Few-Nucleon (elastic) Form Factors

Franz Gross — Jefferson Lab & College of William and Mary

- ★ Background
- ★ Theoretical methods (LF excluded)
  - Covariant Spectator Theory (CST)
  - χEFT; comparison with CST
  - Role of relativity
- ★ Comparison with data
- ★ Summary and Outlook

Reviews:

- ★ Gillman and FG, JPG 28 (2002) R37
- ★ Marcucci, et.al, JPG 43 (2016) 023002

L. Marcucci, FG, M.T. Pena, M. Piarulli, R. Schiavilla, I. Sick, A. Stadler, J.W. Van Orden, M. Viviani

Background — what are few-nucleon form factors?

★ Elastic electron scattering from few-body nuclei (in one photon exchange approximation) can be expressed in terms of 2J+1 scalar functions of Q^2 only (form factors)

deuteron	$J=1$	G_C, G_M, G_Q
^{3}H, ^{3}He,	$J=\frac{1}{2}$	F_C, F_M (similar to N G_E, G_M)
^{4}He,	$J=0$	F_C

★ Nonrelativistic interpretation:

- Fourier transform of EM multiple distributions

Background — measurements of few-nucleon form factors

★ What was the purpose of this experimental program?

1. To understand the electromagnetic shape and structure of few-nucleon systems, and test the predictions of NN force models
2. To study the NN interaction and nuclear current at short range when they are off-shell
3. To learn how to calculate these quantities from field theory, and to define relativistic wave functions
4. To understand (hopefully?) the transition from nucleon to quark degrees of freedom

★ To study items 2, 3 & 4, measurements at high Q were needed

★ Studies of these issues have lead to significant theoretical developments

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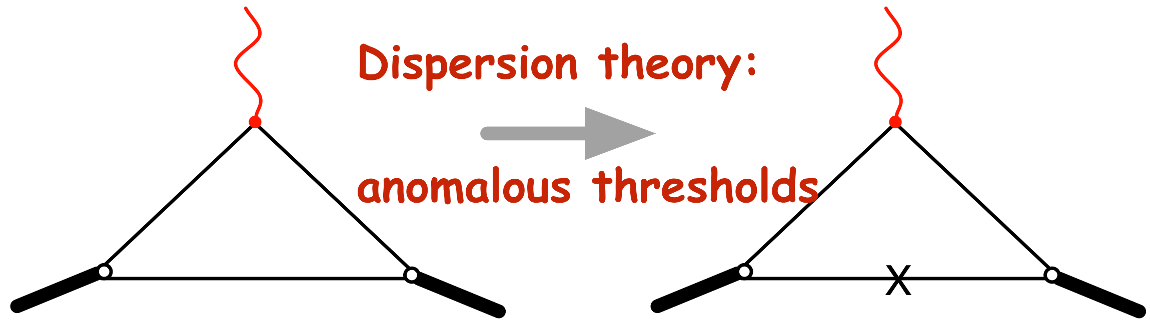
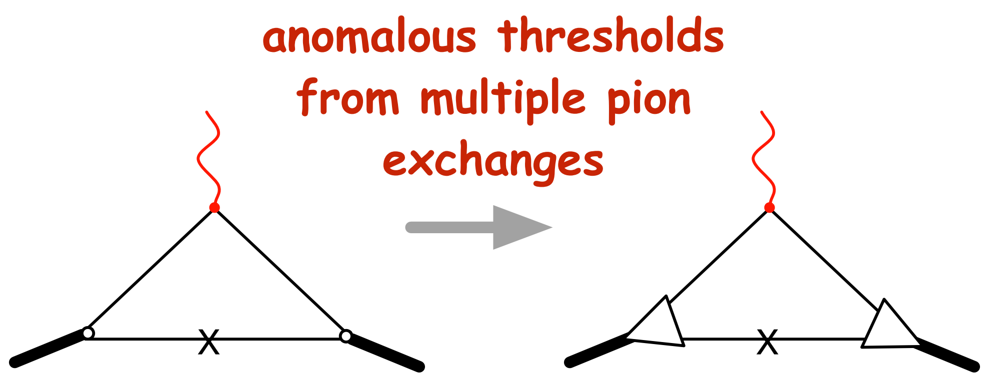
Background — Origin of the CST (1960's)

★ Study of the relativistic triangle diagram for the current of a 2 body loosely bound system:

- full Feynman triangle approximated by spectator on shell



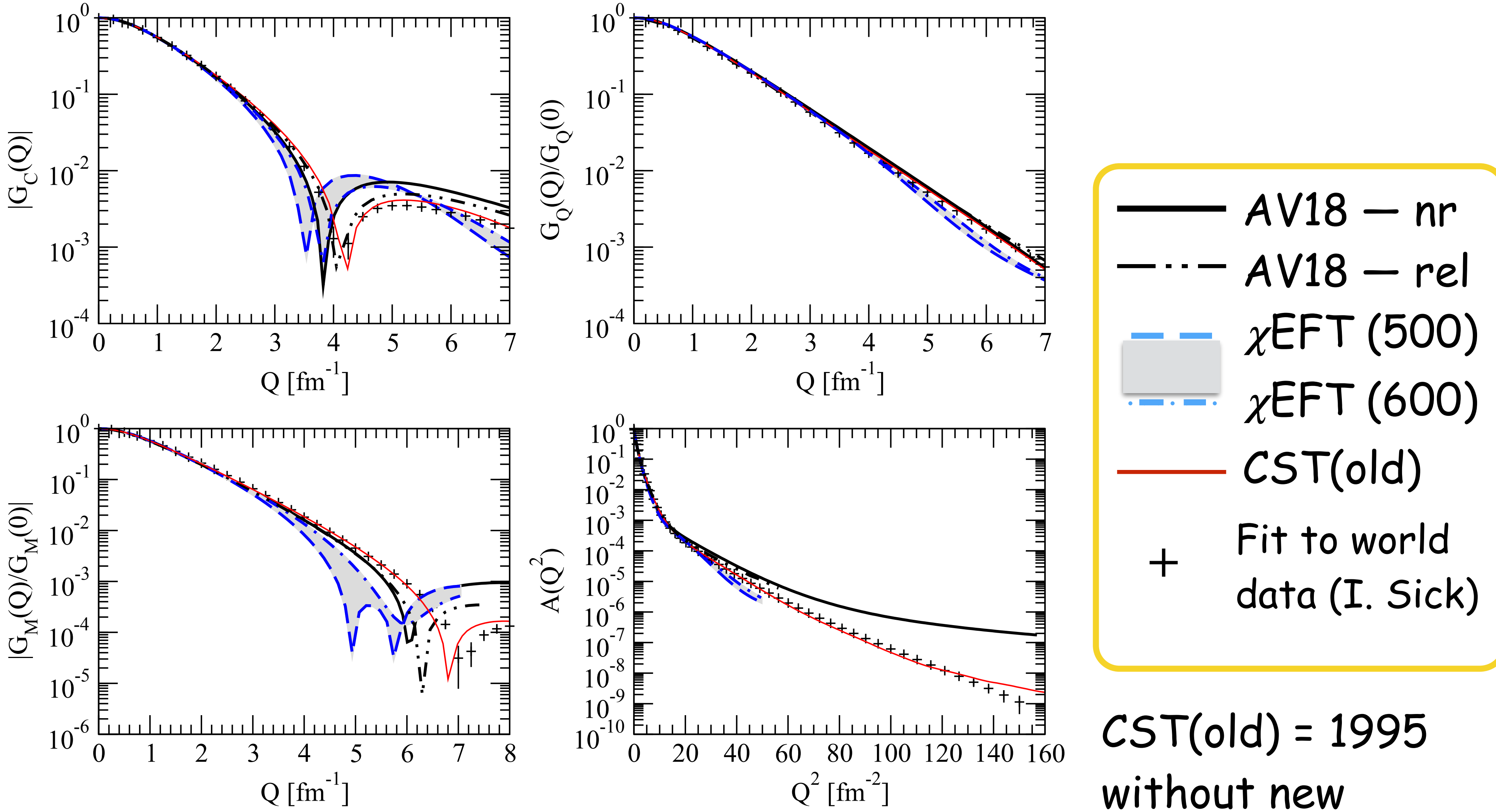
- Multiple pion exchange terms dress the dNN vertex leading to a natural definition of a wave function



Comparison with data

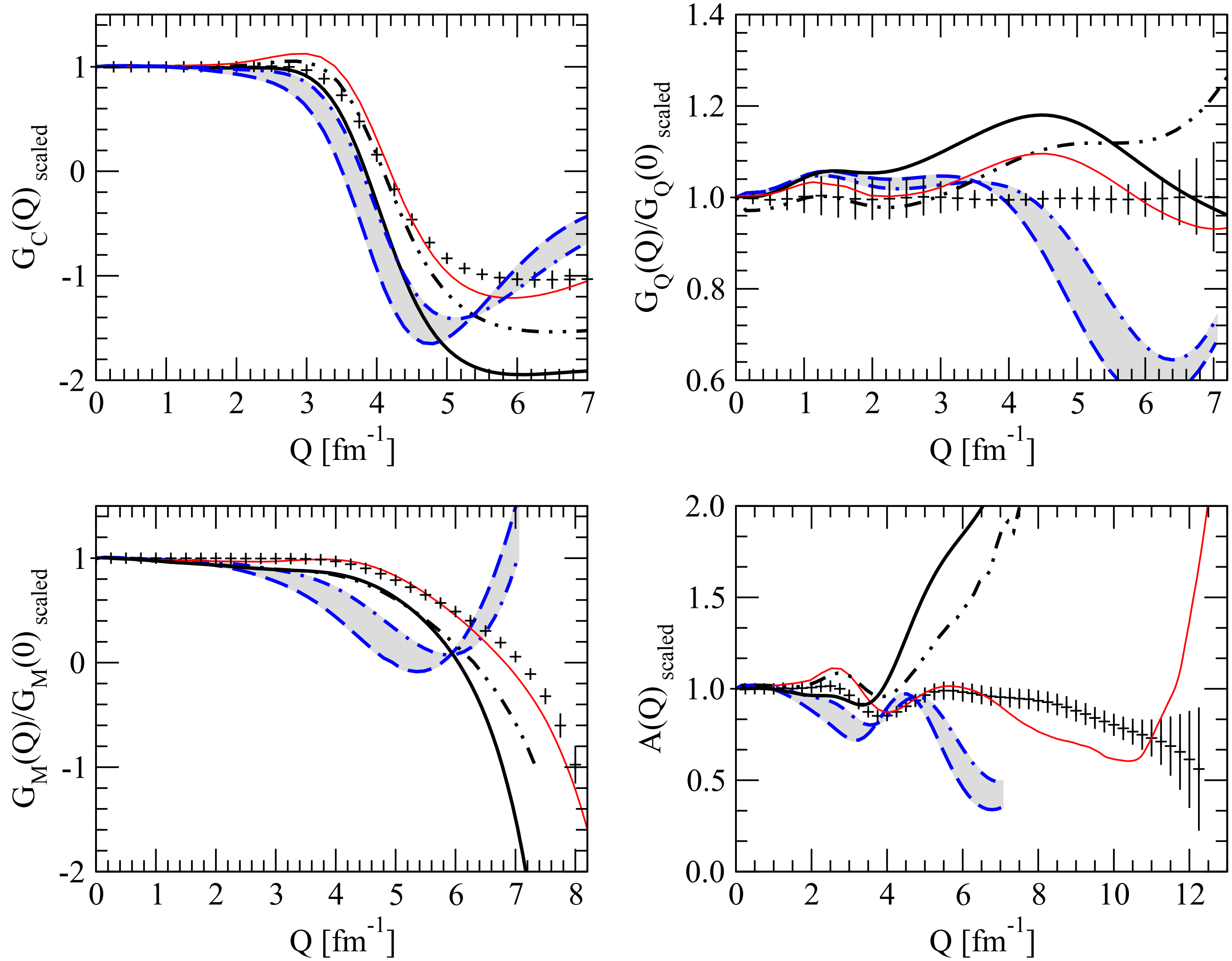
Data represented by Ingo Sick's fits to the world data set

