$N \rightarrow N^*$ Form Factor extraction at large Q^2:

physics issues and challenges

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Nucleon Resonance Structure in Exclusive Electroproduction at High Photon Virtualities with the CLAS 12 Detector Workshop

Jefferson Lab, May 16, 2011

Outline

- Physics encoded in $\gamma^* N \to N^*$ FFs
  and imaging of transition charge densities

- multipole extractions at larger Q^2 :
  reaction mechanism de/pre-scriptions

- testing the model dependence at larger Q^2 :
  role of polarization observables

- two-photon exchange effects at larger Q^2

for more details, see e.g. review on $N \to \Delta$:

Pascalutsa, Vdh, Yang / Phys. Rept. 437 (2007) 125

Physics encoded in $\gamma^* N \rightarrow N^*$ FFs

&

transition charge densities

N -> Δ (1232) transition

- experiment measures multipoles

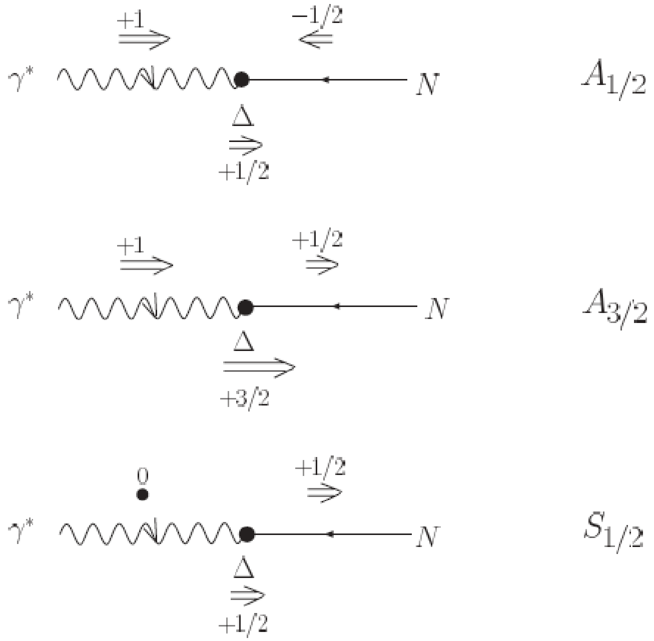
$$\bar{M}_{1+}^{(3/2)}(Q^2) \equiv \sqrt{\frac{2}{3}}\, a_\Delta \, \mathrm{Im} M_{1+}^{(3/2)}(Q^2, W = M_\Delta)$$

- theory calculates helicity form factors

$$A_{3/2} \equiv -\frac{e}{\sqrt{2q_\Delta}} \frac{1}{(4M_N M_\Delta)^{1/2}} \langle\, \Delta(\vec{0}, +3/2) \,|\, \mathbf{J}\cdot\epsilon_{\lambda=+1} \,|\, N(-\vec{q}, +1/2)\,\rangle$$

$$A_{1/2} \equiv -\frac{e}{\sqrt{2q_\Delta}} \frac{1}{(4M_N M_\Delta)^{1/2}} \langle\, \Delta(\vec{0}, +1/2) \,|\, \mathbf{J}\cdot\epsilon_{\lambda=+1} \,|\, N(-\vec{q}, -1/2)\,\rangle$$

$$S_{1/2} \equiv \frac{e}{\sqrt{2q_\Delta}} \frac{1}{(4M_N M_\Delta)^{1/2}} \langle\, \Delta(\vec{0}, +1/2) \,|\, J^0 \,|\, N(-\vec{q}, +1/2)\,\rangle,$$



- define resonance properties

$$A_{3/2} = -\frac{\sqrt{3}}{2}\left\{\bar{M}_{1+}^{(3/2)} - \bar{E}_{1+}^{(3/2)}\right\}$$

$$A_{1/2} = -\frac{1}{2}\left\{\bar{M}_{1+}^{(3/2)} + 3\,\bar{E}_{1+}^{(3/2)}\right\}$$

$$S_{1/2} = -\sqrt{2}\,\bar{S}_{1+}^{(3/2)},$$

N -> Δ magnetic dipole (M1) form factor

Unpolarized e N -> e (π N) cross section, integrated over pion angles

$$\frac{d\sigma}{d\Omega_e dE_f} = \Gamma_V (\sigma_T + \varepsilon \sigma_L)$$

cross section at the resonance peak (Ash, 1967)

$$\left.\frac{d\sigma}{d\Omega_e dE_f}\right|_{peak} = \Gamma_V \frac{4\pi\alpha}{W_R \Gamma_R} \frac{1}{\kappa_{R,l}} (1+\tau^*) \frac{Q^2}{2M} \; G_M^{*\,2}(Q^2)$$

inclusive cross section in narrow width approximation

$$\frac{d\sigma}{d\Omega_e} = \frac{\sigma_{NS}}{\varepsilon} \frac{Q^2}{4M^2} G_{(M)}^{*2}(Q^2)$$

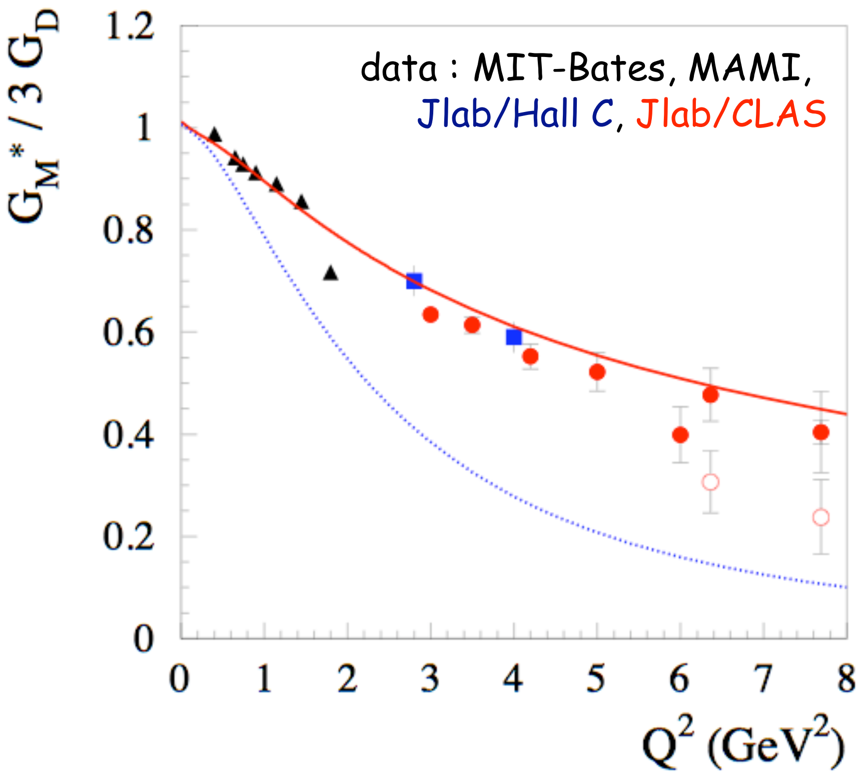
in fact, $G_{(M)}^{*\,2}(Q^2)$ contains also electric and charge contributions

$$G^{*2}(Q^2) \equiv G_{(M)}^{*\,2}(Q^2) = G_M^{*\,2}(Q^2) + 3\,G_E^{*\,2}(Q^2) + \varepsilon \frac{Q^2}{W_\Delta^2} G_C^{*\,2}(Q^2)$$

N -> Δ magnetic dipole form factor

large N_c limit

$$G^*_M(t) = \frac{G^*_M(0)}{\kappa_V} \int_{-1}^{+1} dx \left\{ E^u(x,\xi,t) - E^d(x,\xi,t) \right\} = \frac{G^*_M(0)}{\kappa_V} \left\{ F_2^p(t) - F_2^n(t) \right\}$$



