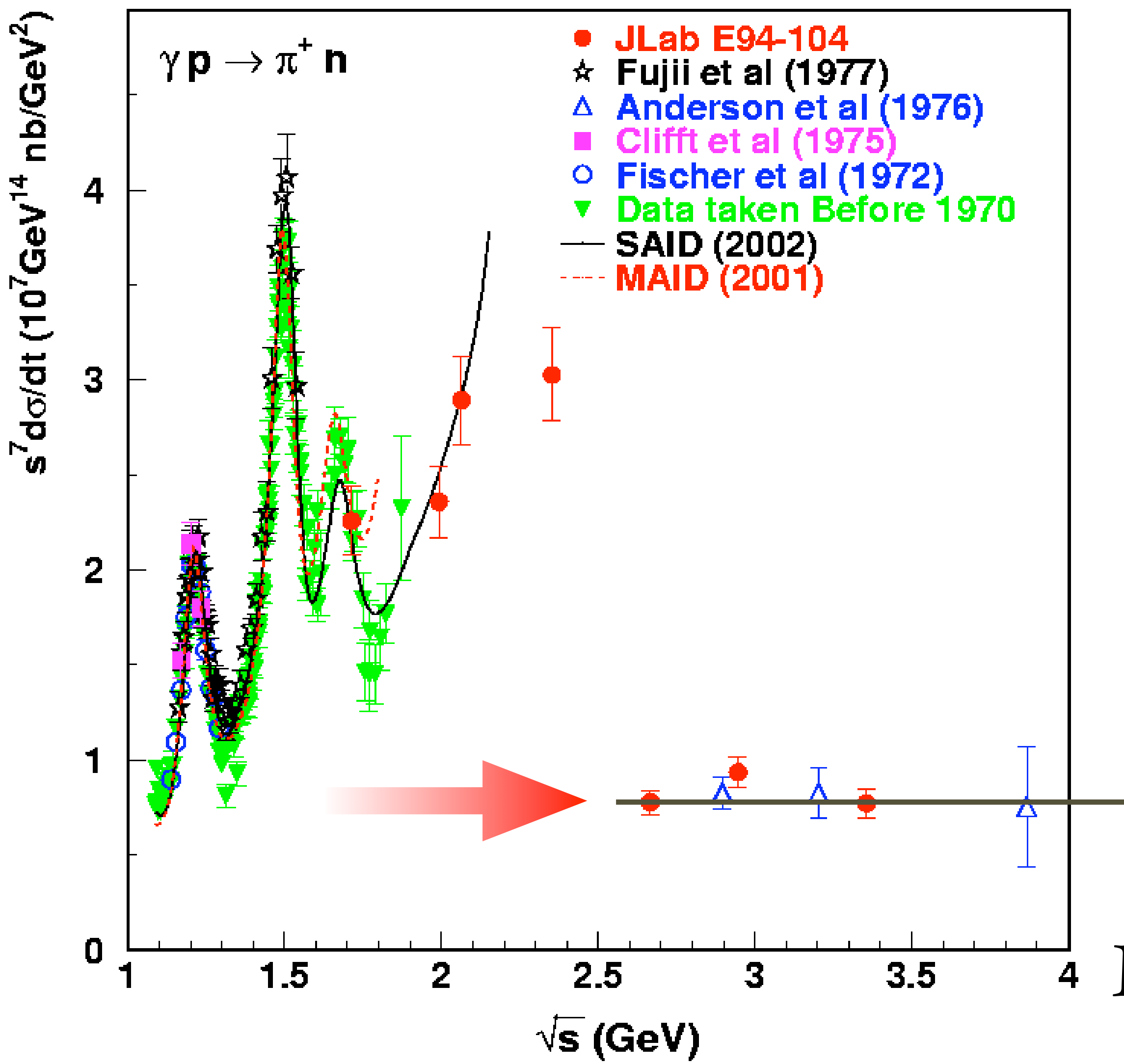
Features of Hard Exclusive Processes in PQCD

- Factorization of perturbative hard scattering subprocess amplitude and nonperturbative distribution amplitudes $M = \int T_H \times \Pi \phi_i$
- **Dimensional counting rules reflect conformal invariance**: $M \sim \frac{f(\theta_{CM})}{Q^{N_{tot}-4}}$
- Hadron helicity conservation: $\sum_{initial} \lambda_i^H = \sum_{final} \lambda_j^H$ Lepage, sjb
- Color transparency Mueller, sjb
- Hidden color Ji, Lepage, sjb
- Evolution of Distribution Amplitudes Lepage, sjb; Efremov, Radyushkin

Test of PQCD Scaling

Constituent counting rules

Farrar, sjb; Muradyan, Matveev, Tavkelidze



$s^7 d\sigma/dt(\gamma p \to \pi^+ n) \sim const$

fixed θ_{CM} scaling

PQCD and AdS/CFT:

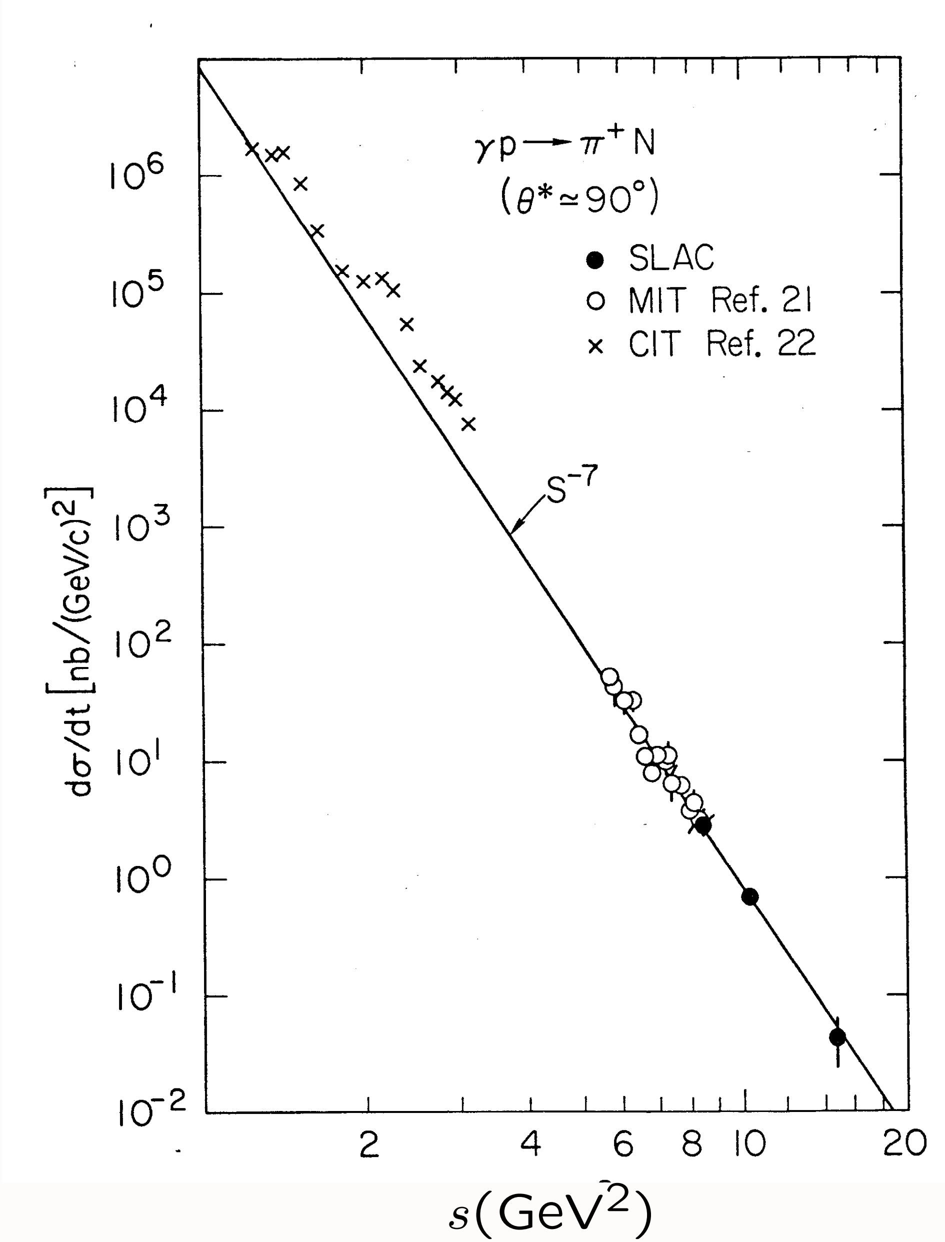
$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B \to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

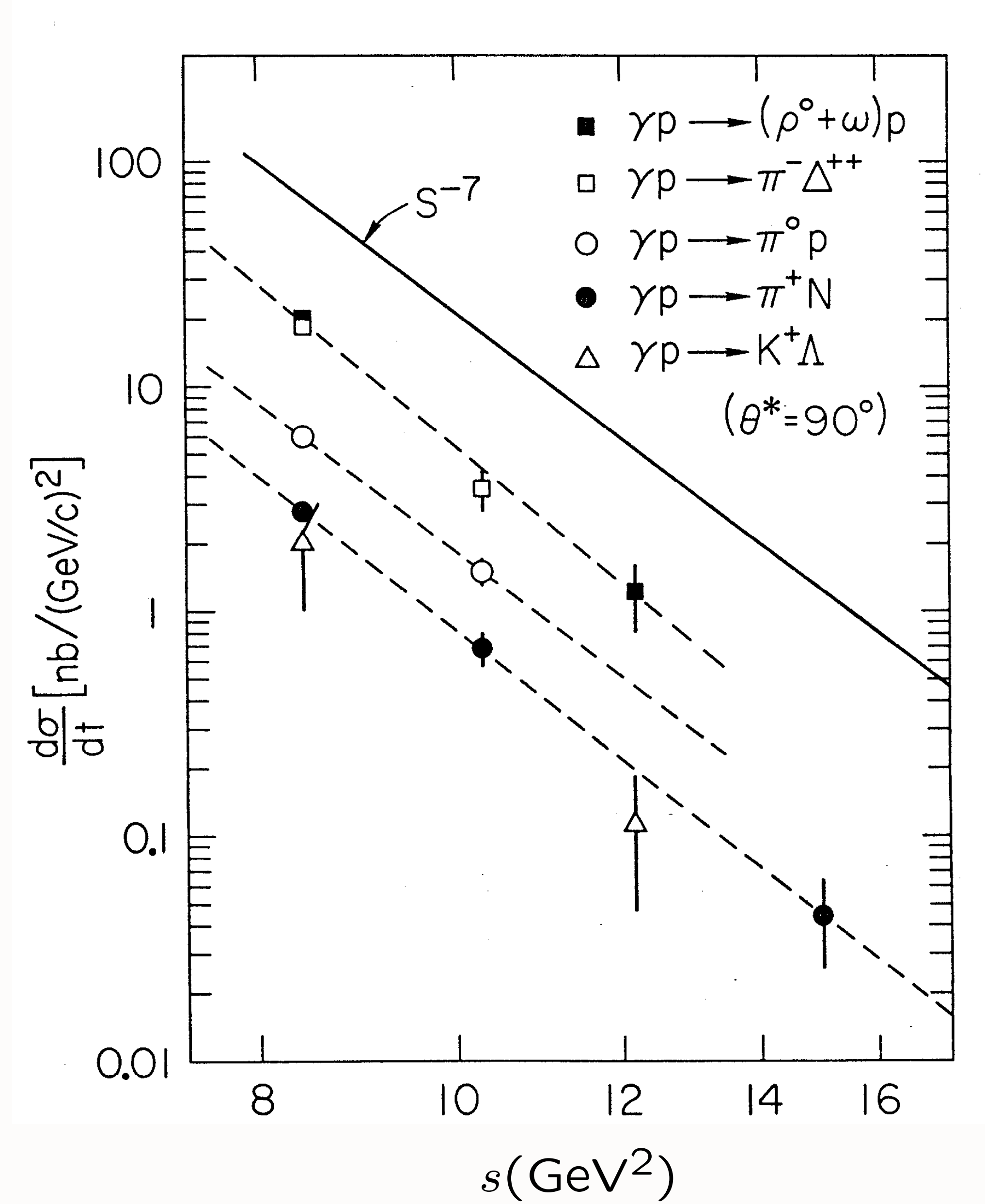
$$s^7\frac{d\sigma}{dt}(\gamma p \to \pi^+ n) = F(\theta_{CM})$$

$$n_{tot} = 1+3+2+3 = 9$$

No sign of running coupling

Conformal invariance

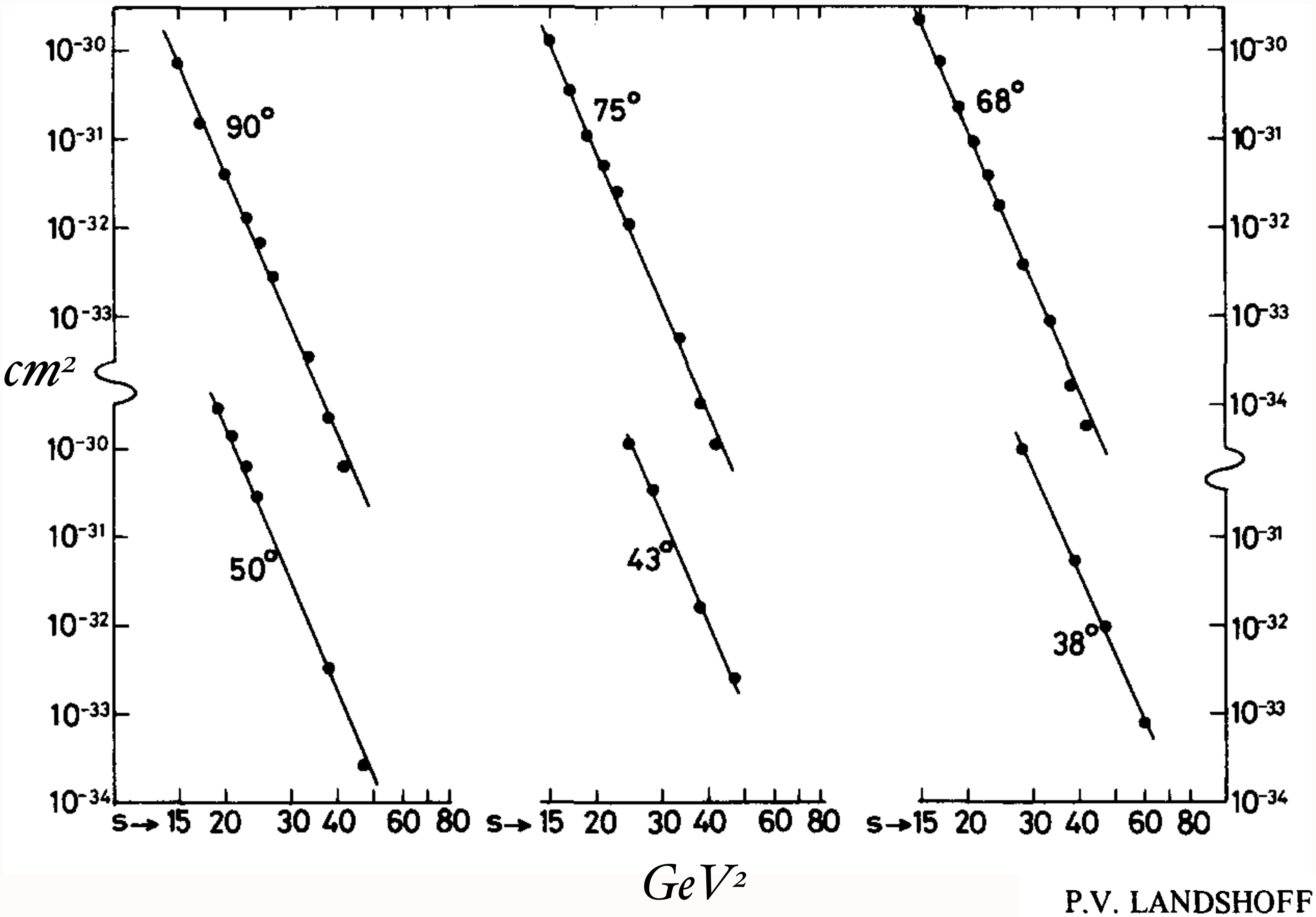




Conformal Invariance:

$$\frac{d\sigma}{dt}(\gamma p \to MB) = \frac{F(\theta_{cm})}{s^7}$$

Quark-Counting : $\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{CM})}{s^{10}}$ $n = 4 \times 3 - 2 = 10$



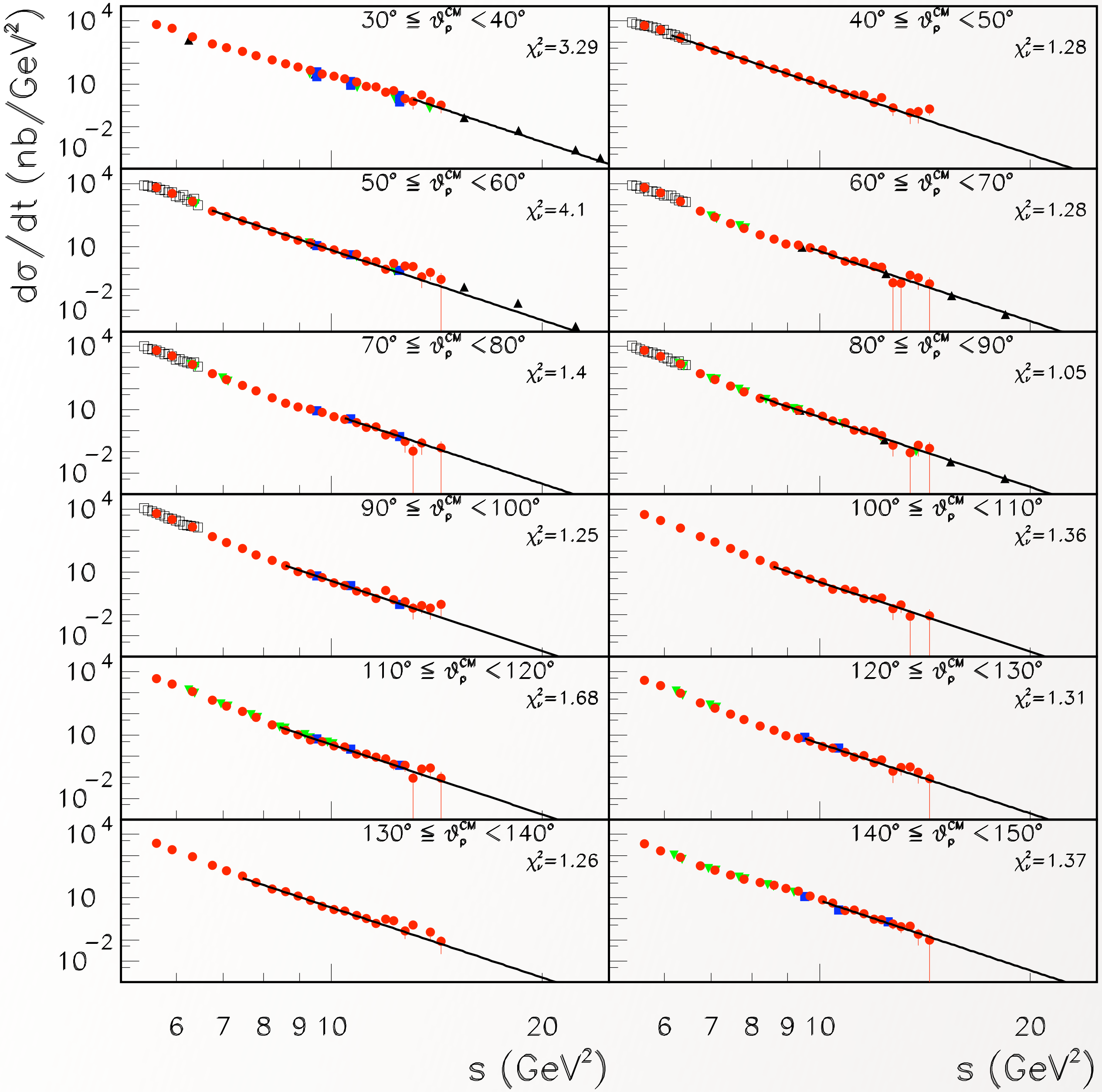
Best Fit

$n = 9.7 \pm 0.5$

Reflects underlying conformal scale-free interactions

P.V. LANDSHOFF and J.C. POLKINGHORNE

Deuteron Photodisintegration



J-Lab

PQCD and AdS/CFT:

$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B\to C+D) =$$
$$F_{A+B\to C+D}(\theta_{CM})$$

$$s^{11}\frac{d\sigma}{dt}(\gamma d\to np) = F(\theta_{CM})$$

$$n_{tot}-2 =$$
(1 + 6 + 3+ 3) - 2 = 11

Reflects conformal invariance

Check of CCR

Fit of $d\sigma/dt$ data for the central angles and $P_T \geq 1.1$ GeV/c with

$A\ s^{-11}$

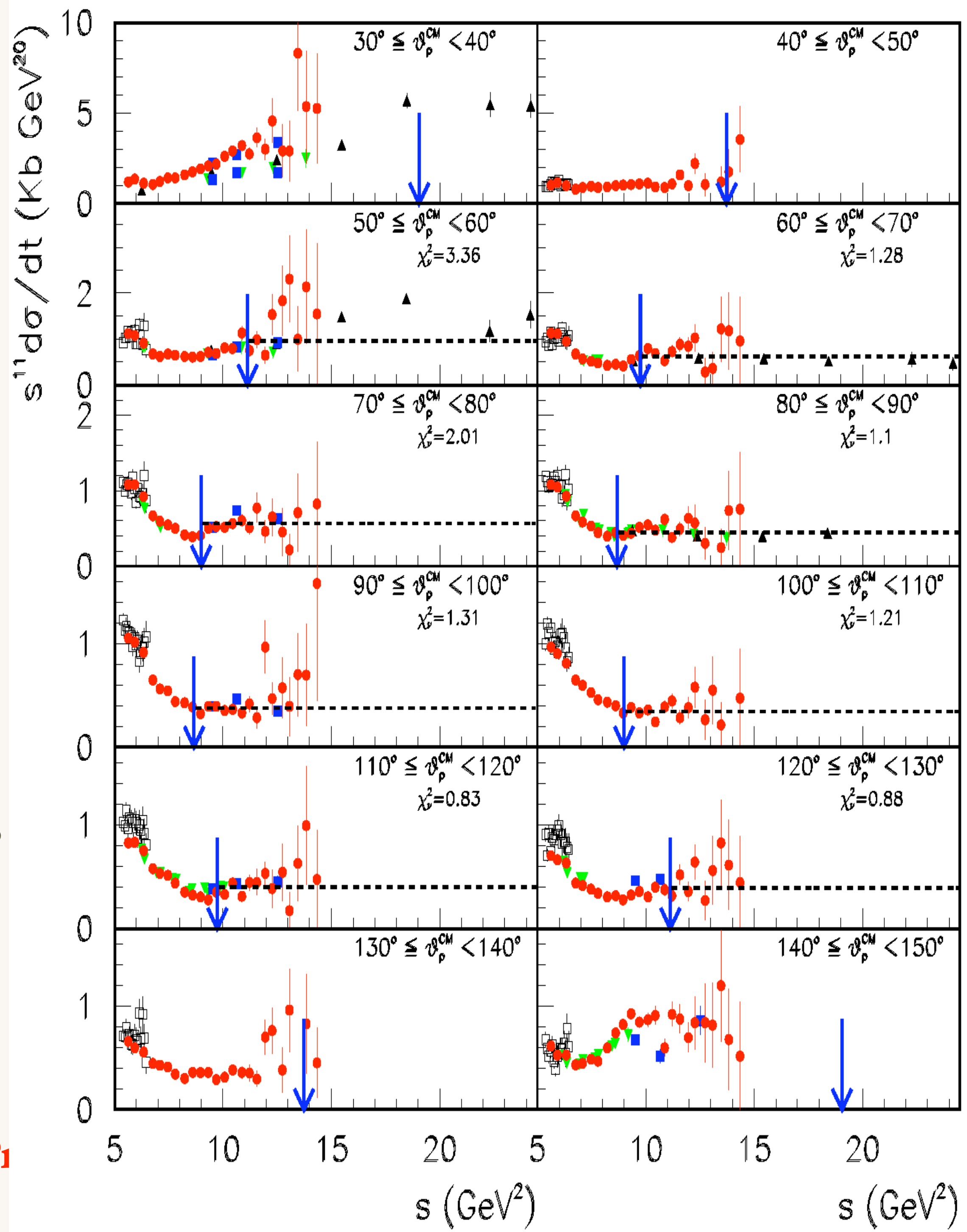
For all but two of the fits

$\chi^2 \leq 1.34$

- Better χ^2 at 55° and 75° if different data sets are renormalized to each other
- No data at $P_T \geq 1.1$ GeV/c at forward and backward angles
- Clear s^{-11} behaviour for last 3 points at 35°

Data consistent with CCR

P.Rossi et al, P.R.L. 94, 012301 (2005)



- Remarkable Test of Quark Counting Rules
- Deuteron Photo-Disintegration $\gamma d \to np$
- $$\frac{d\sigma}{dt} = \frac{F(t/s)}{s^{n_{tot}-2}}$$
- $$n_{tot} = 1 + 6 + 3 + 3 = 13$$

Scaling characteristic of
scale-invariant theory at short distances

Conformal symmetry

Hidden color: $\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$

at high p_T

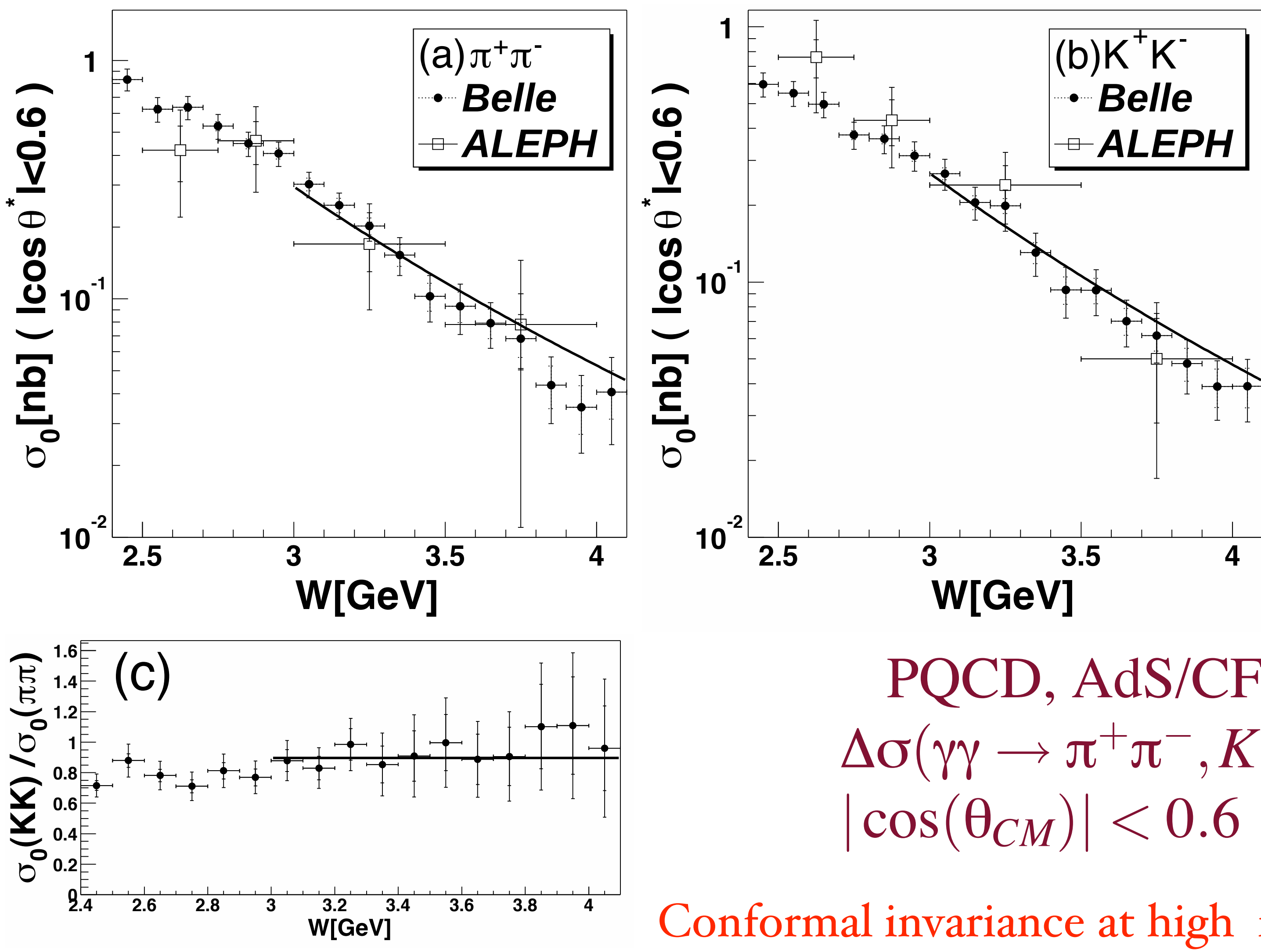


Fig. 5. Cross section for (a) $\gamma\gamma\to\pi^+\pi^-$, (b) $\gamma\gamma\to K^+K^-$ in the c.m. angular region $|\cos\theta^*| < 0.6$ together with a W^{-6} dependence line derived from the fit of $s|R_M|$. (c) shows the cross section ratio. The solid line is the result of the fit for the data above 3 GeV. The errors indicated by short ticks are statistical only.

Two-Photon Reactions

Hard Exclusive Processes: Fixed angle

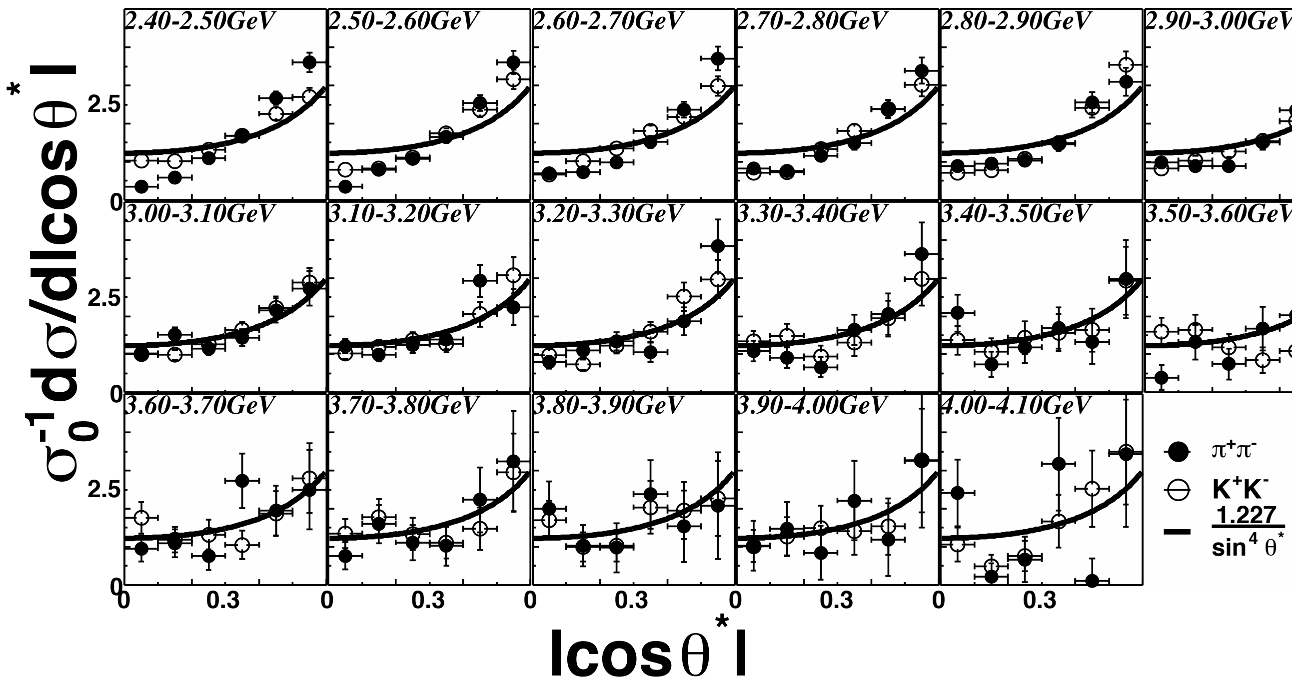
PQCD, AdS/CFT:

$$\Delta\sigma(\gamma\gamma \to \pi^+\pi^-, K^+, K^-) \sim 1/W^6$$

$$|\cos(\theta_{CM})| < 0.6$$

Conformal invariance at high momentum transfers!

PQCD: $$\frac{d\sigma}{d|\cos\theta^*|}(\gamma\gamma \to M^+M^-) \approx \frac{16\pi\alpha^2}{s}\frac{|F_M(s)|^2}{\sin^4\theta^*},$$

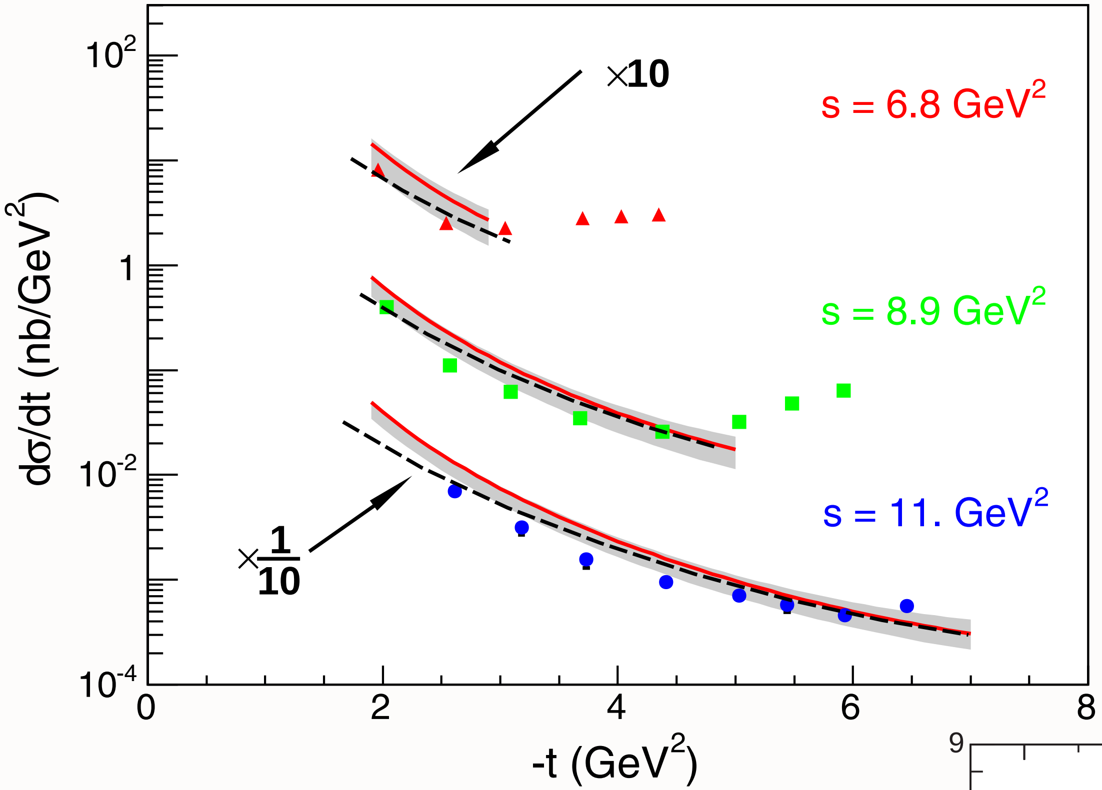


4. Angular dependence of the cross section, $\sigma_0^{-1} d\sigma/d|\cos\theta^*|$, for the $\pi^+\pi^-$(closed circles) and K^+K^-(open circles) processes. The curves are $1.227 \times \sin^{-4}\theta^*$. The errors are statistical only.

