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Deep Inelastic Pion Electroproduction at Threshold

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Outline

Motivation:

- Interesting interplay of the $Q \to \infty$ and $m_\pi \to 0$ limits
- Directly measurable e.g. at JLab
- Background to form factor measurents

Theory:

- General framework
- Soft-pion limit
- Light-cone sum rule approach

Phenomenology:

- Qualitative picture
- Preliminary results

Outlook

General Framework

we consider

$$\gamma^*(q) + p(P_1) \quad \to \quad \pi^+(k) + n(P_2)\,,$$
$$\gamma^*(q) + p(P_1) \quad \to \quad \pi^0(k) + p(P_2)\,,$$

at threshold, i.e. $k_\mu = \delta\, P_{2,\mu}\,, \qquad \delta = m_\pi/m_N \simeq 0.15$

Generalized Form Factors

$$M_\mu = \langle \pi N | j_\mu^{\mathrm{em}} | p \rangle$$

$$M_\mu^{\pi N} = -\frac{i}{f_\pi} \bar{N}(P_2)\gamma_5 \left\{ \left(\gamma_\mu q^2 - q_\mu \not{q}\right) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^\nu}{2m_N} G_2^{\pi N}(Q^2) \right\} N(P_2)$$

General Framework — *continued (1)*

For example:

Deep Inelastic Structure Functions (S-wave)

$$F_1 = \frac{\beta(W)}{(4\pi f_\pi)^2}\frac{Q^2+(2m_N+m_\pi)^2}{2m_N^3(m_N+m_\pi)}\left(G_1Q^2-\frac{1}{2}G_2m_N\,m_\pi\right)^2$$

$$F_2 = \frac{\beta(W)}{(4\pi f_\pi)^2}\frac{Q^2(Q^2+m_\pi(2m+m_\pi))}{m_N^3(m_N+m_\pi)}\left(G_1^2Q^2+\frac{1}{4}G_2^2m_N^2\right)$$

where

$$\beta(W)=\frac{2|\vec{k}_f|}{W}\,, \qquad \vec{k}_f^2=\frac{W^2}{4}\left(1-\frac{(m_N+m_\pi)^2}{W^2}\right)\left(1-\frac{(m_N-m_\pi)^2}{W^2}\right)$$

$\vec{k}_f$ **is the c.m.s. momentum of the pion-nucleon system in the final state**

Soft Pion Limit

Kroll, Ruderman '54... Vainshtein, Zakharov '72...

If $q^2, qk, m_\pi^2 \to 0$, then conservation of the axial current implies

$$\begin{aligned} Q^2 G_1^{\pi^+ n} &\simeq G_A(Q^2)/\sqrt{2} + \ldots + \mathcal{O}(m_\pi/\Lambda) \\ Q^2 G_1^{\pi^0 p} &\simeq 0 \quad + \ldots + \mathcal{O}(m_\pi/\Lambda) \end{aligned}$$

and

$$G_2^{\pi^+ n} = G_2^{\pi^0 p} = 0 + \ldots + \mathcal{O}(m_\pi/\Lambda)$$

- Many more π^+ are produced compared to π^0
- Everything is reduced to the proton axial form factor

$$\langle N(P')|A_\mu|N(P)\rangle = \bar{N}(P')\left[\gamma_\mu G_A(Q^2) - \frac{q_\mu}{2m_N} G_P(Q^2) - i\frac{\sigma_{\mu\nu}q^\nu}{2m_N} G_T(Q^2)\right]\gamma_5 N(P)$$

- Vast literature on the calculation of $\mathcal{O}(m_\pi)$ and $\mathcal{O}(m_\pi^2)$ corrections

Light Cone Sum Rules

consider

Balitsky, V.B., Kolesnichenko '88

$$T_\nu^{\pi N}(P,q) \quad = \quad i\int d^4x\, e^{iqx}\langle 0|T\{\eta_p(0) j_\nu^{\text{em}}(x)|N(P)\pi^a(k)\rangle$$