large N_c : $G^*_M(0) = \kappa_V / \sqrt{2} = 2.62$

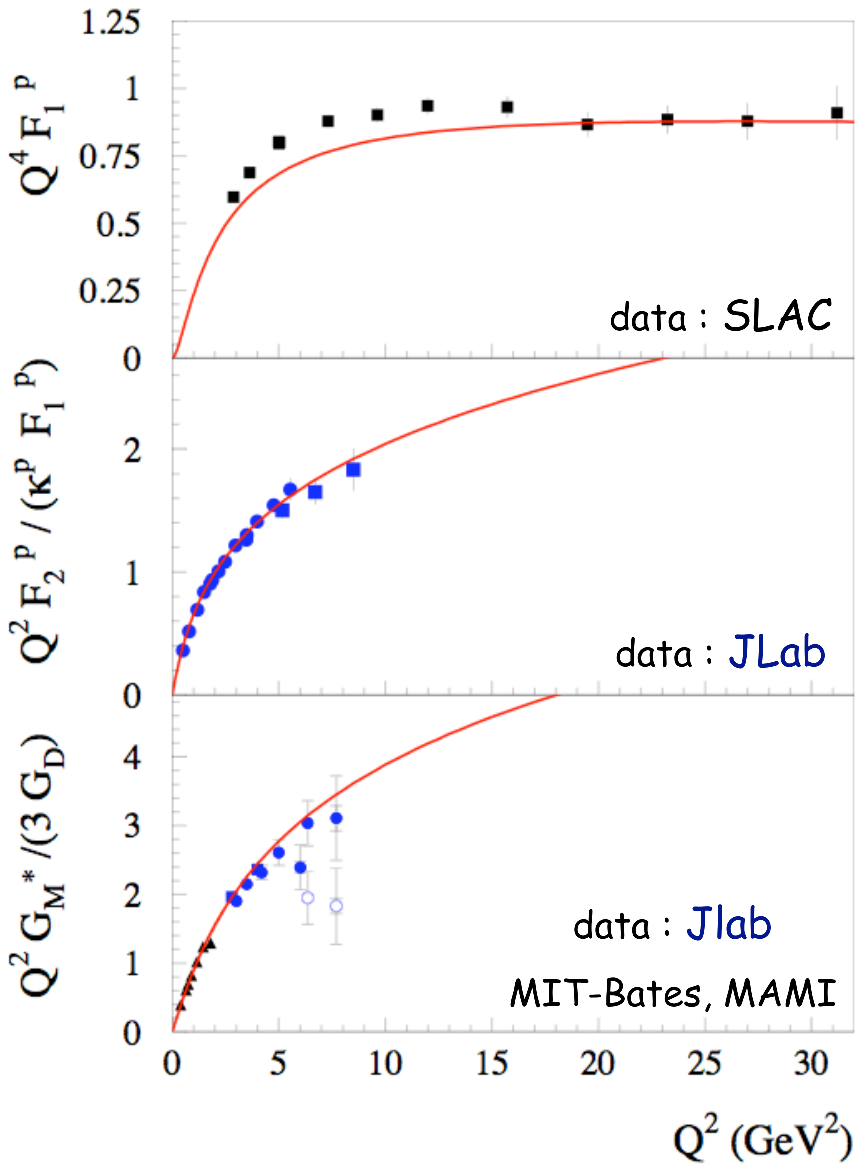
EXP : $G^*_M(0) = 3.02$

→ modified Regge model

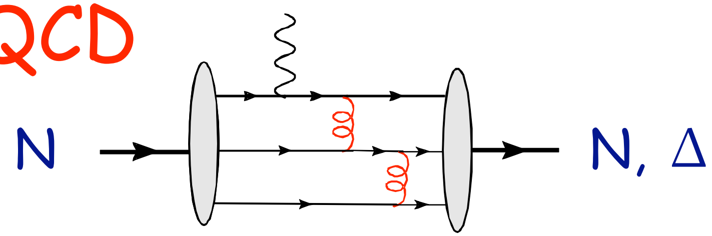
→ Regge model

Guidal, Polyakov, Radyushkin, Vdh (2005)

scaling behavior of N and N -> Δ F.F.



PQCD



+ collinear quarks

$F_1^p \sim 1/Q^4$

$F_2^p / F_1^p \sim 1/Q^2$

$G_M^* \sim 1/Q^4$

GPD ——

modified Regge model

Guidal, Polyakov, Radyushkin, Vdh (2005)

N -> Δ E2 and C2 form factors

- large N_c limit of QCD :

$$Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}} r_n^2 \frac{N_c}{N_c+3} \sqrt{\frac{N_c+5}{N_c-1}}$$

Buchmann, Hester, Lebed (2002)

- $N_c = 3$

$$Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}} r_n^2$$

$$G_E^*(0) = -\frac{1}{6} r_n^2 \frac{1}{\sqrt{2}} \frac{(M_\Delta^2 - M_N^2)}{2}$$

EXP : $r_n^2 = -\,0.113\ (3)\ \mathrm{fm}^2$

large N_c : $Q_{p\to\Delta^+} = -\,0.080\ \mathrm{fm}^2$

EXP : $Q_{p\to\Delta^+} = -\,0.085\ (3)\ \mathrm{fm}^2$

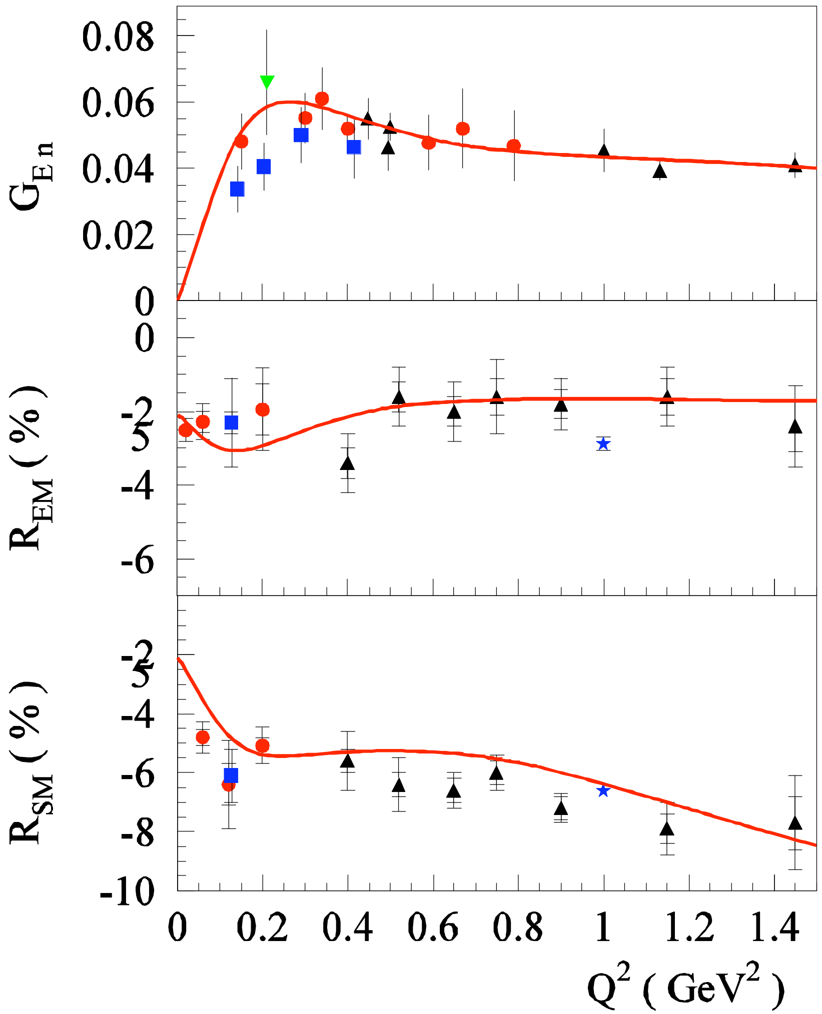
- finite (low) Q^2 : $G_E^n(Q^2) \approx -r_n^2\, Q^2/6$

$$G_E^*(Q^2) \simeq \frac{1}{\sqrt{2}} \frac{(M_\Delta^2 - M_N^2)}{2} \frac{G_E^n(Q^2)}{Q^2}$$

$$G_C^*(Q^2) \simeq \frac{4M_\Delta^2}{(M_\Delta^2 - M_N^2)} G_E^*(Q^2)$$

Pascalutsa, Vdh (2006)

N -> Δ E2 and C2 form factors



G_{En} fit :

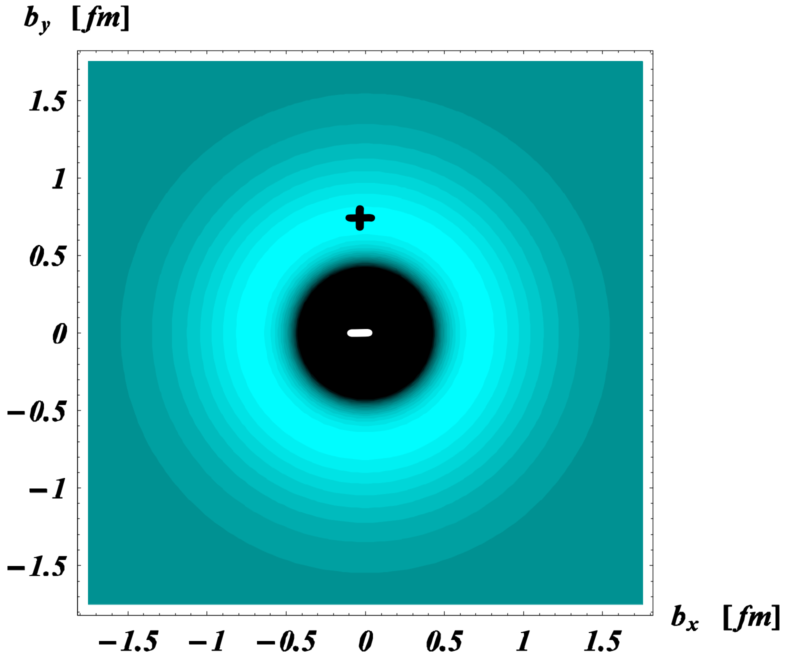
Bradford, Bodek, Budd, Arrington (2006)

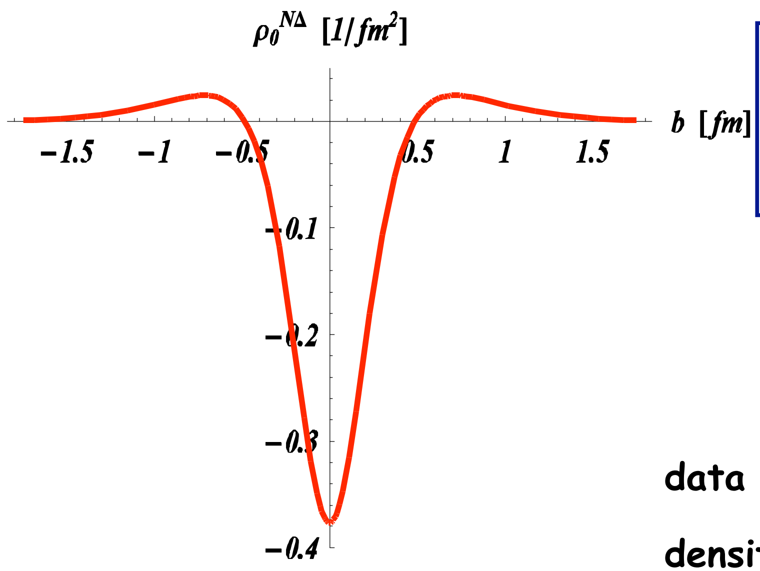
large N_c limit

Pascalutsa, Vdh (2006)

data :

MAMI, BATES, JLab/CLAS, JLab/Hall A





empirical transverse transition densities for N -> Δ excitation

$$\langle P^+, \frac{\vec{q}_\perp}{2}, \lambda_\Delta | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda_N \rangle$$
$$= (2P^+) e^{i(\lambda_N - \lambda_\Delta)\phi_q} \, G^+_{\lambda_\Delta \, \lambda_N}(Q^2)$$

$$\rho_0^{N\Delta}(b) = \int_0^\infty \frac{dQ}{2\pi} Q \, J_0(b\, Q) \, G^+_{+\frac{1}{2}\,+\frac{1}{2}}(Q^2)$$

combination of **M1**, **E2**, **C2** FFs

data : MAID 2007 Drechsel, Kamalov, Tiator (2007)

densities : Carlson, Vdh (2007)

