Two-Photon Exchange in Electron-Proton Elastic Scattering

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Exclusive Reactions at High Momentum Transfer

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Plan of talk

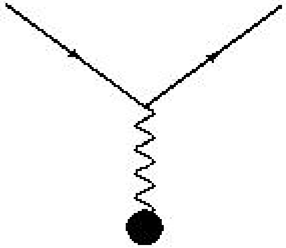
Radiative corrections for elastic electron scattering

- Model-independent and model-dependent; soft and hard photons
- Refined bremsstrahlung calculations

Two-photon exchange effects in the process $e+p \to e+p$

- Models for two-photon exchange
    - Cross sections
    - Polarization transfer
    - Single-spin asymmetries

Elastic Nucleon Form Factors



- Based on one-photon exchange approximation

$$M_{fi} = M_{fi}^{1\gamma}$$

$$M_{fi}^{1\gamma} = e^2 \bar{u}_e \gamma_\mu u_e \bar{u}_p \left(F_1(t)\gamma_\mu - \frac{\sigma_{\mu\nu} q_\nu}{2m} F_2(t)\right) u_p$$

- Two techniques to measure

$$\sigma = \sigma_0 (G_M{}^2 \tau + \varepsilon \cdot G_E{}^2) \quad : \textit{Rosenbluth technique}$$

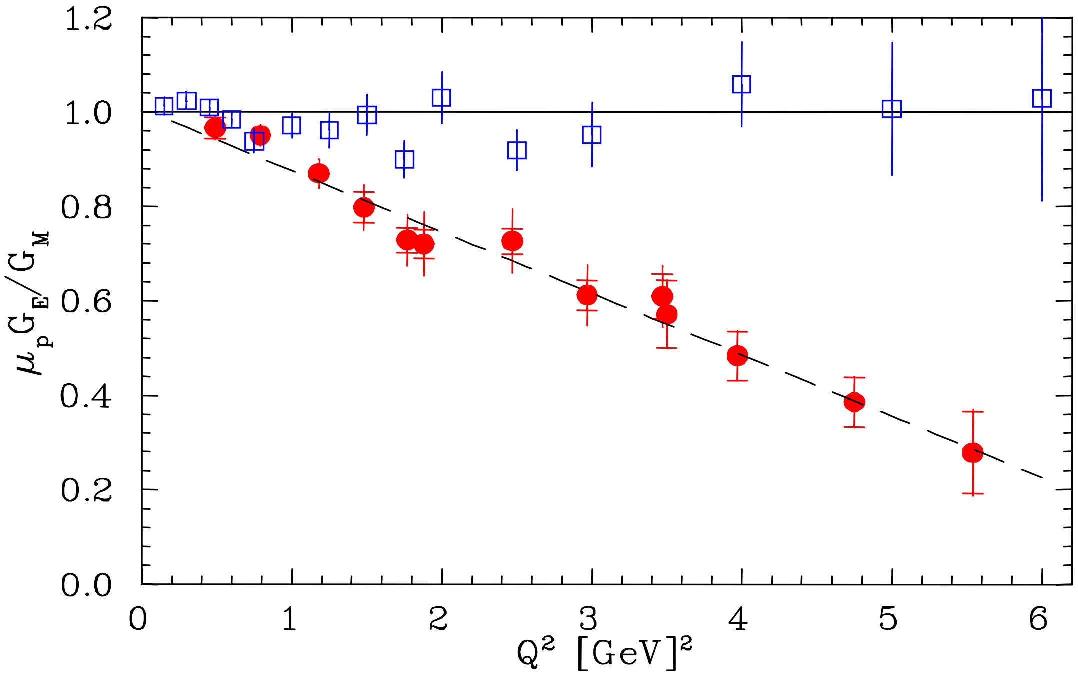
$$\frac{P_x}{P_z} = -\frac{A_x}{A_z} = -\frac{G_E \sqrt{\tau}\sqrt{2\varepsilon(1-\varepsilon)}}{G_M \ \tau\sqrt{1-\varepsilon^2}} \quad : \textit{Polarization technique}$$

$$G_E = F_1 - \tau F_2, \quad G_M = F_1 + F_2$$

$$(P_y = 0)$$

Latter due to: Akhiezer, Rekalo; Arnold, Carlson, Gross

Do the techniques agree?



SLAC/Rosenbluth

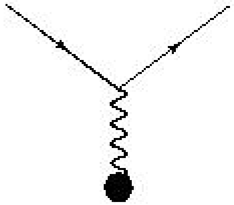
~5% difference in cross-section
x5 difference in polarization

JLab/Polarization

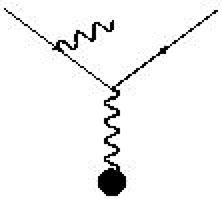
- **Both early SLAC and Recent JLab experiments on (super)Rosenbluth separations followed Ge/Gm~const, see I.A. Quattan et al., Phys.Rev.Lett. 94:142301,2005**
- **JLab measurements using polarization transfer technique give different results (Jones'00, Gayou'02)**

Radiative corrections, in particular, a short-range part of 2-photon exchange is a likely origin of the discrepancy

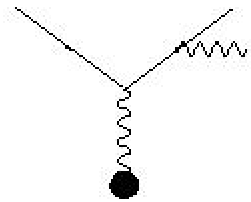
Basics of QED radiative corrections



(First) Born approximation

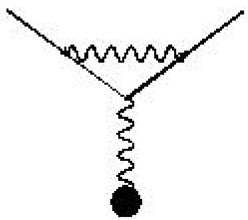


Initial-state radiation



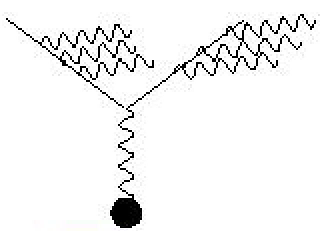
Final-state radiation

Cross section ~ dω/ω => integral diverges logarithmically: **IR catastrophe**



Vertex correction => cancels divergent terms; Schwinger (1949)

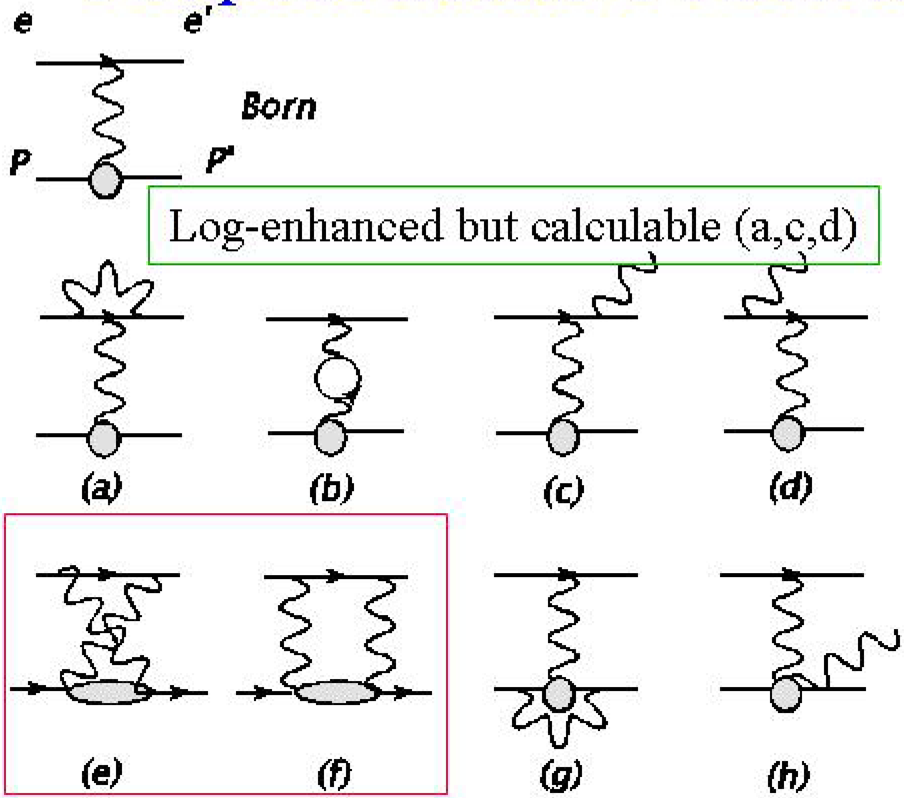
$$\sigma_{\exp} = (1+\delta)\sigma_{Born}, \quad \delta = \frac{-2\alpha}{\pi}\{(\ln\frac{E}{\Delta E} - \frac{13}{12})(\ln\frac{Q^2}{m_e^2} - 1) + \frac{17}{36} + \frac{1}{2}f(\theta)\}$$



Multiple soft-photon emission: solved by exponentiation, Yennie-Frautschi-Suura (YFS), 1961

$$(1+\delta) \to e^{\delta}$$

Complete radiative correction in O(α_{em})



Radiative Corrections:

- Electron vertex correction (a)
- Vacuum polarization (b)
- Electron bremsstrahlung (c,d)
- Two-photon exchange (e,f)
- Proton vertex and VCS (g,h)
- Corrections (e-h) depend on the nucleon structure

•Meister&Yennie; Mo&Tsai

•Further work by Bardin&Shumeiko; Maximon&Tjon; AA, Akushevich, Merenkov;

•Guichon&Vanderhaeghen'03:
Can (e-f) account for the Rosenbluth vs. polarization experimental discrepancy? Look for ~3% ...

