Covariant quark-diquark model for the $N \to N^*$ electromagnetic transitions

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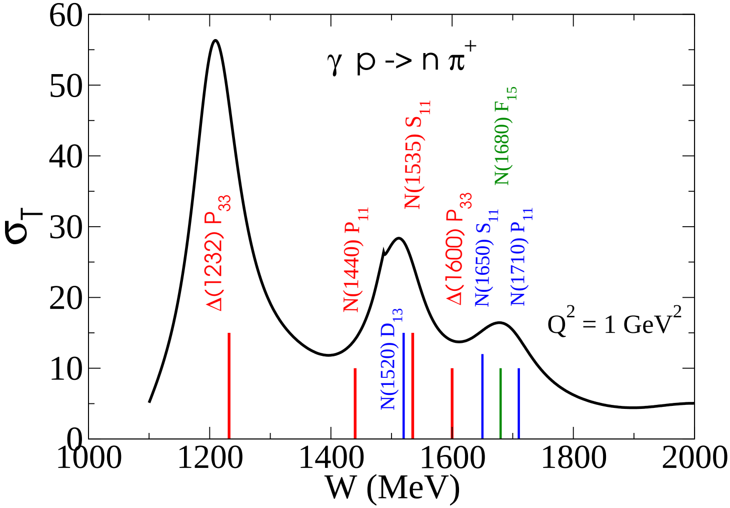
Nucleon Resonance Structure in Exclusive Electroproduction at High Photon Virtualities with the CLAS 12 Detector Workshop

Jlab, Newport News, VA, USA

May 16, 2011

Nucleon Resonance Structure



Frameworks to study $\gamma N \to N^*$ at high Q^2

Study the N^* electroproduction

- (Constituent) Quark Models ... $\surd$
- Coupled-channels reaction models (Dynamical models) $\surd$
  baryon bare core structure (input)
  with meson dressing (meson-baryon interaction)
  [EBAC, Sato-Lee, Mainz (DMT), Julich, Bonn, ...]
- χ-Perturbation Theory, χEFT ✕
  Baryons and pions as d.o.f. - low Q^2 regime
  [Pascalutsa, Vanderhaghen, Gail, Hermert, ...]
- pQCD ... very high Q^2 ✕
- Hybrid models (CBM, soliton, ...) $\surd$

Motivation and goals [$\gamma N \to N^*$]

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    Nucleon resonances (baryons): internal structure ruled by QCD
    Internal degrees of freedom: (light) quarks and gluons
  - Experiments:
    We detect baryons and mesons
    possible decays: πN, ηN, ρN, $\pi\Delta$, ...
    Effective degrees of freedom: mesons $\oplus$ resonant core (N^*)

Framework for high Q^2: Spectator quark model

Covariant Spectator Quark Model© - Franz Gross (CST)

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    Simple way of describing N^* quantum numbers:
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- Ingredients:
  - Wave functions (qqq)
  - Quark current [constituent quarks e.m. form factors]:
    [**dressing** by gluons interactions and some quark-antiquark states]

Program to study $\gamma N \to N^*$ reactions

Goal:
Study Valence Quark content of N^* structure

$N^* = N(939), N(1440), N(1535), \Delta(1232), \Delta(1600), \ ...$

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    $\Rightarrow$ Callibration of the model [Quark current & wave functions]

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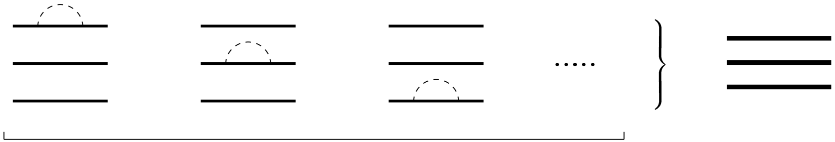
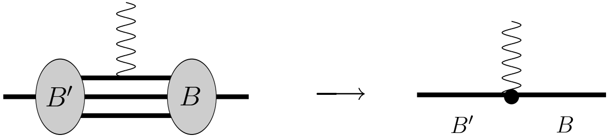
- Output:
  - Valence quark contributions for the $\gamma N \to N^*$ form factors
    Dominant at high Q^2
  - Using complementar information: estimate of meson cloud
    [Low Q^2 data; large-N_c relations for meson cloud, ...]

Quark structure and electromagnetic interaction (I)



γ coupling:



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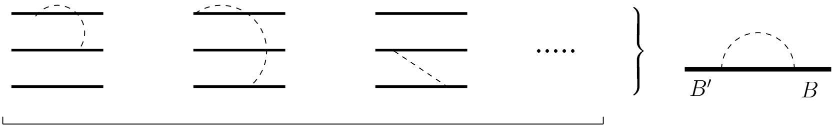

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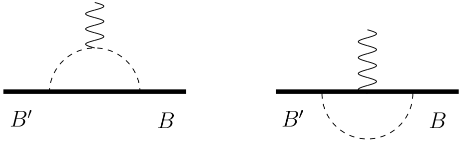

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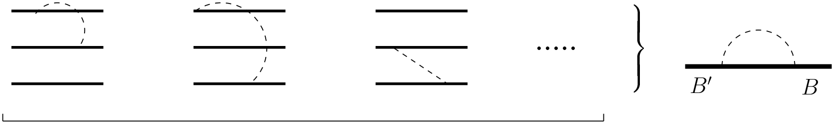

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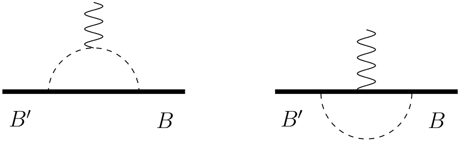

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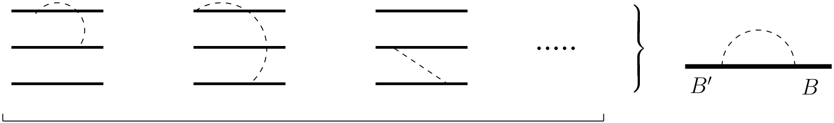

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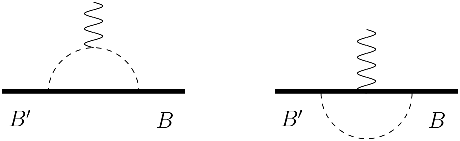
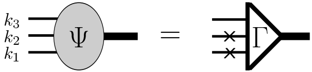


- Not important at high Q^2 [pQCD: supression $1/Q^4$]
- Assume NO interference with quark dressing processes

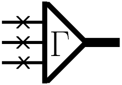
$$G_X = G_X^B + G_X^{mc}$$

- Baryon: 3 constituent quark system
- Covariant Spectator Theory©: wave function Ψ defined in terms of a 3-quark vertex Γ with 2 on-mass-shell quarks



$$\Psi_\alpha(P,k_3) = \left(\frac{1}{m_q - \not{k}_3 - i\varepsilon} \right)_{\alpha\beta} \Gamma^\beta(P,k_1,k_2)$$

- Confinement insures that vertex Γ vanishes when the 3 quarks are on-shell [Γ cancels the quark propagator singularity]



$$= 0$$

Stadler, Gross and Frank PRC 56, 2396 (1998); Savkli and Gross PRC 63, 035208 (2001)

- Ψ **free** of singularities
  Instead of modulate Γ $\Rightarrow$ modulate directly Ψ

- Integrating over the on-mass-shell quark momenta: $k = k_1 + k_2$, $r = \frac{1}{2}(k_1 - k_2)$; reduce current integrals to the integration in $\mathbf{k}$ and $s = (k_1 + k_2)^2$ Gross and Agbakpe, PRC 73, 015203 (2006); PRC 77, 015202 (2008):

$$\int \frac{d^3 k_1}{2E_{k_1}} \int \frac{d^3 k_2}{2E_{k_2}} = \frac{\pi}{4} \int d\Omega_{\hat{\mathbf{r}}} \int_{4m_q^2}^{+\infty} ds \sqrt{\frac{s - 4m_q^2}{s}} \int \frac{d^3 \mathbf{k}}{2E_k}$$

with $E_k = \sqrt{s + \mathbf{k}^2}$ as the energy of the diquark.