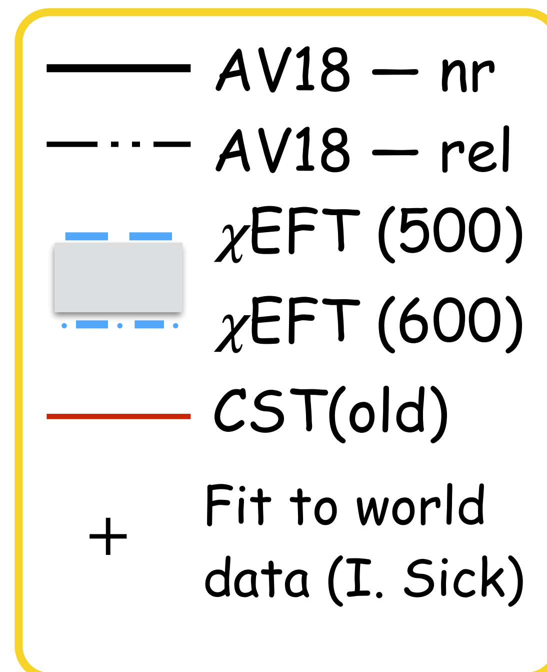
Deuteron form factors — comparison to data



CST(old) = 1995 without new exchange currents (with van Orden)

Scaled deuteron form factors — comparison to data





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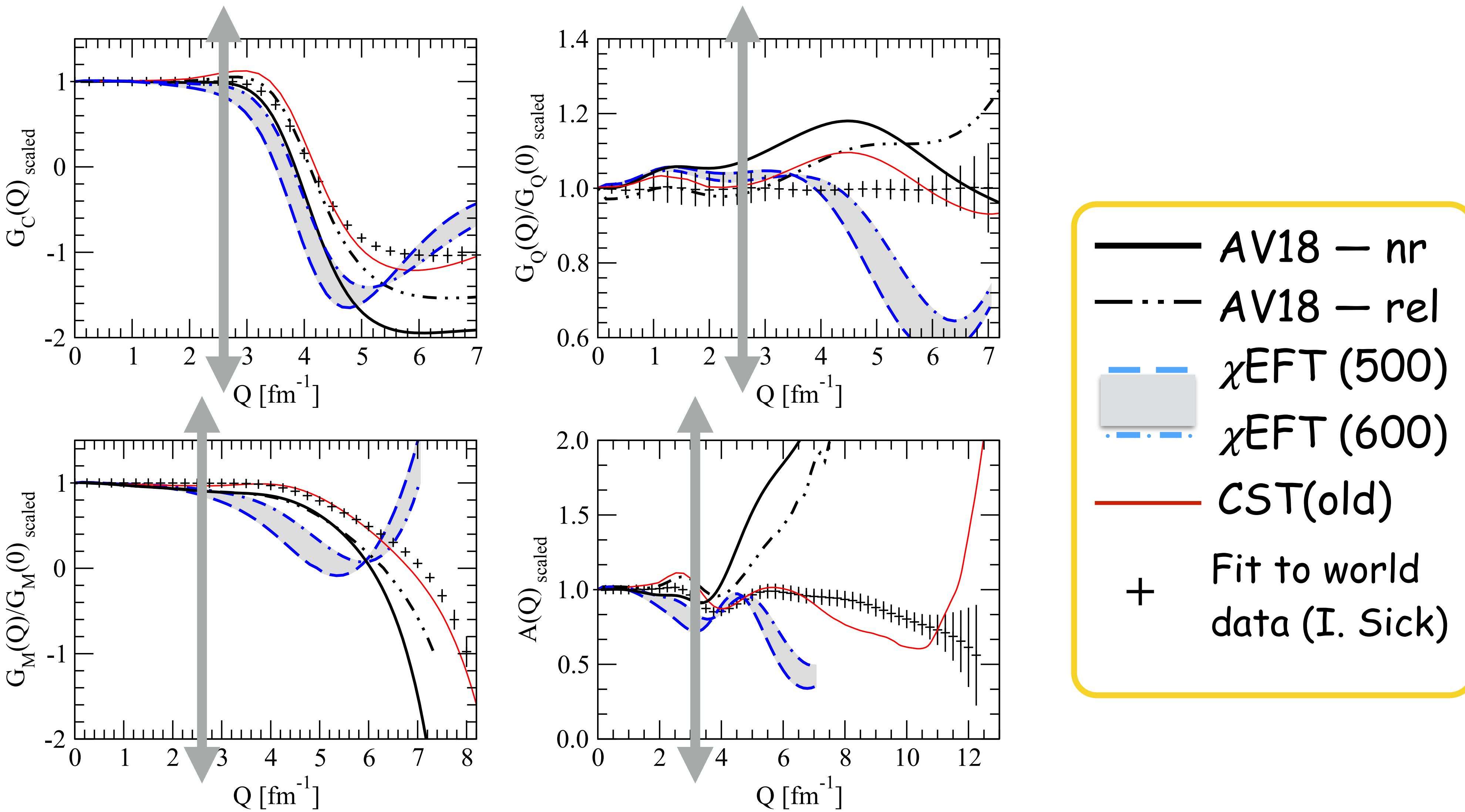


All approaches agree out to 2.6 fm^{-1} ~ 0.5 GeV

Only CST agrees over the whole range

Scaled 3N form factors — comparison to data

- ★ χEFT
  - successful (with some cutoff dependence) !
  - breaks down at about 4 fm^{-1}
- ★ CST calculation is incomplete; OK for what it includes

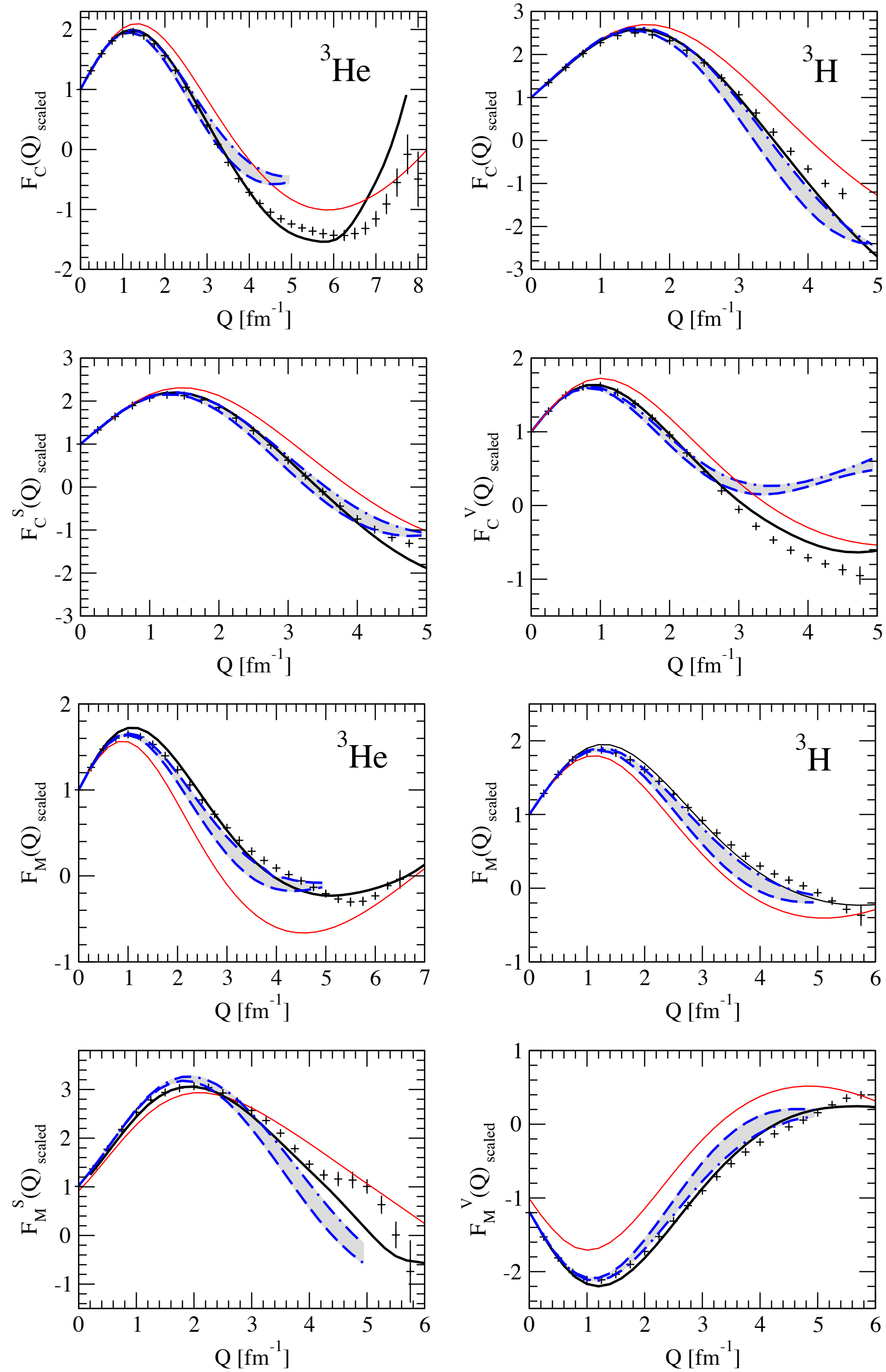
AV18 — nr

χEFT (500)