Main issue: Corrections dependent on nucleon structure

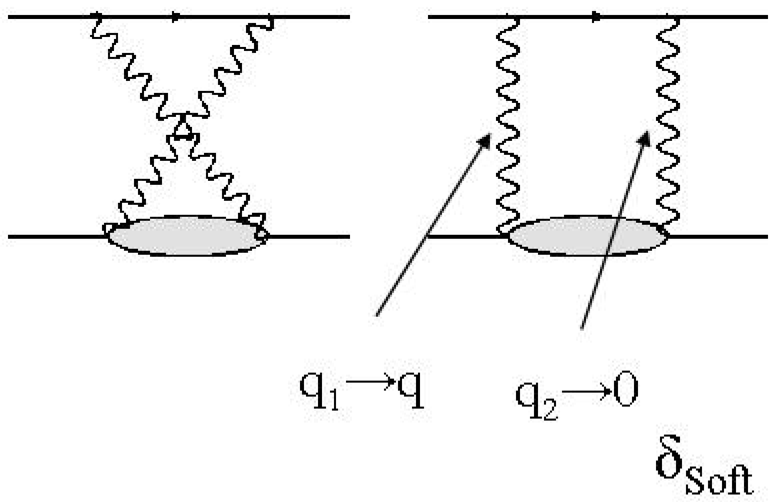
Model calculations:

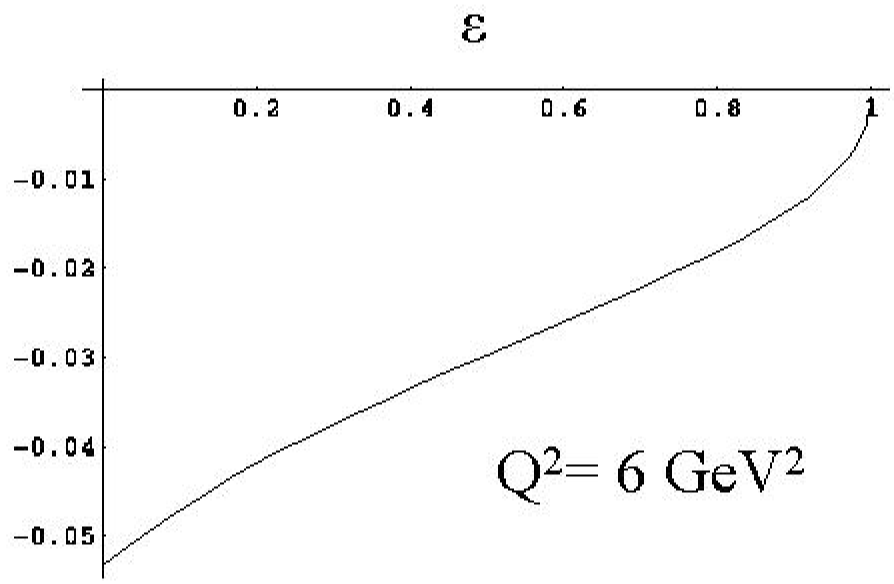
•Blunden, Melnitchouk,Tjon, Phys.Rev.Lett.**91**:142304,2003

•Chen, AA, Brodsky, Carlson, Vanderhaeghen, Phys.Rev.Lett.**93**:122301,2004

Separating *soft* 2-photon exchange

- Tsai; Maximon & Tjon (k→0); similar to Coulomb corrections at low Q^2
- Grammer &Yennie prescription PRD 8, 4332 (1973) (also applied in QCD calculations)
- Shown is the resulting (soft) QED correction to cross section
- **Already included in experimental data analysis**
- NB: Corresponding effect to polarization transfer and/or asymmetry is zero





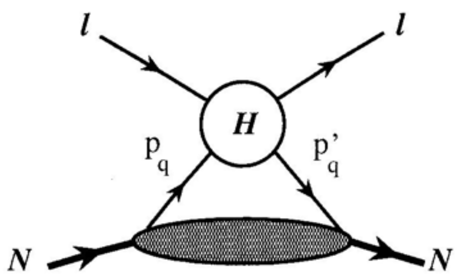
What is missing in the calculation?

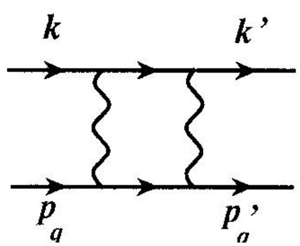
- 2-photon exchange contributions for non-soft intermediate photons
  - Can estimate based on a text-book example from *Berestetsky, Lifshitz, Pitaevsky: Quantum Electrodynamics*
  - Double-log asymptotics of electron-quark backward scattering

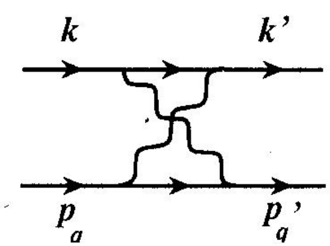
$$\delta = -\frac{e_q e}{8\pi^3}\log^2\frac{s}{m_q^2}.$$

  - Negative sign for backward ep-scattering; zero for forward scattering → Can (at least partially) mimic the electric form factor contribution to the Rosenbluth cross section
  - Numerically ~3-4% (for SLAC kinematics and m_q~300 MeV)
  - **Motivates a more detailed calculation of 2-photon exchange at quark level**

Calculations using Generalized Parton Distributions







Model schematics:

- Hard eq-interaction
- GPDs describe quark emission/absorption
- Soft/hard separation
  - Use Grammer-Yennie prescription

Hard interaction with a quark

AA, Brodsky, Carlson, Chen, Vanderhaeghen, Phys.Rev.Lett.**93**:122301,2004; Phys.Rev.D**72**:013008,2005

Short-range effects; on-mass-shell quark (AA, Brodsky, Carlson, Chen, Vanderhaeghen)

Two-photon probe directly interacts with a (massless) quark
Emission/reabsorption of the quark is described by GPDs

$$A^{2\gamma}_{eq\to eq} = \frac{e_q^2}{t}\frac{\alpha_{em}}{2\pi}(V^e_\mu \otimes V^q_\mu \times f_V + A^e_\mu \otimes A^q_\mu \times f_A),$$

$$V^{e,q}_\mu = \overline{u}_{e,q}\gamma_\mu u_{e,q},\ A^{e,q}_\mu = \overline{u}_{e,q}\gamma_\mu\gamma_5 u_{e,q}$$

$$f_V = -2[\log(-\frac{u}{s}) + i\pi]\log(-\frac{t}{\lambda^2}) - \frac{t}{2}[\frac{1}{s}(\log(\frac{u}{t}) + i\pi) - \frac{1}{u}\log(-\frac{s}{t})] +$$

$$+\frac{(u^2-s^2)}{4}[\frac{1}{s^2}(\log^2(\frac{u}{t}) + \pi^2) + \frac{1}{u^2}\log(-\frac{s}{t})(\log(-\frac{s}{t}) + i2\pi)] + i\pi\frac{u^2-s^2}{2su}$$

$$f_A = -\frac{t}{2}[\frac{1}{s}(\log(\frac{u}{t}) + i\pi) + \frac{1}{u}\log(-\frac{s}{t})] +$$

$$+\frac{(u^2-s^2)}{4}[\frac{1}{s^2}(\log^2(\frac{u}{t}) + \pi^2) - \frac{1}{u^2}\log(-\frac{s}{t})(\log(-\frac{s}{t}) + i2\pi)] + i\pi\frac{t^2}{2su}$$

Note the additional effective (axial-vector)^2 interaction; absence of mass terms

`Hard' contributions to generalized form factors

GPD integrals

$$A \equiv \int_{-1}^{1} \frac{dx}{x} \frac{\left[(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3\right]}{(s-u)} \sum_q e_q^2 \left(H^q + E^q\right),$$

$$B \equiv \int_{-1}^{1} \frac{dx}{x} \frac{\left[(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3\right]}{(s-u)} \sum_q e_q^2 \left(H^q - \tau E^q\right)$$

$$C \equiv \int_{-1}^{1} \frac{dx}{x} \tilde{f}_1^{hard} \, \mathrm{sgn}(x) \sum_q e_q^2 \tilde{H}^q$$