- Mean value theorem: average in diquark mass $\sqrt{s} \to m_D$

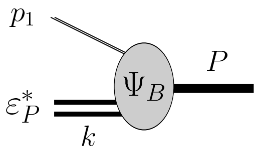
$$\int \frac{d^3 k_1}{2E_{k_1}} \int \frac{d^3 k_2}{2E_{k_2}} \to \int \frac{d^3 \mathbf{k}}{2\sqrt{m_D^2 + \mathbf{k}^2}}$$

m_D=eff. mass; covariant integration in diquark **on-shell** momentum

Spectator QM: Baryon wave functions (III)

- Baryon wave functions: $B =$ **diquark** $\oplus$ **quark**
  Combination of **diquark** (12) and single **quark** (3) states,
  using $SU(6) \otimes O(3)$:

$$\Psi_B = \sum \text{(flavor)} \otimes \text{(spin)} \otimes \text{(orbital)} \otimes \underbrace{\psi_B(P,k)}_{\text{radial}}$$



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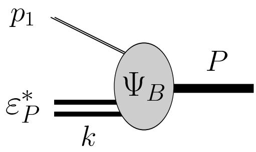

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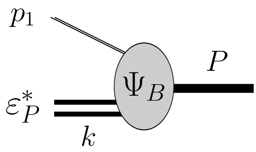

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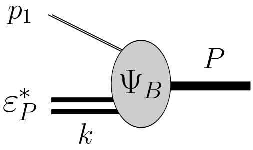
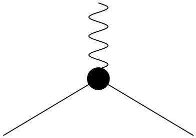


  - Ψ_B in **rest frame** using quark states
  - **Covariant** generalization of Ψ_B in terms **baryon** properties
  - Ψ_B can be used on **any** frame and/or Q^2 regime

Spectator QM: Photon-Quark coupling

- Quark current [$f_{i\pm}$ quark form factors]

$$
\begin{aligned}
j_I^\mu &= \left[\frac{1}{6} f_{1+} + \frac{1}{2} f_{1-} \tau_3\right] \left(\gamma^\mu - \frac{\not q q^\mu}{q^2}\right) + \\
&\quad \left[\frac{1}{6} f_{2+} + \frac{1}{2} f_{2-} \tau_3\right] \frac{i\sigma^{\mu\nu} q_\nu}{2M_N}
\end{aligned}
$$



Quarks with anomalous magnetic moments κ_u, κ_d
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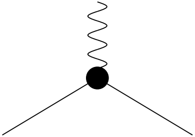
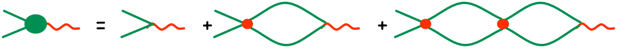


Quarks with anomalous magnetic moments κ_u, κ_d fixed by nucleon magnetic moments: μ_p, μ_n

- Vector meson dominance parameterization:



2 poles:

- Light vector meson: $m_v = m_\rho$
- Effective heavy meson: M_h (=$2M_N$):

4 adjustable coefficients ← Nucleon data

Spectator QM: Transition currents ($\gamma N \to N^*$)

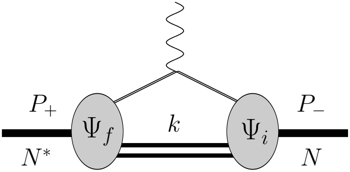
Quark current $j_I^\mu \bigoplus$ Baryon wave function $\Psi_B \Rightarrow J^\mu$

Transition current J^μ in spectator formalism

Franz Gross et al PR 186 (1969); PRC 45, 2094 (1992)

Relativistic impulse approximation:

$$J^\mu = 3 \sum_\lambda \int_k \bar{\Psi}_f(P_+, k) j_I^\mu \Psi_i(P_-, k)$$



diquark on-shell

$$q = P_+ - P_-, \qquad P = \tfrac{1}{2}(P_+ + P_-), \qquad Q^2 = -q^2$$

Spin 1/2 resonances: transition currents

Nucleon:

$$J^\mu = \bar{u}(P_+) \left[F_1 \gamma^\mu + F_2 \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} \right] u(P_-)$$

$\gamma N \to N(1440)$ (R):

$$J^\mu = \bar{u}_R(P_+) \left[F_1^* \left(\gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) + F_2^* \frac{i\sigma^{\mu\nu} q_\nu}{M_R + M_N} \right] u(P_-)$$

$\gamma N \to N(1535)$ (S):

$$J^\mu = \bar{u}_S(P_+) \left[F_1^* \left(\gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) + F_2^* \frac{i\sigma^{\mu\nu} q_\nu}{M_S + M_N} \right] \gamma_5 u(P_-)$$

Form factors - exclusive functions of Q^2

Spin 1/2 resonances: wave functions

Nucleon: S-state approximation (quark-diquark)

$$\Psi_N(P,k) = \frac{1}{\sqrt{2}}\left[\Phi_I^0\Phi_S^0 + \Phi_I^1\Phi_S^1\right]\psi_N(P,k)$$

$N(1440)$:

$$\Psi_R(P,k) = \frac{1}{\sqrt{2}}\left[\Phi_I^0\Phi_S^0 + \Phi_I^1\Phi_S^1\right]\psi_R(P,k)$$

$N(1535)$:

$$\Psi_{S11}(P,k) = \frac{1}{\sqrt{2}}\left[\Phi_I^0 X_\rho - \Phi_I^1 X_\lambda\right]\psi_{S11}(P,k)$$

$\Phi_I^{0,1}$: isospin states; $\Phi_S^{0,1}, X_\rho, X_\lambda$: spin states

Scalar wave function: Nucleon

Scalar wave functions deppendent of $(P-k)^2$= (quark momentum)^2

$$\chi_B = \frac{(M_B - m_D)^2 - (P-k)^2}{M_B m_D} \xrightarrow{NR} \frac{\mathbf{k}^2}{m_D^2}$$

M_B = baryon mass; m_D = diquark mass

Nucleon scalar wave function:

$$\psi_N(P,k) = \frac{N_0}{m_D}\frac{1}{(\beta_1+\chi_N)(\beta_2+\chi_N)} = \frac{N_0}{m_D}\frac{1}{\beta_2-\beta_1}\left[\frac{1}{\beta_1+\chi_N} - \frac{1}{\beta_2+\chi_N}\right]$$

$$\xrightarrow{NR} \frac{N_0}{m_D}\frac{1}{\beta_2-\beta_1}\left[\frac{1}{\beta_1+\frac{\mathbf{k}^2}{m_D^2}} - \frac{1}{\beta_2+\frac{\mathbf{k}^2}{m_D^2}}\right]$$

Position space:

$$\psi_N(P,k) \xrightarrow{FT} \frac{e^{-m_D\sqrt{\beta_1}r}}{r} - \frac{e^{-m_D\sqrt{\beta_2}r}}{r}$$

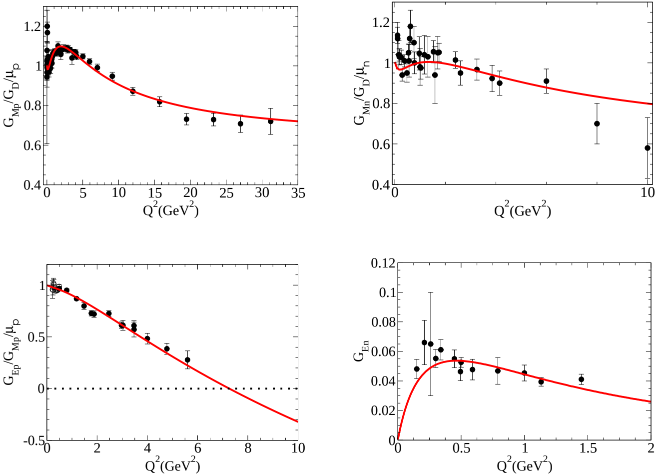
β_1, β_2 momentum range parameters; $\beta_2 > \beta_1$:

β_1 long spatial range; β_2 short spatial range

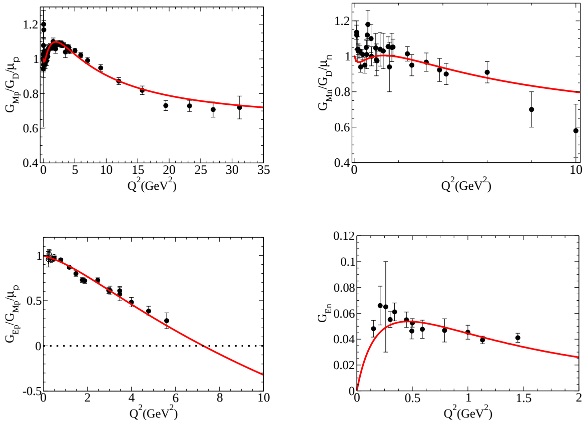
Nucleon form factors (I)

F Gross, GR and MT Peña, PRC 77, 015202 (2008) – model II

Nucleon form factors: $G_E = F_1 - \tau F_2$, $G_M = F_1 + F_2$; $\tau = \frac{Q^2}{4M_N^2}$



Nucleon form factors (II) [PRC 77, 015202 (2008)]



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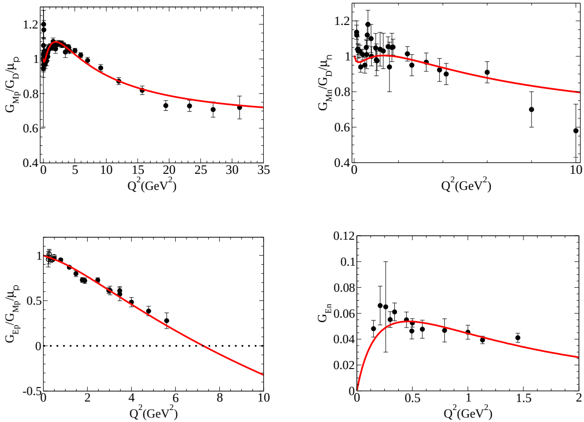

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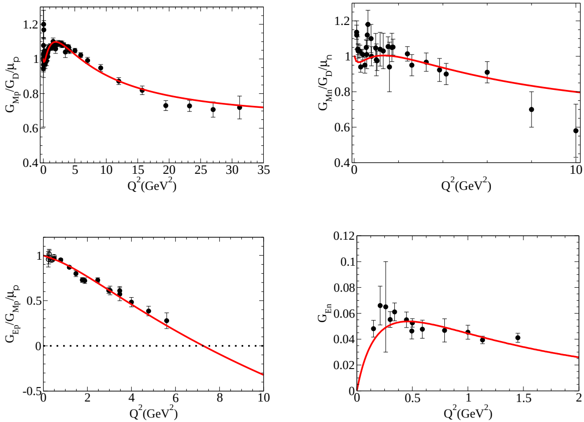

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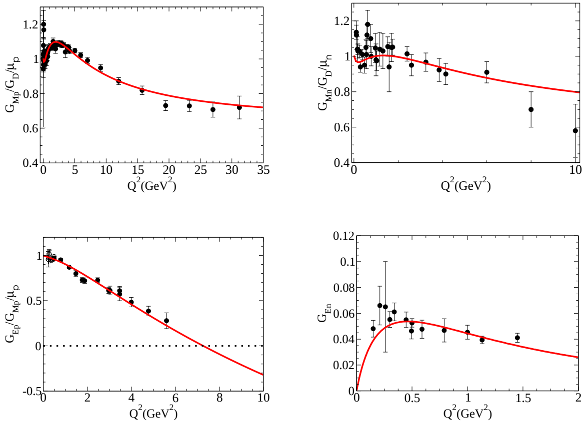

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- How **can we** test the **valence quark** parametrizarion? Lattice

Extension of the model for lattice QCD regime

GR and MT Peña JPG 36, 115011 (2009)

- Quark current:
  $j_I^\mu(M_N; m_\rho, M_h = 2M_N) \to j_I^\mu(M_N^{latt}; m_\rho^{latt}, 2M_N^{latt})$
- Wave functions:
  $\Psi_B(\{M_B\}) \to \Psi_B(\{M_B^{latt}\})$

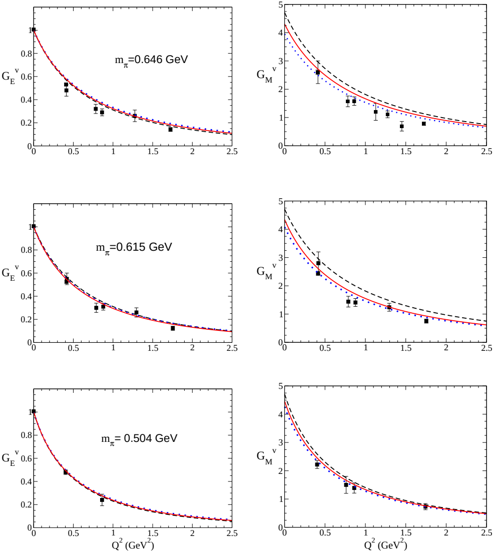
$\Rightarrow$ Implicit m_π dependence in G_X [Form factors]

G_X include only valence quark (bare) contributions $\to G_X^B$

Meson cloud effects suppressed for large m_π:

Compare G_X^B with lattice data

Nucleon form factors on lattice [JPG 36, 115011 (2009)] G_X^{p-n}



Data from Gockeler at al, PRD 71, 034508 (2005)

$N(1440)$ wave function [$N(1440) \equiv R$] (Roper)

- $N(1440)$ is the 1st radial excitation of the nucleon
- **Same** spin a isospin structure
- Ψ_R orthogonal to Ψ_N
  Orthogonality given by scalar wave functions

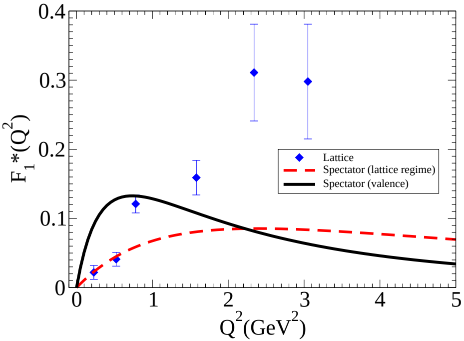
$$\int_k \psi_R(P_+, k)\psi_N(P_-, k)\Big|_{Q^2=0} = 0$$

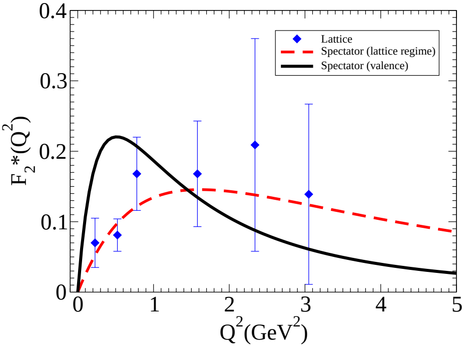
- Wave function:

$$\psi_R\left(\frac{(P-k)^2}{m_D M_R}\right) = N_1 \frac{\overbrace{\beta_3' - 2\frac{(P-k)^2}{m_D M_R}}^{\text{excitation}}}{\beta_1' + 2\frac{(P-k)^2}{m_D M_R}} \times \psi_N\left(\frac{(P-k)^2}{m_S M_R}\right)$$

- β_1' fixed by ψ_N; β_3' determined by the orthogonality condition
  No adjustable parameters $\rightarrow$ predictions