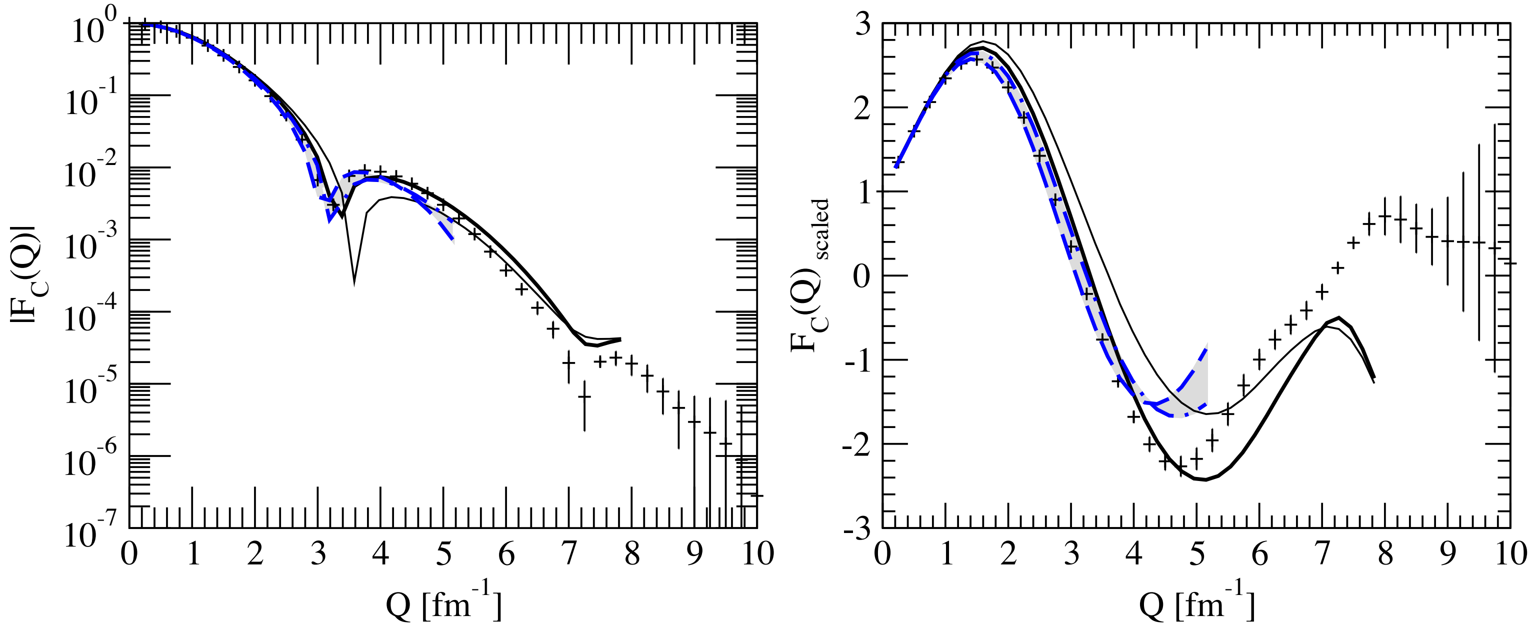
χEFT (600)

CST

+ Fit to world data (I. Sick)



^{4}He form factors — comparison to data



AV18/UIX — 1+2 body
AV18/UIX — 1 body only
χEFT (500)
χEFT (600)
+ Fit to world data (I. Sick)

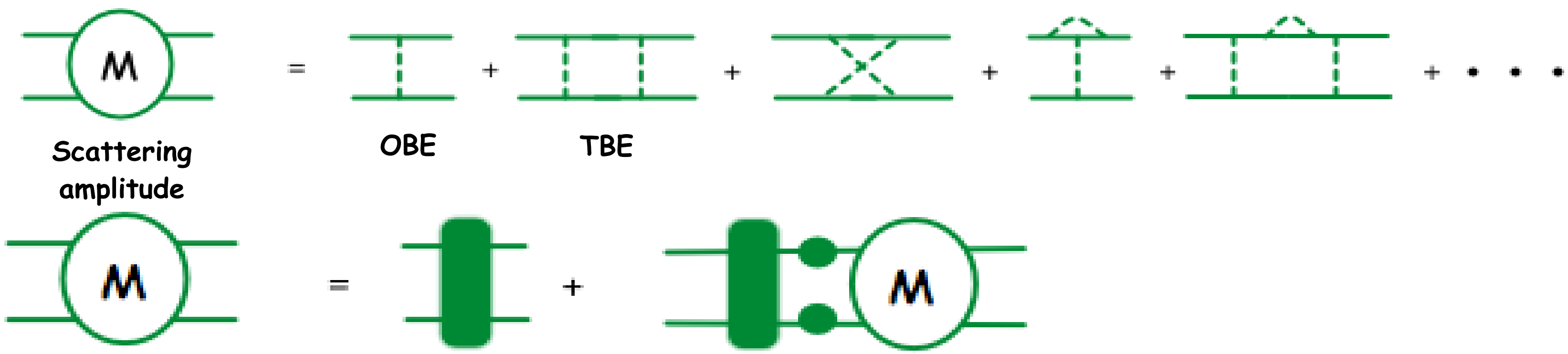
- ★ First 4 body χEFT calculation !
- ★ χEFT breaks down (again) at about 4 fm^{-1}
- ★ AV18/UIX: 2 body currents — significant improvement

Covariant Spectator Theory

Overview of the CST — what is it?

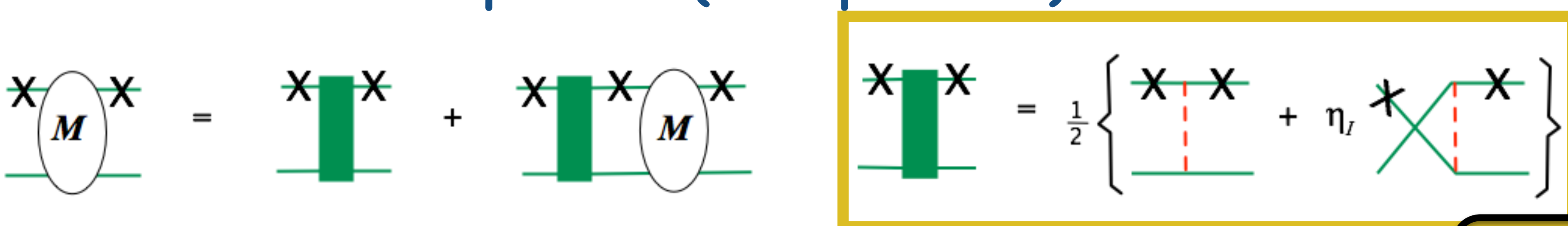
- ★ To study bound states, need to sum infinite number of diagrams to generate the bound-state pole

- ★ 1951 — Bethe-Salpeter equation: both nucleons off-shell



irreducible kernel cannot be computed in closed form

- ★ 1969 — CST: one particle (the spectator) is on-shell



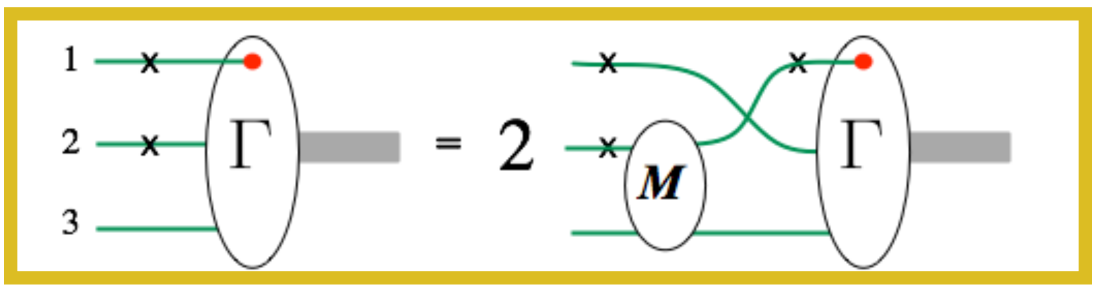
CST is a more convergent version of the BS equation

OBE

Overview of the CST

- ★ 1987 (with D. O. Riska): Derived a 2-body current that is conserved, even in the presence of phenomenological form factors

- ★ 1997 (with A. Stadler): CST OBE requires NO 3 or many-body forces:



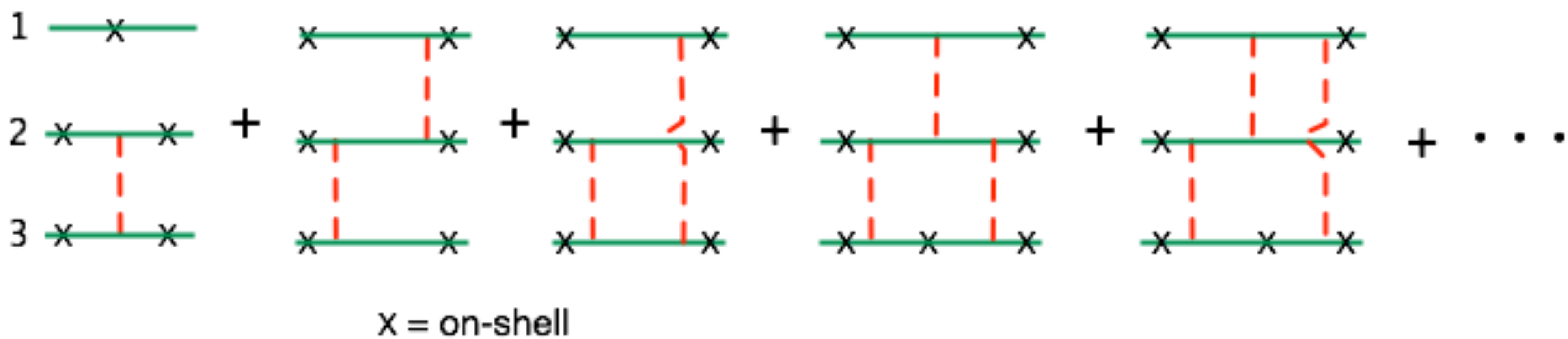
x = on-shell

  - three body equation driven by the same two body scattering amplitude — NO 3-body forces

NO 3-body forces in a CST OBE model

★ Diagrammatic demonstration:

- CST ladder sums are NOT time-ordered — can always be separated into successive 2-body interactions



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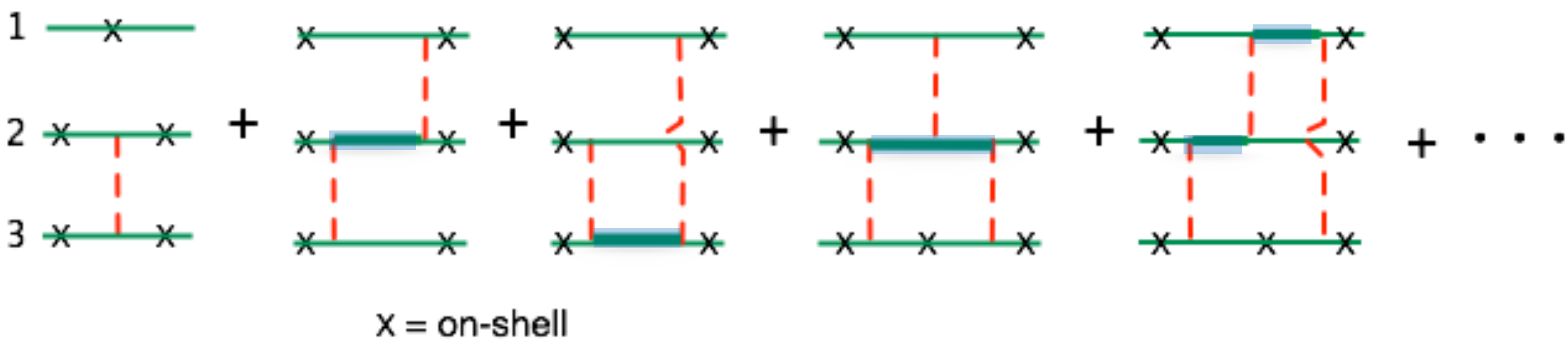
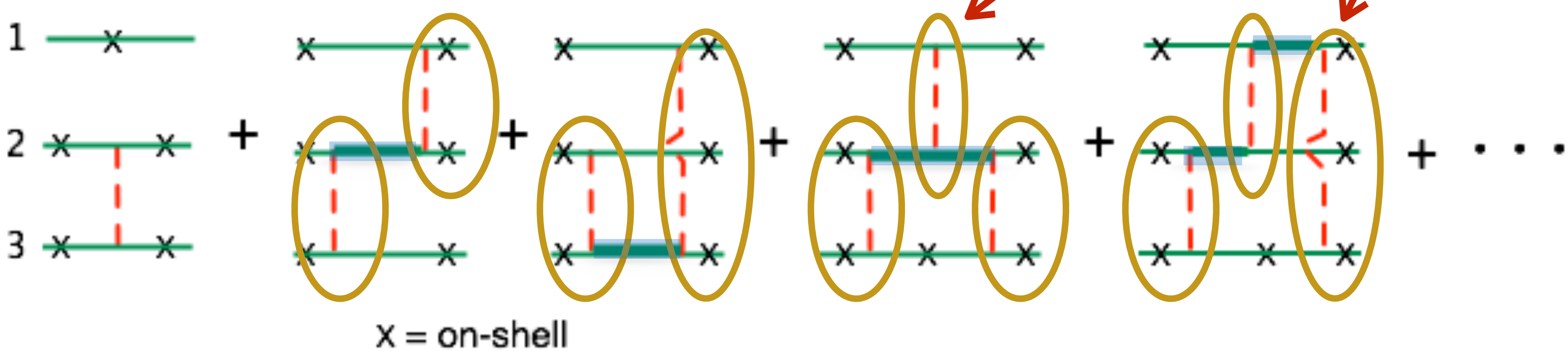