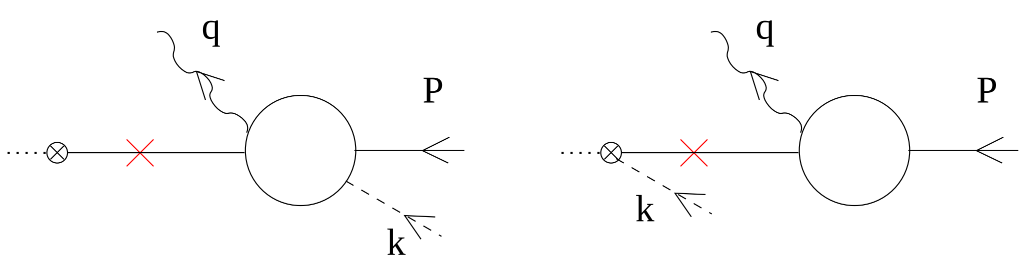
with

$$\eta_p(x) \quad = \quad \epsilon^{ijk}\left[u^i(x) C\gamma_\mu u^j(x)\right]\gamma_5\gamma^\mu d^k(x)\,, \qquad \langle 0|\eta_p|N(P)\rangle = \lambda_p m_N N(P)$$

- In the limit $|k| \to 0$ for fixed q^2 and $(P')^2 = (P+k-q)^2$

$$T_\nu^{\pi N}(P,q) \quad = \quad -\frac{i}{f_\pi}\left[i\int d^4x\, e^{iqx}\langle 0|T\{[Q_5^a,\eta_p(0)] j_\nu^{\text{em}}(x)|N(P)\rangle\right.$$
$$\left. +i\int d^4x\, e^{iqx}\langle 0|T\{\eta_p(0)[Q_5^a, j_\nu^{\text{em}}(x)]|N(P)\rangle\right] + \qquad \text{bremsstrahlung}$$

- An extra term with the chiral rotation of η_p
- An extra term in the dispersion relation in the vicinity of $(P')^2 \to m_N^2$



Light Cone Sum Rules — *continued (1)*

For example for $p\pi^0$ the commutator $[Q_5^3, j^{\text{em}}] = 0$ and one obtains

$$\text{LHS} = -\frac{i\lambda_p}{f_\pi}\frac{m_N + \not{P}'}{m_N^2 - P'^2}\gamma_5\left\{(\gamma_\nu q^2 - q_\nu \not{q})\frac{G_1^{p\pi^0}}{m_N^2} - \frac{i\sigma_{\nu\mu}q^\mu}{2m_N}G_2^{p\pi^0}\right\}N(P)$$

$$+ \lambda_{p\pi^0}\frac{m_N + \not{P}' - \not{k}}{m_N^2 - (P'-k)^2}\left\{\gamma_\nu F_1^p - \frac{i\sigma_{\nu\mu}q^\mu}{2m_N}F_2^p\right\}N(P)$$

$$\text{RHS} = -\frac{i}{f_\pi}\left(-\frac{1}{2}\right)\gamma_5\lambda_p\frac{m_N + \not{P}'}{m_N^2 - (P')^2}\left\{\gamma_\nu F_1^p - \frac{i\sigma_{\nu\mu}q^\mu}{2m_N}F_2^p\right\}N(P)$$

To the same accuracy $\lambda_{p\pi^0} = -\frac{i}{f_\pi}\left(-\frac{1}{2}\right)\gamma_5$ and $(P'-k)^2 \simeq P'^2$ so that

- The two "extra" terms cancel against each other
- The standard result $G_1^{p\pi^0} = 0$ is reproduced (as expected)

◊ If the order of limits $k \to 0$, $q \to \infty$ is reversed, $(P'-k)^2$ moves away from the pole at m_N^2.