Two-photon-exchange form factors from GPDs

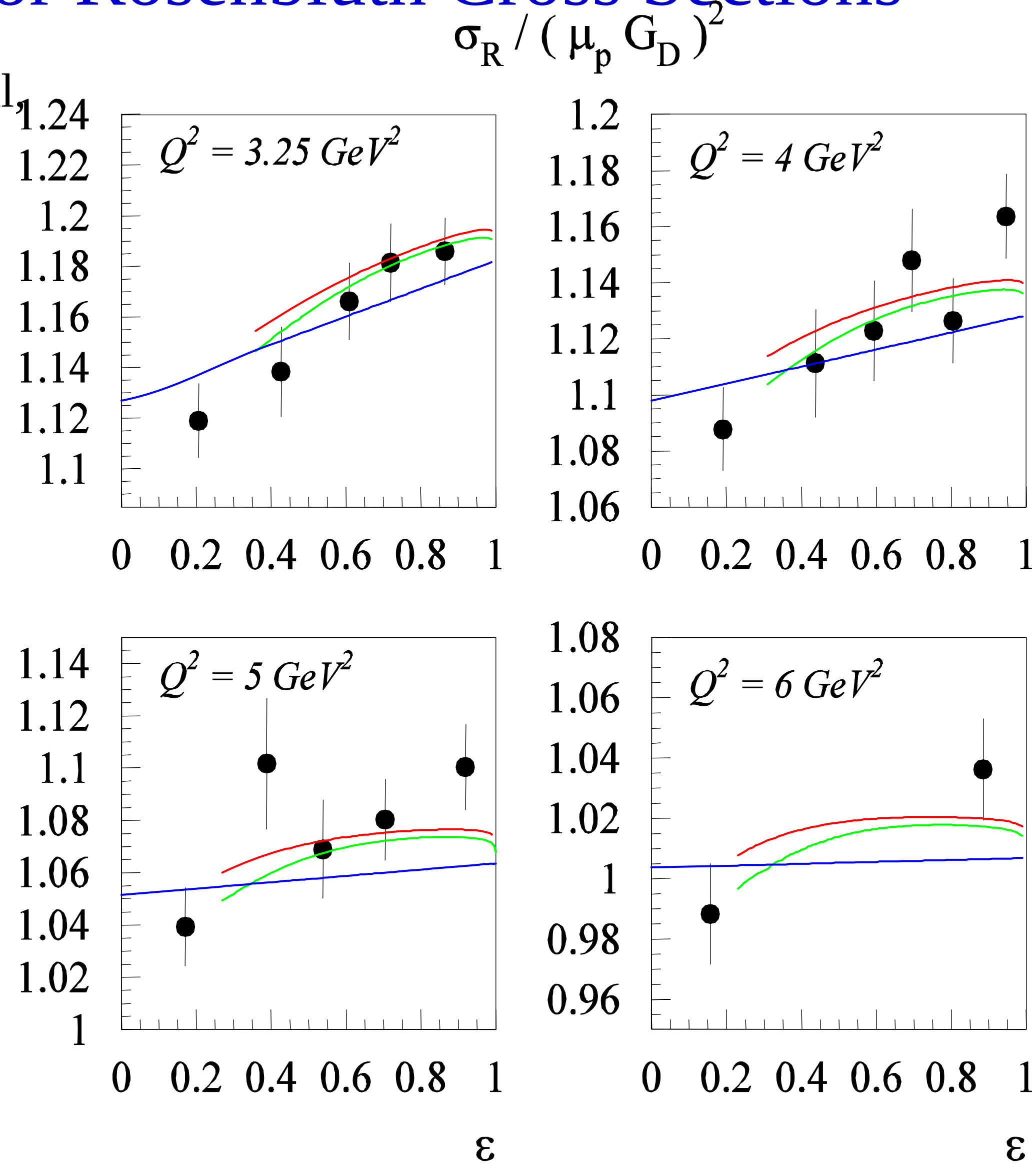
$$\delta\tilde{G}_M^{hard} = C$$

$$\delta\tilde{G}_E^{hard} = -\left(\frac{1+\varepsilon}{2\varepsilon}\right)(A-C) + \sqrt{\frac{1+\varepsilon}{2\varepsilon}}\, B$$

$$\tilde{F}_3 = \frac{M^2}{\nu}\left(\frac{1+\varepsilon}{2\varepsilon}\right)(A-C)$$

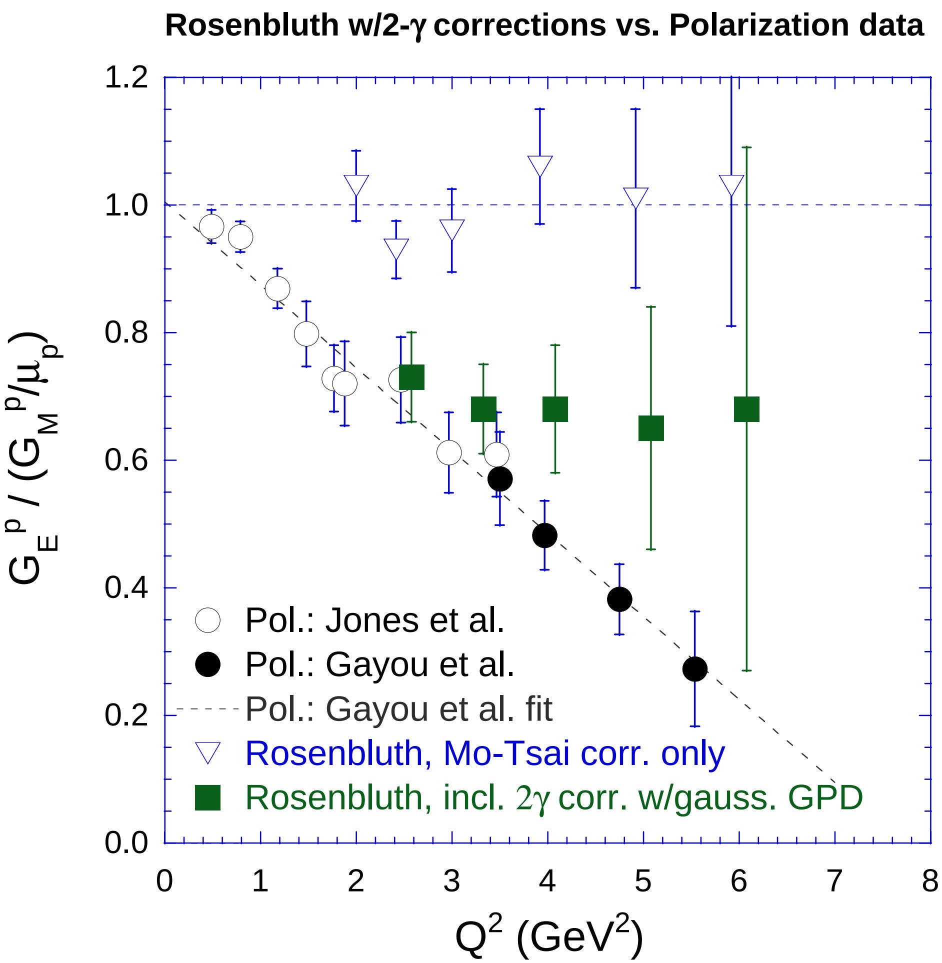
Two-Photon Effect for Rosenbluth Cross Sections

- Data shown are from Andivahis et al, PRD 50, 5491 (1994)
- Included GPD calculation of two-photon-exchange effect
- Qualitative agreement with data:
- Discrepancy likely reconciled



Updated Ge/Gm plot

AA, Brodsky, Carlson, Chen, Vanderhaeghen,
Phys.Rev.Lett.93:122301, 2004; Phys.Rev.D72:013008, 2005



Full Calculation of Bethe-Heitler Contribution

*Additional work by AA et al., using MASCARAD (**Phys.Rev.D64:113009,2001**)*
Full calculation including soft and hard bremsstrahlung