NO 3-body forces in a CST OBE model

★ **Diagrammatic demonstration:**

- CST ladder sums are NOT time-ordered — can always be separated into successive 2-body interactions

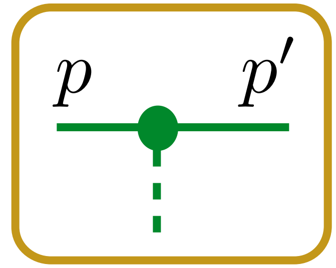
Same OBE kernel used in the 2-body problem



Overview of the CST

- ★ **2004** (with A. Stadler and M. T. Pena): using OBE models, derived a conserved 3-body current using the same 1-body and 2-body interaction currents needed for the 2-body system
- ★ **2008** (with A. Stadler): An OBE model WJC-2 was found that gave a precise fit to np data ($\chi^2/N_{data} \sim$ 1.1) requiring only **15 parameters** from the exchange of only **6 bosons** with $I(J^P)$:
  - pion $1(0^-)$, eta $0(0^-)$, sigma $0(0^+)$, delta $1(0^+)$, omega $0(1^-)$, rho $1(1^-)$ (and photon)
- ★ **2014**: Unique 2-body isoscalar interaction current derived

Off-shell couplings at meson-NN vertices



★ **Examples**

- **Scalars**

$$\Lambda^{\sigma}(p,p') = g_{\sigma}\mathbf{1} - \nu_{\sigma}[\Theta(p) + \Theta(p')], \qquad \Theta(p) = \frac{m - \not{p}}{2m}$$

- **Pseudo-scalars (if pure pseudo-vector)**

$$\Lambda^{\pi}(p,p') = g_{\pi}[\gamma^5 - \Theta(p)\gamma^5 - \gamma^5\Theta(p')] = \frac{g_{\pi}}{2m}\,\not{q}\gamma^5$$

- **Vectors (similar terms)**

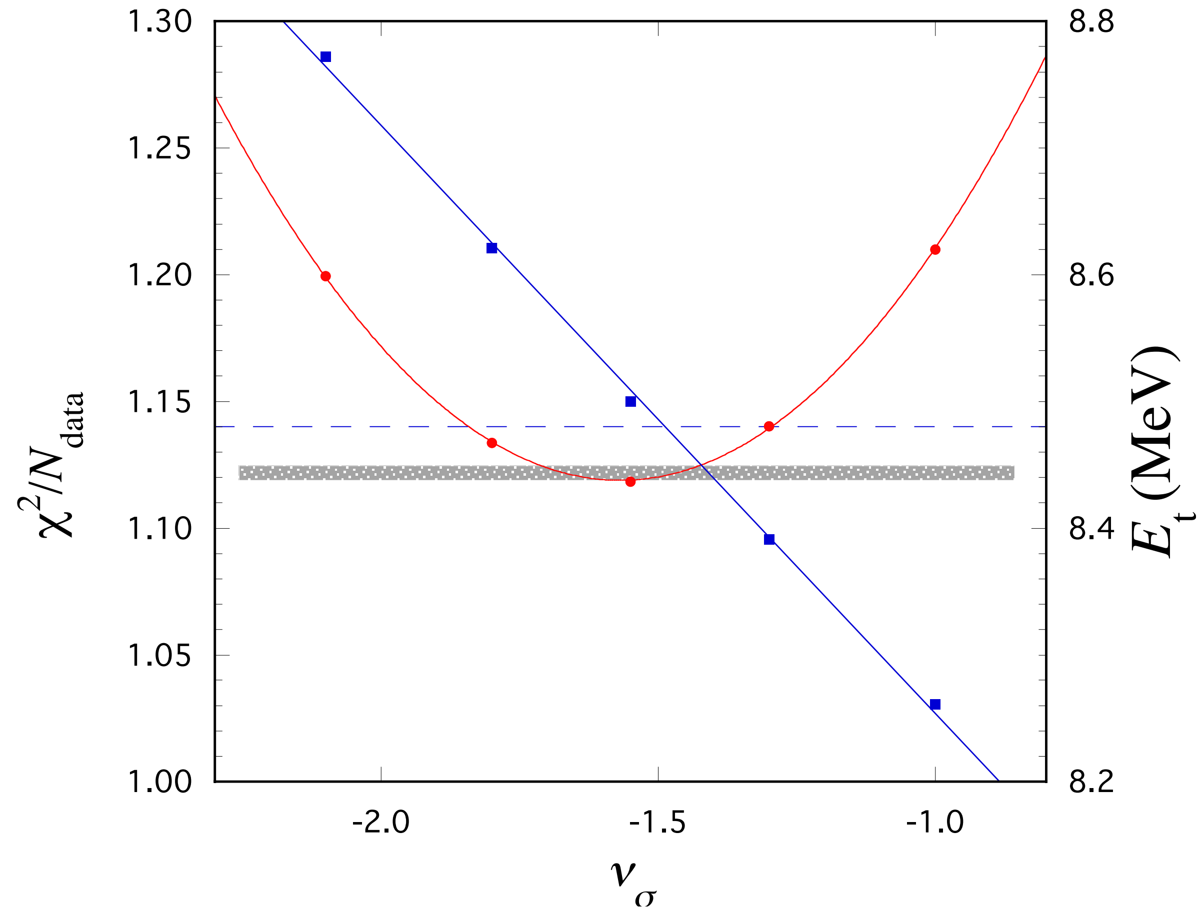
★ **All vertices have product form factors: $h_N(p)h_N(p')f_b(p-p')$**

★ **1997 discovery (with A. Stadler): The fit to the NN data and the 3-body binding energy is very sensitive to ν_{σ} and the value determined by NN fits gives the correct E_T!**

Best fit gives correct 3-body binding energy

χ^2 vs ν_σ

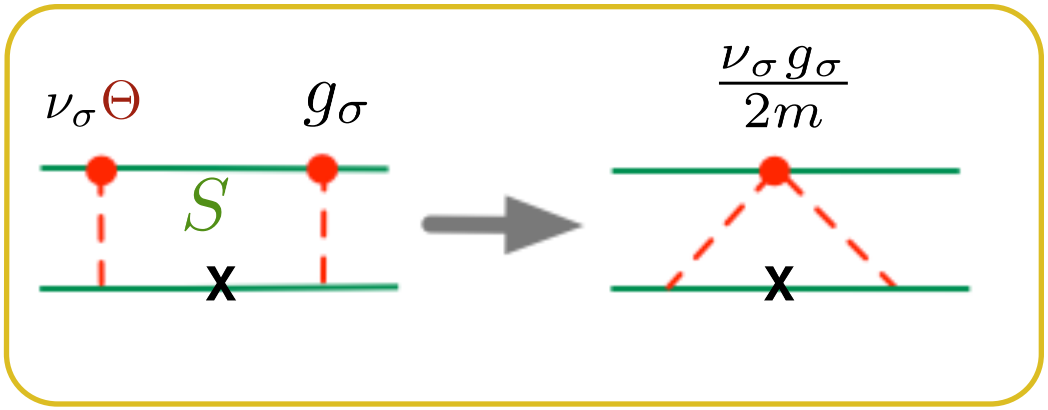
E_t vs ν_σ

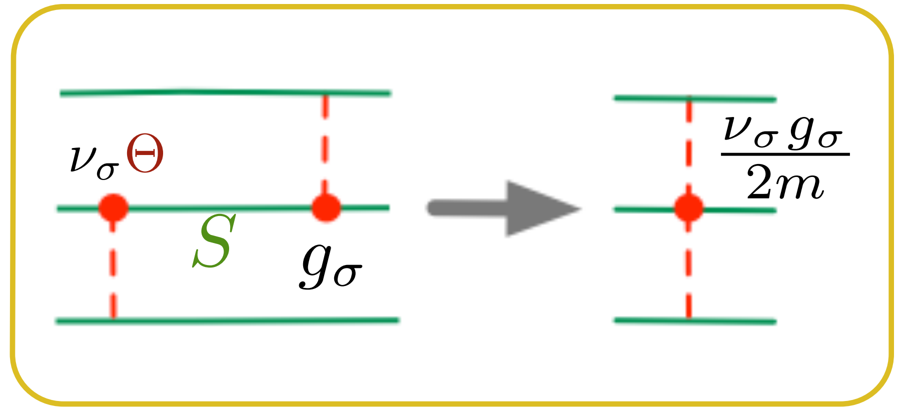


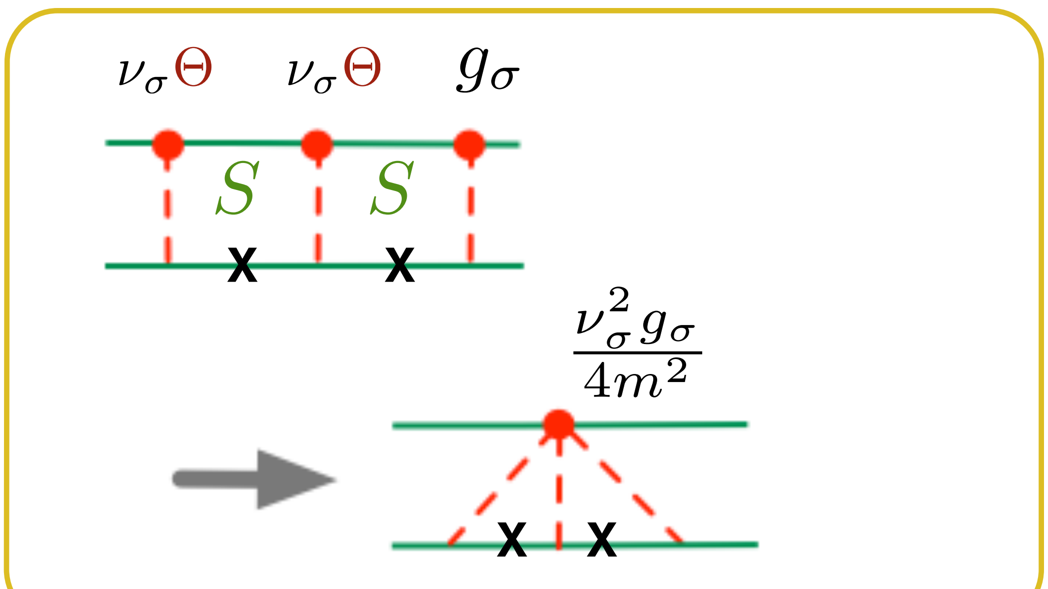
Implications of off-shell couplings:
1. Cancellations of the propagators