Measurement of the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to K^+K^-$ processes at energies of 2.4–4.1 GeV

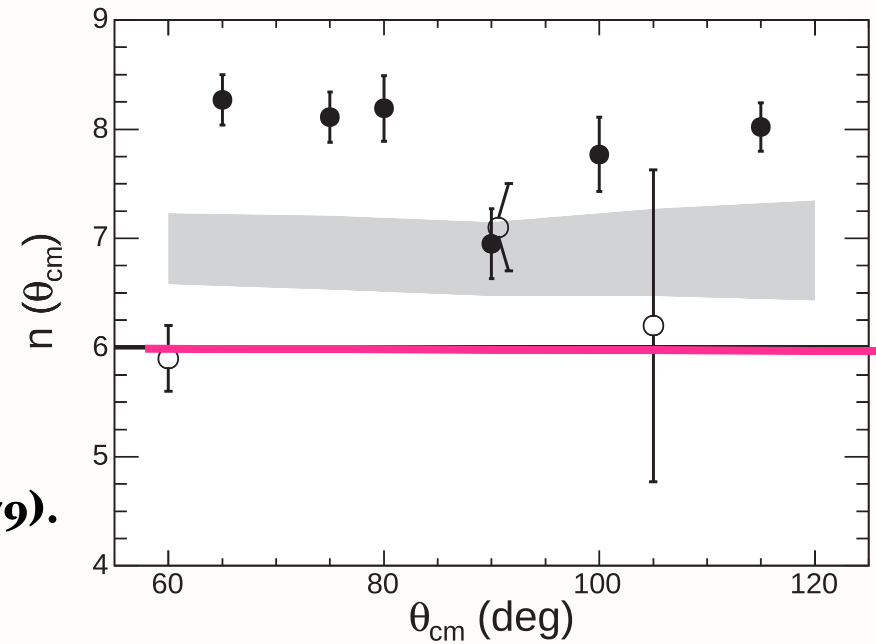
Belle Collaboration

Compton-Scattering Cross Section on the Proton at High Momentum Transfer



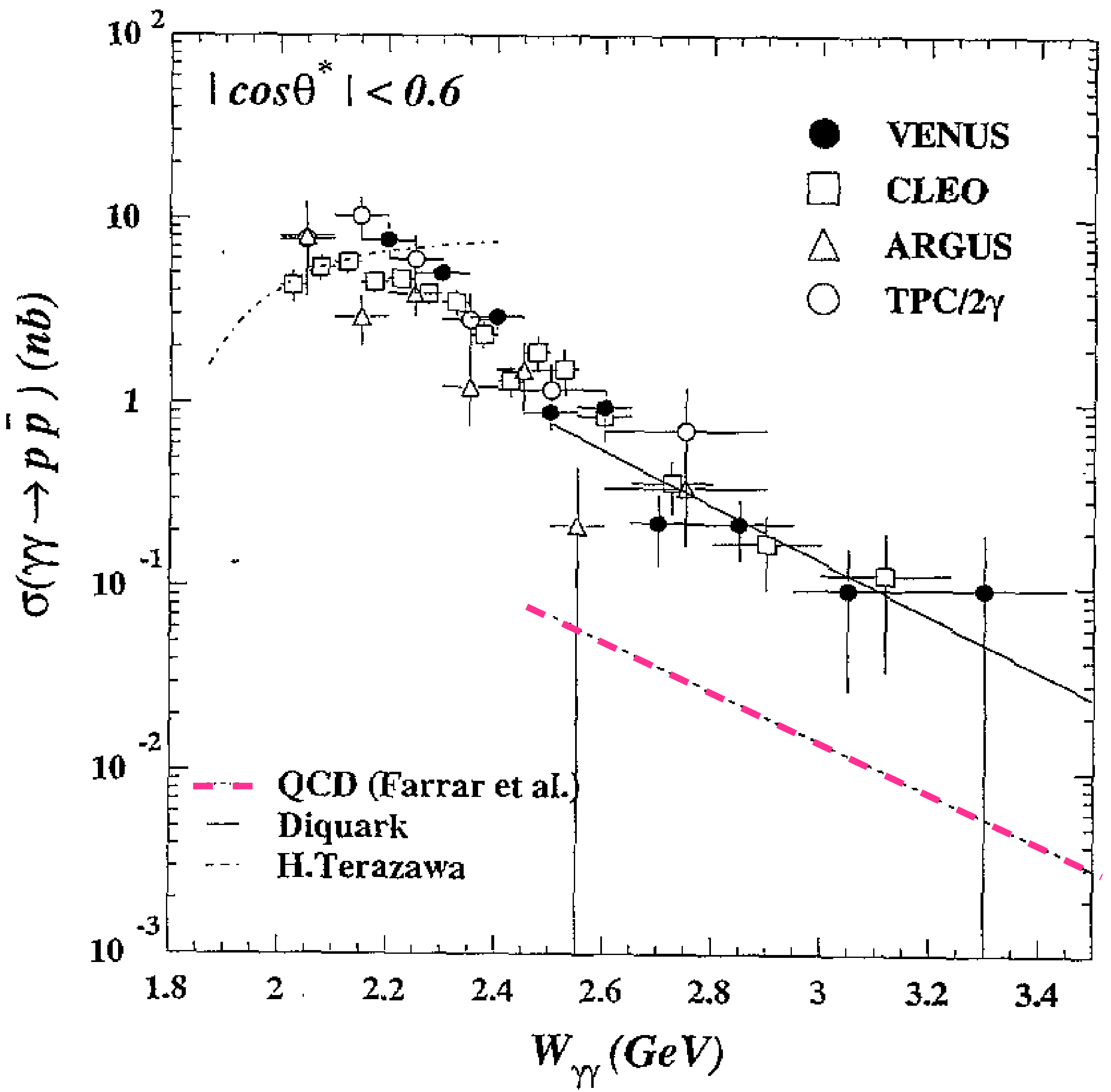
Jefferson Lab
Hall A
Collaboration

Compton at fixed angles falls faster than photoproduction!



Open points: Cornell measurement
M. A. Shupe et al., Phys. Rev. D 19, 1921 (1979).

VENUS Collaboration / Physics Letters B 407 (1997) 185–192



Power fall-off consistent with PQCD

Why do dimensional counting rules work so well?

- **PQCD predicts log corrections from powers of α_s, logs, pinch contributions** Lepage, sjb; Efremov, Radyushkin; Landshoff; Mueller, Duncan

- **DSE: QCD coupling (mom scheme) has IR Fixed point** Alkofer, Fischer, von Smekal et al.

- **Lattice results show similar flat behavior** Furui, Nakajima

- **PQCD exclusive amplitudes dominated by integration regime where α_s is large and flat**

Conformal symmetry: Template for QCD

- Take conformal symmetry as initial approximation; then correct for non-zero beta function and quark masses

- Eigensolutions of ERBL evolution equation for distribution amplitudes

V. Braun et al;
Frishman, Lepage, Sachrajda, sjb

- Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation

- Fix Renormalization Scale (BLM)

- Use AdS/CFT

Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances

- de Teramond, sjb: AdS/QCD Holographic Model: Initial "semi-classical" approximation to QCD. Predict light-quark hadron spectroscopy, form factors.

- Karch, Katz, Son, Stephanov: Harmonic Oscillator Confinement

- Mapping of AdS amplitudes to 3+ 1 Light-Front equations, wavefunctions

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing H^{LF}_{QCD}; variational methods

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$ ← *invariant measure*

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

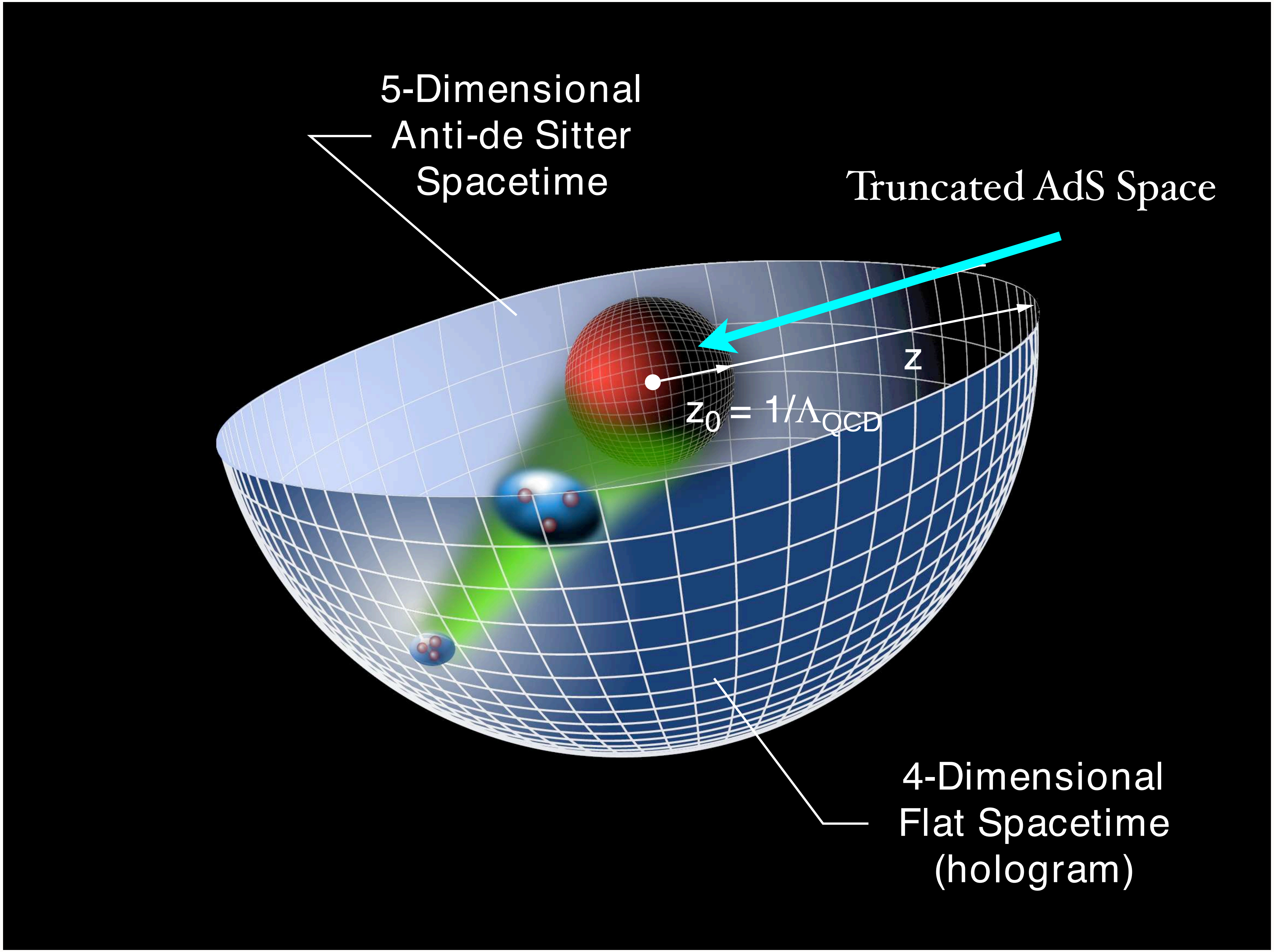
- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.



5-Dimensional
Anti-de Sitter
Spacetime
Truncated AdS Space
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)

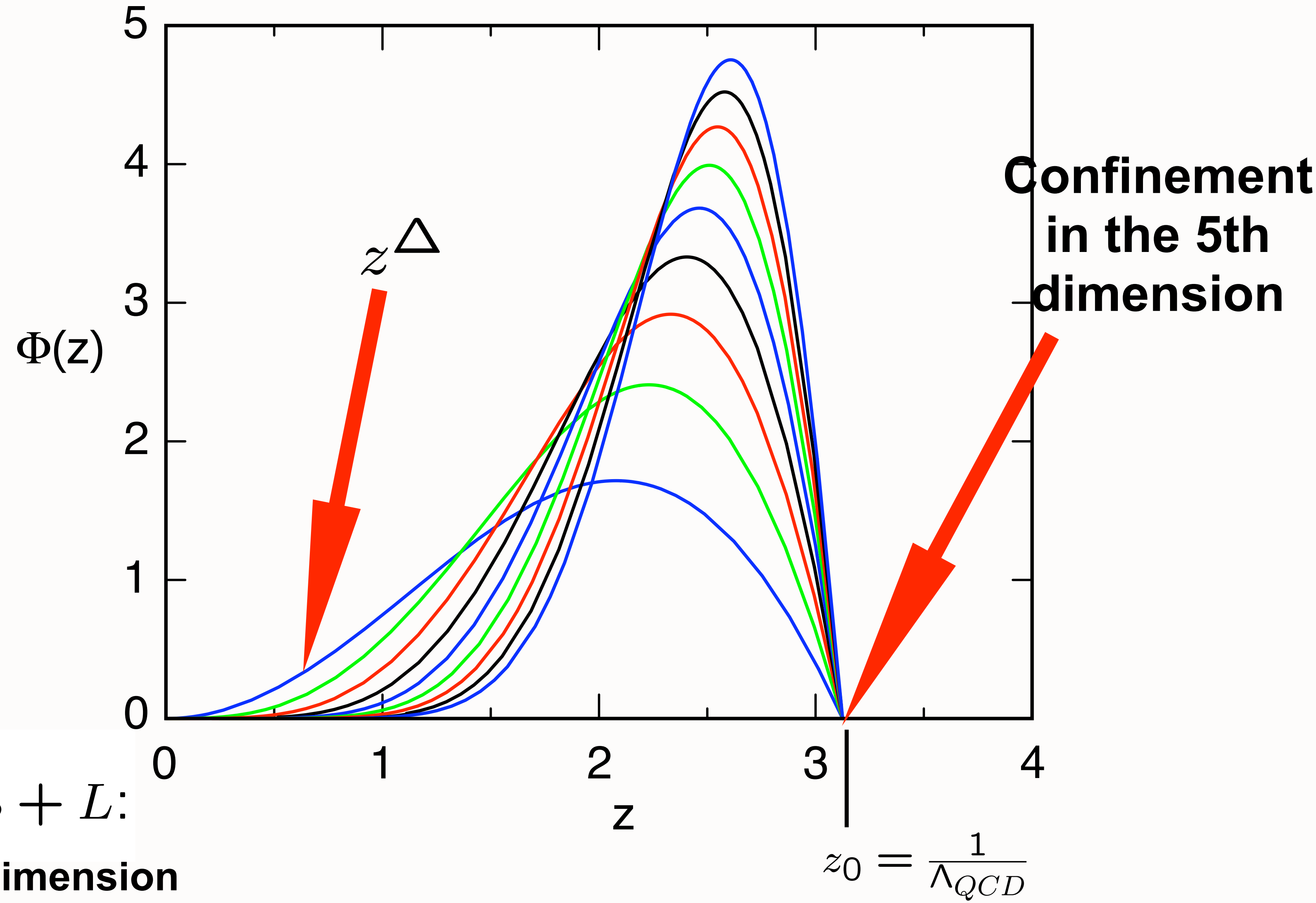
Guy de Teramond
SJB

AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5
- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2$ $z \to \lambda z$
- Holographic model: Confinement at large distances and conformal symmetry in interior $0 < z < z_0$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Truncated space simulates "bag" boundary conditions

$$\psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

Identify hadron by its interpolating operator at z -- > 0



$\Delta = 3 + L$:

Twist dimension of baryon

de Teramond, sjb

$$\Phi(z) = z^{3/2}\phi(z)$$

AdS Schrodinger Equation for bound state of two scalar constituents

$$[-\frac{d^2}{dz^2} + V(z)]\phi(z) = M^2\phi(z)$$

Truncated space

$$V(z) = -\frac{1-4L^2}{4z^2}$$

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

Alternative: Harmonic oscillator confinement

$$V(z) = -\frac{1-4L^2}{4z^2} + \kappa^4 z^2$$

Karch, et al.

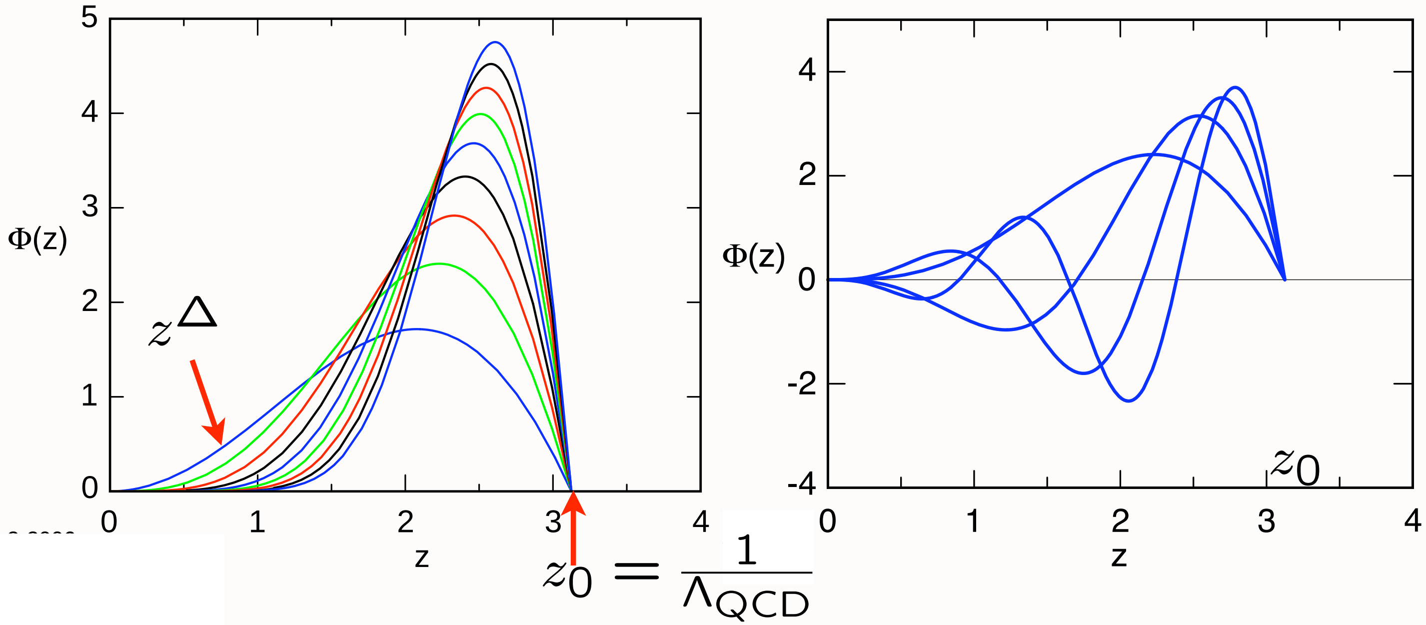
Derived from variation of Action in AdS5

Match fall-off at small z to conformal twist dimension at short distances

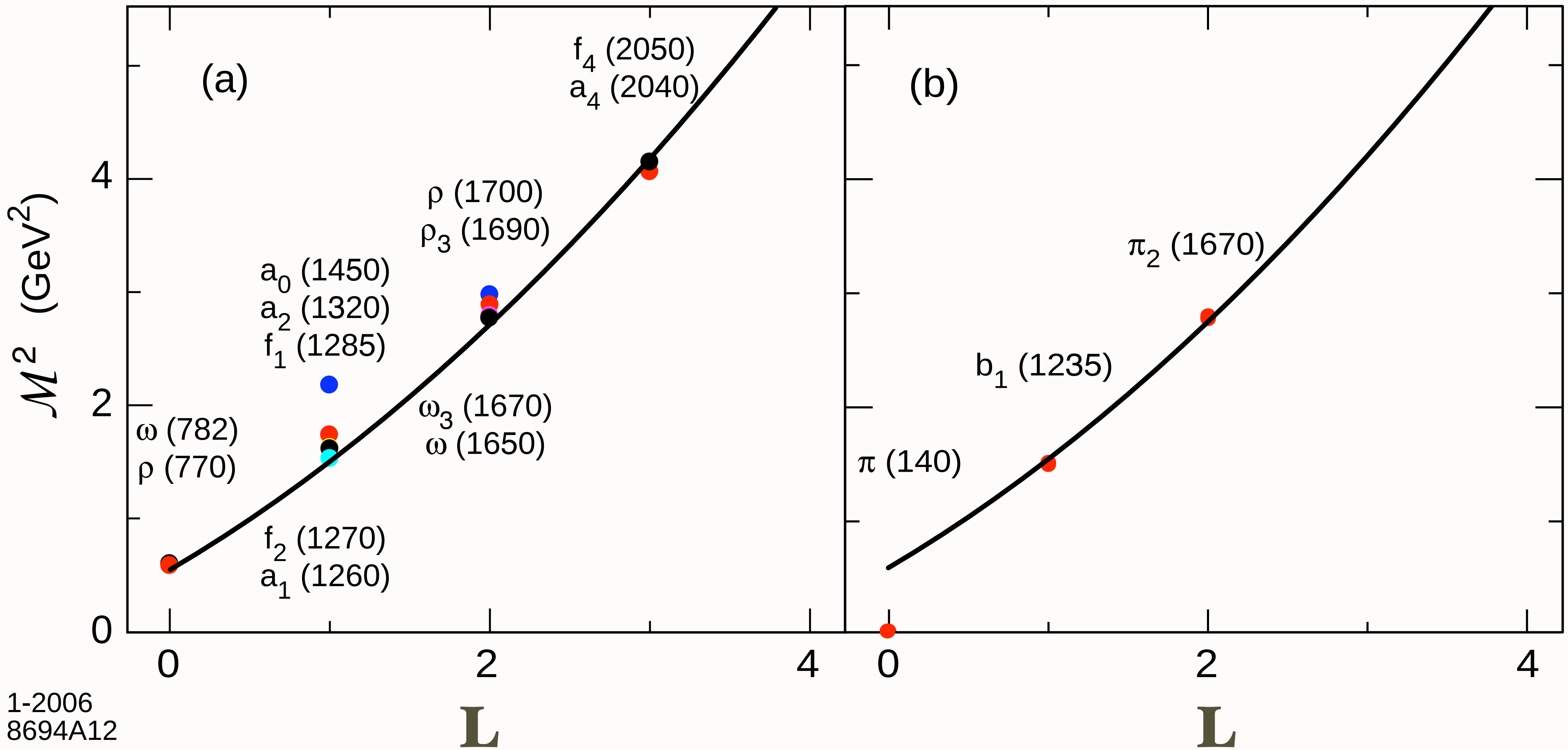
- Pseudoscalar mesons: $\mathcal{O}_{3+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge).

- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$



Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.



Light meson orbital spectrum $\Lambda_{QCD} = 0.32 \text{ GeV}$

Guy de Teramond
SJB

Baryon Spectrum

- Baryon: twist-three, dimension $\frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

Wave Equation: $\boxed{\left[z^2\, \partial_z^2 - 3z\, \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_\pm^2 + 4\right] f_\pm(z) = 0}$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x, z) = C e^{-iP\cdot x} z^2 \Big[J_{1+L}(z\mathcal{M})\, u_+(P) + J_{2+L}(z\mathcal{M})\, u_-(P)\Big].$$

- 4-d mass spectrum $\Psi(x, z_o)^\pm = 0 \quad \Longrightarrow$ parallel Regge trajectories for baryons !

$$\mathcal{M}^+_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}, \qquad \mathcal{M}^-_{\alpha,k} = \beta_{\alpha+1,k} \Lambda_{QCD}.$$

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions !

Prediction from AdS/QCD

Only one parameter!

Entire light quark baryon spectrum

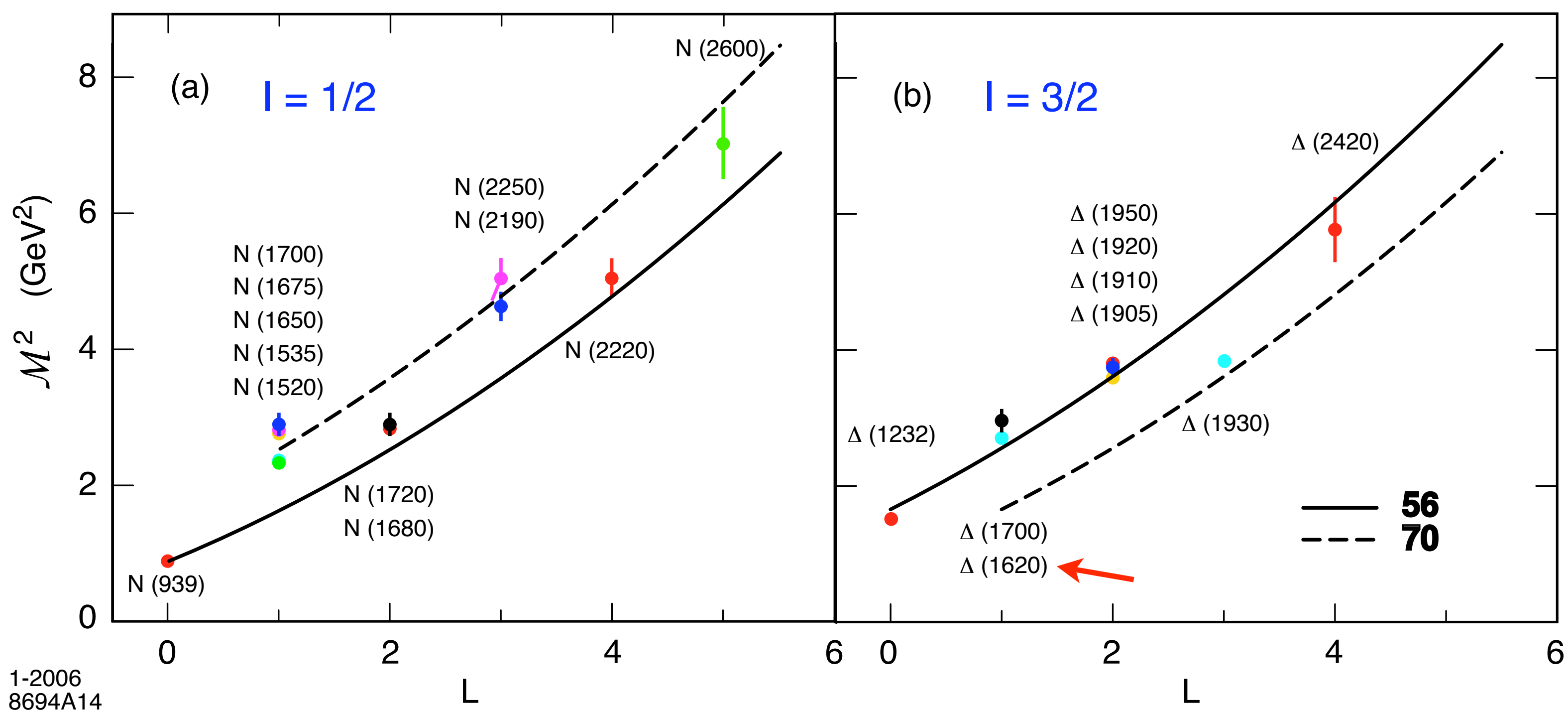


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The **56** trajectory corresponds to L even $P = +$ states, and the **70** to L odd $P = -$ states.

Guy de Teramond
SJB

- $SU(6)$ multiplet structure for N and Δ orbital states, including internal spin S and L.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

Holographic Harmonic Oscillator Model: Baryons

$$\left(\alpha\Pi(\zeta) - \mathcal{M}\right)\psi(\zeta) = 0,$$

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right)$$

$$\Pi^\dagger_\nu(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 + \kappa^2\zeta\gamma_5\right)$$

$$\left(H_{LF} - \mathcal{M}^2\right)\psi(\zeta) = 0, \qquad H_{LF} = \Pi^\dagger\Pi$$

Uncoupled Schrodinger Equations

Harmonic Oscillator Potential!

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2(\nu+1)\kappa^2 + \mathcal{M}^2\right)\psi_+(\zeta) = 0,$$

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4(\nu+1)^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\nu\kappa^2 + \mathcal{M}^2\right)\psi_-(\zeta) = 0,$$

Solution

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),$$

$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2),$$

Same eigenvalue!

$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1)$$

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

↓

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

↓ Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

↓ Integrable!

Hadron Spectra, Wavefunctions, Dynamics

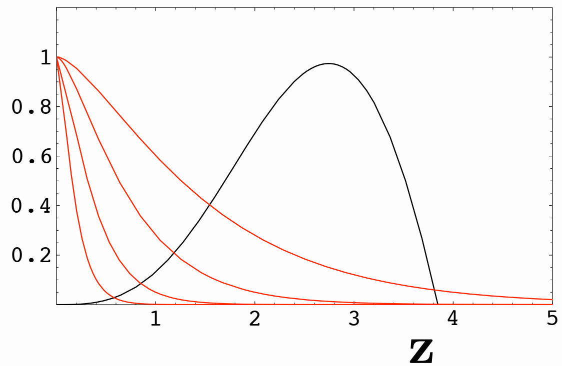
Hadron Form Factors from AdS/CFT

- Propagation of external perturbation suppressed inside AdS. $J(Q,z) = zQK_1(zQ)$

- At large Q^2 the important integration region is $z \sim 1/Q$.

$$F(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q,z) \Phi_I(z)$$

$\mathbf{J(Q,z)}, \ \mathbf{\Phi(z)}$



Polchinski, Strassler
de Teramond, sjb

- Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules: General result from AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} \Phi_{P'}(z) J(Q,z) \Phi_P(z).$$

$$\Phi(z) = \frac{\sqrt{2}\kappa}{R^{3/2}} z^2 e^{-\kappa^2 z^2/2}. \qquad J(Q,z) = zQK_1(zQ).$$

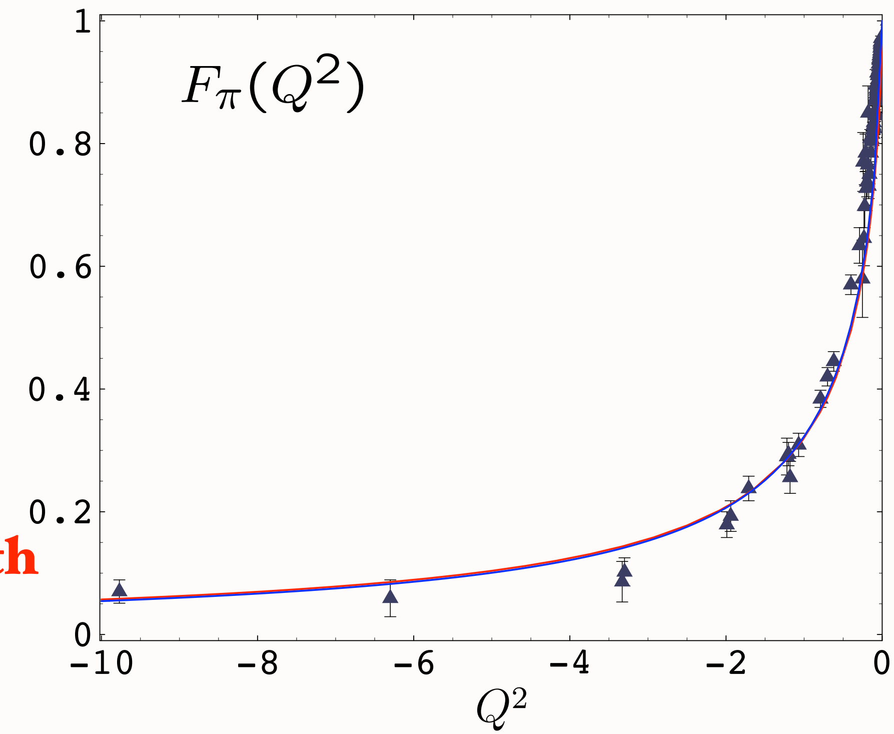
$$F(Q^2) = 1 + \frac{Q^2}{4\kappa^2} \exp\left(\frac{Q^2}{4\kappa^2}\right) Ei\left(-\frac{Q^2}{4\kappa^2}\right) \qquad Ei(-x) = \int_\infty^x e^{-t} \frac{dt}{t}.$$

Space-like Pion Form Factor

$\kappa = 0.4$ GeV

$\Lambda_{QCD} = 0.2$ GeV.