$\gamma N \rightarrow N(1440)$ on lattice [PRD 81, 074020 (2010)]



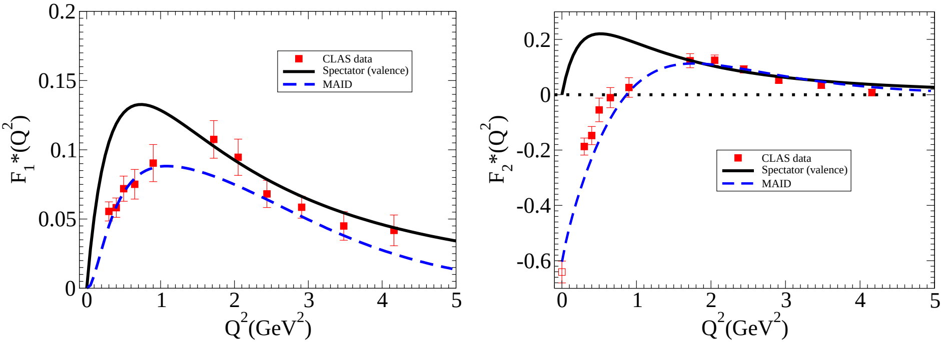


Data: H.W. Lin et al PRD 78, 114508 (2008)

Good agreement with Lattice data

GR and K Tsushima, PRD 81, 074020 (2010)

$\gamma N \to N(1440)$ form factors [PRD 81, 074020 (2010)]



- CLAS data - Aznauryan et al PRC 80, 055203 (2009), MAID fit
- Good agreement for $Q^2 > 1.5$ GeV^2
- Difference for $Q^2 < 1.5$ GeV^2 –manifestation of meson cloud

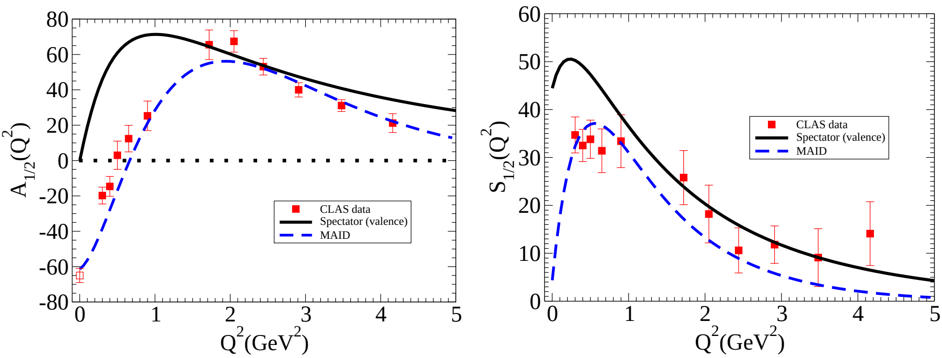
$\gamma N \to N(1440)$ helicity amplitudes

$$A_{1/2}(Q^2) = \mathcal{R}\left[F_1^*(Q^2) + F_2^*(Q^2)\right]$$
$$S_{1/2}(Q^2) = \frac{\mathcal{R}}{\sqrt{2}}\frac{M_R + M}{Q^2}|\mathbf{q}|\left[F_1^*(Q^2) - \tau F_2^*(Q^2)\right]$$

$$\mathcal{R} = \sqrt{\frac{\pi\alpha\left[(M_R - M)^2 + Q^2\right]}{M_R M K}}, \quad K = \frac{M_R^2 - M^2}{2M_R}, \quad \tau = \frac{Q^2}{(M_R + M)^2}$$

$|\mathbf{q}|$ = photon momentum in Roper rest frame

$\gamma N \rightarrow N(1440)$ helicity amplitudes [PRD 81, 074020 (2010)]



GR and K Tsushima PRD 81, 074020 (2010)

Good description of the data ($Q^2 > 1.5$ GeV^2)

$\rightarrow N(1535)$

$N(1535)$ wave function $[N(1535) = S11 \simeq S]$

Approximations: See **NSTAR2011**

- Pointlike diquark $k_\rho = \frac{1}{\sqrt{2}}(k_1 - k_2) \to 0$ [no internal diquark P-states]
- Pure spin 1/2 core:

$$
\begin{aligned}
|N(1535)\rangle &= \cos\theta_S |S = 1/2\rangle - \sin\theta_S |S = 3/2\rangle \\
&\to |S = 1/2\rangle
\end{aligned}
$$

[Karl-Isgur model: $\cos\theta_S \approx 0.85$]

Spin states (diquark-quark system with $L = 1$, $P = -1$): $1 \oplus \frac{1}{2}$

$$X_\rho\left(+\tfrac{1}{2}\right) \approx \sum_m \langle 1m; \tfrac{1}{2}, \tfrac{1}{2} | \tfrac{1}{2}, \tfrac{1}{2}\rangle Y_{1m}(\hat{k}_\lambda) |m - \tfrac{1}{2}\rangle_\rho \qquad [MA]$$

$$X_\lambda\left(+\tfrac{1}{2}\right) \approx \sum_m \langle 1m; \tfrac{1}{2}, \tfrac{1}{2} | \tfrac{1}{2}, \tfrac{1}{2}\rangle Y_{1m}(\hat{k}_\lambda) |m - \tfrac{1}{2}\rangle_\lambda \qquad [MS]$$

$$\psi_{S11}(P,k) = \frac{N_S}{m_D\left(\beta_1' + 2\frac{P\cdot k}{m_D M_S}\right)\left(\beta_2' + 2\frac{P\cdot k}{m_D M_S}\right)} \approx \psi_N(P,k)$$

No adjustable parameters $\to$ predictions

Form factors:

$$F_1^*, F_2^* \propto \mathcal{I}_0(Q^2)$$

Overlap integral (S11 rest frame): $|\mathbf{q}|_0 = \frac{M_S^2 - M^2}{2M_S}$: photon moment

$$\begin{aligned} \mathcal{I}_0(Q^2) &= \int_k \frac{k_z}{|\mathbf{k}|} \psi_{S11}(P_{S11}, k)\, \psi_N(P_N, k) \\ &= \text{const} \times |\mathbf{q}|_0 \quad (Q^2 \to 0) \end{aligned}$$

$\gamma N \to N(1535)$ form factors [arXiv:1105.2223 [hep-ph]]

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- If $M_S \neq M \Rightarrow \mathcal{I}_0(0) \neq 0 \Rightarrow$ **No orthogonality**
  [Consequence of relativistic generalization (boost of a state)]

$\gamma N \to N(1535)$ form factors [arXiv:1105.2223 [hep-ph]]

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$\gamma N \to N(1535)$ form factors [arXiv:1105.2223 [hep-ph]]

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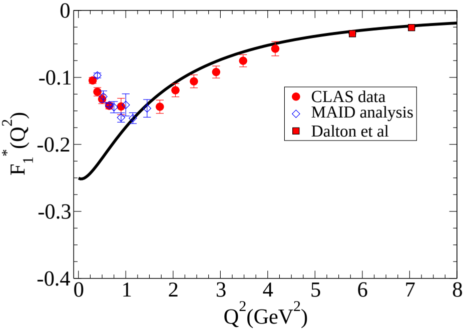
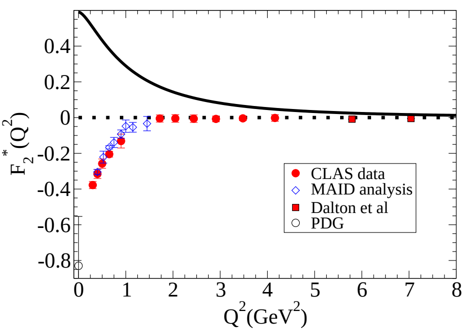
- Range of application of the model ? ($\mathcal{I}_0(0) \approx 0$)
  $|\mathbf{q}|_0$ defines the momentum scale
- If $Q^2 \gg |\mathbf{q}|_0^2 =$ 0.23 GeV^2 $\Rightarrow \mathcal{I}_0(0) \approx 0$
  Model valid for $Q^2 > 2.3$ GeV^2