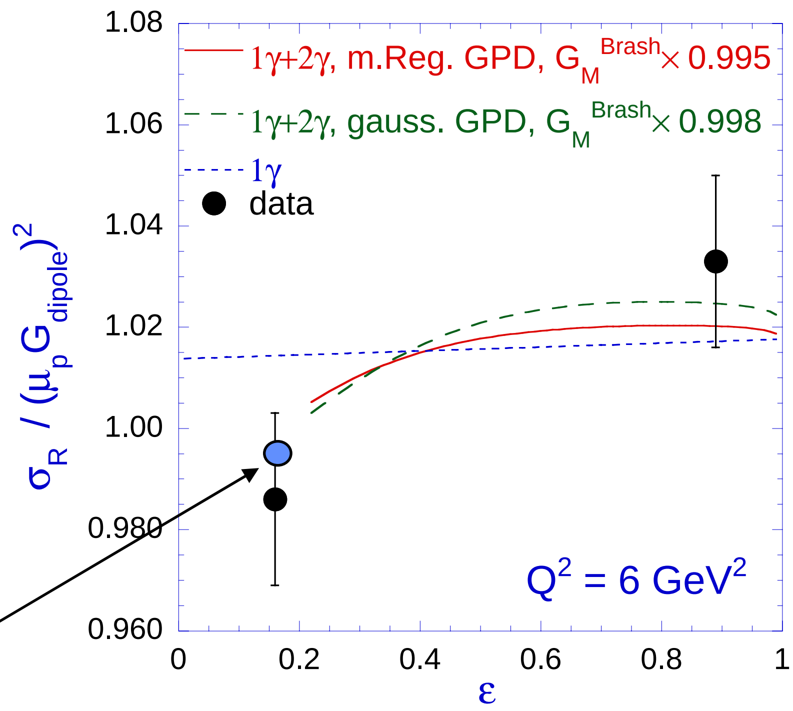
Radiative leptonic tensor in full form
AA et al, ***PLB 514, 269 (2001)***

$$L^{r}_{\mu\nu} = -\frac{1}{2} Tr(\hat{k}_2 + m)\Gamma_{\mu\alpha}(1+\gamma_5\hat{\xi}_e)(\hat{k}_1 + m)\overline{\Gamma}_{\alpha\nu}$$

$$\Gamma_{\mu\alpha} = \left(\frac{k_{1\alpha}}{k\cdot k_1} - \frac{k_{2\alpha}}{k\cdot k_2}\right)\gamma_\mu - \frac{\gamma_\mu \hat{k}\gamma_\alpha}{2k\cdot k_1} - \frac{\gamma_\alpha \hat{k}\gamma_\mu}{2k\cdot k_2}$$

$$\Gamma_{\alpha\nu} = \left(\frac{k_{1\alpha}}{k\cdot k_1} - \frac{k_{2\alpha}}{k\cdot k_2}\right)\gamma_\nu - \frac{\gamma_\alpha \hat{k}\gamma_\nu}{2k\cdot k_1} - \frac{\gamma_\nu \hat{k}\gamma_\alpha}{2k\cdot k_2}$$

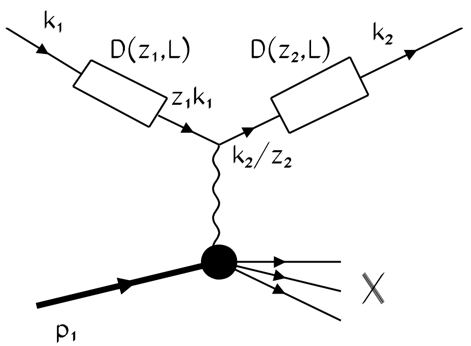
Cross section for ep elastic scattering



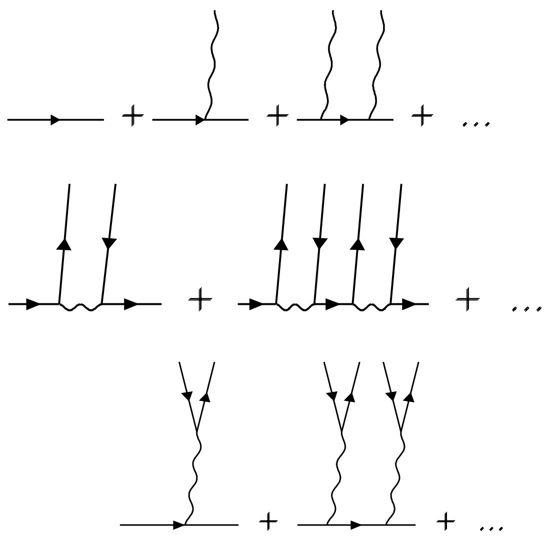
Additional effect of full soft+hard brem → +1.2% correction to ε-slope

Resolves additional ~25% of Rosenbluth/polarization discrepancy!

Electron Structure Functions



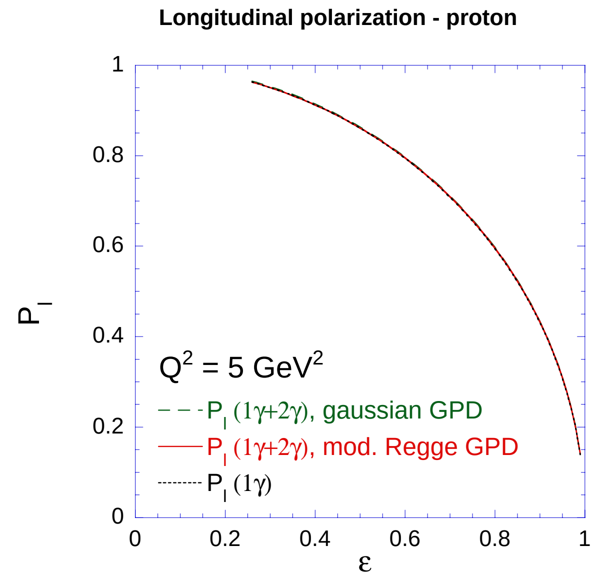
- For polarized ep->e'X scattering, AA et al, JETP 98, 403 (2004); elastic ep: AA et al. PRD 64, 113009 (2001).
  - Resummation technique for collinear photons (=peaking approx.)
  - Difference <0.5% from previous calculation including hard brem

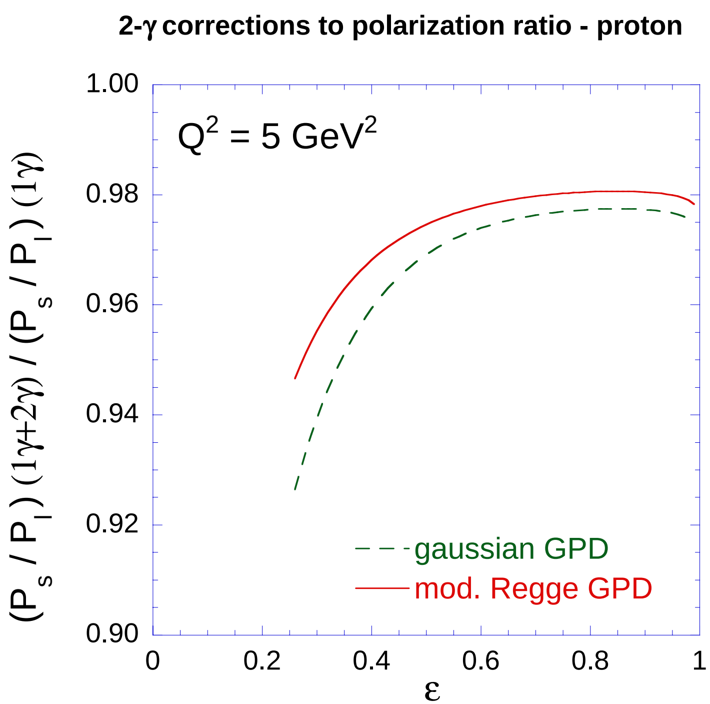


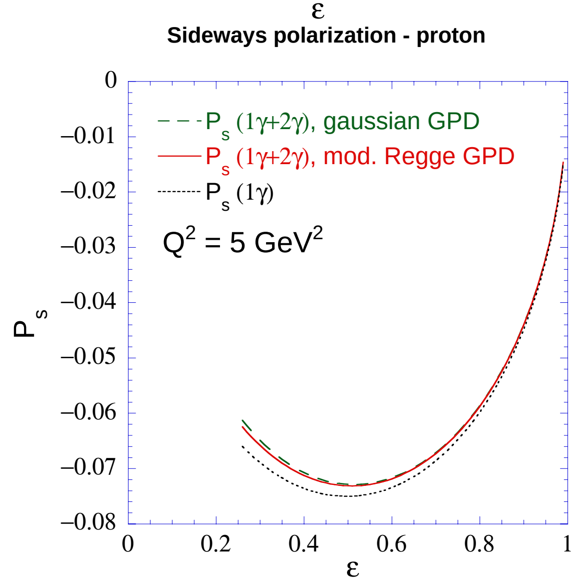
- Bystritskiy, Kuraev, Tomasi-Gustafson (2007) claimed this approach resolves Rosenbluth vs polarization discrepancy… but used incorrect energy cutoff ΔE/E of 3% (instead of e.g. 1.5%) =>miscalculated rad.correction by ~5% (absolute)