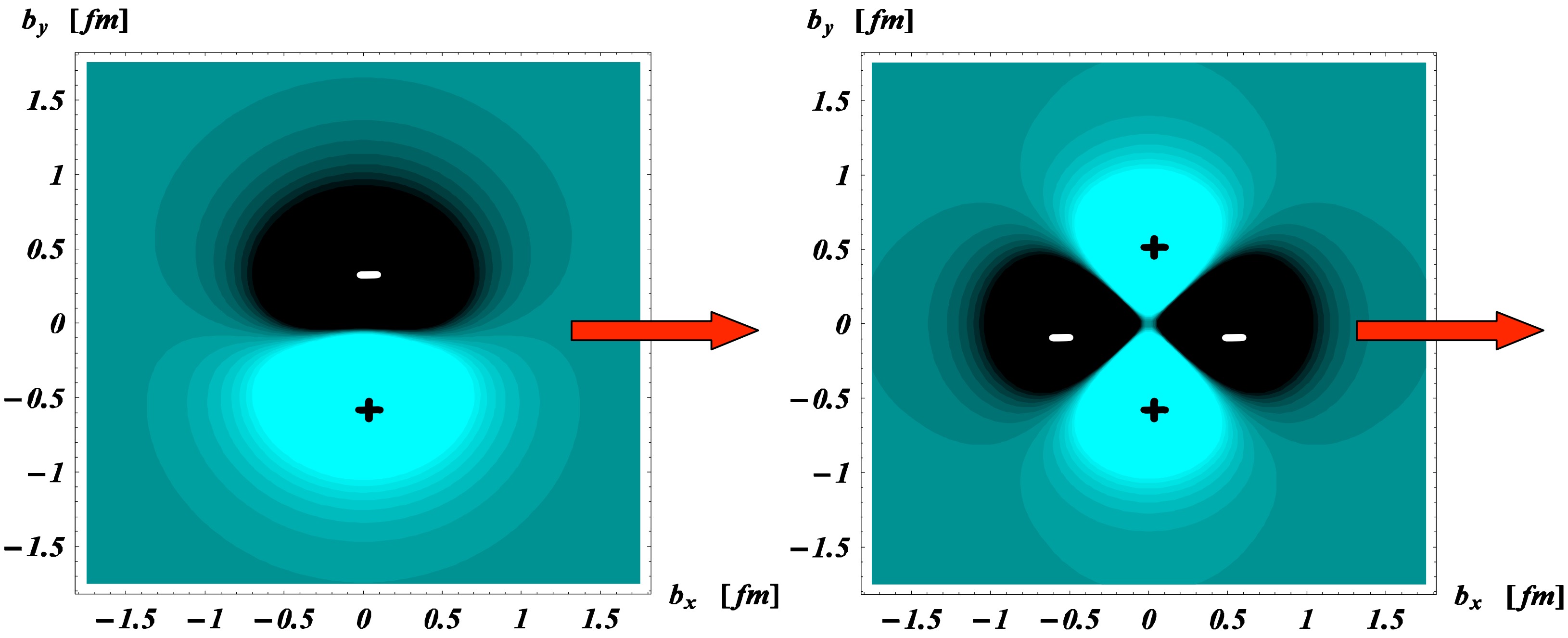
N -> Δ(1232) transition densities in transverse spin state

$$\rho_T^{N\Delta}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\,\vec{q}_\perp\cdot\vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp^\Delta = +\frac{1}{2} \,|\, J^+(0) \,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp^N = +\frac{1}{2}\rangle$$

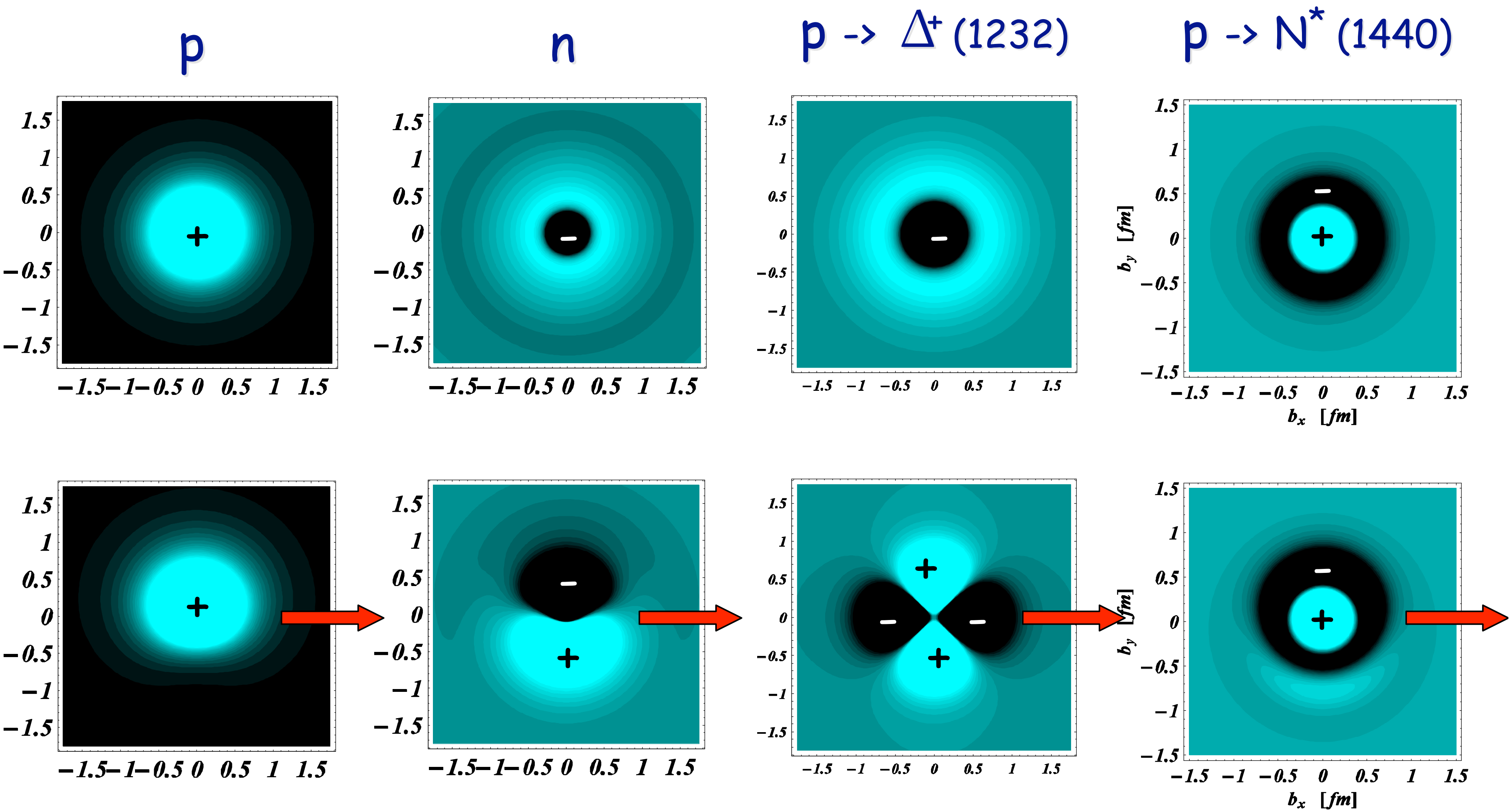
$$= \int_0^\infty \frac{dQ}{2\pi} \frac{Q}{2} \left\{ J_0(b\,Q)\, G^+_{+\frac{1}{2}\,+\frac{1}{2}} \right. \longrightarrow \text{monopole}$$

$$- \sin(\phi_b - \phi_S)\, J_1(b\,Q) \left[\sqrt{3} G^+_{+\frac{3}{2}\,+\frac{1}{2}} + G^+_{+\frac{1}{2}\,-\frac{1}{2}} \right] \longrightarrow \text{dipole}$$

$$\left. - \cos 2(\phi_b - \phi_S)\, J_2(b\,Q) \sqrt{3}\, G^+_{+\frac{3}{2}\,-\frac{1}{2}} \right\} \longrightarrow \text{quadrupole}$$



empirical transverse transition densities



Carlson, Vdh (2007)

quadrupole pattern

Tiator, Vdh (2008)

multipole extractions
at larger Q^2 :

reaction mechanism
de/pre-scriptions

Multipole truncation method

- Unpolarized e N -> e π N cross section

$$\frac{d\sigma^{\gamma^*}}{d\Omega_\pi^*} = A(\cos\theta^*) + \epsilon B(\cos\theta^*)\cos 2\phi^* + \sqrt{2\epsilon(1+\epsilon)}C(\cos\theta^*)\cos\phi^*$$

- Truncate the partial wave expansion

(over) simplified picture in Δ(1232) region :

Make fit to angular dependence by keeping only s- and p-waves (πN system)

$$A(\cos\theta^*) \equiv A_0 + A_1\cos\theta^* + A_2\cos^2\theta^*$$
$$B(\cos\theta^*) \equiv B_0\sin^2\theta^*$$
$$C(\cos\theta^*) \equiv (C_0 + C_1\cos\theta^*)\sin\theta^*$$

A_0, A_1, A_2, B_0, C_0, C_1

only functions of Q^2 and W

- under M1 dominance assumption :

$$A_0 = \frac{2W|\mathbf{k}_\pi^*|}{W^2 - m_p^2}|M_{1+}|^2\left[\frac{5}{2} - 3\frac{\Re(E_{1+}^* M_{1+})}{|M_{1+}|^2} + \frac{\Re(M_{1+}^* M_{1-})}{|M_{1+}|^2}\right]$$

$$A_1 = \frac{2W|\mathbf{k}_\pi^*|}{W^2 - m_p^2}|M_{1+}|^2 2\frac{\Re(E_{0+}^* M_{1+})}{|M_{1+}|^2}$$

$$A_2 = \frac{2W|\mathbf{k}_\pi^*|}{W^2 - m_p^2}|M_{1+}|^2\left[-\frac{3}{2} + 9\frac{\Re(E_{1+}^* M_{1+})}{|M_{1+}|^2} - 3\frac{\Re(M_{1-}^* M_{1+})}{|M_{1+}|^2}\right]$$

$$B_0 = \frac{2W|\mathbf{k}_\pi^*|}{W^2 - m_p^2}|M_{1+}|^2\left[-\frac{3}{2} - 3\frac{\Re(E_{1+}^* M_{1+})}{|M_{1+}|^2} - 3\frac{\Re(M_{1-}^* M_{1+})}{|M_{1+}|^2}\right]$$

