N to Δ electromagnetic and axial form factors in Lattice QCD

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Exclusive Reactions at High Momentum Transfer
Jefferson Lab
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Outline

Motivation

Main focus: Form factors of the nucleon-Δ system

- describe structure of hadrons e.g. quadrupole N to Δ transition form factors may indicate deformation in the nucleon and/or Δ
- provide important input for phenomenological models builders and for chiral effective theories
- make direct contact with experiment e.g.

1. provide a prediction for the parity violating asymmetry in axial N to Δ transition
2. evaluate the EMR and CMR at low q^2

- test the diagonal and non-diagonal Goldberger-Treiman relations

Evaluation of Observables

Calculate vacuum expectation value of gauge invariant operators in Euclidean time:

$$< \Omega|\hat{O}|\Omega >= \frac{1}{Z}\int d[U]d[\bar\psi]d[\psi]\;\; O[U,\bar\psi,\psi]e^{-S_g[U]-S_F[U,\bar\psi,\psi]}$$

Integrate over the fermionic degrees of freedom

$$\longrightarrow < \Omega|\hat{O}|\Omega >= \frac{1}{Z}\int d[U]\;\; \det(D[U])O[U,D^{-1}[U]]e^{-S_g[U]}$$

$\rightarrow D^{-1}_{jn}[U]$ substitutes each appearance of $-\bar\psi_n\psi_j$ - valence quarks
$\rightarrow \det(D[U])$ - sea quarks

- Put on a 4-D lattice: many ways to do this $\rightarrow$ Wilson, staggered, Domain wall fermions
- Do numerically by stochastically generating a representative ensemble of U according to the probability

$$P[U]=\frac{1}{Z}\exp\left\{-S_g[U]+\ln\left(\det(D[U])\right)\right\}$$

- Then compute $< \Omega|\hat{O}|\Omega >= \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}O[U^k,D^{-1}[U^k]]$

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Lattice caveats

- **q^2-values:** Fourier transform of lattice results in coordinate space taken numerically $\rightarrow$ for large values of momentum transfer results are too noisy
  $\Longrightarrow$ Limited to $-q^2 \sim 2$ GeV^2.
- **Finite Volume:** Only discrete values of momentum in units of $2\pi/L$ are allowed. Take box sizes such that $Lm_\pi \gtrsim 4.5$.
- **Finite lattice spacing a** (Ultra-violet cut-off): Use two different formulations:
  1. Wilson fermions: $\mathcal{O}(a)$ errors
  2. Staggered fermions with Asqtad action and Domain wall fermions (hybrid approach): $\mathcal{O}(a^2)$ errors
  $\Longrightarrow$ agreement between them provides a consistency check of lattice formulation.
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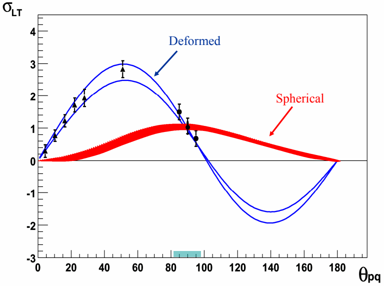
$\gamma^* N \to \Delta$

- A dominant magnetic dipole, M1
- An electric quadrupole, E2 and a Coulomb, C2 signal a deformation in the nucleon/Δ
- Experimental evidence for non-zero deformation in nucleon/Δ *



- Precise data strongly "suggesting" deformation in the Nucleon/Δ
- EMR=$(-2.00 \pm 0.40_{\rm stat+sys} \pm 0.27_{\rm mod})$%, CMR=$(-6.27 \pm 0.32_{\rm stat+sys} \pm 0.10_{\rm mod})$%

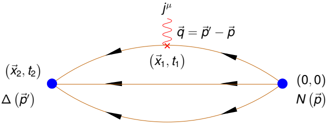
$$R_{EM}(\mathrm{EMR}) = -\frac{G_{E2}(q^2)}{G_{M1}(q^2)},$$

$$R_{SM}(\mathrm{CMR}) = -\frac{|\vec{q}|}{2m_\Delta}\frac{G_{C2}(q^2)}{G_{M1}(q^2)},$$

in lab frame of the Δ.

* C. N. Papanicolas, Eur. Phys. J. A18, 141 (2003); N. Sparveris *et al.*, PRL **94**, 022003 (2005)

$N\gamma^* \rightarrow \Delta$ on the Lattice



Sachs form factors:

$$G_{M1}(q^2),\; G_{E2}(q^2),\; G_{C2}(q^2)$$

The standard decomposition of the N to Δ electromagnetic matrix element:

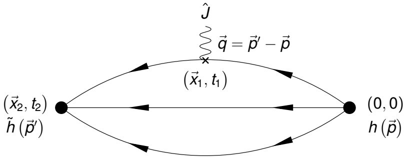
$$\langle \Delta(\vec{p}', s')|j_\mu|N(\vec{p}, s)\rangle = \imath\sqrt{\frac{2}{3}}\left(\frac{m_\Delta m_N}{E_\Delta(p')E_N(p)}\right)^{1/2} \bar{u}^\sigma(\vec{p}', s')\mathcal{O}_{\sigma\mu}u(\vec{p}, s)\,,$$

with

$$\mathcal{O}_{\sigma\mu} = G_{M1}(q^2)K^{M1}_{\sigma\mu} + G_{E2}(q^2)K^{E2}_{\sigma\mu} + G_{C2}(q^2)K^{C2}_{\sigma\mu}\,,$$

Use the lattice conserved current for Wilson fermions and the local current for DWF

Three-point functions



$$G^{\tilde{h}Jh}(t_2, t_1; \mathbf{q}) \quad = $$
$$< \Omega| \sum_{\mathbf{x_1}, \mathbf{x_2}} e^{i\mathbf{q}.\mathbf{x_1}}\, \hat{T} \hat{J}_{\tilde{h}}(\mathbf{x_2}, t_2) \hat{J}(\mathbf{x_1}, t_1) \hat{J}_h^\dagger(0)|\Omega >$$

where the final hadron is produced at rest.