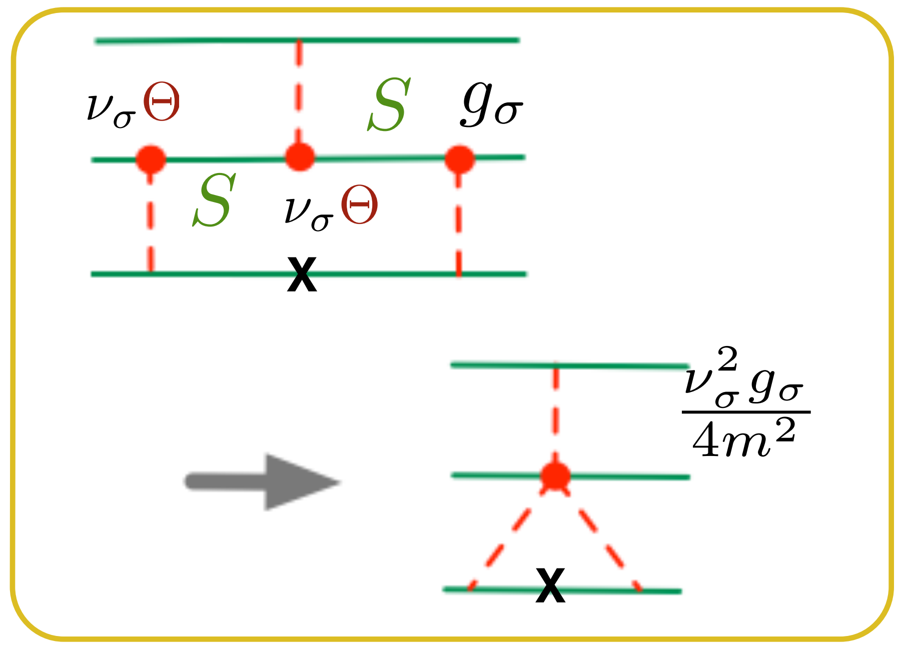
$$\Lambda^{\sigma}(p,p') = g_{\sigma}\mathbf{1} - \nu_{\sigma}[\Theta(p) + \Theta(p')], \qquad \Theta(p)S(p) = \frac{1}{2m}$$

Examples at 4th and 6th order





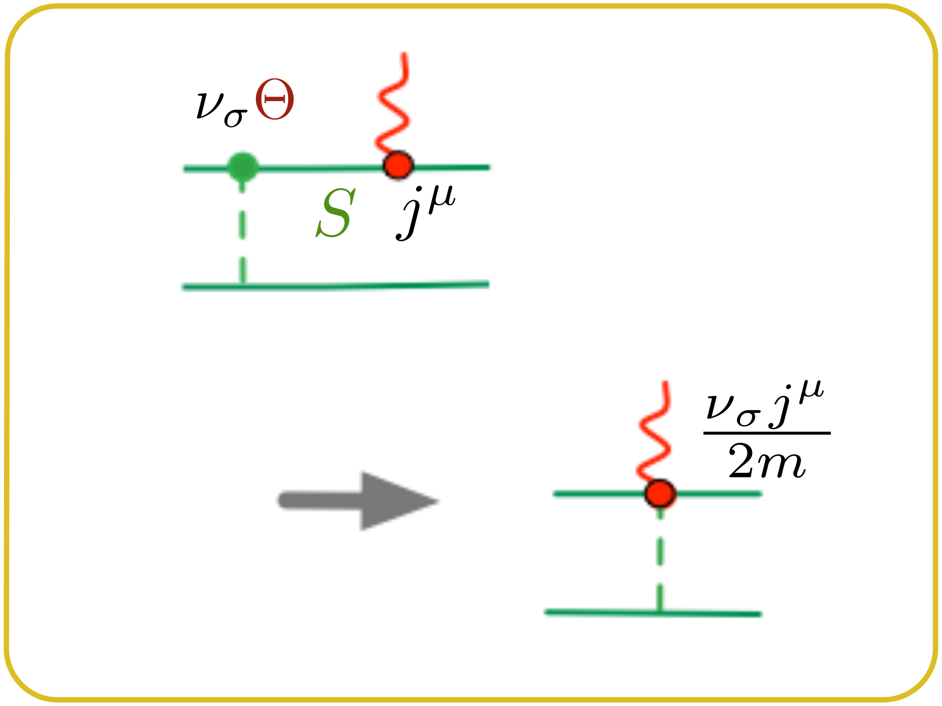


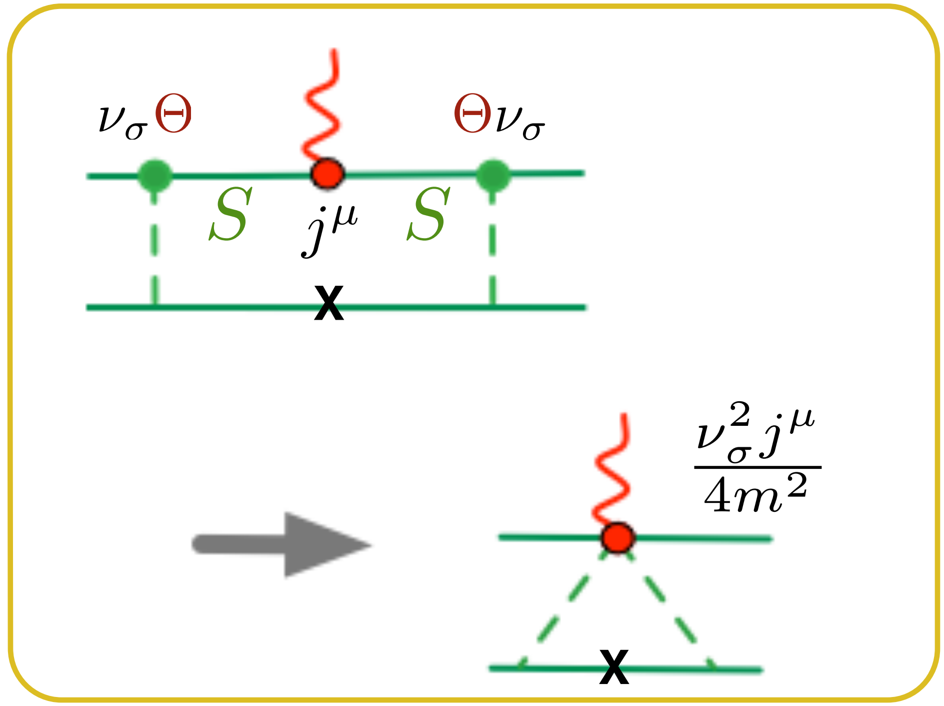


Implications of off-shell couplings
2. New effective interaction currents

$$\Lambda^\sigma(p,p') = g_\sigma \mathbf{1} - \nu_\sigma[\Theta(p) + \Theta(p')], \qquad \Theta(p)S(p) = \frac{1}{2m}$$

Examples at 2nd and 4th order





The interaction currents generated by ν_σ require a new generation of form factor calculations, underway but not finished!

Equivalence theorem

- ★ **PictureA:** a CST OBE model **WITH** off-shell couplings is equivalent to
- ★ **Picture B:** a CST model with OBE terms **WITHOUT** off-shell couplings **PLUS** an infinite sum of specific non-OBE terms, many body forces, and non-OBE interaction currents with couplings depending on combinations of only a few parameters

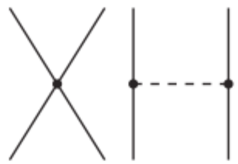
- ★ So.. Are there three body forces?
  - using picture A: NO
  - using picture B: YES (but are all fixed by picture A)
- ★ I am reminded of a statement by Peter Sauer: "Three-body forces are not made by God"

Comparison with χEFT

Comparison with χEFT (NN)

★ χEFT*:

LO
$(Q/\Lambda_\chi)^0$



NLO
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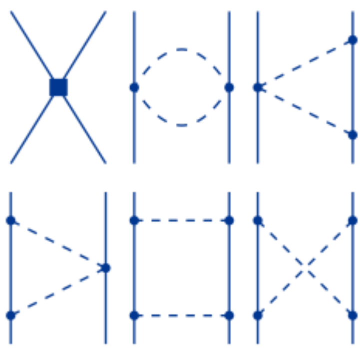
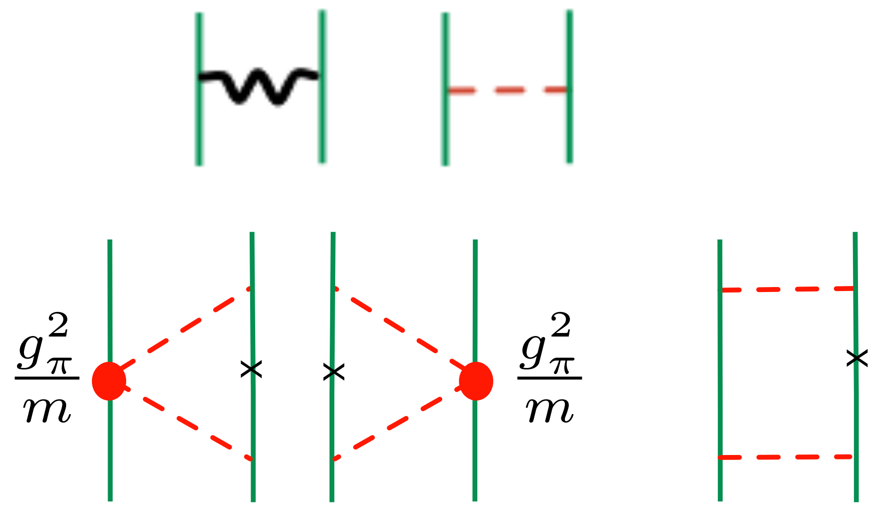


★ CST (picture B):



Machleidt, in "NN and 3N Interactions," Blokhintsev & Strakovsky, eds, (2014)

Expand the heavy meson propagators

$$\frac{1}{m_V^2 - q^2} \simeq \frac{1}{m_V^2} + \frac{q^2}{m_V^4} + \cdots$$

Comparison with χEFT (NN)

★ χEFT*:

LO
$(Q/\Lambda_\chi)^0$

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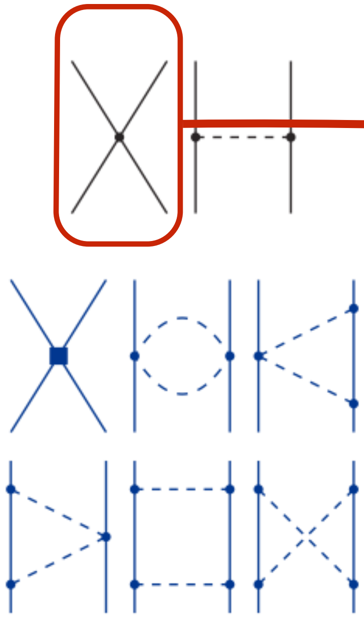