◊ The chiral rotation of the nucleon current is no more compensated

◊ This happens when $(P'-k)^2 - m_N^2 \sim \Lambda^2$, or $Q^2 \geq \frac{\Lambda^3}{m_\pi}$

Light Cone Sum Rules — *continued (2)*

Since we cannot calculate at $P'^2 \to m_N^2$, take $P'^2 \sim -1$ GeV^2 and make a matching between

(a) **The Operator Product Expansion in terms of pion-nucleon DAs**

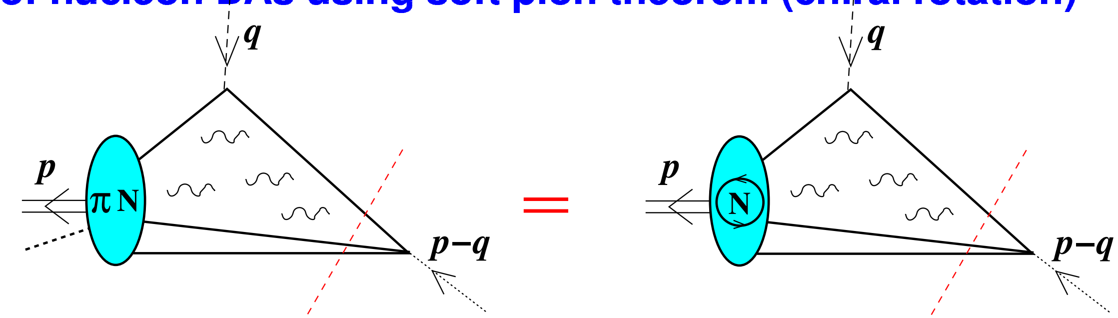
$$\langle 0|T\{\eta_p(0) j_\nu^{\rm em}(x)|N(P)\pi^a(k)\rangle \quad = \quad \sum_{\rm twist} C_\nu(x^2, px) \otimes \langle 0|q(x_1)q(x_2)q(x_3)|N(P)\pi^a(k)\rangle$$

The OPE goes in pion-nucleon DAs of increasing twist 3,4,5 $\quad\to\quad$ **next slide**

$$4\langle 0|\epsilon^{ijk} u_\alpha^i(a_1 z) u_\beta^j(a_2 z) d_\gamma^k(a_3 z)|N(P)\pi\rangle_{\rm twist-3} =$$

$$= (\gamma_5)_{\gamma\delta} \frac{-i}{f_\pi} \left[V_1^{\pi N} (\not p C)_{\alpha\beta} \left(\gamma_5 N^+\right)_\gamma + A_1^{\pi N} (\not p \gamma_5 C)_{\alpha\beta} N_\gamma^+ + T_1^{\pi N} \left(i\sigma_{\perp p} C\right)_{\alpha\beta} \left(\gamma^\perp \gamma_5 N^+\right)_\gamma \right].$$

calculated in terms of nucleon DAs using soft pion theorem (chiral rotation)



◇ **We do not include operators with a pion field**

Pion-Nucleon Distribution Amplitudes

$$|p\uparrow\rangle = \frac{\phi_s(x)}{\sqrt{6}}|2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle + \frac{\phi_a(x)}{\sqrt{2}}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|p\uparrow\pi^0\rangle = \frac{\phi_s(x)}{2\sqrt{6}f_\pi}|6u_\uparrow d_\downarrow u_\uparrow + u_\uparrow u_\downarrow d_\uparrow + d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2\sqrt{2}f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|n\uparrow\pi^+\rangle = \frac{\phi_s(x)}{\sqrt{12}f_\pi}|2u_\uparrow d_\downarrow u_\uparrow - 3u_\uparrow u_\downarrow d_\uparrow - 3d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

Pobylitsa, Polyakov, Strikman ’01

Equivalent representation

$$V_1^{n\pi^+}(1,2,3) = \frac{1}{\sqrt{2}}\Big\{V_1^n(1,3,2) + V_1^n(1,2,3) + V_1^n(2,3,1) + A_1^n(1,3,2) + A_1^n(2,3,1)\Big\},$$

$$A_1^{n\pi^+}(1,2,3) = -\frac{1}{\sqrt{2}}\Big\{V_1^n(3,2,1) - V_1^n(1,3,2) + A_1^n(2,1,3) + A_1^n(2,3,1) + A_1^n(3,1,2)\Big\},$$