The interpolating fields for N and Δ (sink and source Wuppertal smeared):

$$\begin{aligned} J^p(x) &= \epsilon^{abc}[u^{Ta}(x)C\gamma_5 d^b(x)]u^c(x), \\ J_\sigma^{\Delta^+}(x) &= \frac{1}{\sqrt{3}}\epsilon^{abc}\{2[u^{Ta}(x)C\gamma_\sigma d^b(x)]u^c(x) \\ &+ [u^{Ta}(x)C\gamma_\sigma u^b(x)]d^c(x)\} \end{aligned}$$

- HYP-smearing applied to the links for the interpolating fields for the case of unquenched Wilson fermions
- HYP-smeared MILC configurations

- Sequential inversion : fixed quantum numbers at sink and source
- fixed sink time t_2 and variable insertion time t_1
- this allows any operator to be inserted at t_1
- sum over all $\vec{x}_1$ and $\vec{x}_2$ and vary t_1 in search for a plateau

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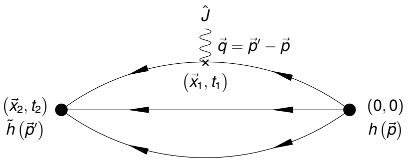

$$G^{\tilde{h}Jh}(t_2, t_1; \mathbf{q}) = < \Omega| \sum_{\mathbf{x_1}, \mathbf{x_2}} e^{i\mathbf{q}.\mathbf{x_1}} \hat{T} \hat{J}_{\tilde{h}}(\mathbf{x_2}, t_2) \hat{J}(\mathbf{x_1}, t_1) \hat{J}_h^\dagger(0)|\Omega >$$

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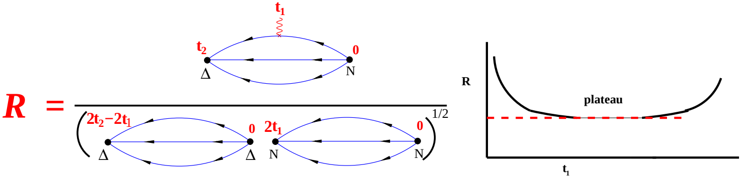
The exponential time dependence and unknown overlaps of the interpolating fields with the physical states cancel by dividing the three-point function with appropriate combinations of two-point functions. For example

$$R = \frac{G^{\tilde{h}j_\mu h}(t_2,t_1;\mathbf{q})}{\sqrt{G^{\tilde{h}}(2t_2-2t_1;\mathbf{0})G^{h}(2t_1;\mathbf{q})}} \xrightarrow{t_1>>1,t_2-t_1>>1} <\tilde{h}|j_\mu|h>$$



- Wuppertal and HYP-smearing filters ground state efficiently i.e. t_1 and $t_2 - t_1$ small
- Optimize R so that two-points functions with the shortest possible time separation are involved $\longrightarrow$ less noisy signal

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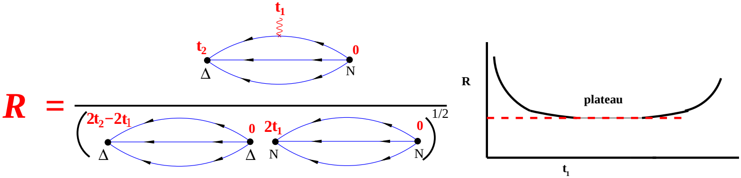

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Lattice parameters

Wilson fermions

number of confs	κ	m_π (GeV)	m_N (GeV)
Quenched $32^3 \times 64$, $\beta = 6.0$, $a^{-1} = 2.14(6)$ GeV ($a = 0.09$ fm) from nucleon mass at chiral limit			
200	0.1554	0.563(4)	1.267(11)
200	0.1558	0.490(4)	1.190(13)
200	0.1562	0.411(4)	1.109(13)
	κ_c =0.1571	0.	0.938(9)
Unquenched* $24^3 \times 40$, $\beta = 5.6$, $a^{-1} = 2.56(10)$ GeV ($a = 0.08$ fm)			
185	0.1575	0.691(8)	1.485(18)
157	0.1580	0.509(8)	1.280(26)
Unquenched† $24^3 \times 32$, $\beta = 5.6$, $a^{-1} = 2.56(10)$ GeV			
200	0.15825	0.384(8)$\leftarrow Lm_\pi = 3.6$	1.083(18)
	κ_c = 0.1585	0.	0.938(33)

Hybrid scheme $a^{-1} = 1.58$ GeV ($a = 0.125$ fm) from MILC collaboration

number of confs	Volume	$(am_{u,d})^{\rm sea}$	$(am_s)^{\rm sea}$	$(am_q)^{DW}$	m_π^{DW} (GeV)	m_N (GeV)
150	$20^3 \times 64$	0.03	0.05	0.0478	0.589(2)	1.392(9)
198	$20^3 \times 64$	0.02	0.05	0.0313	0.501(4)	1.255(19)
100	$20^3 \times 64$	0.01	0.05	0.0138	0.362(5)	1.138(25)
150	$28^3 \times 64$	0.01	0.05	0.0138	0.354(2)	1.210(24)

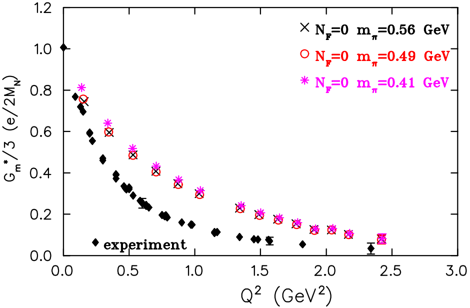
For Wilson fermions we have consistency with determination of scale using the Sommer scale.

* SESAM collaboration (TχL), B. Orth *et al.*, Phys. Rev. D72(2005)014503

† DESY-Zeuthen group, C.Urbach *et al.*, Comput. Phys. Commun. 174(2006)87.

Magnetic dipole form factor§

Ash parametrization: $G_m^* = \frac{1}{\sqrt{1+\frac{Q^2}{(m_N+m_\Delta)^2}}} G_{M1}$, $Q^2 = -q^2$ is the momentum transfer squared



- Results in units of Bohr magnetons using the nucleon mass determined on the lattice
- Almost no dependence on quark mass for this range of pion masses

§C.A. Ph. de Forcrand, th. Lippert H. Neff, J. W. Negele, K. Schilling, W. Schroers, A. Tsapalis, PRL 94, 021601 (2005)

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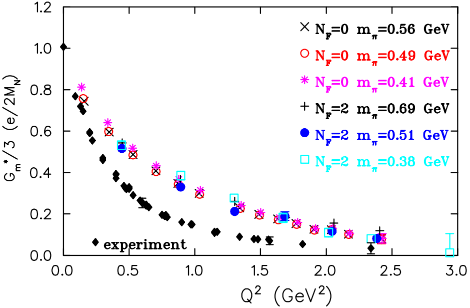