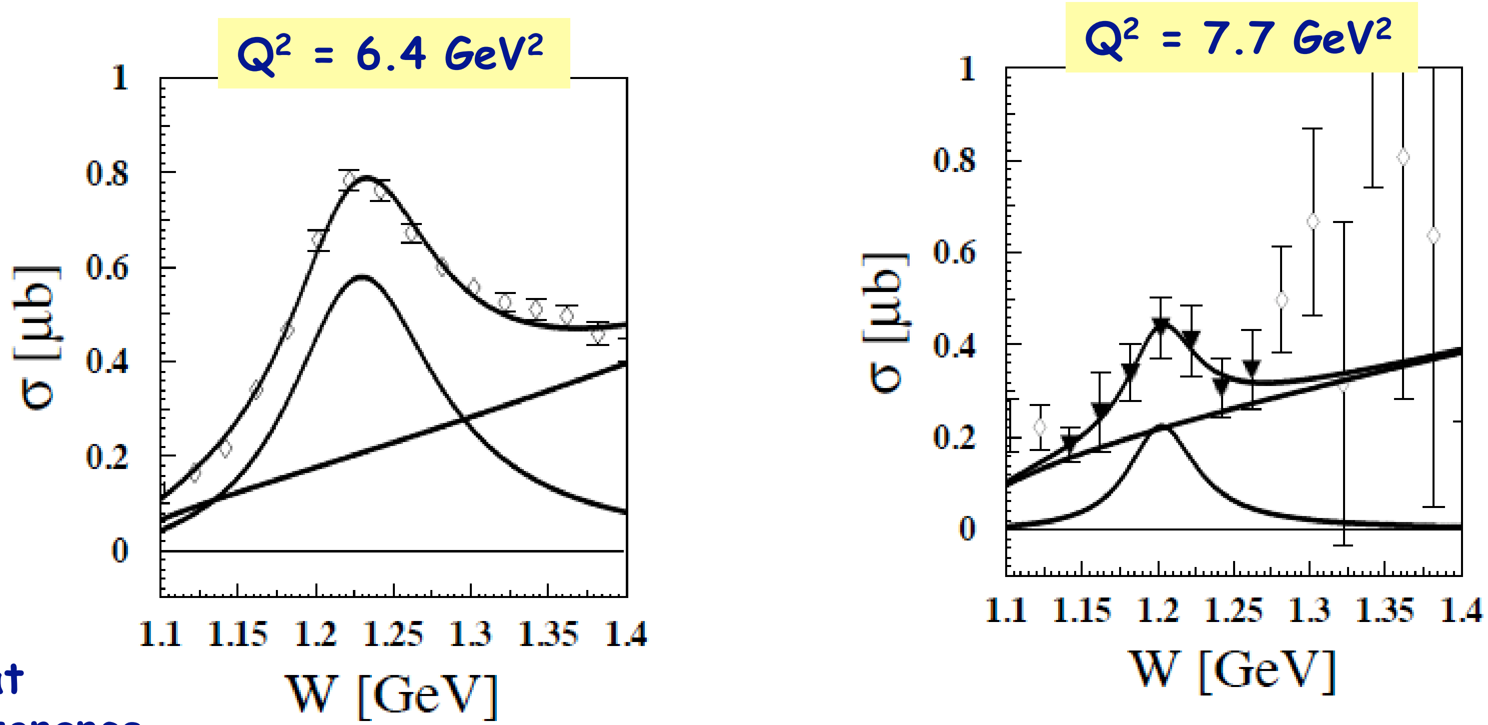
$$C_0 = \frac{2W|\mathbf{k}_\pi^*|}{W^2 - m_p^2}|M_{1+}|^2\sqrt{\frac{2Q^2}{|\mathbf{q}^*|^2}}\frac{\Re(S_{0+}^* M_{1+})}{|M_{1+}|^2}$$

$$C_1 = \frac{2W|\mathbf{k}_\pi^*|}{W^2 - m_p^2}|M_{1+}|^2 6\sqrt{\frac{2Q^2}{|\mathbf{q}^*|^2}}\frac{\Re(S_{1+}^* M_{1+})}{|M_{1+}|^2}$$

Resonance/background separation at large Q^2

CLAS data : Villano et al.(2009)

Fit with resonance + (polynomial) background



at resonance position : Backg/Res ≈ 50 % Backg/Res ≈ 100 %

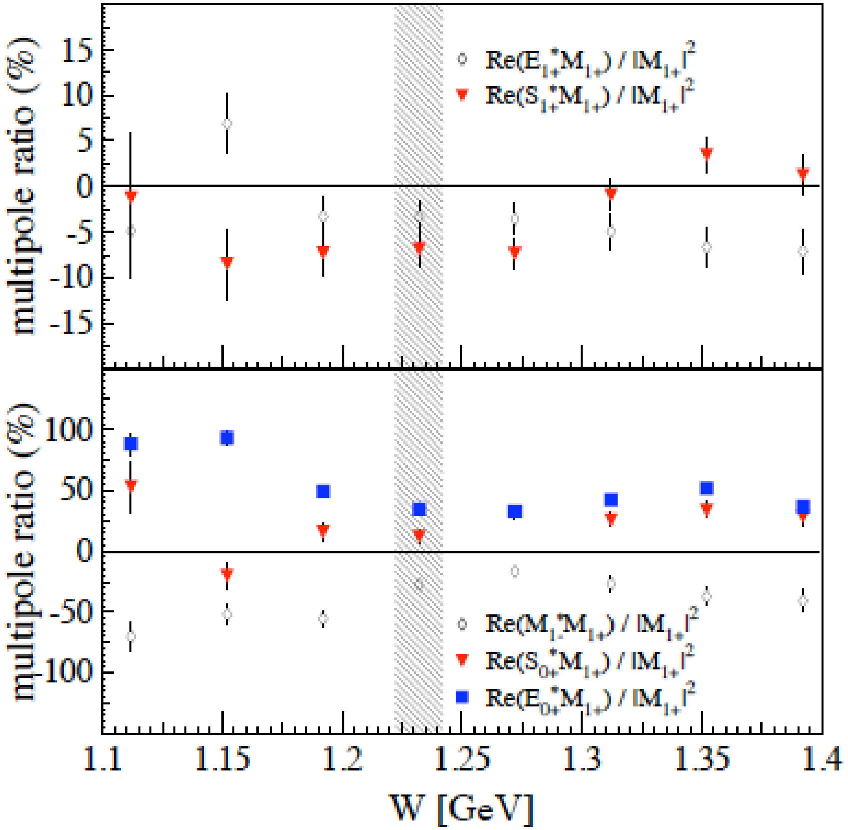
⇒ Δ(1232) drops faster with Q^2 than background and other resonances

Multipole truncation method at large Q^2

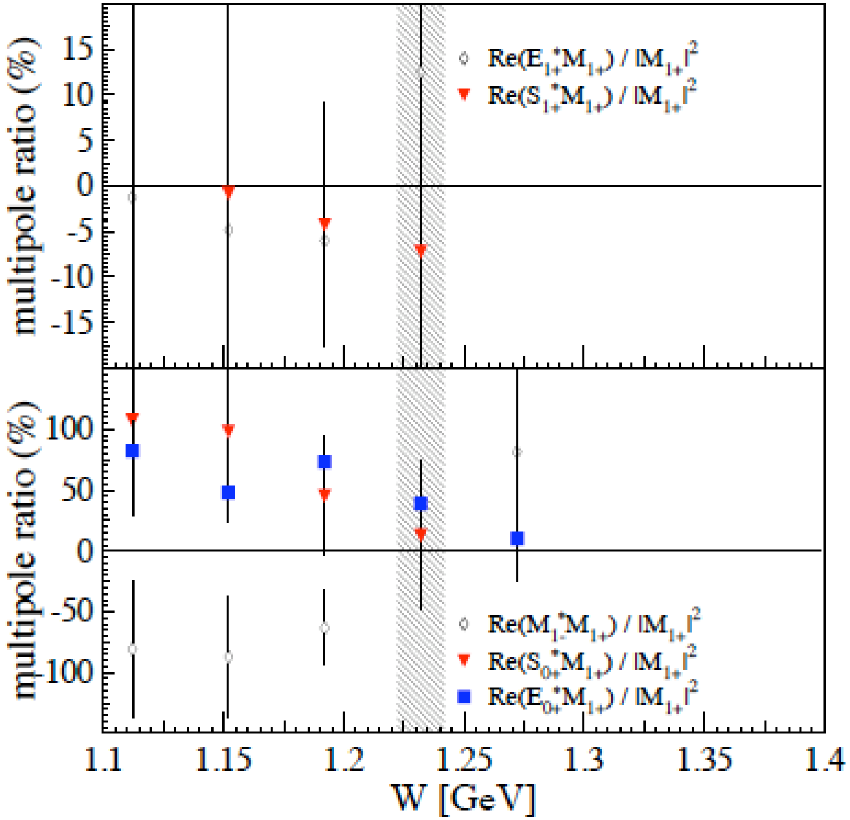
CLAS data

Villano et al.(2009)

