The Electroweak Model and Constraints on New Physics from a High-Energy Perspective, Electroweak Radiative Corrections

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Workshop on the Scientific Impact and Feasibility of an Ultra-precise 12 GeV Møller Experiment to Test the Standard Model
Jefferson Lab, Newport News, VA, December 11-13, 2006

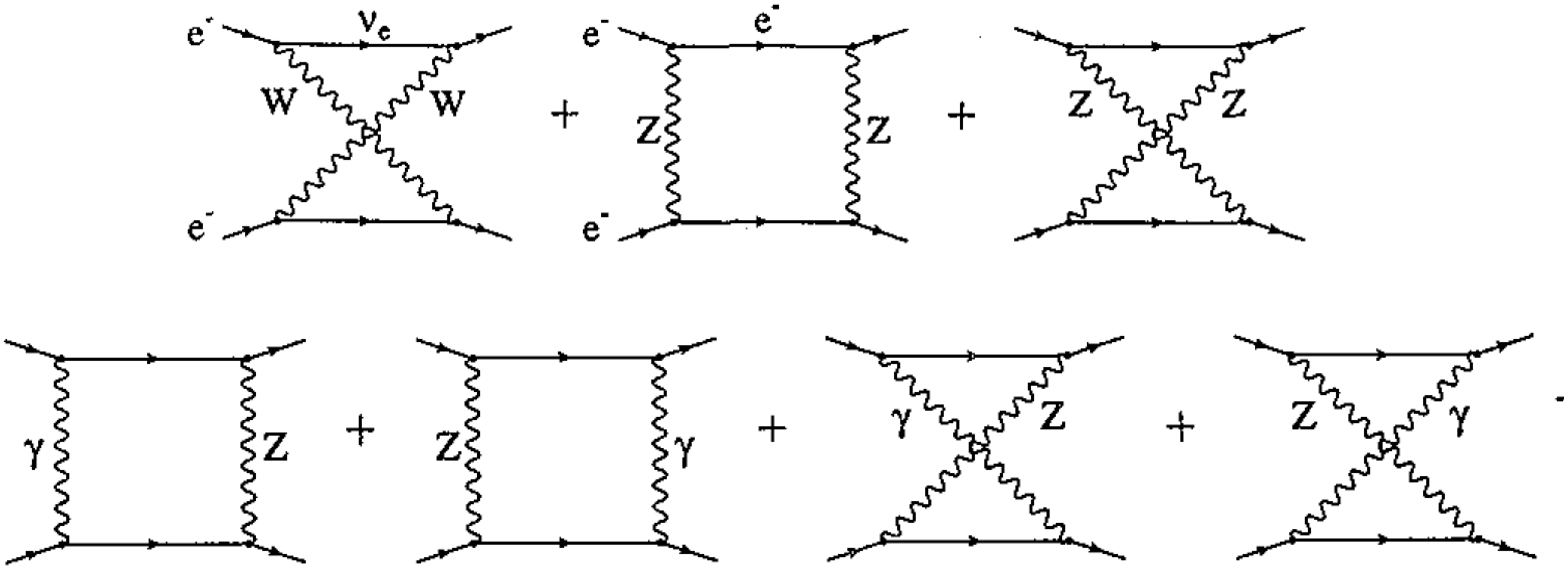
Outline

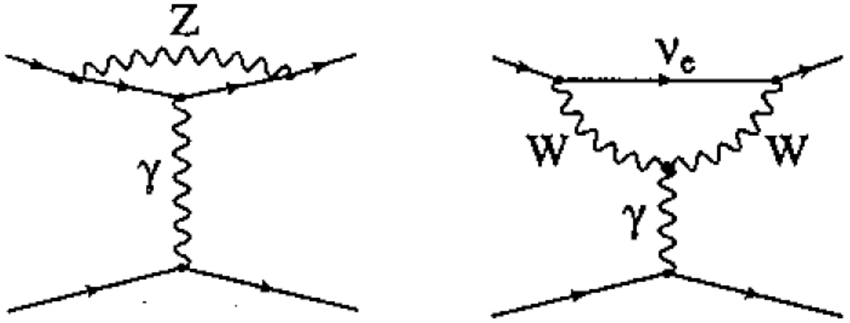
- Radiative Corrections:
  comparison of electron and proton weak charges; weak mixing angle at low Q^2
- New Physics (only brief):
  reach of low-energy measurements;
  ST parameters; comment on Z'
- Electroweak Model:
  status; implications of 12 GeV Møller

Electroweak
Radiative Corrections

Radiative Corrections to Q_W^e

Czarnecki & Marciano





All Z graphs suppressed by

$$1 - 4\sin^2\theta_W$$

Radiative Corrections to Q_W^p

Marciano & Sirlin, Ramsey-Musolf & JE

$$Q_W^p = [\rho_{NC} + \Delta_e][1 - 4\sin^2\hat{\theta}_W(0) + \Delta_e'] + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$

- Similar structure as for Møller scattering.
- γZ box: long-distance QCD, suppressed by 1 - 4 sin²θ.
- WW-box has factor 7 over Møller ⇒ 26% effect!
- ⇒ need 2-loop mixed electroweak-QCD corrections.

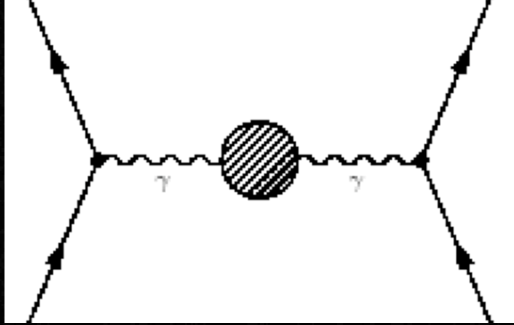
$$\Box_{WW} = \frac{\hat{\alpha}}{4\pi\sin^2\hat{\theta}_W}\left[2 + 5\left(1 - \frac{\alpha_s(M_W^2)}{\pi}\right)\right]$$

$$\hat{s}^2(\mu) = \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\hat{s}^2(\mu_0) + \lambda_1\left[1 - \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\right] +$$

$$\hat{\alpha}(\mu)\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4}\ln\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \tilde{\sigma}(\mu_0) - \tilde{\sigma}(\mu)\right].$$

energy range	λ_1	λ_2	λ_3	λ_4
$\bar{m}_t \leq \mu$	9/20	289/80	14/55	9/20
$M_W \leq \mu < \bar{m}_t$	21/44	625/176	6/11	3/22
$\bar{m}_b \leq \mu < M_W$	21/44	15/22	51/440	3/22
$m_\tau \leq \mu < \bar{m}_b$	9/20	3/5	2/19	1/5
$\bar{m}_c \leq \mu < m_\tau$	9/20	2/5	7/80	1/5
$\bar{m}_s \leq \mu < \bar{m}_c$	1/2	1/2	5/36	0
$\bar{m}_d \leq \mu < \bar{m}_s$	9/20	2/5	13/110	1/20
$\bar{m}_u \leq \mu < \bar{m}_d$	3/8	1/4	3/40	0
$m_\mu \leq \mu < \bar{m}_u$	1/4	0	0	0
$m_e \leq \mu < m_\mu$	1/4	0	0	0
$\mu < m_e$	0	0	0	0