- Results in units of Bohr magnetons using the nucleon mass determined on the lattice
- Unquenching effects small for this range of pion masses

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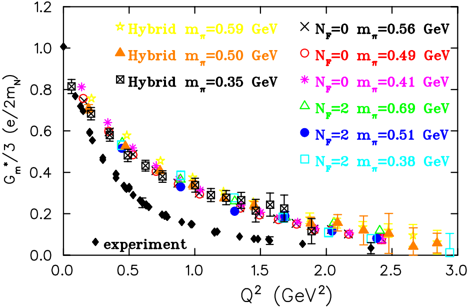
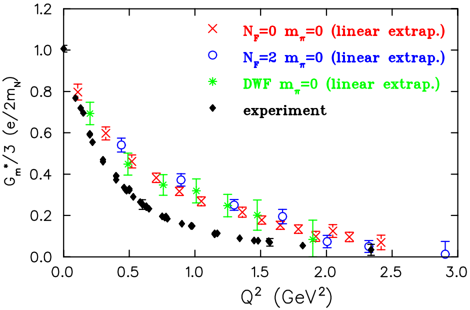


- Results in the hybrid approach in agreement with results using Wilson fermions $\Longrightarrow$ since these two lattice formulations have different systematics (e.g. different dependence on the lattice spacing a) agreement between them is non-trivial $\rightarrow$ small lattice artifacts?

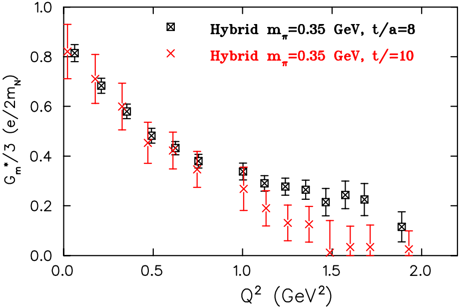
§C.A., R. Edwards, G. Koutsou, Th. Leontiou, H. Neff, J. W. Negele, W. Schroers, A. Tsapalis, PoS LAT2005, 091 (2006)

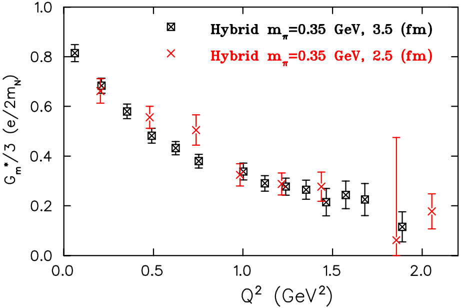
Results for magnetic dipole at the physical limit



- What could explain the discrepancy with experiment?
- Finite lattice spacing, finite volume??

Check lattice calculation





- Increasing source-sink separation t_2 by 25% fm does not change the results. Good plateaus $\rightarrow$ ground state dominance.
- Increasing volume from 2.5 fm to 3.5 fm does not change the results.
- For Wilson fermions we have $\mathcal{O}(a)$ errors, in the hybrid approach we have $\mathcal{O}(a^2)$ errors
- Extrapolation to physical limit???

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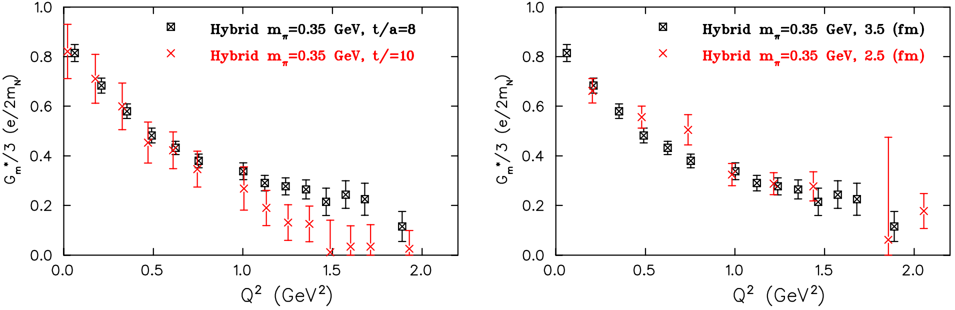

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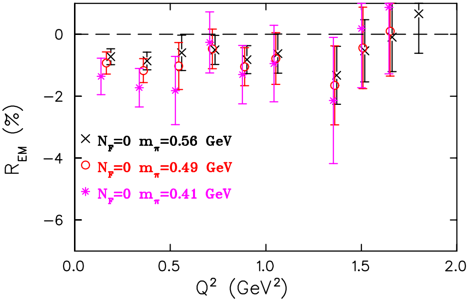
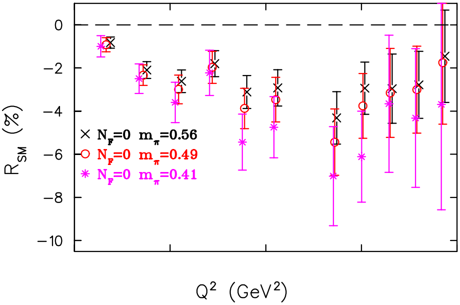
Results for EMR and CMR

$$R_{EM}(\text{EMR}) = -\frac{G_{E2}(q^2)}{G_{M1}(q^2)}, \quad R_{SM}(\text{CMR}) = -\frac{|\vec{q}|}{2m_\Delta}\frac{G_{C2}(q^2)}{G_{M1}(q^2)},$$

in lab frame of the Δ.





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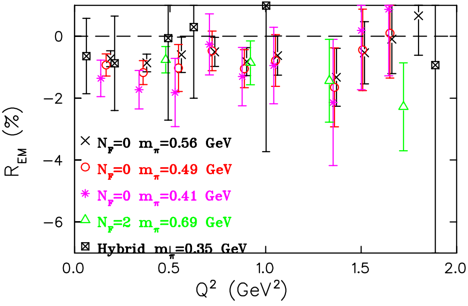
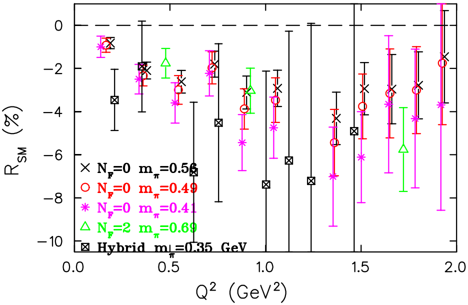
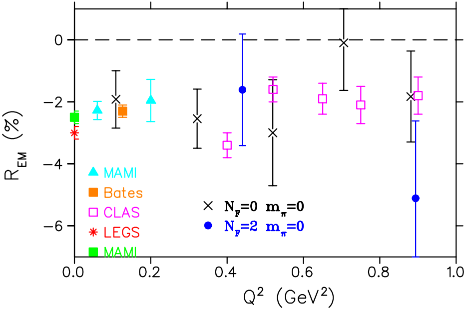