Q^2 = 6.4 GeV^2



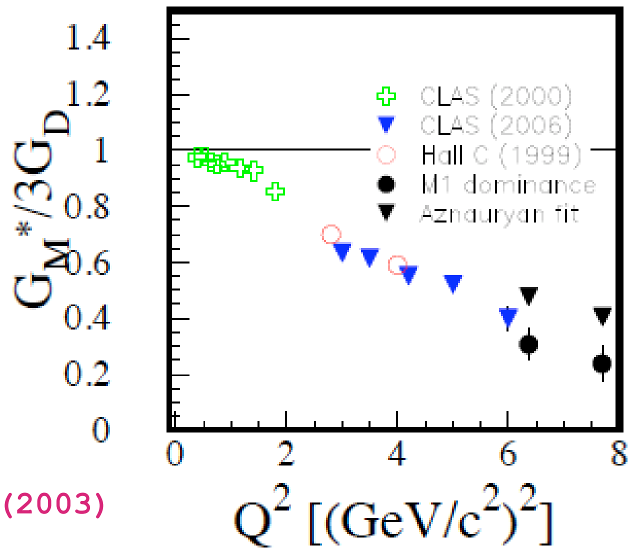
Q^2 = 7.7 GeV^2



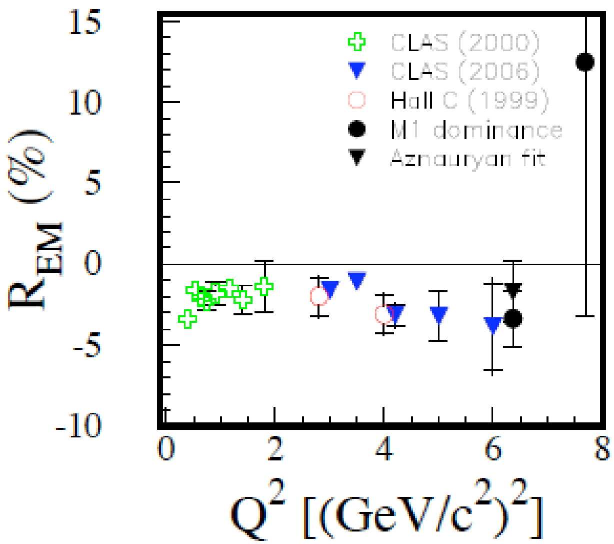
M1 dominance is not a good assumption any more at large Q^2

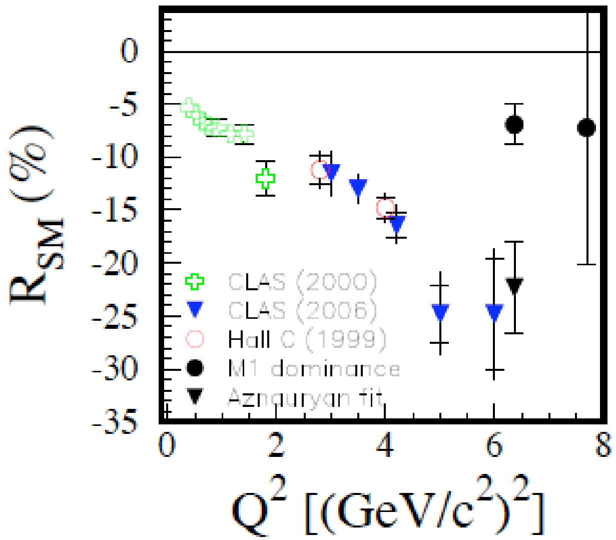
Extraction using Unitary Isobar Model (I)

- **Model background**
  - -> using Born terms (+ FFs)
  - -> VM (ρ, ω) (reggeized) exchange
- **πN unitarity implemented**
  - background & resonances separately unitarized

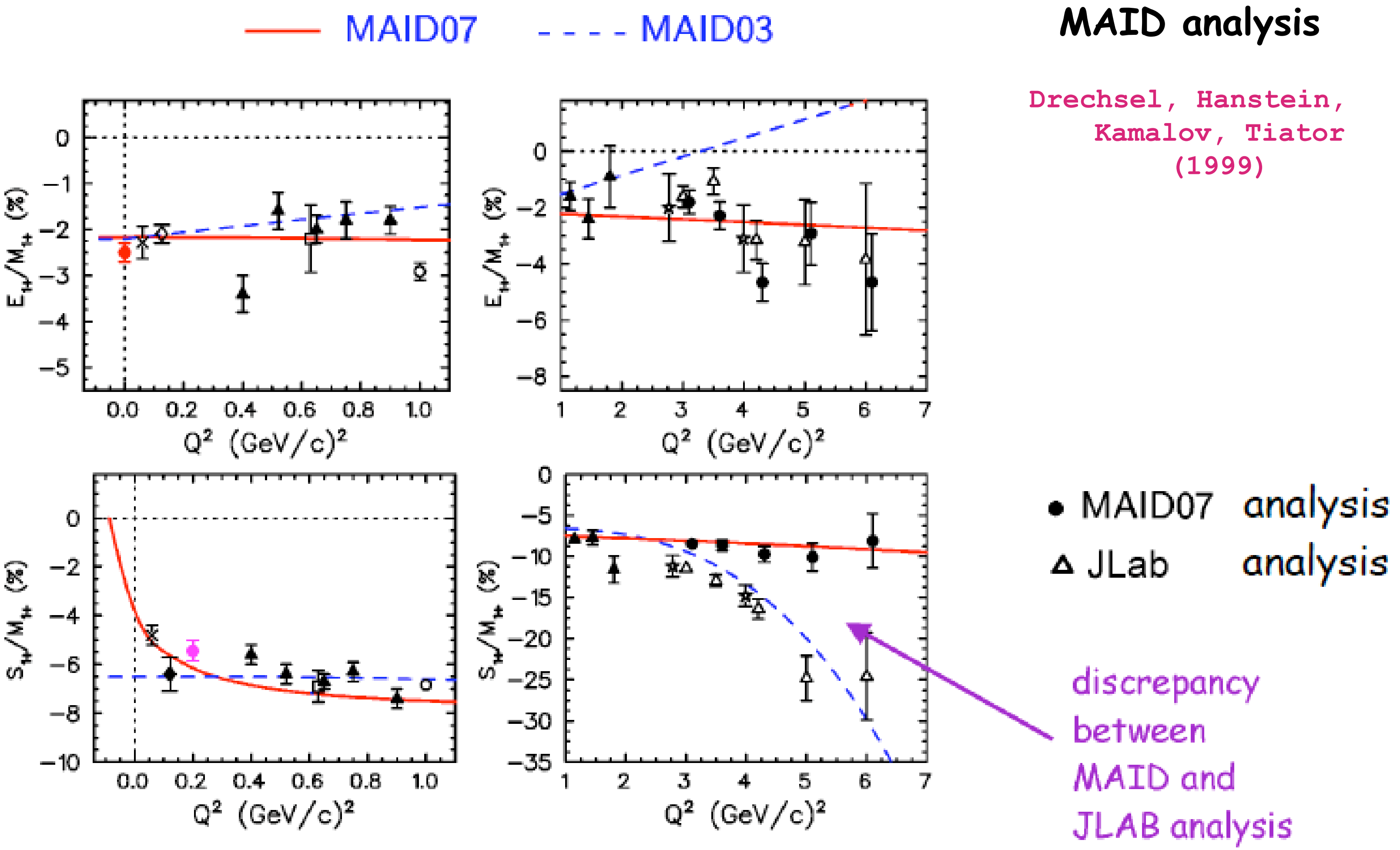


Aznauryan (2003)





Extraction using Unitary Isobar Model (II)



MAID analysis

Drechsel, Hanstein, Kamalov, Tiator (1999)

the analyses are based on π^0 data from JLab, Mainz, Bonn and Bates

Comparison with Dynamical Models (I)

Unitarity implemented solving 3dim Lippmann-Schwinger eq. (for multipole α)

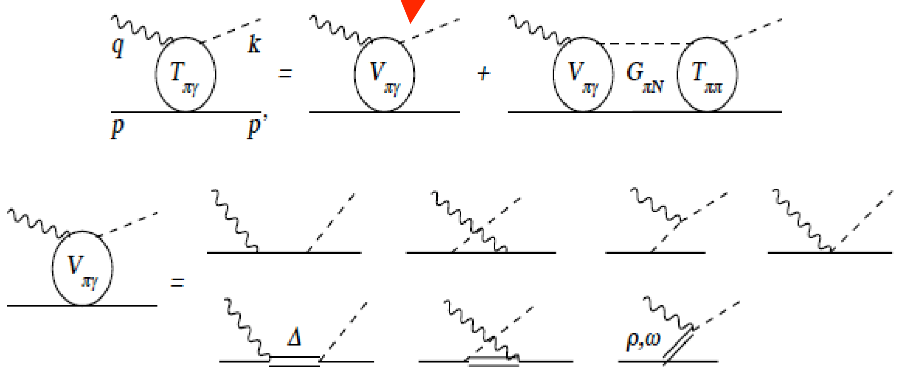
πN phase shift

$$t^{\alpha}_{\gamma\pi}(k_E, q; E + i\varepsilon) = e^{i\delta_\alpha}\cos\delta_\alpha \times \left[v^{\alpha}_{\gamma\pi}(k_E, q) + P\int_0^\infty dk' \frac{k'^2 R^{\alpha}_{\pi N}(k_E, k')\, v^{\alpha}_{\gamma\pi}(k', q)}{E - E_{\pi N}(k')} \right]$$

π on-shell momentum corr. with energy E

πN reaction matrix

solve integral eq. with meson exchange potential as driving term



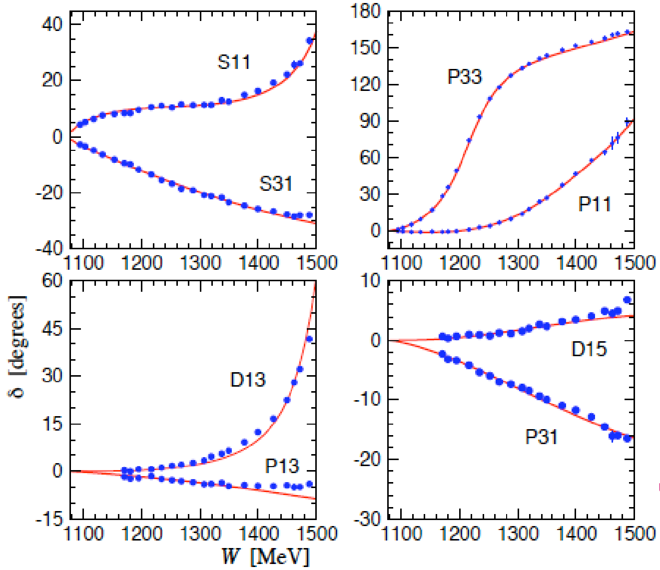


illustration : Utrecht-Ohio model

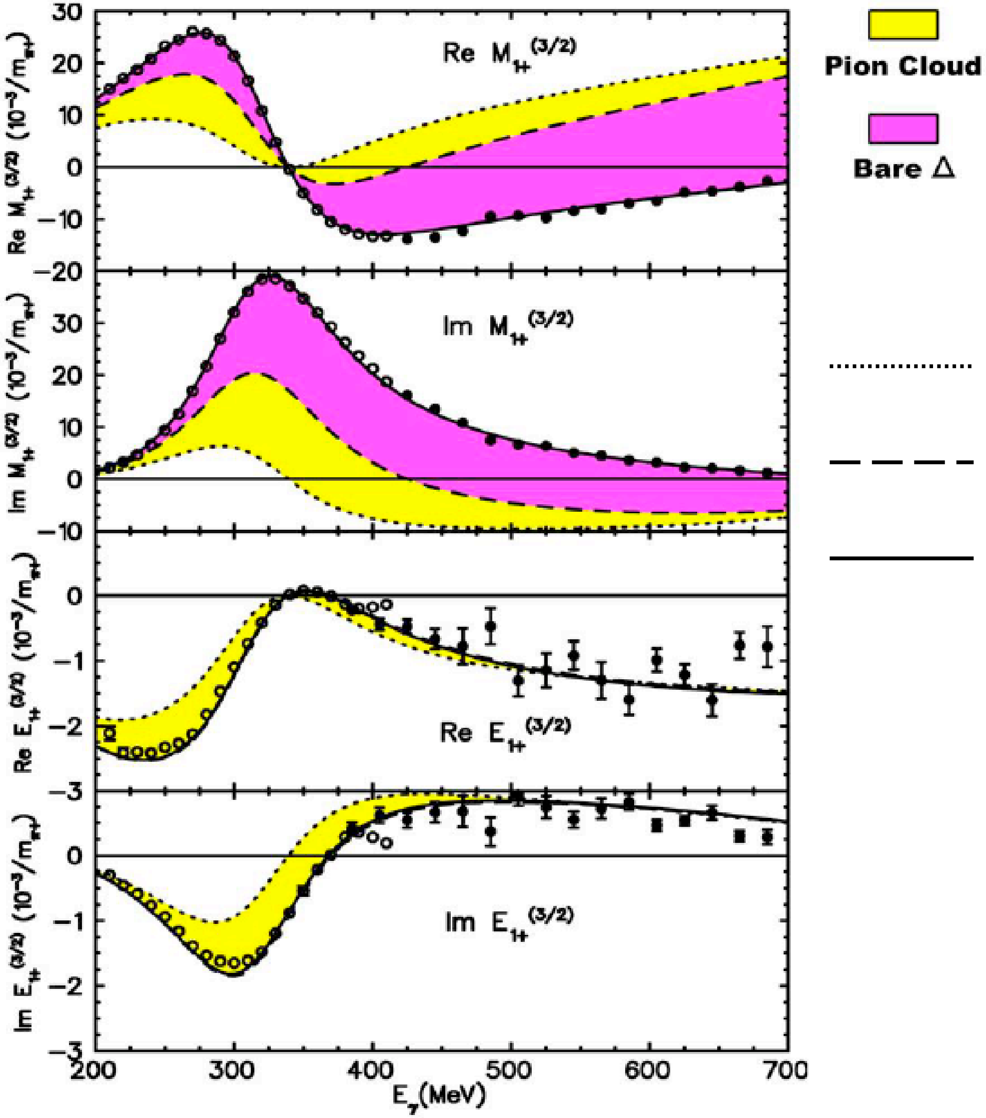
Pascalutsa, Tjon (2004)

gauge invariance implemented by

$$J^\mu \to J'^\mu(Q^2) = J^\mu + [F(Q^2) - 1]\, O^{\mu\nu} J_\nu$$

$$O^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu / q^2$$

Comparison with Dynamical Models (II)



Resonant multipoles for $\gamma N \to \pi N$

Background without rescattering (P.V.)

Background + rescattering (P.V. term)

previous + bare Δ

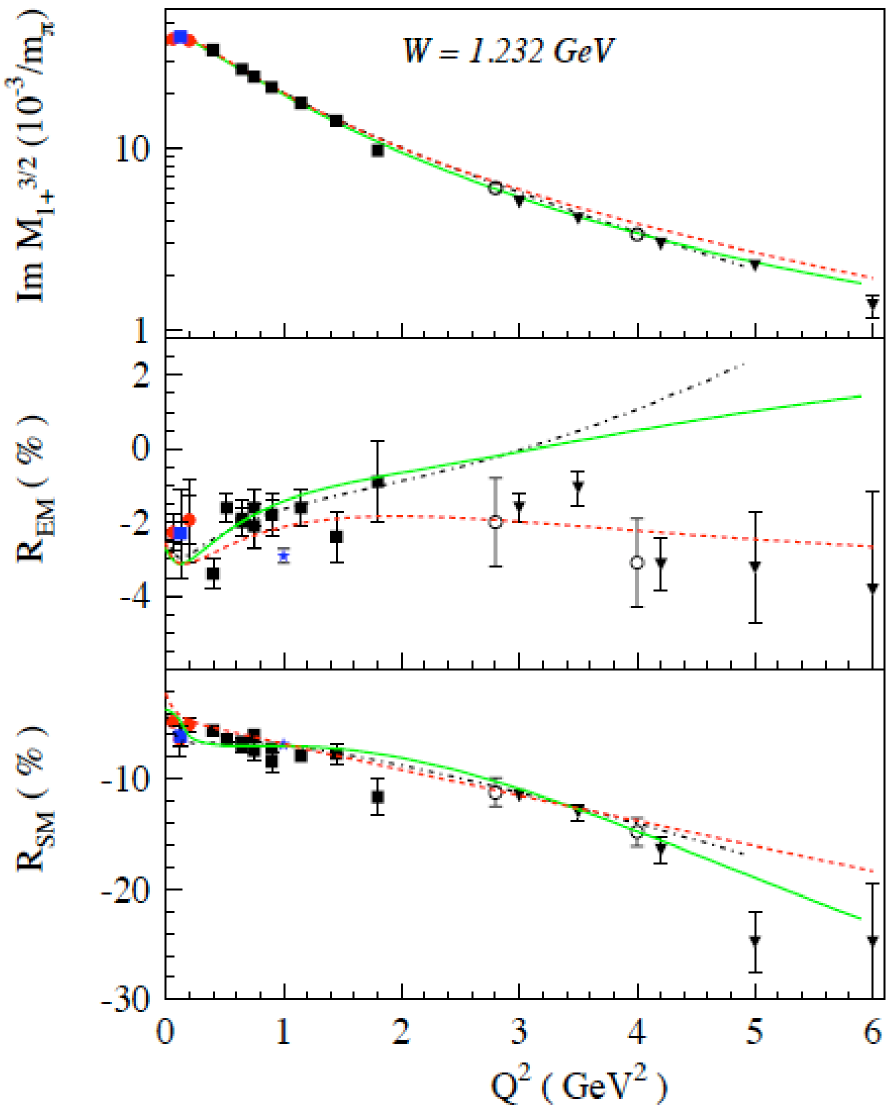
illustration :

Dubna-Mainz-Taipei (DMT)

Kamalov, Yang (1999)

Kamalov, Yang, Drechsel, Hanstein, Tiator (2001)

Comparison with Dynamical Models (III)



Sato-Lee (SL)

Sato, Lee(1996, 2001)

Dubna-Mainz-Taipei (DMT)

Kamalov, Yang (1999)

Kamalov, Yang, Drechsel, Hanstein, Tiator (2001)

Dynamical Utrecht-Ohio model (DUO)

Pascalutsa, Tjon (2004)

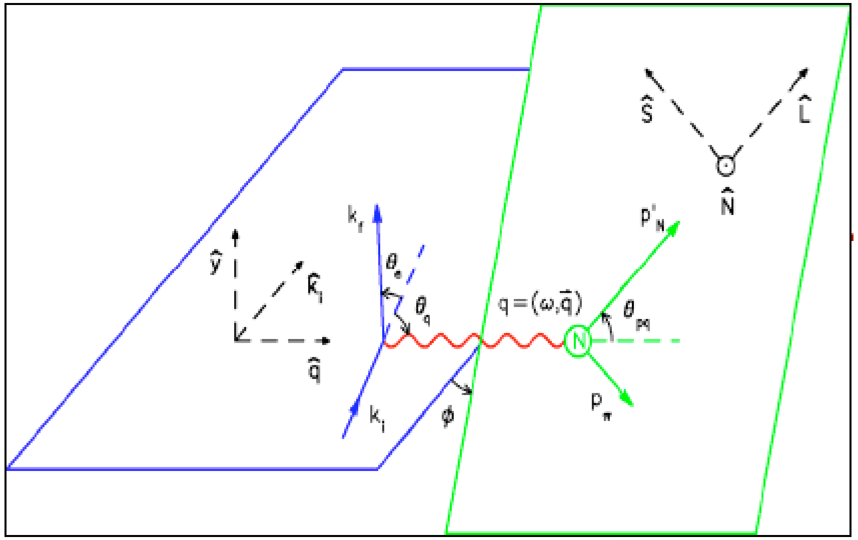
Caia, Wright, Pascalutsa (2004)

testing the model dependence at larger Q^2 :

role of polarization observables

Complete experiment

for e p -> e π^0 p



described by

18 response functions

$$\frac{d^5\sigma}{d\varepsilon_f d\Omega_e d\Omega_{cm}} = \frac{p_{cm}}{k_{\gamma cm}}\Gamma_\gamma\,\bar\sigma_0\left[1 + hA + \boldsymbol{\mathcal{S}}\cdot(\boldsymbol{P} + h\boldsymbol{P}')\right]$$

$$\bar\sigma_0 = \nu_L R_L + \nu_T R_T + \nu_{LT} R_{LT}\cos\phi + \nu_{TT} R_{TT}\cos 2\phi$$

$$A\bar\sigma_0 = \nu'_{LT} R'_{LT}\sin\phi$$

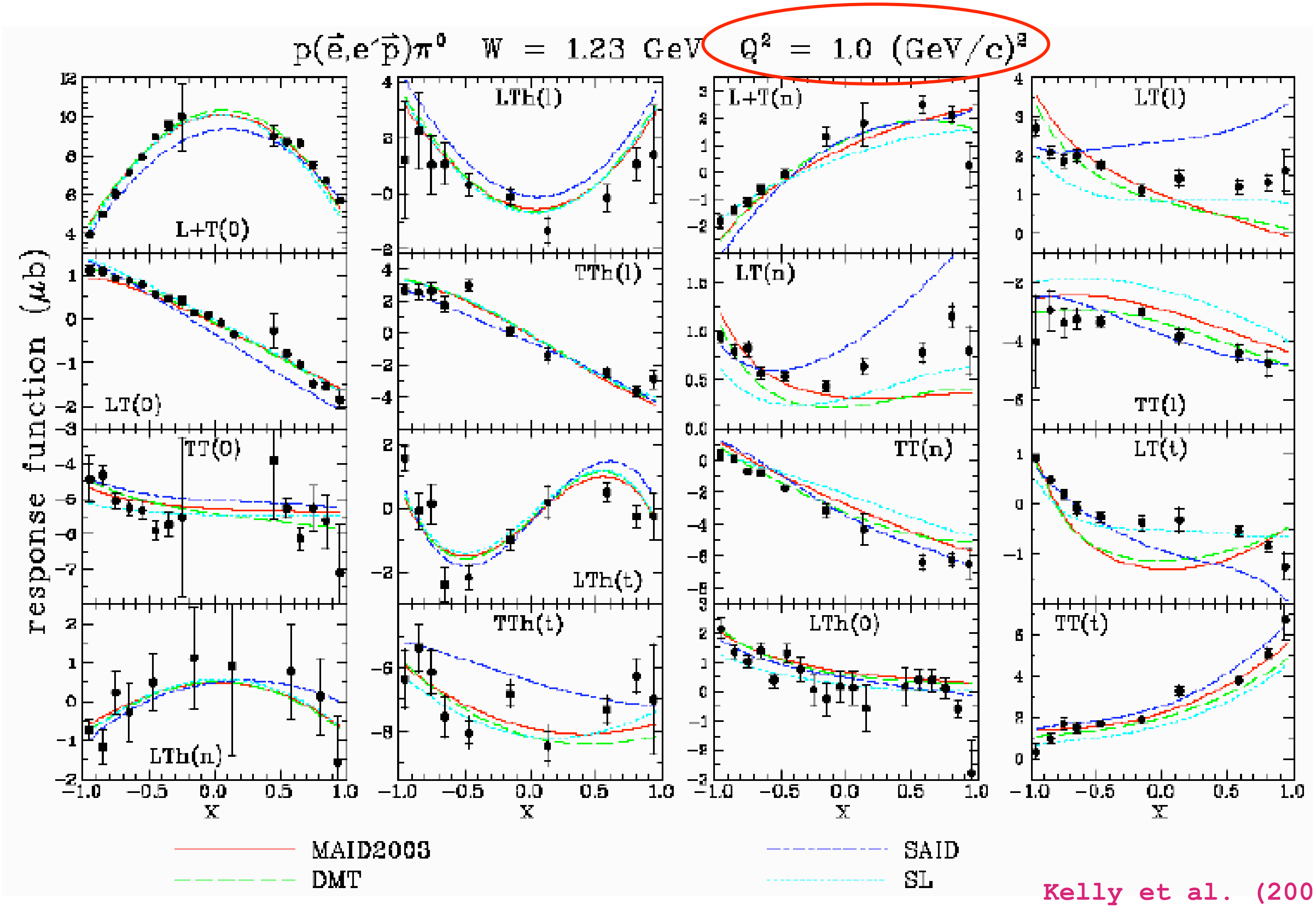
$$P_N\bar\sigma_0 = \left[\nu_L R_L^N + \nu_T R_T^N + \nu_{LT} R_{LT}^N\cos\phi + \nu_{TT} R_{TT}^N\cos 2\phi\right]$$

$$P_m\bar\sigma_0 = \left[\nu_{LT} R_{LT}^m\sin\phi + \nu_{TT} R_{TT}^m\sin 2\phi\right] \qquad (m\in\{L,S\})$$

$$P'_N\bar\sigma_0 = \nu'_{LT} R'^N_{LT}\sin\phi$$

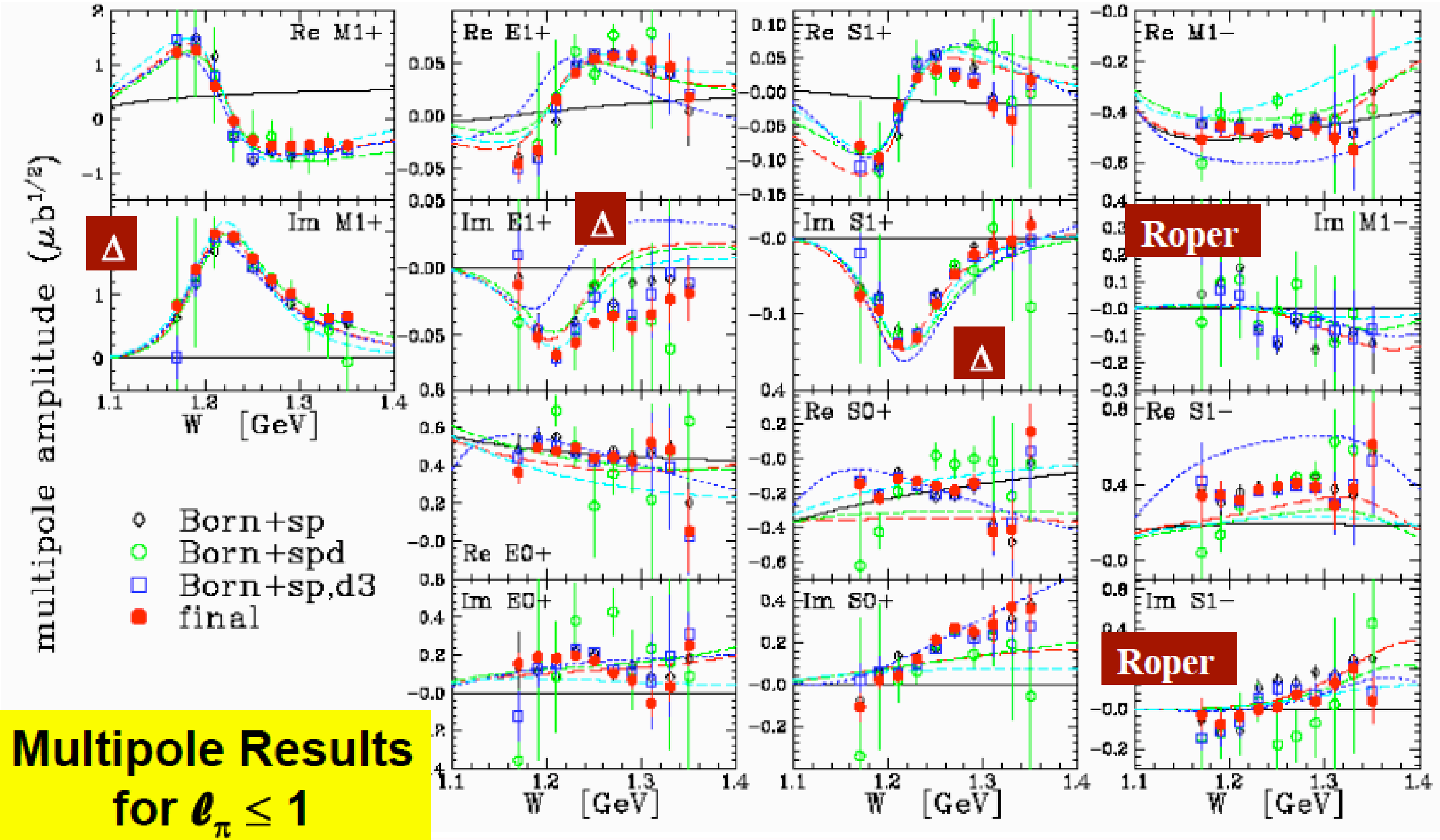
$$P'_m\bar\sigma_0 = \left[\nu'_{LT} R'^m_{LT}\cos\phi + \nu'_{TT} R'^m_{TT}\right] \qquad (m\in\{L,S\})$$

Tour de force : Jlab/Hall A E91011



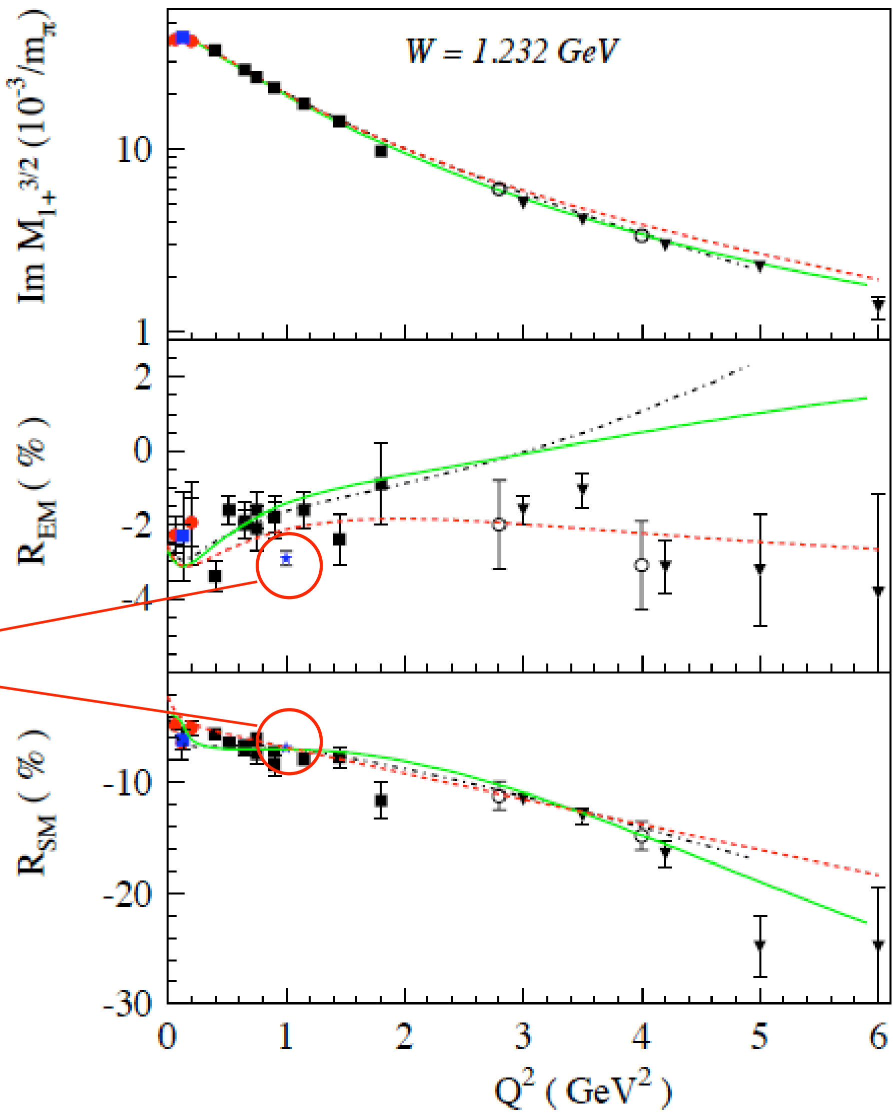
Kelly et al. (2005)