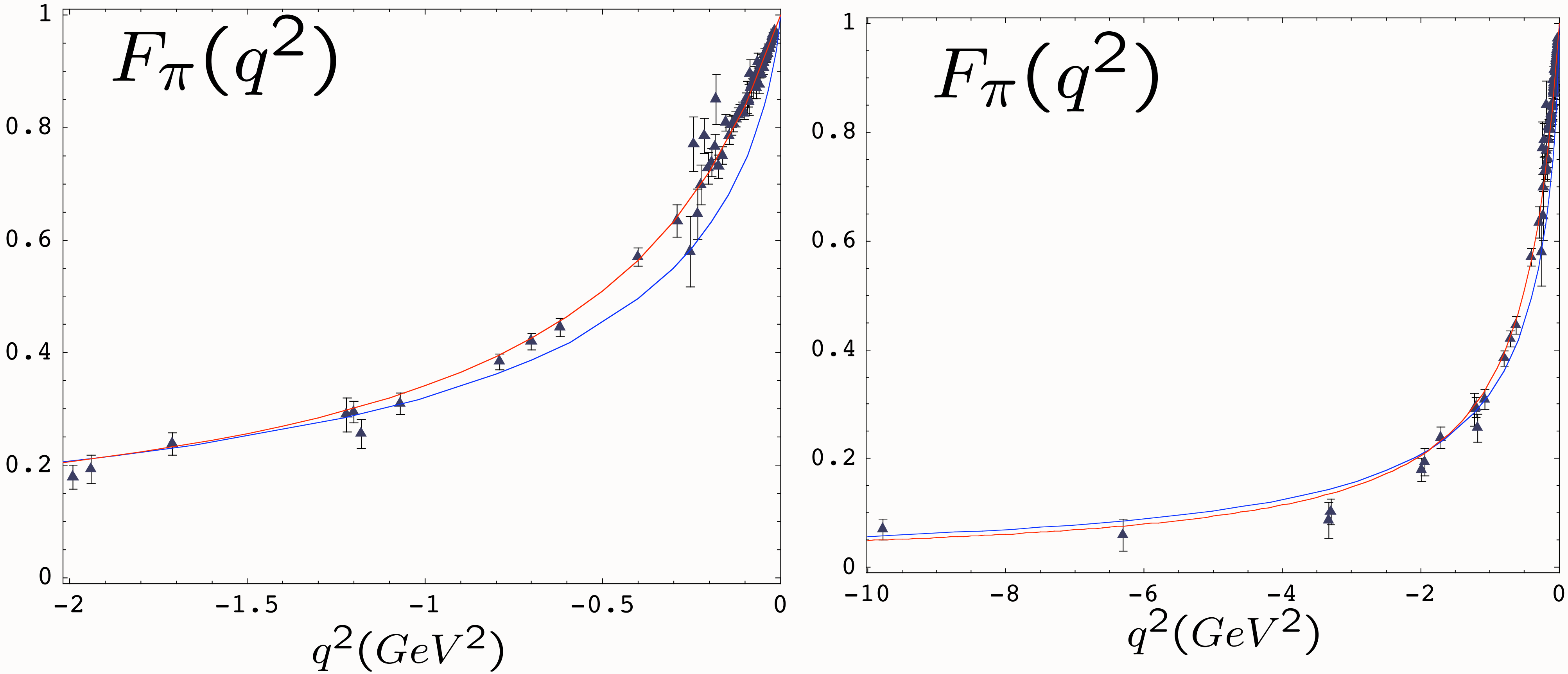
Identical Results for both confinement models



$$F(Q^2) \to \frac{4\kappa^2}{Q^2}$$

$\kappa = 2\Lambda_{QCD}$

Spacelike pion form factor from AdS/CFT



Data Compilation from Baldini, Kloe and Volmer

Harmonic Oscillator Confinement

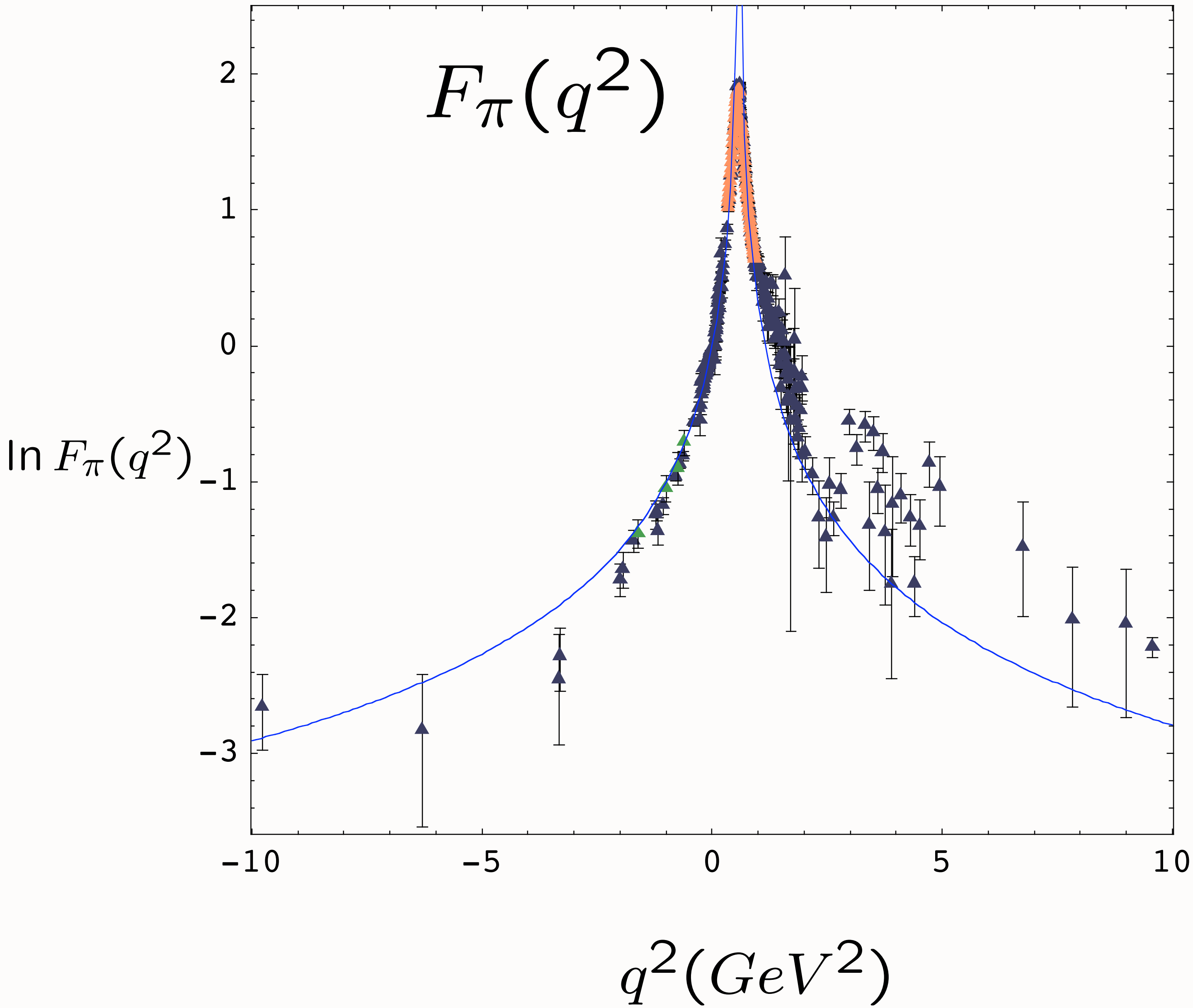
Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
Grigorian, Radyushkin

Spacelike and Timelike Pion form factor from AdS/CFT

G. de Teramond, sjb



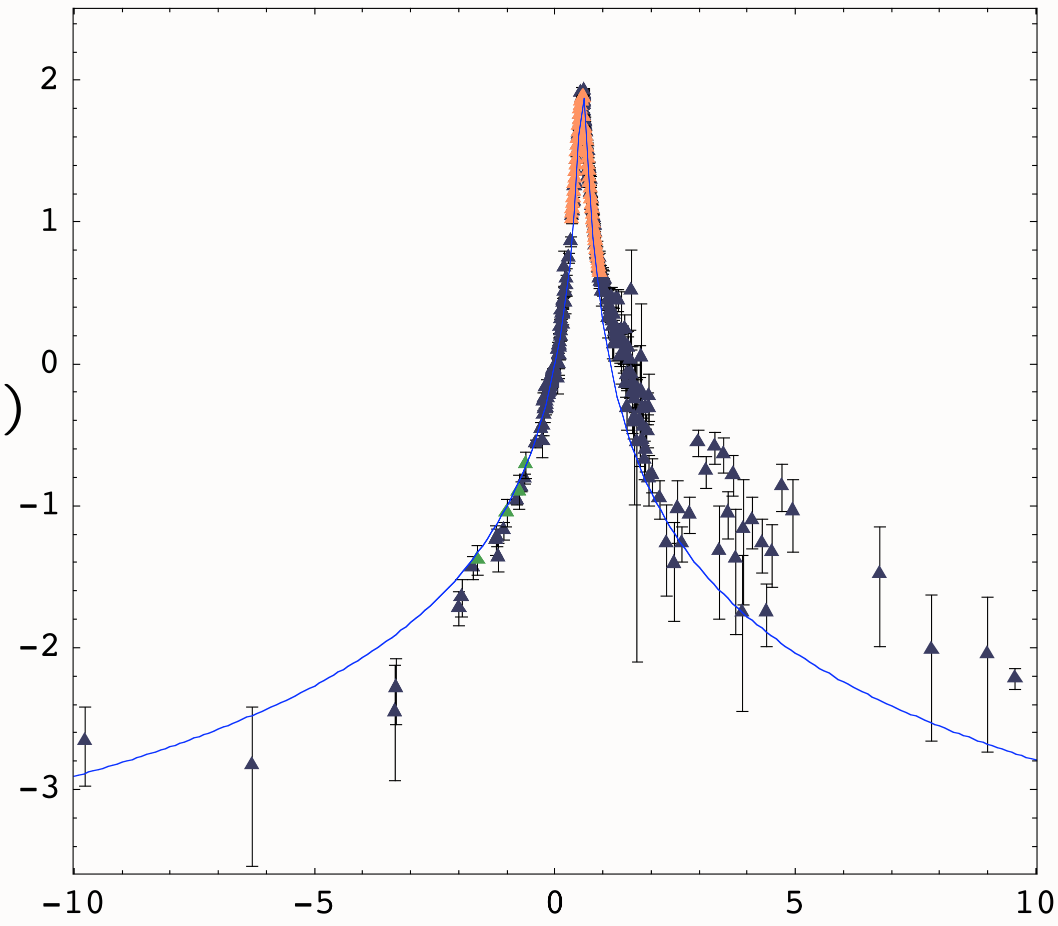
Harmonic Oscillator Confinement scale set by pion decay constant

$\kappa = 0.38$ GeV

Spacelike and Timelike Pion form factor from AdS/CFT

G. de Teramond, sjb

$F_\pi(q^2)$



$q^2(GeV^2)$

Harmonic Oscillator Confinement

$\kappa = 0.38$ GeV

Analytic continue to timelike momenta and introduce width

$$q^2 \to q^2 + i\epsilon \to q^2 + iM\Gamma$$

Fit to height, predict width

$\Gamma_\rho = 111$ MeV

$\Gamma_\rho^{exp} = 150.3 \pm 1.6$ MeV

Baryon Form Factors

- Coupling of the extended AdS mode with an external gauge field $A^\mu(x,z)$

$$ig_5 \int d^4x \, dz \, \sqrt{g} \, A_\mu(x,z) \, \overline{\Psi}(x,z) \gamma^\mu \Psi(x,z),$$

where

$$\Psi(x,z) = e^{-iP\cdot x} \left[\psi_+(z) u_+(P) + \psi_-(z) u_-(P)\right],$$

$$\psi_+(z) = C z^2 J_1(zM), \qquad \psi_-(z) = C z^2 J_2(zM),$$

and

$$u(P)_\pm = \frac{1 \pm \gamma_5}{2} u(P).$$

$$\psi_+(z) \equiv \psi^\uparrow(z), \quad \psi_-(z) \equiv \psi^\downarrow(z),$$

the LC $\pm$ spin projection along $\hat{z}$.

- Constant C determined by charge normalization:

$$C = \frac{\sqrt{2}\Lambda_{\rm QCD}}{R^{3/2}\left[-J_0(\beta_{1,1}) J_2(\beta_{1,1})\right]^{1/2}}.$$

Nucleon Form Factors

- Consider the spin non-flip form factors in the infinite wall approximation

$$F_+(Q^2) = g_+ R^3 \int \frac{dz}{z^3} J(Q,z)\, |\psi_+(z)|^2,$$
$$F_-(Q^2) = g_- R^3 \int \frac{dz}{z^3} J(Q,z)\, |\psi_-(z)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(z)$ and $\psi_-(z)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = R^3 \int \frac{dz}{z^3} J(Q,z) |\psi_+(z)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} R^3 \int \frac{dz}{z^3} J(Q,z) \left[|\psi_+(z)|^2 - |\psi_-(z)|^2 \right],$$

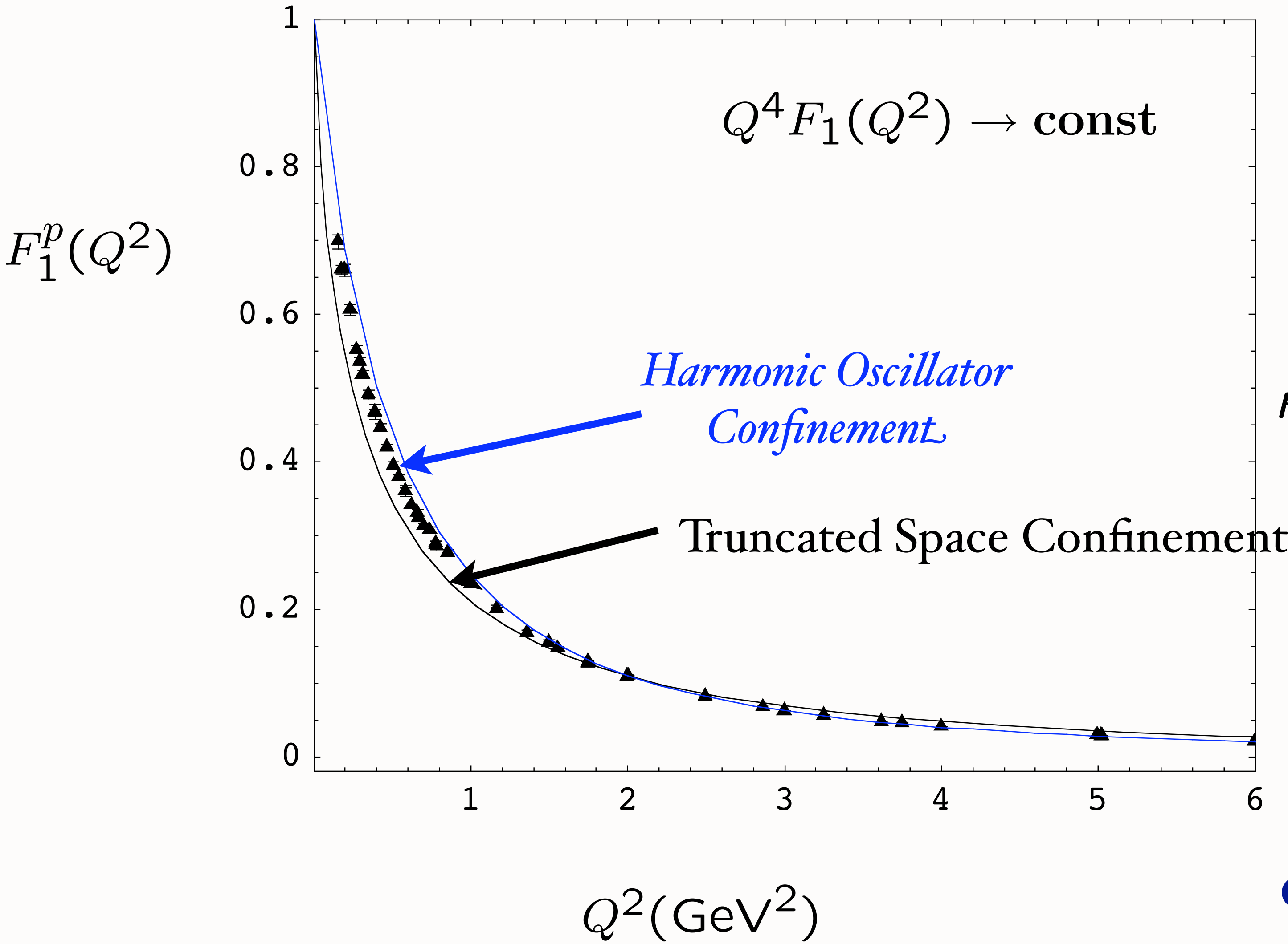
where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Large Q power scaling: $F_1(Q^2) \to \left[1/Q^2\right]^2$.

G. de Teramond, sjb

G. de Teramond, sjb

Preliminary



$\kappa = 0.424 \;\text{GeV}$

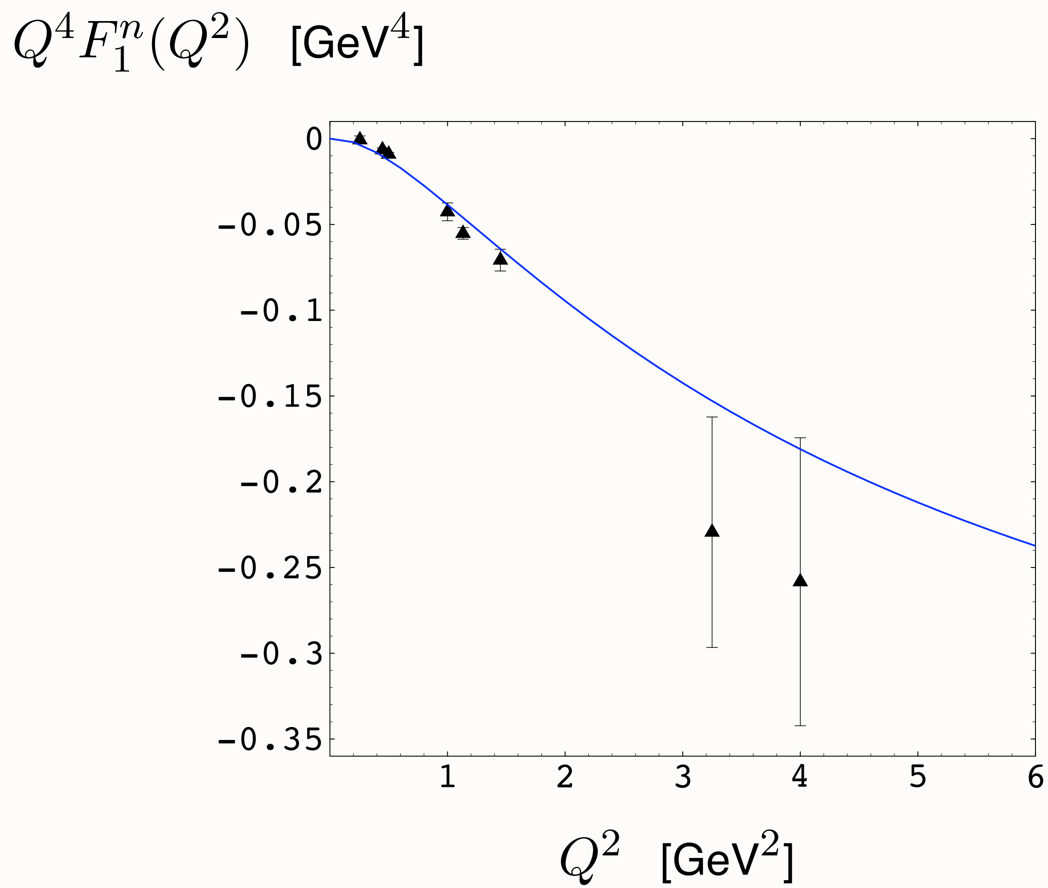
$\Lambda = 0.2 \;\text{GeV}$

Current modified by metric

$$F_1(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi^{\uparrow}_F(z) J(Q,z) \Phi^{\uparrow}_I(z)$$

Dirac Neutron Form Factor
(Valence Approximation)

Truncated Space Confinement



Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Hadronic Form Factor in Space and Time-Like Regions

- The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron Φ_I and Φ_F and the non-normalizable mode J, dual to the external source (hadron spin σ):

$$F(Q^2)_{I\to F} = R^{3+2\sigma}\int_0^\infty \frac{dz}{z^{3+2\sigma}}\, e^{(3+2\sigma)A(z)}\Phi_F(z)\, J(Q,z)\, \Phi_I(z)$$
$$\simeq R^{3+2\sigma}\int_0^{z_o} \frac{dz}{z^{3+2\sigma}}\, \Phi_F(z)\, J(Q,z)\, \Phi_I(z),$$

- $J(Q,z)$ has the limiting value 1 at zero momentum transfer, $F(0)=1$, and has as boundary limit the external current, $A^\mu = \epsilon^\mu e^{iQ\cdot x} J(Q,z)$. Thus:

$$\lim_{Q\to 0} J(Q,z) = \lim_{z\to 0} J(Q,z) = 1.$$

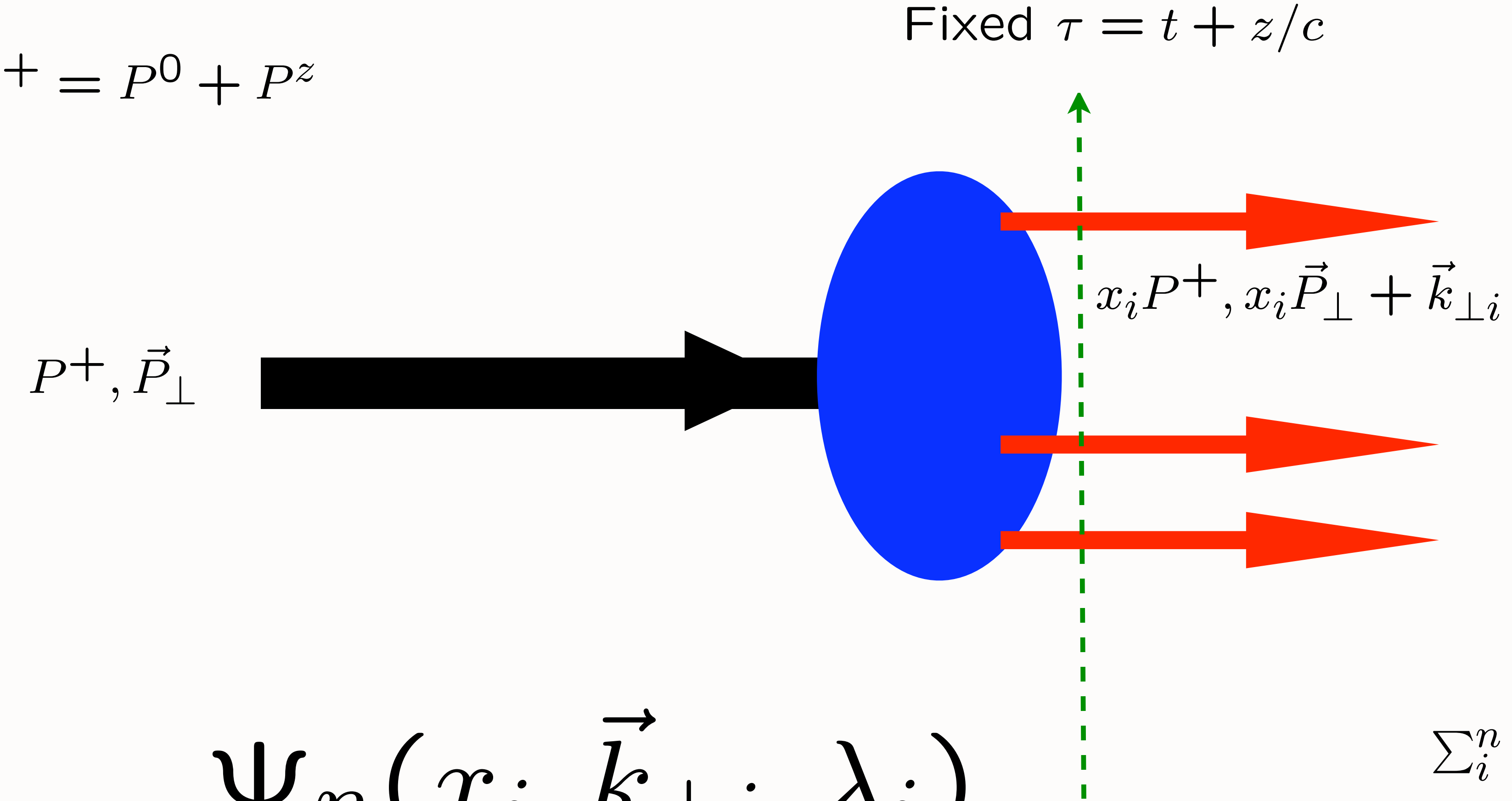
- Solution to the AdS Wave equation with boundary conditions at $Q=0$ and $z\to 0$:

$$J(Q,z) = zQK_1(zQ).$$

Polchinski and Strassler, hep-th/0209211; Hong, Yong and Strassler, hep-th/0409118.

Light-Front Wavefunctions

$P^+ = P^0 + P^z$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of $\mathbf{P^\mu}$

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$|p,S_z> = \sum_{n=3} \Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)|n;\vec{k}_{\perp_i},\lambda_i>$$

sum over states with n=3, 4, ... constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)$$

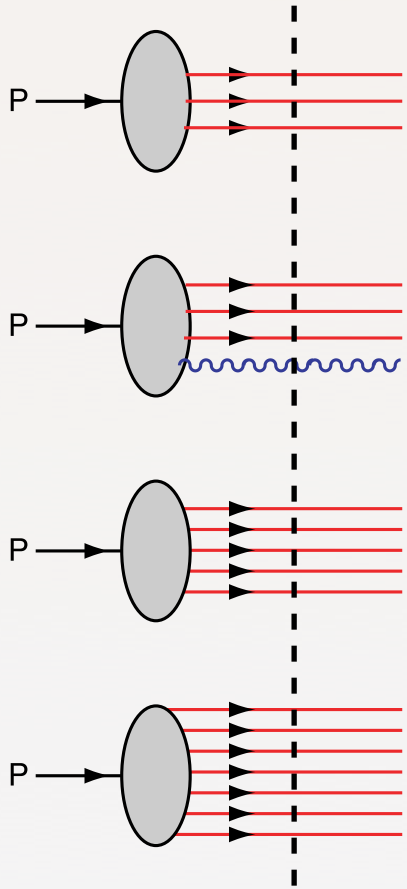
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0+k_i^z}{P^0+P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks, $\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

Remarkable Features of Hadron Structure

- Valence quarks carry less than half of the proton's spin and momentum
- Non-zero quark orbital angular momentum
- Asymmetric sea: $\bar{u}(x) \neq \bar{d}(x)$ relation to meson cloud
- Non-symmetric strange and antistrange sea $\bar{s}(x) \neq s(x)$
- Intrinsic charm and bottom at high x
- Hidden-Color Fock states of the Deuteron

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp) \, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp \, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find $(b = |\vec{b}_\perp|)$:

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp \, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0\left(bqx\right) \, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Identical DYW and AdS5 Formulae: Two-parton case

- Change the integration variable $\zeta = |\vec{b}_\perp|\sqrt{x(1-x)}$

$$F(Q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int_0^{\zeta_{max}=\Lambda^{-1}_{\rm QCD}} \zeta \, d\zeta \, J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) \left|\widetilde{\psi}(x,\zeta)\right|^2,$$

- Compare with AdS form factor for arbitrary Q. Find:

Same result for LF and AdS5

$$J(Q,\zeta) = \int_0^1 dx J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) = \zeta Q K_1(\zeta Q),$$

$\zeta \leftrightarrow \mathbf{z}$

the solution for the electromagnetic potential in AdS space, and

$$\widetilde{\psi}(x,\vec{b}_\perp) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi}J_1(\beta_{0,1})}\sqrt{x(1-x)}J_0\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{0,1}\Lambda_{QCD}\right)\theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right)$$

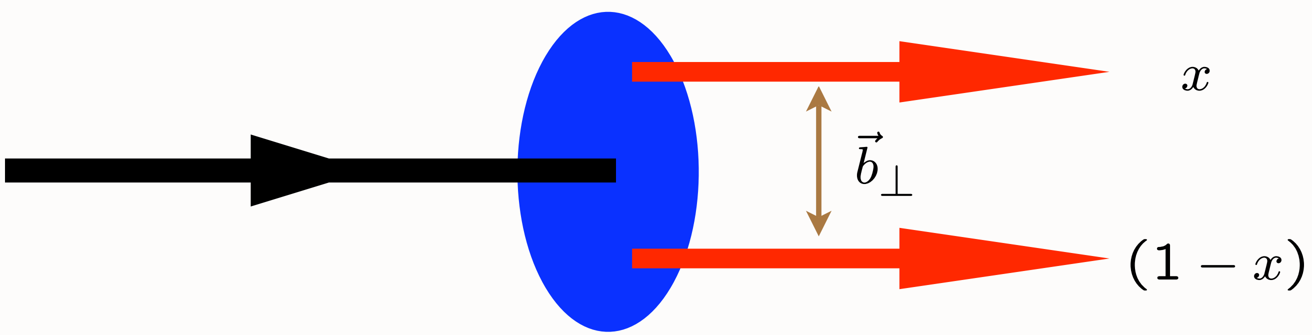
the holographic LFWF for the valence Fock state of the pion $\psi_{\overline{q}q/\pi}$.

- The variable ζ, $0 \le \zeta \le \Lambda^{-1}_{QCD}$, represents the scale of the invariant separation between quarks and is also the holographic coordinate $\zeta = z$!

LF(3+1)   AdS$_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)}\ \phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

N-parton case

- Define effective single particle transverse density by (Soper, Phys. Rev. D **15**, 1141 (1977))

$$F(q^2) = \int_0^1 dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp)$$

- From DYW expression for the FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\vec{b}_{\perp j} \, \delta(1 - x - \sum_{j=1}^{n-1} x_j) \, \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp) |\psi_n(x_j, \vec{b}_{\perp j})|^2$$

- Compare with the the form factor in AdS space for arbitrary Q:

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) \, J(Q, z) \, \Phi_P(z)$$

- Holographic variable z is expressed in terms of the average transverse separation distance of the spectator constituents $\vec{\eta} = \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j}$

$$z \rightarrow \sqrt{\frac{x}{1-x}} \Big| \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j} \Big|$$

Holography: Map AdS/CFT to 3+1 LF Theory

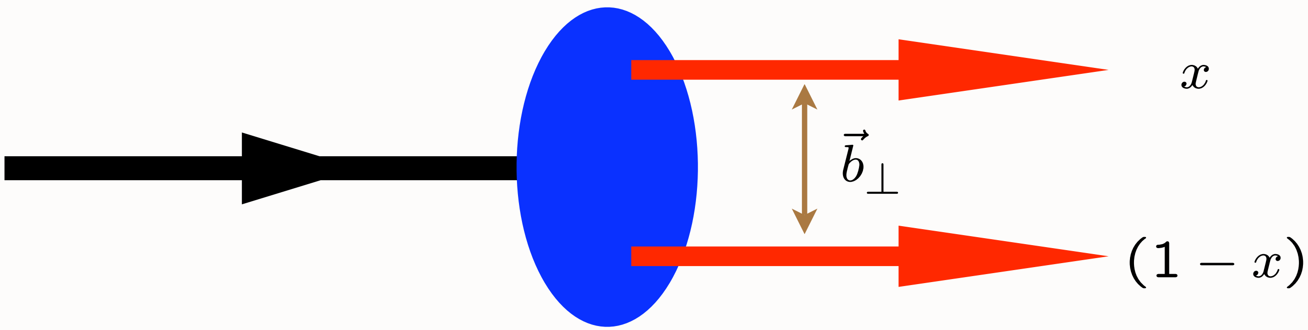
Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

Induced by conformal metric

G. de Teramond and sjb

Map AdS/CFT to 3+1 LF Theory

Effective radial equation:

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Effective conformal potential:

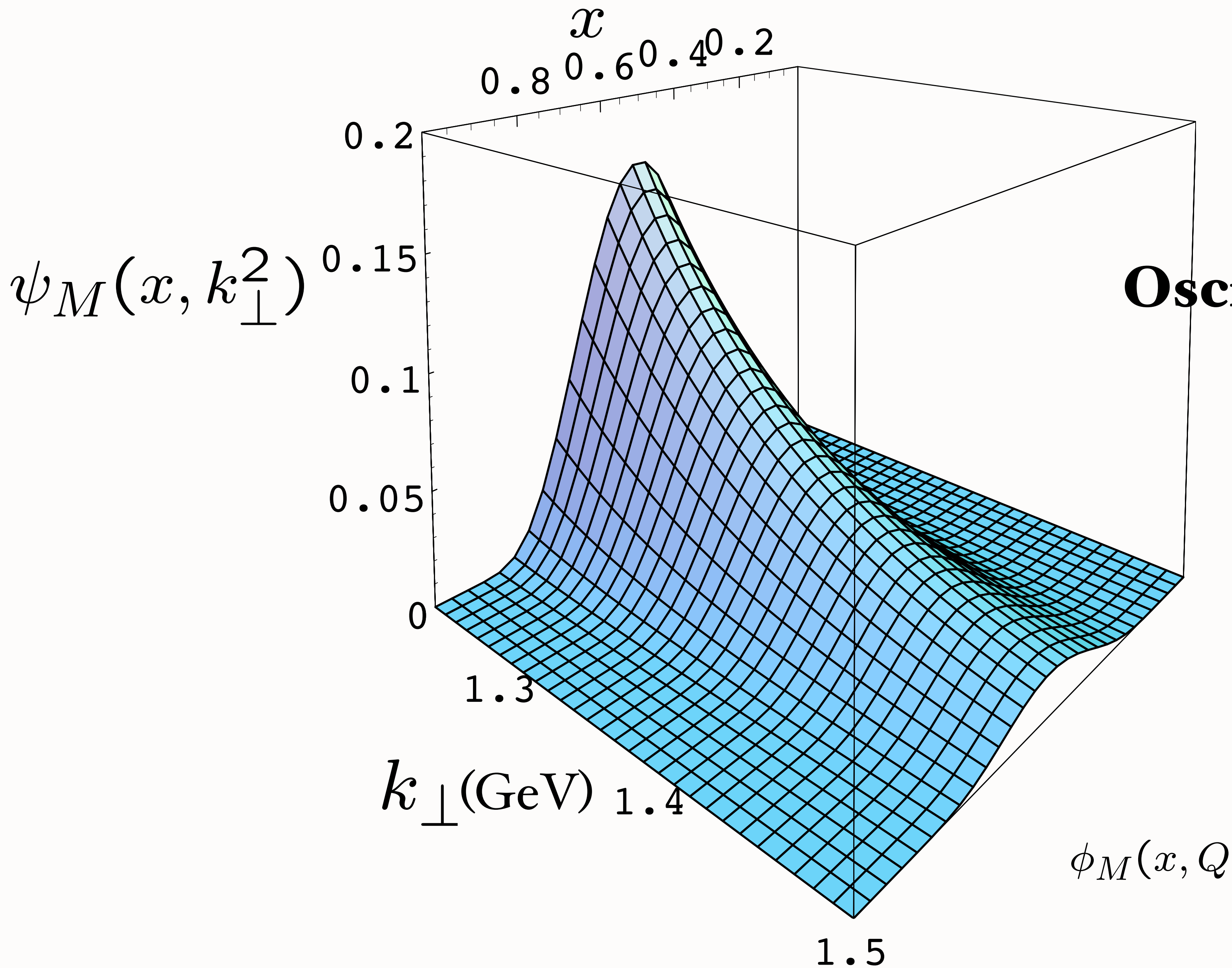
$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

General solution:

$$\widetilde{\psi}_{L,k}(x,\vec{b}_\perp) = B_{L,k}\sqrt{x(1-x)}$$

$$J_L\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda_{\rm QCD}^{-2}}{x(1-x)}\right),$$

Prediction from AdS/CFT: Meson LFWF

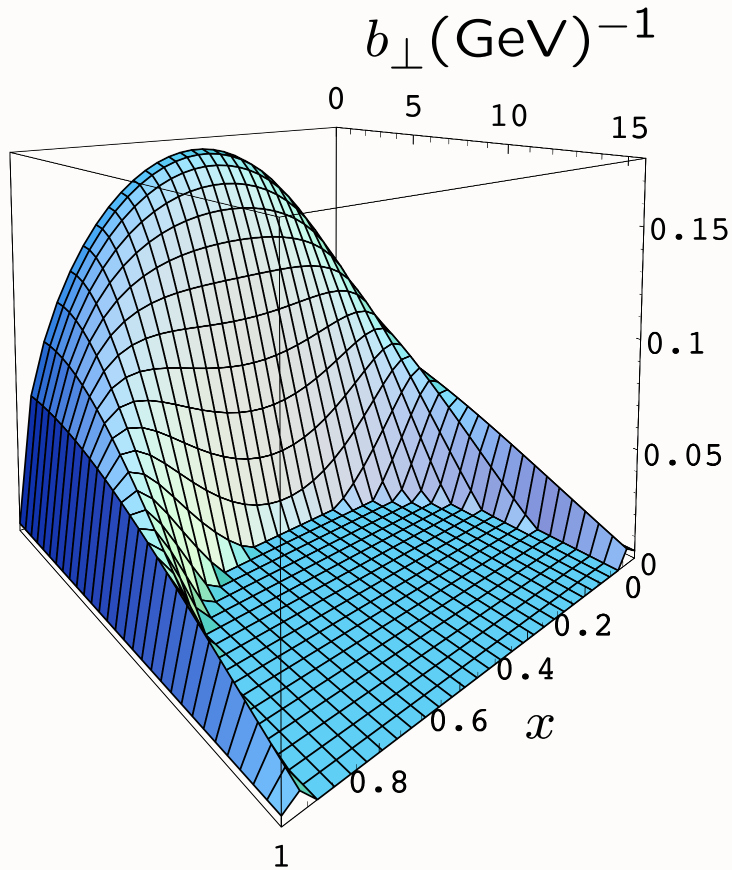


Harmonic Oscillator model

de Teramond, sjb

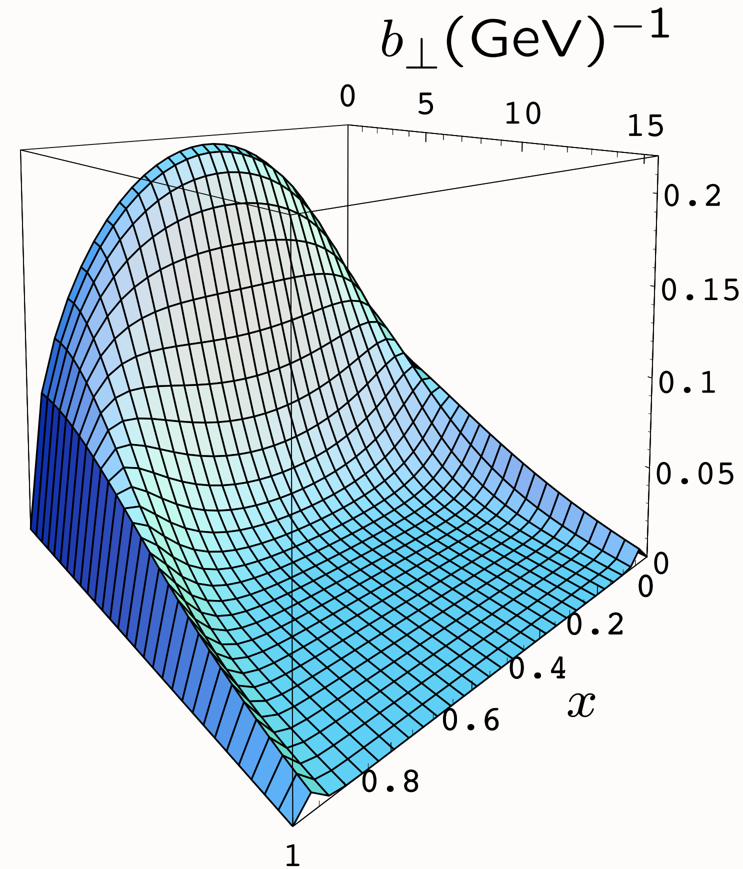
$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$

AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$



$\Lambda_{QCD} = 0.32 \text{ GeV}$

Truncated Space



$\kappa = 0.76 \text{ GeV}$

Harmonic Oscillator

AdS/CFT Prediction for Meson LFWF

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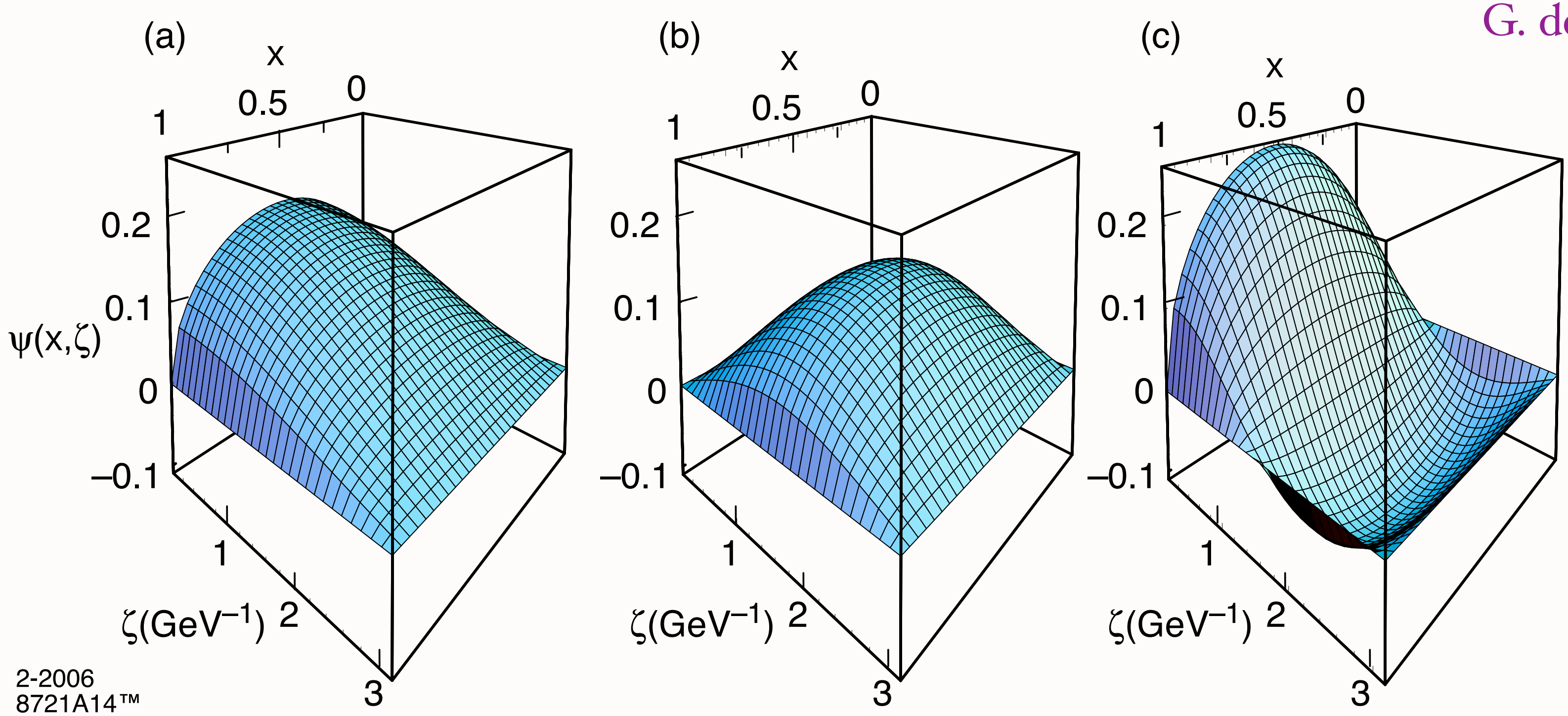

Two-parton holographic LFWF in impact space $\widetilde{\psi}(x,\zeta)$ for $\Lambda_{QCD} = 0.32$ GeV: (a) ground state $L = 0,\ k = 1$; (b) first orbital exited state $L = 1,\ k = 1$; (c) first radial exited state $L = 0,\ k = 2$. The variable ζ is the holographic variable $z = \zeta = |b_\perp|\sqrt{x(1-x)}$.

$$\widetilde{\psi}(x,\zeta) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\zeta \beta_{0,1} \Lambda_{QCD}\right) \theta\left(z \le \Lambda^{-1}_{\rm QCD}\right)$$

AdS/CFT Holographic Model

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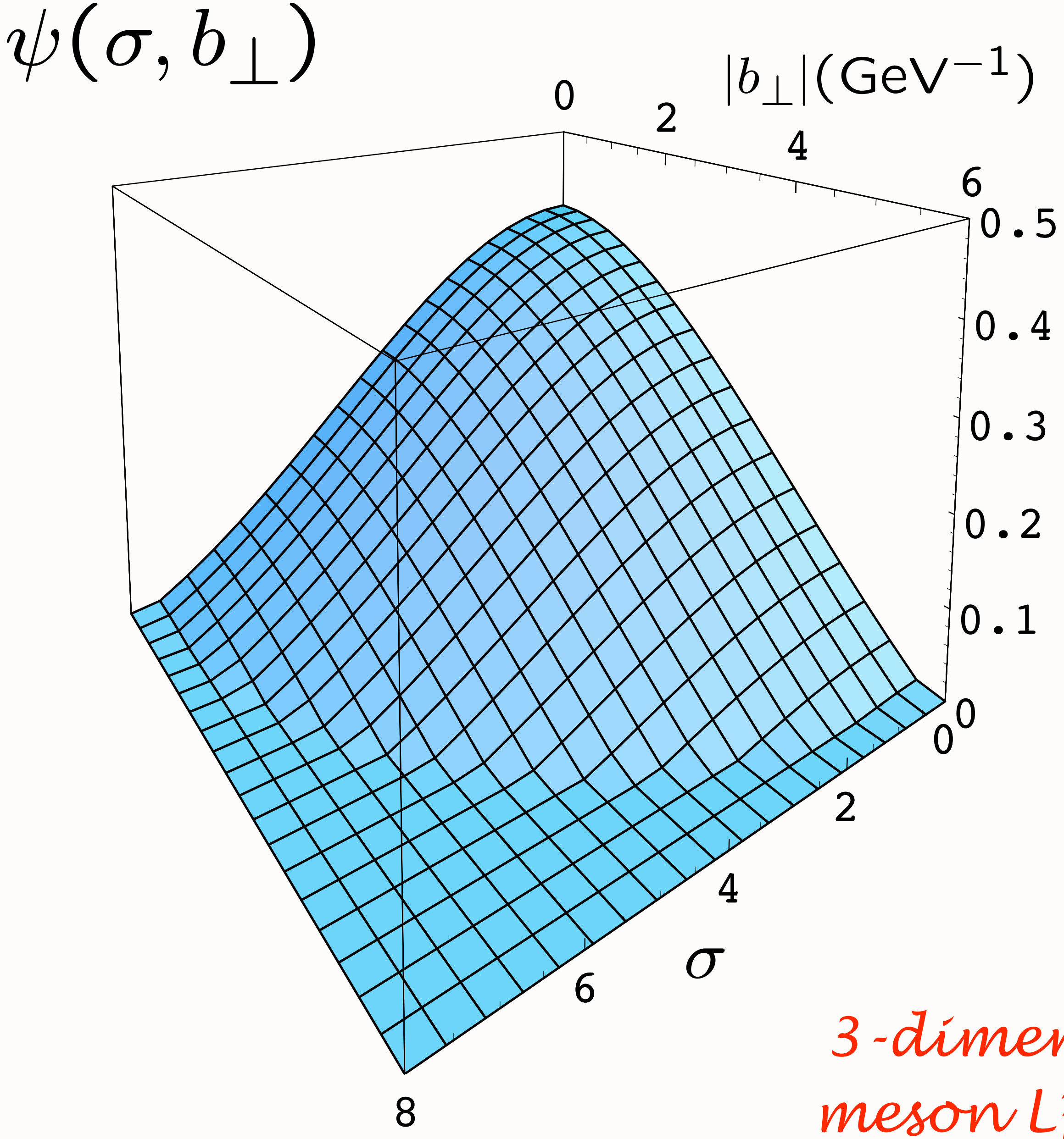
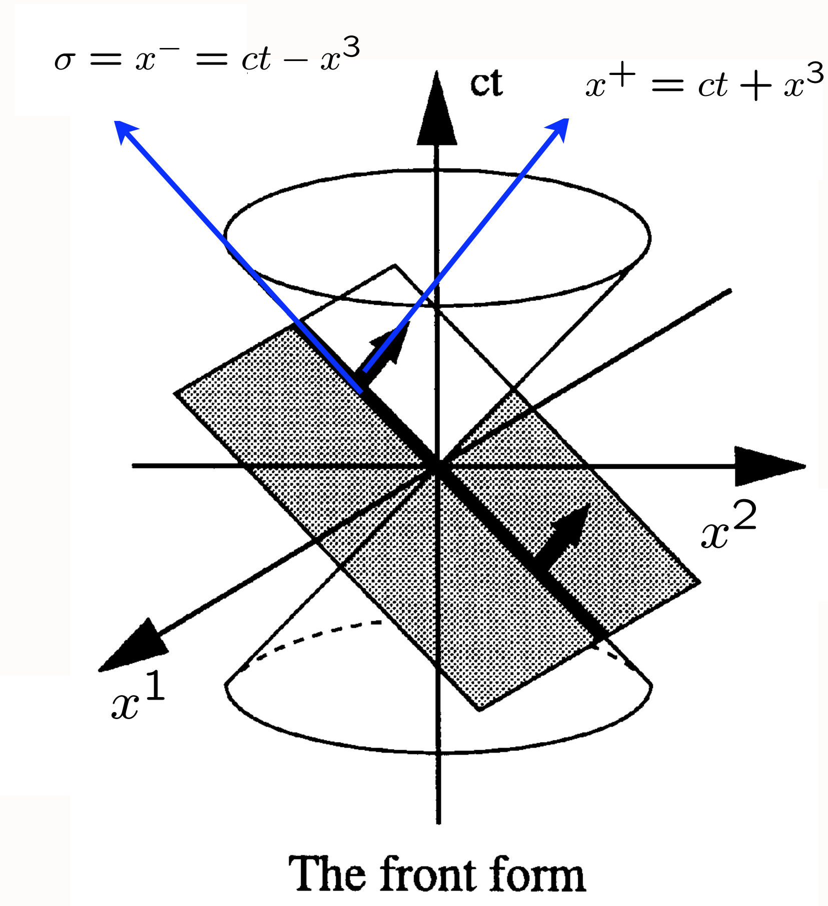
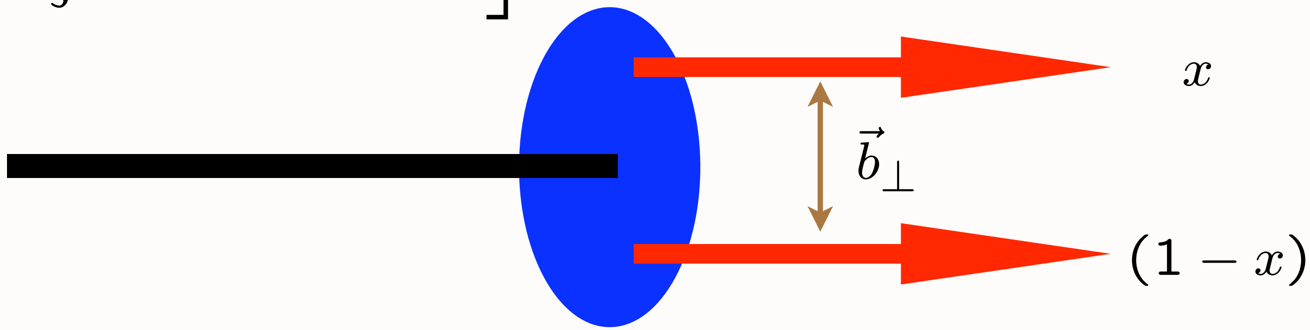
SJB





3-dimensional photograph: meson LFWF at fixed LF Time

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Holographic Variable

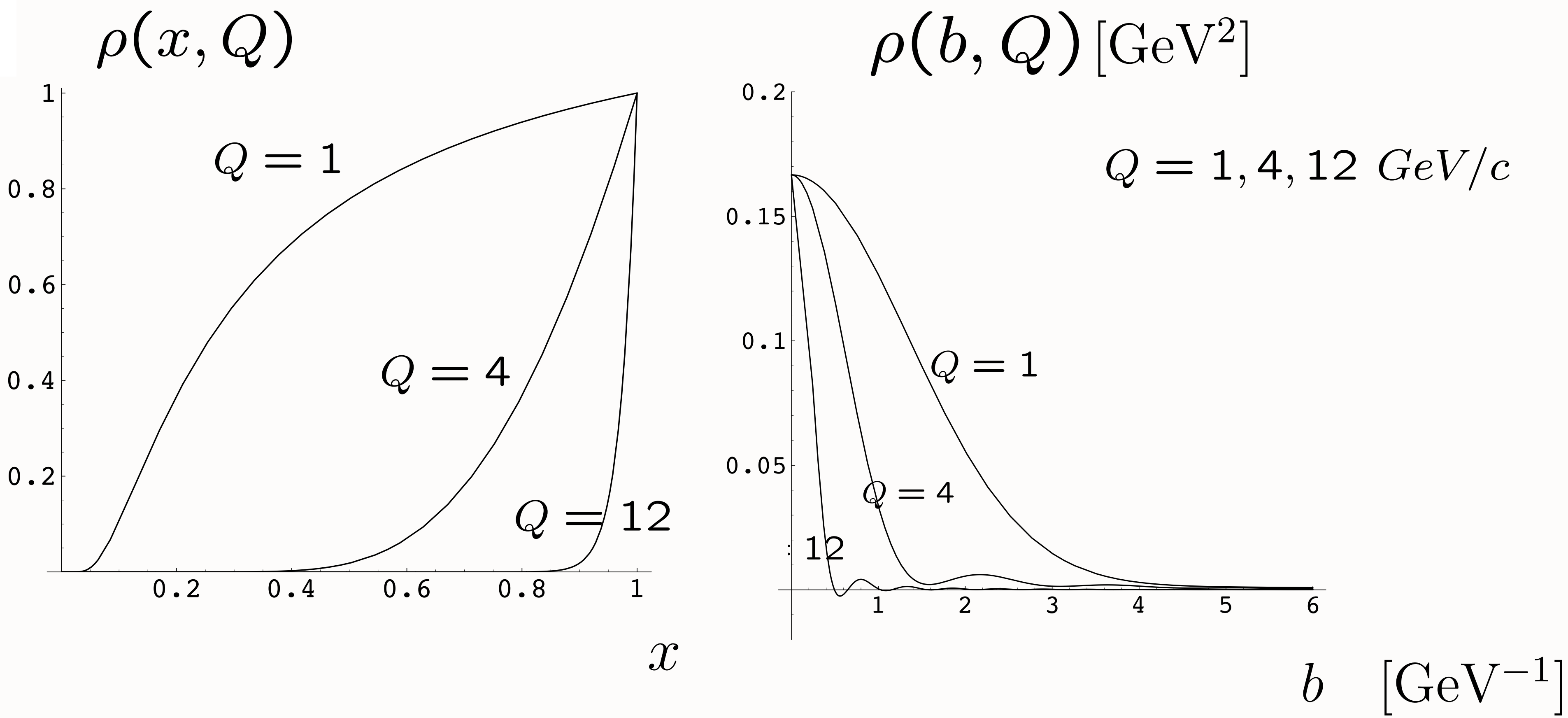
$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

LF Kinetic Energy in momentum space

Conjecture for mesons with massive quarks

$$-\frac{d}{d\zeta^2} \to -\frac{d}{d\zeta^2} + \frac{m_a^2}{x} + \frac{m_b^2}{1-x} \equiv \frac{k_\perp^2 + m_a^2}{x} + \frac{k_\perp^2 + m_b^2}{1-x}$$

AdS/QCD HO Model for Meson Form Factors



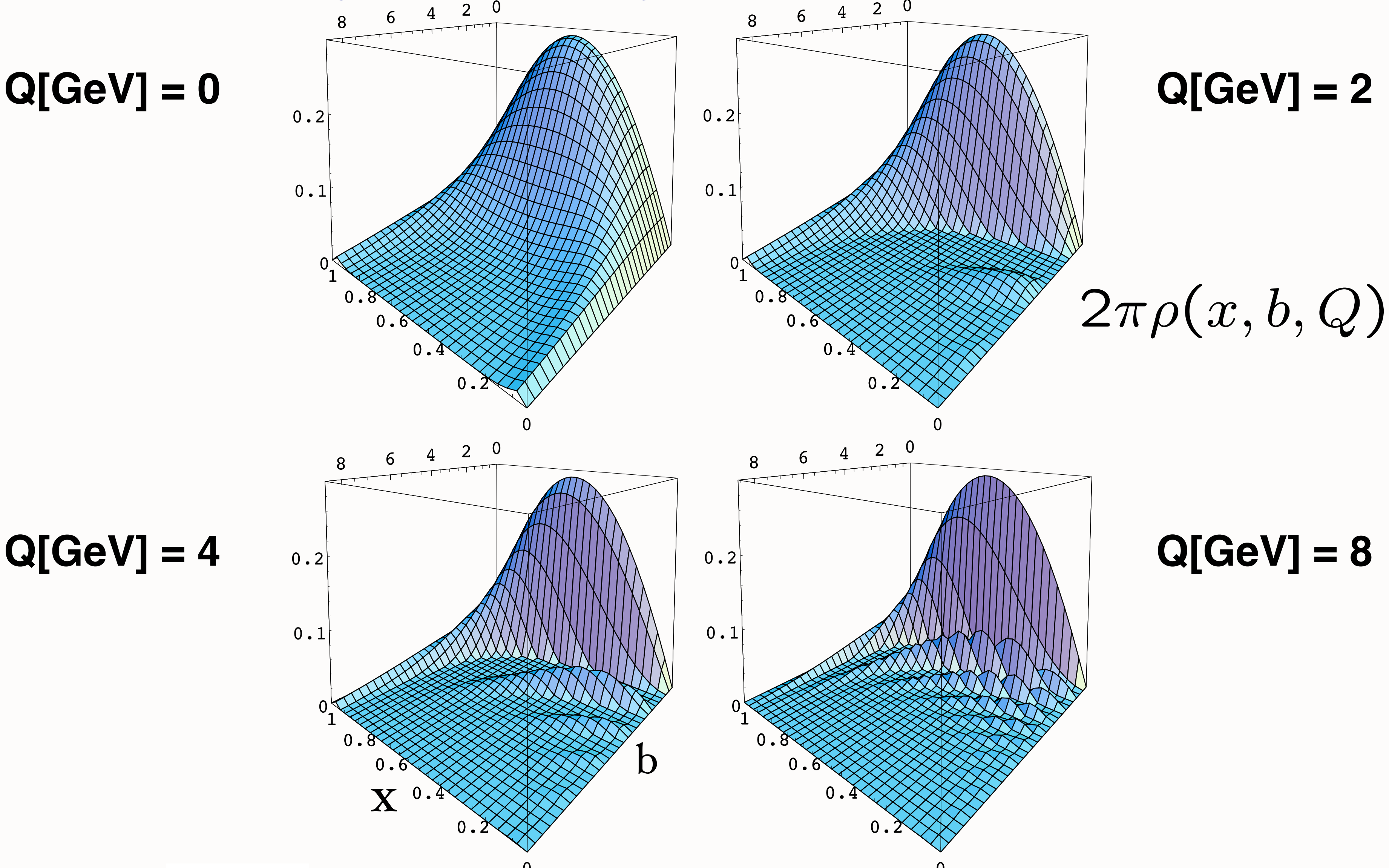
$$F(Q^2) = \int_0^1 dx \int db^2 \kappa^2 x(1-x) J_0(bQ(1-x)) e^{-\kappa^2 x(1-x) b^2}$$

$$F(Q^2) = \int_0^1 dx \rho(x, Q) = \int db^2 \rho(b, Q)$$

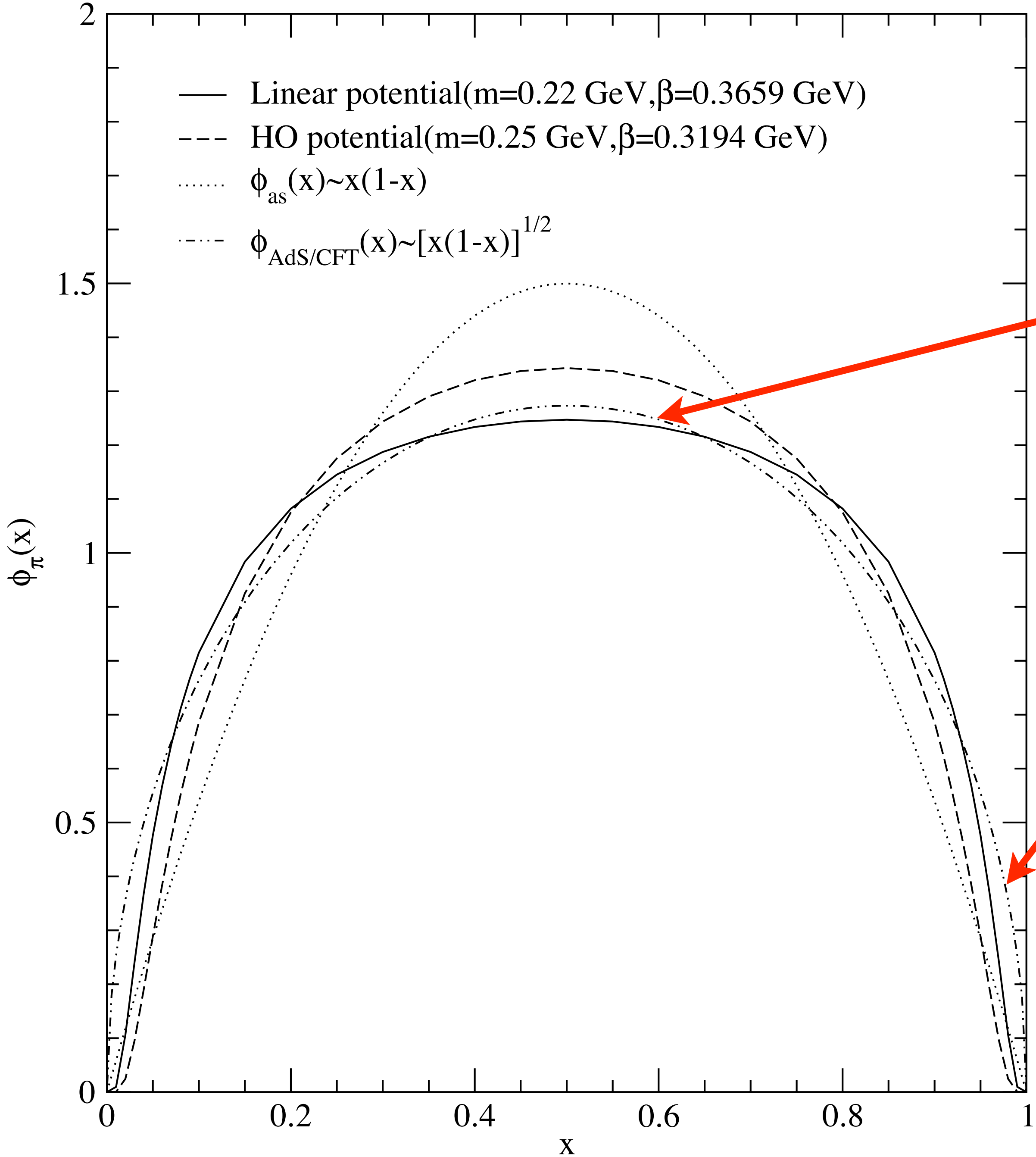
Large Q form factor derives from small b *and* small 1-x

$$x \rightarrow 1 \equiv k_z \rightarrow -\infty$$

AdS/QCD HO Model for Meson Form Factors



Effective partonic density $2\pi\rho(x, b_\perp, Q)$ in terms of the longitudinal momentum fraction x, the transverse relative impact variable $b_\perp$ and momentum transfer Q for the harmonic oscillator model. The figure corresponds to $\kappa = 0.67$ GeV. The values of Q are 0, 2, 4 and 8 GeV/c. As Q increases the distribution becomes increasingly important near $x = 1$ and $b_\perp = 0$. At very large Q the distribution is peaked towards $b_\perp = 0$.



AdS/CFT:

$$\phi(x, Q_0) \propto \sqrt{x(1-x)}$$

Increases PQCD leading twist predictio
$F_\pi(Q^2)$ by factor 16/9

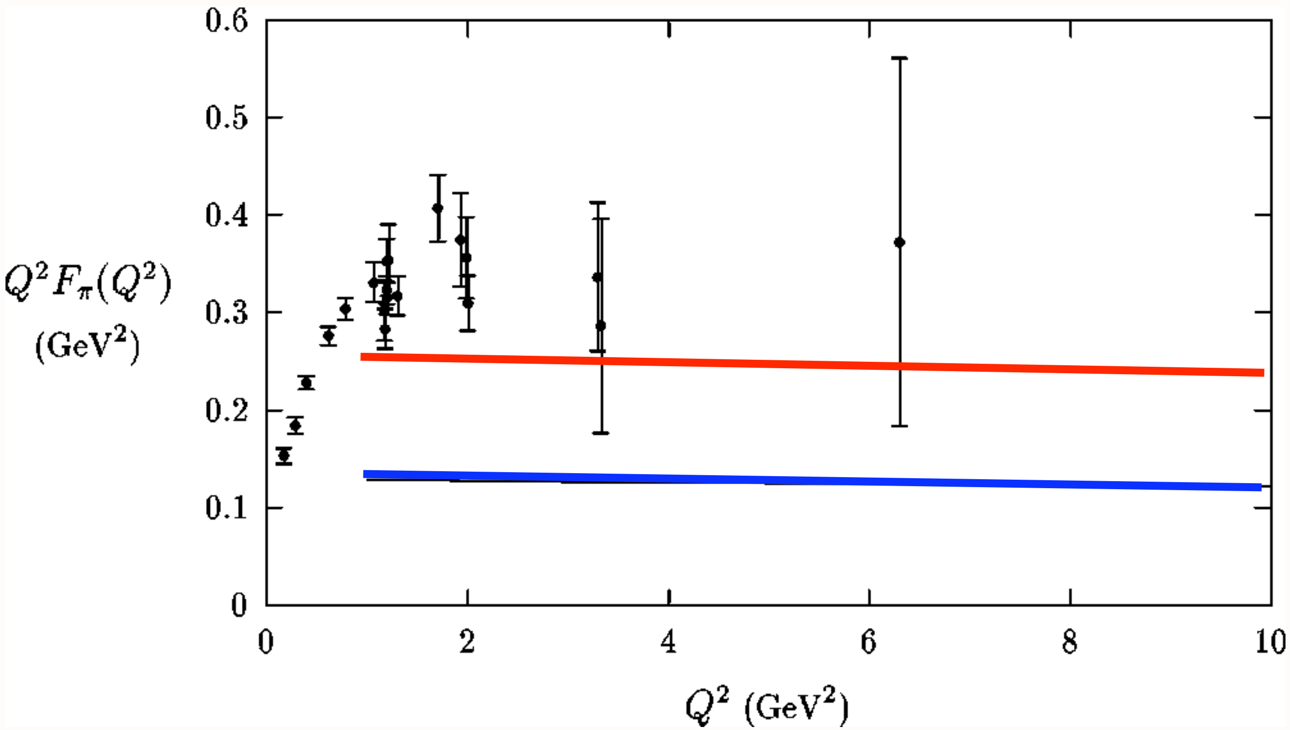
QCD

Stan Brodsky, SLAC

Lepage, sjb

C. Ji, A. Pang, D. Robertson, sjb
Choi, Ji

$$F_\pi(Q^2) = \int_0^1 dx\,\phi_\pi(x) \int_0^1 dy\,\phi_\pi(y) \frac{16\pi C_F \alpha_V(Q_V)}{(1-x)(1-y)Q^2}$$

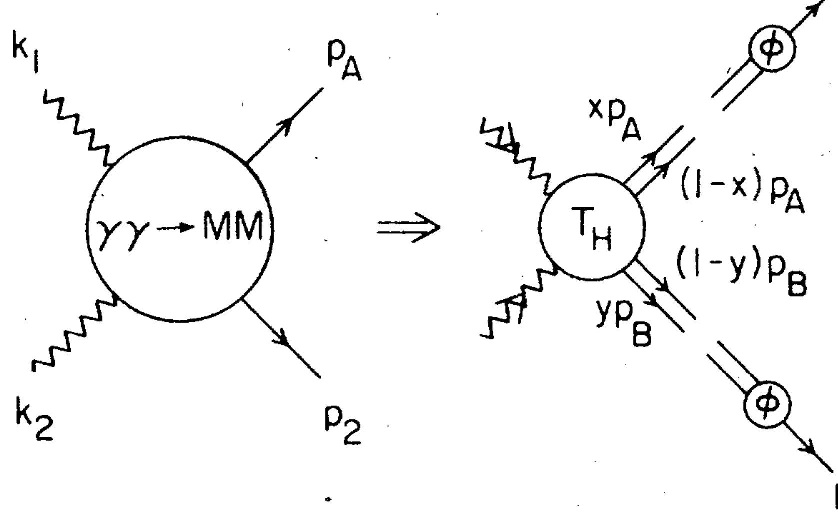


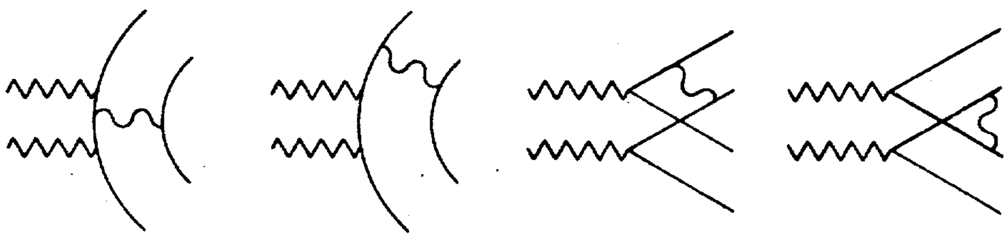
$\phi(x, Q_0) \propto \sqrt{x(1-x)}$

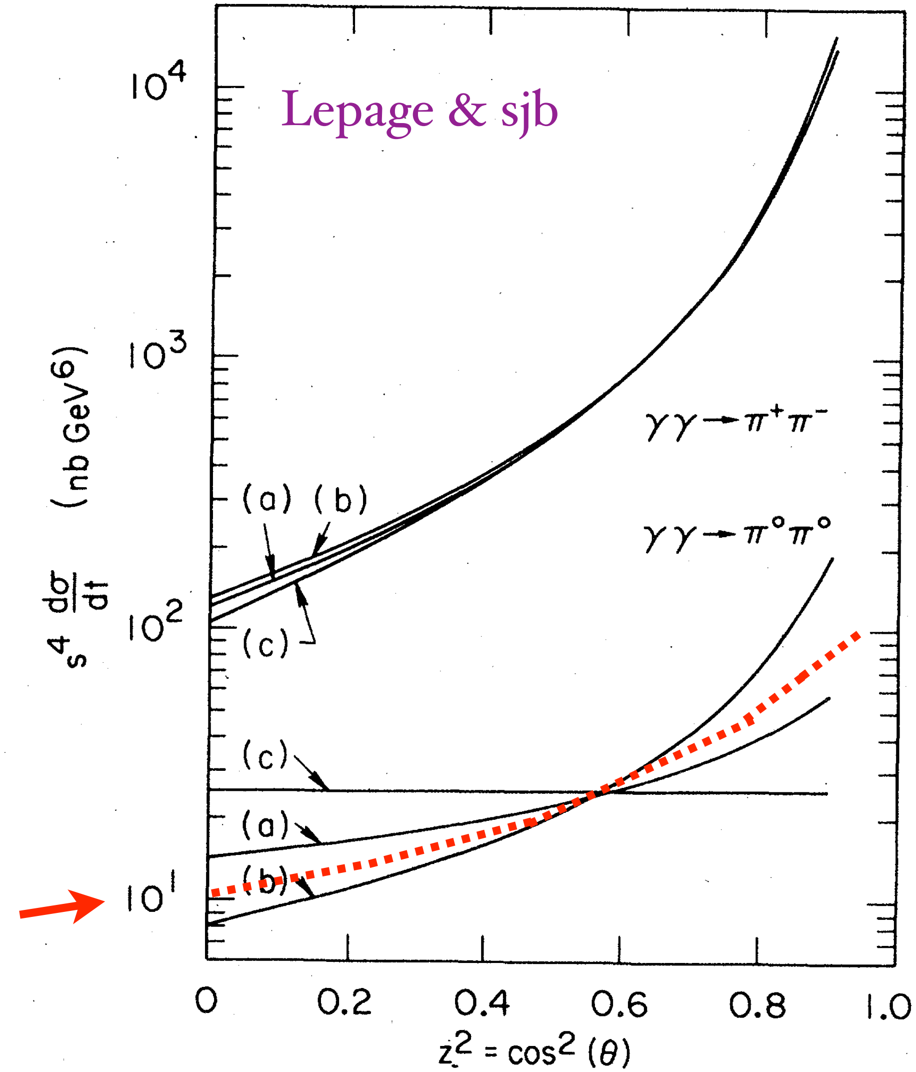
$\phi_{asymptotic} \propto x(1-x)$

Normalized to f_π

AdS/CFT: Increases PQCD leading twist prediction for $F_\pi(Q^2)$ by factor 16/9







Neutral pair angular distribution sensitive to AdS/CFT distribution!

$$\phi_\pi^{AdS/QCD}(x) \propto [x(1-x)]^{1/2}$$

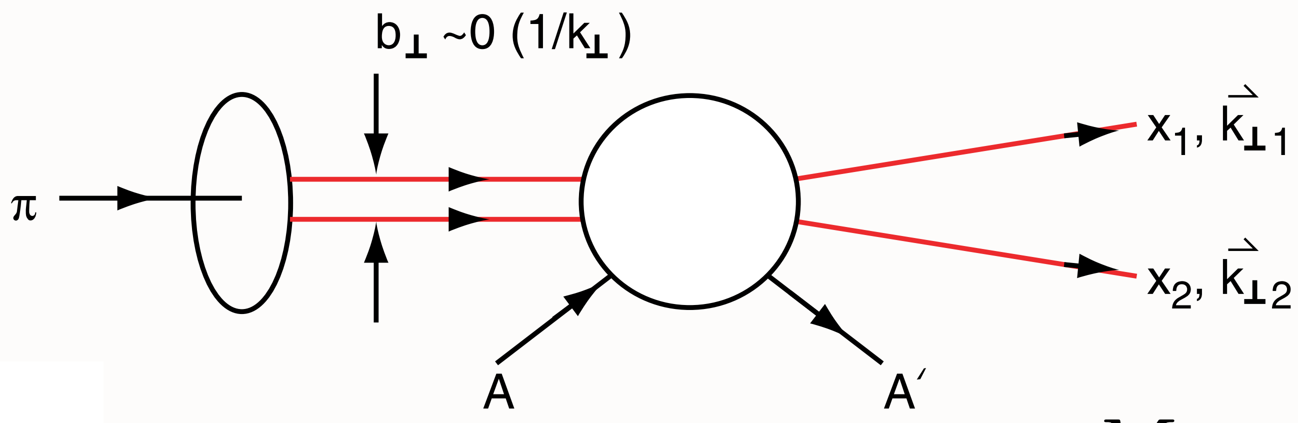
(a): $\phi_\pi(x) \propto x(1-x)$

(b): $\phi_\pi(x) \propto [x(1-x)]^{1/4}$

(c): $\phi_\pi(x) \propto \delta(x - 1/2)$

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.



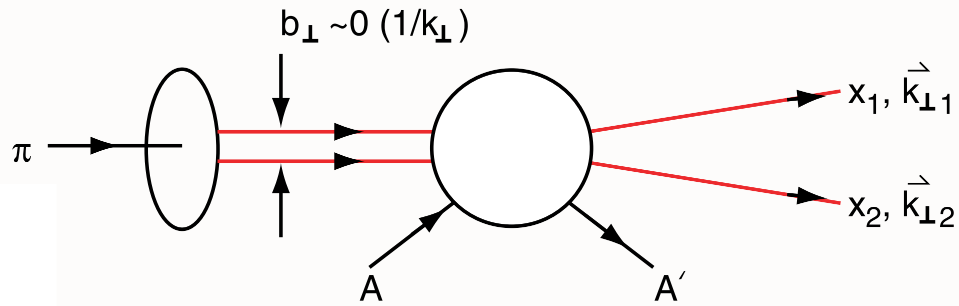
$$M \propto \frac{\partial^2}{\partial^2 k_\perp}\psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

Nucleus left Intact!