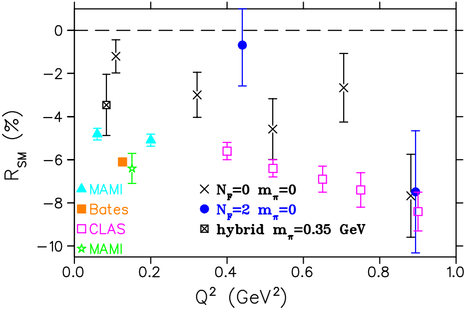


Results for EMR and CMR at physical limit

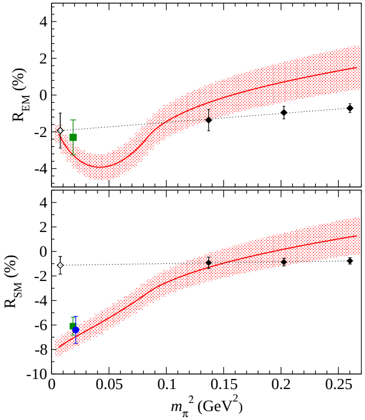
$$R_{EM}(\mathrm{EMR}) = -\frac{G_{E2}(q^2)}{G_{M1}(q^2)}, \quad R_{SM}(\mathrm{CMR}) = -\frac{|\vec{q}|}{2m_\Delta}\frac{G_{C2}(q^2)}{G_{M1}(q^2)}.$$

in lab frame of the Δ





Extrapolation in m_π^2



- We used a linear extrapolation in order to approach the physical limit
- Calculation within a chiral effective-field theory, using an expansion where $\frac{m_\Delta - m_N}{m_N} \sim \mathcal{O}(\delta)$ and $\frac{m_\pi}{m_N} \sim \mathcal{O}(\delta^2)$, has shown strong dependence on m_π. Only done at the lowest Q^2 *

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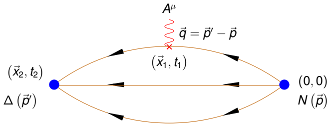
*V. Pascalutsa and M. Vanderhaeghen PRL 95 (2005) 232001

N to Δ axial form factors

Any operator can be inserted at t_1 $\longrightarrow$ with no additional inversions we can evaluate the N-N and N-Δ matrix elements for any operator.



- Vector current: $j_\mu = \bar{\psi}\gamma_\mu\psi$
- Pseudoscalar current: $P^a = \bar{\psi}i\gamma_5\frac{\tau^a}{2}\psi$
- Axial current: $A^a_\mu = \bar{\psi}\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi$: axial N to Δ transition form factors and asymmetry to be measured at JLab (G0 experiment)

The optimal ratios: shortest possible time separation

$$R_\sigma(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma;\mu) = \frac{\langle G_\sigma^{\Delta A^\mu N}(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma)\rangle}{\langle G_{ii}^{\Delta\Delta}(t_2;\mathbf{p}';\Gamma_4)\rangle}\left[\frac{\langle G^{NN}(t_2-t_1;\mathbf{p};\Gamma_4)\rangle\langle G_{ii}^{\Delta\Delta}(t_1;\mathbf{p}';\Gamma_4)\rangle\langle G_{ii}^{\Delta\Delta}(t_2;\mathbf{p}';\Gamma_4)\rangle}{\langle G_{ii}^{\Delta\Delta}(t_2-t_1;\mathbf{p}';\Gamma_4)\rangle\langle G^{NN}(t_1;\mathbf{p};\Gamma_4)\rangle\langle G^{NN}(t_2;\mathbf{p};\Gamma_4)\rangle}\right]^{1/2}$$

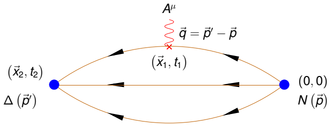
$$\stackrel{t_2-t_1\gg 1,\,t_1\gg 1}{\Rightarrow} \Pi_\sigma(\mathbf{p}',\mathbf{p};\Gamma;\mu).$$

σ is the spin index of the Δ field and the projection matrices Γ are given

$$\Gamma_i = \frac{1}{2}\begin{pmatrix}\sigma_i & 0\\ 0 & 0\end{pmatrix} \quad \Gamma_4 = \frac{1}{2}\begin{pmatrix}I & 0\\ 0 & 0\end{pmatrix}$$

N to Δ axial form factors

Any operator can be inserted at t_1 $\longrightarrow$ with no additional inversions we can evaluate the N-N and N-Δ matrix elements for any operator.



- Vector current: $j_\mu = \bar{\psi}\gamma_\mu\psi$
- Pseudoscalar current: $P^a = \bar{\psi}i\gamma_5\frac{\tau^a}{2}\psi$
- Axial current: $A^a_\mu = \bar{\psi}\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi$: axial N to Δ transition form factors and asymmetry to be measured at JLab (G0 experiment)

The optimal ratios: shortest possible time separation

$$R_\sigma(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma; \mu) = \frac{\langle G_\sigma^{\Delta A^\mu N}(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma)\rangle}{\langle G_{ii}^{\Delta\Delta}(t_2; \mathbf{p}'; \Gamma_4)\rangle}\left[\frac{\langle G^{NN}(t_2 - t_1; \mathbf{p}; \Gamma_4)\rangle\langle G_{ii}^{\Delta\Delta}(t_1; \mathbf{p}'; \Gamma_4)\rangle\langle G_{ii}^{\Delta\Delta}(t_2; \mathbf{p}'; \Gamma_4)\rangle}{\langle G_{ii}^{\Delta\Delta}(t_2 - t_1; \mathbf{p}'; \Gamma_4)\rangle\langle G^{NN}(t_1; \mathbf{p}; \Gamma_4)\rangle\langle G^{NN}(t_2; \mathbf{p}; \Gamma_4)\rangle}\right]^{1/2}$$

$$\stackrel{t_2 - t_1 \gg 1, t_1 \gg 1}{\Rightarrow} \Pi_\sigma(\mathbf{p}', \mathbf{p}; \Gamma; \mu).$$