parametrizing $\hat{\alpha}(\mu)$

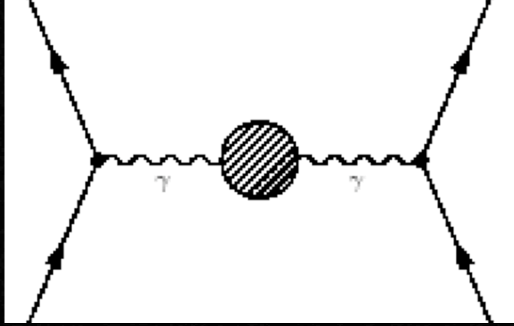


- use PQCD as much as possible:

  heavy quarks: 4-loop RGE + 3-loop matching

  light quarks: 4-loop analytic results above 1.8 GeV
- c and b masses QCD from sum rules (S resonances)
- absorb higher order QCD corrections into effective "threshold masses": $\bar{m}_q$
- for light quarks below 1.8 GeV use dispersive result and approximate isospin symmetry, $\bar{m}_u = \bar{m}_d$

parametrizing $\hat{\alpha}(\mu)$



Define $\xi_q = 2\bar{m}_q / M_{1S}$ and take $\xi_s > \xi_u = \xi_d$

and $\xi_s < \xi_c$ as limiting cases

(singlet = QCD annihilation diagrams negligible)

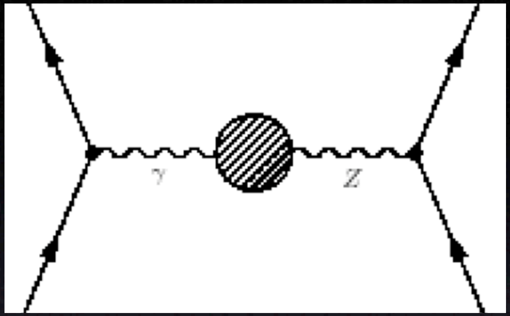
$$\bar{m}_u = \bar{m}_d = 176 \pm 9 \text{ MeV}$$

$$\bar{m}_s = 305^{-65}_{+82} \text{ MeV}$$

$$\bar{m}_c = 1.18 \pm 0.04 \text{ GeV}$$

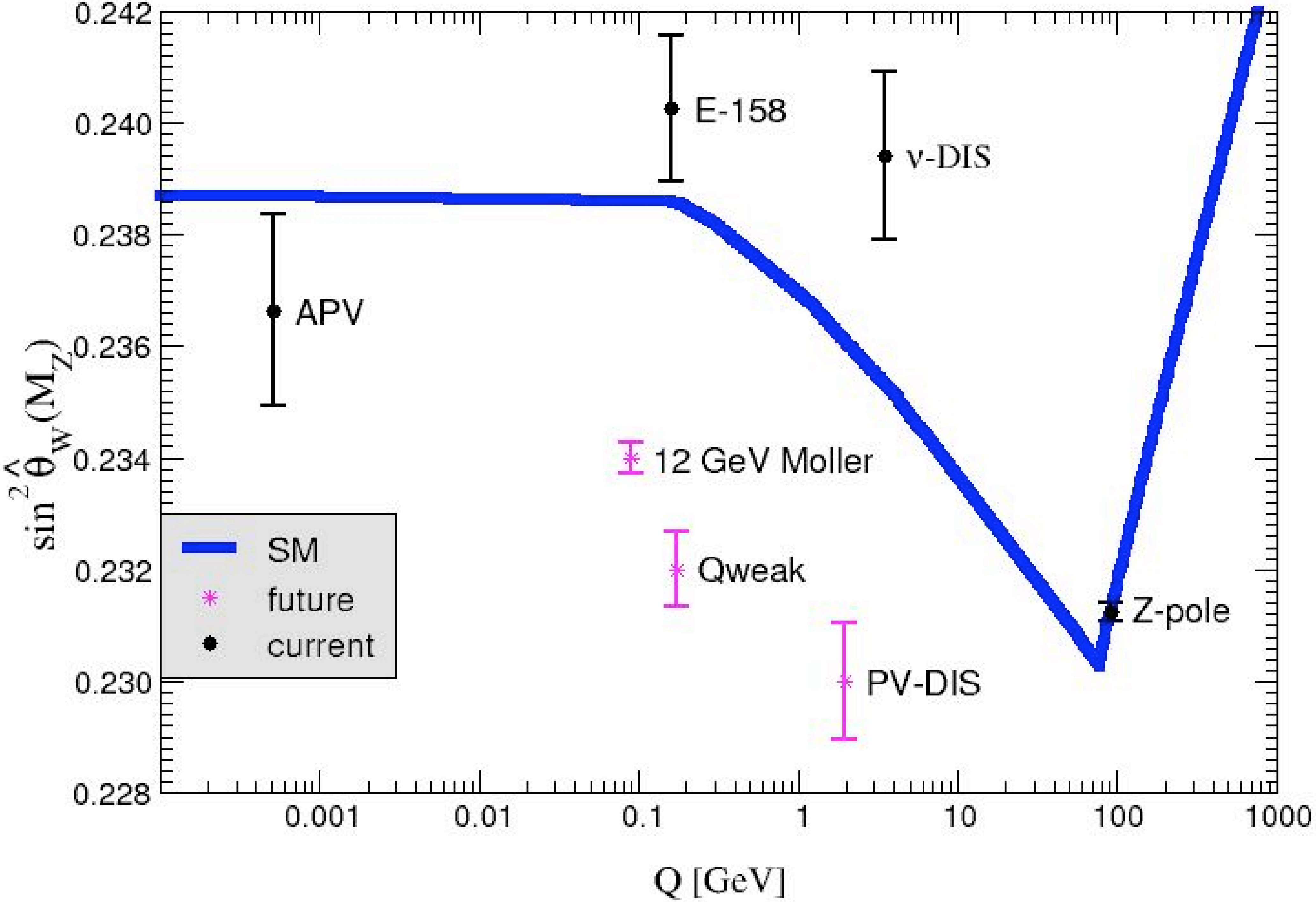
$$\bar{m}_b = 4.00 \pm 0.03 \text{ GeV}$$

application to $\sin^2\hat{\theta}_W(\mu)$



- rigorous (same spectral and kernel functions) once a set of quark masses have been accepted.
- $\sin^2\hat{\theta}_W(0)$ important input for low energy observables, like APV, E158, Qweak, 12 GeV Møller, ...

source	uncertainty
s quarks	0.00005
OZI rule	0.00003
isospin	0.00001
non-param.	0.00006
data/OPE	0.00003
α_s	0.00004
m_c, m_b	0.00004
sub-total	0.00009
$\sin^2\hat{\theta}_W(M_Z)$	0.00015
TOTAL	0.00016



(uncertainty not Q^2-independent)

Future improvements

- s quarks: currently very conservative. In future use τ spectral functions? 0.00005→0.00003
- e+e- data already improved, and will continue to improve: 0.00003→0.00002
- strong coupling will incrementally improve in the future: 0.00004→0.00002
- sum rule error on charm quark mass currently inflated; expect 0.00004→0.00003
- in total: ±0.00009 → ±0.00006