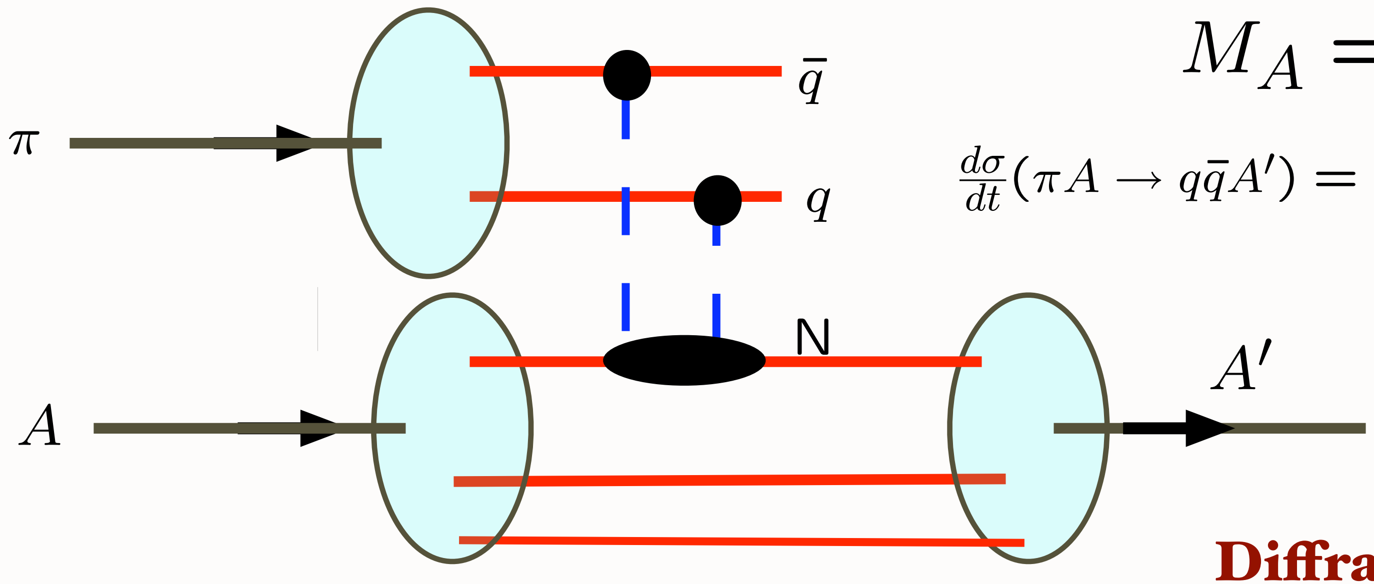
Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller Strikman

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

QCD COLOR Transparency



$$M_A = A\ M_N$$

$$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2\ \frac{d\sigma}{dt}(\pi N \to q\bar{q}N')\ F_A^2(t)$$

Target left intact

Diffraction, Rapidity gap

Color Transparency

Bertsch, Gunion, Goldhaber, sjb
A. H. Mueller, sjb

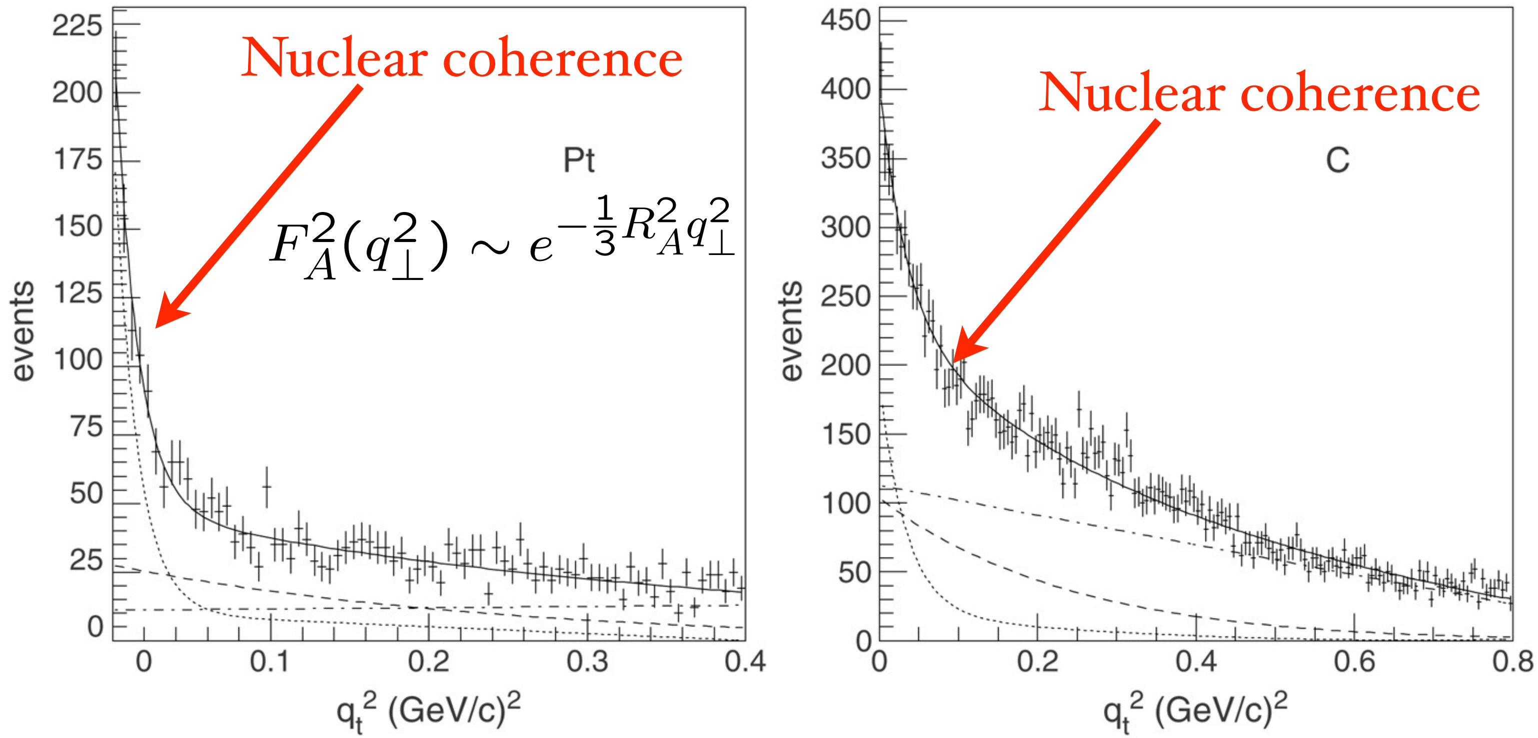
- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets

- Fully coherent interactions between pion and nucleons.
- Emerging Di-Jets do not interact with nucleus.

$$\mathcal{M}(\mathcal{A}) = \mathcal{A} \cdot \mathcal{M}(\mathcal{N})$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\sigma \propto A^{4/3}$$



Mueller, sjb; Bertsch et al;
Frankfurt, Miller, Strikman

Measure pion LFWF in diffractive dijet production
Confirmation of color transparency

A-Dependence results: $\sigma \propto A^{\alpha}$

k_t range (GeV/c)	α	α (CT)
$1.25 < k_t < 1.5$	1.64 +0.06 -0.12	1.25
$1.5 < k_t < 2.0$	1.52 ± 0.12	1.45
$2.0 < k_t < 2.5$	1.55 ± 0.16	1.60

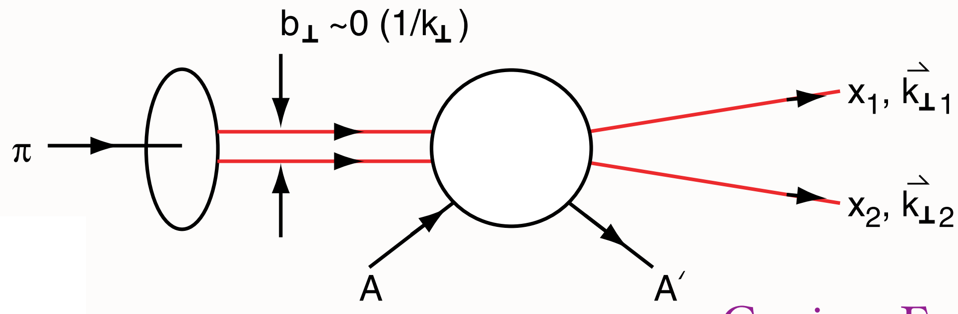
Ashery E791

α (Incoh.) = 0.70 ± 0.1

Conventional Glauber Theory Ruled Out !

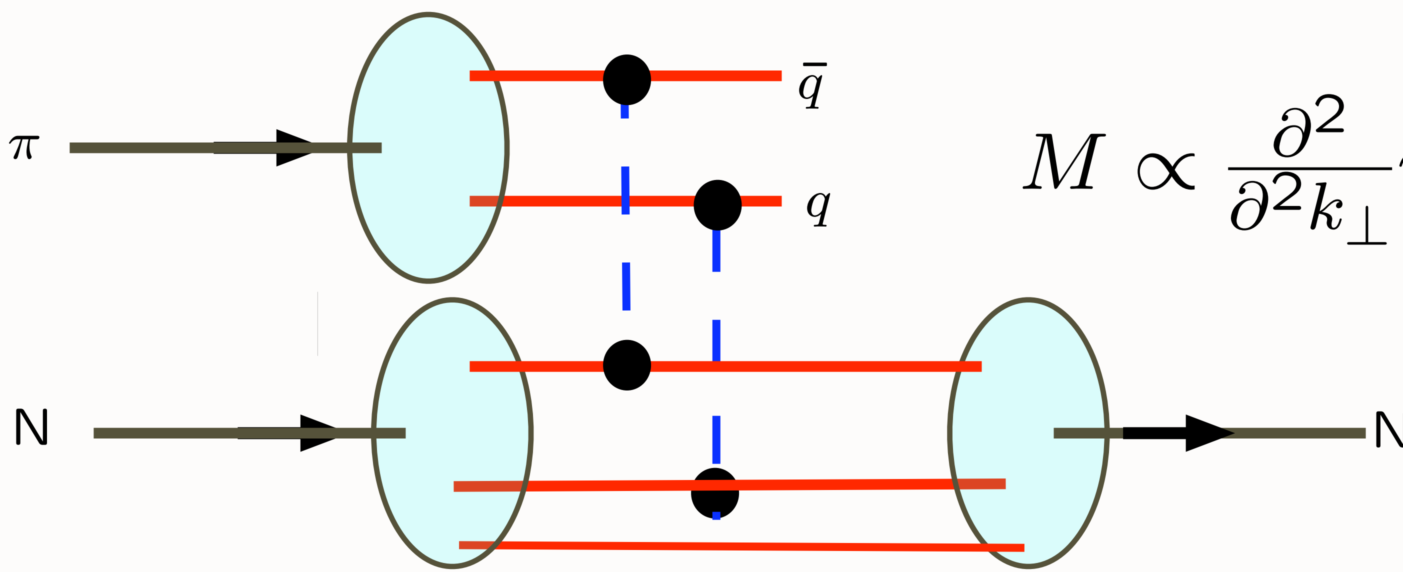
Factor of 7

Key Ingredients in Ashery Experiment



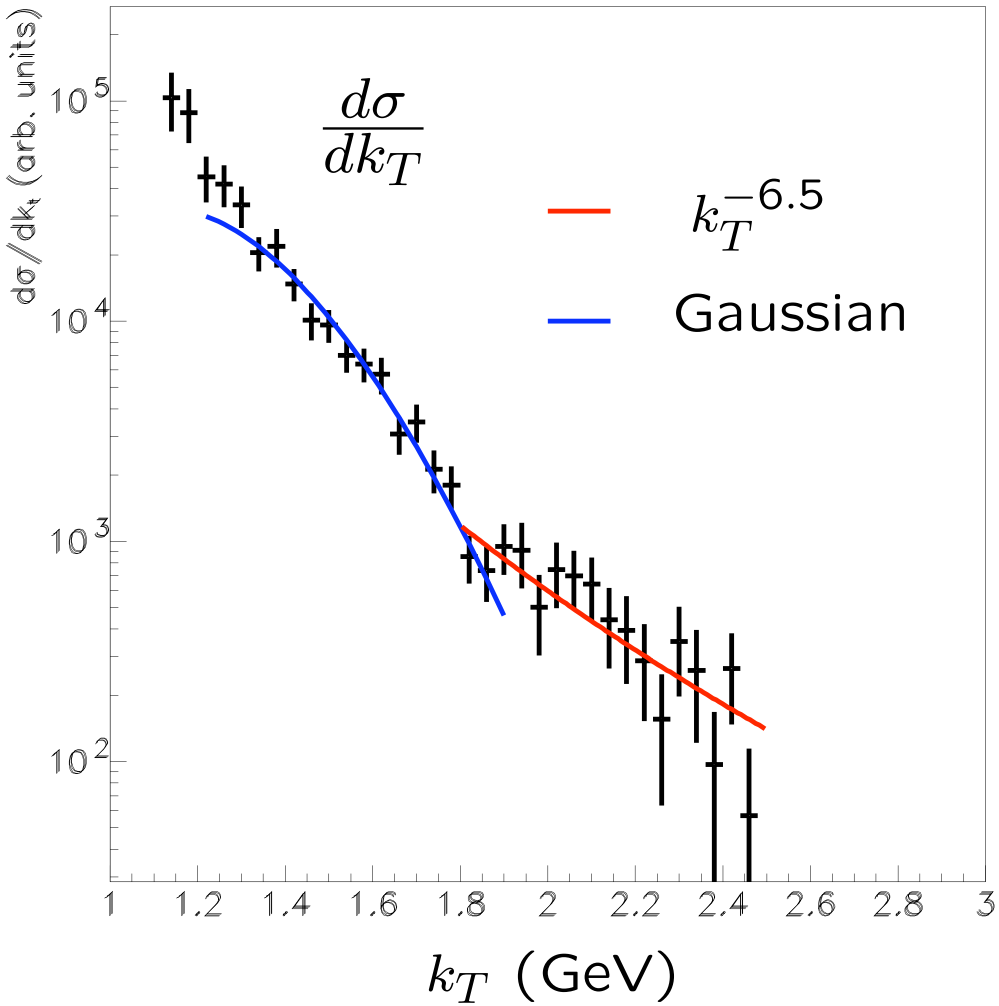
Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction



$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

E791 Diffractive Di-Jet transverse momentum distribution

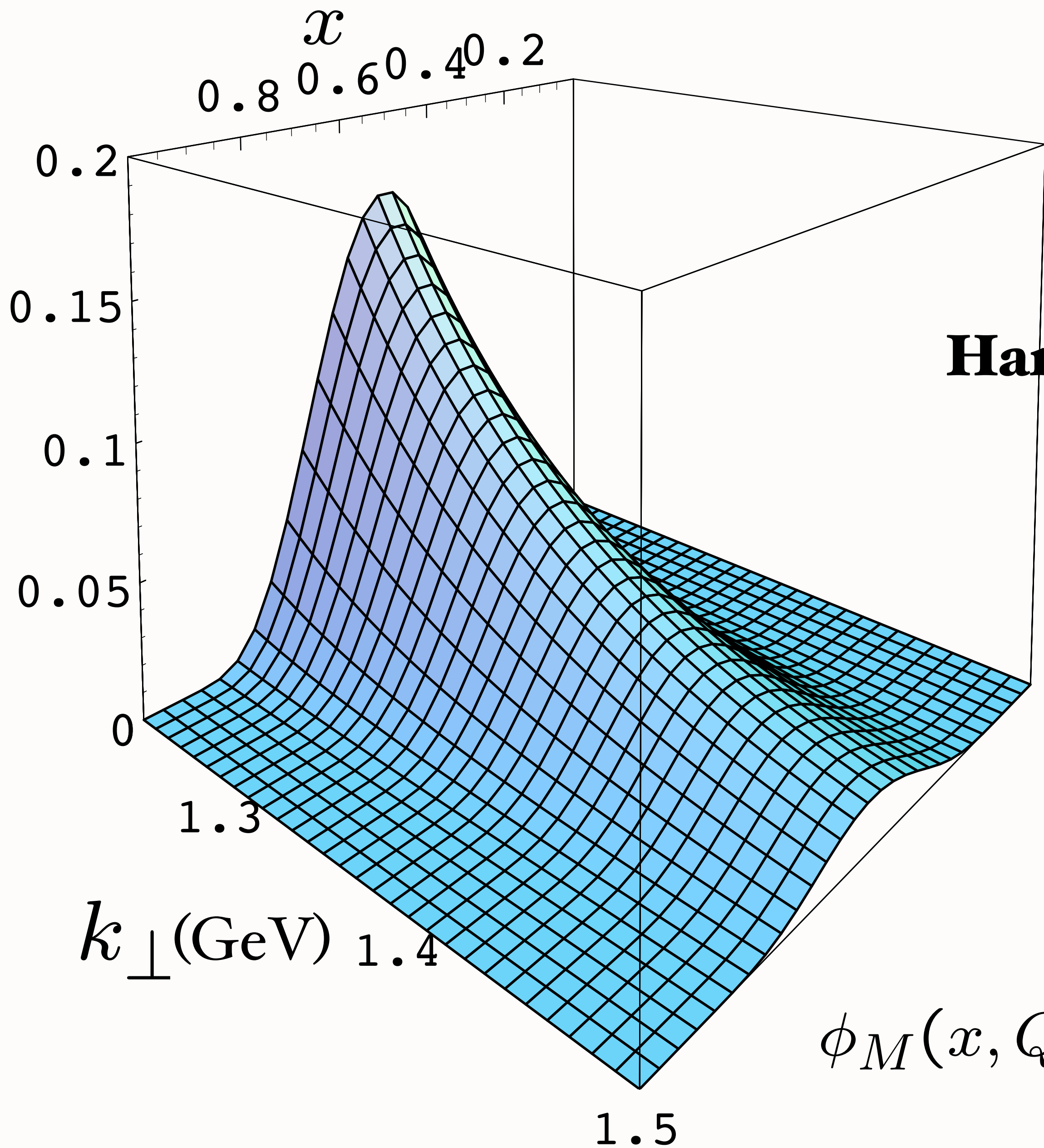


Two Components

High Transverse momentum dependence $k_T^{-6.5}$ *consistent with PQCD, ERBL Evolution*

Gaussian component similar to AdS/CFT HO LFWF

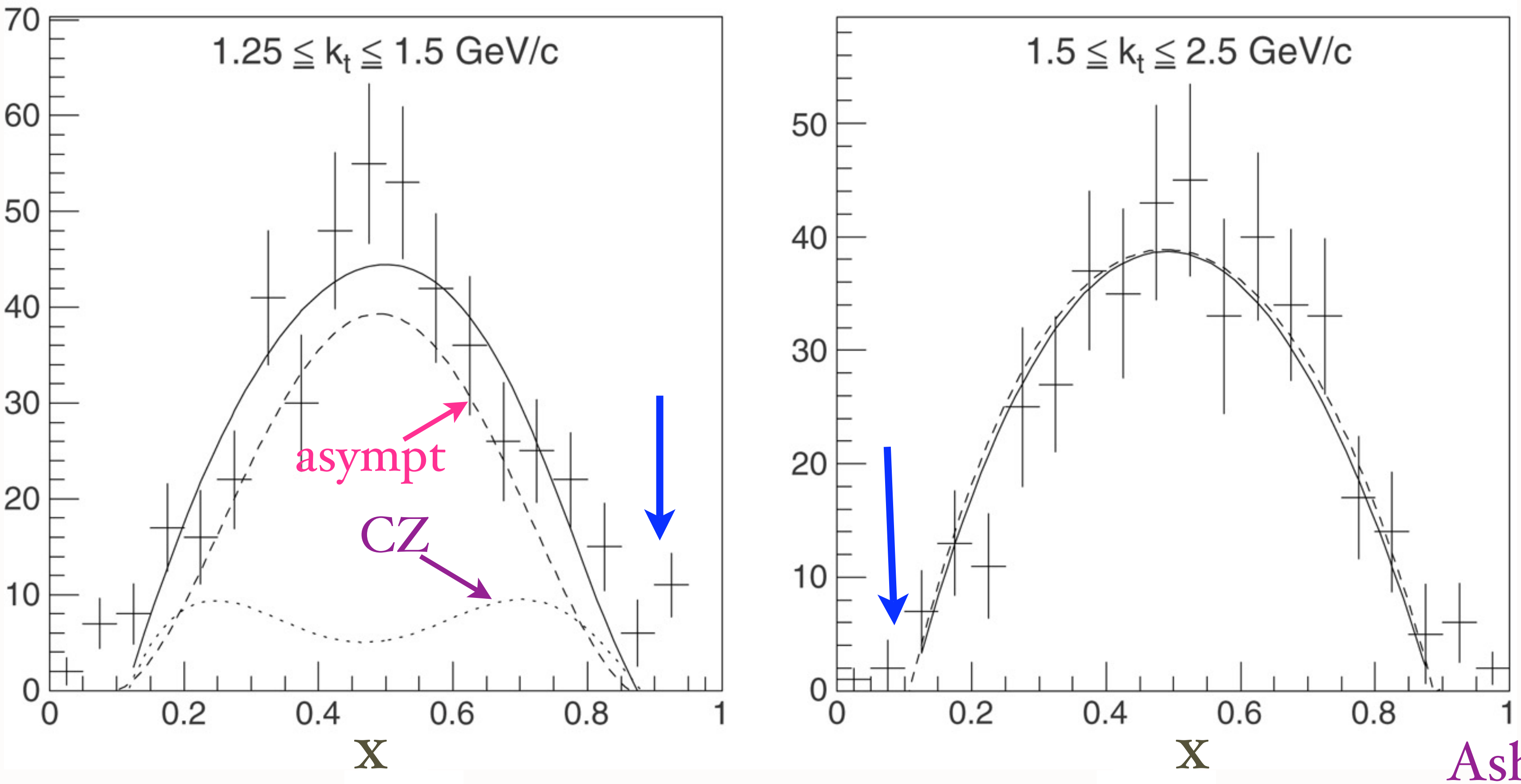
Prediction from AdS/CFT: Meson LFWF



Harmonic oscillator model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$



Ashery E791

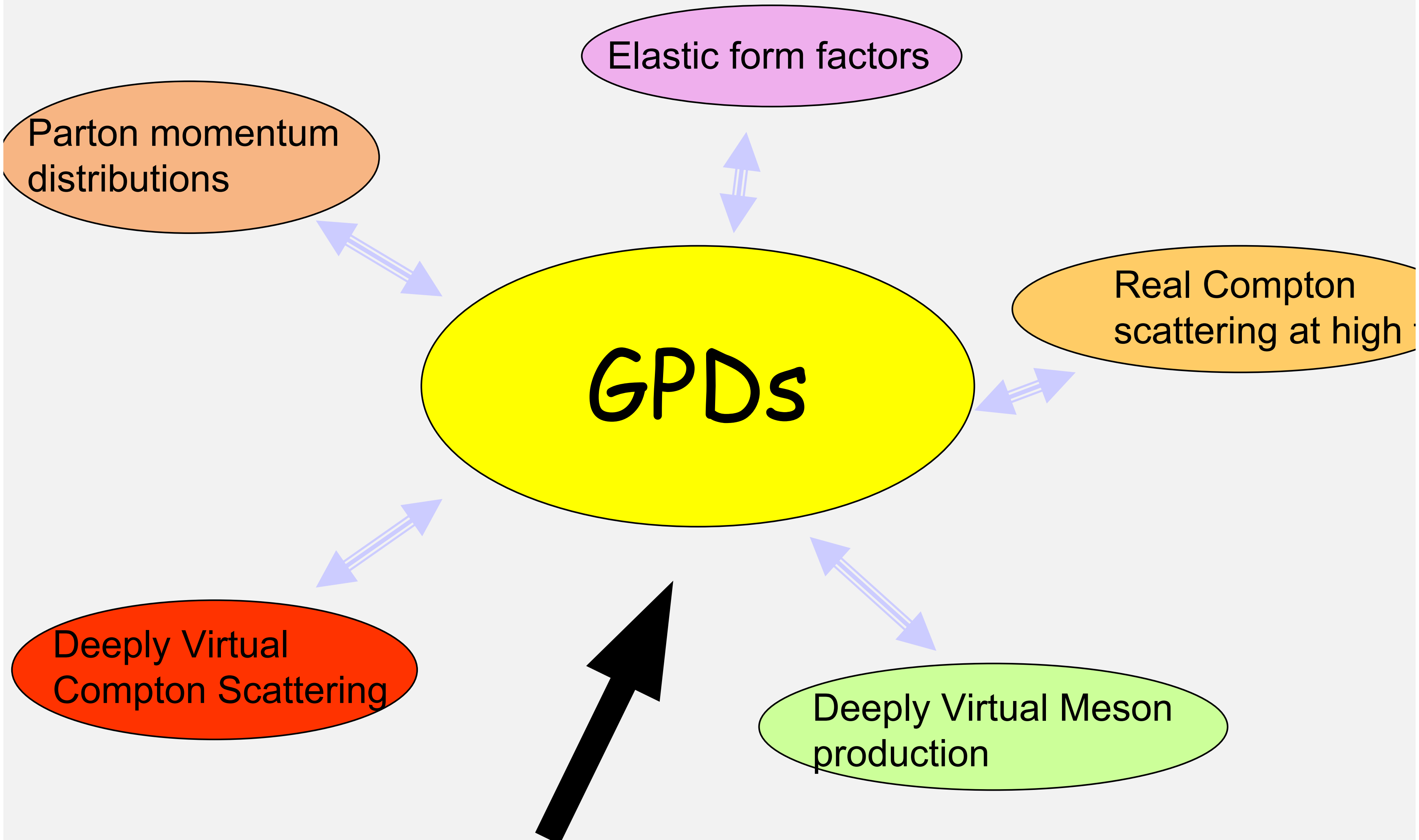
Narrowing of x distribution at higher jet transverse momentum

x distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/c (left) and for $1.5 \leq k_t \leq 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components:
Nonperturbative (AdS/CFT) and
Perturbative (ERBL)
Evolution to asymptotic distribution

$$\phi(x) \propto \sqrt{x(1-x)}$$

A Unified Description of Hadron Structure



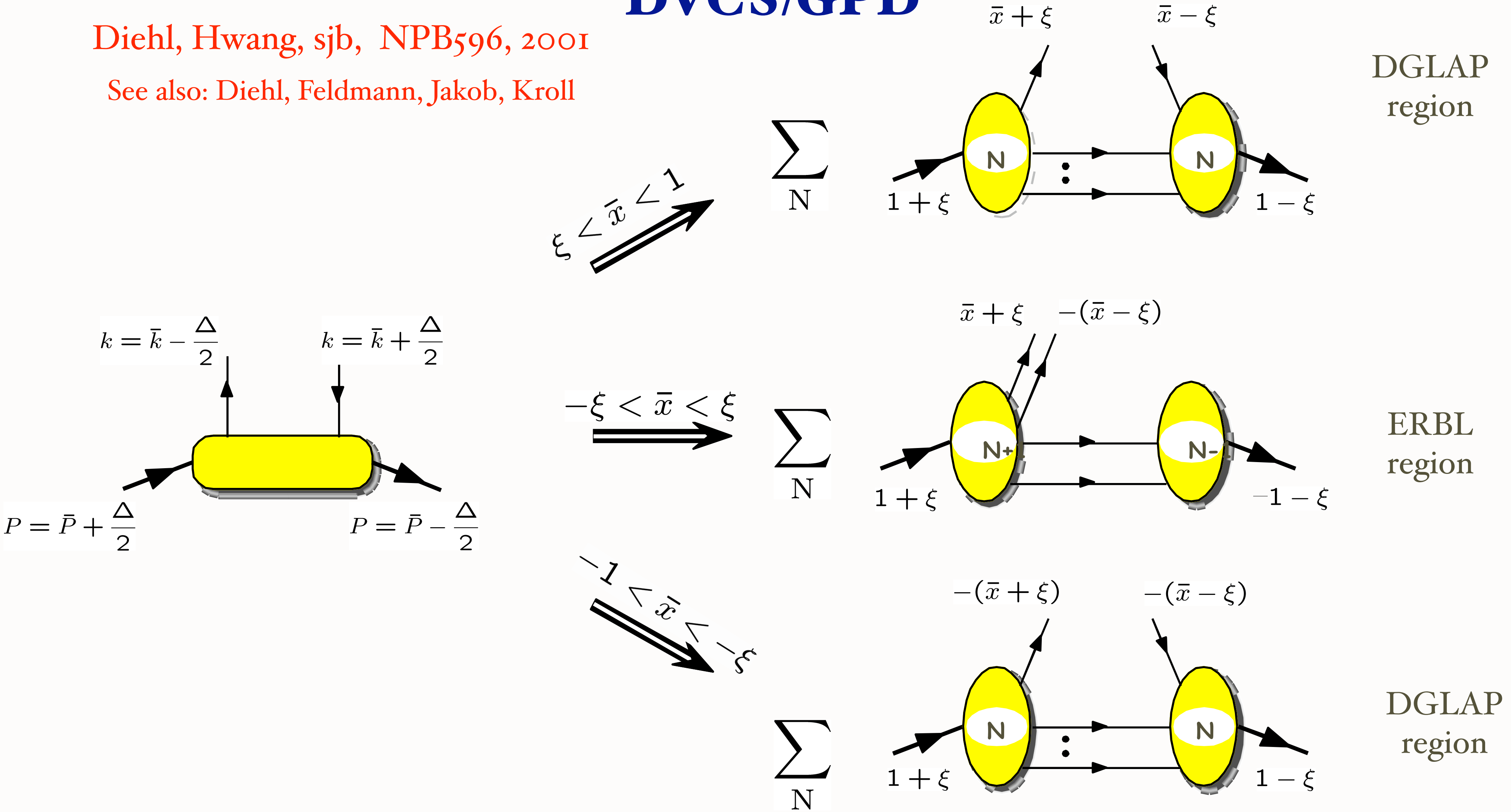
Light Front Wavefunctions

Light-Front Wave Function Overlap Representation

DVCS/GPD

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll



N=3 VALENCE QUARK ⇒ Light-cone Constituent quark model

N=5 VALENCE QUARK + QUARK SEA ⇒ Meson-Cloud model

Example of LFWF representation of GPDs (n => n)

Diehl, Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\,\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\, 16\pi^3 \delta\left(1-\sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$

$$\times\, \delta(x-x_1)\, \psi^{\uparrow *}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i),$$

where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1-\zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2, \ldots, n.$$

Example of LFWF representation of GPDs (n+1 => n-1)

Diehl, Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\Delta^2}{2M} E_{(n+1\to n-1)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{3-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n+1} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3} 16\pi^3 \delta\left(1 - \sum_{j=1}^{n+1} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n+1} \vec{k}_{\perp j}\right)$$

$$\times 16\pi^3 \delta(x_{n+1} + x_1 - \zeta) \delta^{(2)}(\vec{k}_{\perp n+1} + \vec{k}_{\perp 1} - \vec{\Delta}_\perp)$$

$$\times \delta(x - x_1) \psi^{\uparrow *}_{(n-1)}(x'_i, \vec{k}'_{\perp i}, \lambda_i) \psi^{\downarrow}_{(n+1)}(x_i, \vec{k}_{\perp i}, \lambda_i) \delta_{\lambda_1 - \lambda_{n+1}},$$

where $i = 2, \ldots, n$ label the $n-1$ spectator partons which appear in the final-state hadron wavefunction with

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta} \vec{\Delta}_\perp.$$

Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$

$$\widetilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int_{-1}^{1} dx \sum_q \left[H^q(x,\xi,t) \right] = F_1(t) \text{ Dirac f.f.}$$

$$\int_{-1}^{1} dx \sum_q \left[E^q(x,\xi,t) \right] = F_2(t) \text{ Pauli f.f.}$$

$$\int_{-1}^{1} dx\, \widetilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^{1} dx\, \widetilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

$$H^q, E^q, \widetilde{H}^q, \widetilde{E}^q(x,\xi,t)$$

Verified using LFWFs

Diehl, Hwang, sjb

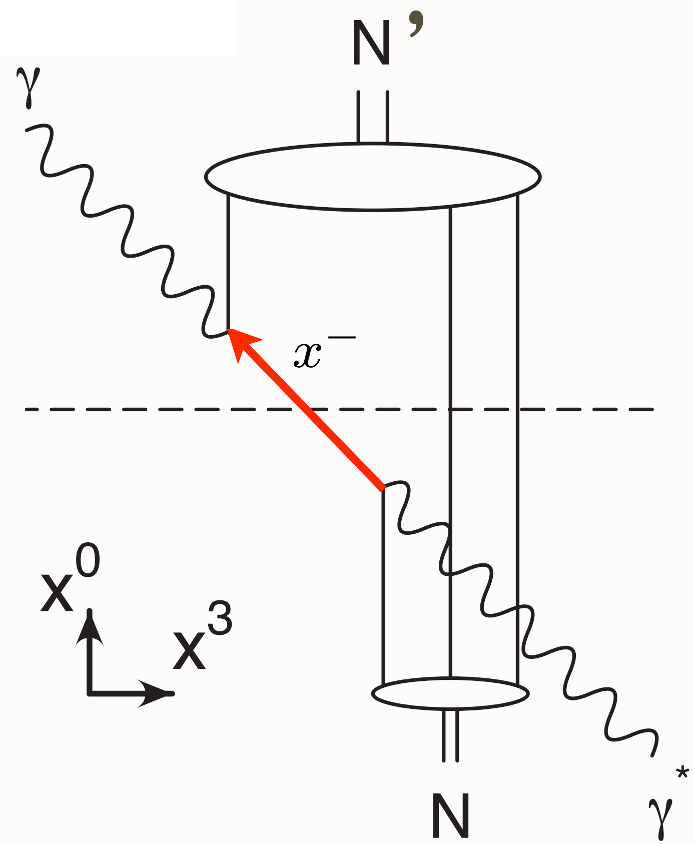
Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2} \int_{-1}^{1} x\,dx \left[H^q(x,\xi,0) + E^q(x,\xi,0) \right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

Space-time picture of DVCS

P. Hoyer



$x^+ = \mathbf{x}_\perp = 0$)

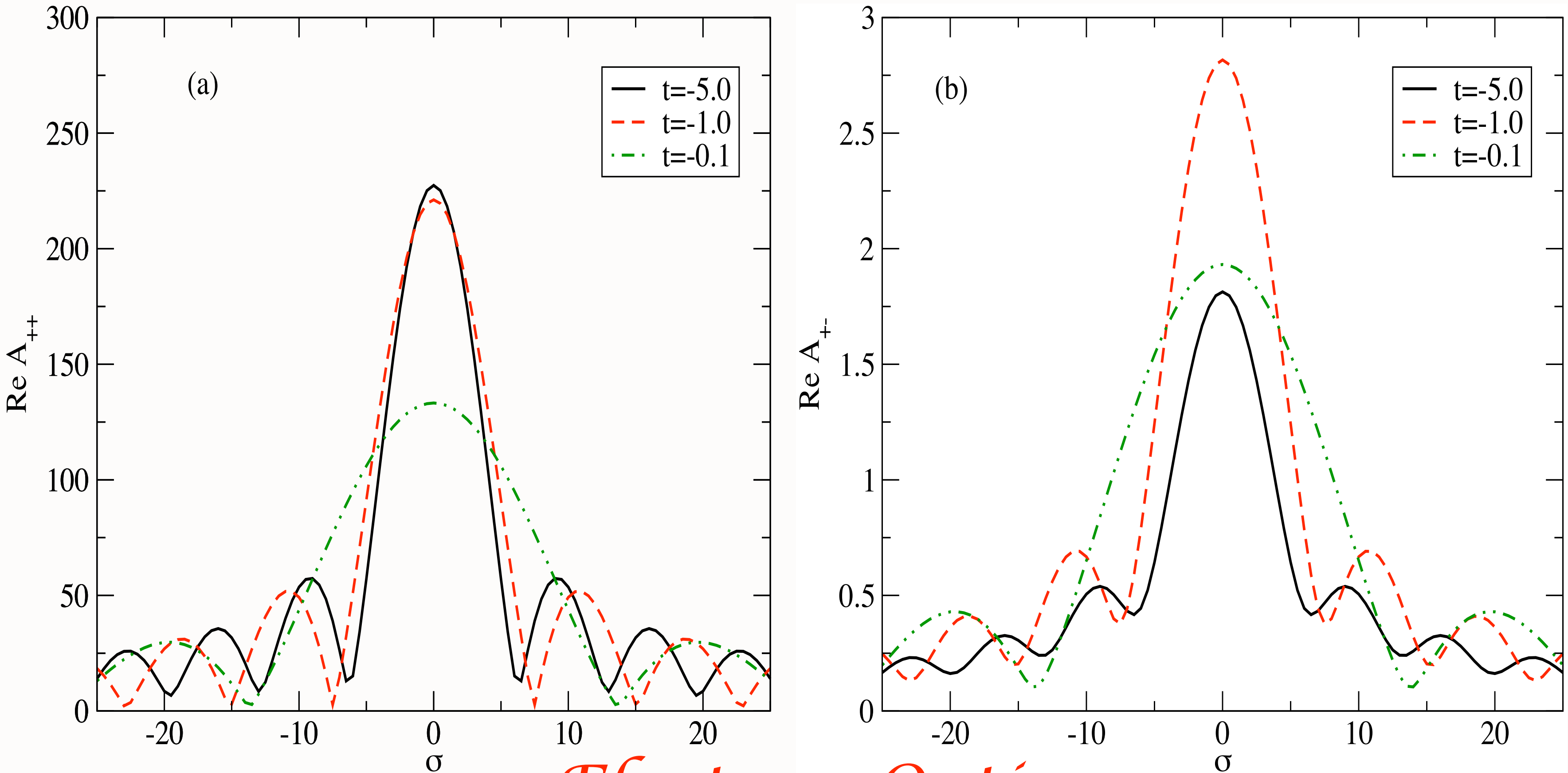
$\sigma = \frac{1}{2} x^- P^+$

The position of the struck quark differs by x^- in the two wave functions

Measure x- distribution from DVCS:
Take Fourier transform of skewness, $\xi = \frac{Q^2}{2p.q}$
the longitudinal momentum transfer

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]



Electron Optics

Fourier spectrum of the real part of the DVCS amplitude of an electron vs. σ for $M = 0.51$ MeV, $m = 0.5$ MeV, $\lambda = 0.02$ MeV, (a) when the electron helicity is not flipped; (b) when the helicity is flipped. The parameter t is in MeV^2.

$$\sigma = \frac{1}{2}x^- P^+$$

$$A(\sigma, \vec{\Delta}_\perp) = \frac{1}{2\pi} \int d\xi e^{i\frac{1}{2}\xi\sigma} M(\xi, \vec{\Delta}_\perp)$$

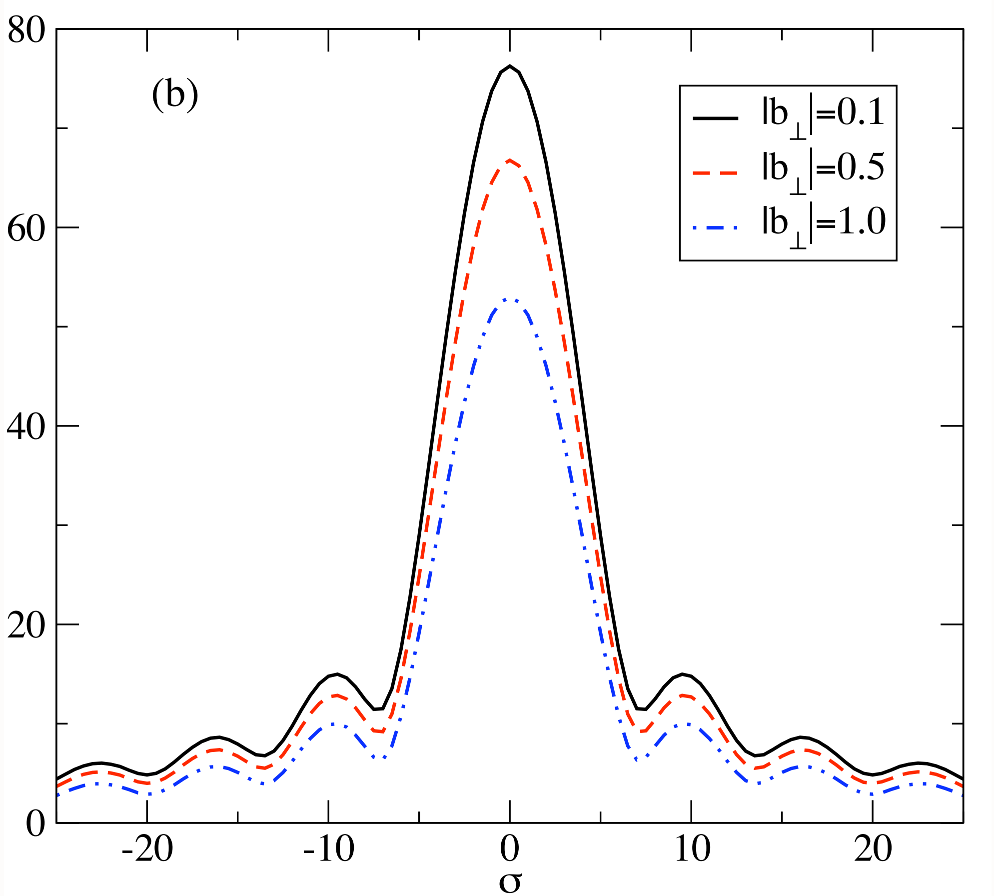
$$\xi = \frac{Q^2}{2p.q}$$

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

Hadron Optics

$$A(\sigma, \vec{b}_\perp) = \frac{1}{2\pi} \int d\xi e^{i\frac{1}{2}\xi\sigma} \tilde{A}(\xi, \vec{b}_\perp)$$

$$\sigma = \frac{1}{2} x^- P^+ \quad \xi = \frac{Q^2}{2p.q}$$

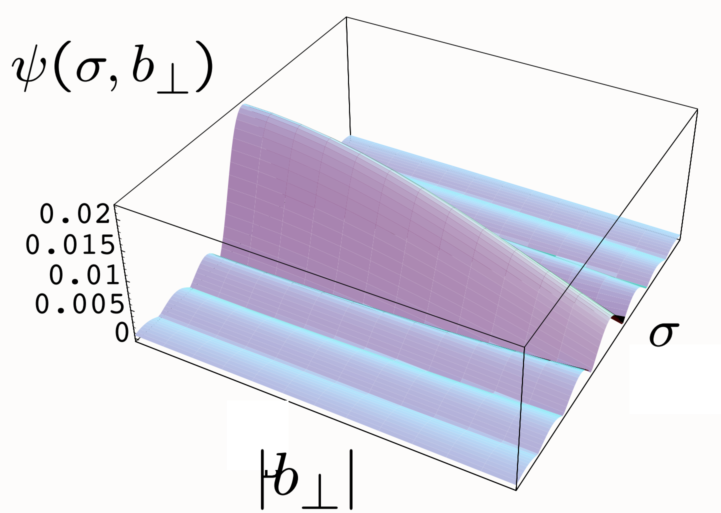


The Fourier Spectrum of the DVCS amplitude in σ space for different fixed values of $|b_\perp|$.