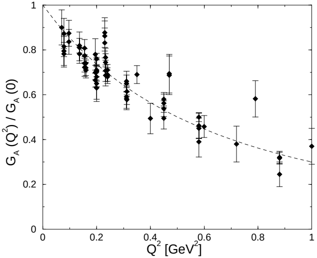
σ is the spin index of the Δ field and the projection matrices Γ are given

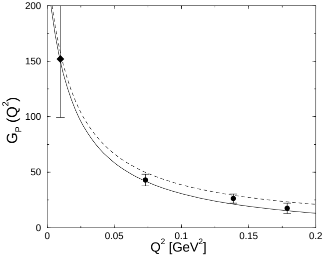
$$\Gamma_i = \frac{1}{2}\begin{pmatrix}\sigma_i & 0\\ 0 & 0\end{pmatrix} \quad \Gamma_4 = \frac{1}{2}\begin{pmatrix}I & 0\\ 0 & 0\end{pmatrix}$$

Nucleon axial form factors

Decomposition of the nucleon weak matrix element:

$$\langle N(p')|A^3_\mu|N(p)\rangle = \imath\left(\frac{m_N^2}{E_N(p')E_N(p)}\right)^{1/2}\bar{u}(p')\left[\left(G_A(q^2)\gamma_\mu\gamma_5 + \frac{q_\mu}{2m_N}G_p(q^2)\right)\right]\frac{\tau^3}{2}u(p)$$





From V. Bernard, L. Elouadrhiri and U. Meissner, hep-ph/0107088

Lattice studies:

-$G_A(0)$ LHP collaboration PRL 96 052001 (2006) and QCDSF PRD 74 094508 (2006)

- $G_A(q^2)$ and $G_p(q^2)$ K.F. Liu, S.J. Dong and T. Drapper, PRL 74 (1995) 2172 and LHP Collaboration, hep-lat/0610007.

N to Δ axial transition form factors*

Decomposition of the N to Δ weak matrix element in terms of four transition form factors†

$$\langle \Delta(p')|A^3_\mu|N(p)\rangle = \imath\sqrt{\frac{2}{3}}\left(\frac{m_\Delta m_N}{E_\Delta(p')E_N(p)}\right)^{1/2}\bar{u}^\lambda(p')\Bigg[$$

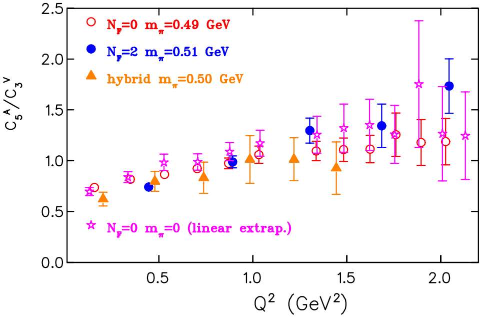
$$\left(\frac{C_3^A}{m_N}\gamma^\nu + \frac{C_4^A}{m_N^2}p'^\nu\right)(g_{\lambda\nu}g_{\rho\nu} - g_{\lambda\rho}g_{\mu\nu})q^\rho + C_5^A g_{\lambda\mu} + \frac{C_6^A}{m_N^2}q_\lambda q_\mu\Bigg]u(p)$$

* C.A., Th. Leontiou, J. W. Negele and A. Tsapalis, PRL 98 052003 (2007)

† S.P. Wells (PAVI 2002); L. S. Alder, Ann. Phys. 50, 189 (1968); L. Smith, Phys. Rep. 3C(1972) 261

Parity violating asymmetry

- Under the assumptions that $C_3^A \sim 0$ and $C_4^A \ll C_5^A$ the parity violating asymmetry is proportional to the ratio C_5^A/C_3^V (analog of g_A/g_V) *
- C_3^V can be evaluated from the electromagnetic N to Δ transition
- G0 collaboration plans to measure PV asymmetry at Jefferson Lab. **



- Non-zero when $Q^2 = 0$
- Increases with Q^2 up to about $Q^2 \sim 1.5$ GeV^2
- Unquenching effects small for this range of quark masses
- Weak quark mass dependence $\rightarrow$ results can be taken as a physical prediction for the ratio

*N.C. Mukhopadhyay *et al.* NP A633(1998) 481
**S.P. Wells, PAVI 2002

PCAC

- Partial conservation of axial current:

$$\partial^\mu A^a_\mu = f_\pi m^2_\pi \pi^a$$

- f_π determined from the two-point function

$$< 0|A^a_\mu|\pi^b(p) >= ip_\mu \delta^{ab} f_\pi$$

  with $f_\pi = 92$ MeV.

- Axial Ward Identity:

$$\partial^\mu A^a_\mu = 2m_q P^a$$

  $\Longrightarrow$ relate the pion field π^a with the pseudoscalar density: $\pi^a = \left(2m_q P^a / f_\pi m^2_\pi\right)$

- Compute m_q from the matrix element:

$$m_q = \frac{m_\pi < 0|A^a_0|\pi^a(0) >}{2 < 0|P^a|\pi^a(0) >}$$

Pion-nucleon (Δ) form factors

- Obtain couping of the nucleon with the pion field using the relation

$$2m_q < N(p')|P^3|N(p)> = \frac{f_\pi m_\pi^2 G_{\pi NN}}{m_\pi^2 - q^2}\bar{u}(p') i\gamma_5 u(p)$$

- Similarly for $G_{\pi N\Delta}$ we have

$$2m_q < \Delta(p')|P^3|N(p)> = \sqrt{\frac{2}{3}}\frac{f_\pi m_\pi^2 G_{\pi N\Delta}}{m_\pi^2 - q^2}\bar{u}_\nu(p')\frac{q^\nu}{2m_N}u(p)$$

- PCAC relates axial form factors G_A and G_p with the couping constant $G_{\pi NN}$ and equivalently C_5^A and C_6^A with $G_{\pi N\Delta}$ $\Longrightarrow$ Goldberger Treiman relations (GTR)

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$\Longrightarrow$ Goldberger Treiman relations (GTR)

The Goldberger-Treiman relations

- Diagonal and non-diagonal GTR:

$$\begin{aligned} G_A(q^2) + \frac{q^2}{4m_N^2} G_p(q^2) &= \frac{1}{2m_N}\frac{2G_{\pi NN}(q^2) f_\pi m_\pi^2}{m_\pi^2 - q^2} \\ C_5^A(q^2) + \frac{q^2}{m_N^2} C_6^A(q^2) &= \frac{1}{2m_N}\frac{G_{\pi N\Delta}(q^2) f_\pi m_\pi^2}{m_\pi^2 - q^2} \end{aligned}$$