Polarization transfer

- Also corrected by two-photon exchange, but with little impact on Gep/Gmp extracted ratio



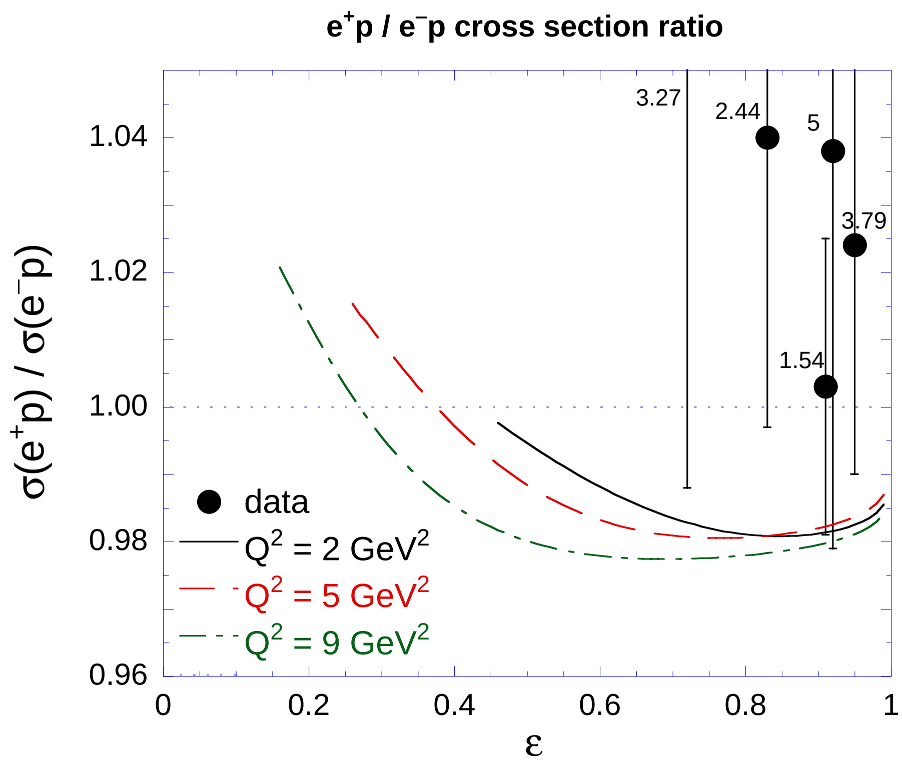




Charge asymmetry

- Cross sections of electron-proton scattering and positron-proton scattering are equal in one-photon exchange approximation
  - Different for two- or more photon exchange

$$\sigma_{\pm}=\sigma_{1\gamma}\pm\sigma_{2\gamma}$$



Measured in JLab Experiment 04-116,
Spokepersons: W. Brooks, AA, J. Arrington, K.Joo, B.Raue, L.Weinstein
First run scheduled for Fall 2006

Single-Spin Asymmetries in Elastic Scattering

Parity-conserving

- Observed spin-momentum correlation of the type:

$$\vec{s} \cdot \vec{k}_1 \times \vec{k}_2$$

where $k_{1,2}$ are initial and final electron momenta, s is a polarization vector of a target OR beam

- For elastic scattering asymmetries are due to *absorptive part* of 2-photon exchange amplitude

Parity-Violating

$$\vec{s} \cdot \vec{k}_1$$

Quark+Nucleon Contributions to A_n

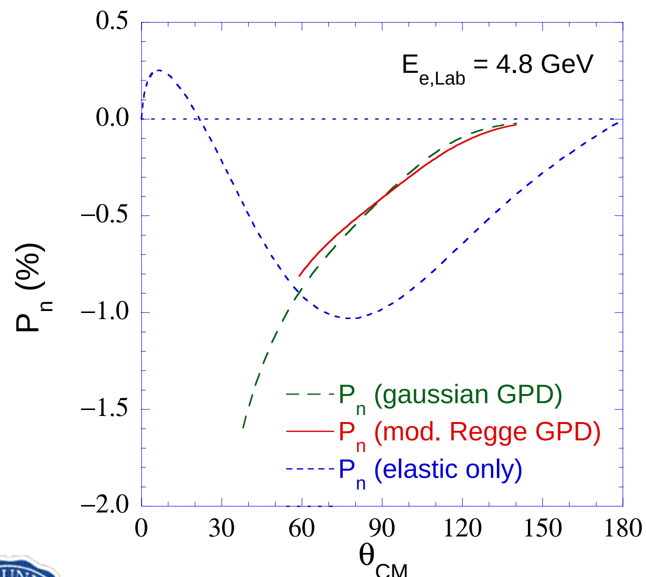
- Single-spin asymmetry or polarization normal to the scattering plane
- Handbag mechanism prediction for single-spin asymmetry/polarization of elastic ep-scattering on a polarized proton target

$$A_n = \sqrt{\frac{2\varepsilon(1+\varepsilon)}{\tau}}\,\frac{1}{\sigma_R}\left[G_E\,\mathrm{Im}(A) - \sqrt{\frac{1+\varepsilon}{2\varepsilon}}G_M\,\mathrm{Im}(B)\right]$$

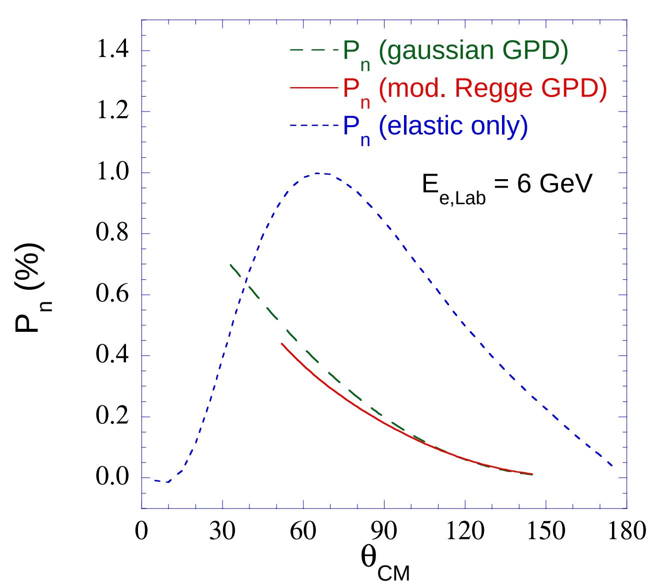
Only minor role of quark mass

No dependence on GPD $\widetilde{H}$

Normal Polarization or Analyzing Power - Neutron



Normal Polarization or Analyzing Power - Proton



Proton Mott Asymmetry at Higher Energies

Spin-orbit interaction of electron moving in a Coulomb field

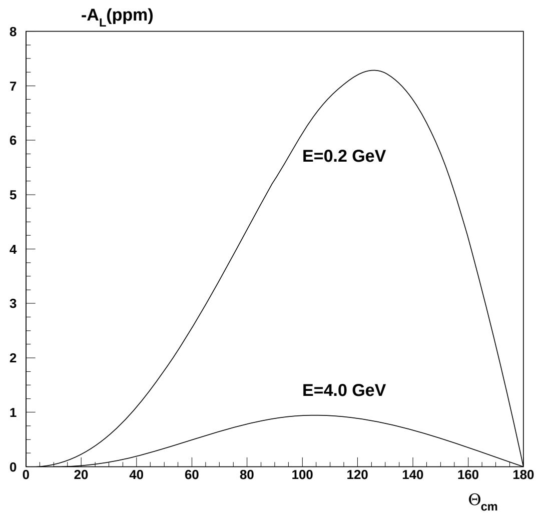
N.F. Mott, Proc. Roy. Soc. London, Set. A **135**, 429 (1932);

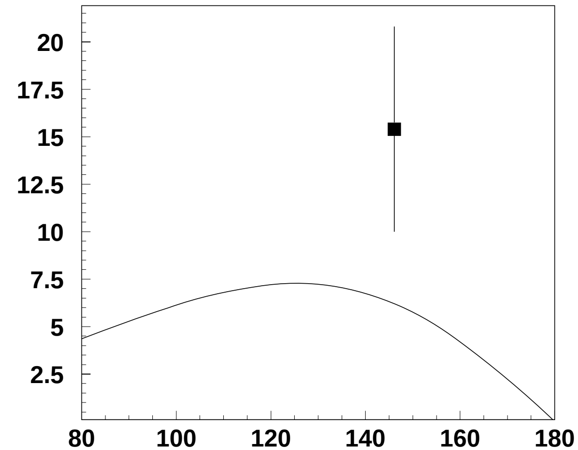
BNSA for electron-muon scattering: Barut, Fronsdal, Phys.Rev.120, 1871 (1960);

BNSA for electron-proton scattering: Afanasev, Akushevich, Merenkov, hep-ph/0208260

Transverse beam SSA, units are parts per million

Figures from AA et al, hep-ph/0208260





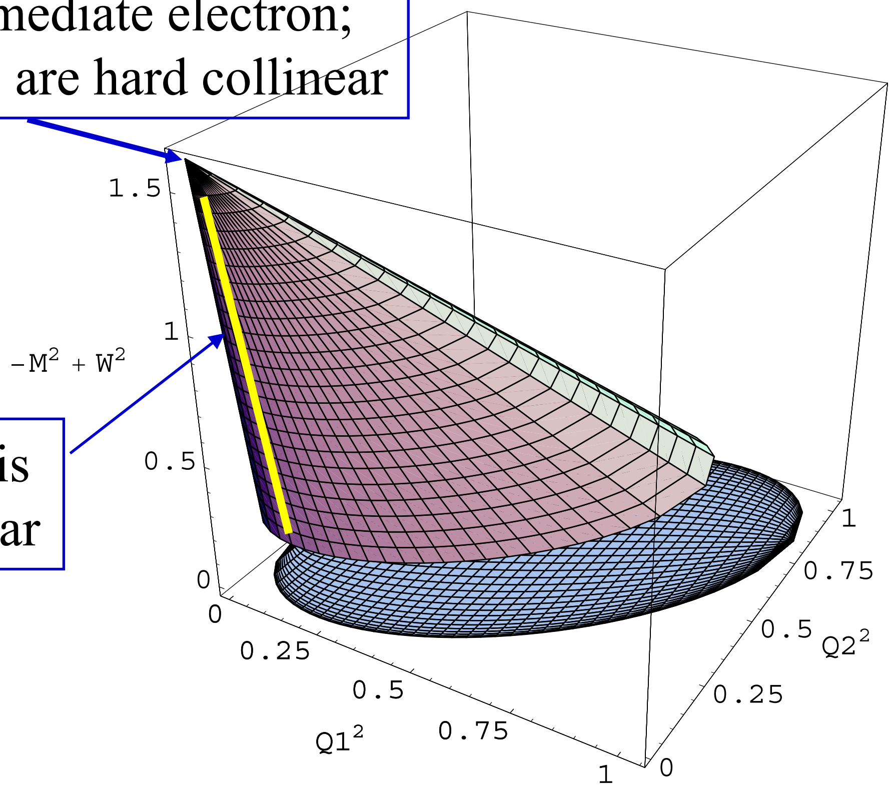
- **Due to absorptive part of two-photon exchange amplitude; shown is elastic contribution**
- **Nonzero effect observed by SAMPLE Collaboration (S.Wells et al., PRC63:064001,2001) for 200 MeV electrons**

Phase Space Contributing to the absorptive part of 2γ-exchange amplitude

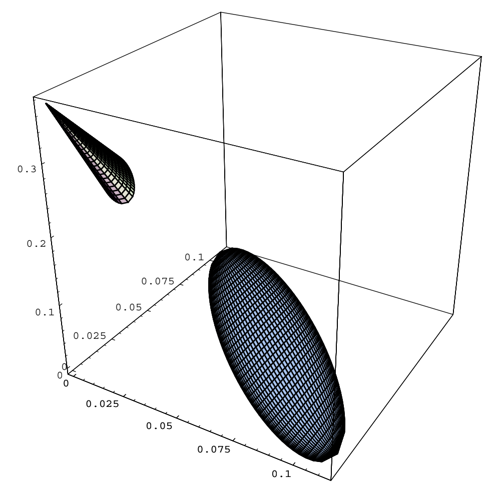
- 2-dimensional integration (Q_1^2, Q_2^2) for the elastic intermediate state
- 3-dimensional integration (Q_1^2, Q_2^2, W^2) for inelastic excitations

`Soft' intermediate electron;
Both photons are hard collinear

Examples: MAMI A4
E= 855 MeV
Θcm= 57 deg;
SAMPLE, E=200 MeV



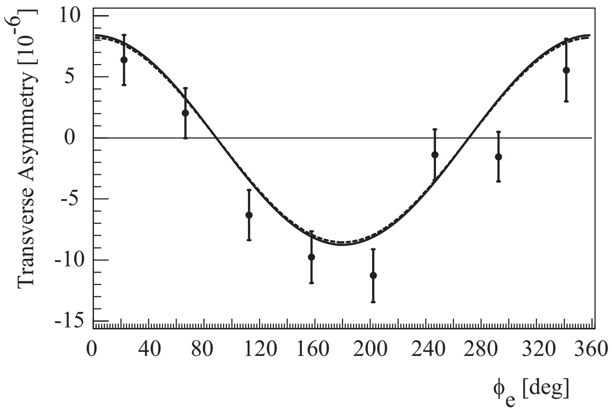
One photon is
Hard collinear



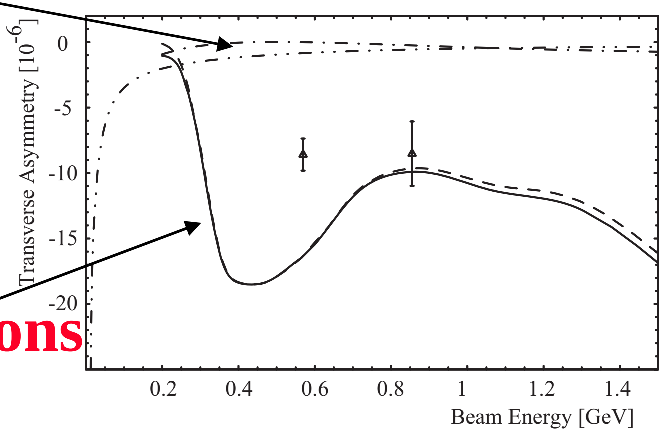
MAMI data on Mott Asymmetry

- F. Maas et al., [MAMI A4 Collab.] Phys.Rev.Lett.**94**:082001, 2005
- Pasquini, Vanderhaeghen: Phys.Rev.C**70**:045206,2004

Surprising result: Dominance of inelastic intermediate excitations

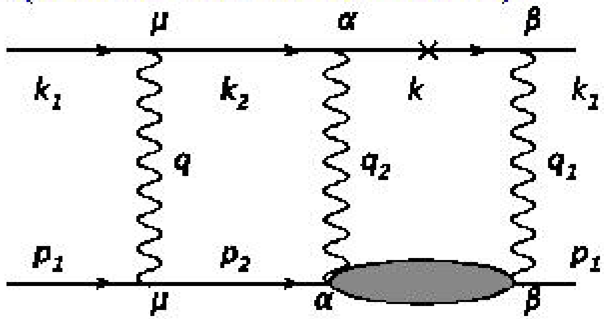


Elastic intermediate state



Inelastic excitations dominate

Beam Normal Asymmetry (=Mott asymmetry) (AA, Merenkov)



$$A_n^{e,P} = -\frac{\alpha Q^2}{\pi^2 D(s,Q^2)} \mathrm{Im} \int \frac{d^3 k}{2k_0} \cdot \frac{L_{\mu\alpha\beta} H_{\mu\alpha\beta}}{Q_1^2 Q_2^2}$$

$$L_{\mu\alpha\beta} = \frac{1}{4} Tr(\hat{k}_2 + m_e)\gamma_\mu(\hat{k}_1 + m_e)(1-\gamma_5\hat{\xi}_e)\gamma_\beta(\hat{k} + m_e)\gamma_\alpha$$

$$H_{\mu\alpha\beta} = \frac{1}{4} Tr(\hat{p}_2 + M)\Gamma_\mu(\hat{p}_1 + M)(1-\gamma_5\hat{\xi}_p)T_{\beta\alpha}$$

$$\hat{a} \equiv a_\mu \gamma_\mu$$

$$L_{\mu\alpha\beta} q_\mu = L_{\mu\alpha\beta} q_{2\alpha} = L_{\mu\alpha\beta} q_{1\beta} = H_{\mu\alpha\beta} q_\mu = H_{\mu\alpha\beta} q_{2\alpha} = H_{\mu\alpha\beta} q_{1\beta} = 0$$

EM gauge invariance important for cancellation of collinear singularity for target asymmetry

$$L_{\mu\alpha\beta} H_{\mu\alpha\beta} \to 0 \quad if \quad Q_1^2 \; and/or \; Q_2^2 \to 0$$

Feature of the normal beam asymmetry: After m_e is factored out, the remaining expression is singular when virtuality of the photons reach zero in the loop integral.