GeV units

DVCS Amplitude using holographic QCD meson LFWF

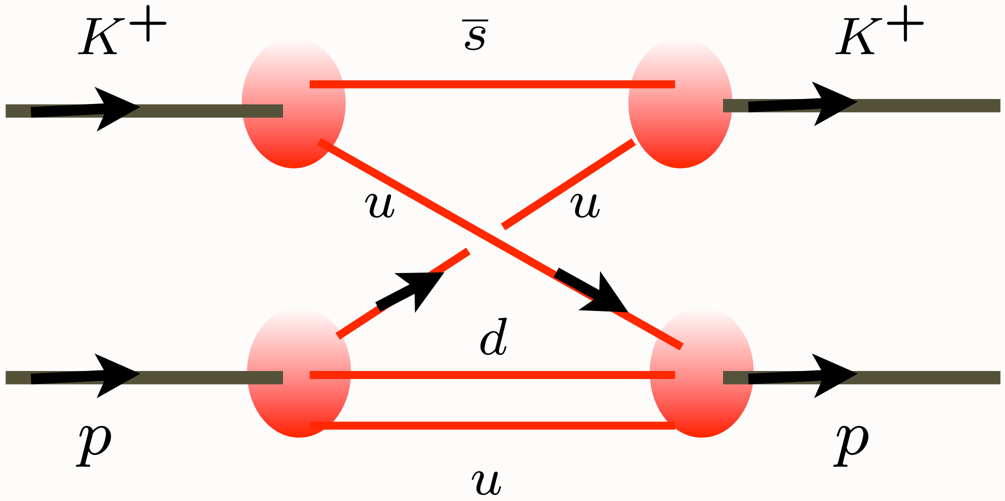
$$\Lambda_{QCD} = 0.32$$



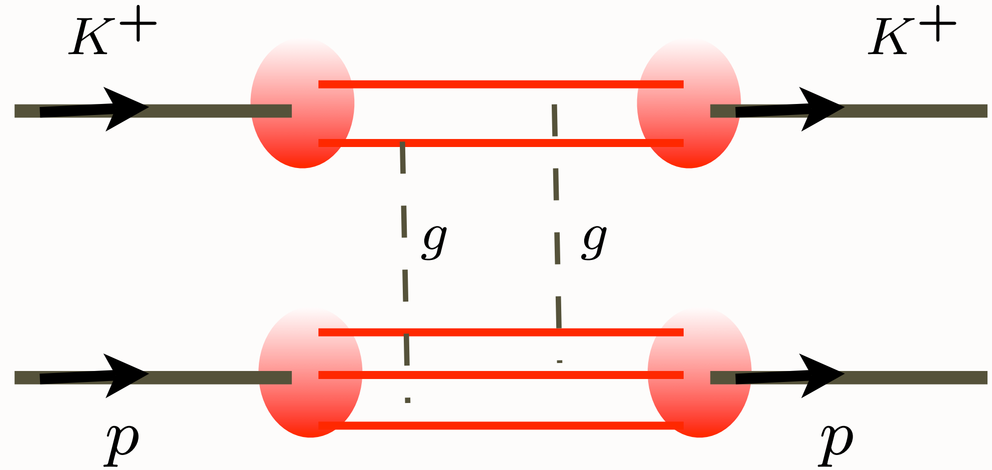
New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

CIM: Blankenbecler, Gunion, sjb



Quark Interchange
(Spin exchange in atom-atom scattering)



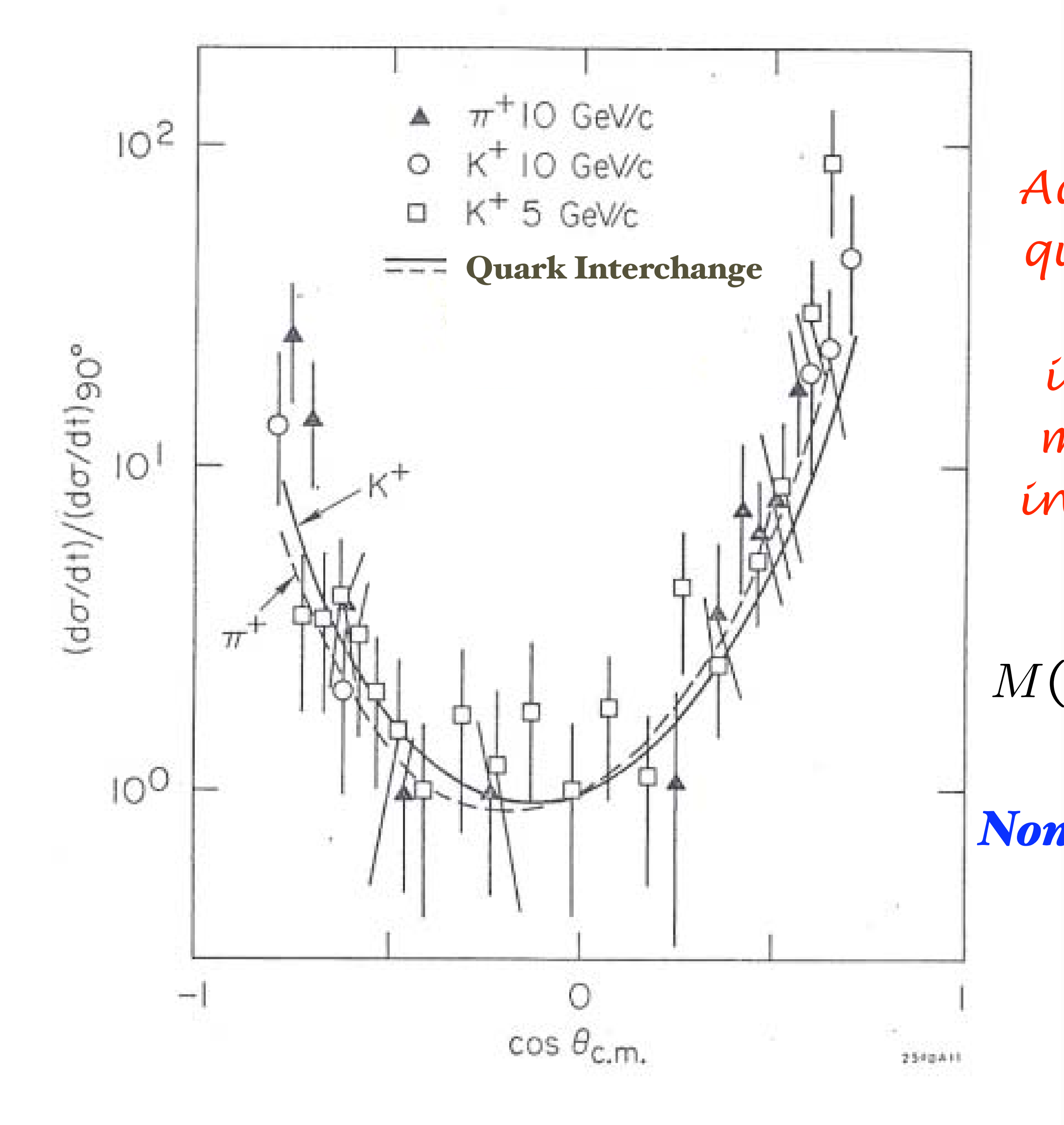
Gluon Exchange
(Van der Waal -- Landshoff)

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$

$M(s,t)_{\text{gluonexchange}} \propto sF(t)$

MIT Bag Model (de Tar), large N_C, ('t Hooft), AdS/CFT all predict dominance of quark interchange:



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$

Why is quark-interchange dominant over gluon exchange?

Example: $M(K^+p \to K^+p) \propto \frac{1}{ut^2}$

Exchange of common u quark

$M_{QIM} = \int d^2k_\perp dx \ \psi^\dagger_C \psi^\dagger_D \Delta \psi_A \psi_B$

Holographic model (Classical level):

Hadrons enter 5th dimension of AdS_5

Quarks travel freely within cavity as long as separation $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.

Comparison of Exclusive Reactions at Large t

B. R. Baller,[a] G. C. Blazey,[b] H. Courant, K. J. Heller, S. Heppelmann,[c] M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl[d]
University of Minnesota, Minneapolis, Minnesota 55455

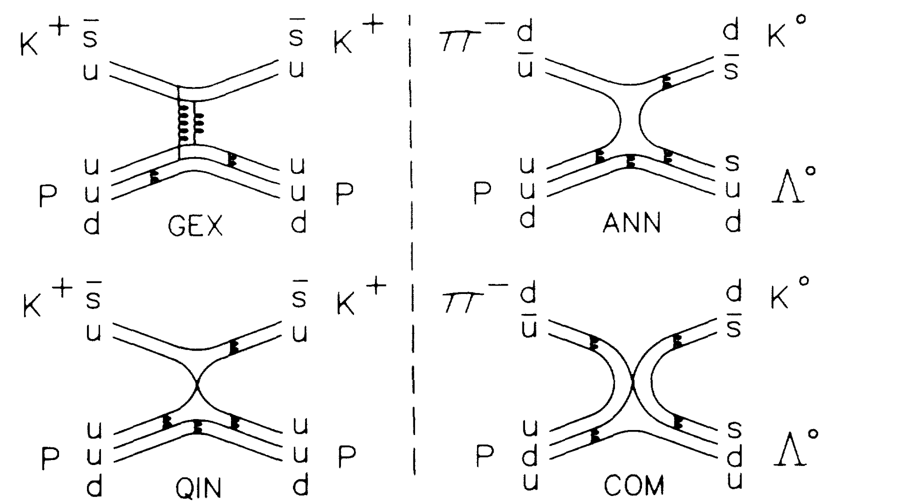
D. S. Barton, G. Bunce, A. S. Carroll, and Y. I. Makdisi
Brookhaven National Laboratory, Upton, New York 11973

and

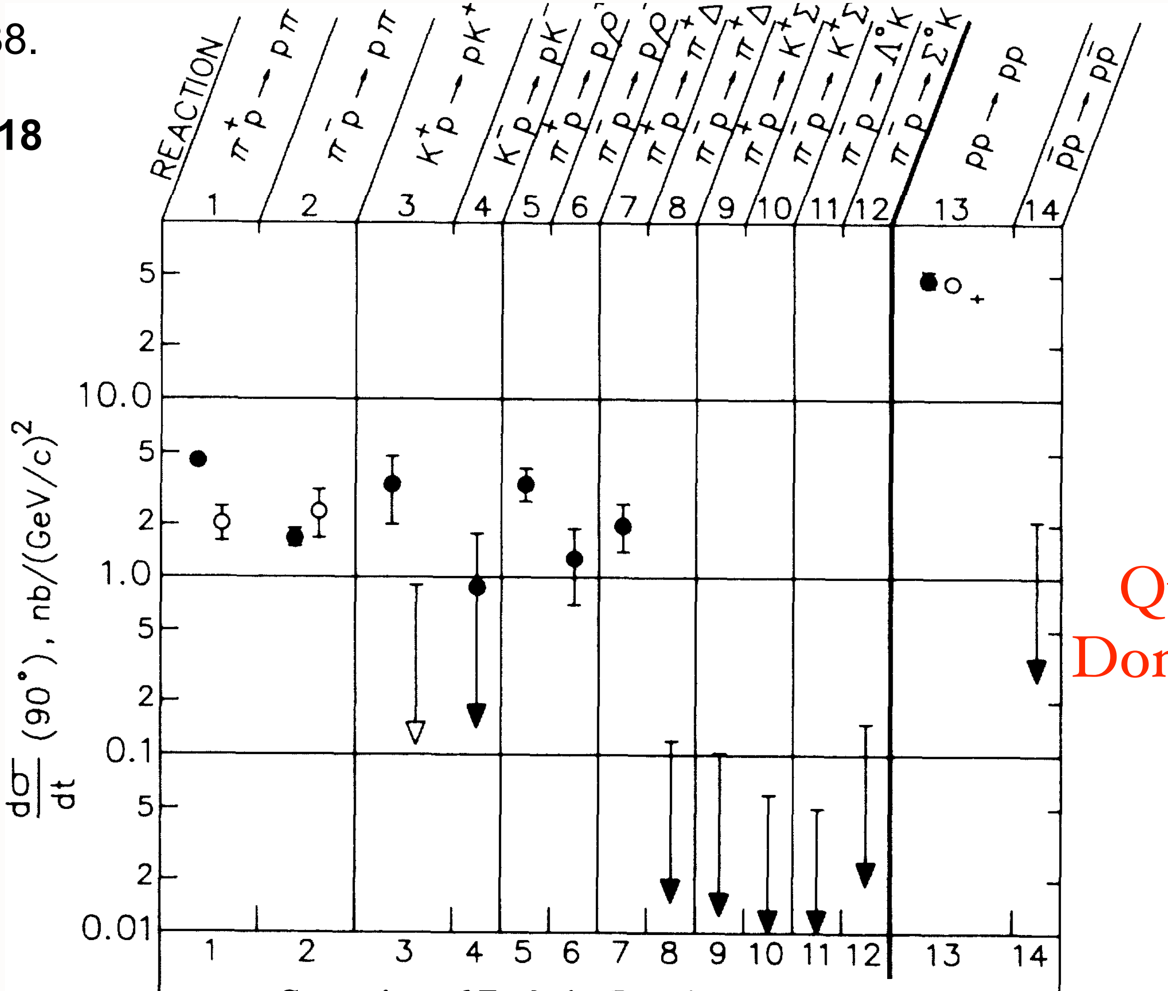
S. Gushue[e] and J. J. Russell
Southeastern Massachusetts University, North Dartmouth, Massachusetts 02747
(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^{\pm}p\to p\pi^{\pm}, p\rho^{\pm}, \pi^{+}\Delta^{\pm}, K^{+}\Sigma^{\pm}, (\Lambda^0/\Sigma^0)K^0$; $K^{\pm}p\to pK^{\pm}$; $p^{\pm}p\to pp^{\pm}$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

$$\pi^{\pm}p\to p\pi^{\pm},$$
$$K^{\pm}p\to pK^{\pm},$$
$$\pi^{\pm}p\to p\rho^{\pm},$$
$$\pi^{\pm}p\to \pi^{+}\Delta^{\pm},$$
$$\pi^{\pm}p\to K^{+}\Sigma^{\pm},$$
$$\pi^{-}p\to \Lambda^0K^0, \Sigma^0K^0,$$
$$p^{\pm}p\to pp^{\pm}.$$



B.R. Baller *et al.*. 1988. Published in **Phys.Rev.Lett.60:1118-1121,1988**



Comparison of Exclusive Reactions at Large *t*

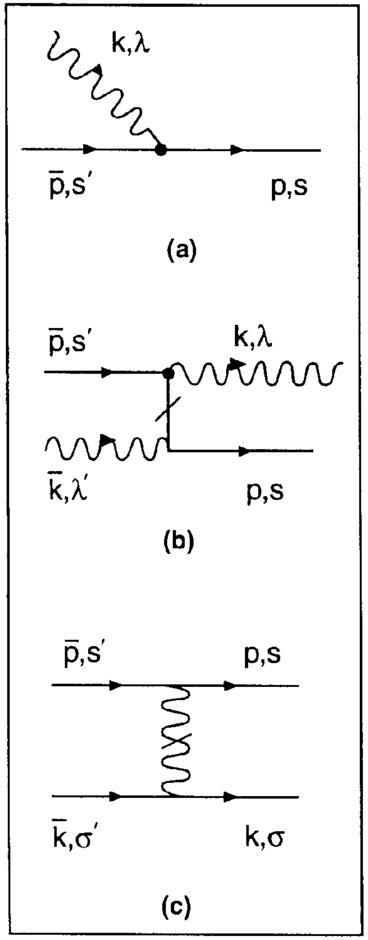
The cross section and upper limits (90% confidence level) measured by this experiment are indicated by the filled circles and arrowheads. Values from this experiment and from previous measurements represent an average over the angular region of $-0.05 < \cos\theta_{c.m.} < 0.10$. The other measurements were obtained from the following references: $\pi^+ p$ and $K^+ p$ elastic, Ref. 5; $\pi^- p \to p\pi^-$, Ref. 6; $pp \to pp$, Ref. 7: Allaby, open circle; Akerlof, cross. Values for the cross sections [(Reaction), cross section in nb/(GeV/c)^2] are as follows: (1), 4.6 ± 0.3; (2), 1.7 ± 0.2; (3), 3.4 ± 1.4; (4), $0.9^{+0.7}_{-0.9}$; (5), 3.4 ± 0.7; (6), 1.3 ± 0.6; (7), 2.0 ± 0.6; (8), <0.12; (9), <0.1; (10), <0.06; (11), <0.05; (12), <0.15; (13), 48 ± 5; (14), <2.1.

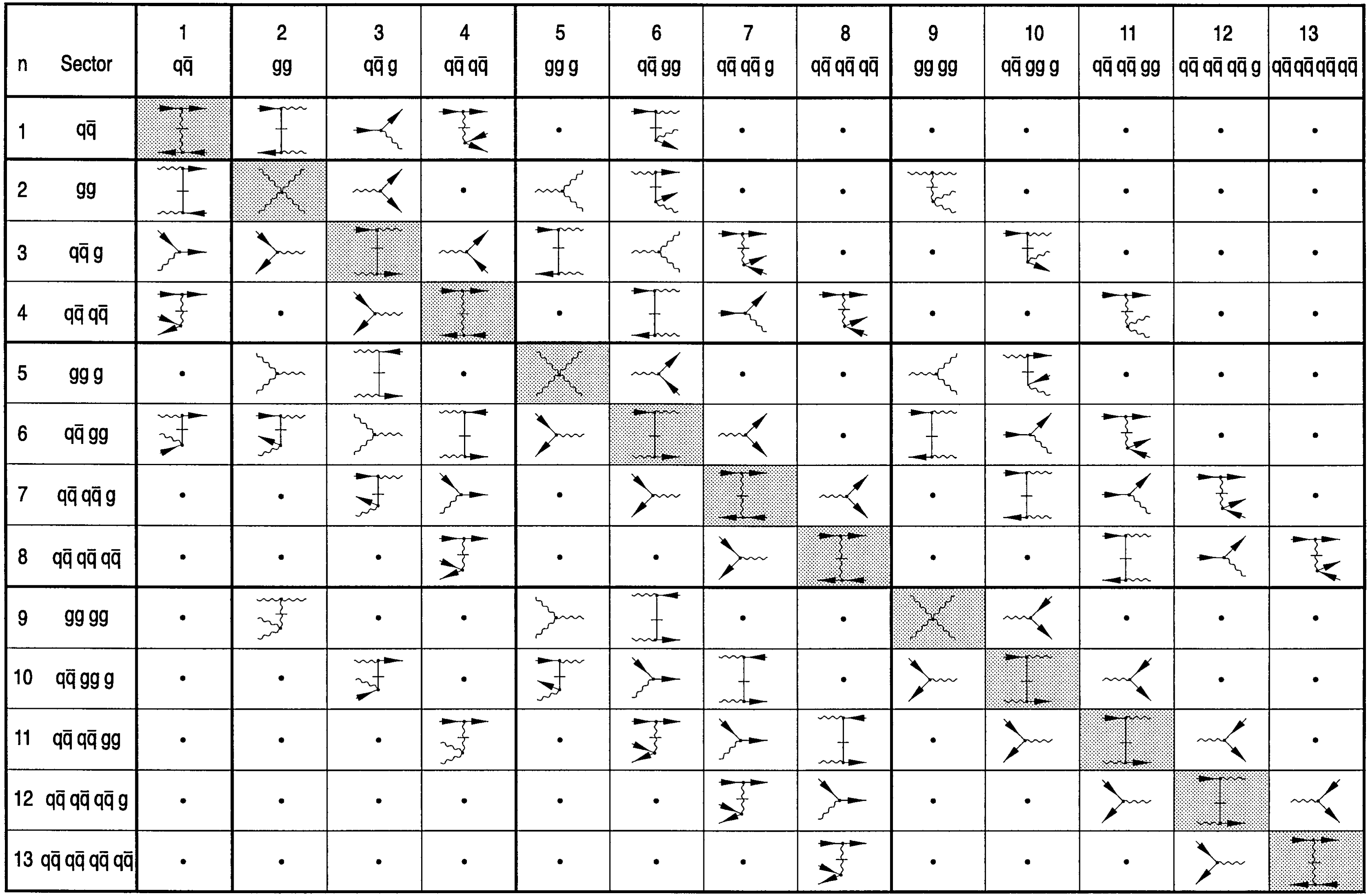
Quark Interchange: Dominant Dynamics at large t, u

Relative Rates Correct

Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\;|\Psi_h\rangle$$





Use AdS/QCD basis functions

Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis — Pauli, Hornbostel, Hiller, McCartor, sjb
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion — Vary, Harinandrath, Maris, sjb
- Similar to Shell Model calculations

Maris, Vary, sjb

$\alpha = \frac{e_e^2}{4\pi}, Z\alpha = \frac{e_\mu e_e}{4\pi}$

Semi-Classical LF Hamiltonian

Precision QED calculation of muonium and hydrogenic atom spectroscopy

$$E_n = E_n(Z\alpha, \alpha)$$

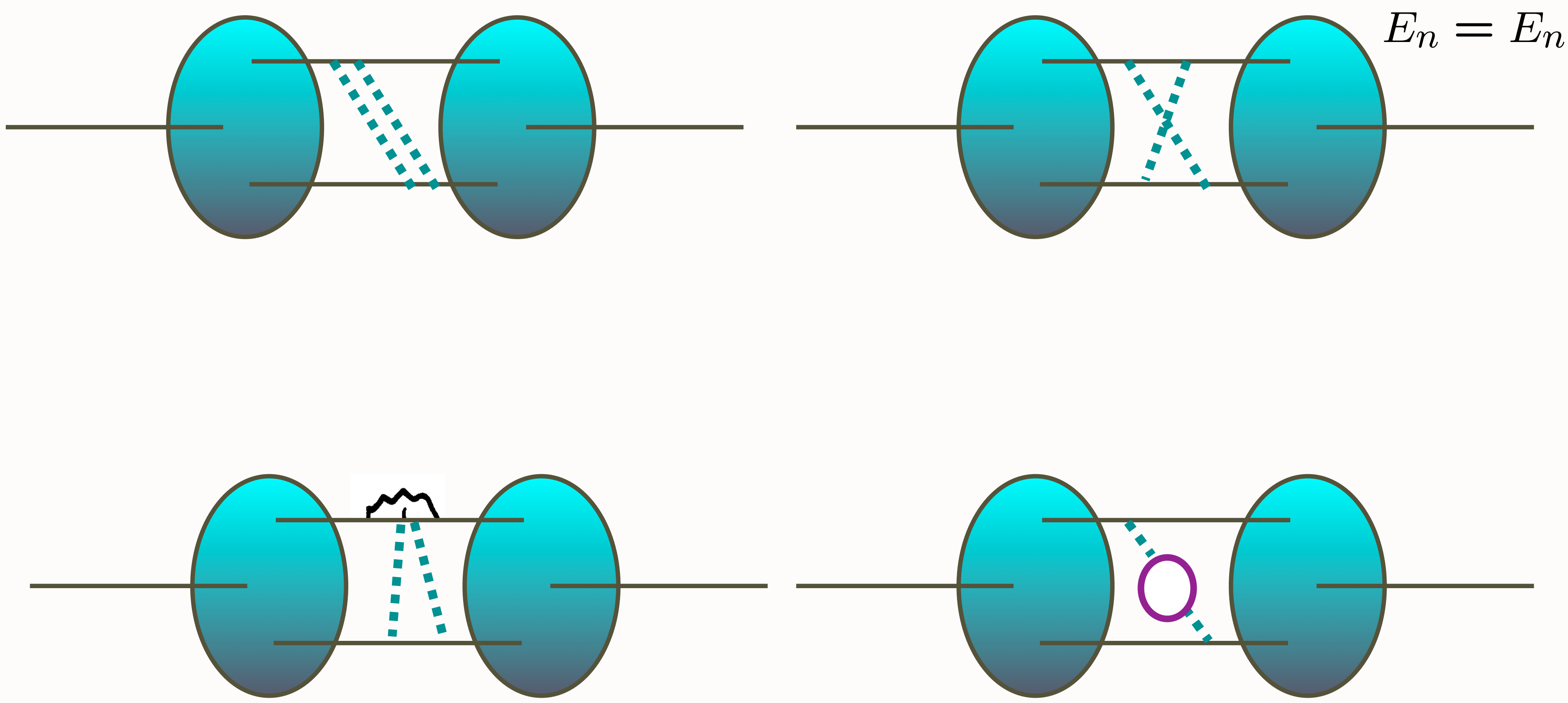
Semiclassical theory

$$E_n = E_n(Z\alpha, \alpha = 0)$$

No Lamb Shift, Renormalization

Muonium and Hydrogenic Atoms

$$E_n = E_n(Z\alpha, \alpha)$$



Lamb Shift

Vacuum Polarization

Hiller, Hwang, Karmanov, sjb

Angular Momentum on the Light-Front

A^+=0 gauge: No unphysical degrees of freedom

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

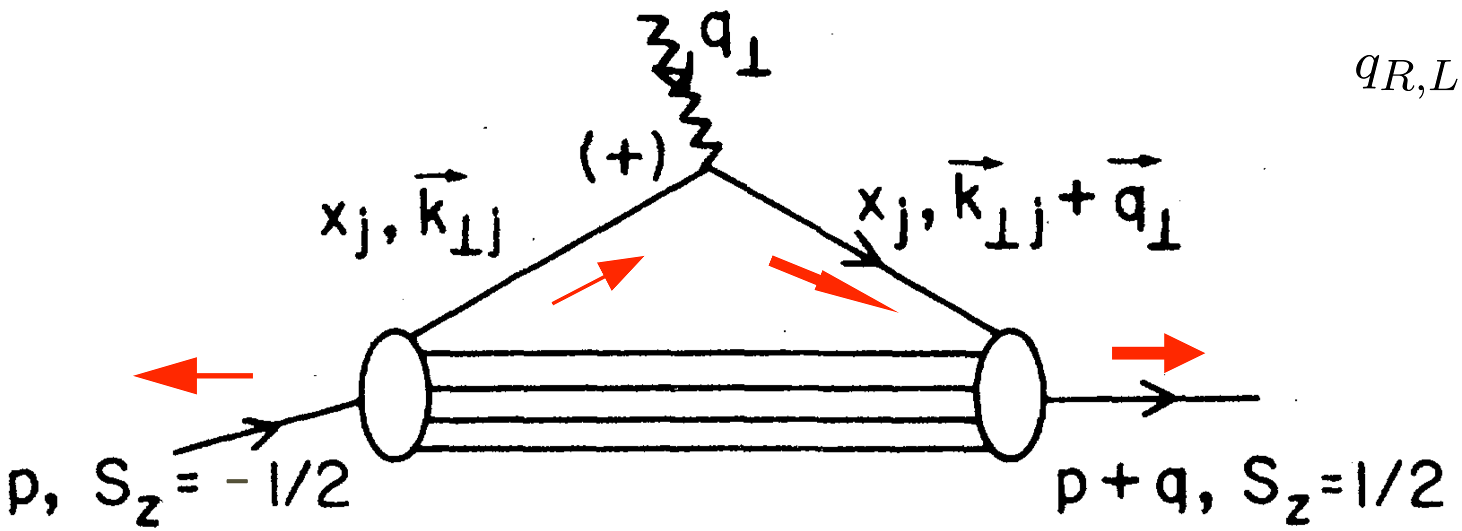
Nonzero Anomalous Moment requires
Nonzero orbital angular momentum

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$



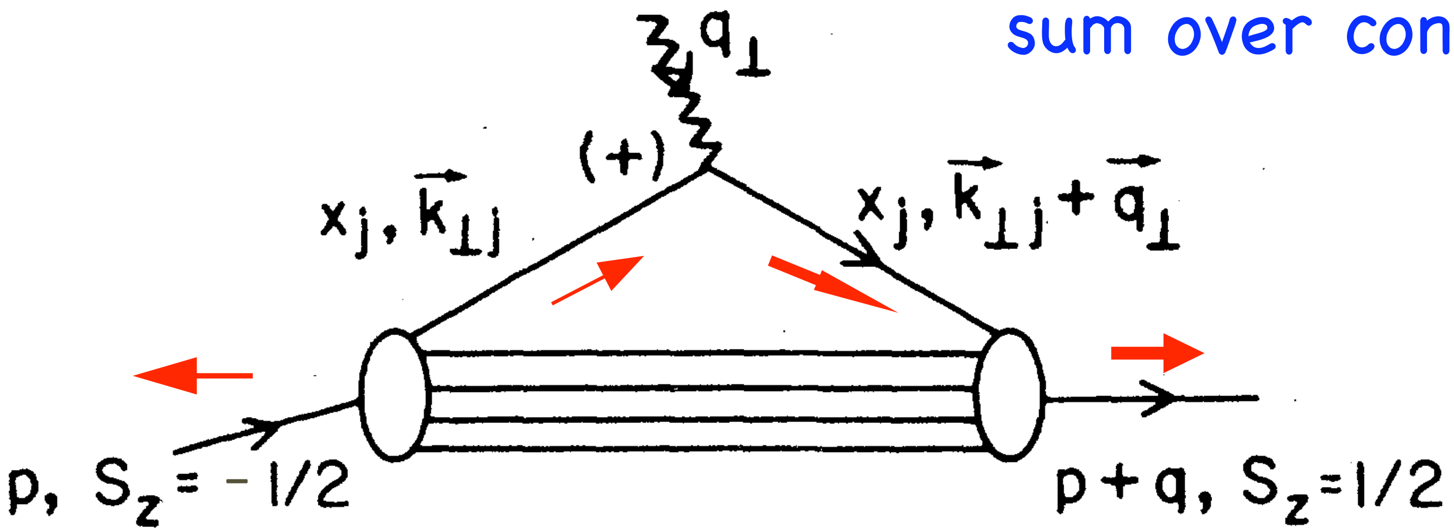
$$q_{R,L} = q^x \pm i q^y$$

Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Anomalous gravitomagnetic moment B(0)

Okun et al: *B(0) Must vanish because of Equivalence Theorem*

graviton



sum over constituents

Hwang, Schmidt, sjb; Holstein et al

B(0) = 0

Each Fock State

The Anomalous Magnetic Moment in Light-Front QCD

Each Fock state of the light-front wave function for a nucleon of spin J^z obeys

$$J^z = \sum_{i=1}^{n} S_i^z + \sum_{i=1}^{n-1} L_i^z$$

There are n-1 orbital angular momenta in a Fock state of n constituents.

Recall [Brodsky, Drell,1980]

$$\kappa = -\sum_a \sum_j e_j \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp]\psi_a^*(x_i, \mathbf{k}_{\perp i}, \lambda_i)\mathbf{S}_\perp \cdot \mathbf{L}_\perp^{q_j}\psi_a(x_i, \mathbf{k}_{\perp i}, \lambda_i)\,,$$

with $\mathbf{S}_\perp \cdot \mathbf{L}_\perp^{q_j} \equiv (S_+ L_-^{q_j} + S_- L_+^{q_j})/2$

where $S_\pm = S_1 \pm iS_2$ and $L_\pm^{q_j} = \sum_{i \neq j} x_i(\partial/\partial k_{1i} \mp i\partial/\partial k_{2i})$

Empirically, $\kappa_n = -1.91\mu_N$ and $\kappa_p = 1.79\mu_N$.

- The $\mathbf{S}_\perp \cdot \mathbf{L}_\perp^{q_j}$ matrix element is large!
- $\kappa_p + \kappa_n \ll \kappa_p - \kappa_n$
  $\Longrightarrow$ The isoscalar anomalous magnetic moment is *very* small.