- Assuming pion pole dominance for G_p and C_6^A:

$$\begin{aligned} \frac{1}{2m_N} G_p(q^2) &\sim \frac{2G_A(q^2)}{m_\pi^2 - q^2} \sim \frac{2G_{\pi NN}(q^2) f_\pi}{m_\pi^2 - q^2} \\ \frac{1}{m_N^2} C_6^A(q^2) &\sim \frac{C_5^A(q^2)}{m_\pi^2 - q^2} \sim \frac{1}{2m_N}\frac{G_{\pi N\Delta}(q^2) f_\pi}{m_\pi^2 - q^2} \end{aligned}$$

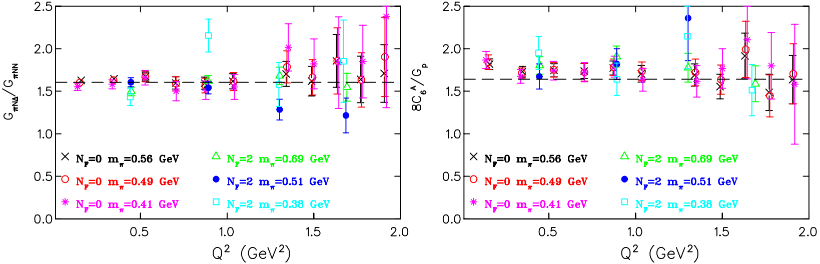
$\Longrightarrow$ GTR: $G_{\pi NN} f_\pi = m_N G_A$ and $G_{\pi N\Delta} f_\pi = 2m_N C_5^A$

Ratios

Advantages of taking ratios:

- Renormalization constants cancel
- Weaker dependence on quark mass
- Requires no knowledge of m_q which can have large lattice artifacts
- Finite volume and lattice spacing effects?

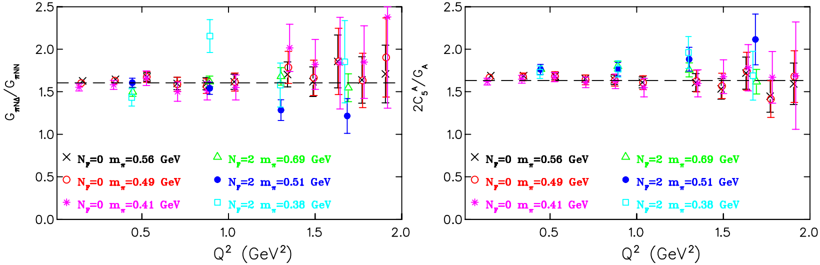
Ratios of $G_{\pi NN}$ and $G_{\pi N\Delta}$



- The ratio $G_{\pi N\Delta}/G_{\pi NN}$ comes out independent of q^2 and quark mass. Furthermore the value of 1.6 is what is expected.
- The ratio $8C_6^A/G_p \sim 1.6 \sim G_{\pi N\Delta}/G_{\pi NN} \Longrightarrow$ pion pole dominance:

$$\frac{1}{2m_N}G_p \sim \frac{2G_{\pi NN}f_\pi}{m_\pi^2 - q^2} \qquad \frac{1}{m_N^2}C_6^A \sim \frac{1}{2m_N}\frac{G_{\pi N\Delta}f_\pi}{m_\pi^2 - q^2}$$

Ratios of $G_{\pi NN}$ and $G_{\pi N\Delta}$



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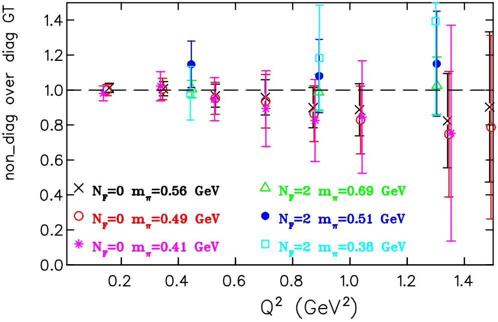
- The ratio $8C_6^A/G_p \sim 1.6 \sim G_{\pi N\Delta}/G_{\pi NN} \Longrightarrow$ pion pole dominance
- The ratio $2C_5^A/G_A \sim 1.6 \sim G_{\pi N\Delta}/G_{\pi NN} \Longrightarrow$ imply the Goldberger-Treiman relations with the assumption of pole dominance

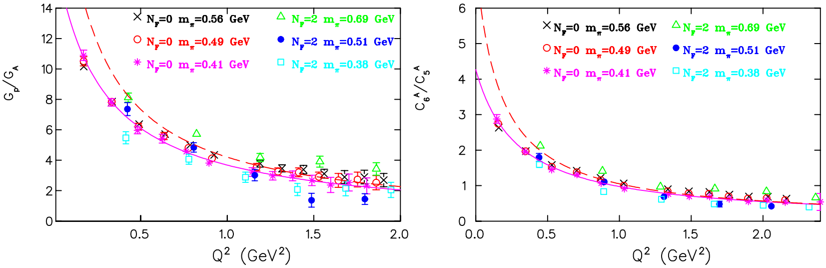
Results for the Goldberger-Treiman relation

Ratio of non-diagonal to diagonal GTR



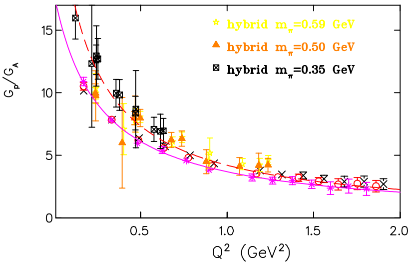
Ratio is unity as expected but...

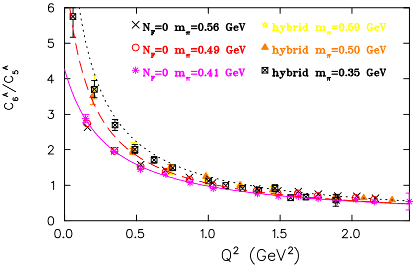
Pion pole dominance



- Pion dominance: $G_p/G_A = 4m_N^2/(Q^2 + m_\pi^2)$ and $C_6^A/C_5^A = m_N^2/(Q^2 + m_\pi^2) \Longrightarrow$ ratios should be described by the dashed lines
- Solid curves are fits to a monopole $\frac{g_0}{(Q^2+m^2)}$. Fit yields $m > m_\pi$.

