# ELECTROPRODUCTION OF NUCLEON RESONANCES AT LARGE MOMENTUM TRANSFERS

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# What are big issues?

- **Chiral symmetry in hard processes**
  - Conjectured: chiral symmetry restored in excited states.
    All discussions so far have been about degeneracies in mass spectra

    **Are the form factors of excited parity partner states equal to each other?**

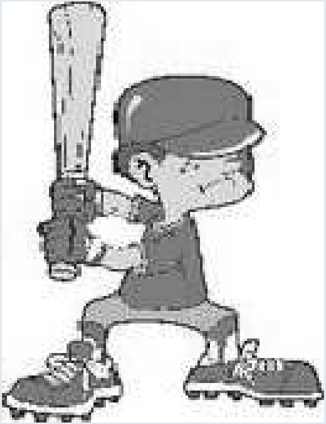
  - Breakdown of chiral perturbation theory at miserable $Q^2 \sim 0.01$ does not imply that chiral symmetry is broken strongly

    **What happens with classical low energy theorems?**
    **How large is the $SU(3)$ flavor breaking? Does it follow any simple pattern?**

- **Valence quark momentum fraction distributions**
  - How do the resonances look like at short distances?
    Are they the same as the nucleon or different?

    **The difference is mainly in pion cloud or it affects the quark core?**

# How to transfer large momentum to a fragile system?



an accelerator

$e^-$, 12 GeV

proton target

# Final state

**Elastic scattering or excitation**

## Final state



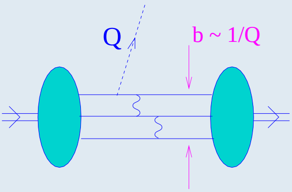
**Elastic scattering or excitation**

**Deep-inelastic scattering**

# How to transfer a large momentum to a weekly bound system?

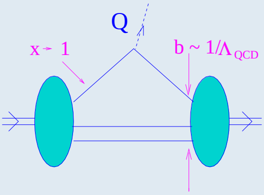
## Heuristic picture:

- quarks can acquire large transverse momenta when they exchange gluons
- "hard" gluon exchanges can be separated from "soft" nonperturbative wave functions
- hard gluons can only be exchanged at small transverse separations




**Hard rescattering:**

**Small** $b$
**Average** $0 < x < 1$




**Soft (Feynman):**

**Average** $b$
**Large** $x \to 1$

## In practice three-quark states indeed seem to dominate, however

- "Squeesing" to small transverse separations occurs very slowly
- Helicity selections rules do not work. Orbital angular momentum?
- $\Rightarrow$ More complicated nonperturbative input needed

# Wave functions and Distribution amplitudes

- **Nucleon light-cone wave function** — Brodsky, Lepage

$$|P\uparrow\rangle^{\ell_z=0} = \int \frac{[dx][d^2\vec{k}]}{12\sqrt{x_1x_2x_3}}\, \psi^{L=0}(x_i,\vec{k}_i) \times$$
$$\times\Big\{\Big|u^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)d^\uparrow(x_3,\vec{k}_3)\Big\rangle - \Big|u^\uparrow(x_1,\vec{k}_1)d^\downarrow(x_2,\vec{k}_2)u^\uparrow(x_3,\vec{k}_3)\Big\rangle\Big\}$$

- **Leading-twist-three distribution amplitude**

Brodsky, Lepage, Peskin, Chernyak, Zhitnitsky

$$\Phi_3(x_1,x_2,x_3;\mu) = 2\int^{\mu}[d^2\vec{k}]\;\psi^{L=0}(x_1,x_2,x_3;\vec{k}_1,\vec{k}_2,\vec{k}_3)$$

**can be studied using the OPE**

$$\Phi_3(x_i;\mu) = 120 f_N x_1x_2x_3\Big\{1 + c_{10}(x_1-2x_2+x_3)L^{\frac{8}{3\beta_0}}$$
$$+\, c_{11}(x_1-x_3)L^{\frac{20}{9\beta_0}} + c_{20}\left[1+7(x_2-2x_1x_3-2x_2^2)\right]L^{\frac{14}{3\beta_0}}$$
$$+\, c_{21}\,(1-4x_2)\,(x_1-x_3)\,L^{\frac{40}{9\beta_0}} + c_{22}\left[3-9x_2+8x_2^2-12x_1x_3\right]L^{\frac{32}{9\beta_0}} + \ldots\Big\}$$

- $f_N(\mu_0)$: **wave function at the origin**
- $c_{nk}(\mu_0)$: **shape parameters**

$L \equiv \alpha_s(\mu)/\alpha_s(\mu_0)$

Braun, Manashov, Rohwild

# Wave functions and Distribution amplitudes (II)

- **Contributions of orbital angular momentum** Ji, Ma, Yuan, '03

$$|P\uparrow\rangle^{\ell_z=1} = \int \frac{[dx][d^2\vec{k}]}{12\sqrt{x_1x_2x_3}} \left[k_1^+\psi_1^{L=1}(x_i,\vec{k}_i) + k_2^+\psi_2^{L=1}(x_i,\vec{k}_i)\right] \times$$
$$\times\left\{\left|u^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)d^\downarrow(x_3,\vec{k}_3)\right\rangle - \left|d^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)u^\downarrow(x_3,\vec{k}_3)\right\rangle\right\}$$

**are related to higher-twist-four distribution amplitudes**

Belitsky, Ji, Yuan, '03

$$\Phi_4(x_2,x_1,x_3;\mu) = 2\int^\mu \frac{[d^2\vec{k}]}{m_N x_3}\, k_3^-\left[k_1^+\psi_1^{L=1} + k_2^+\psi_2^{L=1}\right](x_i,\vec{k}_i)$$

$$\Psi_4(x_1,x_2,x_3;\mu) = 2\int^\mu \frac{[d^2\vec{k}]}{m_N x_2}\, k_2^-\left[k_1^+\psi_1^{L=1} + k_2^+\psi_2^{L=1}\right](x_i,\vec{k}_i)$$

$k^\pm = k_x \pm i k_y$

**and, again, can be studied using OPE** Braun, Fries, Mahnke, Stein '00

$$\Phi_4(x_i;\mu) = 12\lambda_1 x_1x_2 + 12 f_N x_1x_2\left[1 + \frac{2}{3}(1-5x_3)\right] + \ldots$$

$$\Psi_4(x_i;\mu) = 12\lambda_1 x_1x_3 + 12 f_N x_1x_3\left[1 + \frac{2}{3}(1-5x_2)\right] + \ldots$$

- **to this accuracy only one new nonperturbative constant** $\lambda_1(\mu)$

# So what?

The moral of the story so far:

- **Hard (Brodsky-Lepage) and soft (Feynman) contributions to baryon form factors are additive; this can be formalized using "strategy of regions" or SCET**
- **Both must be taken into account**
- **Feynman contribution is complicated because it involves large transverse distances, hence all orbital angular momenta (all twists)**

What can be done:

- **Hope in Sudakov suppression of Feynman, $k_T$ factorization**

  unfortunately seems to be too weak

- **Models for complete baryon wave functions quark models, AdS/QCD, Dyson-Schwinger**

  a model is not a theory

- **Estimate Feynman in terms of DA using dispersion relations and duality LCSR**

  an estimate does not have error bars

# What we are doing:

Braun *et al.* Phys.Rev.Lett.103:072001,2009

- Calculate moments of distribution amplitudes (DAs)
  $\Leftarrow$ **lattice QCD**
  - expensive
  - many technical problems still need to be solved
  - only limited information
  - studies of parity partners look most promising, e.g.

$$\langle 0|qqq|N(p)\rangle = f_N N(p) \qquad \langle 0|qqq|N^*(p)\rangle = f_{N^*}\gamma_5 N(p)$$

- Calculate electroproduction cross sections (transition form factors) in terms of DAs
  $\Leftarrow$ **light-cone sum rules (LCSRs)**
  - based on analyticity and quark-hadron duality
  - well-known and tested technique for mesons, less so for baryons
  - irredicible uncertainty of 20%(?) – need confirmation
  - NLO calculations so far not available

**A large-scale long-term research project within QCDSF:**

**First principles calculation of lowest moments of baryon distribution amplitudes**

**with emphasize on the comparison of states with opposite parity**

## Wilson gauge action, Wilson clover fermions

**Status: May 2011**

- **new data $N_f = 2$:**

| $\beta$ | $\kappa$ | $m_\pi$[GeV] | volume | $a$[fm] | $L$[fm] | $m_\pi L$ |
|---|---|---|---|---|---|---|
| 5.29 | 0.13632 | 0.270 | $24^3 \times 48$ | 0.075 | 1.8 | 2.5 |
| 5.29 | 0.13632 | 0.270 | $32^3 \times 64$ | 0.075 | 2.4 | 3.3 |
| 5.29 | 0.13632 | 0.270 | $40^3 \times 64$ | 0.075 | 3.0 | 4.1 |

- **in progress $N_f = 2$:**

| $\beta$ | $\kappa$ | $m_\pi$[GeV] | volume | $a$[fm] | $L$[fm] | $m_\pi L$ |
|---|---|---|---|---|---|---|
| 5.29 | 0.13640 | 0.170 | $48^3 \times 64$ | 0.075 | 3.6 | 3.1 |

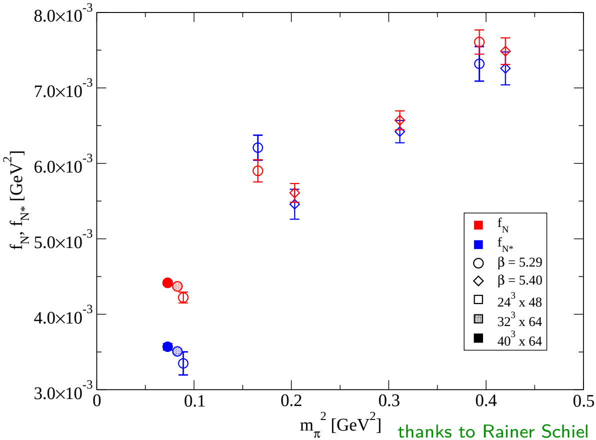
- **starting $N_f = 2 + 1$ (PRACE proposal):** $\Lambda(1116), \Lambda(1405)$

| $\beta$ | $\kappa_l$ | $m_\pi$[GeV] | volume | $a$[fm] | $L$[fm] | $m_\pi L$ |
|---|---|---|---|---|---|---|
| 5.5 | 0.121095 | 0.290 | $32^3 \times 64$ | 0.079 | 2.5 | 3.7 |
| 5.5 | 0.121145 | 0.241 | $32^3 \times 64$ | 0.079 | 2.5 | 3.1 |
| 5.5 | 0.121193 | 0.180 | $32^3 \times 64$ | 0.079 | 2.5 | 2.3 |
| 5.5 | 0.121193 | 0.180 | $48^3 \times 96$ | 0.079 | 3.8 | 3.5 |

## Wave functions at the origin, $L = 0$

**new:**

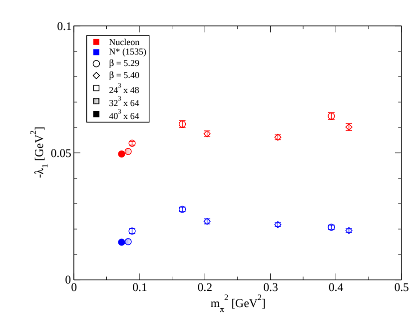
$$a = 0.075 \text{ fm}$$
$$m_\pi = 270 \text{ MeV}$$
$$m_\pi L = 2.5, 3.3, 4.1$$

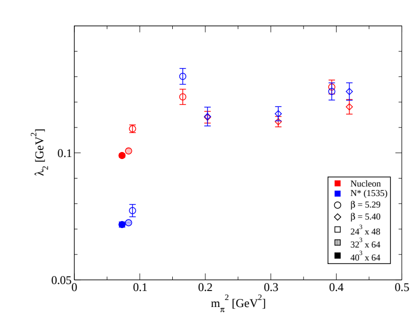



thanks to Rainer Schiel

- **All results preliminary, statistical errors only**

# Orbital angular momentum $L = 1$

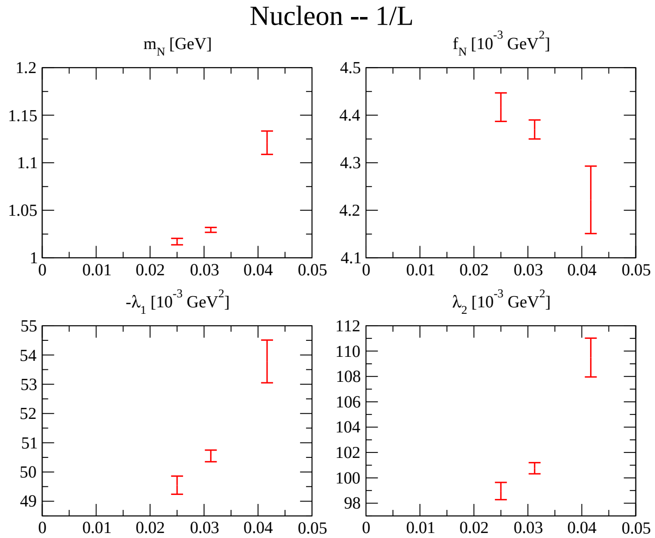



thanks to Rainer Schiel

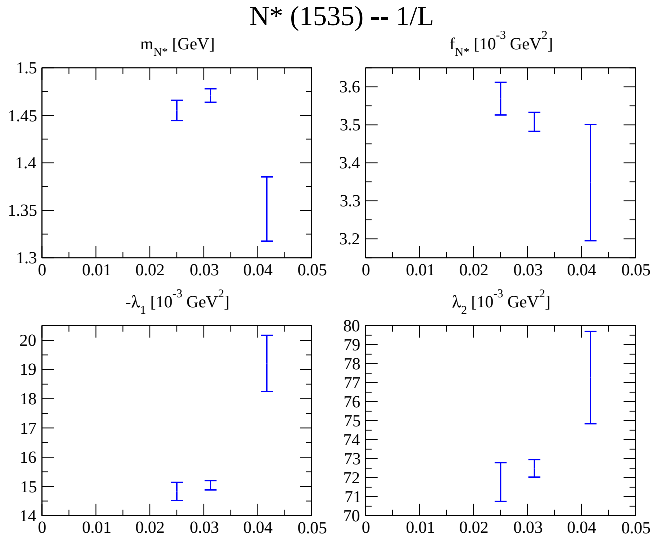
- **All results preliminary, statistical errors only**

## Finite size effects



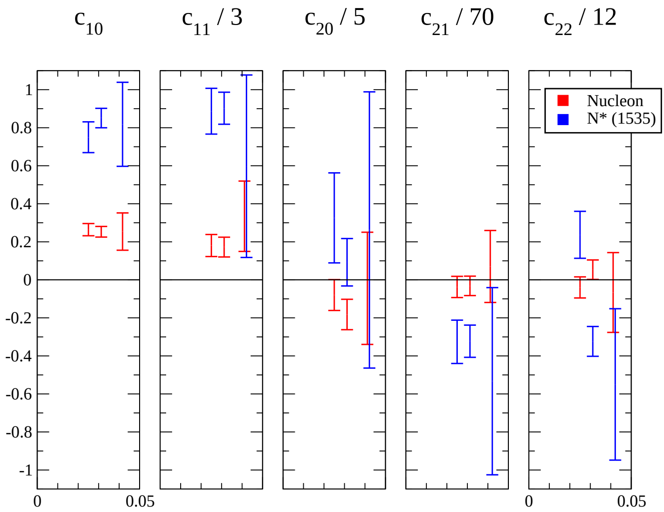

thanks to Rainer Schiel

## Finite size effects




thanks to Rainer Schiel

## Shape parameters

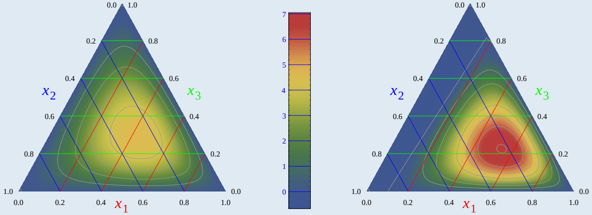



thanks to Rainer Schiel

## Valence quark distributions

**"Mandelstam plot":** $s + t + u = 4m^2 \Rightarrow x_1 + x_2 + x_3 = 1$

$N(940)$ $N^*(1535)/N^*(1650)$



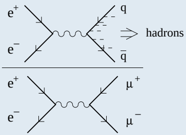

thanks to Rainer Schiel

**Momentum fractions carried by valence quarks:**

| | $N$ | $N^*$ |
|---|---|---|
| $u^\uparrow$ | $0.37 \pm (?)$ | $0.49 \pm (?)$ |
| $q^\downarrow$ | $0.31 \pm (?)$ | $0.26 \pm (?)$ |
| $q^\uparrow$ | $0.32 \pm (?)$ | $0.25 \pm (?)$ |

# From distribution amplitides to form factors: Duality

$R(s = E_{\rm cm}^2) =$



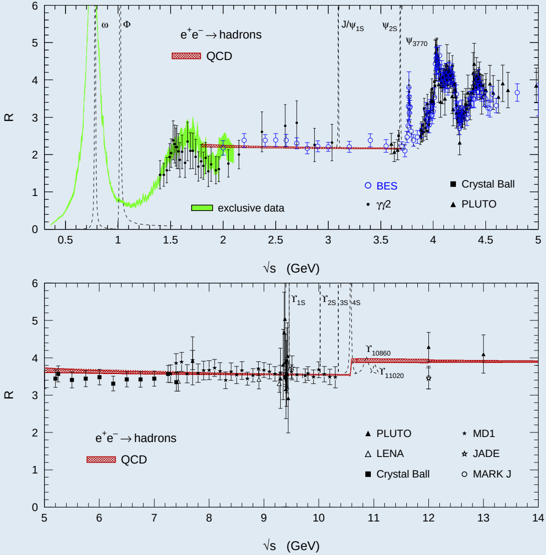

observe

$$R^{\rm QCD}(s) \neq R^{\rm exp}(s)$$

in the resonance region $\sqrt{s} < 1.5 GeV$, but

$$\int_0^{s_0} ds\, R^{\rm QCD}(s) = \int_0^{s_0} ds\, R^{\rm exp}(s)$$

$s_0$ is called interval of duality




*Davier et al., Eur.Phys.J.C27:497-521,2003*

## a consequence of two major principles:

- **unitarity** ← probability interpretation of wave functions

$$R(s) \quad = \quad \frac{1}{\pi}\mathrm{Im}\ \Pi(s=q^2)$$

where

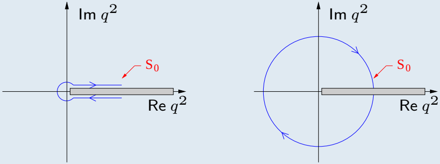
$$i\int d^4x\, e^{iqx}\langle 0|\,T\{j_\mu^{\rm em}(x)j_\nu^{\rm em}(0)\}|0\rangle \;=\; (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi(q^2)$$

- **analyticity** ← causality

$$R(s) \;=\; \frac{1}{2\pi i}[\Pi(q^2+i\epsilon)-\Pi(q^2-i\epsilon)]$$

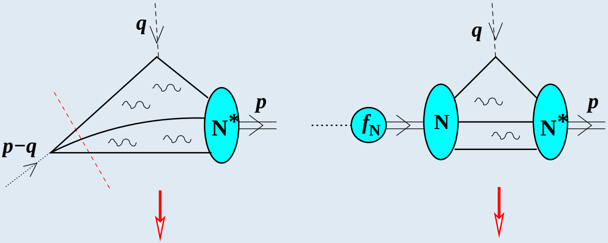
$$\int_0^{s_0} ds\, R(s) \;=\; \frac{1}{2\pi i}\oint dq^2\; R(q^2)$$

$$\simeq \frac{1}{2i}\oint dq^2\; R^{\rm pQCD}(q^2)$$




because the region of $q^2 \sim \Lambda^2_{\rm QCD}$ is avoided

- a development of this idea ⇒ Light-Cone Sum Rules:




$$\frac{1}{\pi}\int_0^{s_0} ds\, \mathrm{Im}\, T(p,q) \quad \stackrel{duality}{=} \quad f_N\, F_{N\to N^*}(Q^2)$$

- $T(p,q)$ is calculated in terms of $N^*$ distribution amplitudes
  *Balitsky, Braun, Kolesnichenko, Nucl.Phys.B312:509-550,1989*
  *Braun, Halperin, Phys.Lett.B328:457-465,1994*

- This is a Feynman (soft) contribution; hard terms can be added systematically and without double counting

**Question:**

**Can one use LCSRs to extract the distribution amplitutudes from experimental data?**

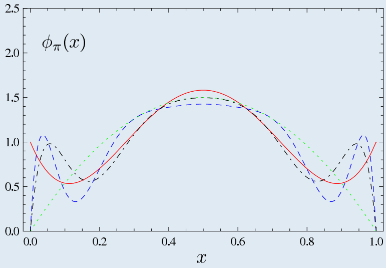
**To what detail?**

**Defensible error bars?**

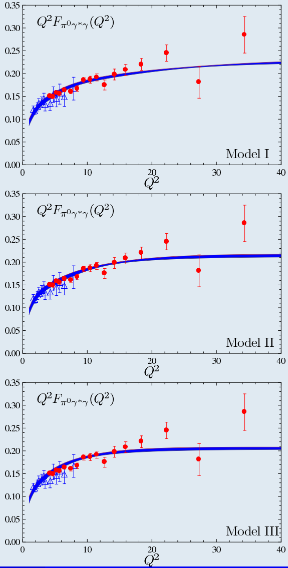
## Example: $\gamma^* \to \pi\gamma$ form factor

B. Aubert *et al.* [The BABAR Collaboration], Phys. Rev. **D80**, 052002 (2009)

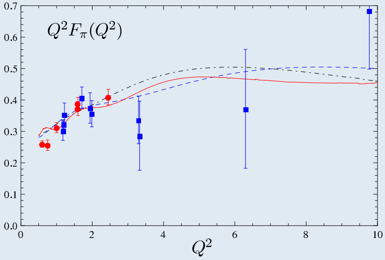
- Two-component models

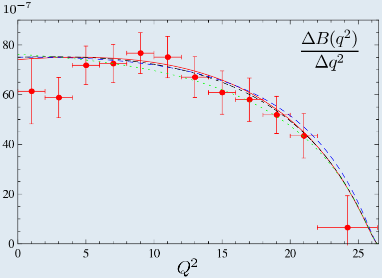



*S.S. Agaev, V.M. Braun, N. Offen, F.A. Porkert, Phys. Rev.* ***D83*** *(2011) 054020*



- The same models describe pion form EM factor and $B \to \pi \ell \nu_\ell$ width



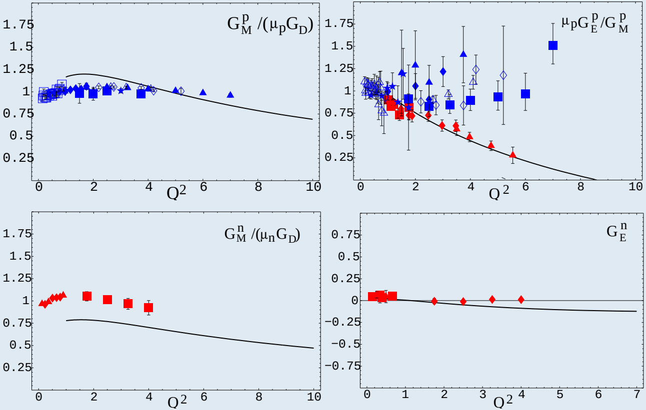

- However, these two can be described with much simpler models as well

- **state-of-the-art NLO LCSRs in all cases**

## Light-Cone Sum Rules: Nucleon Electromagnetic Formfactors

**proton**

**neutron**




*Braun, Lenz, Wittmann; PRD73:094019,2006*

- **Nucleon DAs fitted to the $G_E^p/G_M^p$ ratio**

## Towards baryon LCSRs with NLO corrections

Passek-Kumericki, Peters, Phys.Rev.D78:033009,2008

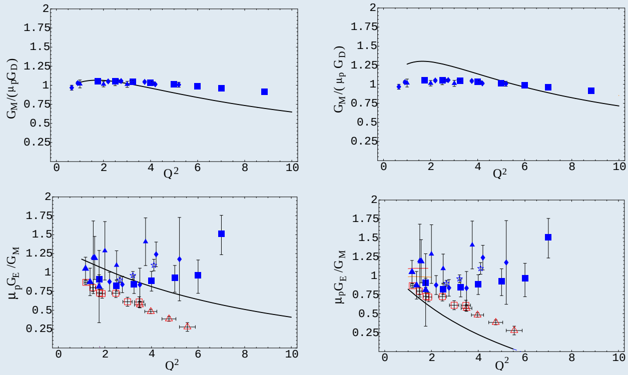


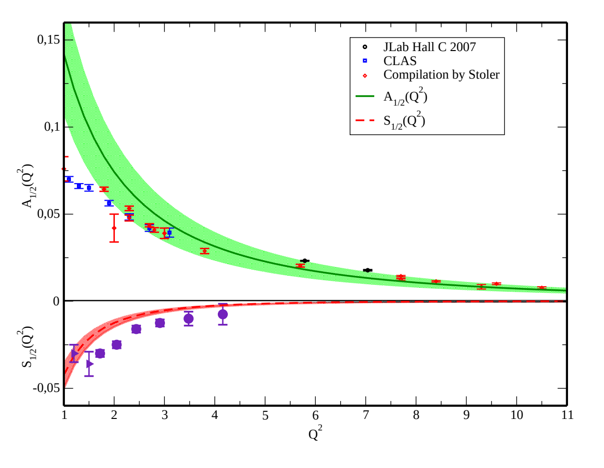

Figure: LCSR results for the electromagnetic proton form factors for a realistic model of nucleon distribution amplitudes. Left panel: Leading order (LO); right panel: next-to-leading order (NLO) for twist-three contributions. Figure adapted from [PassekKumericki:2008sj].

- NEW **a consistent renormalization scheme for three-quark operators (S. Kränkl, A. Manashov, in preparation)**

## $\gamma^* N \to N^*(1535)$: helicity amplitudes

- **A pilot project:** Braun *et al.* Phys.Rev.Lett.103:072001,2009

**Electroproduction of $N^*(1535)$ with lattice-constrained $N^*$ distribution amplitides**




CLAS data: *I.G. Aznauryan et al., Phys.Rev.C80:055203,2009*

## Electroproduction with $Q^2$ in a few GeV$^2$ range:

- **Tradition: excitation of nucleon resonances (transition form factors)**

$$
\begin{aligned}
e(l) + p(P) &\rightarrow e(l') + \Delta(1232)(P') \\
e(l) + p(P) &\rightarrow e(l') + N(1440)(P')
\end{aligned}
$$

- **Proposal: pion electroproduction close to threshold** $W \to W_{\rm th}$

$$
\begin{aligned}
e(l) + p(P) &\rightarrow e(l') + \pi^+(k) + n(P') \\
e(l) + p(P) &\rightarrow e(l') + \pi^0(k) + p(P')
\end{aligned}
$$

$$
\begin{aligned}
W^2 &= (P' + k)^2 \\
W_{\rm th} &= m_N + m_\pi \\
Q^2 &= -q^2 = -(\ell - \ell')^2
\end{aligned}
$$

# Generalized Form Factors = S-wave Multipoles at Threshold

**at the threshold**

$$\langle \pi N | j_\mu^{\rm em} | p \rangle = -\frac{i}{f_\pi} \bar{N}(P_2)\gamma_5 \left\{ \left(\gamma_\mu q^2 - q_\mu \not{q}\right) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} G_2^{\pi N}(Q^2) \right\} N(P_2)$$

**related to S-wave multipoles in the PWA, e.g. for** $m_\pi = 0$

$$E_{0+}^{\pi N}(Q^2, W_{\rm th}) = \frac{\sqrt{4\pi\alpha_{\rm em}}}{8\pi} \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_1^{\pi N}$$

$$L_{0+}^{\pi N}(Q^2, W_{\rm th}) = \frac{\sqrt{4\pi\alpha_{\rm em}}}{32\pi} \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_2^{\pi N}$$

**e.g. the differential cross section at threshold is given by**

$$\left.\frac{d\sigma_{\gamma^*}}{d\Omega_\pi}\right|_{\rm th} = \frac{2|\vec{k}_f| W}{W^2 - m_N^2} \left[ (E_{0+}^{\pi N})^2 + \epsilon \frac{Q^2}{(\omega_\gamma^{\rm th})^2} (L_{0+}^{\pi N})^2 \right]$$

# Chiral rotation

- **In the chiral limit, $m_\pi/m_N \to 0$, the pion can be "rotated" away:**

$$
\begin{aligned}
|p\uparrow\rangle &= \frac{\phi_s(x)}{\sqrt{6}}|2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle + \frac{\phi_a(x)}{\sqrt{2}}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle \\
|p\uparrow\pi^0\rangle &= \frac{\phi_s(x)}{2\sqrt{6}f_\pi}|6u_\uparrow d_\downarrow u_\uparrow + u_\uparrow u_\downarrow d_\uparrow + d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2\sqrt{2}f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle \\
|n\uparrow\pi^+\rangle &= \frac{\phi_s(x)}{\sqrt{12}f_\pi}|2u_\uparrow d_\downarrow u_\uparrow - 3u_\uparrow u_\downarrow d_\uparrow - 3d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle
\end{aligned}
$$

Pobylitsa, Polyakov, Strikman; PRL87(2001)022001

- **allows one to "look" at the proton from a different "angle"**

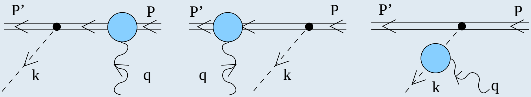
# Low-Energy Theorems

- *predate ChPT and QCD*

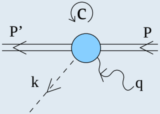
**Chiral symmetry:**

1. pion mass $m_\pi \to 0$
2. pion coupling $\sim |k| \to 0$

- **Pion emission from external legs**




- **Chiral Rotation**




$$\langle \pi^a N | j_\mu^{\rm em} | N \rangle \sim \frac{i}{f_\pi} \langle N | [j_\mu^{\rm em}, Q_5^a] | N \rangle$$

Kroll, Ruderman ’54

## Low-Energy Theorems – *continued*

**PCAC + current algebra:**

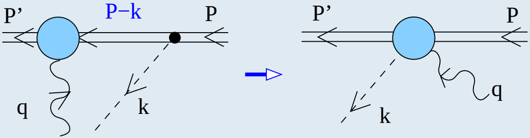
$$\frac{Q^2}{m_N^2} G_1^{\pi^0 p} = \frac{g_A}{2} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^p\,, \qquad G_2^{\pi^0 p} = \frac{2 g_A m_N^2}{(Q^2 + 2m_N^2)} G_E^p\,,$$

$$\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^n + \frac{1}{\sqrt{2}} G_A\,, \qquad G_2^{\pi^+ n} = \frac{2\sqrt{2} g_A m_N^2}{(Q^2 + 2m_N^2)} G_E^n\,,$$

- ◇ **The $\mathcal{O}(m_\pi)$ corrections can be added**
- ◇ **but, no systematic way to treat $\mathcal{O}(m_\pi^2)$ terms (ChPT)**

- expected to fail for $Q^2 \sim \frac{m_N^3}{m_\pi}$

**since $\pi$ cannot have small momentum w.r.t. the initial and final state protons simultaneously**




**at threshold**

$$m_N^2 - (P-k)^2 = \frac{m_\pi}{m_N}\left[Q^2 + 2m_N^2\right]$$

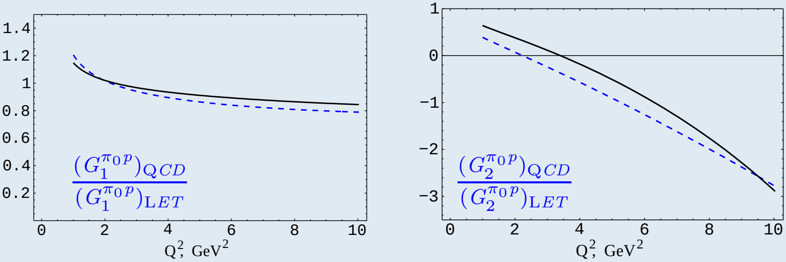
⇒ **phenomenological Lagrangians to take into account nucleon resonances**

⇒ **or go over to quark-gluon description**

**LCSR: Deviation from LET:**

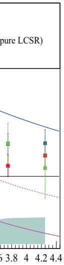
*Braun, Ivanov, Lenz, Peters, PRD75:014021,2007*

*Braun, Ivanov, Peters, PRD77:034016,2008*




♡ **LCSR reproduce LET for $Q^2 \sim 1$ GeV$^2$**

*Puneet Khetarpal, PhD Thesis, Aug. 2010*




pole formula $G_D$ are plotted as
using three methods (red, blue
tistical uncertainties added in
he bottom. The LCSR based
ncertainties (magenta-dashed)

98

♡ **Nice confirmation of LET in $\pi^0$ electroproduction**

# Nucleon axial form factor from $\pi^+$ electroproduction at threshold

- **a very old idea**

$$\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^n + \frac{1}{\sqrt{2}} G_A$$

? **need $S$-wave very close to threshold**

? **may help that $P$-wave is rather well understood**

- **may work better for large $Q^2$ as for small $Q^2$!**

## Once more, physical issues:

- **Chiral symmetry in hard processes**
  - Conjectured: chiral symmetry restored in excited states.
    All discussions so far have been about degeneracies in mass spectra

    **Are the form factors of excited parity partner states equal to each other?**

  - Breakdown of chiral perturbation theory at miserable $Q^2 \sim 0.01$ does not imply that chiral symmetry is broken strongly

    **What happens with classical low energy theorems?**
    **How large is the $SU(3)$ flavor breaking? Does it follow any simple pattern?**

- **Valence quark momentum fraction distributions**
  - How do the resonances look like at short distances?
    Are they the same as the nucleon or different?

    **The difference is mainly in pion cloud or it affects the quark core?**

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