Extracted multipoles from fit to 14+2 response functions $p(\vec{e}, e'\vec{p})\pi^0$



Multipole Results for $\ell_\pi \leq 1$

Results for E2/M1 and C2/M1 ratios



Jlab/Hall A data

Kelly et al. (2005)

2γ-exchange effects at larger Q^2

e N -> e Δ beyond one-photon exchange

- The unpolarized pion electroproduction cross-section is, *in general*, written as:

$$\frac{d\sigma}{(dE_e'\, d\Omega_e')^{lab}\, d\Omega_\pi} \equiv \Gamma_v \frac{d\sigma}{d\Omega_\pi}\,, \qquad \text{Flux: } \Gamma_v = \frac{e^2}{(2\pi)^3}\left(\frac{E_e'}{E_e}\right)^{lab} \frac{(s_{\pi N} - M_N^2)/(2M_N)}{Q^2\,(1-\varepsilon)}.$$

$$\frac{d\sigma}{d\Omega_\pi} = \frac{d\sigma_0}{d\Omega_\pi} + \varepsilon\, \cos(2\Phi)\frac{d\sigma_{TT}}{d\Omega_\pi} + \sqrt{2\,\varepsilon\,(1+\varepsilon)}\, \cos\Phi\, \frac{d\sigma_{LT}}{d\Omega_\pi} + \varepsilon\, \sin(2\Phi)\frac{d\sigma_{TTi}}{d\Omega_\pi} + \sqrt{2\,\varepsilon\,(1-\varepsilon)}\, \sin\Phi\, \frac{d\sigma_{LTi}}{d\Omega_\pi}.$$

- at the Δ-resonance, these cross-sections are expressed in terms of 8 e N -> e Δ helicity amplitudes

$$\begin{aligned}
\frac{d\sigma}{d\Omega_\pi} &= \frac{1}{\pi}\frac{9\,Q^2\,(1-\varepsilon)}{16\,M_\Delta\,(M_\Delta^2 - M_N^2)\,\Gamma_\Delta} \\
&\times \left\{ \frac{1}{2}\sin^2\theta_\pi\, \left[\, |T_1|^2 + |T_2|^2 + |T_7|^2 + |T_8|^2 \,\right] + \frac{1}{6}(1 + 3\,\cos^2\theta_\pi)\, \left[\, |T_3|^2 + |T_4|^2 + |T_5|^2 + |T_6|^2 \,\right] \right. \\
&+ \cos\Phi\, \frac{1}{\sqrt{3}}\, \sin(2\theta_\pi)\, \Re\left[T_1\, T_3^* + T_2\, T_4^* - T_7\, T_5^* - T_8\, T_6^* \right] - \cos(2\Phi)\, \frac{1}{\sqrt{3}}\, \sin^2\theta_\pi\, \Re\left[T_1\, T_5^* + T_2\, T_6^* + T_7\, T_3^* + T_8\, T_4^* \right] \\
&+ \left. \sin\Phi\, \frac{1}{\sqrt{3}}\, \sin(2\theta_\pi)\, \Im\left[T_1\, T_3^* + T_2\, T_4^* + T_7\, T_5^* + T_8\, T_6^* \right] - \sin(2\Phi)\, \frac{1}{\sqrt{3}}\, \sin^2\theta_\pi\, \Im\left[T_1\, T_5^* + T_2\, T_6^* - T_7\, T_3^* - T_8\, T_4^* \right] \right\}.
\end{aligned}$$

analysis beyond one-photon exchange

$$\sigma_0 = A_0 + \frac{1}{2}(3\cos^2\theta_\pi - 1)\, A_2$$

$$\sigma_{TT} = \sin^2\theta_\pi\, C_0 \qquad \sigma_{LT} = \frac{1}{2}\sin(2\theta_\pi)\, D_1$$

$$\sigma_{TTi} = \sin^2\theta_\pi\, C_{0i} \qquad \sigma_{LTi} = \frac{1}{2}\sin(2\theta_\pi)\, D_{1i}$$

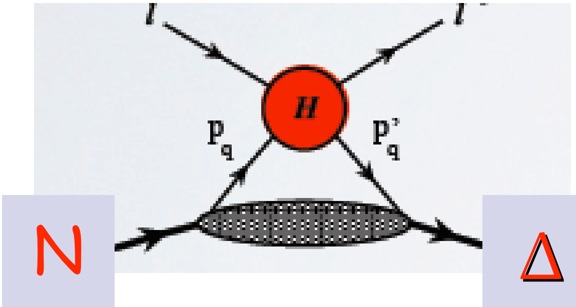
$$A_0 = \mathcal{I}\,\frac{e^2}{4\pi}\,\frac{Q_-^2}{4\,M_N^2}\,\frac{(M_\Delta + M_N)}{(M_\Delta - M_N)}\,\frac{1}{M_\Delta\,\Gamma_\Delta}(G_M^*)^2\,\sigma_R$$

$$R_{EM}^{exp,I} = \frac{3A_2 - 2C_0}{12A_0} \overset{1\gamma}{=} R_{EM} + \varepsilon\,\frac{4M_\Delta^2 Q^2}{Q_+^2\, Q_-^2} R_{SM}^2 + \ldots$$

$$R_{SM}^{exp} = \frac{Q_+\, Q_-}{Q\, M_\Delta}\,\frac{D_1}{6A_0} \overset{1\gamma}{=} R_{SM} - R_{SM} R_{EM} + \ldots$$

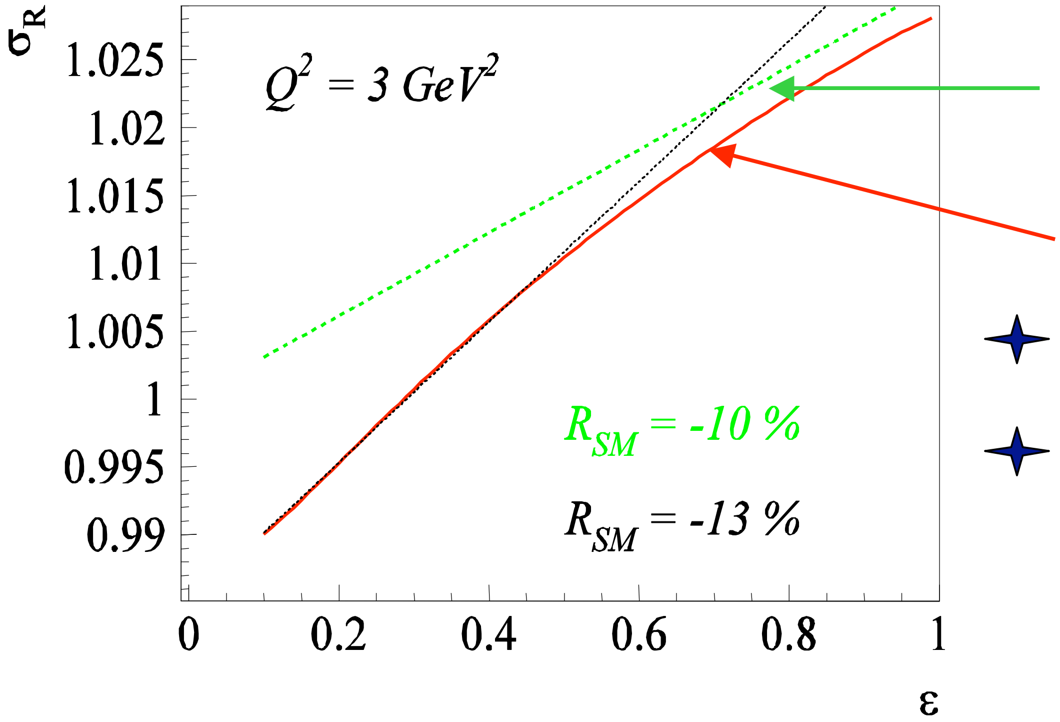
$$R_{EM}^{exp,II} = \frac{-(A_0 - A_2) - 2\,C_0}{3(A_0 - A_2) - 2\,C_0} \overset{1\gamma}{=} R_{EM}$$

Two-photon exchange in N -> Δ transition



- General formalism for eN -> e Δ has been worked out
- Model calculation for large Q^2 in terms of N -> Δ GPDs

Pascalutsa, Carlson, Vdh (2006)



- R_{EM} little affected < 1 %
- R_{SM} mainly affected when extracted through Rosenbluth method

Summary

- $\gamma N\Delta$ form factors :
  - -> large N_c : G^*_M : relation with F_2 , G^*_E and G^*_C related with G_{En}
  - -> large Q^2 : allows to compare scaling behavior for N with N -> Δ
  - -> 2D Fourier transform of γNN^* FFs in light-front frame : maps out transition charge densities

- multipole extractions at larger Q^2 :
  - -> sp-multipole truncation + M1 dominance does **not** work well for N -> Δ
  - -> models of reaction mechanism : unitary isobar, dynamical, DR (?), …
  - -> Complete experiments (incl. double pol.) can reduce the model dependence in the multipole extraction (e.g as shown by Jlab/Hall A expt.) practical : use as check at intermediate Q^2 values

- 2γ-exchange effects at larger Q^2
  - -> formalism developed for e N -> e Δ -> e π N, leads to new angular dependencies
  - -> $\gamma N\Delta$ transition : effect on R_{SM} when using Rosenbluth method
  - -> polarization observables allow to constrain the 2γ-effects further