Pion pole dominance





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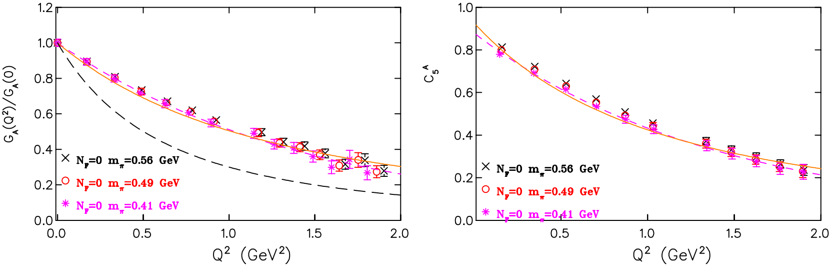
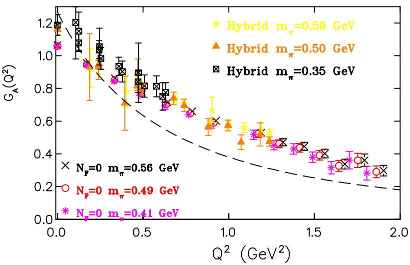
- Solid curves are fits to a monopole $\frac{g_0}{(Q^2+m^2)}$. Fit yields $m > m_\pi$.
- Dynamical QCD results in hybrid approach higher at small Q^2
- For the nucleon form factors results in the hybrid approach are by the LHP collaboration (Thanks J. Negele).

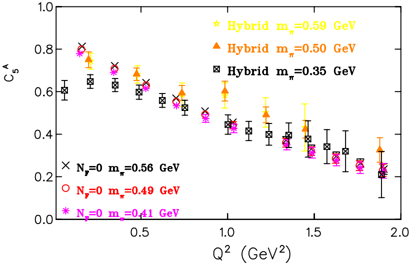
Q^2-dependence of G_A and C_5^A



- Fit: G_A and C_5^A to a dipole $g_0/(\frac{Q^2}{m_A^2}+1)^2$ as done for experimental data. Note that a good description is also provided by an exponential $\tilde{g}_0 e^{-Q^2/\tilde{m}_A^2}$.

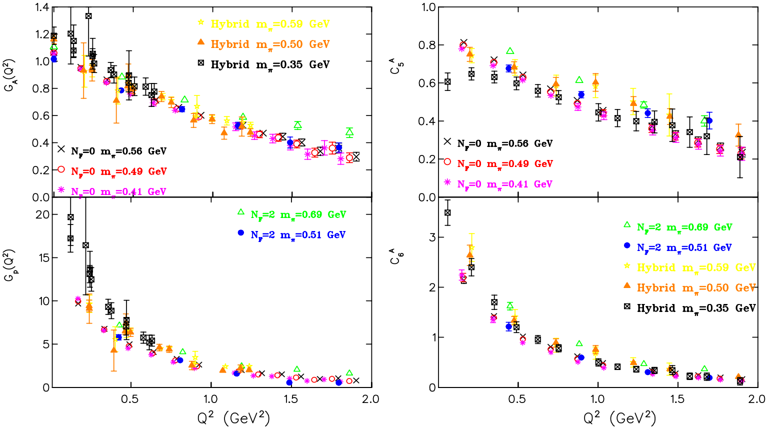
Q^2-dependence of G_A and C_5^A





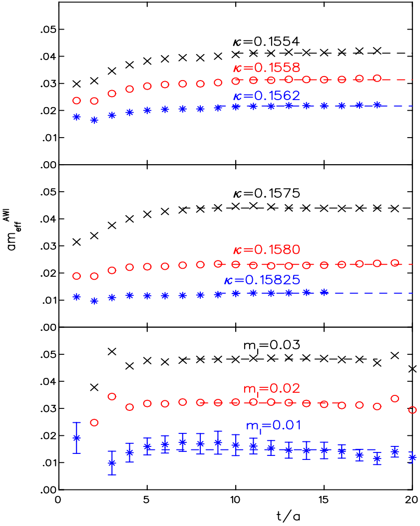
- Hybrid results on G_A from LHP collaboration (Thanks: J. Negele).
- Dynamical QCD results in the hybrid approach deviate at smallest pion mass at low Q^2.

Q^2-dependence of G_A and C_5^A



- Hybrid results on G_A from LHP collaboration (Thanks: J. Negele).
- Lattice results in the hybrid approach show deviations at low Q^2.

Renormalized quark mass

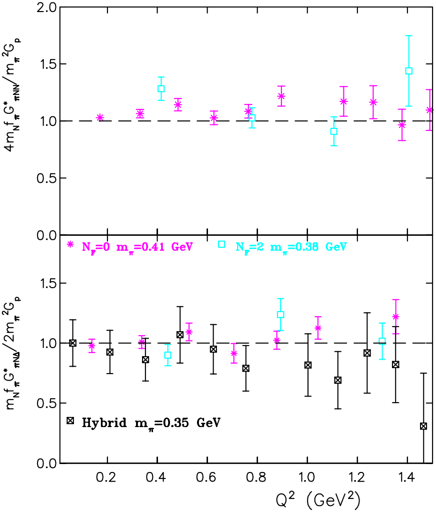


- Compute m_q from the matrix element:

$$m_q = \frac{m_\pi < 0|A_0^a|\pi^a(0) >}{2 < 0|P^a|\pi^a(0) >}$$

- Needed for the extraction of the strong coupling constants
- Good plateaus lead to accurate determination of the quark mass.