$\gamma N \to N(1535)$ form factors [arXiv:1105.2223 [hep-ph]]





- Model compared with CLAS and MAID data

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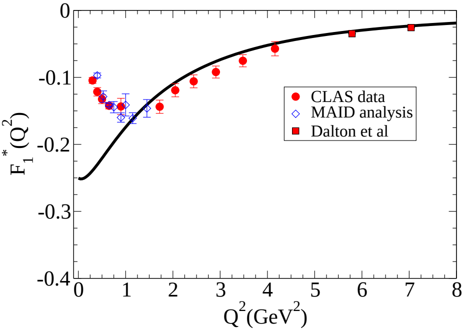

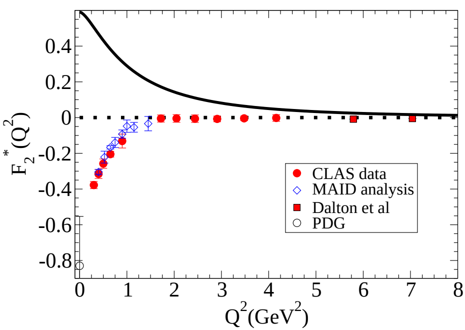

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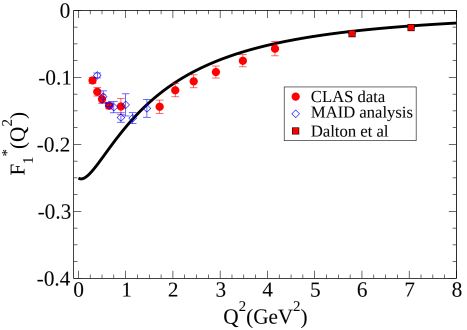

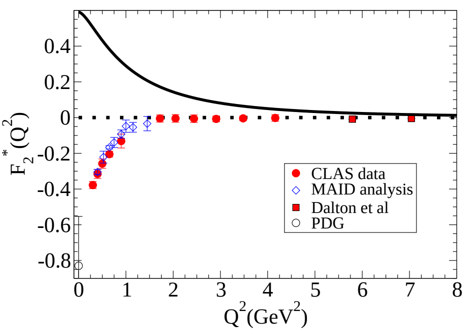

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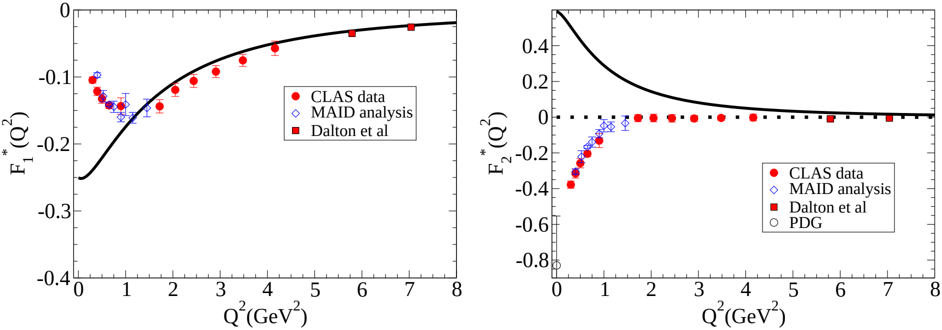
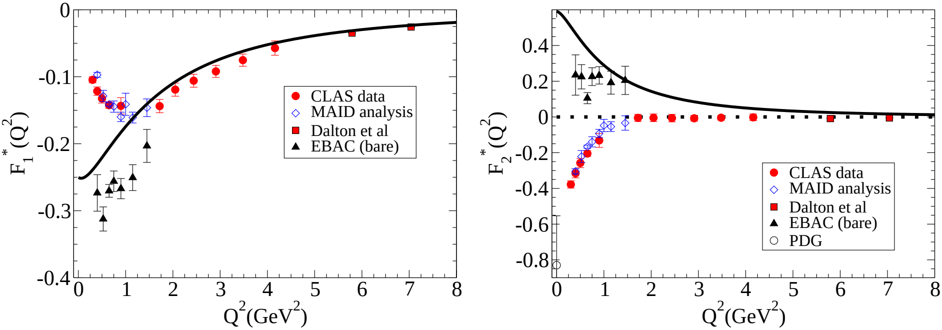
$\gamma N \to N(1535)$ form factors [arXiv:1105.2223 [hep-ph]]



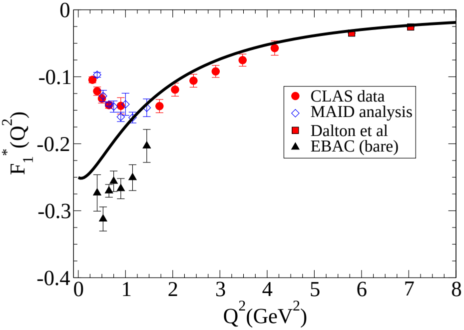
- Model compared with CLAS and MAID data
- F_1^* OK; F_2^* wrong sign
- ... There is also estimates of **valence** contributions (EBAC)

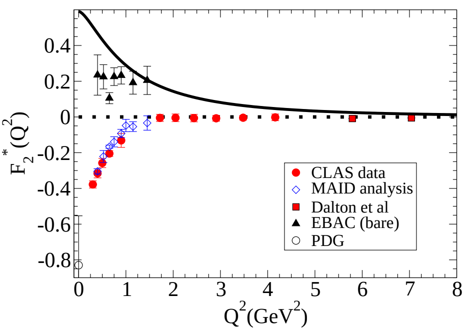
$\gamma N \to N(1535)$ form factors [arXiv:1105.2223 [hep-ph]]



- Model compared with EBAC: J. Diaz et al PRC 60, 025207 (2009)

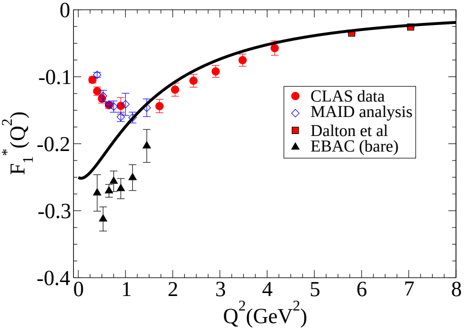
$\gamma N \to N(1535)$ form factors [arXiv:1105.2223 [hep-ph]]

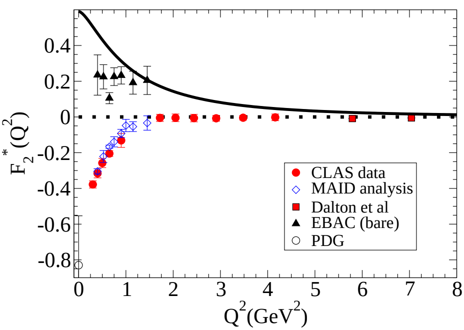




- Model compared with EBAC: J. Diaz et al PRC 60, 025207 (2009)
- F_1^* close to EBAC (valence quark core) ($Q^2 < 2$ GeV^2)

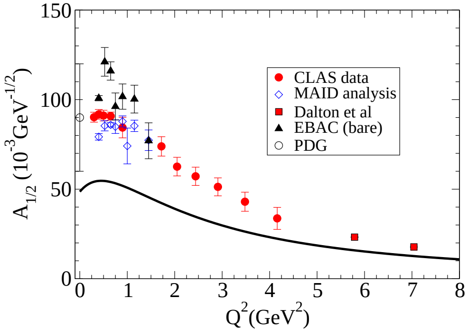
$\gamma N \to N(1535)$ form factors [arXiv:1105.2223 [hep-ph]]