The Generalized Parton Distribution $E(x, \zeta, t)$

The generalized form factors in virtual Compton scattering $\gamma^*(q) + p(P) \to \gamma^*(q') + p(P')$ with $t = \Delta^2$ and $\Delta = P - P' = (\zeta P^+, \boldsymbol{\Delta}_\perp, (t + \boldsymbol{\Delta}_\perp^2)/\zeta P^+)$, have been constructed in the light-front formalism. [Brodsky, Diehl, Hwang, 2001]

We find, under $\boldsymbol{q}_\perp \to \boldsymbol{\Delta}_\perp$, for $\zeta \le x \le 1$,

$$\frac{E(x,\zeta,0)}{2M} = \sum_a (\sqrt{1-\zeta})^{1-n} \sum_j \delta(x - x_j) \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \times \psi_a^*(x_i', \mathbf{k}_{\perp i}, \lambda_i) \mathbf{S}_\perp \cdot \mathbf{L}_\perp^{\mathbf{q_j}} \psi_a(x_i, \mathbf{k}_{\perp i}, \lambda_i)\,,$$

with $x_j' = (x_j - \zeta)/(1-\zeta)$ for the struck parton j and $x_i' = x_i/(1-\zeta)$ for the spectator parton i.

The E distribution function is related to a $\mathbf{S}_\perp \cdot \mathbf{L}_\perp^{\mathbf{q_j}}$ matrix element at finite ζ as well.

Electric Dipole Form Factor on the Light Front

We consider the electric dipole form factor $F_3(q^2)$ in the light-front formalism of QCD, to complement earlier studies of the Dirac and Pauli form factors. [Drell, Yan, PRL 1970; West, PRL 1970; Brodsky, Drell, PRD 1980]

Recall

$$\langle P', S_z' | J^\mu(0) | P, S_z \rangle =$$
$$\bar{U}(P', \lambda') \left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2M}\sigma^{\mu\alpha}q_\alpha + F_3(q^2)\frac{-1}{2M}\sigma^{\mu\alpha}\gamma_5 q_\alpha \right] U(P, \lambda)$$

$$\kappa = \frac{e}{2M}[F_2(0)] \; , \qquad d = \frac{e}{M}[F_3(0)]$$

We will find a close connection between κ and d, as long anticipated. [Bigi, Uralstev, NPB 1991]

Gardner, Hwang, sjb,

Electromagnetic Form Factors on the Light Front

Interaction picture for $J^+(0)$, $q^+ = 0$ frame,
imply ($q^{R/L} \equiv q^1 \pm iq^2$):

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right],$$

$$\frac{F_3(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{i}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) - \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right],$$

$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$ for the struck constituent j and $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$ for each spectator ($i \neq j$). $q^+ = 0 \Longrightarrow$ only $n' = n$.

Both $F_2(q^2)$ and $F_3(q^2)$ are helicity-flip form factors.

Gardner, Hwang, sjb,

A Universal Relation for $F_2(q^2)$ and $F_3(q^2)$

β_a violates $\mathcal{P}_\perp$ and $\mathcal{T}_\perp$.

$$\psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp\, i}, \lambda_i) = \phi_a^{\uparrow}(x_i, \mathbf{k}_{\perp\, i}, \lambda_i) e^{+i\beta_a/2} \,, \\ \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp\, i}, \lambda_i) = \phi_a^{\downarrow}(x_i, \mathbf{k}_{\perp\, i}, \lambda_i) e^{-i\beta_a/2}$$

$$\frac{F_2(q^2)}{2M} = \sum_a \cos(\beta_a) \Xi_a \quad ; \quad \frac{F_3(q^2)}{2M} = \sum_a \sin(\beta_a) \Xi_a \,,$$

$$\Xi_a = \int \frac{\left[\mathrm{d}^2 \vec{k}_\perp \mathrm{d}x \right]}{16\pi^3} \sum_j e_j \frac{1}{-q^1 + iq^2} \left[\phi_a^{\uparrow *}(x_i, \vec{k}'_{\perp\, i}, \lambda_i)\, \phi_a^{\downarrow}(x_i, \vec{k}_{\perp\, i}, \lambda_i) \right] .$$

For Fock component a:

$$[F_3(q^2)]_a = (\tan \beta_a)[F_2(q^2)]_a$$

$$d_a = (\tan \beta_a) 2\kappa_a \quad \text{or} \quad d_a = 2\kappa_a \beta_a \quad \text{as} \quad q^2 \to 0$$

Both the EDM and anomalous magnetic moment should be calculated within a given method, to test for consistency.

CP-violating phase

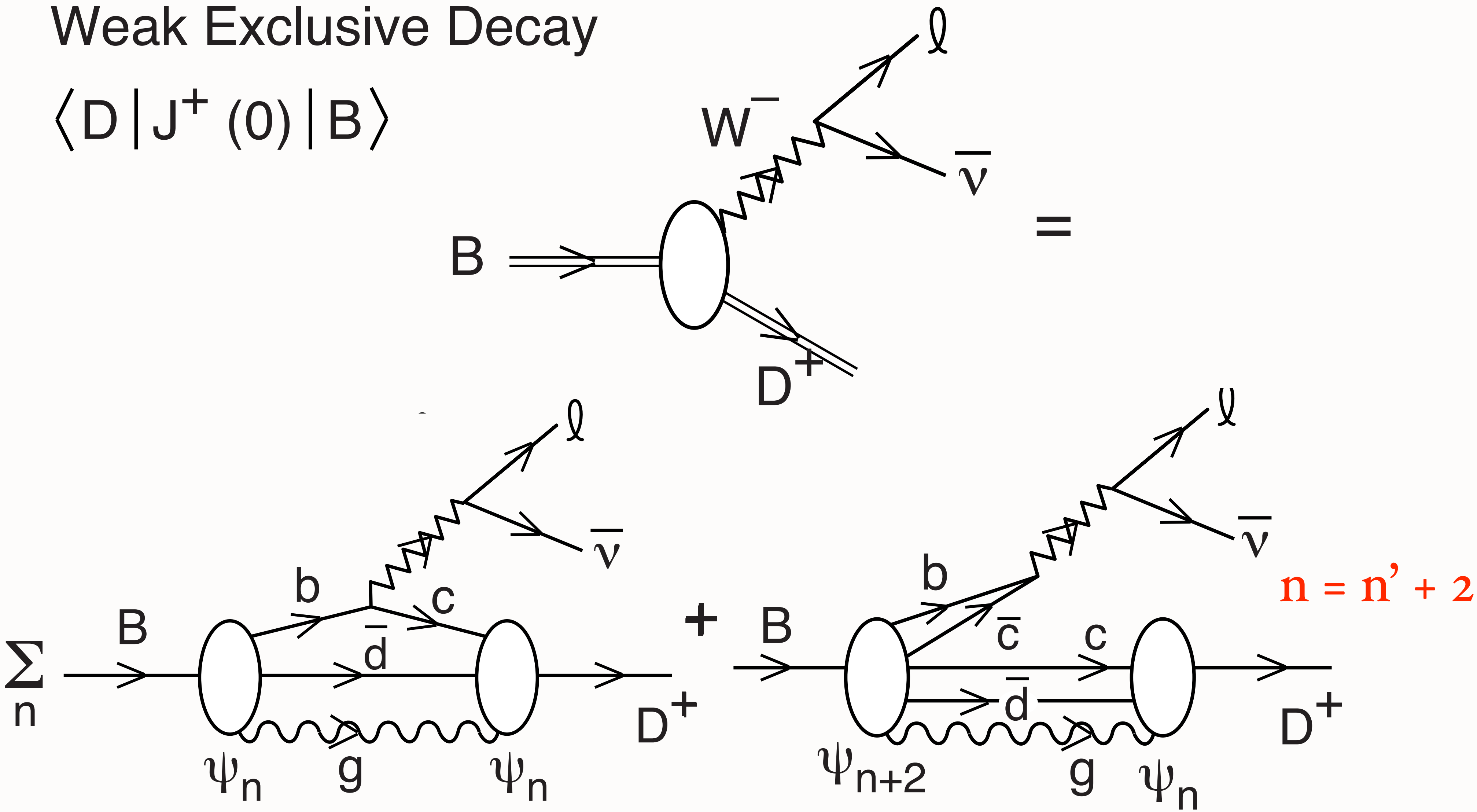
$$F_3(q^2) = F_2(q^2) \times \tan\phi$$

Fock state by Fock state

Gardner, Hwang, sjb,

Weak Exclusive Decay

$\langle D | J^+(0) | B \rangle$

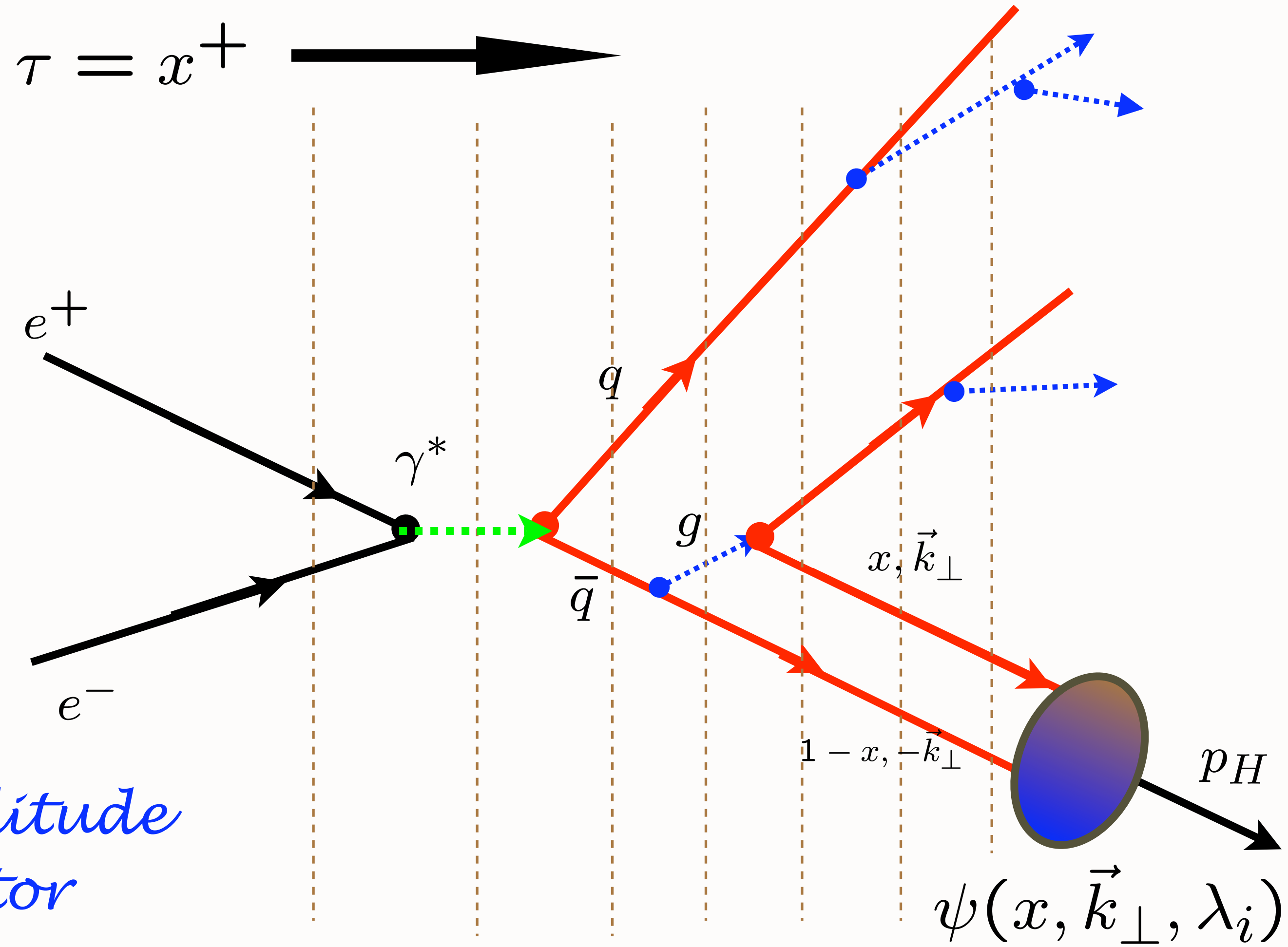


Annihilation amplitude needed for Lorentz Invariance

Exact Formula

Hwang, SJB

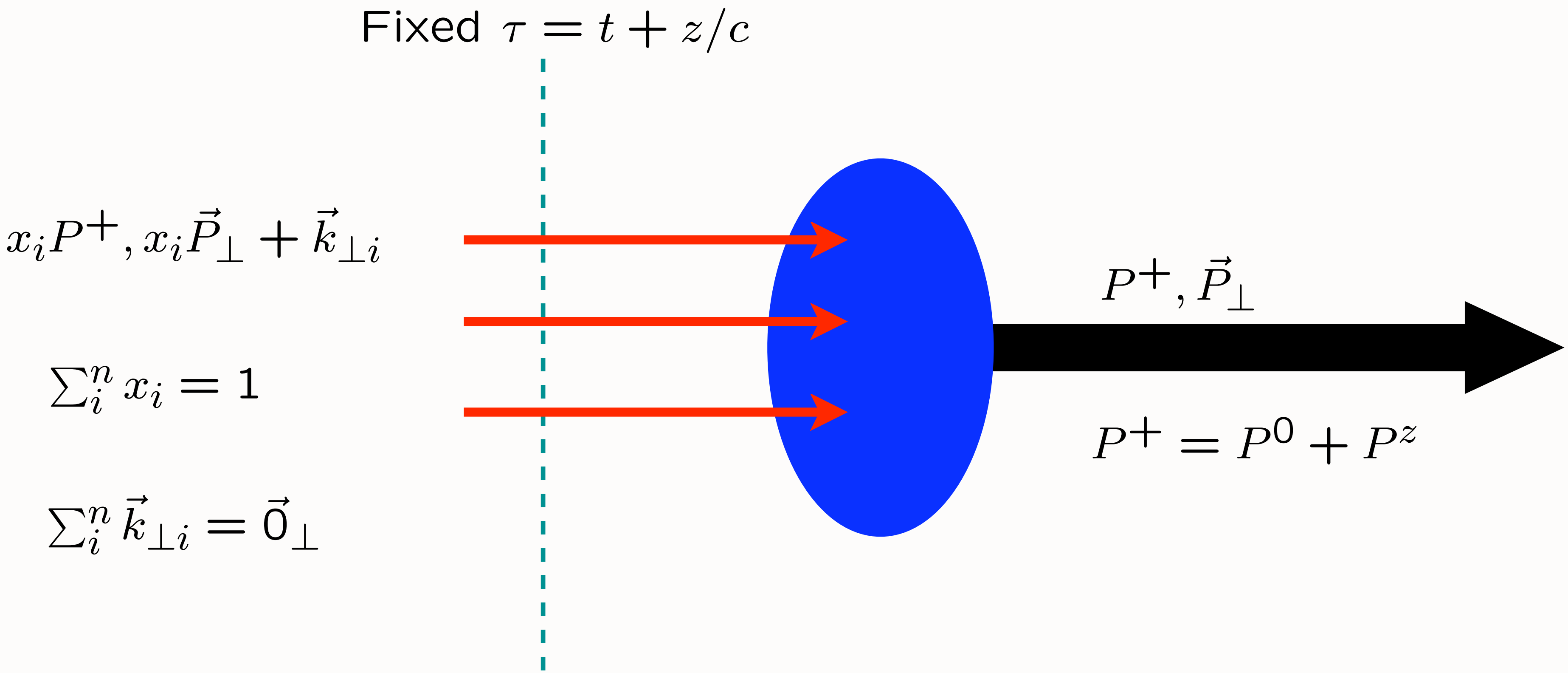
Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

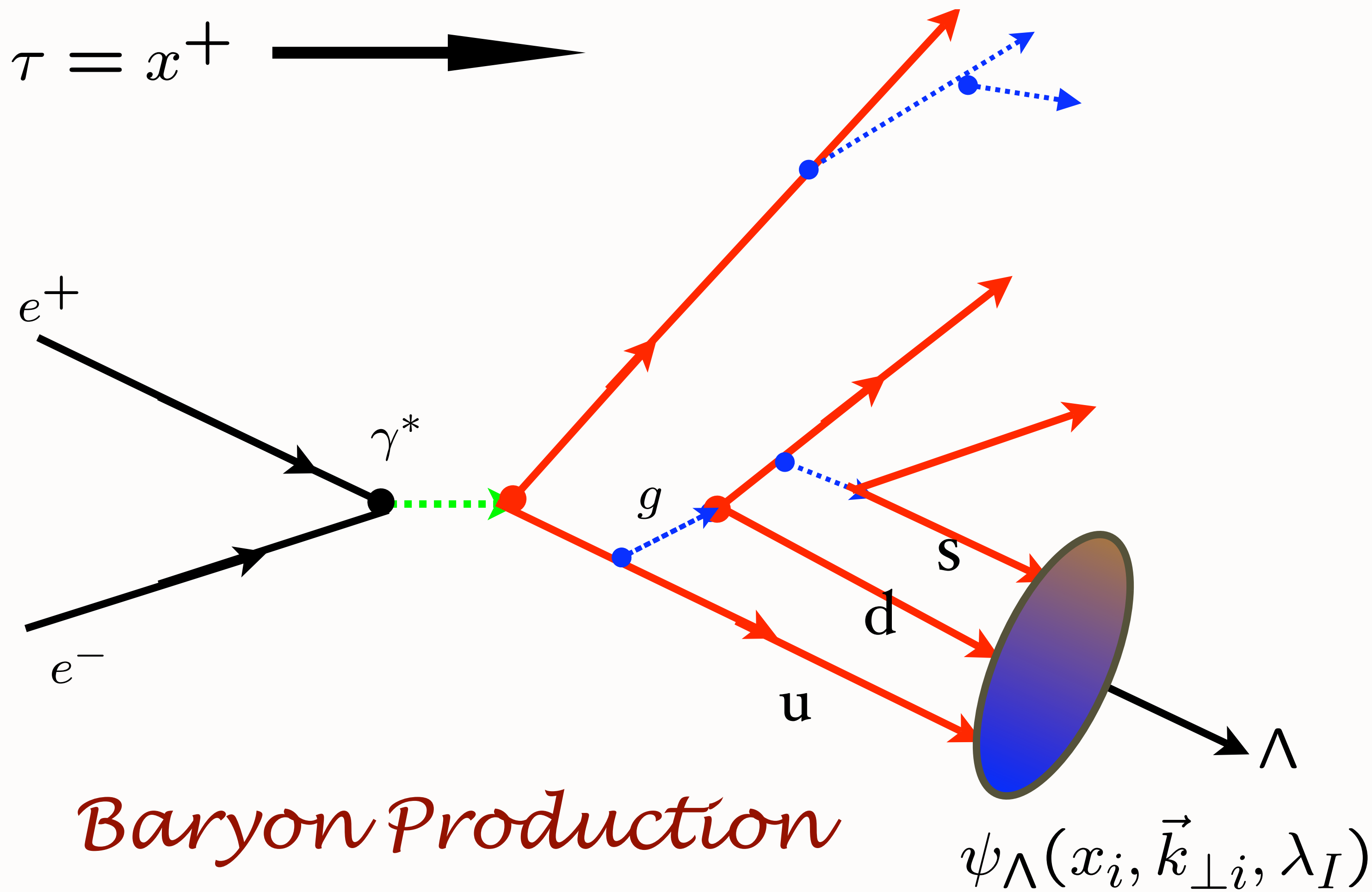
Light-Front Wavefunctions



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Invariant under boosts! Independent of P^μ

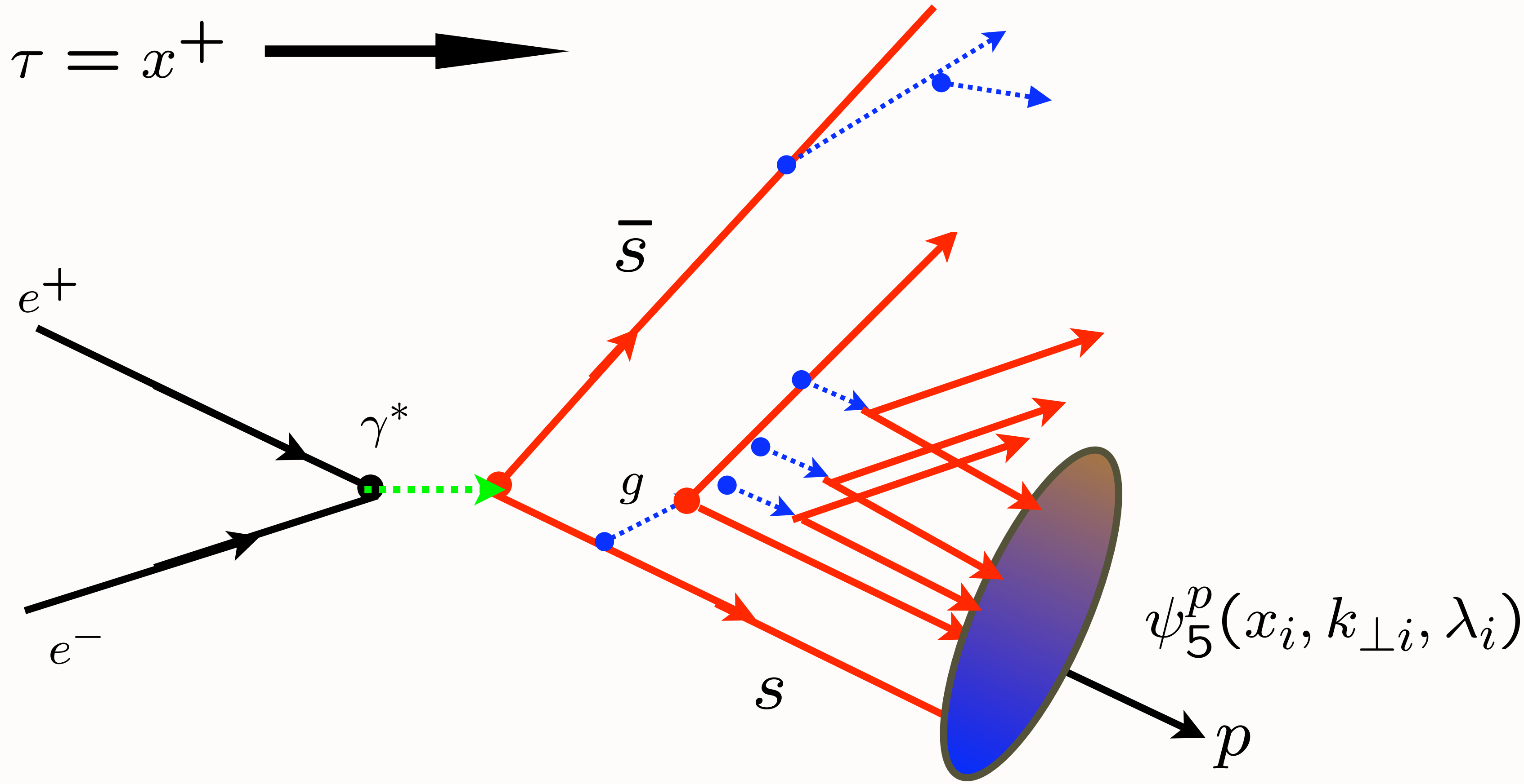
Hadronization at the Amplitude Level



Baryon Production

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



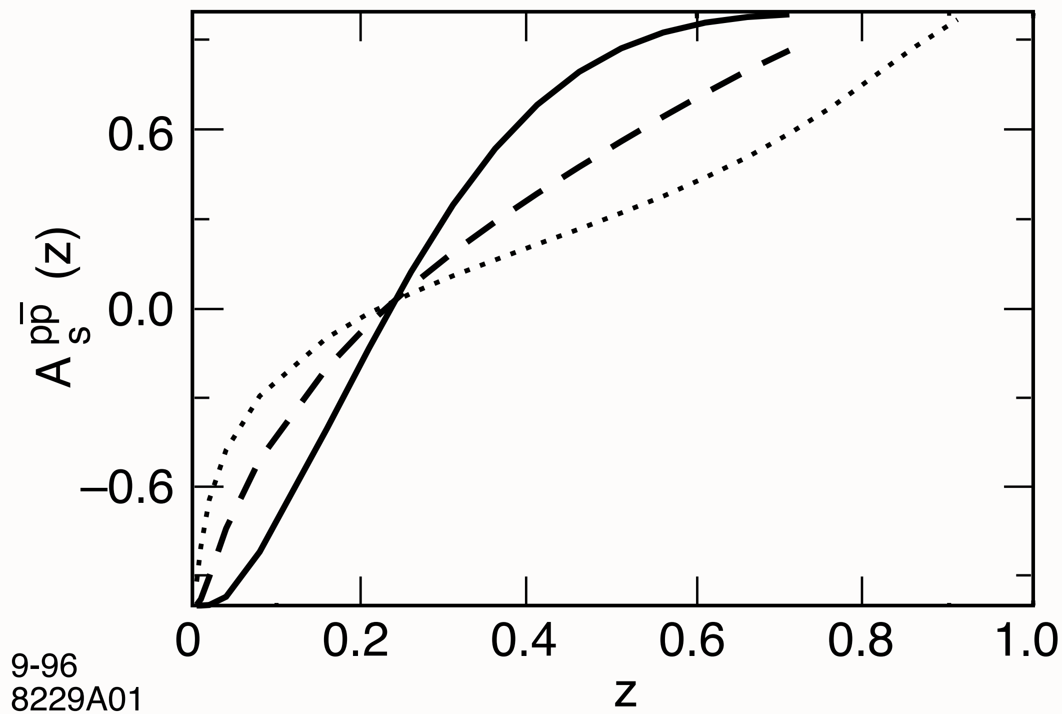
Higher Fock State Coalescence $|uuds\bar{s}>$

Asymmetric Hadronization ! $D_{s\to p}(z) \neq D_{s\to \bar{p}}(z)$

B-Q Ma, sjb

$$D_{s\to p}(z) \neq D_{s\to \bar{p}}(z)$$

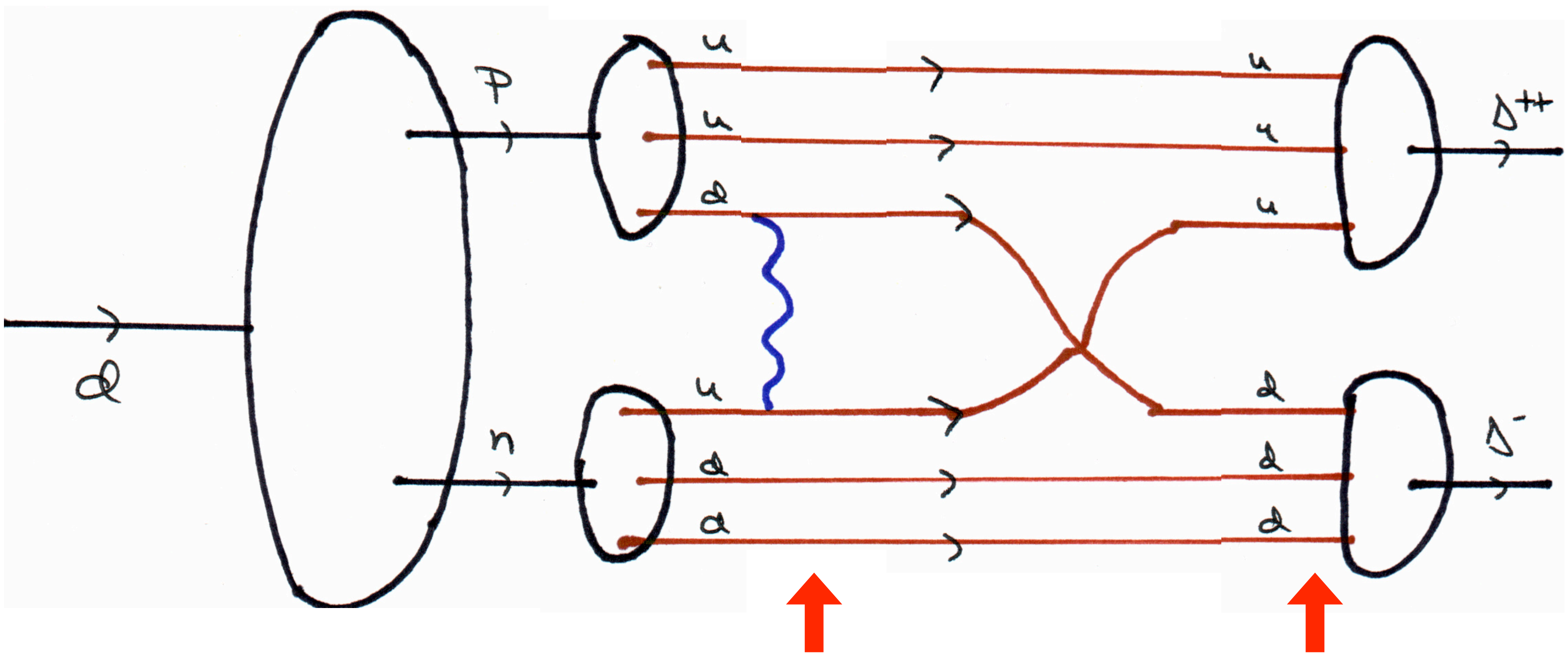
B-Q Ma, sjb



$$A_s^{p\bar{p}}(z) = \frac{D_{s\to p}(z) - D_{s\to \bar{p}}(z)}{D_{s\to p}(z) + D_{s\to \bar{p}}(z)}$$

Consequence of $s_p(x) \neq \bar{s}_p(x)$ $\quad |uuds\bar{s} > \simeq |K^+\Lambda >$

Structure of Deuteron in QCD



Hidden Color Fock State

Delta-Delta Fock State

Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six-quark wavefunction
- 5 color-singlet combinations of 6 color-triplets -- only one state is |n p>
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict

$$\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn) \text{ at high } Q^2$$

The evolution equation for six-quark systems in which the constituents have the light-cone longitudinal momentum fractions x_i $(i = 1, 2, \ldots, 6)$ can be obtained from a generalization of the proton (three-quark) case.[2] A nontrivial extension is the calculation of the color factor, C_d, of six-quark systems[5] (see below). Since in leading order only pairwise interactions, with transverse momentum Q, occur between quarks, the evolution equation for the six-quark system becomes $\{[dy] = \delta(1 - \sum_{i=1}^{6} y_i)\prod_{i=1}^{6} dy_i$, $C_F = (n_c^2 - 1)/2n_c = \frac{4}{3}$, $\beta = 11 - \frac{2}{3}n_f$, and n_f is the effective number of flavors$\}$

$$\prod_{k=1}^{6} x_k \left[\frac{\partial}{\partial \xi} + \frac{3C_F}{\beta}\right] \tilde{\Phi}(x_i, Q) = -\frac{C_d}{\beta} \int_0^1 [dy] V(x_i, y_i) \tilde{\Phi}(y_i, Q),$$

$$\xi(Q^2) = \frac{\beta}{4\pi} \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \alpha_s(k^2) \sim \ln\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right).$$

$$V(x_i, y_i) = 2 \prod_{k=1}^{6} x_k \sum_{i \neq j}^{6} \theta(y_i - x_i) \prod_{l \neq i,j}^{6} \delta(x_l - y_l) \frac{y_j}{x_j} \left(\frac{\delta_{h_i \bar{h}_j}}{x_i + x_j} + \frac{\Delta}{y_i - x_i}\right)$$

where $\delta_{h_i \bar{h}_j} = 1$ (0) when the helicities of the constituents $\{i, j\}$ are antiparallel (parallel). The infrared singularity at $x_i = y_i$ is cancelled by the factor $\Delta\tilde{\Phi}(y_i, Q) = \tilde{\Phi}(y_i, Q) - \tilde{\Phi}(x_i, Q)$ since the deuteron is a color singlet.

QCD Prediction for Deuteron Form Factor

$$F_d(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2}\right]^5 \sum_{m,n} d_{mn} \left(\ln\frac{Q^2}{\Lambda^2}\right)^{-\gamma_n^d - \gamma_m^d} \left[1 + O\left(\alpha_s(Q^2), \frac{m}{Q}\right)\right]$$

Define "Reduced" Form Factor

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N^{\,2}(Q^2/4)} .$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2} \left(\ln\frac{Q^2}{\Lambda^2}\right)^{-(2/5)\, C_F/\beta}$$

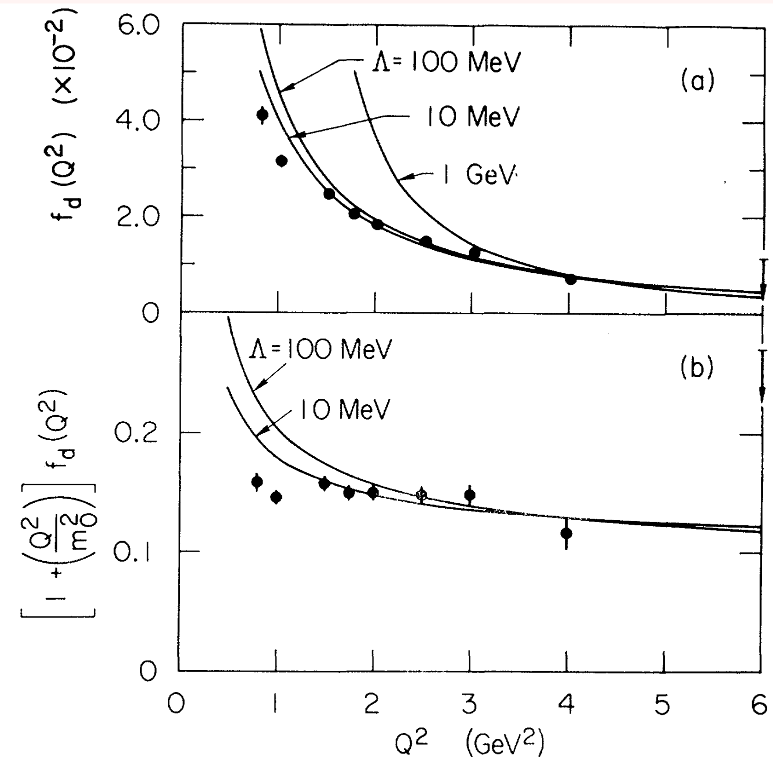
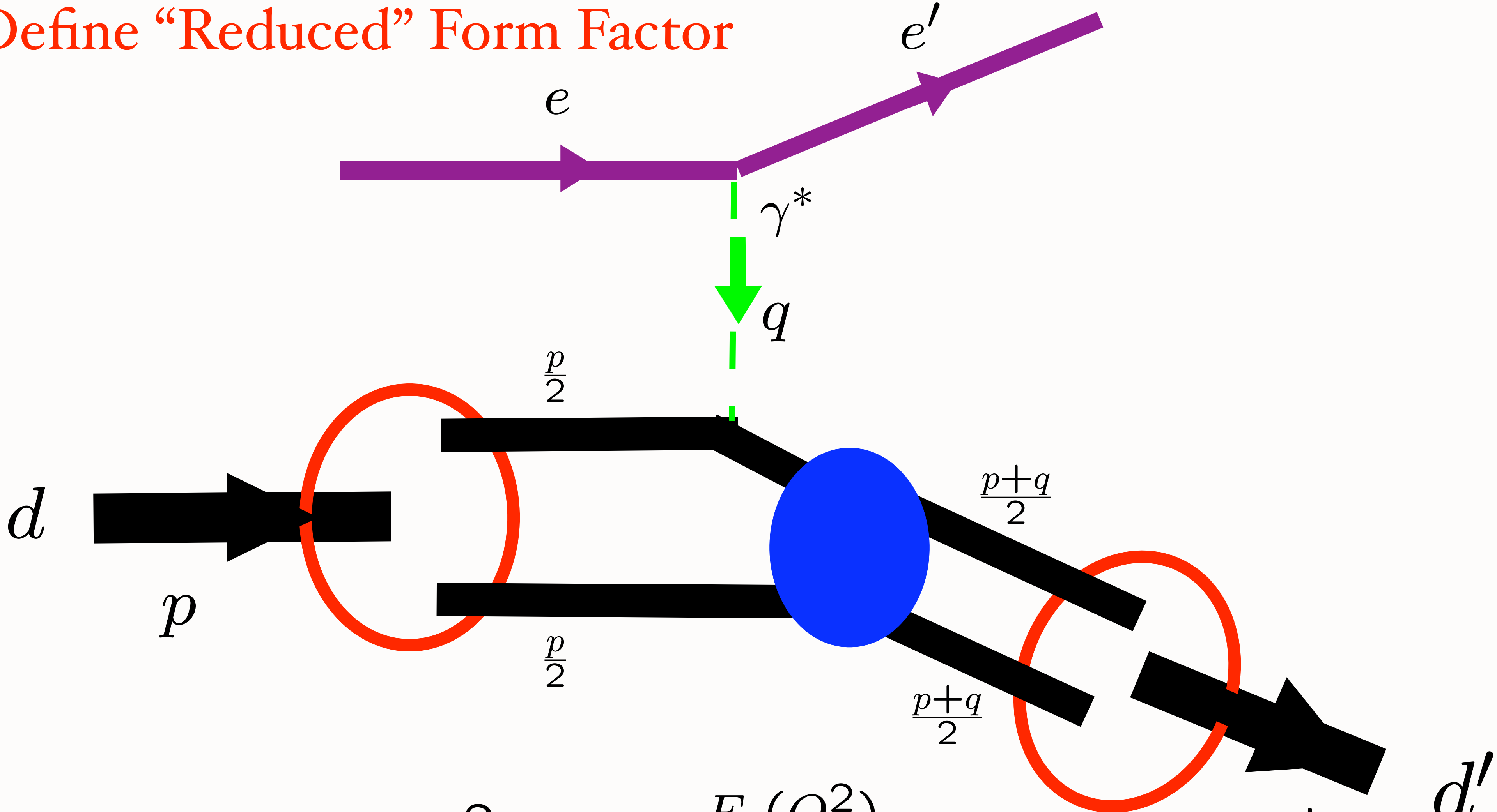


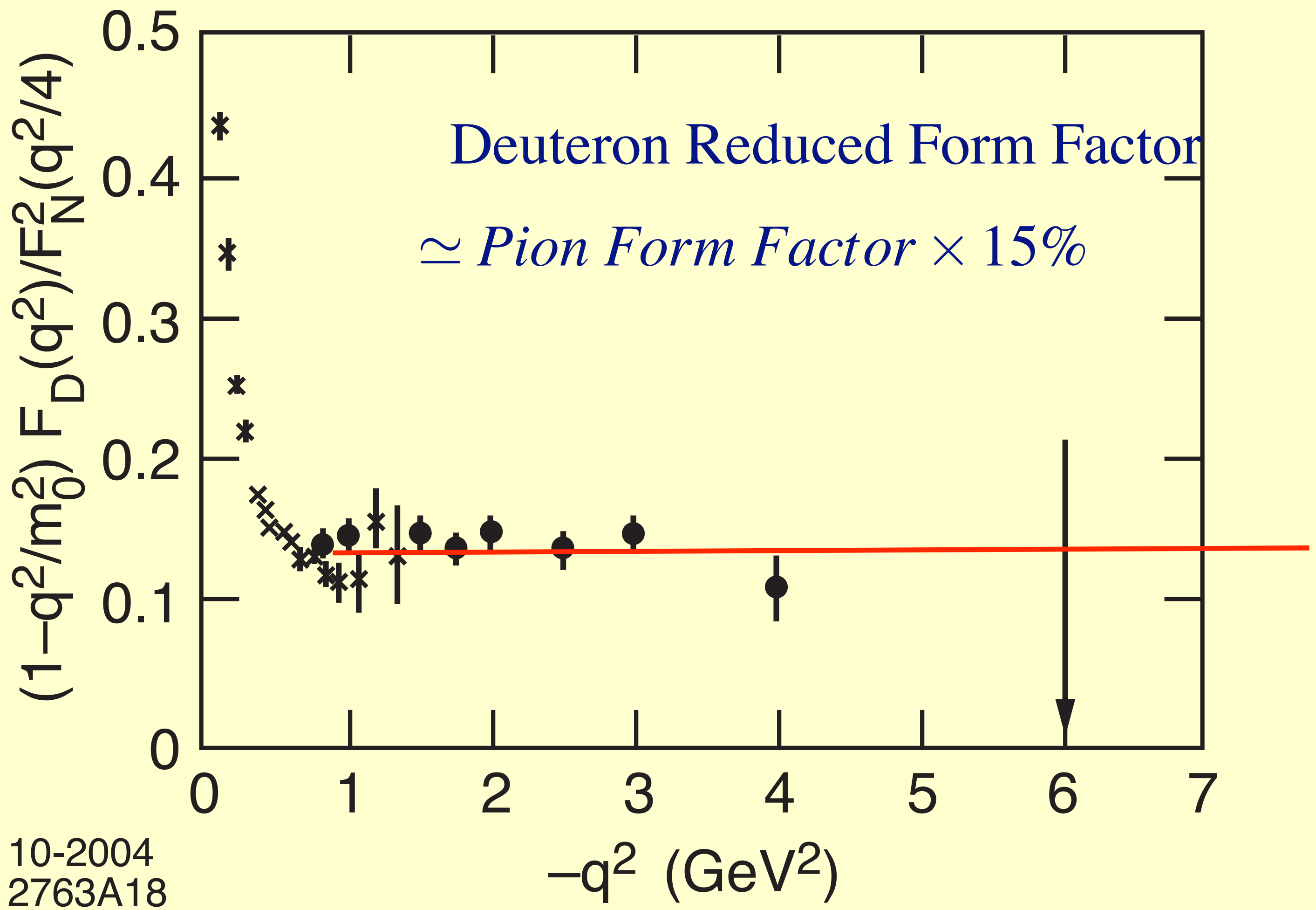
FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto (1/Q^2)[\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2) = [1 + Q^2/(0.71\ \mathrm{GeV}^2)]^{-2}$. The normalization is fixed at the $Q^2 = 4\ \mathrm{GeV}^2$ data point. (b) Comparison of the prediction $[1 + (Q^2/m_0^2)] f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with the above data. The value $m_0^2 = 0.28\ \mathrm{GeV}^2$ is used (Ref. 8).