Pion pole dominance



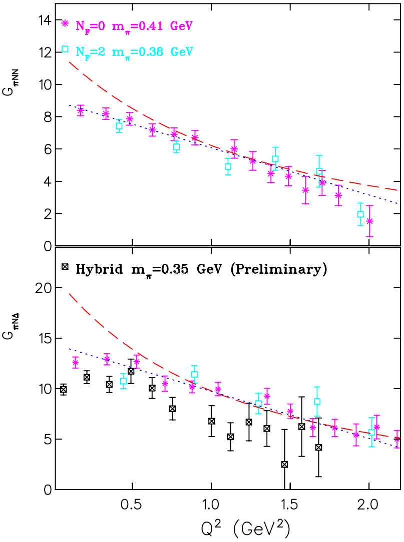
For pole dominance:

$$G^*_{\pi NN} \equiv \frac{G_{\pi NN}}{1 + Q^2/m_\pi^2} = G_p \frac{2m_\pi^2}{m_N f_\pi}$$

$$G^*_{\pi N\Delta} \equiv \frac{G_{\pi N\Delta}}{1 + Q^2/m_\pi^2} = C_6^A \frac{m_\pi^2}{4m_N f_\pi}$$

Well satisfied

$G_{\pi NN}$ and $G_{\pi N\Delta}$



Curves are obtained using:

- Red dash curve:

$$G_{\pi NN}(Q^2) = G_A(Q^2)\,\frac{m_N}{f_\pi}$$

$$G_{\pi N\Delta}(Q^2) = C_5^A(Q^2)\,\frac{2m_N}{f_\pi}$$

Experimental value: $G_{\pi NN}(0) = 13.21(11)$
$\Longrightarrow$ lattice results underestimate $G_{\pi NN}$ as $Q^2 \to 0$

- Blue dotted line:

$$G_{\pi NN}(Q^2) = a\left(1 - \Delta\frac{Q^2}{m_\pi^2}\right),$$

$$G_{\pi N\Delta}(Q^2) = 1.6\,G_{\pi NN}(Q^2)$$

with a, Δ fit parameters.
We find $a \sim 70\%$ what expected and $\Delta \sim 5\%$ at the lightest quenched mass.

Conclusions

- Calculation of Vector, Axial vector and Pseudoscalar form factors in the N to Δ in the quenched approximation + two-flavors of dynamical Wilson fermions + Hybrid scheme.
- Ratios of form factors as expected:
  - quadrupole to dipole ratios EMR and CMR
  - ratios of axial form factors
  - ratios of pion coupling constants $G_{\pi N\Delta}$ and $G_{\pi NN}$
- Predict the ratio C_5^A/C_3^V as a function of Q^2 and hence the leading contribution to the the parity violating asymmetry.
- Deviations from experiment seen for the dipole N-Δ transition form factor G_m^* and the values of $G_{\pi NN}$ and $G_{\pi N\Delta}$ in the limit $Q^2 \to 0$.
- Check finite a, $m_\pi \to 140$ MeV and renormalized quark mass evaluated using AWI.

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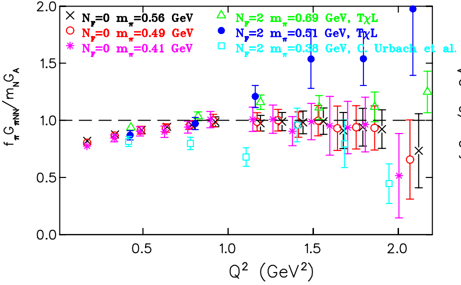
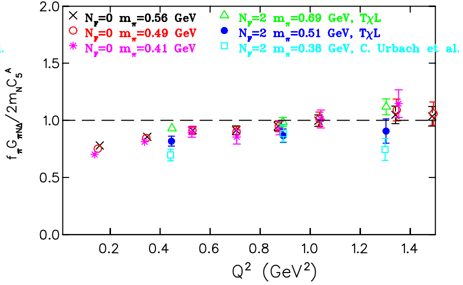
Backup slides

Goldberger Treiman relations

Ratios: $G_{\pi NN} f_\pi / m_N G_A$ and $G_{\pi N\Delta} f_\pi / 2m_N C_5^A$

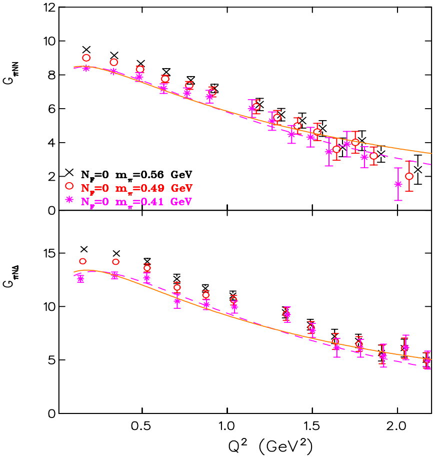




Deviations decrease as q^2 increases

$\Longrightarrow$ axial form factors G_A and C_5^A have different Q^2 dependence at low Q^2.

$G_{\pi NN}$ and $G_{\pi N\Delta}$ in the quenched theory



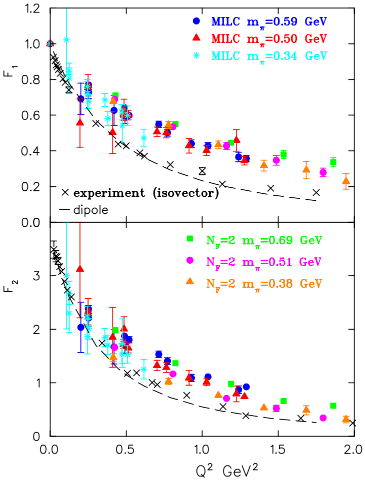
- Curves are fits to

$$\frac{c_0(Q^2+m_\pi^2)}{(Q^2/m_A^2+1)^2(Q^2+m^2)}$$

  at the smallest pion mass.

- The axial mass m_A is determined from fitting G_A or C_5^A
- m from fitting the ratio of G_p/G_A or C_6^A/C_5^A
- c_0 is fitted to the coupling constants $G_{\pi NN}$ or $G_{\pi N\Delta}$

Comparison with results using Domain wall fermions



LHPC/MILC *:

- Hybrid calculation over a range of pion masses using 350, 564, 657 and 270 configurations.
- Lattice spacing: $q\bar{q}$ potential for domain wall fermions and from the nucleon mass at the chiral limit for Wilson fermions.
- Results using and DWF are in agreement.
- $\Longrightarrow$ This agreement is not trivial since these lattice formulations have different lattice artifacts.

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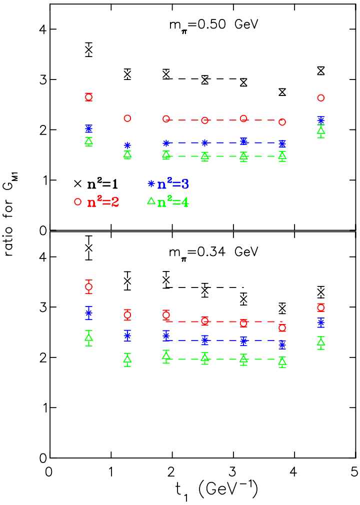
* Thanks J. W. Negele for the DWF results

Check lattice calculation



- Good plateaus → ground state dominance