Constraints on New Physics

New Physics: Reach

Polarized electron scattering and APV

$$\frac{\Lambda_{\rm NEW}}{g} \approx \frac{1}{\sqrt{\sqrt{2}G_F|\Delta Q_W|}} \approx \begin{cases} 1.5 \text{ TeV (PV-DIS)} \\ 4.2 \text{ TeV (APV)} \\ 4.6 \text{ TeV (Qweak)} \\ 3.2 \text{ TeV (E-158)} \\ 7.3 \text{ TeV (e2ePV)} \end{cases}$$

ν-DIS: $$\frac{\Lambda_{\rm NEW}^{L,R}}{g} \approx \sqrt{\frac{\max[\epsilon_{L,R}(u), \epsilon_{L,R}(d)]}{\sqrt{2}G_F|\Delta g_{L,R}^2|}} \approx \begin{cases} 4.7 \text{ TeV (L)} \\ 3.1 \text{ TeV (R)} \end{cases}$$

g−2: $$\Lambda_{\rm NEW} \approx \frac{m_\mu}{\sqrt{|\Delta a_\mu|}} \approx \begin{cases} 3.7 \text{ TeV (E-821)} \\ 5.3 \text{ TeV (E-969)} \end{cases}$$

New Physics: Significance

experiment	reach	significance Λ=2 TeV
PV-DIS	1.5 TeV	0.6 σ
APV	4.2 TeV	4.4 σ
Qweak	4.6 TeV	5.3 σ
E-158	3.2 TeV	2.6 σ
e2ePV	7.3 TeV	13.3 σ
ν-DIS (L)	4.7 TeV	5.5 σ
ν-DIS (R)	3.1 TeV	2.4 σ
E-821	3.7 TeV	3.4 σ
E-969	5.3 TeV	7.0 σ

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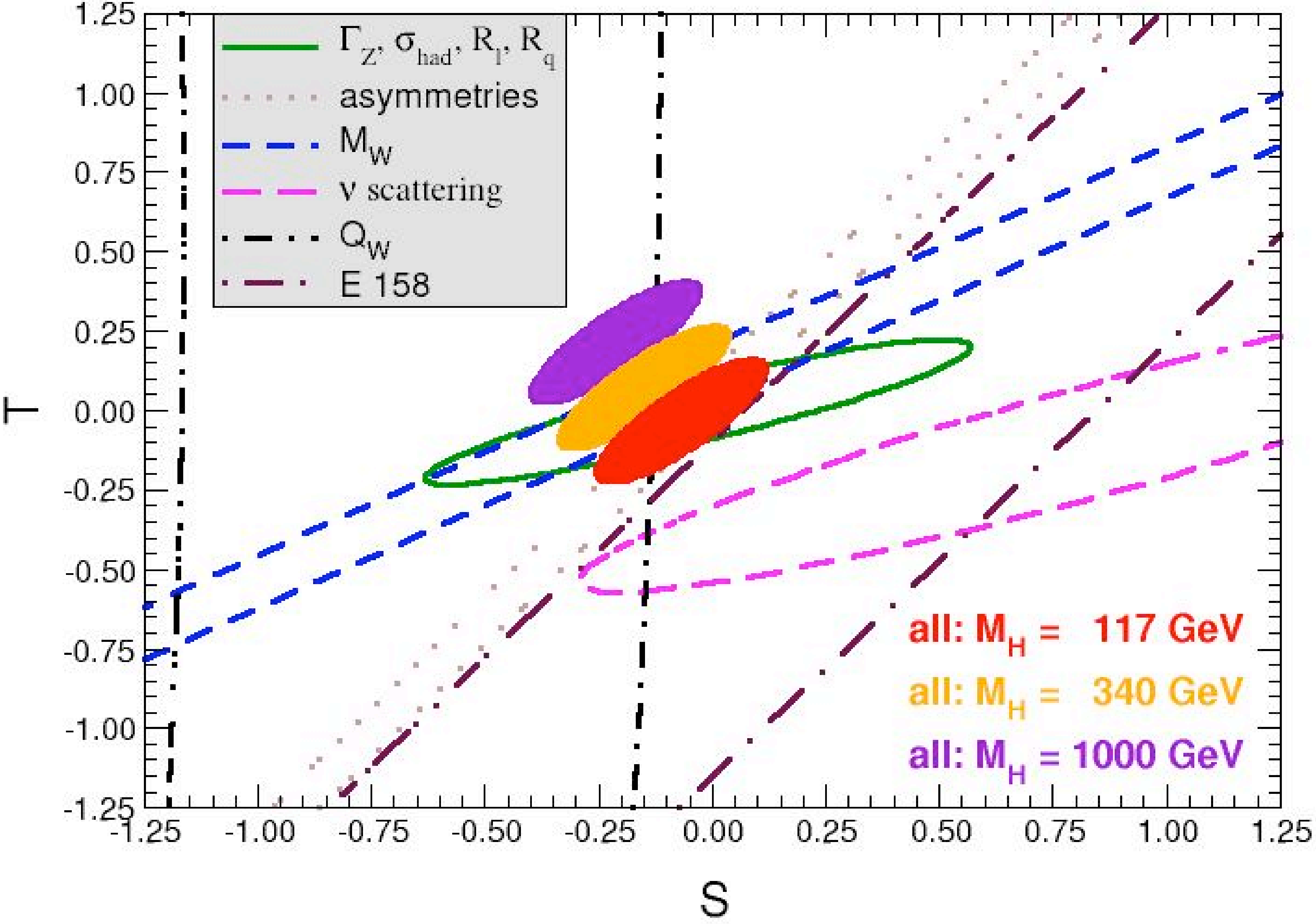
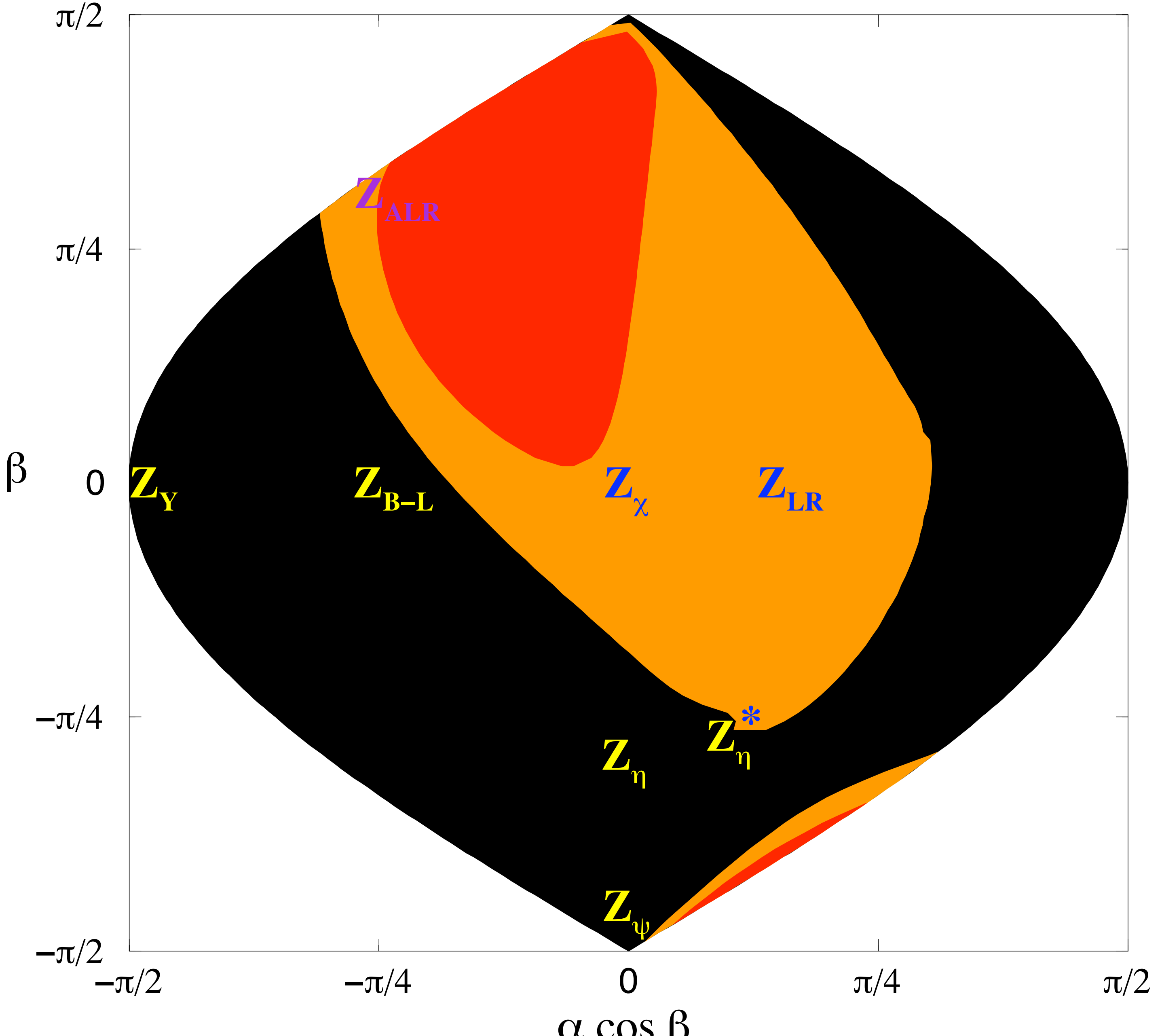


Z' bosons from E_6



The Electroweak Model

Weak Mixing Angle

$$Z^0_\mu = \cos\theta_W\, W^3_\mu - \sin\theta_W\, B_\mu$$

$$\mathrm{A}_\mu = \sin\theta_W\, W^3_\mu + \cos\theta_W\, B_\mu$$

$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2} = 1 - \frac{M_W^2}{M_Z^2}$$

$$e = g\sin\theta_W = g'\cos\theta_W$$

$$A_f = \frac{2 v_f\, a_f}{v_f^2 + a_f^2}$$

$$v_f = t_3^f - 2Q_f\sin^2\theta_W$$

Z pole physics

Lineshape: $M_Z, \Gamma_Z, \sigma^0_{\mathrm{had}}$

Branching ratios: R_ℓ, R_b, R_c

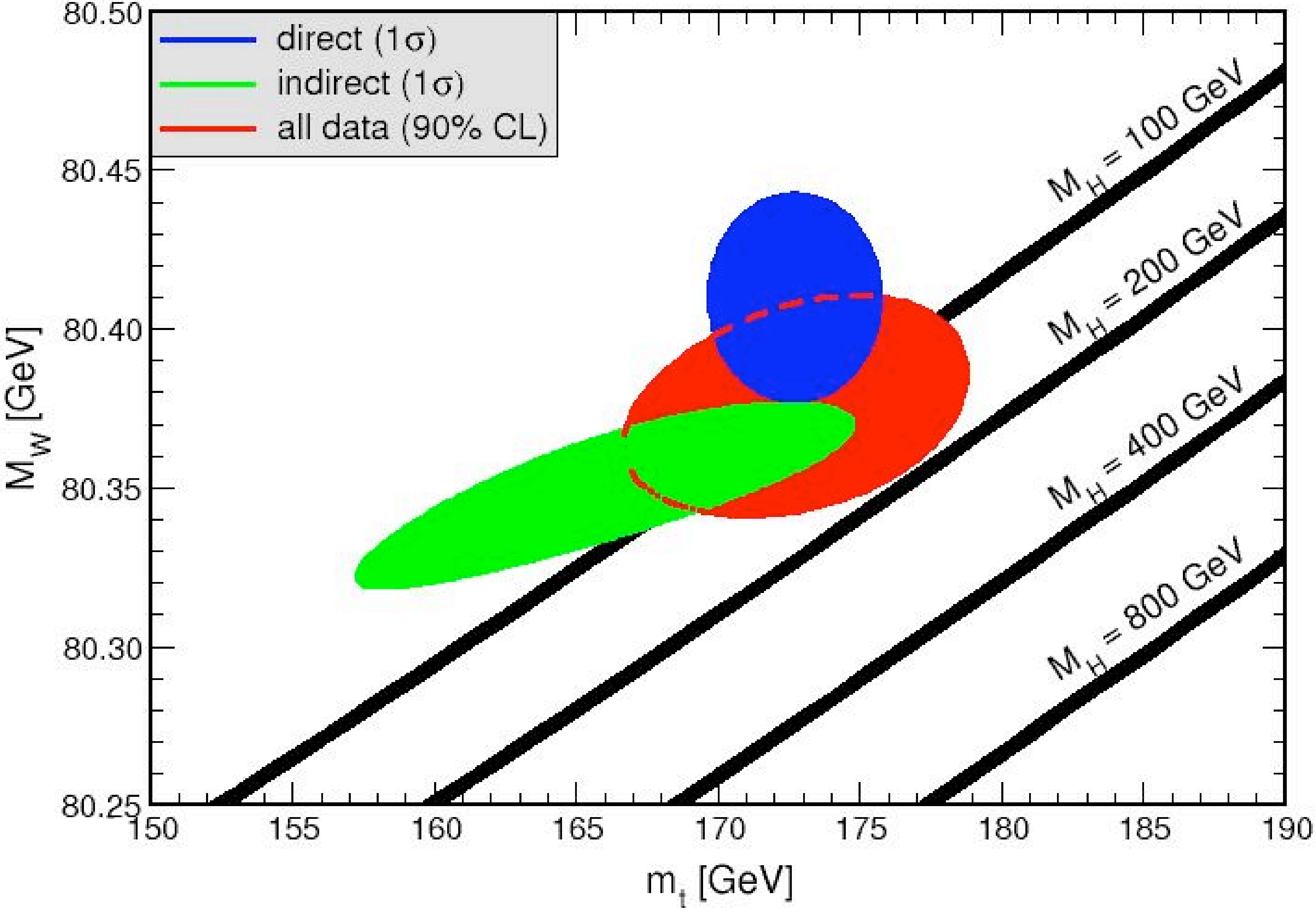
LEP asymmetries: $A_{FB}(\ell, b, c), \mathcal{P}_\tau, \mathcal{P}_\tau^{FB}$

$$A_{FB}(f) = \frac{3}{4} A_e A_f, \mathcal{P}_\tau = A_\tau, \mathcal{P}_\tau^{FB} = A_e$$

SLD asymmetries: $A_{LR}(\mathrm{had}, \ell), A_{LR}^{FB}(\ell, b, c)$

$$A_{LR} = A_e, A_{LR}^{FB}(f) = A_f$$

New Physics: tight constraints on Zff-couplings, but new amplitudes suppressed by Γ_Z^2/M_Z^2



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Global Fit

September 2005 + new top quark and
W boson masses as of October 2006

$$M_H = 84^{+32}_{-25}\ \text{GeV}$$

$$m_t = 171.4 \pm 2.1\ \text{GeV}$$

$$\alpha_s(M_Z) = 0.1216 \pm 0.0017$$

$$\chi^2/\text{d.o.f.} = 47.3/42\,(27\%)$$

indirect only:

$$m_t = 171.0^{+9.5}_{-7.1}\ \text{GeV}$$

Small Deviations

	value	error	SM	pull	comments
$\sigma^0_{\mathrm{had}}[\mathrm{nb}]$	41.541	0.037	41.467	2.0	$N_\nu = 2.986 \pm 0.007$
$A^b_{FB}(\mathrm{LEP})$	0.0992	0.0016	0.1031	-2.4	best $\hat{s}^2$@ LEP
$A_{LR}(\mathrm{SLD})$	0.1514	0.0022	0.1471	2.0	best $\sin^2\theta_W$
$g_L^2(\mathrm{NuTeV})$	0.3001	0.0014	0.3038	2.7	QED, PDFs
$10^9 \frac{g-2-\frac{\alpha}{\pi}}{2}$	4511.07	0.82	4509.82	3.4	w/o τ-data

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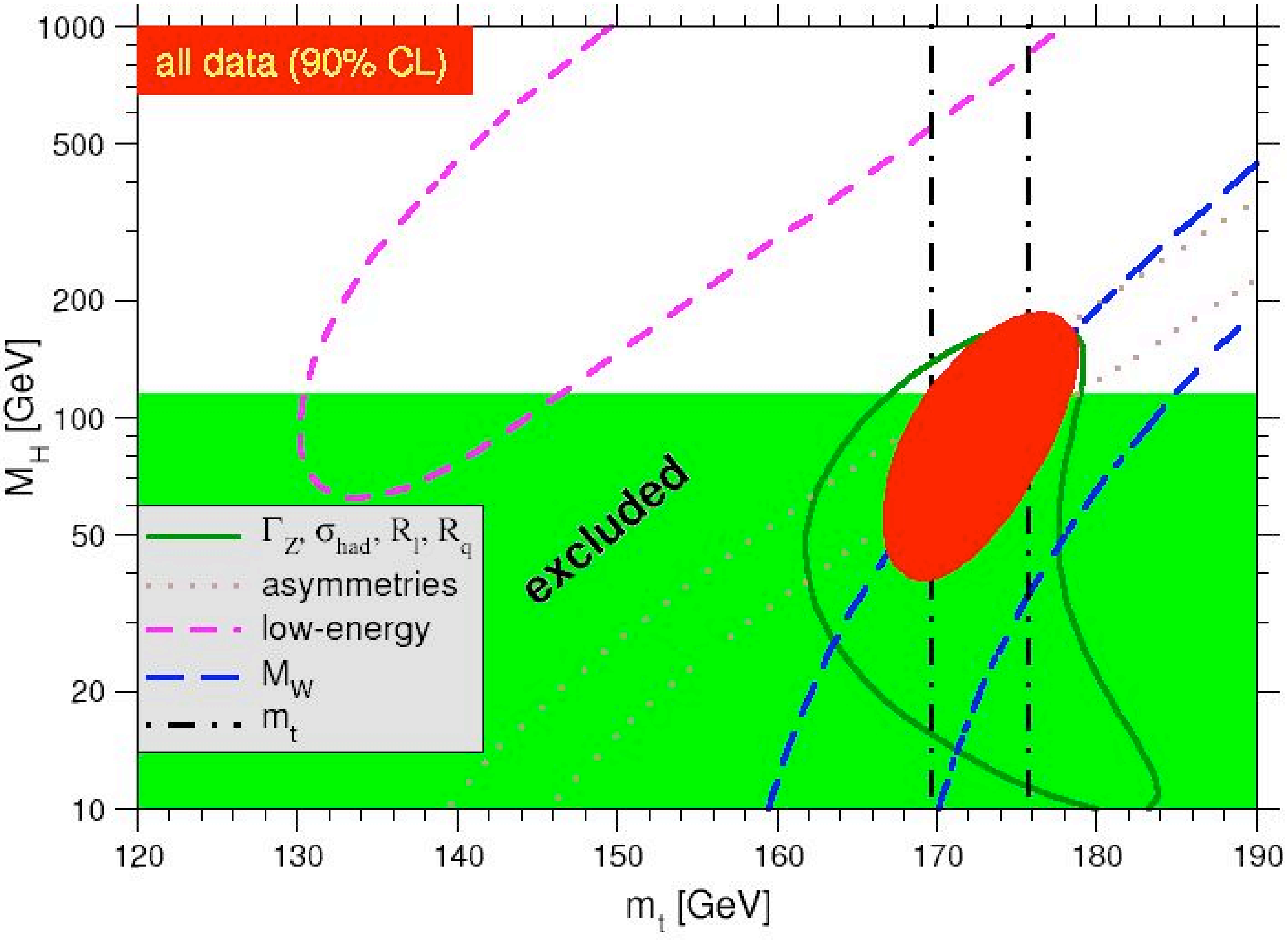
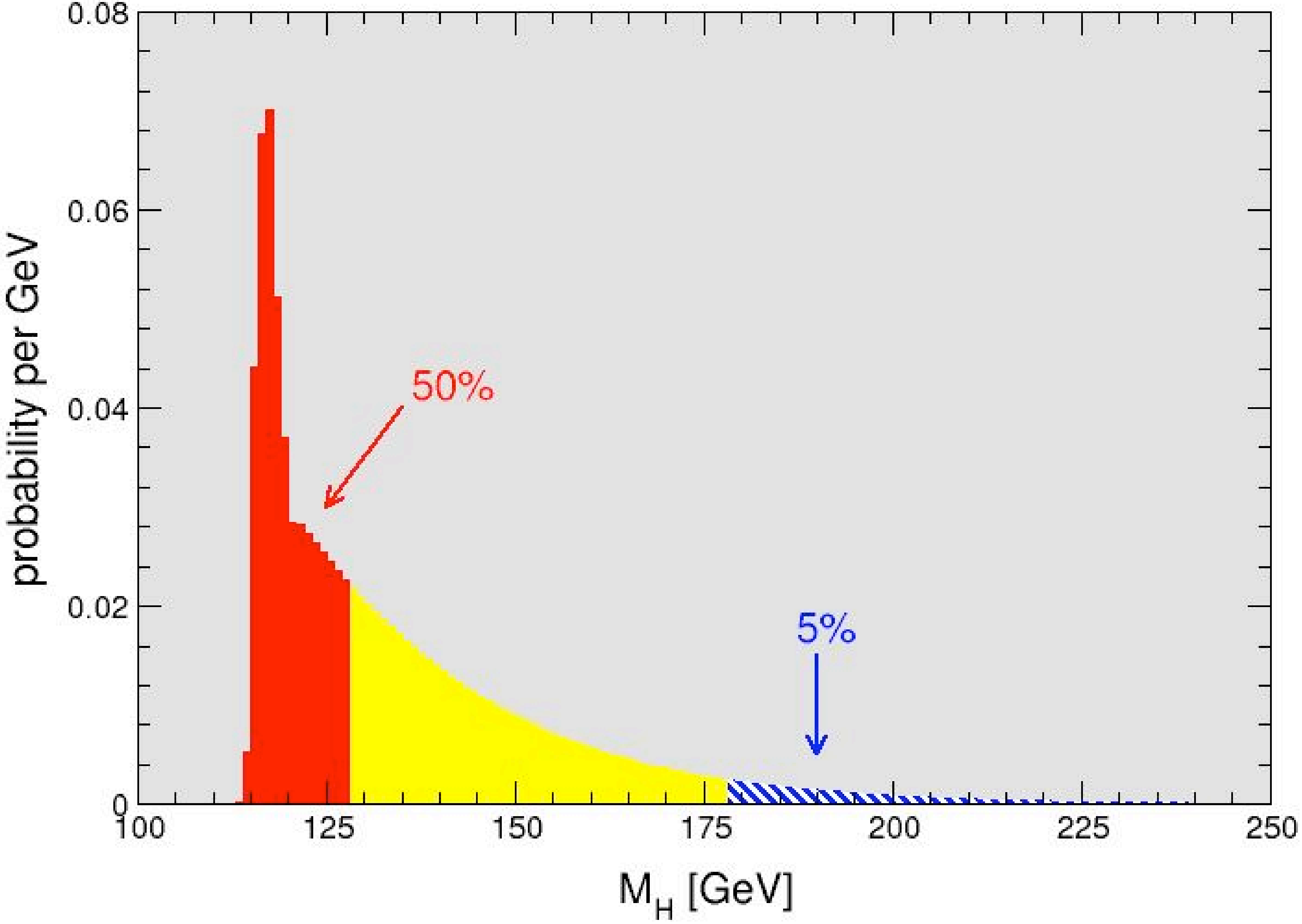


October 2006



Implications of Møller

$$\Delta \sin^2 \theta_W(0) = 0.00030$$

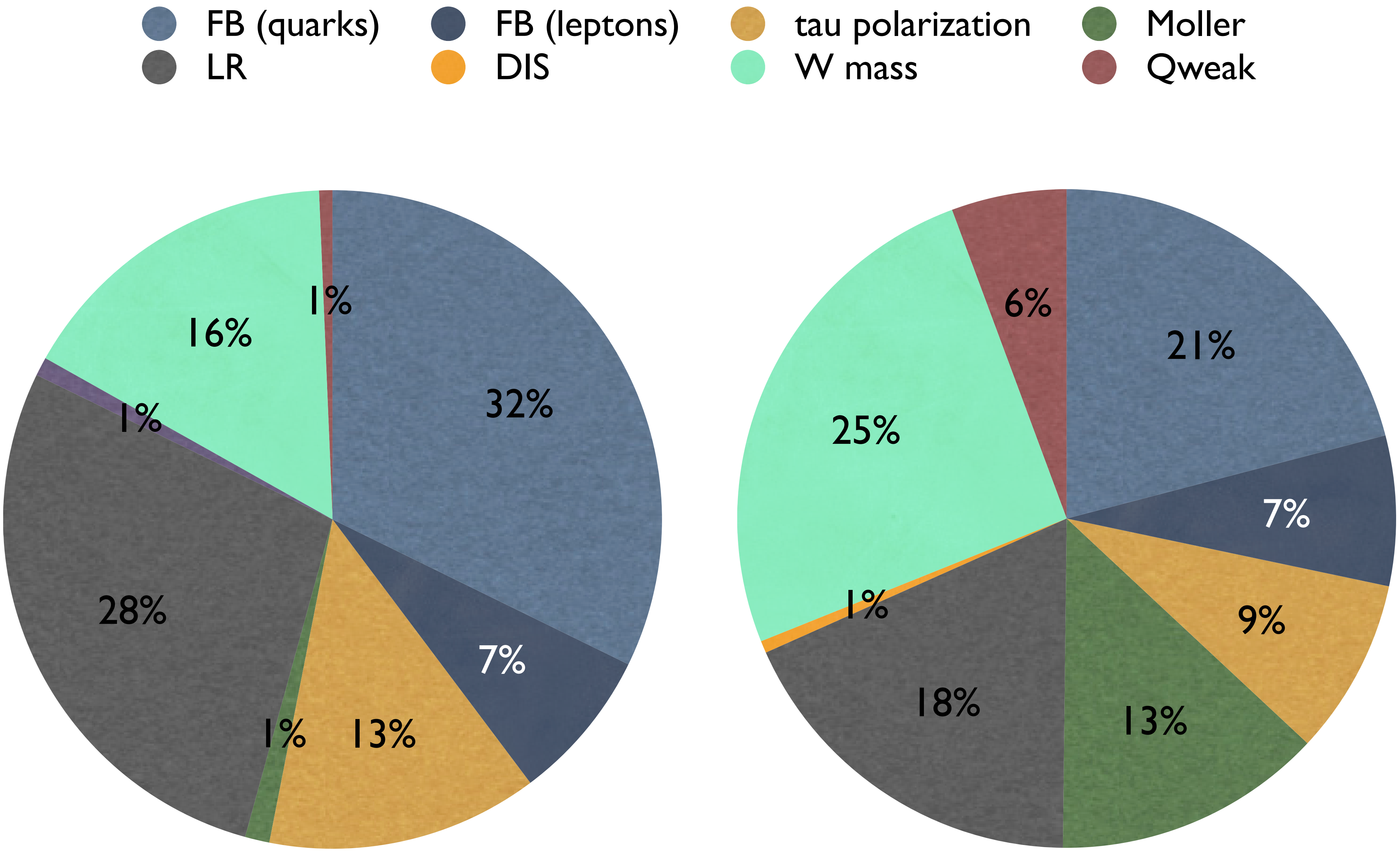
$$\Delta \sin^2 \theta_W(M_Z) = \Delta \sin^2 \theta^{\ell}_{\text{eff.}} = 0.00033$$