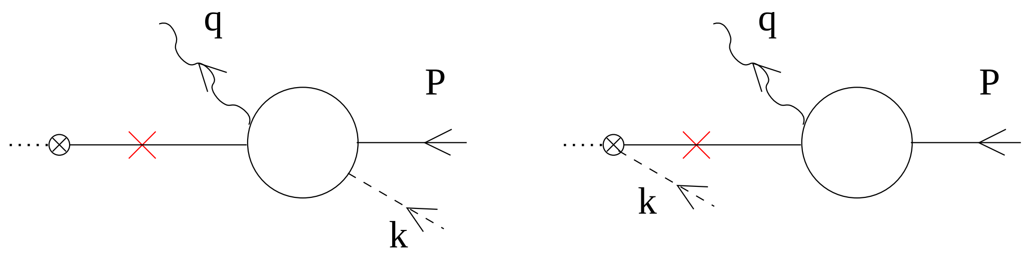
$$T_1^{n\pi^+}(1,2,3) = \frac{1}{2\sqrt{2}}\Big\{A_1^n(2,3,1) + A_1^n(1,3,2) - V_1^n(2,3,1) - V_1^n(1,3,2)\Big\}.$$

Braun, Ivanov, Lenz, A.Peters; PRD75:014021,2007

Extended to twist-4,5,6

Light Cone Sum Rules — *continued (3)*

(b) **The dispersion integral in terms of hadron states**



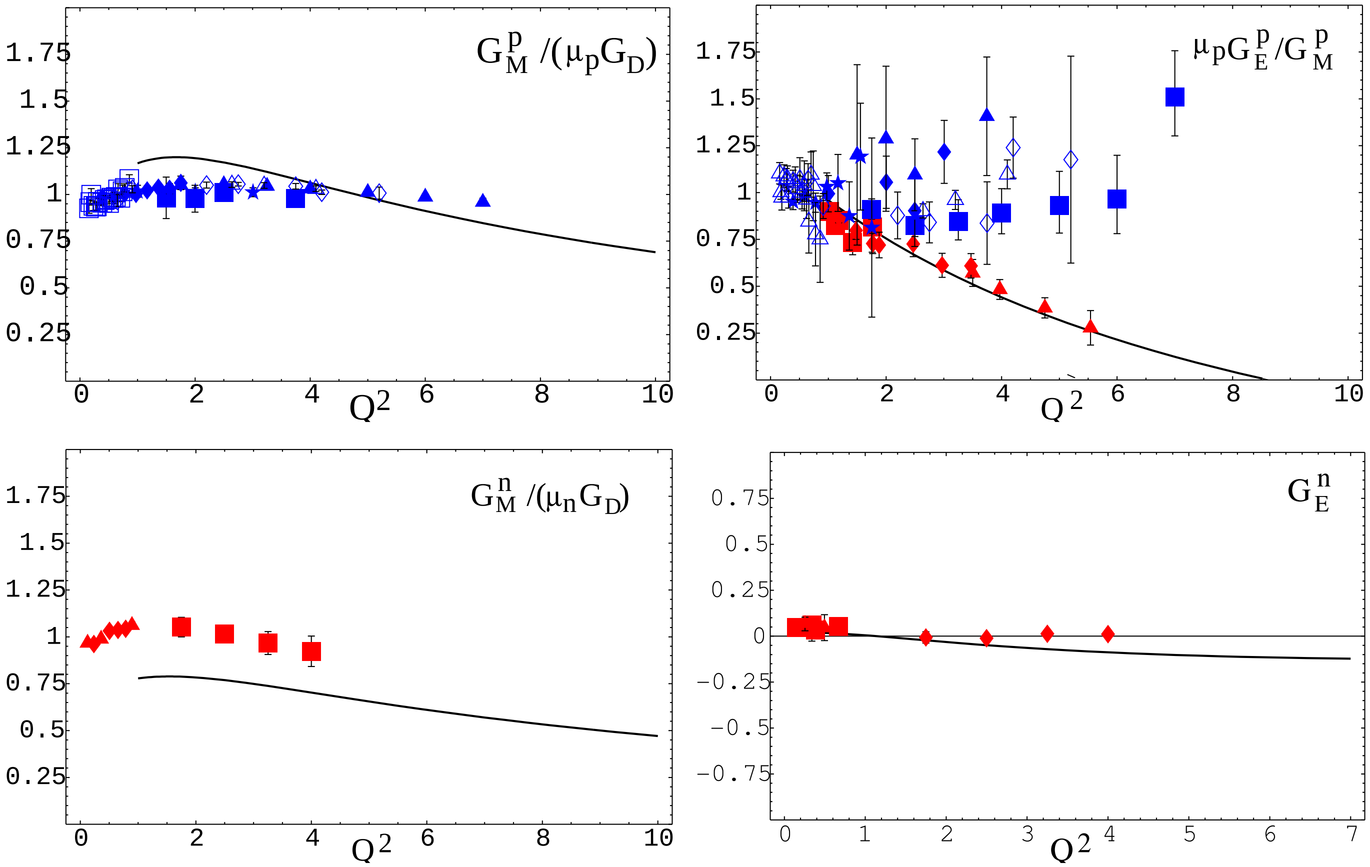
nucleon **pion-nucleon** **continuum**

(b) **Lorentz Structure** $\times \left\{ \frac{2\lambda_p (Q^2/m_N^2) G_1^{\pi N}}{m_N^2 - P'^2} + \frac{2\lambda_{\pi N} F_1^{\rm em}}{m_N^2 - (P'-k)^2} + \int_{s_0}^{\infty} \frac{\rho_{\rm QCD}(s) ds}{s - P'^2} \right\}$

- **The pion-nucleon contribution (semidisconnected) can be included in the continuum, if** $m_\pi Q^2 \geq m_N (s_0 - m_N^2) - 2 m_\pi m_N^2$, **wherefrom** $Q^2 \geq 7$ **GeV**^{2}