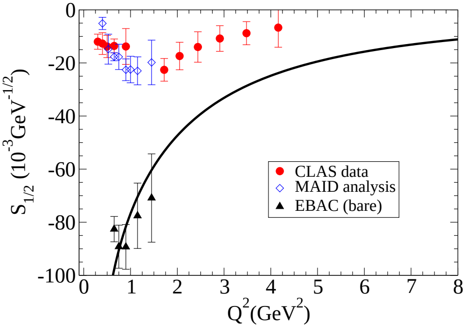




- Model compared with EBAC: J. Diaz et al PRC 60, 025207 (2009)
- F_1^* close to EBAC (valence quark core) ($Q^2 < 2$ GeV^2)
- F_2^* close to valence estimate ($Q^2 \approx 1$ GeV^2) $(F_2^*)^{Sp} \simeq (F_2^*)^{QM}$

$\gamma N \to N(1535)$ helicity amplitudes

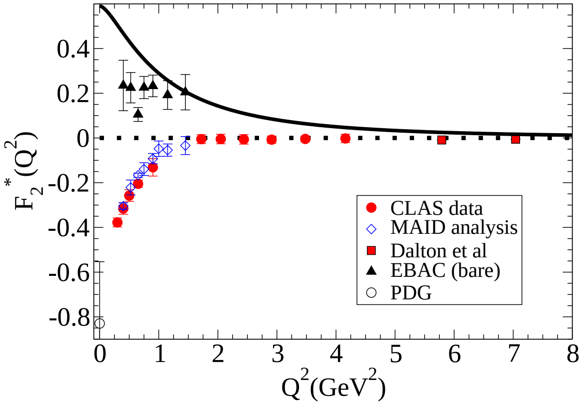




$$A_{1/2} = -2b\left[F_1^* + \frac{M_S - M}{M_S + M}F_2^*\right], \; S_{1/2} = \sqrt{2}b(M_S + M)\frac{|\mathbf{q}|}{Q^2}\left[\frac{M_S - M}{M_S + M}F_1^* - \tau F_2^*\right]$$

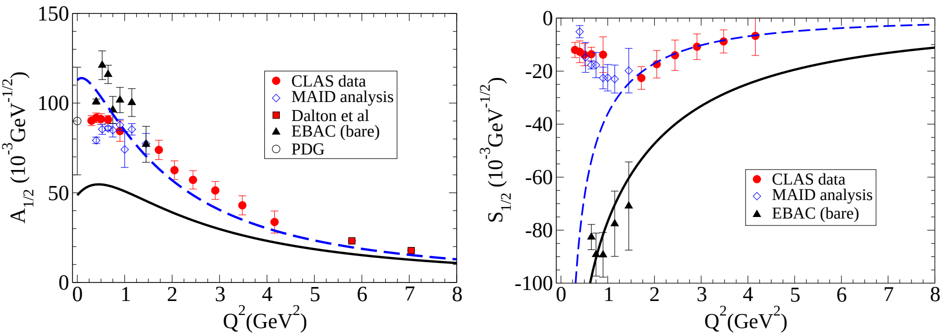
$$|\mathbf{q}| = \frac{\sqrt{[(M_S - M)^2 + Q^2][(M_S + M)^2 + Q^2]}}{2M_S}, \quad b = e\sqrt{\frac{(M_S - M)^2 + Q^2}{8M(M_S^2 - M^2)}}, \quad \tau = \frac{Q^2}{(M_S + M)^2}$$

$\gamma N \to N(1535)$ form factors



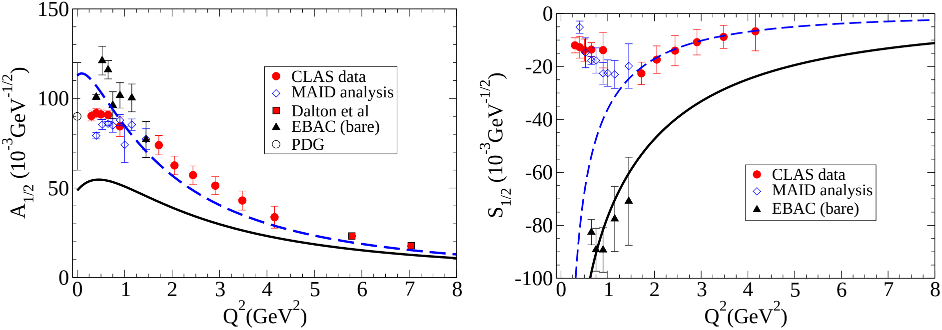
What if we use $F_2^* \approx 0$? $(Q^2 > 1.5 \text{ GeV}^2)$

$\gamma N \to N(1535)$ helicity amplitudes [arXiv:1105.2223 [hep-ph]]



- $F_2^* = 0$ (data), F_1^* from Spectator model - - - - - -

$\gamma N \to N(1535)$ helicity amplitudes [arXiv:1105.2223 [hep-ph]]



- $F_2^* = 0$ (data), F_1^* from Spectator model - - - - - -
- Very good description of $A_{1/2}$ and $S_{1/2}$ for $Q^2 > 2.3$ GeV^2

Implication of $F_2^* = 0$:

- Model with valence quark $\oplus$ meson cloud:

  Cancelation of valence quark contributions and meson cloud contributions

$$F_2^B(Q^2) + F_2^{mc} \simeq 0$$

  F_2^B = valence quarks; F_2^{mc} = meson cloud

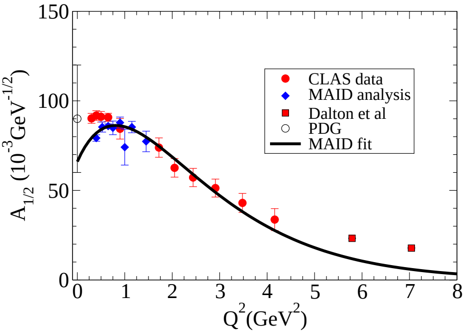
- Consequence: for $Q^2 > 1.8$ GeV^2 $[\, |\mathbf{q}| \simeq Q\sqrt{1+\tau}\,]$:

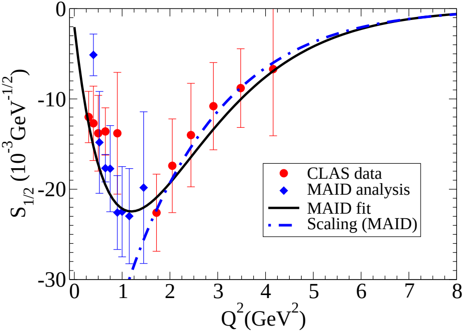
$$S_{1/2} \simeq -\frac{\sqrt{1+\tau}}{\sqrt{2}} \frac{M_S^2 - M^2}{2M_S Q} A_{1/2}$$

  $S_{1/2}$ *scales* with $A_{1/2}$

GR and K Tsushima arXiv:1105.2484 [hep-ph]

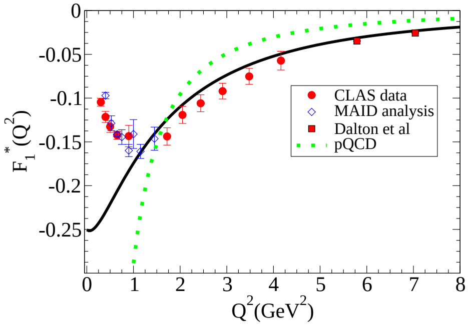
Relation between $A_{1/2}$ and $S_{1/2}$ (MAID)

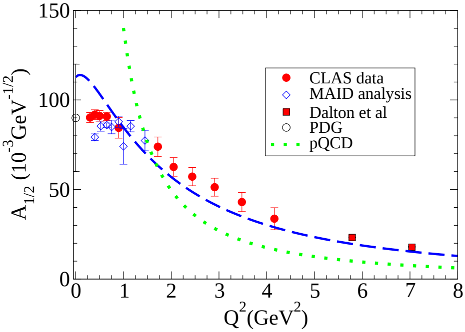




MAID parametrization $A_{1/2}$: $\quad S_{1/2} \simeq -\frac{\sqrt{1+\tau}}{\sqrt{2}} \frac{M_S^2 - M^2}{2 M_S Q} A_{1/2}$