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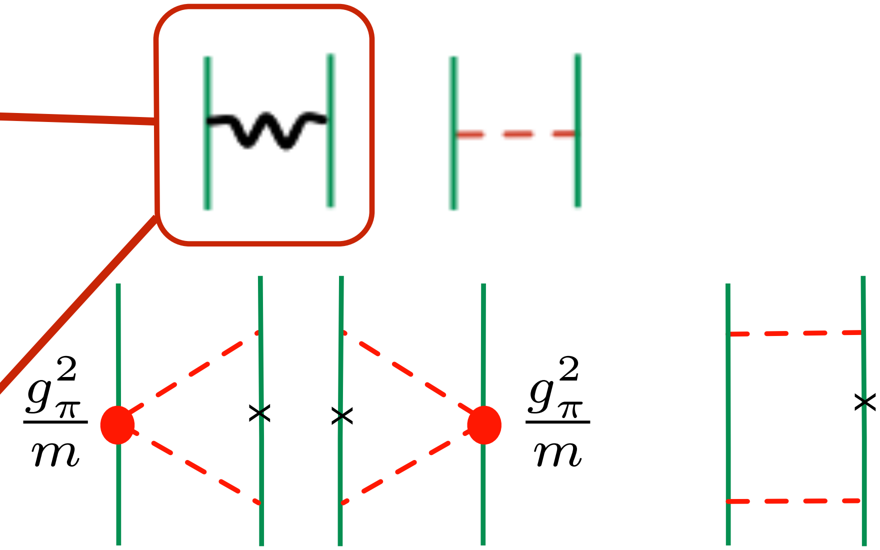

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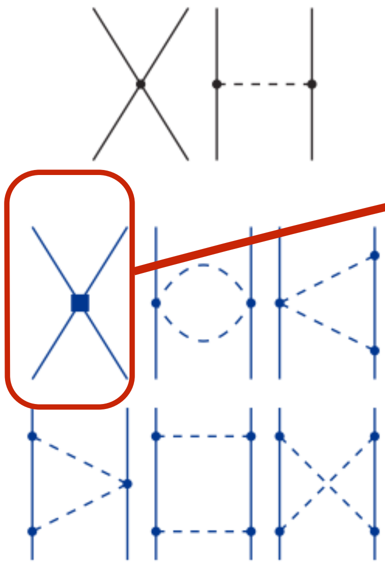

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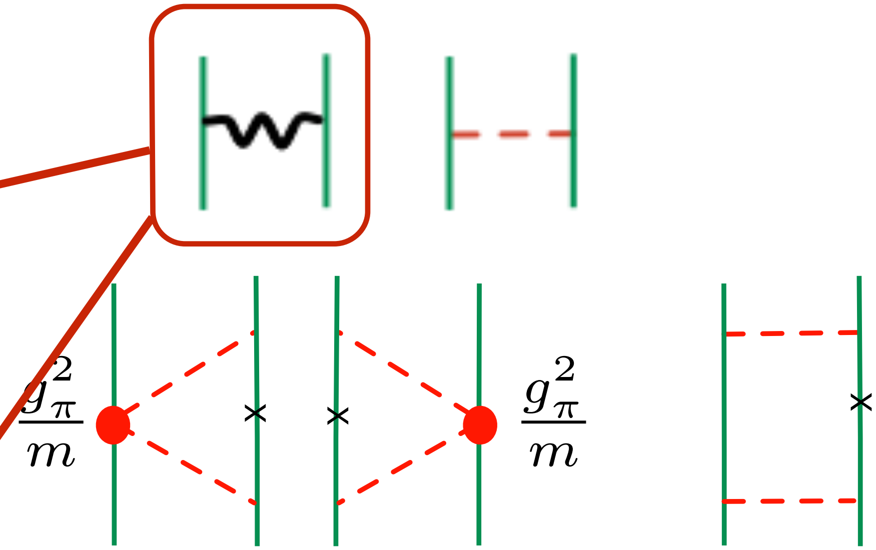

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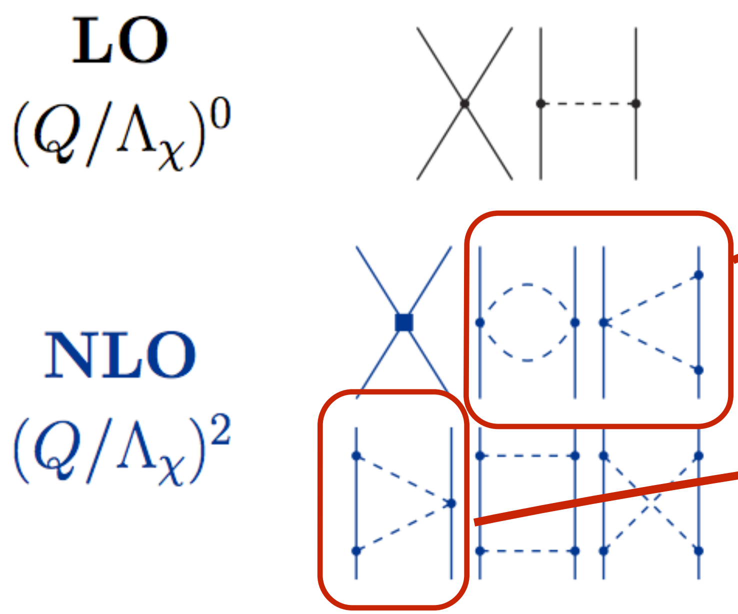

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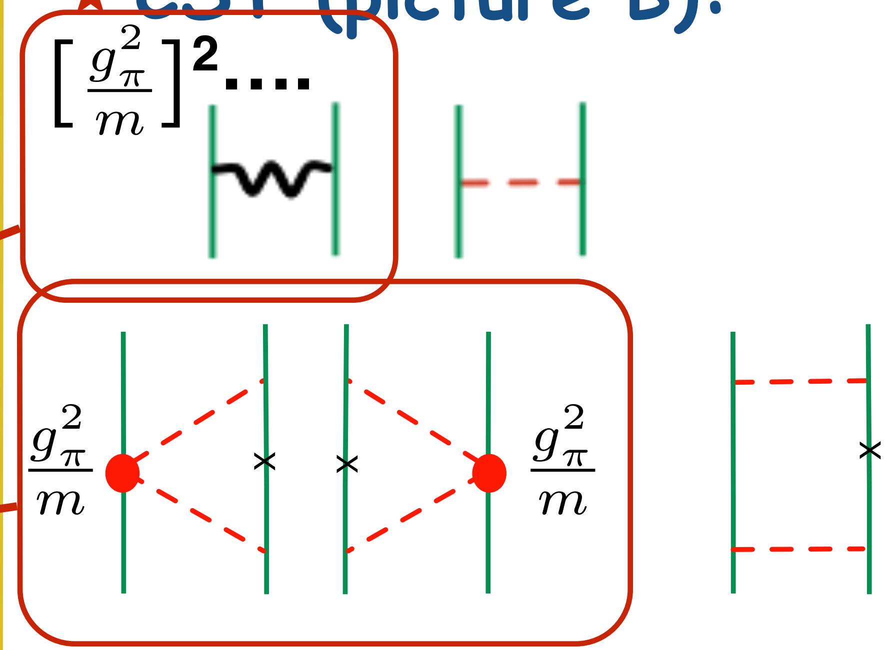

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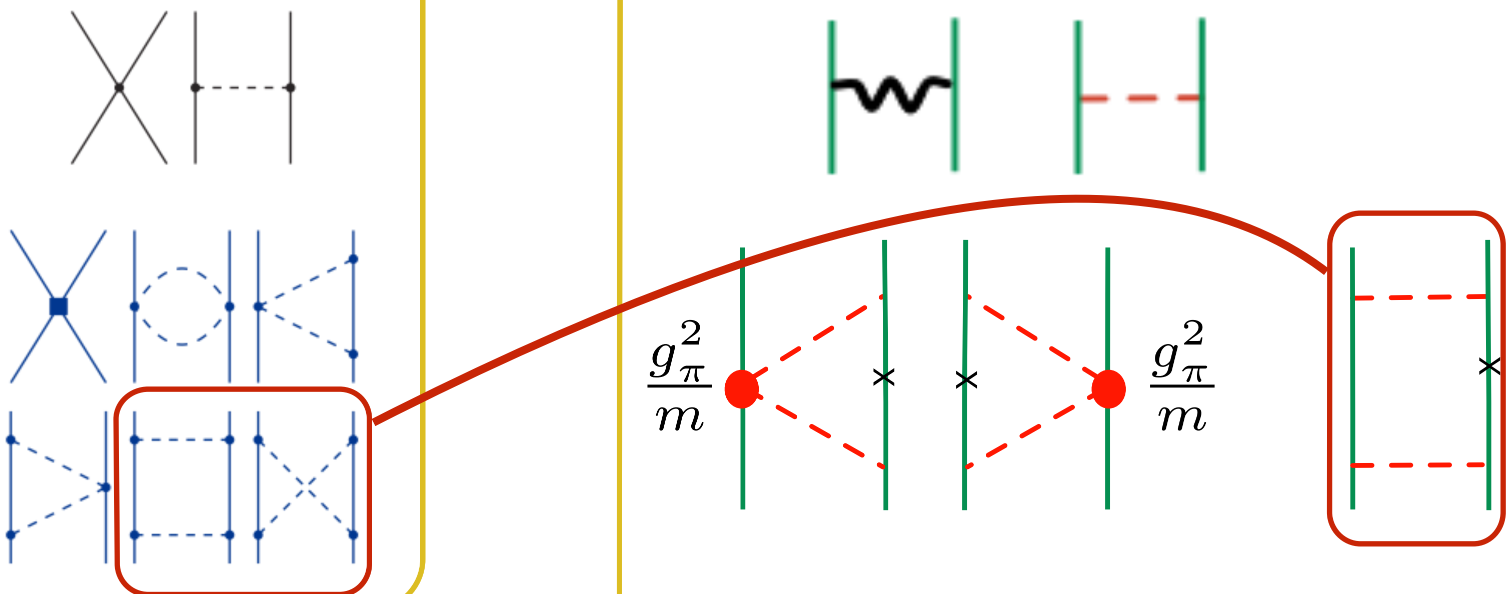

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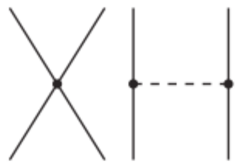

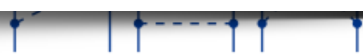

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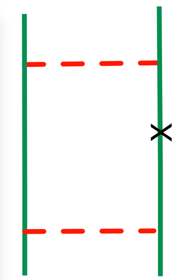

This comparison has yet to be worked out in detail !