$$L_{\mu\alpha\beta} H_{\mu\alpha\beta} \to m_e \cdot const \quad if \quad Q_1^2 \; and/or \; Q_2^2 \to 0 \Rightarrow A \sim m_e \log^2 \frac{Q^2}{m_e^2}, m_e \log \frac{Q^2}{m_e^2}$$

Also calculations by Vanderhaeghen, Pasquini (2004); Gorchtein (2005); Borisyuk&Kobushkin (2005) confirm ***quasi-real photon exchange*** enhancement of inelastic intermediate excitations

Special property of Mott asymmetry

AA, Merenkov, Phys.Lett.B599:48,2004, Phys.Rev.D70:073002,2004; +Erratum (hep-ph/0407167v2)

- Reason for the unexpected behavior: hard collinear quasi-real photons
  - Intermediate photon is collinear to the parent electron
  - It generates a dynamical pole and logarithmic enhancement of inelastic excitations of the intermediate hadronic state
  - For s>>-t and above the resonance region, the asymmetry is given by:

$$A_n^e = \sigma_{\gamma p} \frac{(-m_e)\sqrt{Q^2}}{8\pi^2} \cdot \frac{F_1 - \tau F_2}{F_1^2 + \tau F_2^2} (\log(\frac{Q^2}{m_e^2}) - 2)$$

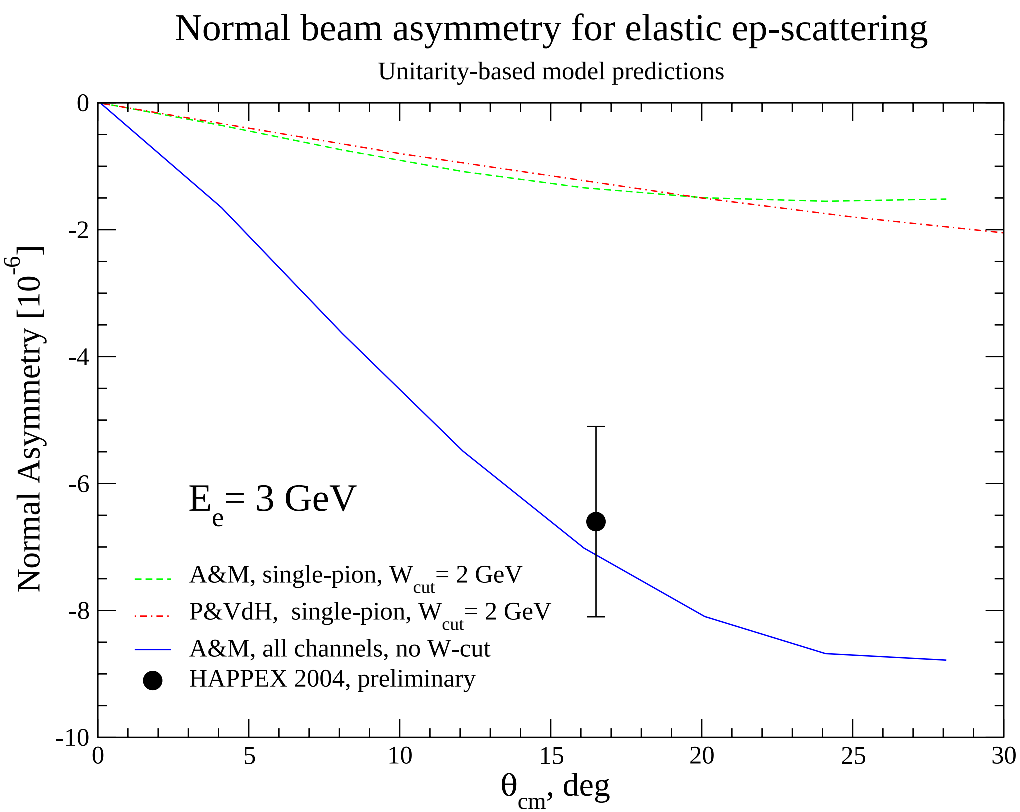
In addition multiply by a standard diffractive factor exp(-BQ^2), where B=3.5-4 GeV^{-2}

Compare with no-inelastic-excitation asymmetry at small θ => **may differ by orders of magnitude depending on kinematics of the measurement**

$$A_n^e \propto \alpha \frac{m_e}{\sqrt{s}} \theta^3$$

Predictions vs experiment for Mott asymmetry

Use fit to experimental data on $\sigma_{\gamma p}$ and exact 3-dimensional integration over phase space of intermediate 2 photons



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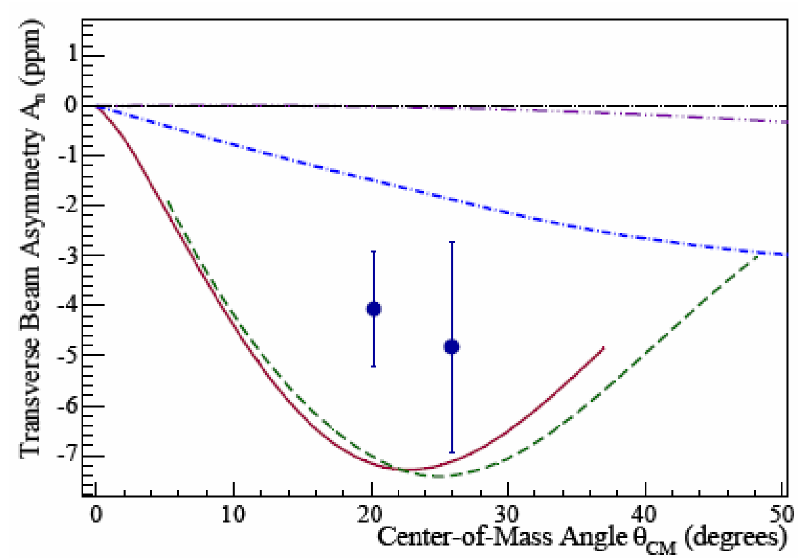
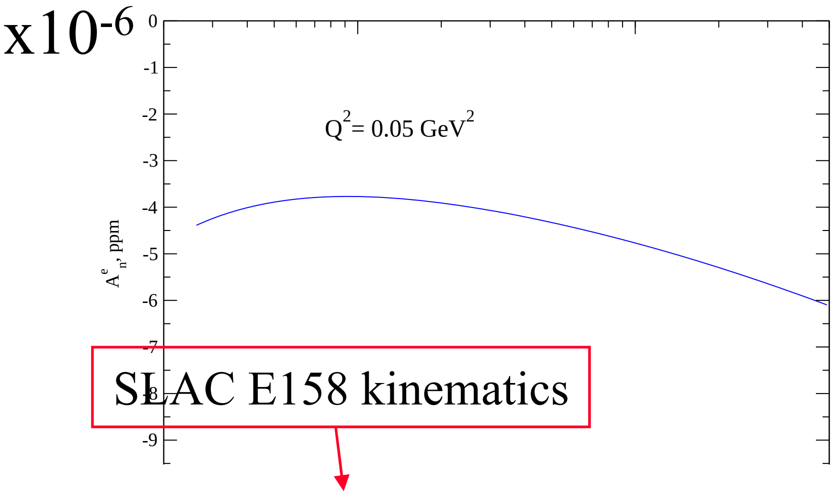
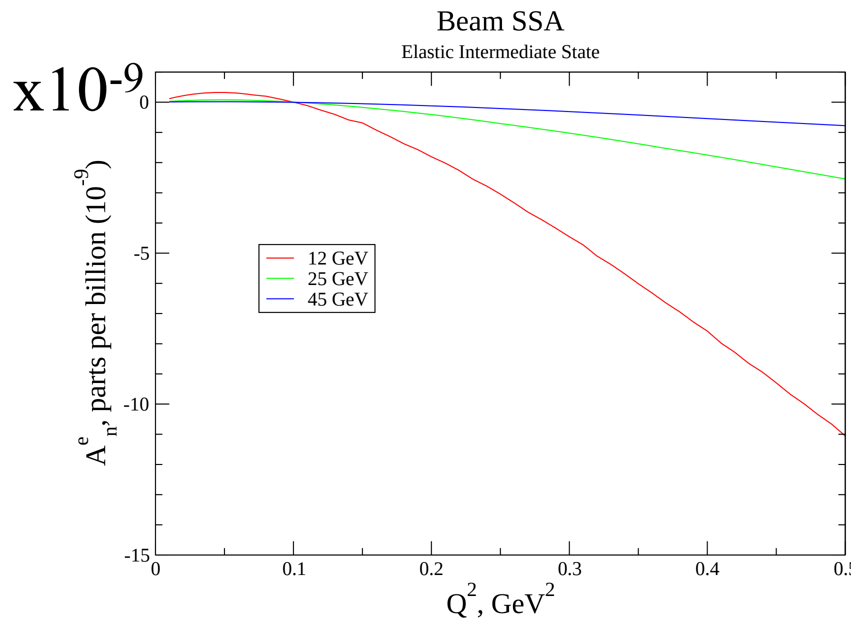
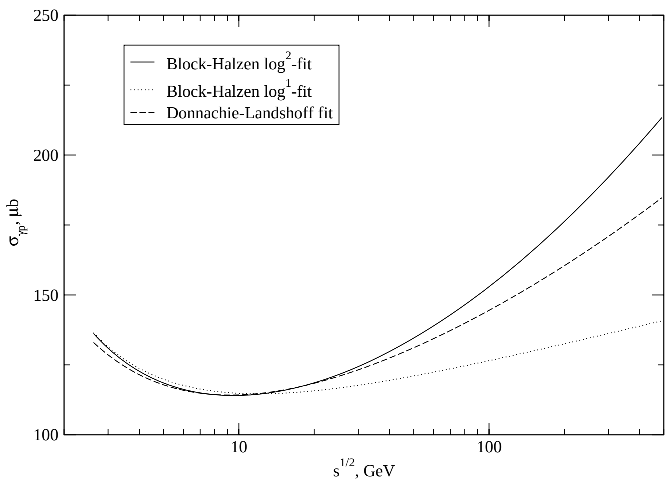