Define "Reduced" Form Factor



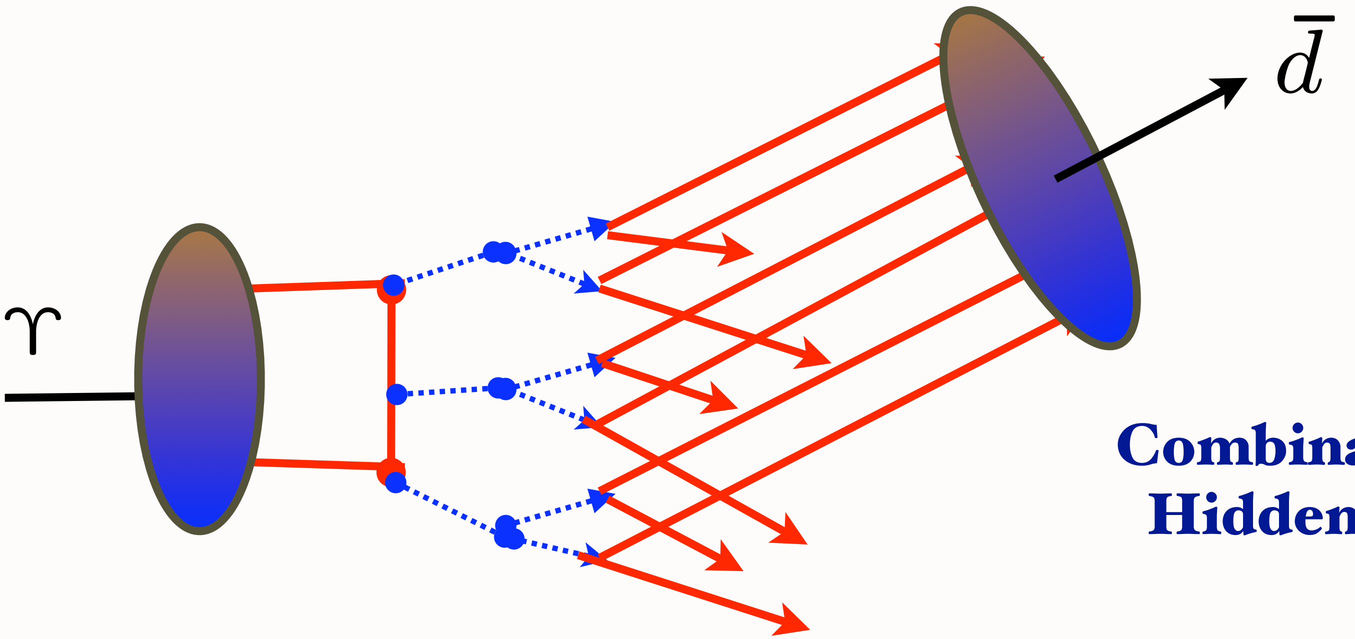
$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_p(\frac{Q^2}{4})F_n(\frac{Q^2}{4})}$$

Elastic electron-deuteron scattering



- Evidence for Hidden Color in the Deuteron

Hadronization at the Amplitude Level



Combinatoric Advantage for Hidden-Color Fock States

$$\Upsilon \to ggg \to q\bar{q}\ q\bar{q}\ q\bar{q}\ q\bar{q}\ q\bar{q}\ q\bar{q} \to \bar{d}\ X$$

Anti-Deuteron vs. double antibaryon production

$$\Upsilon \to ggg \to q\bar{q}\ q\bar{q}\ q\bar{q}\ q\bar{q}\ q\bar{q}\ q\bar{q} \to \bar{p}\ \bar{n}\ X$$

Key Test of Hidden Color

- CLEO measurement: Upsilon decay to anti-deuteron $\Upsilon \to ggg \to \bar{d}X$
- Is ratio of deuteron production to production of anti-nucleon pairs determined by Nuclear Physics?

$$R = \frac{\Gamma(\Upsilon \to \bar{d}X)}{\Gamma(\Upsilon \to \bar{p}\bar{n}X)}$$

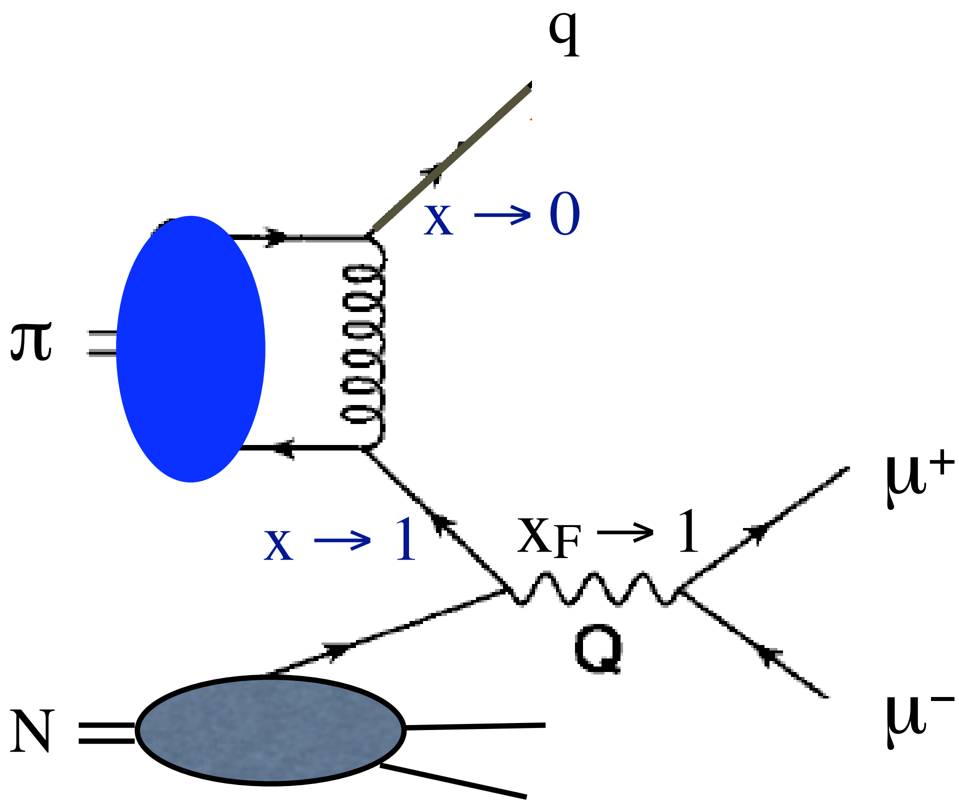
$$\frac{E}{\sigma_{\mathrm{tot}}} \frac{\mathrm{d}^3\sigma(\mathrm{d})}{\mathrm{d}^3 p} = C \left(\frac{E}{\sigma_{\mathrm{tot}}} \frac{\mathrm{d}^3\sigma(\mathrm{p})}{\mathrm{d}^3 p} \right)^2 \qquad C = \tfrac{4\pi}{3} p_0^3 / m_{\mathrm{p}} \qquad p_0 \approx 130\mathrm{MeV}$$

Gustafson, Hakkinen

$\pi N \to \mu^+ \mu^- X$ at high x_F

In the limit where $(1-x_F)Q^2$ is fixed as $Q^2 \to \infty$

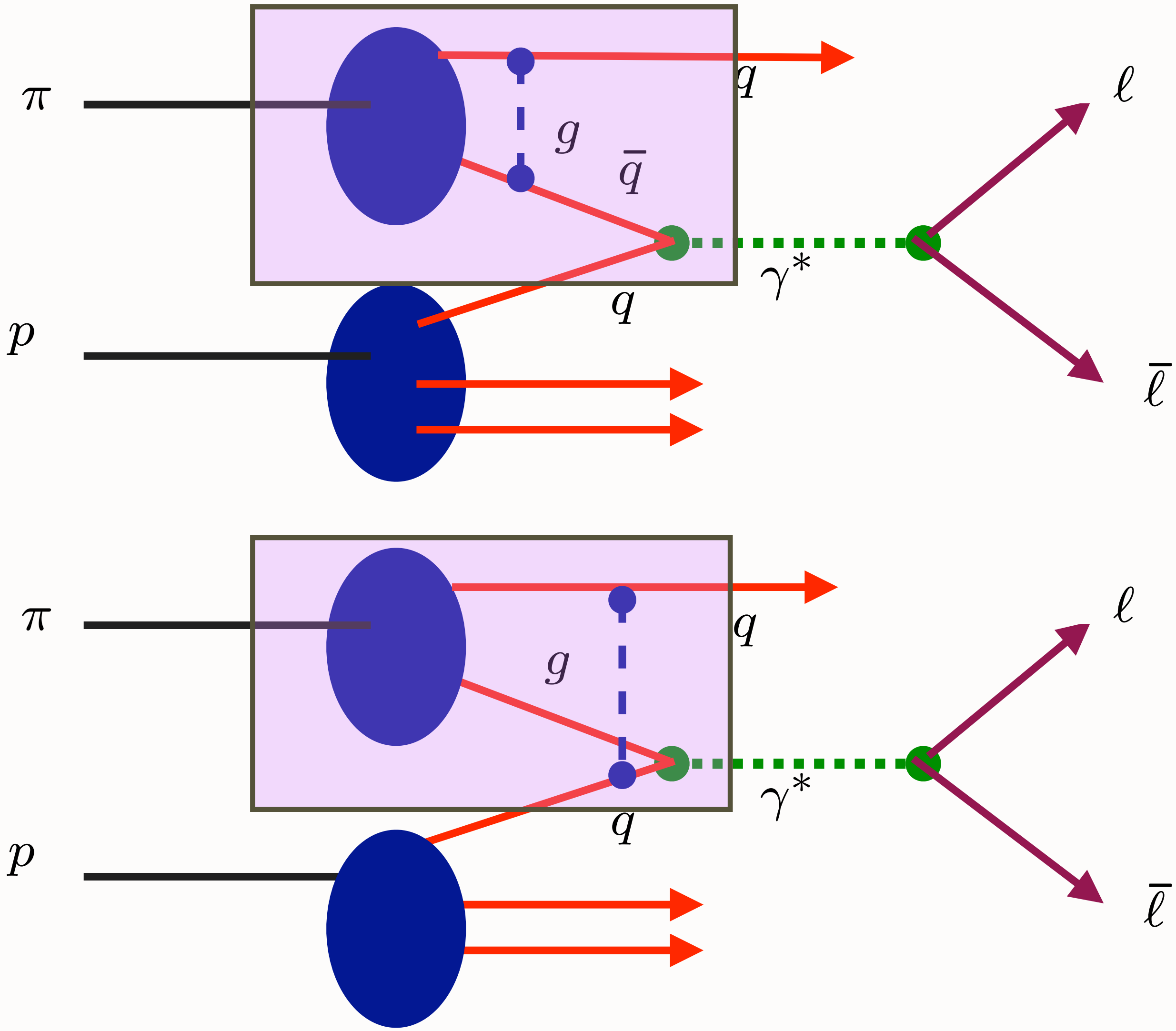
Entire pion wf contributes to hard process



Virtual photon is longitudinally polarized

Berger and Brodsky, PRL 42 (1979) 940

Berger, Lepage, sjb



$$\pi q \rightarrow \gamma^* q$$

Pion appears directly in subprocess at large x_F

All of the pion's momentum is transferred to the lepton pair

Lepton Pair is produced longitudinally polarized

$\pi^- N \rightarrow \mu^+ \mu^- X$ at 80 GeV/c

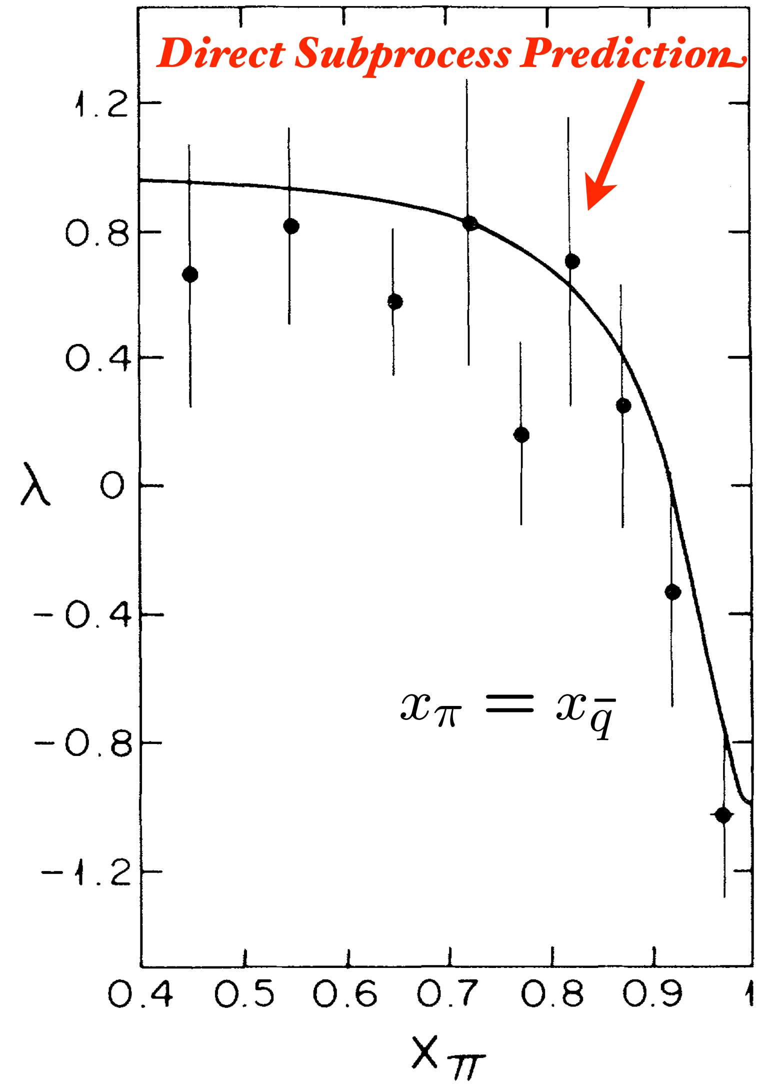
$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi.$$

$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[(1-x_\pi)^2 (1+\cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

$$\langle k_T^2 \rangle = 0.62 \pm 0.16 \ \text{GeV}^2/c^2$$

Dramatic change in angular distribution at large x_F

Example of a higher-twist direct subprocess



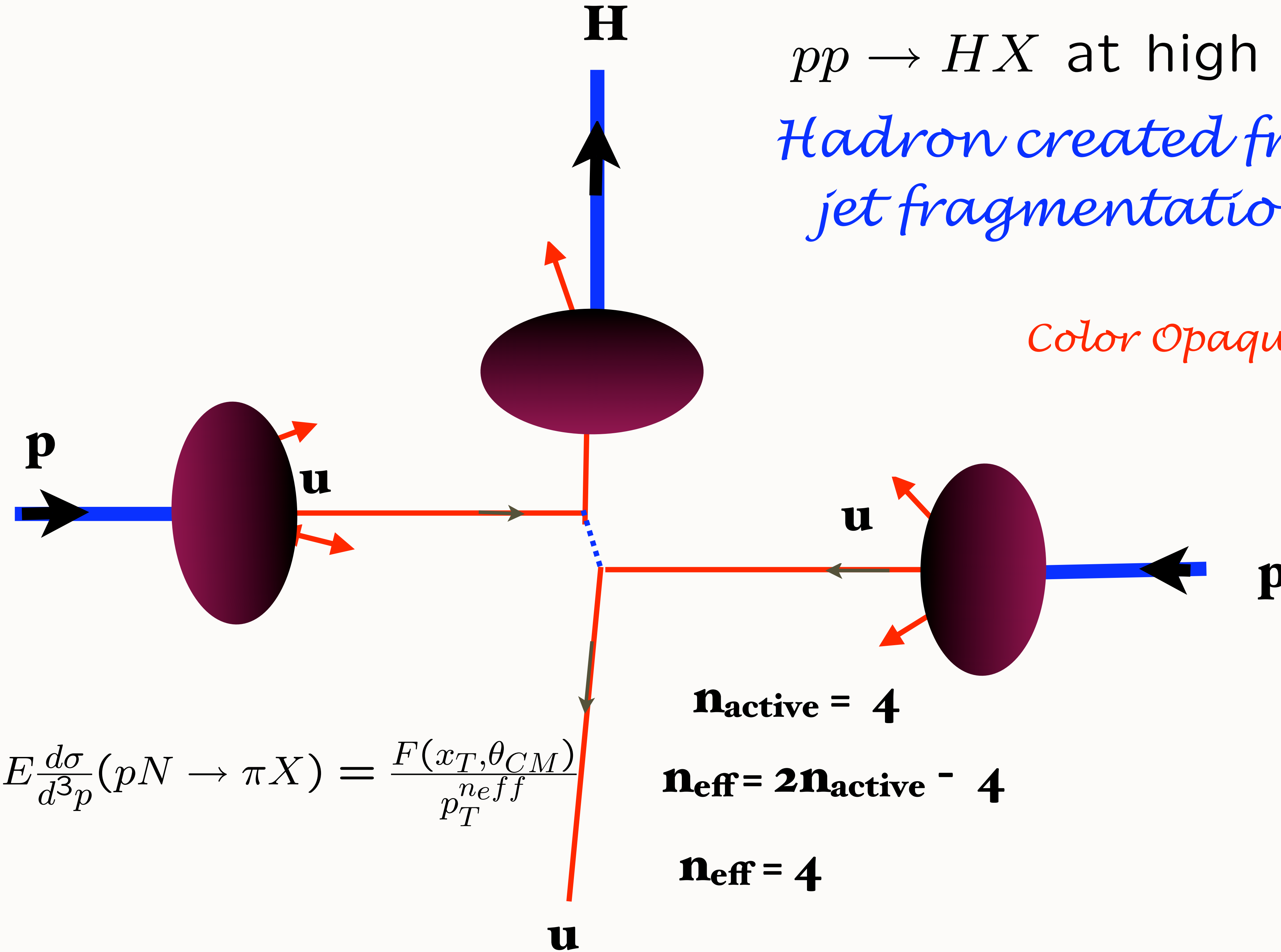
Chicago-Princeton Collaboration

Phys.Rev.Lett.55:2649,1985

$pp \to HX$ at high p_T

Hadron created from jet fragmentation

Color Opaque



$$E\frac{d\sigma}{d^3p}(pN \to \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$

$n_{active} = 4$

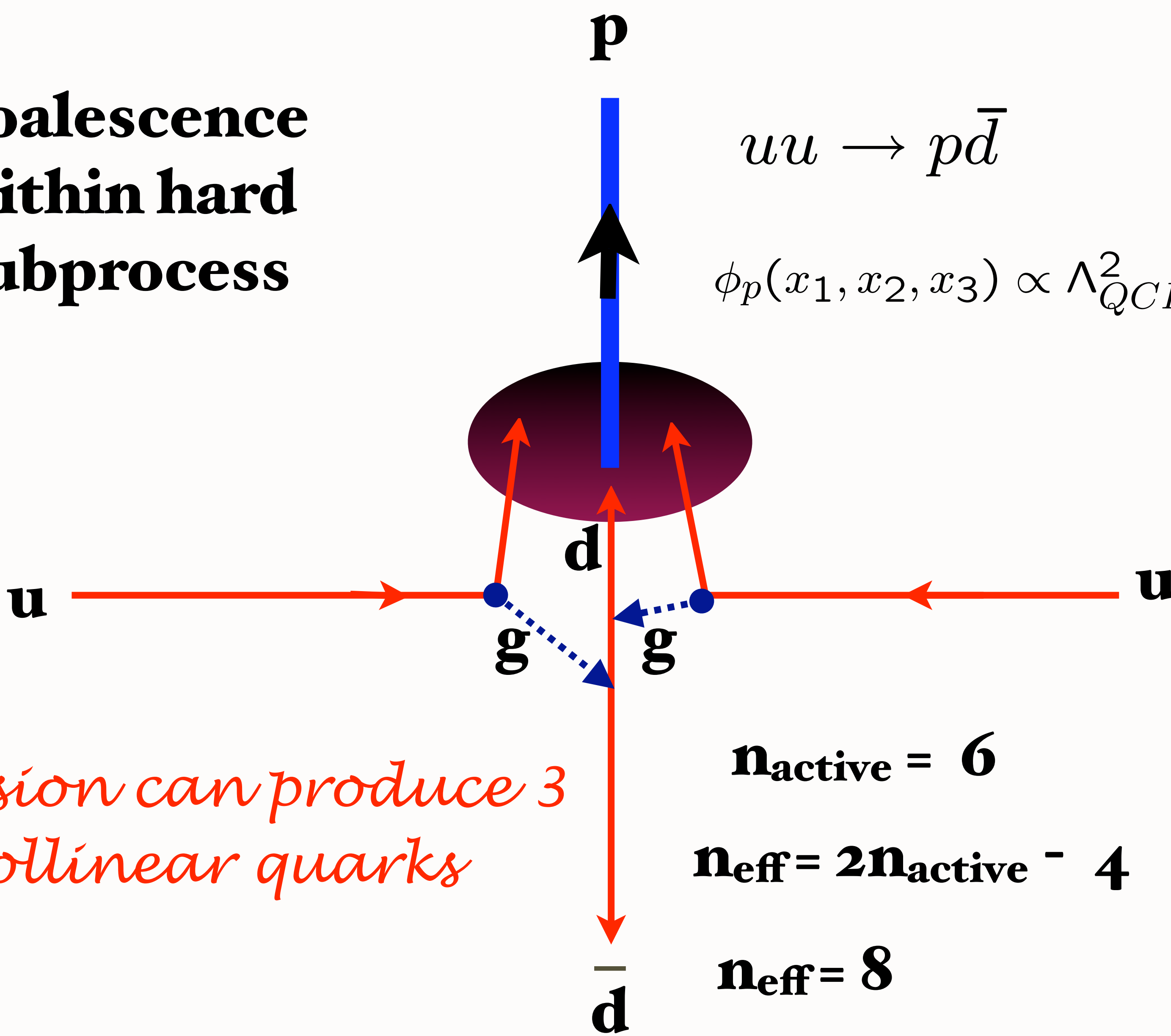
$n_{eff} = 2n_{active} - 4$

$n_{eff} = 4$

Baryon can be made directly within hard subprocess

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Coalescence within hard subprocess



$uu \rightarrow p\bar{d}$

$\phi_p(x_1, x_2, x_3) \propto \Lambda^2_{QCD}$

Collision can produce 3 collinear quarks

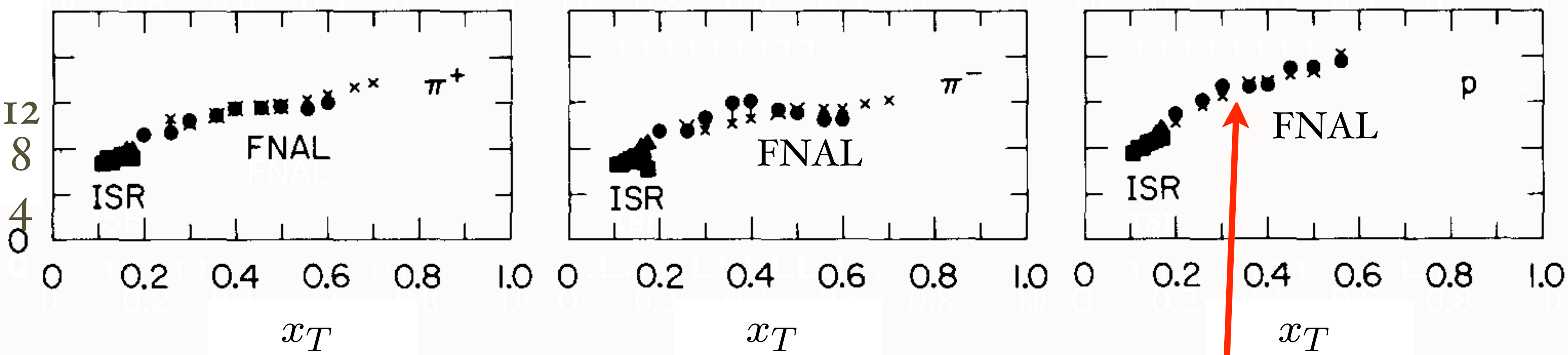
$n_{active} = 6$

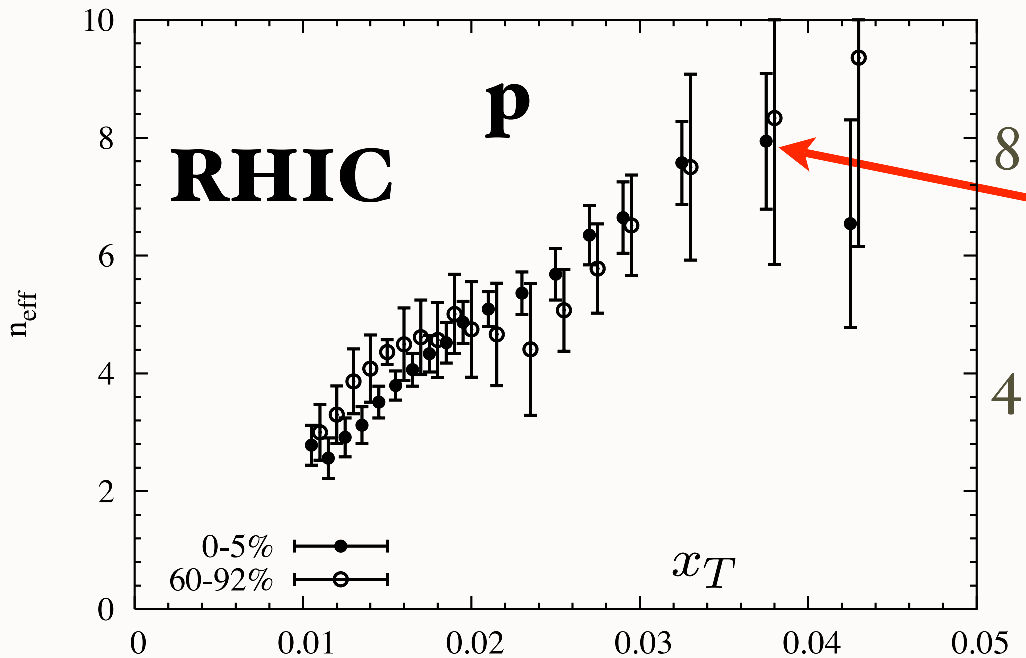
$n_{eff} = 2n_{active} - 4$

$n_{eff} = 8$

$qq \rightarrow B\bar{q}$

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T,\theta_{CM})}{p_T^{n_{eff}}}$$





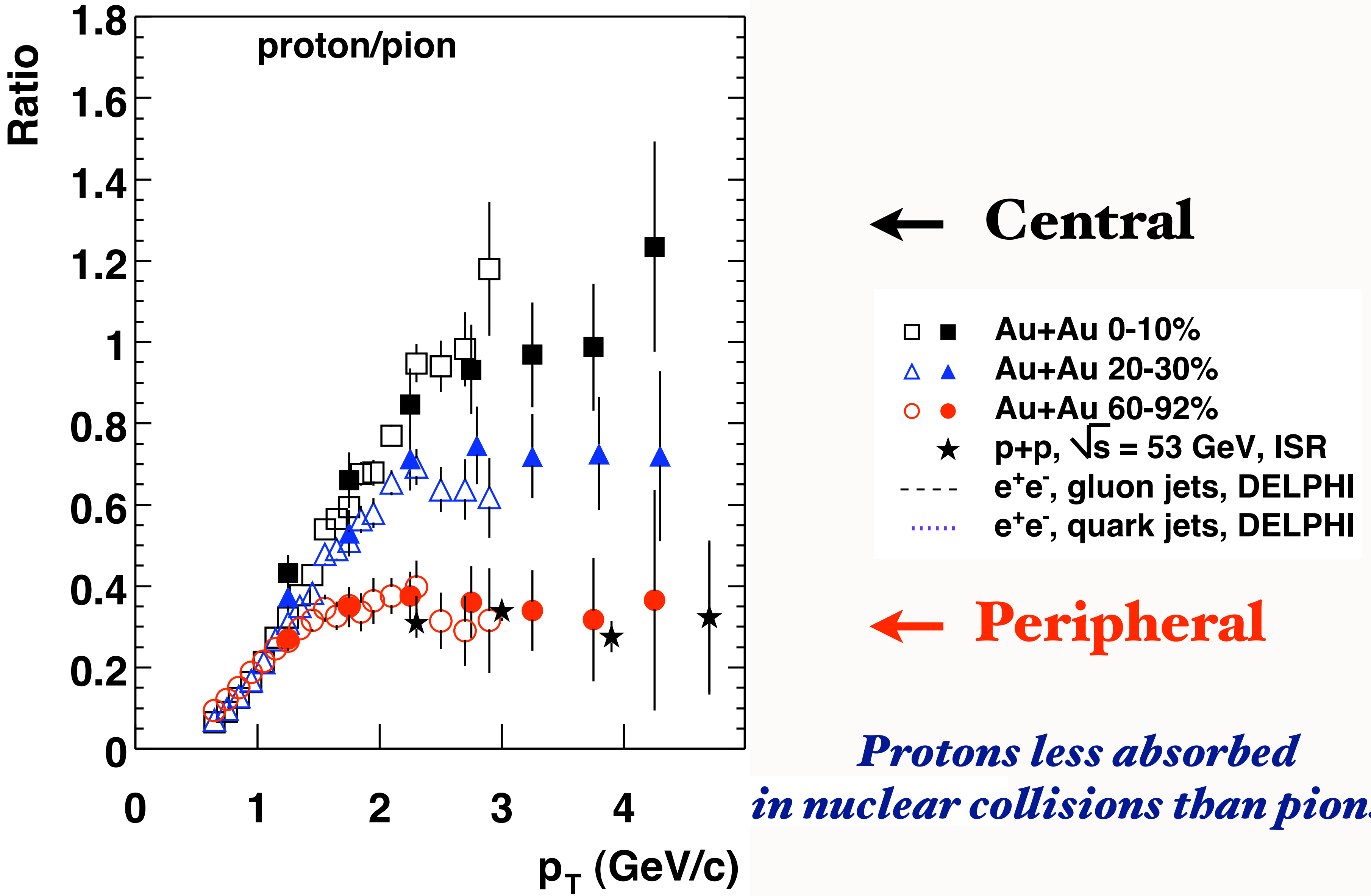
$$E\frac{d\sigma}{d^3p}(pp \to pX) = \frac{F(x_T,\theta_{CM})}{p_T^{12}}$$

$$E\frac{d\sigma}{d^3p}(pp \to pX) = \frac{F(x_T,\theta_{CM})}{p_T^{8}}$$

Trend consistent with RHIC at small x_T

S. S. Adler *et al*. PHENIX Collaboration *Phys. Rev. Lett*. **91**, 172301 (2003).

Particle ratio changes with centrality!



← **Central**

← **Peripheral**

Protons less absorbed in nuclear collisions than pions!

Open (filled) points are for $\pi^{\pm}$ (π^{0}), respectively.

Evidence for Direct, Higher-Twist Subprocesses

- Anomalous power behavior at fixed x_T
- Protons more likely to come from direct subprocess than pions
- Protons less absorbed than pions in central nuclear collisions because of **color transparency**
- Predicts increasing proton to pion ratio in central collisions
- Exclusive-inclusive connection at $x_T = 1$

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Light-Front QCD Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme

Holographic Connnection between LF and AdS/CFT

- Predictions for hadronic spectra, light-front wavefunctions, interactions
- Use AdS/CFT as basis for diagonalizing the LF Hamiltonian
- Deduce meson and baryon wavefunctions, distribution amplitude, structure function from holographic constraint
- Extension to massive quarks
- Implementation of Chiral Symmetry

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

↓

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances + Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

↓ Holography

Boost Invariant 3+1 Light-Front Wave Equations

↓

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics