What did we learn about GPDs from meson electroproduction?

P. Kroll

Fachbereich Physik, Univ. Wuppertal and Univ. Regensburg

Jefferson Lab, May 2011

Outline:

- **Exclusive processes and GPDs**
- **Power corrections?**
- **Extraction and parameterizations of GPDs**
- **Analysis of meson electroproduction**
- **Present knowledge of GPDs from meson production**
- **DVCS**
- **More about pion production**
- **Summary**

Hard exclusive scattering - GPDs

DVCS and meson electroproduction

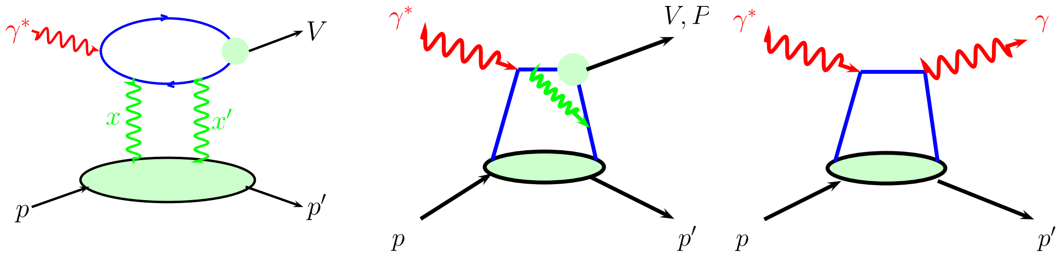
rigorous proofs of collinear factorization for $Q^2 \to \infty$:

Radyushkin, Collins et al, Ji-Osborne

hard subprocesses

$\gamma^* g \to V g\,,$

$\gamma^* q \to V(P,\gamma) q$



and GPDs and meson w.f.

(encode the soft physics)

$$\mathcal{M} \sim \int_{-1}^{1} d\bar{x}\, \mathcal{H}(\bar{x},\xi,t) F(\bar{x},\xi,t)$$

dominant transitions $\gamma_L^* \to V_L(P)$, $\gamma_T^* \to \gamma_T$

others power suppressed but often non-negligible (e.g. $\gamma_T^* \to V_T$ large)

GPDs

D. Müller et al (94), Ji(97), Radyushkin (97)

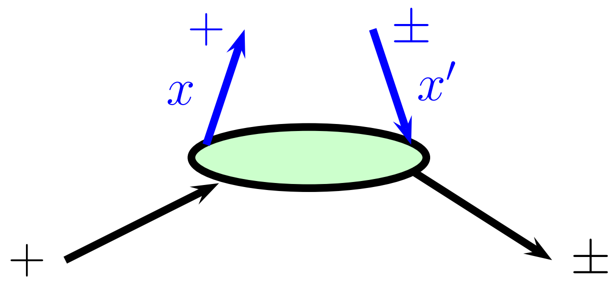
GPDs: $F = F(\bar{x}, \xi, t)$

$$F = H, E, \widetilde{H}, \widetilde{E}, H_T, \ldots$$

$$x = \frac{\bar{x}+\xi}{1+\xi} \quad x' = \frac{\bar{x}-\xi}{1-\xi}$$

for quarks ($\xi < \bar{x} < 1$) and gluons

(antiquarks for $-1 < \bar{x} < -\xi$, $q\bar{q}$ pairs $-\xi < \bar{x} < \xi$)



properties:

reduction formula $H^q(\bar{x}, \xi = t = 0) = q(\bar{x})$, $\widetilde{H}^q \to \Delta q(\bar{x})$, $H_T^q \to \delta^q(\bar{x})$

sum rules (proton form factors): $F_1^q(t) = \int d\bar{x} H^q(\bar{x}, \xi, t)$, $F_1 = \sum e_q F_1^q$

$$E \to F_2,\ \widetilde{H} \to F_A,\ \widetilde{E} \to F_P$$

polynomiality, universality, evolution, positivity constraints

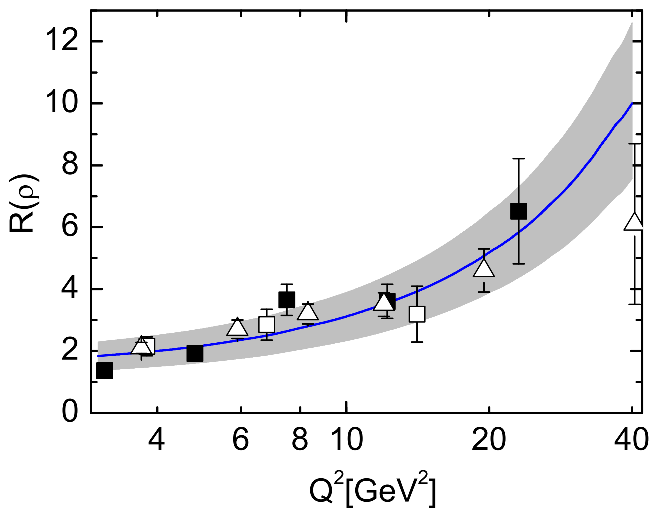
Ji's sum rule $\quad \langle J_q \rangle = \frac{1}{2} \int_{-1}^{1} d\bar{x}\, \bar{x} \left[H^q(\bar{x}, \xi, t=0) + E^q(\bar{x}, \xi, t=0) \right]$

FT $\mathbf{\Delta} \to \mathbf{b}$ ($\Delta^2 = -t$): information on parton localization in trans. position space

$q(x, \xi = 0, \mathbf{b})$ probability to find a q with long. mom. fraction x at transv. position $\mathbf{b}$

Power corrections?

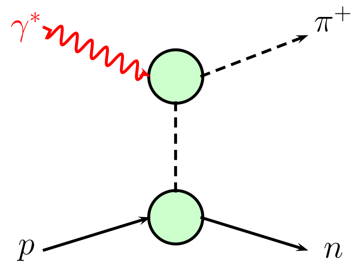
factorization proven for $Q^2 \to \infty$, at finite value there may be power corrections



$$R = \sigma_L/\sigma_T$$

data: HERA $W \simeq 80\,\mathrm{GeV}$

$\gamma_T^* \to V_T$ transitions substantial



$\sim F_\pi(Q^2)$

$\sim g_{\pi NN} F_{\pi NN}(t)$

look only to longitudinal cross section?

but π^+ electroproduction:

$\widetilde{E}^{(3)}_{\text{pole}} = \Theta(|\,\bar{x}\,| \leq \xi)\Phi_\pi \frac{F_P(t)}{\xi}$

$F_P = \frac{2\sqrt{2} g_{\pi NN} f_\pi}{m_\pi^2 - t} F_{\pi NN}(t)$

from usual handbag graph one obtains

ampl. $\propto F_\pi^{\text{pert.}}$ about $1/3$ of $F_\pi^{\exp}$

measured in same reaction

... but we are not able to calculate GPDs as yet

controlled by non-pert. QCD

lattice QCD: a few moments of GPDs Hägler *et al*

models: Overlap of light-cone wave functions Diehl *et al*, Pasquini *et al*

$\gamma_L^* \gamma \to \pi^+ \pi^-$ with NJL Noguera *et al*

H, E in chiral quark soliton model Goeke *et al*

extraction from experiment:

analysis of nucleon FF with help of sum rules

- determination of val. quark GPDs $H, E, \widetilde{H}$ Guidal *et al*, Diehl *et al*

fit parameterizations of GPDs to data on

- meson electroproduction
- DVCS

Parameterizing the GPDs

double distribution ansatz (Mueller *et al* (94), Radyushkin (99))

$$F_i(\bar{x},\xi,t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta+\xi\alpha-\bar{x})\, f_i(\beta,\alpha,t) + D_i\, \Theta(\xi^2-\bar{x}^2)$$

DD: $f_i =$ zero-skewness GPD $\times$ weight fct (generating ξ dep.)

$F(\bar{x},\xi=0,t) = f(\bar{x}) \exp\left[(b_f + \alpha'_f \ln(1/\bar{x}))t\right]$

$f = q, \Delta q, \delta^q$ for $H, \widetilde{H}, H_T$ or $c\bar{x}^{-\alpha_f(0)}(1-\bar{x})^{\beta_f}$

Regge-like t dep. large x, large $-t$ more complicated profile fct Diehl *et al* (04)

advantage: polynomiality and reduction formulas automatically satisfied

dual parameterization (Polyakov(99), Polyakov-Semenov(09))

repres. of GPDs in terms of infinite sum of t-channel resonances - 'duality'

in practice: truncation of partial wave series at small j (very few appl.)

program by Mueller and coll.(08,09): more general t-channel partial waves, and analyticity $\Rightarrow$ LO, leading-twist formalism - GPD at cross-over line:

$\mathrm{Im}\langle F\rangle \sim F(\xi,\xi,t)$ and Disp. Rel. $\mathrm{Re}\langle F\rangle \sim \int_0^1 dx \frac{2x^2}{\xi^2-x^2} F(x,x,t)$

Analysis of meson electroproduction

Goloskokov-K. 06, 07, 08, 09 small $\xi (\simeq x_{Bj}/2)$, small $-t$

subprocess amplitudes: mod. pert. approach (Sterman et al (93))

LO pQCD$+$ quark trans. mom. $+$ Sudakov suppr. $\Rightarrow$ coll. appr. for $Q^2 \to \infty$

emission and absorption of partons from proton collinear to proton momenta

(bears resemblance to color dipole model Frankfurt et al (95))

GPDs constructed from CTEQ6 PDFs through the double distr. ansatz

Gaussian wave fcts for the mesons $\Psi_{Vj}(\tau, \mathbf{k}_\perp) \propto \exp[-a_{Vj}^2 \mathbf{k}_\perp^2/(\tau\bar{\tau})]$

L an T different, free parameters - $a_{L,T}^V$ (transverse size $\langle k_\perp^2 \rangle^{1/2} \propto 1/a_{L,T}^V$)

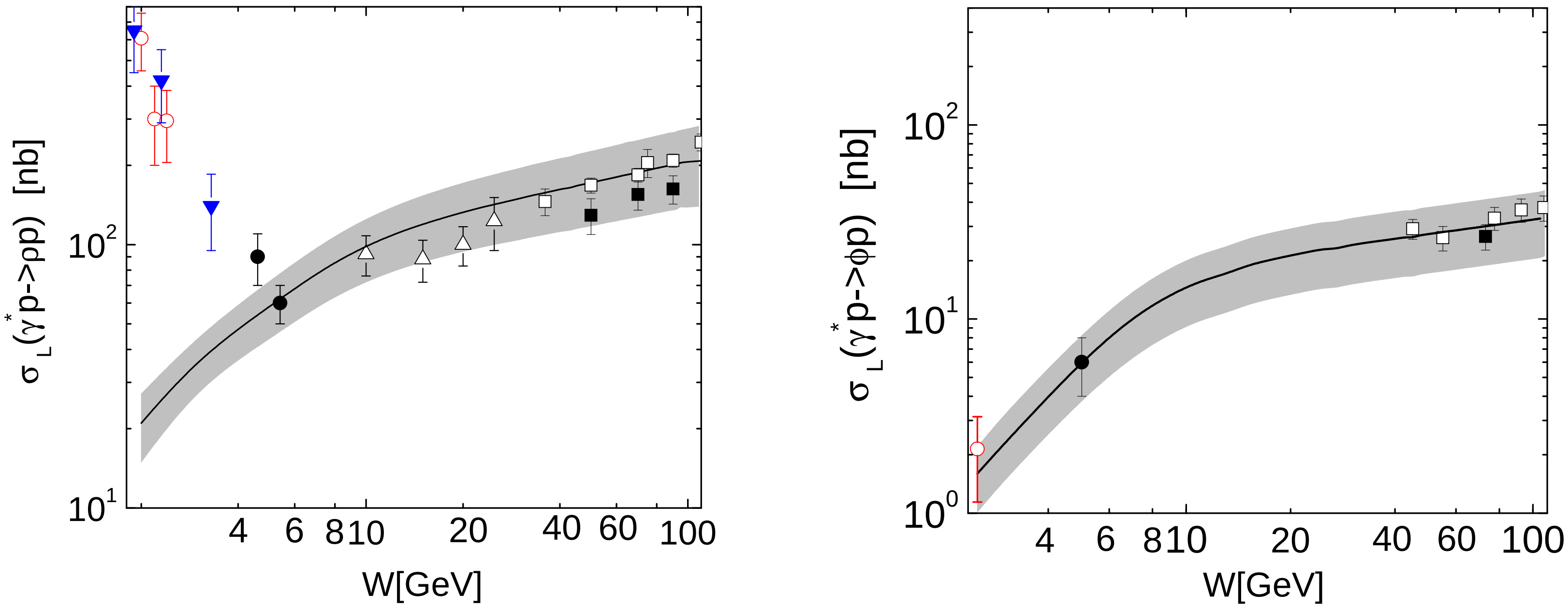
fit to all vector meson data from HERMES, COMPASS, E665, H1, ZEUS

cover large range of kinematics $Q^2 \simeq 3 - 100\,\mathrm{GeV}^2$ $W \simeq 5 - 180\,\mathrm{GeV}$

cross sections and SDMEs probe H A_{UT} probes $\mathrm{Im}\langle E\rangle^* \langle H \rangle$

π^+ production data from HERMES probe $\widetilde{H}, \widetilde{E}, H_T$

ρ^0 and ϕ cross sections



at $Q^2 = 4(3.8)\,\mathrm{GeV}^2$ E665 (△), HERMES (•), CORNELL (▲)
ZEUS (□), H1 (■), CLAS (○)

Goloskokov-K (09)

ω, ρ^+ very large at small W too CLAS (most likely val. quarks responsible)

double distrib. ansatz too simple for valence quarks at large ξ? (resonances?)

breakdown of handbag physics?

JLAB12 may explore region close to minimum

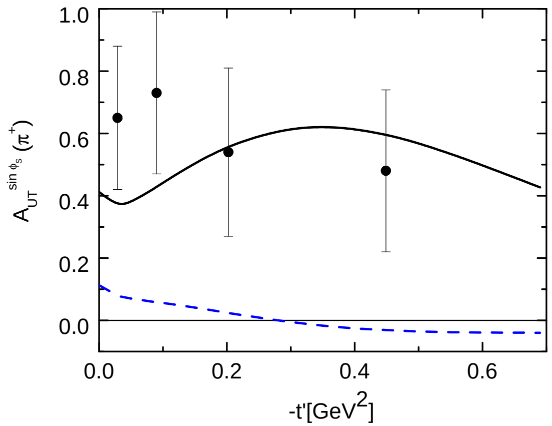
π^+ electroproduction

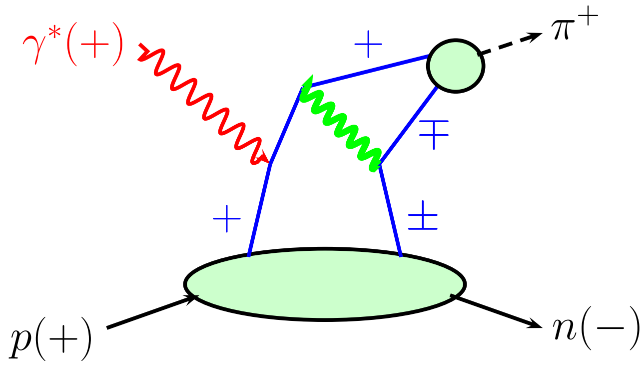
Same approach as for vector mesons Goloskokov-K (09)

contributions from $\widetilde{H}^{(3)} = \widetilde{H}^u - \widetilde{H}^d$ and $\widetilde{E}^{(3)}$ (including pion pole)

for longitudinal photons (full pion FF needed and extra $\widetilde{E}^{n.p.}$)

but contributions from γ_T^* important too:





$Q^2 \simeq 2.5\,\mathrm{GeV}^2$, $W = 3.99\,\mathrm{GeV}$

HERMES(09)

ϕ_S orientation of target-spin vector

does not seem to vanish for $t' \to 0$

$A_{UT}^{\sin\phi_S} \propto \mathrm{Im}\Big[M^*_{0-,++} M_{0+,0+}\Big]$

n-f. ampl. $\mathcal{M}_{0-,++}$ required

twist-3 pion w.f. and

helicity flip GPDs $(H_T, E_T, \ldots)$ required

$\mathcal{M}_{0-,++} \propto \int d\bar{x} \mathcal{H}^{\text{twist}-3}_{0-,++} H_T^{(3)}$

H_T modelled as DD from transversity

PDFs $\delta^a(x)$ Anselmino *et al* (09)

$\sim \mu_\pi/Q \qquad \mu_\pi = m_\pi^2/(m_u + m_d) \simeq 2\mathrm{GeV}$

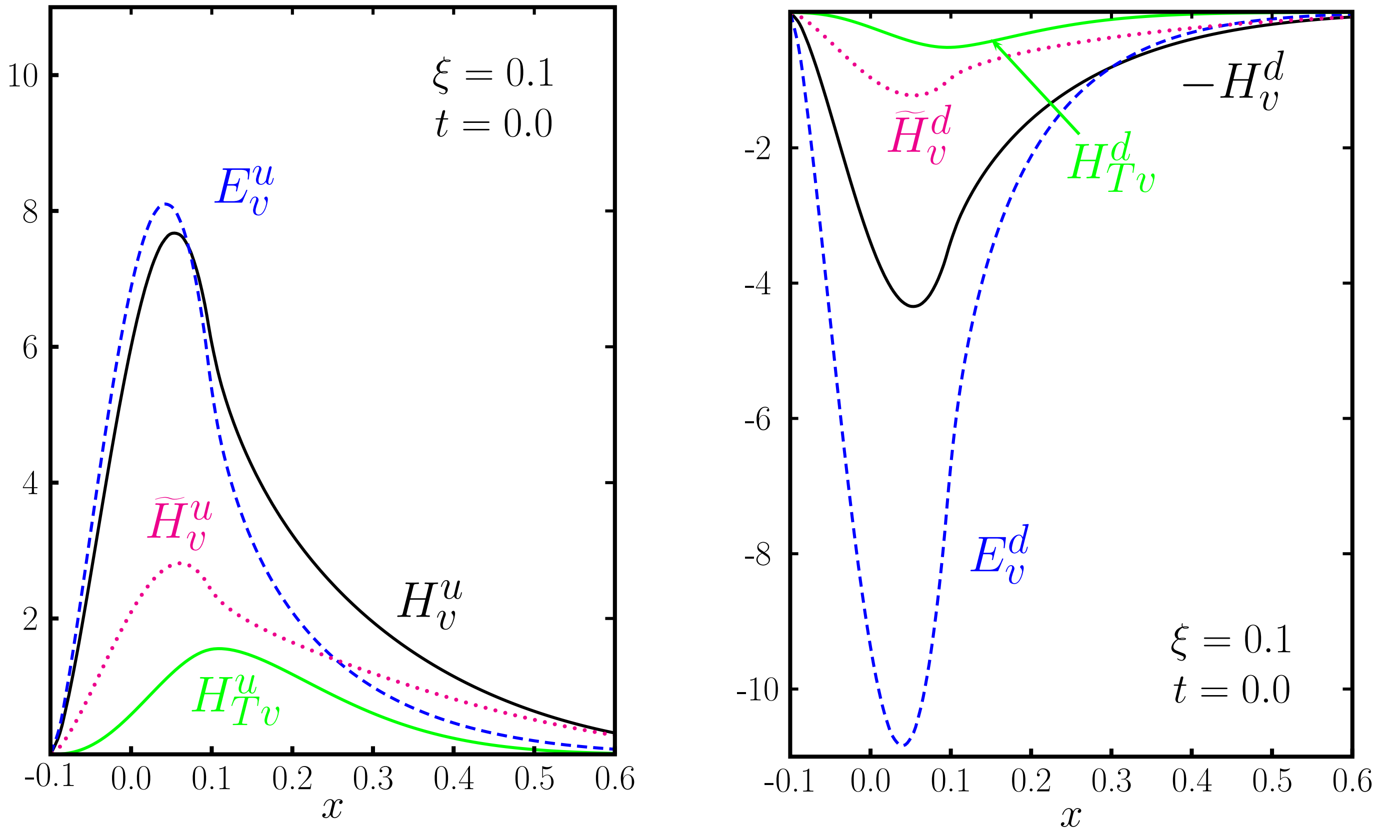
What did we learn about GPDs from meson production?

GPD	probed by	constraints	status
H	ρ^0, ϕ cross sections	PDFs	***
$\widetilde{H}$	$A_{LL}(\rho^0)$	polarized PDFs	*
E	$A_{UT}(\rho^0, \phi)$	sum rule for 2^{nd} moments	*
$\widetilde{E}, H_T, \ldots$	-	-	-
H	ρ^0, ϕ cross sections	PDFs, Dirac ff	***
$\widetilde{H}$	π^+ data	pol. PDFs, axial ff	**
E	$A_{UT}(\rho^0, \phi)$	Pauli ff	*
$\widetilde{E}^{n.p.}$	π^+ data	-	*
H_T	π^+ data	transversity PDFs	*
$\widetilde{H}_T, E_T, \widetilde{E}_T$	-	-	-

Status of small-skewness GPDs as extracted from meson electroproduction data. The upper (lower) part is for gluons and sea (valence) quarks. Except of H for gluons and sea quarks all GPDs are probed for scales of about $4\,\mathrm{GeV}^2$

PDFs *****

Valence quark GPDs



	H	E	$\widetilde{H}$
u_v	2	$\kappa_u = \quad 1.67$	0.93
d_v	1	$\kappa_d = -2.03$	-0.34

lowest moments at $t = 0$

fix signs and rel. sizes

if GPDs have no nodes and

similar t dependence

Applications: Ji's sum rule, transv. location

$$\langle J^a \rangle = \frac{1}{2}\left[q_{20}^a + e_{20}^a\right] \qquad \langle J^g \rangle = \frac{1}{2}\left[g_{20} + e_{20}^g\right]$$

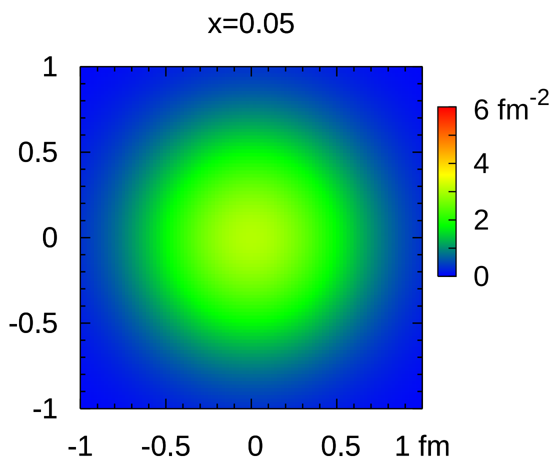
$(\xi = t = 0)$

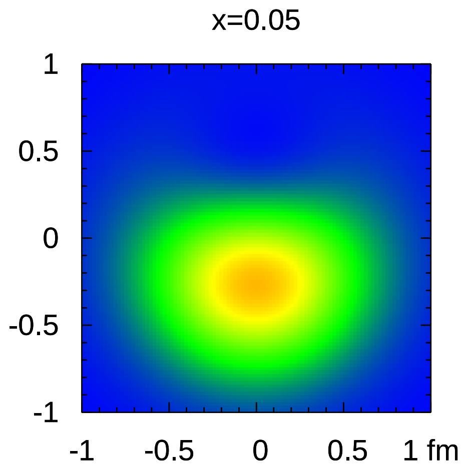
$J^u \simeq 0.250 \quad J^d \simeq 0.020 \quad J^s \simeq 0.015 \quad J^g \simeq 0.214$

J^i quoted at scale $4\,\mathrm{GeV}^2$, $\sum J^i \simeq 1/2$, the spin of the proton

there is no spin crisis

Tomography of d_v quarks



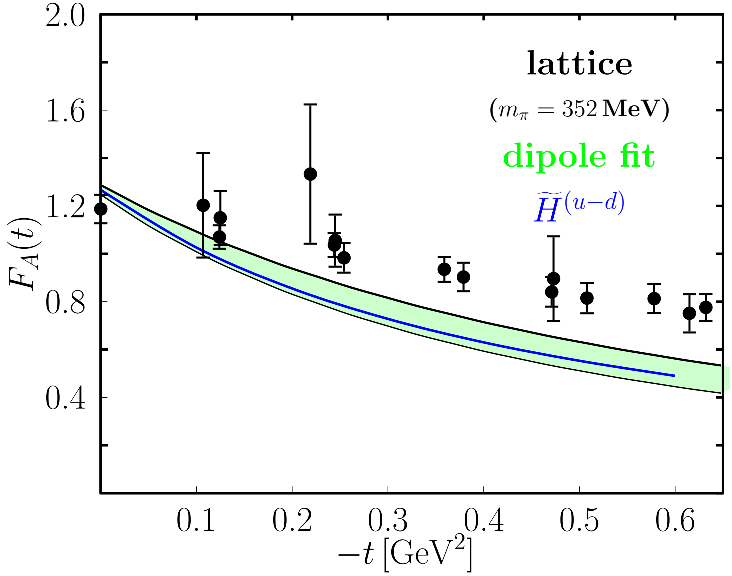


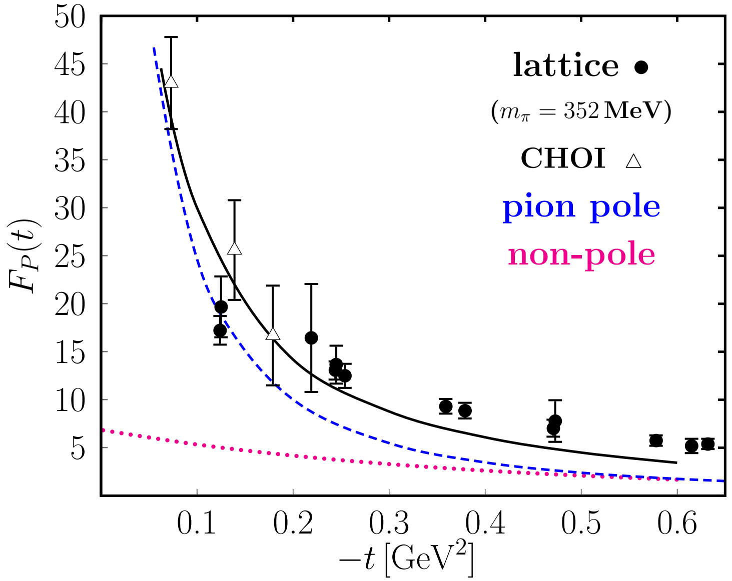
$$q_v^X(x, \mathbf{b}) = q_v(x, \mathbf{b}) - \frac{b^y}{m} \frac{\partial}{\partial \mathbf{b}^2} e_v^q(x, \mathbf{b})$$

e_v contains non-zero orbital angular momentum

Diehl et al (04)

Comparison with lattice results





Hägler (07), Göckeler (05) lowest pion mass 352 MeV, no chiral extrapolation

in general at $t \simeq 0$ reasonable agreement but t dependences flatter than DD ansatz (and form factor data)

relative sizes of moments and relative t dependence in reasonable agreement

H_T lattice moments are larger by about factor of 2 as those constructed from transversity PDFs with help of DD ansatz

DVCS results

Mueller and various collaborators (...08,09,10) partial wave program

more flexible than DD ratio $H^q(\xi,\xi,t=0)/q(2\xi)$ can be adjusted

analysis of DVCS data from CLAS, HERMES and HERA

Moutarde (09) extraction of convolutions ('Compton FF') from CLAS data

$\mathcal{F}(\xi,t) = \sum_a e_a^2 \int_{-1}^{1} dx F^a(x,\xi,t)\left[\frac{1}{\xi-x-i\epsilon} - \frac{\delta}{\xi+x-i\epsilon}\right]$ $(H,E:\delta=1;\ \widetilde{H},\widetilde{E}:=-1)$

H dominance

Guidal-Moutarde (09) extraction of convolutions from HERMES data

H, E and $\widetilde{H}$

Hyde-Guidal-Radyushkin (10) VGG-model for GPD (based on DD) applied to Jlab data - need for improved GPDs claimed

flavor separation not possible

Exploiting universality

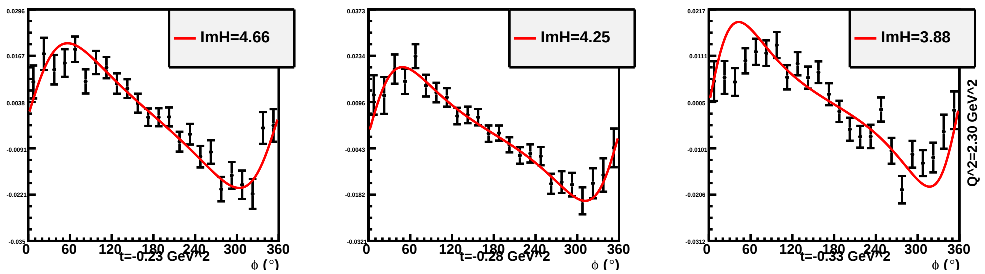
Applying a given set of GPDs determined in either DVCS or meson electroproduction, to the other process

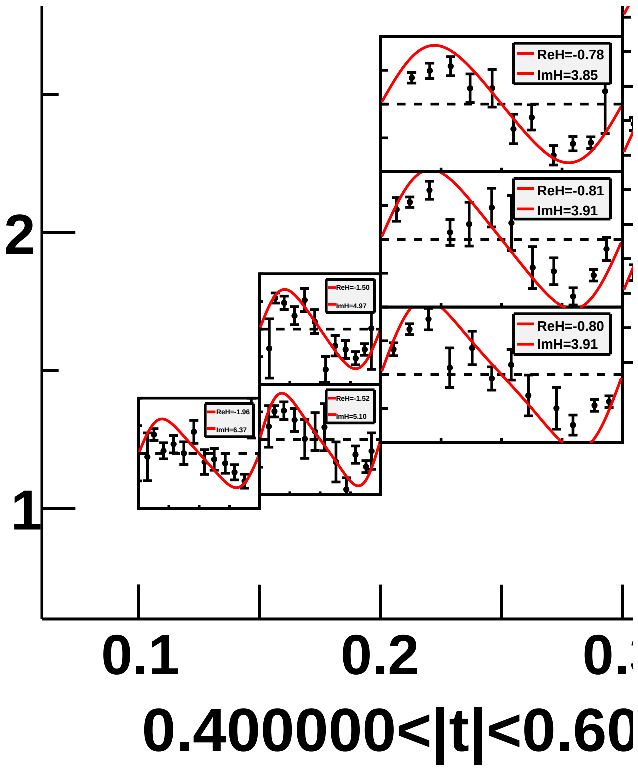
new developments

set of GK GPDs applied to DVCS to LO collinear calculation (prediction)
compatible with GK approach to meson prod.

Kumericky *et al* (11)
Moutarde-Sabatie in progress

first results show reasonable agreement
some difficulties for Jlab kinematics (large skewness, small W, small Q^2)
see plot





vertical: Q^2, horizontal: x_{Bj}

Moutarde-Sabatie prel.

helicity dep. cross sect.

$\sim a + b \sin\phi + c \sin 2\phi$

measures $\mathrm{Im}(BH - DVCS)$

Hall A Munoz *et al* (06)

$Q^2 = 2.3\,\mathrm{GeV}^2$

$-t = 0.23, 0.28, 0.33\mathrm{GeV}^2$

from GK GPDs H, E, $\widetilde{H}$

beam asymmetry

$$A_{LU} = \frac{a \sin\phi}{1 + c\cos\phi + d\cos 2\phi}$$

Hall B Giroz *et al* (08)

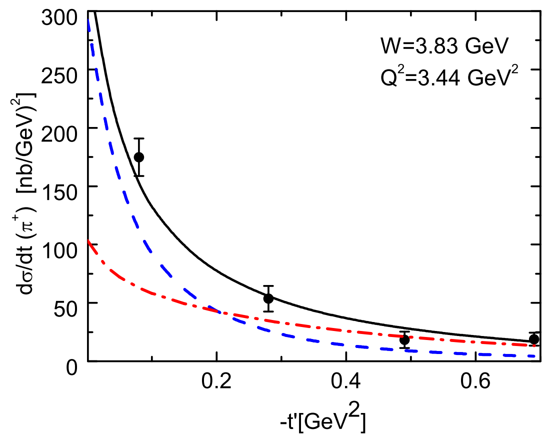
Chiral-odd GPDs

Lattice result for $\bar{E}_T = 2\widetilde{H}_T + E_T$: Large, same sign and almost same size for u and d quarks (as H, $\widetilde{E}^{n.p.}$, others opposite sign) Göckeler et al (06)

Relevant for pion production?

twist-3 effect as for H_T ($\mu = \pm$):

$$\mathcal{M}_{0+,\mu+} = -e_0 \frac{\sqrt{-t'}}{4m} \int_{-1}^{1} d\bar{x} \mathcal{H}^{\text{twist}-3}_{0-,++} \, \bar{E}_T$$

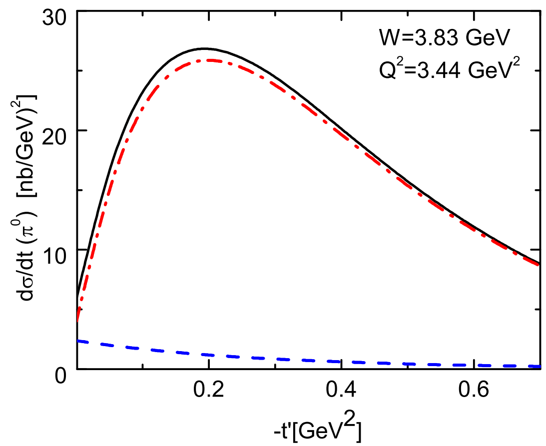


π^+: pion pole and $\propto F^u - F^d$ for all GPDs

π^0: no pion pole and $\propto 2F^u + F^d$

$\Longrightarrow$ $\widetilde{H}$ and H_T large for π^+, small for π^0

$\widetilde{E}^{n.p.}$ and $\bar{E}_T$ small for π^+, large for π^0



$\bar{E}_T$ parameterization:

$e^a_T = \bar{N}^e_T e^{b_{eT} t} x^{-\alpha^e_T(t)} (1-x)^{\beta^a_{eT}}$ as DD

parameters adjusted to lattice results

prominent role of chiral-odd GPDs also claimed by Ahmad et al (08)

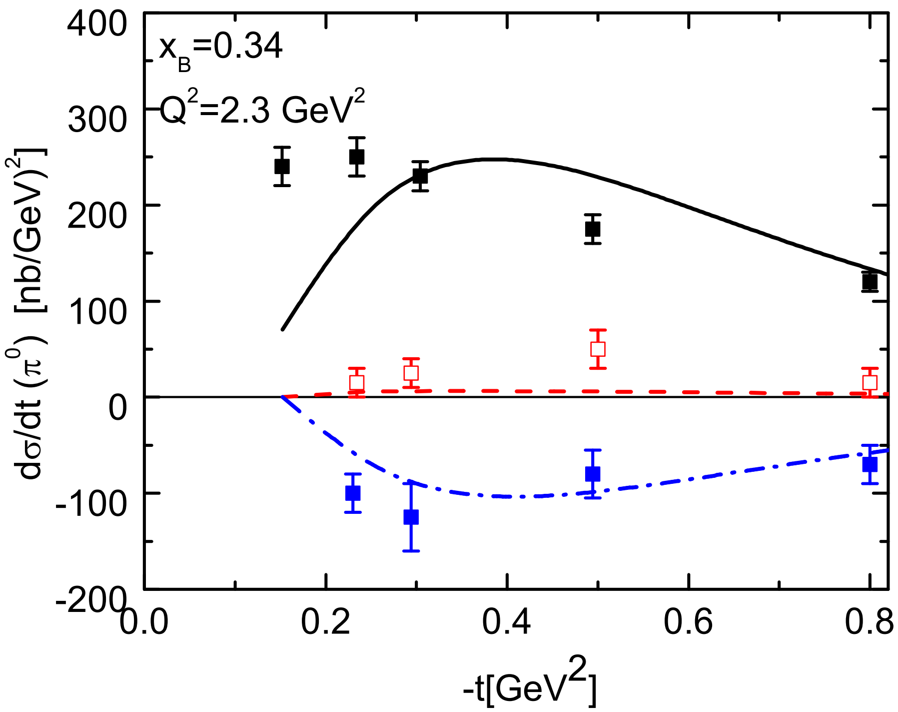
but analysis and results different

CLAS result for π^0 production

CLAS results at low W, i.e. at large skewness

Comparison with our results is to be done with utmost caution

(cf. difficulties with ρ^0 production)

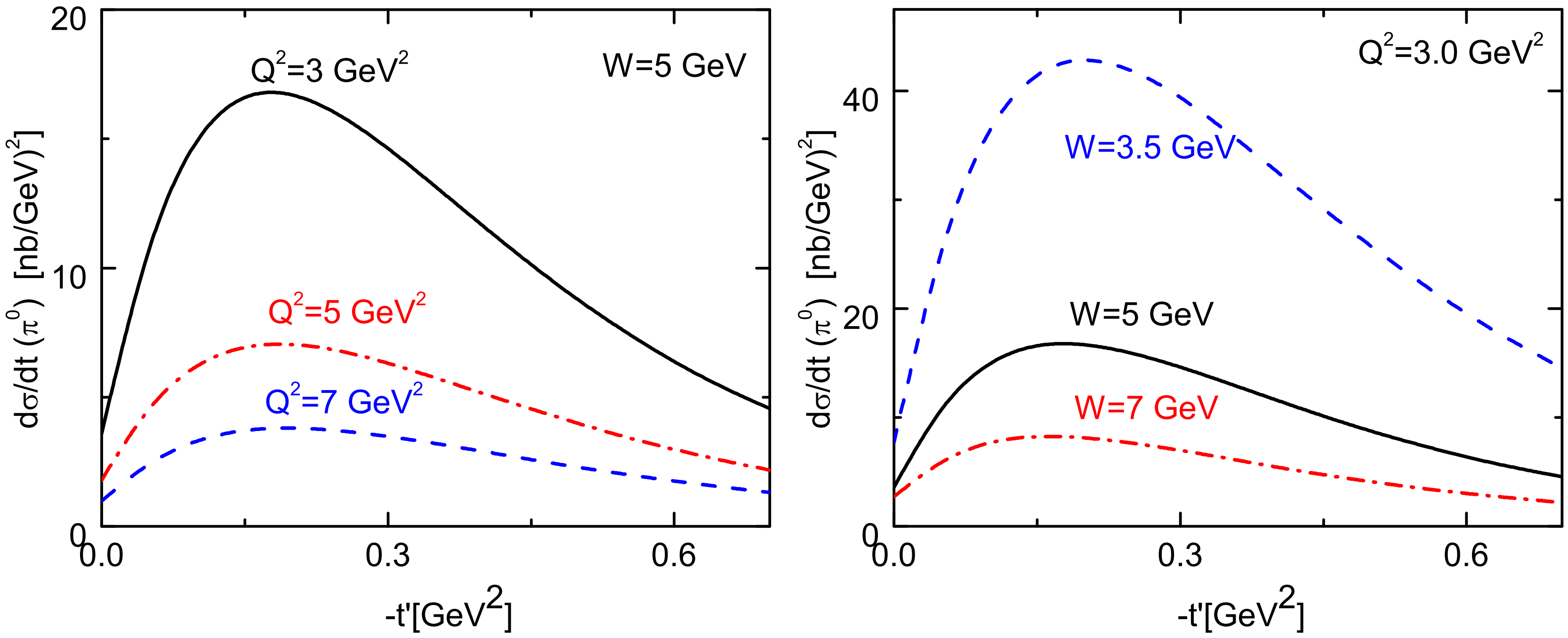


prel. Data: CLAS

unseparated cross section

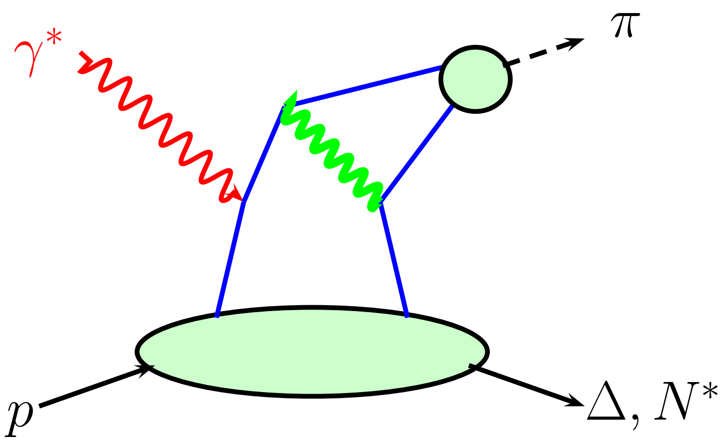
σ_{LT}

σ_{TT}



Results for π^0 cross section versus W at $Q^2 = 3\,\mathrm{GeV}^2$ and versus Q^2 at $W = 5\,\mathrm{GeV}$. The polarization of the photon flux is taken as $\varepsilon = 0.35$ for $W = 3.5\,\mathrm{GeV}$ and 0.8 for $W \geq 5\,\mathrm{GeV}$

Resonance production



new unknown GPDs for
$p \to \Delta, N^*$ transitions

case of Δ (see reviews Goeke et la (01), Belitsky-Radyushkin (04))

in large N_c limit and using SU(3) flavor symmetry:
$p \to \Delta$ GPDs can be related to the proton ones in isovector combination

$$\approx F_i^u - F_i^d$$

available
quality of relations unknown

Summary

- phenomenology of DVME within the handbag approach is complicated, many GPDs contribute, but there are plenty of good data for several mesons
- progress has been made in the determination of a set of GPDs at small ξ; parameterization of GPDs based on double distribution ansatz with a Regge-like t dependence (α', b); for E, $\widetilde{E}$, H_T also forward limit is to be parameterized ($\sim cx^{\alpha}(1-x)^{\beta}$)
- gluon and sea-quark sector almost unknown (exception H), no experimental information as yet
- new devellopment: the set of GPDs is used to predict DVCS
  LO, collinear calculation seems to provide reasonable agreement with experiment
- open question with large ξ region: does handbag physics still apply or have the GPD parameterizations to be improved at large ξ?
  (see failure with $\sigma_L(\rho^0)$)

Strangeness production

e.g. $\gamma^* p \to K^+ \Lambda(\Sigma^0)$

similar to π^+ production
Kaon pole (smaller than pion pole) and
twist-3 effect with $\mu_K = m_K^2/(m_u + m_s) \simeq 1.5\,\mathrm{GeV}$ (also smaller)

would probe $\widetilde{H}$, $\widetilde{E}$ and H_T for flavor symmetry breaking in sea
e.g.

$$F_{p\to\Sigma^0} = -F_v^d + (F^s - F^{\bar{d}})\,,$$

$$F_{p\to\Lambda} = -\tfrac{1}{\sqrt{6}}\Big[2F_v^u - F_v^d + (2F^{\bar{u}} - F^{\bar{d}} - F^s)\Big]$$