No suppression for Mott asymmetry with energy at fixed Q^2



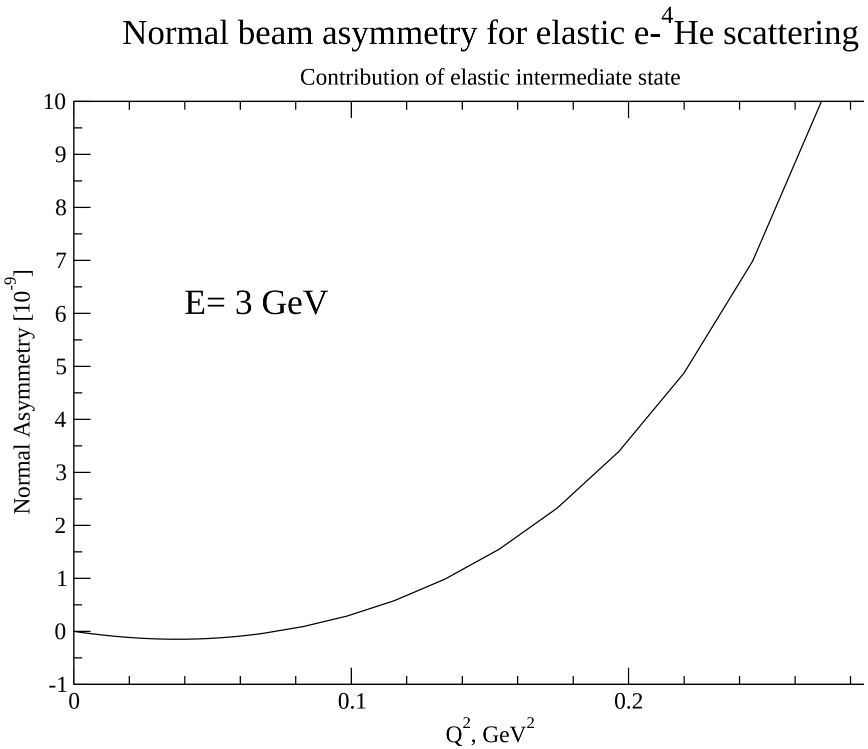


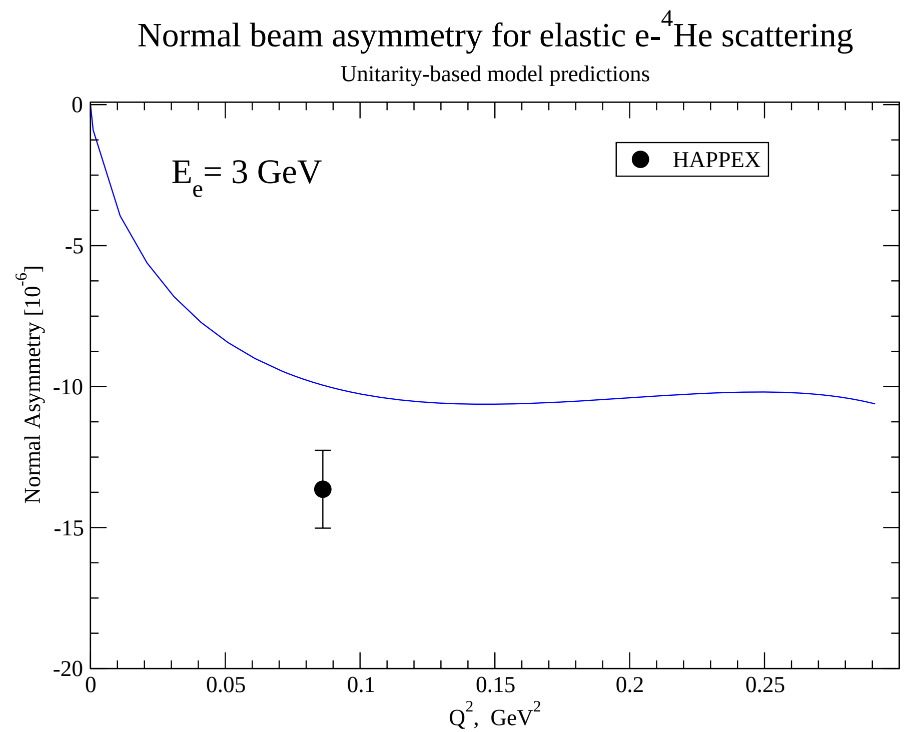


Parts-per-million vs. parts-per billion scales: a consequence of soft Pomeron exchange, and hard collinear photon exchange

Mott Asymmetry on Nuclei

- Important systematic correction for parity-violation experiments (~10ppm for HAPPEX on ^{4}He, ~5ppm for PREX on Pb)
- Measures (integrated) absorptive part of Compton scattering amplitude
- Coulomb distortion: only10^{-10} effect (Cooper&Horowitz, Phys.Rev.C72:034602,2005)





Five orders of magnitude enhancement in HAPPEX kinematics due to excitation of inelastic intermediate states in 2γ-exchange (AA, Merenkov; use Compton data from Erevan)

Summary: SSA in Elastic ep-Scattering

- Collinear photon exchange present in (light particle) beam SSA
- **Warning: Models violating EM gauge invariance** will suffer collinear divergence for target SSA
- VCS amplitude in *beam asymmetry* is enhanced in different kinematic regions compared to *target asymmetry*
- Strong-interaction dynamics for Mott asymmetry in small-angle ep-scattering above the resonance region is ***soft diffraction***
  - ***For the diffractive mechanism A_n is a) not supressed with beam energy and b) does not grow with Z***
  - ***Confirmed experimentally → first observation of diffractive <u>elastic</u> electron-hadron scattering***

Other theoretical developments

- Blunden et al., Phys.Rev.C72:034612, 2005
  Approximate proton Compton amplitude by Born terms
- Kondratyuk et al., nucl-th/0506026
  Add intermediate Δ-excitation to the above
- Carlson, Vanderhaeghen, Pascalutsa, hep-ph/0509055 (PRL'05)
  GPD approach extended to N→Δ transition
- Borisyuk, Kobushkin, Phys.Rev.C72:035207,2005
- Jain et al (2006): Model nonlocal ep-interaction, confirm results of others authors for corrections to Rosenbluth separation
- AA, Strikman, Weiss: Single-spin asymmetries from 2γ-exchange in DIS (see talk by Weiss at this workshop)

Two-photon exchange for electron-nucleon scattering

- Model calculations of 2γ-exchange radiative corrections bring into agreement the results of polarization transfer and Rosenbluth techniques for Gep measurements
- Full treatment of brem corrections removes ~25% of R/P discrepancy **in addition** to 2γ → Important to compute conventional corrections accurately
- **<u>Experimental tests of two-photon exchange</u>**
  - C-violation in electron vs positron elastic scattering (**JLab E04-116, E-07-005**)
  - Measurement of nonlinearity of Rosenbluth plot (**JLab E05-017**)
  - Search for deviation of angular dependence of polarization and/or asymmetries from Born behavior at fixed Q^2 (**JLab E04-019**)
  - Elastic single-spin asymmetry or induced polarization (**JLab E05-015**)
    - Extended to inelastic (e,e') in **E-07-013**
  - 2γ normal beam asymmetry measurements parallel to parity-violating experiments (**HAPPEX, G0, PREX**)

These studies provide

a) Testing precision of the electromagnetic probe and

b) Double-virtual VCS studies