$\gamma N \to N^*(1535)$ asymptotic behavior [arXiv:1105.2223 [hep-ph]]





Comparing with pQCD, Carlson *et al.* PRL 81, 2646 (1998)

Model and Data overestimates pQCD result

Spin 3/2 resonances: transition currents

$\gamma N \to \Delta(1232)$ or $\Delta(1600)$:

$$J^\mu = \bar{u}_\beta(P_+) \left[G_1 q^\beta \gamma^\mu + G_2 q^\beta P^\mu + G_3 q^\beta q^\mu - G_4 g^{\beta\mu} \right] \gamma_5 u(P_-)$$

u_β Rarita-Schwinger spinor

$$q_\mu J^\mu = 0 \Rightarrow G_4 = (M_\Delta + M_N) G_1 + \tfrac{1}{2}(M_\Delta^2 - M_N^2) G_2 - Q^2 G_3$$

$$G_M^* = \tfrac{M_N}{3(M_N+M_\Delta)} \left\{ \left[(3M_\Delta + M_N)(M_\Delta + M) + Q^2 \right] \frac{G_1}{M_\Delta} + (M_\Delta^2 - M_N^2) G_2 - 2Q^2 G_3 \right\}$$

$$G_E^* = \tfrac{M_N}{3(M_N+M_\Delta)} \left\{ (M_\Delta^2 - M_N^2 - Q^2) \frac{G_1}{M_\Delta} (M_\Delta^2 - M_N^2) G_2 - 2Q^2 G_3 \right\}$$

$$G_C^* = \tfrac{M_N}{3(M_N+M_\Delta)} \left\{ 4M_\Delta G_1 + (3M_\Delta^2 + M_N^2 + Q^2) G_2 + 2(M_\Delta^2 - M_N^2 - Q^2) G_3 \right\}$$

$$A_{1/2} = -\mathcal{N} \frac{\sqrt{3}}{2} \left[G_M^* - 3G_E^* \right], \quad A_{3/2} = -\mathcal{N} \frac{\sqrt{3}}{2} \left[G_M^* + G_E^* \right], \quad S_{1/2} \propto G_C^*$$

Spin 3/2 resonances: wave functions

- Δ wave function: EPJA 36, 329 (2008); PRD 78, 114017 (2008); PRD 80, 013008 (2009)

$$\Psi_\Delta = N\left[\Psi_S + a\Psi_{D3} + b\Psi_{D1}\right]$$

$$S = 0 \oplus \frac{3}{2}, \quad D3 = 2 \oplus \frac{3}{2}, \quad D1 = 2 \oplus \frac{1}{2}$$

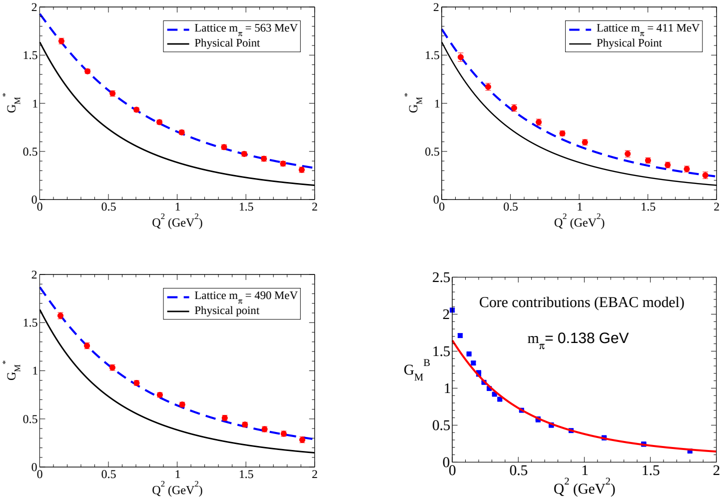
- Wave functions fited to form factor data $[G_M^*, G_E^*, G_C^*]$
- S-state model: good description of G_M^* data EPJA, 36, 329 (2008)
- With D-states: S-state $\Leftarrow G_M^*$ **Fit** physical data $\oplus$ EBAC (core) $G_E^*, G_C^* \Leftarrow$ **Fit** lattice QCD data (bare contribution)

Extracting valence quark contributions from lattice QCD:

$$G_X^{latt}(m_\pi^{latt}) \to \overbrace{G_X^B(m_\pi^{latt})}^{\text{Model}} \xrightarrow{m_\pi^{latt} \to m_\pi^{phys}} G_X^B(physical)$$

GR and MT Peña PRD 80, 013008 (2009)

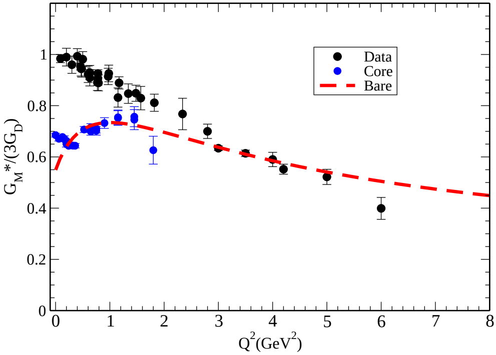
$\gamma N \to \Delta$: $G_M^*(Q^2)$ on lattice [PRD 80, 013008 (2009)]



Lattice: Alexandrou et al, PRD 77, 085012 (2008)

⊕ EBAC: J. Diaz et al, PRC 75, 015205 (2007)

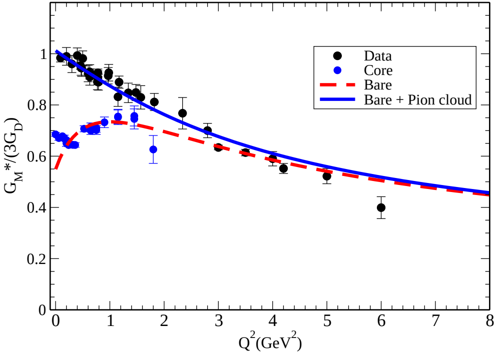
GR and MT Peña PRD 80, 013008 (2009)



- Bare $\approx$ EBAC model

$\gamma N \to \Delta$: $G_M^*(Q^2)$ (valence + pion cloud [phenomenological])

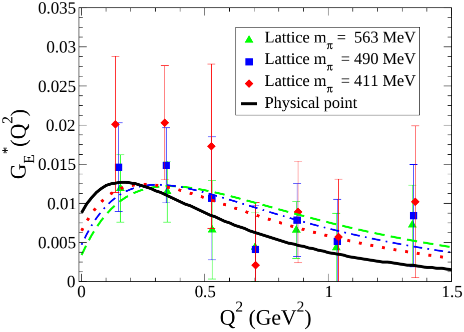
GR and MT Peña PRD 80, 013008 (2009)

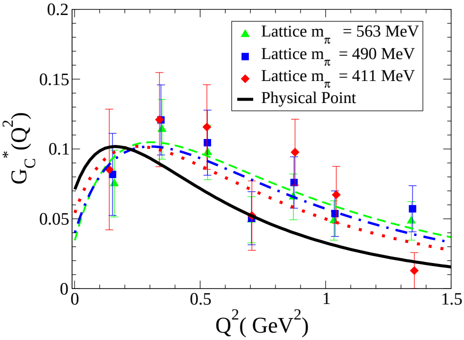


- Bare ≈ EBAC model ⊕ $G_M^\pi = \lambda_\pi \left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2+Q^2}\right)^2 (3G_D)$ $\qquad \frac{G_M^B(0)}{3G_D} \le 0.7$

$\gamma N \to \Delta$: $G_E^*(Q^2)$, $G_C^*(Q^2)$ on lattice [PRD 80, 013008 (2009)]