- **Borel transformation** $P'^2 \to M^2$**:** $\int \frac{\rho(s) ds}{s - P'^2} \to \int \rho(s) ds \, \exp[-s/M^2]$

Nucleon electromagnetic form factors

$$\langle N(P')|j_\mu^{em}(0)|N(P)\rangle = \bar{N}(P')\left[\gamma_\mu F_1(Q^2) - i\frac{\sigma_{\mu\nu}q^\nu}{2m_N}F_2(Q^2)\right]N(P)$$



proton

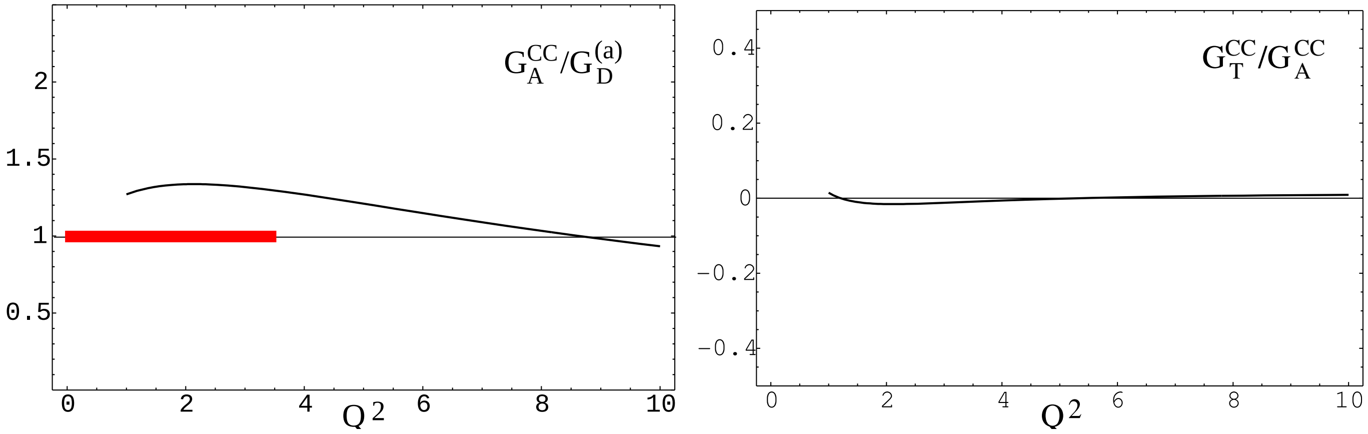
neutron

- **Leading order LCSR, BLW distribution amplitudes**

Braun, Lenz, Wittmann; PRD73(2006)094019

Nucleon axial vector form factors

$$\langle N(P')|A_\mu(0)|N(P)\rangle = \bar{N}(P')\left[\gamma_\mu G_A(Q^2) - \frac{q_\mu}{2m_N} G_P(Q^2) - i\frac{\sigma_{\mu\nu}q^\nu}{2m_N} G_T(Q^2)\right]\gamma_5 N(P)$$