$$\Delta M_W = 40 \ MeV$$

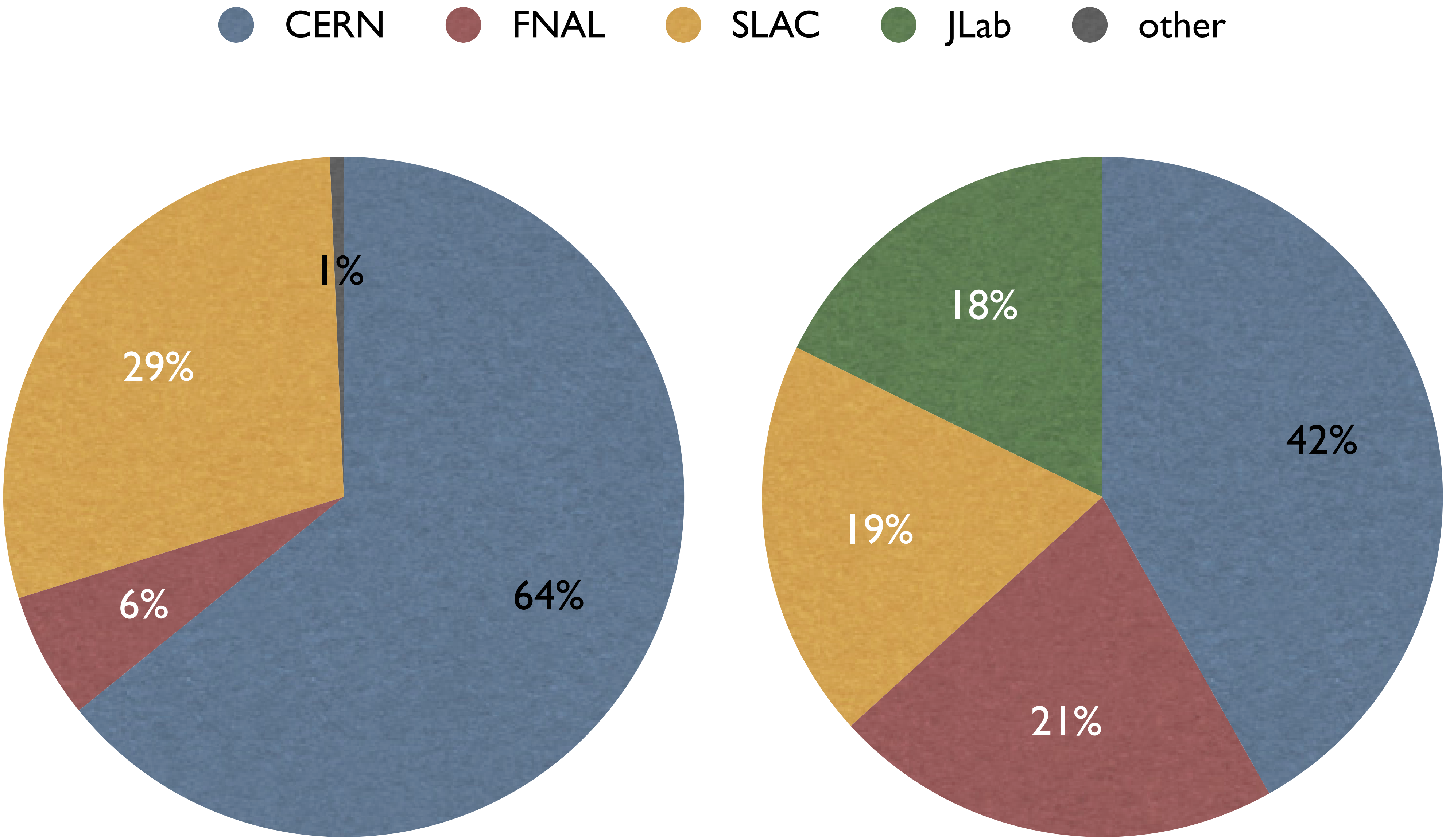
compare: Tevatron Run I: 58 MeV, LEP 2: 33 MeV

(the last two include the theory uncertainty
from running weak mixing angle)

World Knowledge of $\sin^2\theta_W^{\mathrm{eff.}}$



$\sin^2\theta_W^{\text{eff.}}$ by Lab



$\sin^2\theta_W^{\mathrm{eff.}}$	fb^{-1} per experiment	value	goal	√L-scaling
Tevatron Run I	0.072	0.2238	0.0050	-
SLC	0.05	0.23098	0.00026	-
LEP I	0.20	0.23187	0.00021	-
currently		0.23152	0.00016	-
Tevatron Run IIA	2		0.0008	0.0009
Tevatron Run IIB	8		0.0003	0.0005
JLab	$\vec{e}e, \vec{e}p$		0.0003	-
LHC low lumi	30		0.00066	0.00024
LHC high lumi	400		0.00014	0.00007
ILC	Møller		0.00008	0.00004
GigaZ	70		0.000013	0.000016

Extracting Higgs mass from Møller asymmetry

1. Define $Q_W(e)$ (electron weak charge) as pseudo-observable. Useful to do this independent of energy transfer, y, and at $Q^2=0$ (compare Z pole). Is this possible? If not, specify kinematic reference values.
2. Extract $Q_W(e)$ from A_{PV} and quote as main result rather than $\sin^2\hat{\theta}_W$ or A_{PV} itself (model independence).
3. Compute $\sin^2\hat{\theta}_W(0)$ from $\sin^2\hat{\theta}_W(M_Z)$.
4. $Q_W(e)$ is function of $\sin^2\hat{\theta}_W(0)$ (tree-level) and $\sin^2\hat{\theta}_W(M_Z)$ (loops) → fit

Impact of 12 GeV Møller on Higgs mass

If in agreement with SM:

$$M_H = 86^{+79}_{-44}\ \mathrm{GeV}$$

$$M_H = 84^{+32}_{-25}\ \mathrm{GeV} \rightarrow M_H = 84^{+30}_{-23}\ \mathrm{GeV}$$

With a 2 σ upward fluctuation (direction of E-158):

$$M_H = 69^{+26}_{-20}\ \mathrm{GeV}$$

which at the 90 (95)% CL is excluded (only barely consistent) with lower Higgs mass limit of 114.4 GeV.

LEP-SLD conflict

$\sin^2\hat{\theta}_W(M_Z)$

$$A_{LR}(\text{had.}) + M_Z = 0.23067 \pm 0.00032$$

$$A_{FB}^{b} + M_Z = 0.23193 \pm 0.00029$$

2.9 σ between most precise single determinations

$$\text{SLD} + M_Z = 0.23067 \pm 0.00029$$

$$A_{FB}^{b,c} + M_Z = 0.23193 \pm 0.00028$$

3.1 σ between measurement classes

$$12\ \text{GeV Møller} + M_Z : \pm\ 0.00033$$

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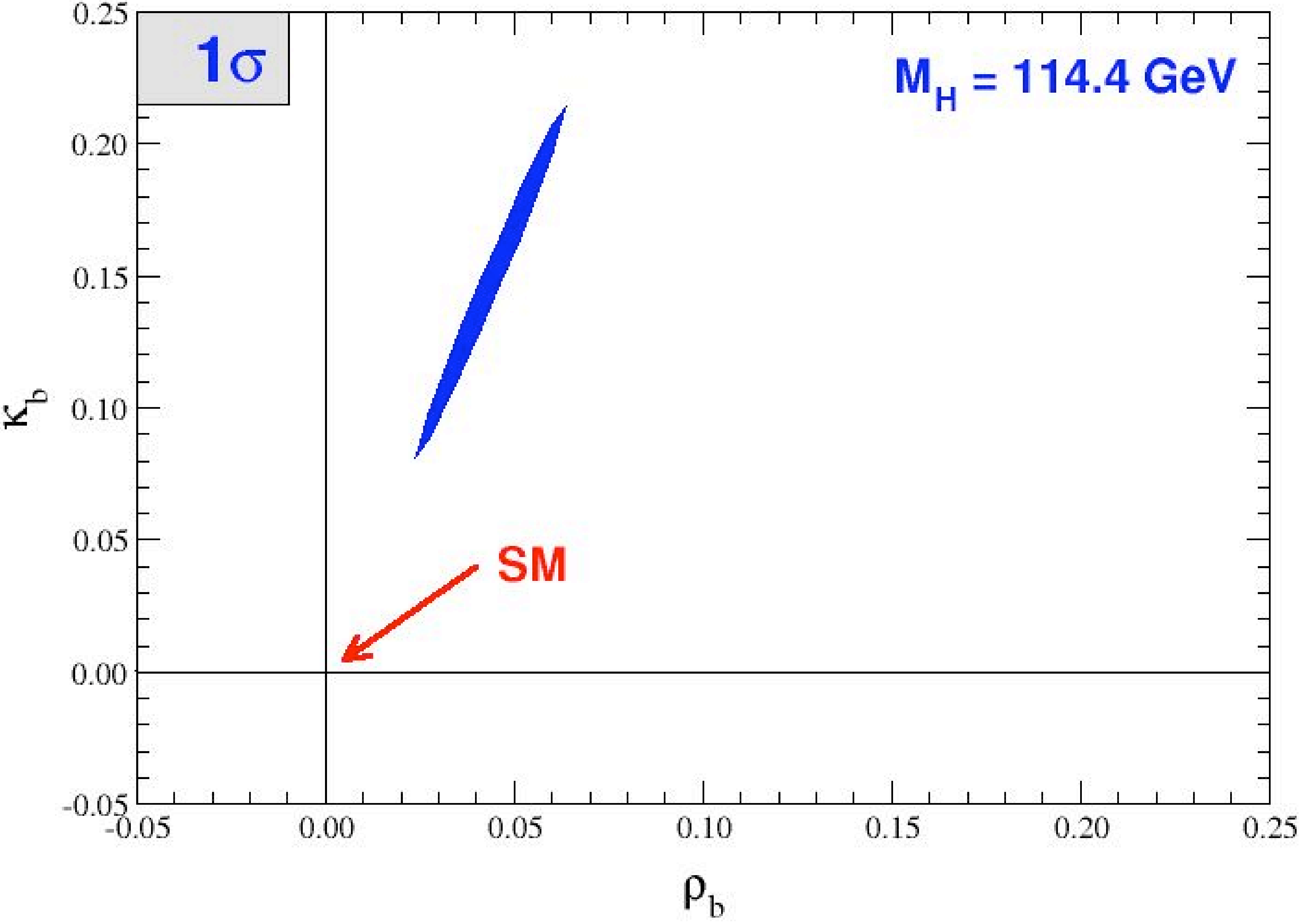

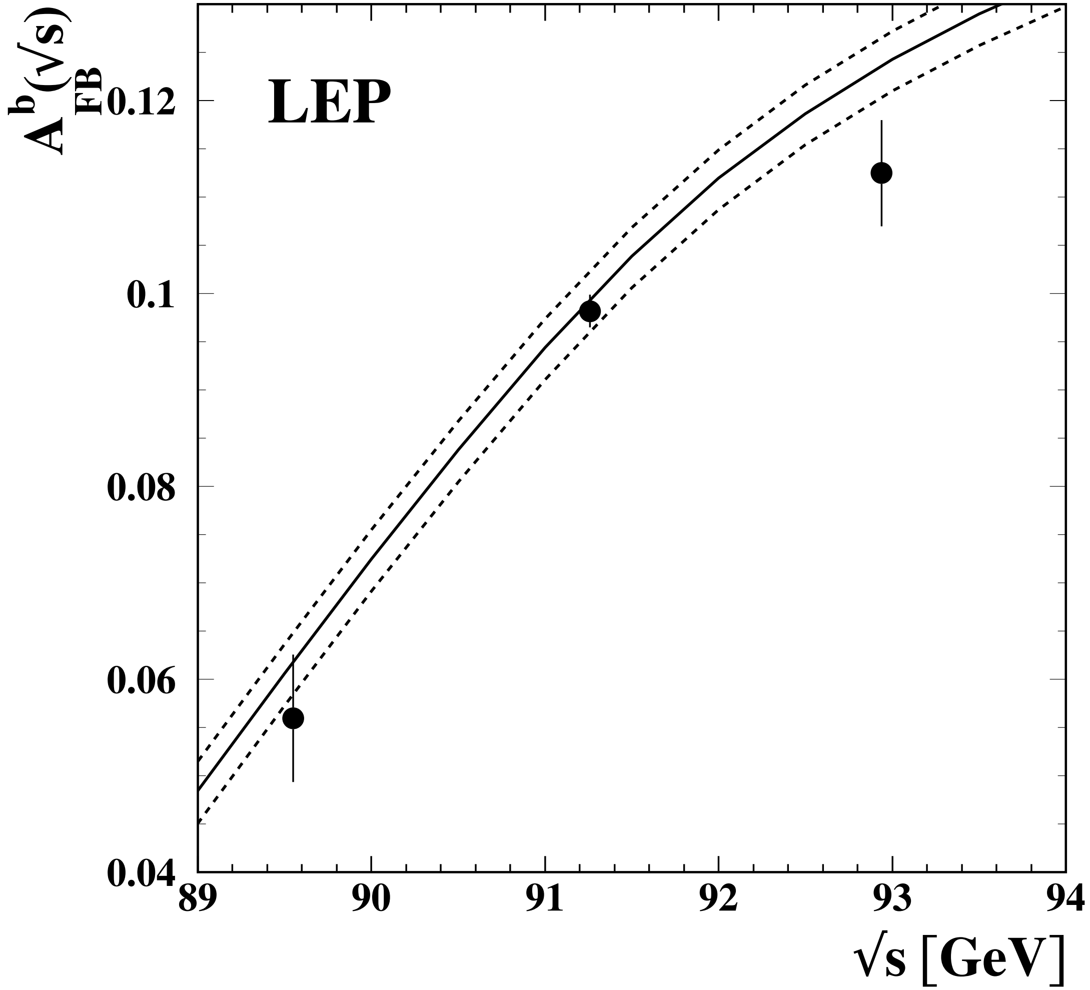

Statistics dominated!

$$\chi^2/d.o.f. = 53/91$$

Impact on LEP-SLD conflict

With a 2 σ upward fluctuation (direction of E-158)
(confirming SLD):

b-asymmetry grows from 2.4 to 2.8 σ,
while SLD decreases from to 2.0 to 1.7 σ

With a 2 σ downward fluctuation
(confirming b-asymmetry):

b-asymmetry decreases from 2.4 to 2.1 σ,
while SLD grows from to 2.0 to 2.4 σ

Conclusions

- The 12 GeV Møller asymmetry will be the most important low-energy measurement of any SM-allowed process since the Prescott DIS experiment.
- The environment is clean and free of any QCD, hadronic, or nuclear issues, except for calculation of low-energy weak mixing angle. This can be dealt with.
- The level of precision should make it 3rd in line of any weak mixing angle result, and within 10% to the best one (b-asymmetry).
- Worth to aggressively push Møller to its limits.

Very long term

- Can an order of magnitude increase in investment cut the systematic uncertainty in half? If yes, 1.6% → 0.8% (syst.)
- A $\sqrt{10}$ gain in stat. precision ⇒ 1.9% → 0.6% (stat.)
- Total: 2.5% → 1.0%
- error on weak mixing angle: ±0.00033 → ±0.00013; better than current global fit and the most optimistic LHC projection, close to Møller @ ILC (±0.00008).
- New physics reach: 7.3 TeV → 11.5 TeV