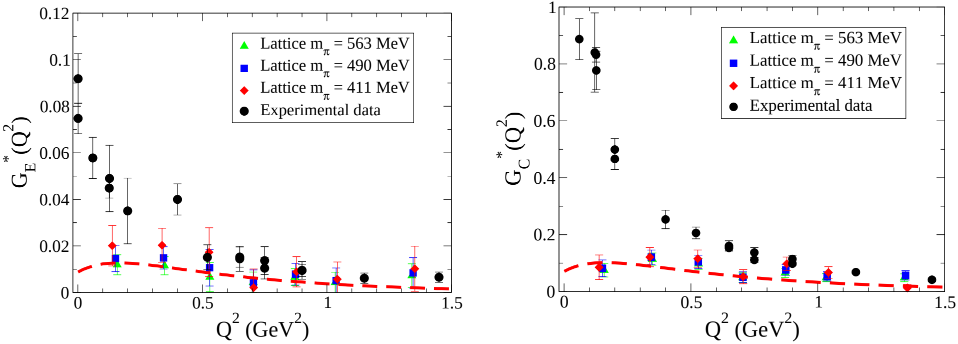
Fit to lattice QCD data (bare contribution)
Alexandrou et al, PRD, 77, 085012 (2008)





D3 state: 0.72%   D1 state: 0.72%

$\gamma N \to \Delta$: $G_E^*(Q^2)$, $G_C^*(Q^2)$ (bare) [PRD 80, 013008 (2009)]

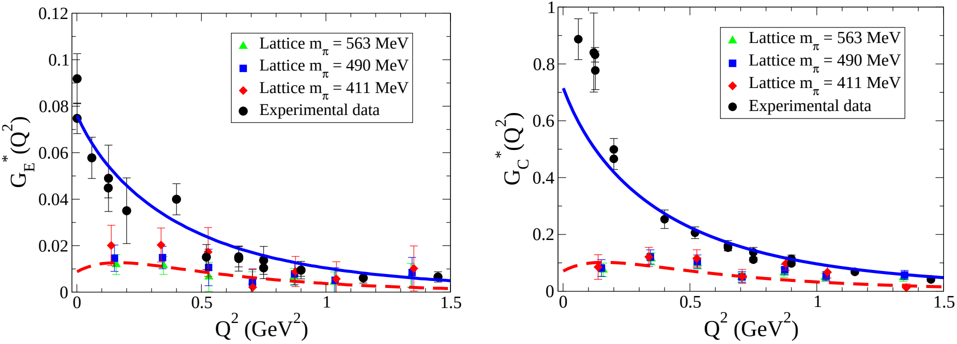


Compare with **Physical data**

Small valence quark contributions

GR, MT Peña PRD 80, 013008 (2009)

$\gamma N \to \Delta$: $G_E^*(Q^2)$, $G_C^*(Q^2)$ (bare + pion cloud)



Pion cloud [Large N_c; no additional parameters]

Pion cloud dominant; Good global description ($Q^2 < 1.5$ GeV^2)

$\Rightarrow$ callibration valence quark contribution (all Q^2)

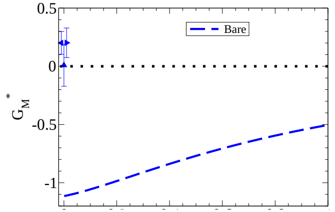
$\Delta(1600)$ as the 1st radial excitation of $\Delta(1232)$ EPJA, 36, 329 (2008) [S-state]

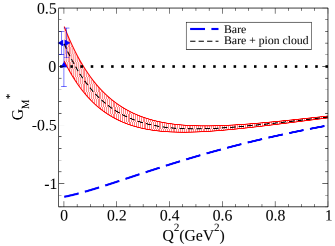
$G_E^* \equiv 0$, $G_C^* \equiv 0$

$$\text{Bare}: \quad G_M^B(0) = -1.113$$

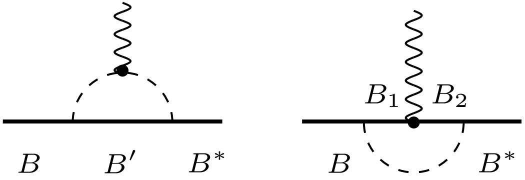
Effects of π cloud?

Decay	BR
$\Delta(1600) \to \pi N$	0.153±0.019
$\Delta(1600) \to \pi \Delta$	0.590±0.100
$\Delta(1600) \to \pi N(1440)$	0.130±0.040





GR and K.Tsushima, PRD 82, 073007 (2010)



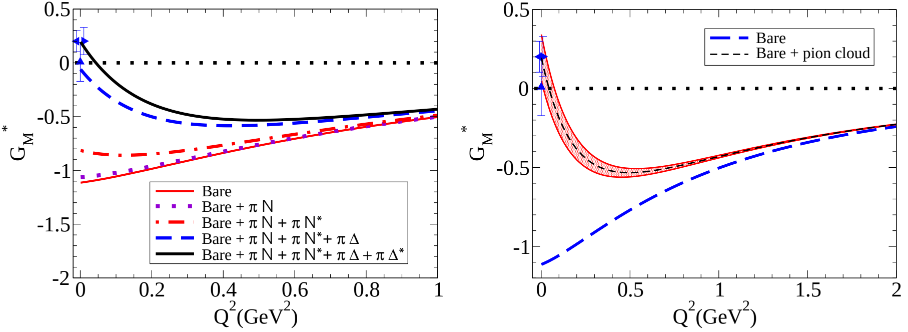
(a) (b)

- Dominance of diagram (a): leading order in χPT
- Pion cloud generalization of $\gamma N \to \Delta(1232)$
  Including channels: $\pi N, \pi\Delta, \pi N^*, \pi\Delta^*$
  Assuming equal masses in the loops:

$$G_M^\pi = (\lambda_\pi^N + \lambda_\pi^{N^*} + \lambda_\pi^\Delta + \lambda_\pi^{\Delta^*}) \left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2 + Q^2} \right)^2 (3G_D)$$

$\lambda_\pi^B \leftarrow f_{\pi B' B^*}$ **determined by the Data** (Γ and BR)

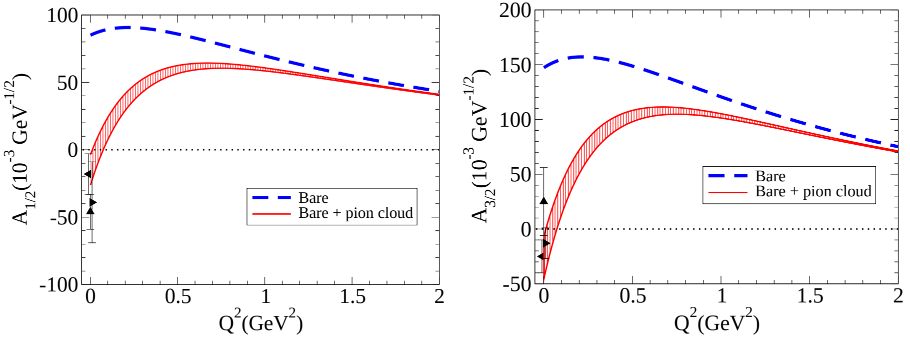
$\gamma N \to \Delta(1600)$ form factors [PRD 82, 073007 (2010)]



Valence quark dominance for high Q^2

GR and K.Tsushima, PRD 82, 073007 (2010)

$\gamma N \to \Delta(1600)$ helicity amplitudes [PRD 82, 073007 (2010)]



Valence quark dominance for high Q^2

GR and K.Tsushima, PRD 82, 073007 (2010)

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    Lattice data $[\Delta(1232)]$
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- Prespective of extension to other resonances
  $P_{11}(1710)$, $D_{13}(1520)$, $S_{11}(1650)$, ...

Nucleon Resonance Structure