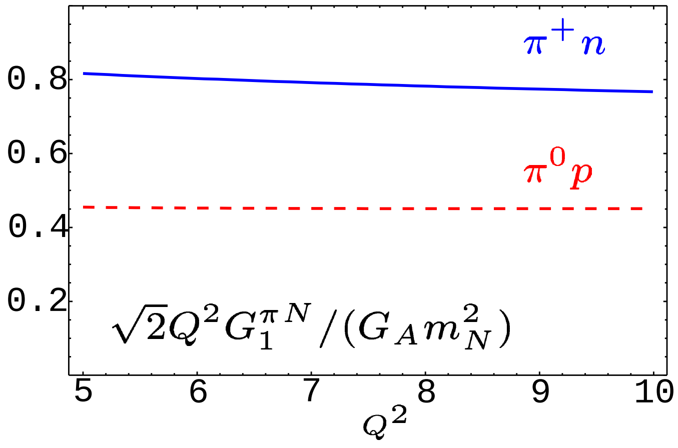
charged current

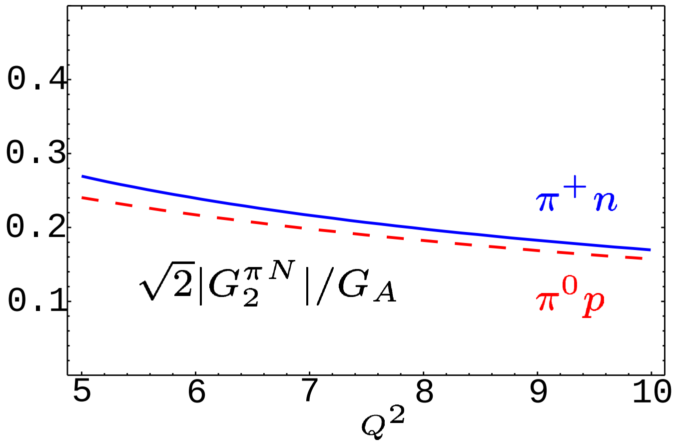
- **Leading order LCSR, BLW distribution amplitudes**

Braun, Lenz, Wittmann; PRD73(2006)094019

Generalized pion-nucleon form factors

$$M_\mu^{\pi N} = -\frac{i}{f_\pi}\bar{N}(P_2)\gamma_5\left\{\left(\gamma_\mu q^2 - q_\mu \not{q}\right)\frac{1}{m_N^2}G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}G_2^{\pi N}(Q^2)\right\}N(P_2)$$





- **Leading order LCSR, BLW distribution amplitudes**

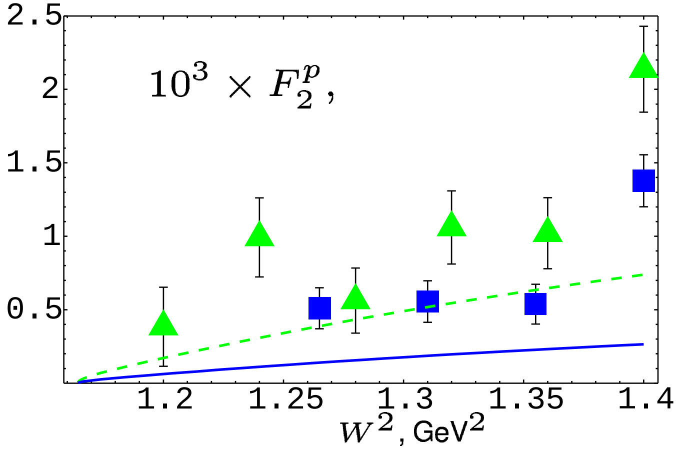
Braun, Ivanov, Lenz, A.Peters; PRD75:014021,2007