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Expand the heavy meson propagators

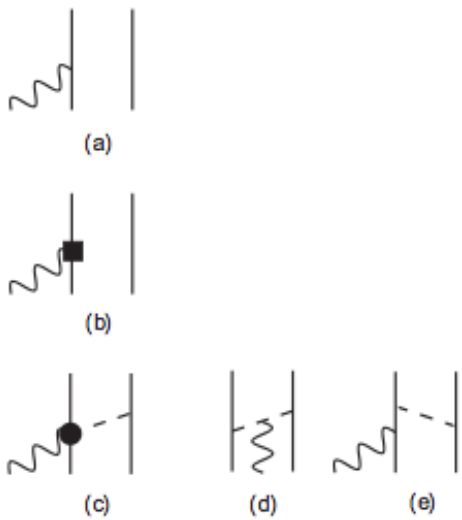
$$\frac{1}{m_V^2 - q^2} \simeq \frac{1}{m_V^2} + \frac{q^2}{m_V^4} + \cdots$$

Comparison with χEFT (charge and current)

Q^0 LO:

Q^2 N2LO:

Q^3 N3LO:

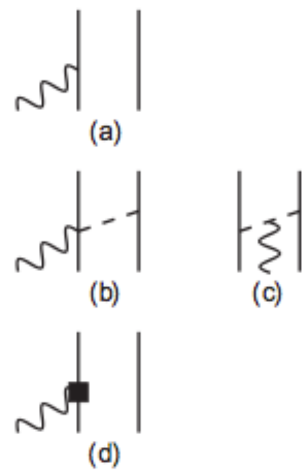


charge

Q^1 LO:

Q^2 N1LO:

Q^3 N2LO:



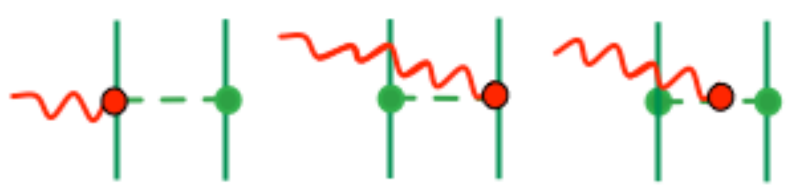
3-current

CST combines charge and 3-current into a single off-shell nucleon 4-current

CST 1-body

$$\bar{u}(\mathbf{p},\lambda_f)\left[j_N^\mu(p,p')\right]u(\mathbf{p}',\lambda_i) = \begin{cases} F_1(Q^2) + \text{rel corrections} & \mu = 0 \quad \text{charge} \\ F_2(Q^2)\frac{(\boldsymbol{\sigma}\times\mathbf{q})^i}{2m} + \text{rel corrections} & \mu = i \quad 3\,\text{current} \end{cases}$$

CST interaction currents



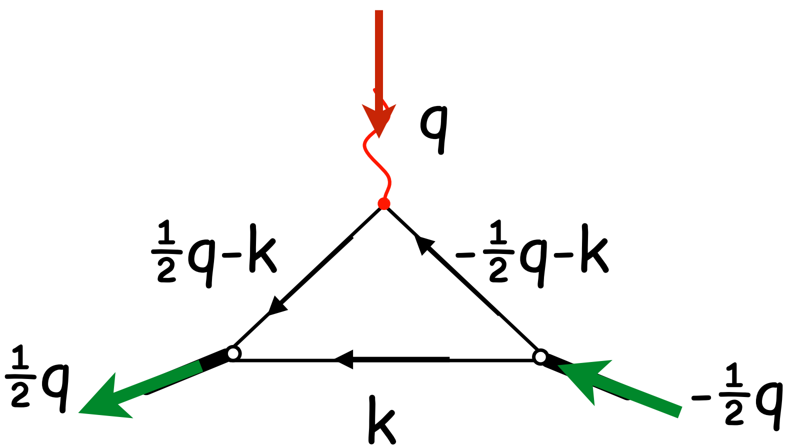
Intermediate summary: CST vs χEFT

- ★ Conjecture: CST and χEFT include the same physics
- ★ Comparing the two leads to an understanding of
  - the content of CST OBE diagrams, and the off-shell couplings in particular
  - how χEFT parameters might be determined from fewer OBE parameters (will it work? — still to be done)
- ★ Currents: both approaches use measured nucleon form factors, and are therefore incomplete

Role of Relativity

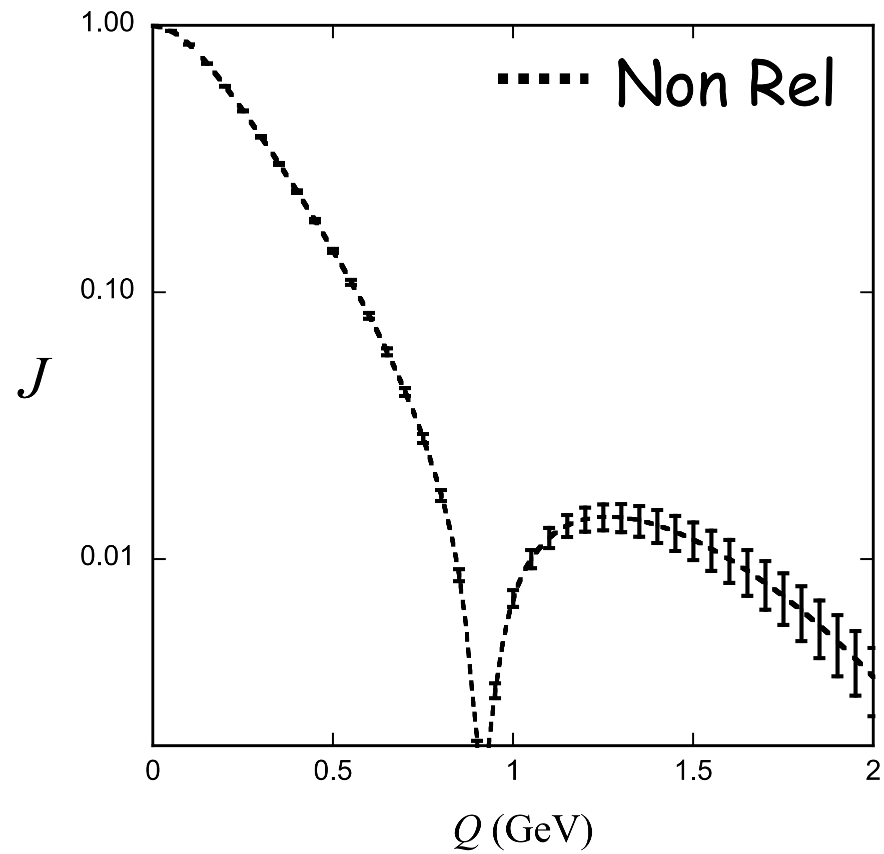
Role of relativity at high Q

- ★ Example: scalar model
- ★ Non-relativistic Δ diagram



wave function relative momentum squared $= (k \pm \frac{1}{4}q)^2$

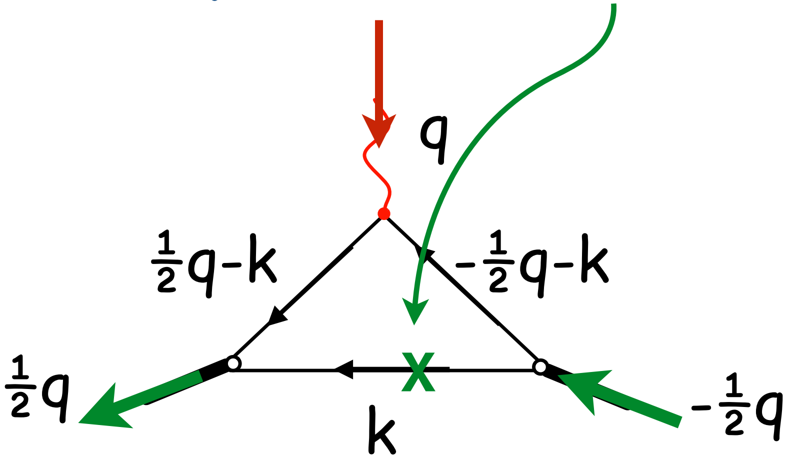
example uses S-state deuteron wave function

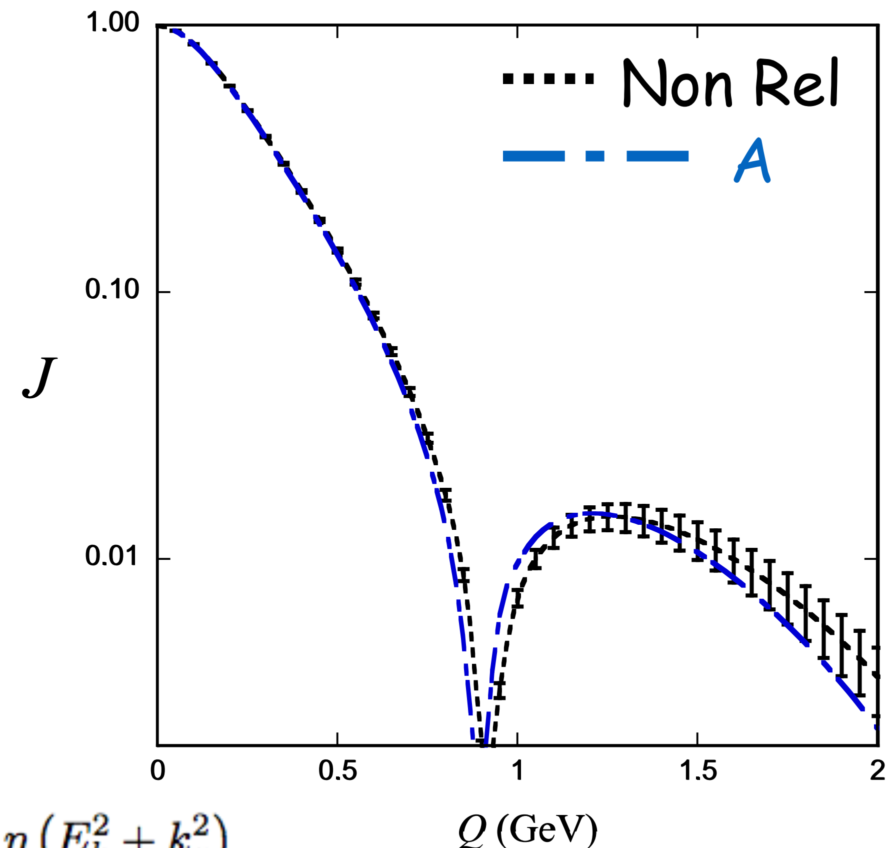


error bars are $\pm \dfrac{Q^2}{4\ m_d^2}$

Role of relativity at high Q

- ★ Example: scalar model
- ★ Relativistic Δ diagram with spectator on shell





wave function relative momentum squared

$$R_\pm^2 = \frac{(P_\pm \cdot k)^2}{m_d^2} - m^2$$

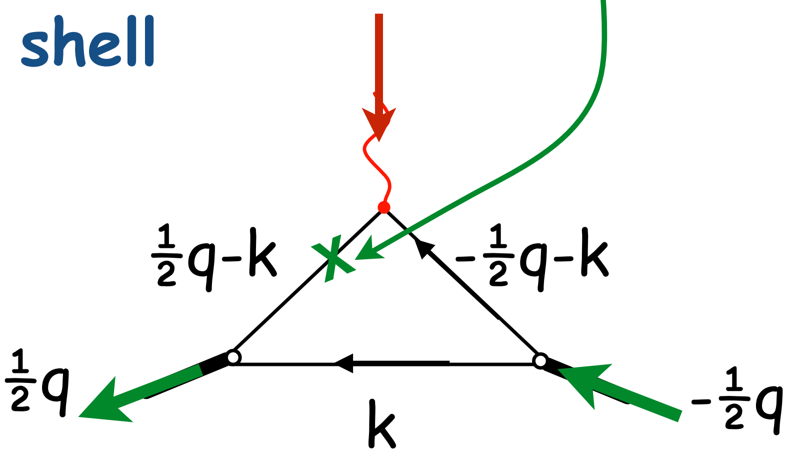
$$= k^2 \mp k_z\, Q \frac{D_0 E_k}{m_d^2} + \eta \left(E_k^2 + k_z^2\right)$$

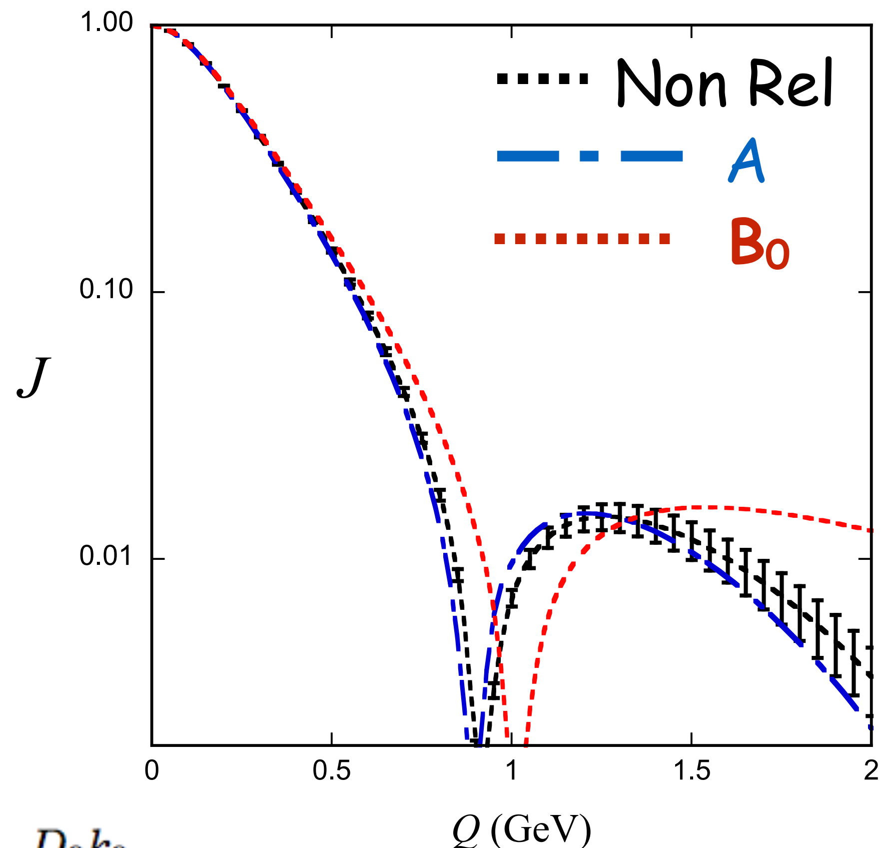
$$\to \left(\mathbf{k} \mp \frac{1}{4}\mathbf{q}\right)^2 \qquad m, m_d \to \infty \text{ limit}$$

Role of relativity at high Q

B₀ -> both incoming and outgoing struck particles on-shell !

- ★ Example: scalar model
- ★ Relativistic Δ diagram with struck particle on shell





wave function relative momentum squared

$$\widetilde{R}^2_\pm = \frac{\left(P_\pm \cdot \widetilde{k}_\pm\right)^2}{m_d^2} - \widetilde{k}^2_\pm$$

$$= k_\perp^2 + (k_\pm)_z^2 \mp (k_\pm)_z Q \frac{D_0 k_0}{m_d^2} + \eta[k_0^2 + (k_\pm)_z^2]$$

$$\to \left(\mathbf{k} \pm \frac{1}{4}\mathbf{q}\right)^2 \qquad m, m_d \to \infty, \frac{k_0 - m}{m} \to 0$$

Role of relativity at high Q

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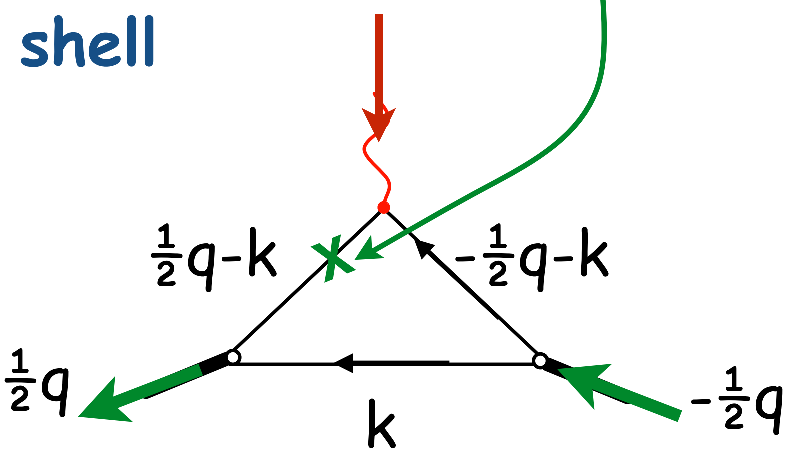

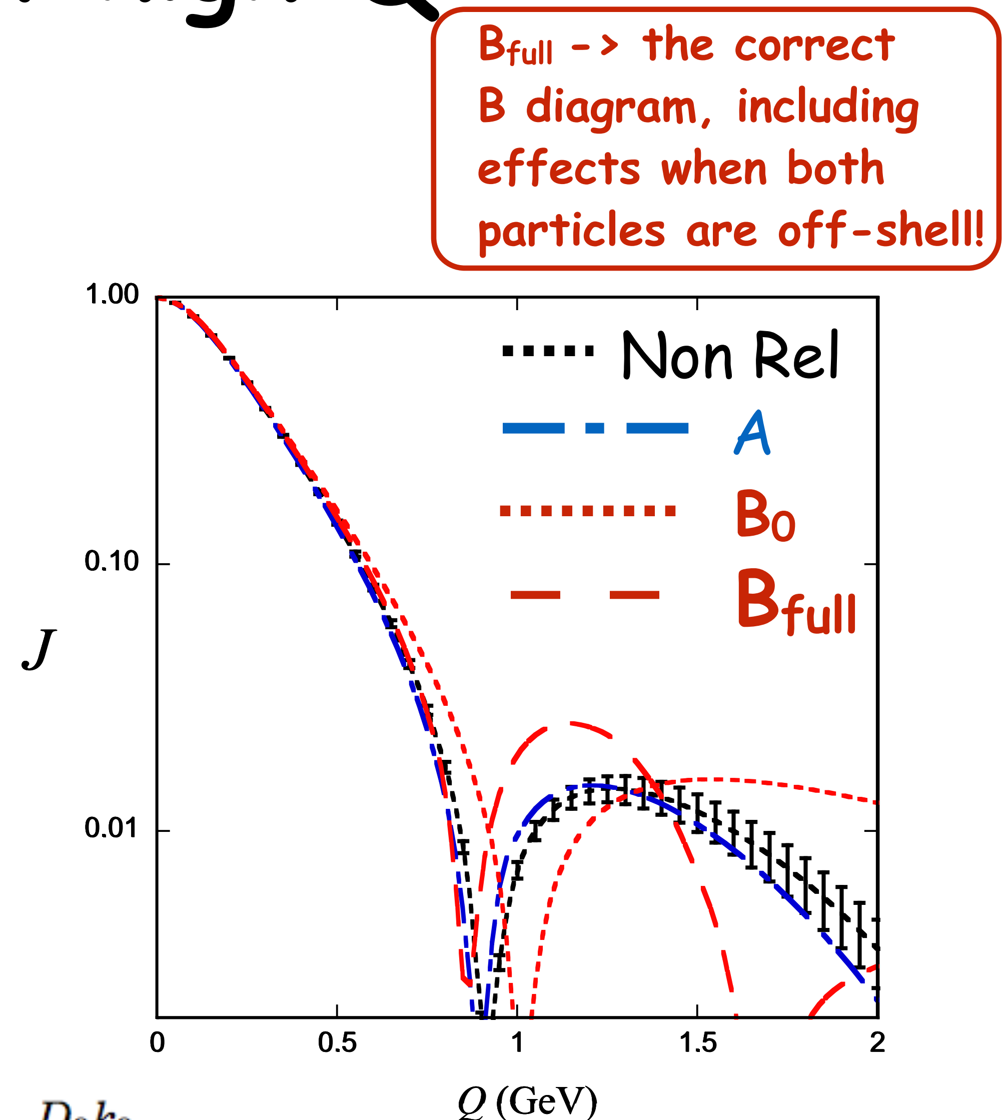

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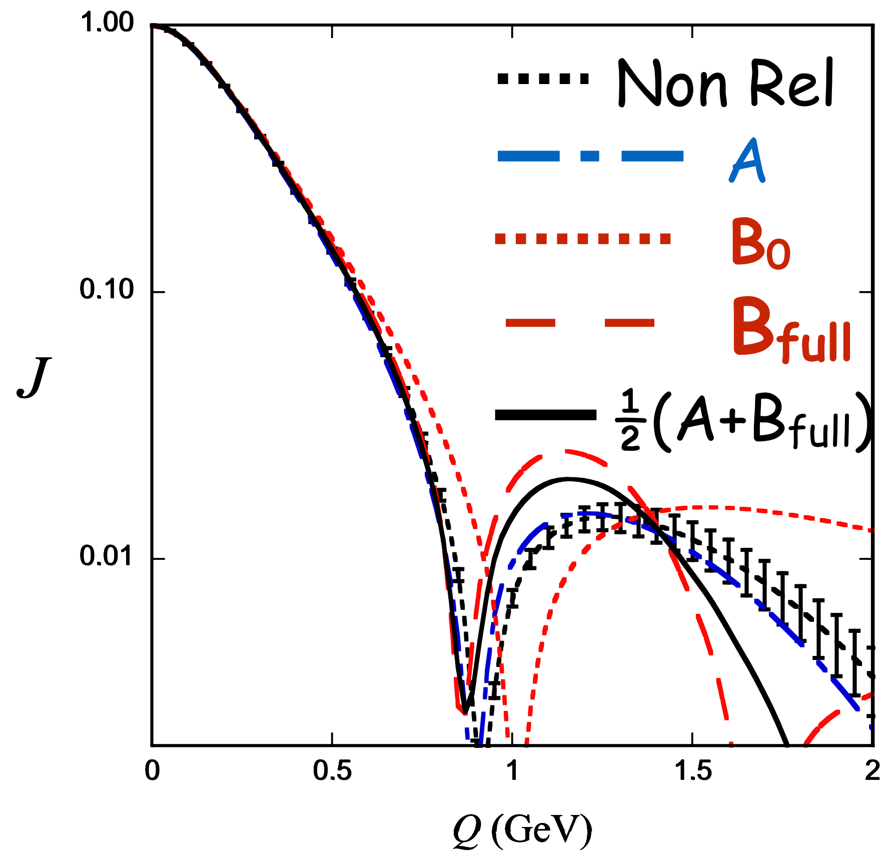
Role of relativity at high Q

- ★ Example: scalar model
- ★ Relativistic Δ diagram with contributions from both the spectator on-shell (A) and the struck particles on shell ($B = B_{full}$)
- ★ Relativistic effects from argument shifts (only) are significant
- ★ Realistic case has spin and off-shell effects from the current



Summary and Outlook

- ★ NN interaction — when derived from on-shell scattering using either χEFT or the CST-OBE model — also explain electron scattering at low Q when the correct current operator is included
- ★ to describe the data beyond Q ~ 0.5 GeV, a fully covariant description (CST) is needed — but there is NO evidence for the explicit appearance of quark degrees of freedom
- ★ CST(old) describes the deuteron form factors to the highest Q measured — but calculation of deuteron form factors with new interaction currents is in progress and final assessment awaits completion of this work
- ★ Unfortunately, CST has not been extend to A=4, and this is unlikely
- ★ It is possible to believe that a relativistic description of 3 and 4 body nuclei, when available, will fully explain the form factors!
- ★ Studies of form factors have been an invaluable tool in teaching us how describe bound states using field theory