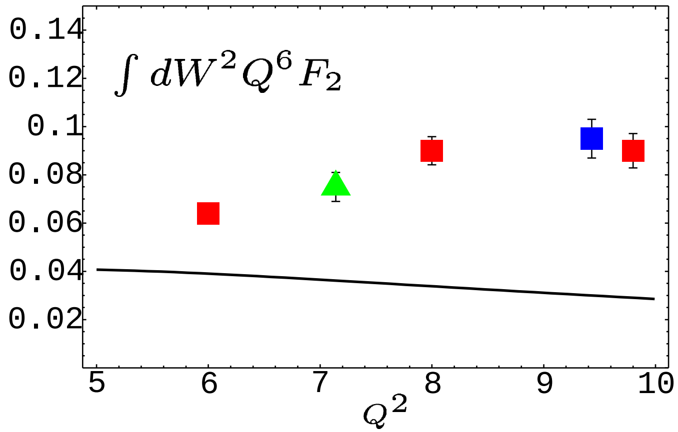
- π^0/π^+ **production ratio** $\sim 1/3$ **und roughly** Q^2**-independent**
- **Significant "tensor" form factors, similar size and opposite sign for** π^0 **and** π^+
- **No significant enhancement for the DIS total cross section**

Structure function $F_2^p(x, Q^2)$

S-wave + P-wave

$$F_2^p(W,Q^2) \simeq \frac{Q^2\beta(W)}{(4\pi f_\pi)^2}\left[\frac{Q^4}{m_N^4}\left((G_1^{\pi N})^2 Q^2 + \frac{1}{4}(G_2^{\pi N})^2 m_N^2\right) + \frac{3g_A^2\beta^2(W)W^4}{4(W^2-m_N^2)^2}(G_M^p)^2\right]$$





A.Peters; work in progress

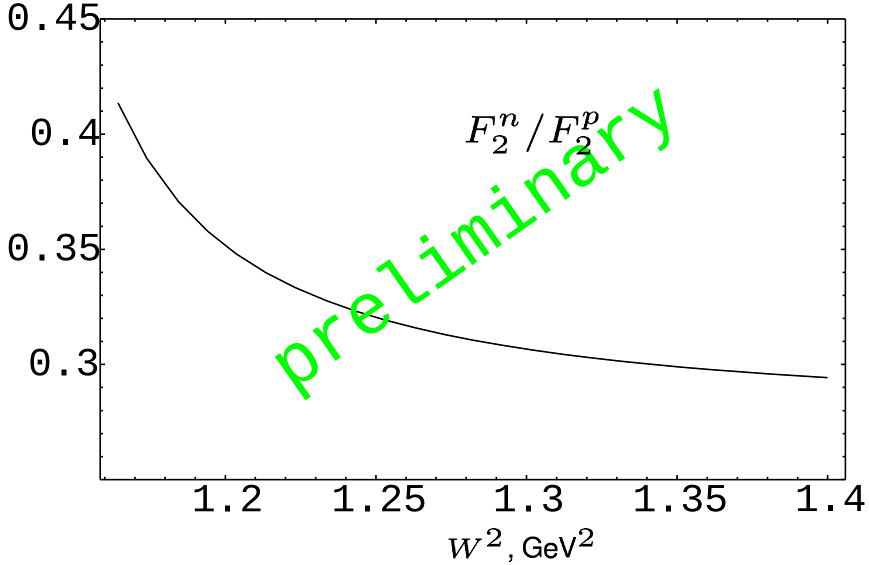
- **SLAC E136, $Q^2 = 7.14$ and $Q^2 = 9.43$**

vs.

- **Leading order LCSR, BLW distribution amplitudes**

- **No significant enhancement for the DIS total cross section**

$F_2^n(x, Q^2)/F_2^p(x, Q^2)$ Ratio



$$\lim_{W \to W_{th}} \frac{F_2^n(W, Q^2)}{F_2^p(W, Q^2)} = 0.41 \pm 0.08$$

- **Leading order LCSR, BLW distribution amplitudes**

A.Peters; work in progress

- **In agreement with parton model prediction** $F_2^n/F_2^p = 3/7$

Outlook

- LCSR approach is a natural candidate; however, theoretical understanding is not yet complete
- P-wave contributions (pole terms) have to be included for comparison with the data
- Can be done: radiative corrections to LCSR; also the energy dependence of pion-nucleon DAs
- Many potential applications