Hyperons in nuclei and neutron stars

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FRIB Theory Fellow

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✓ Alessandro Lovato, ANL

✓ Francesco Pederiva, Trento

✓ Francesco Catalano, Uppsala

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SciDAC
Scientific Discovery through Advanced Computing

MICHIGAN STATE
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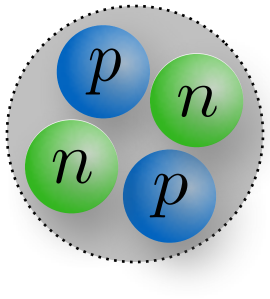
NUCLEI
Nuclear Computational Low-Energy Initiative

NERSC

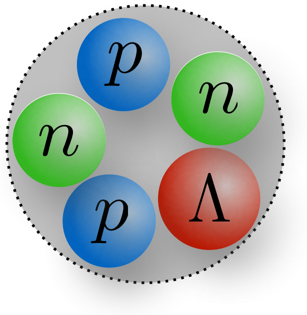
Marciana Marina, June 30, 2016

Outline

- ✓ Introduction
  - interest and motivations
  - hyperon puzzle
- ✓ Quantum Monte Carlo: AFDMC
- ✓ Hyperons in nuclei
- ✓ Hyperons in neutron stars
- ✓ Conclusions

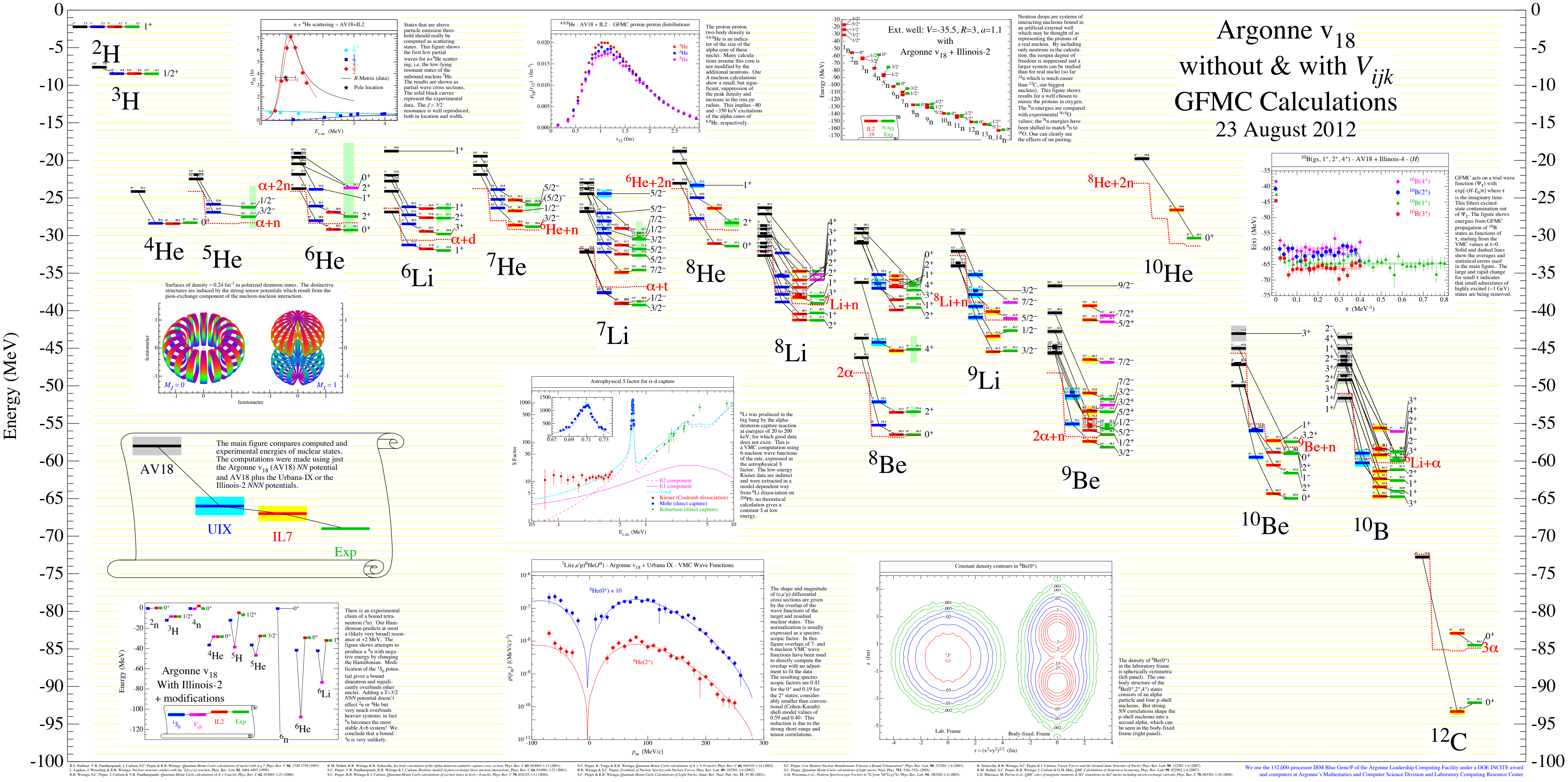


^{A-1}Z : $^{4}\mathrm{He}$



$^{A}_{\Lambda}Z$: $^{5}_{\Lambda}\mathrm{He}$

Introduction: non-strange sector

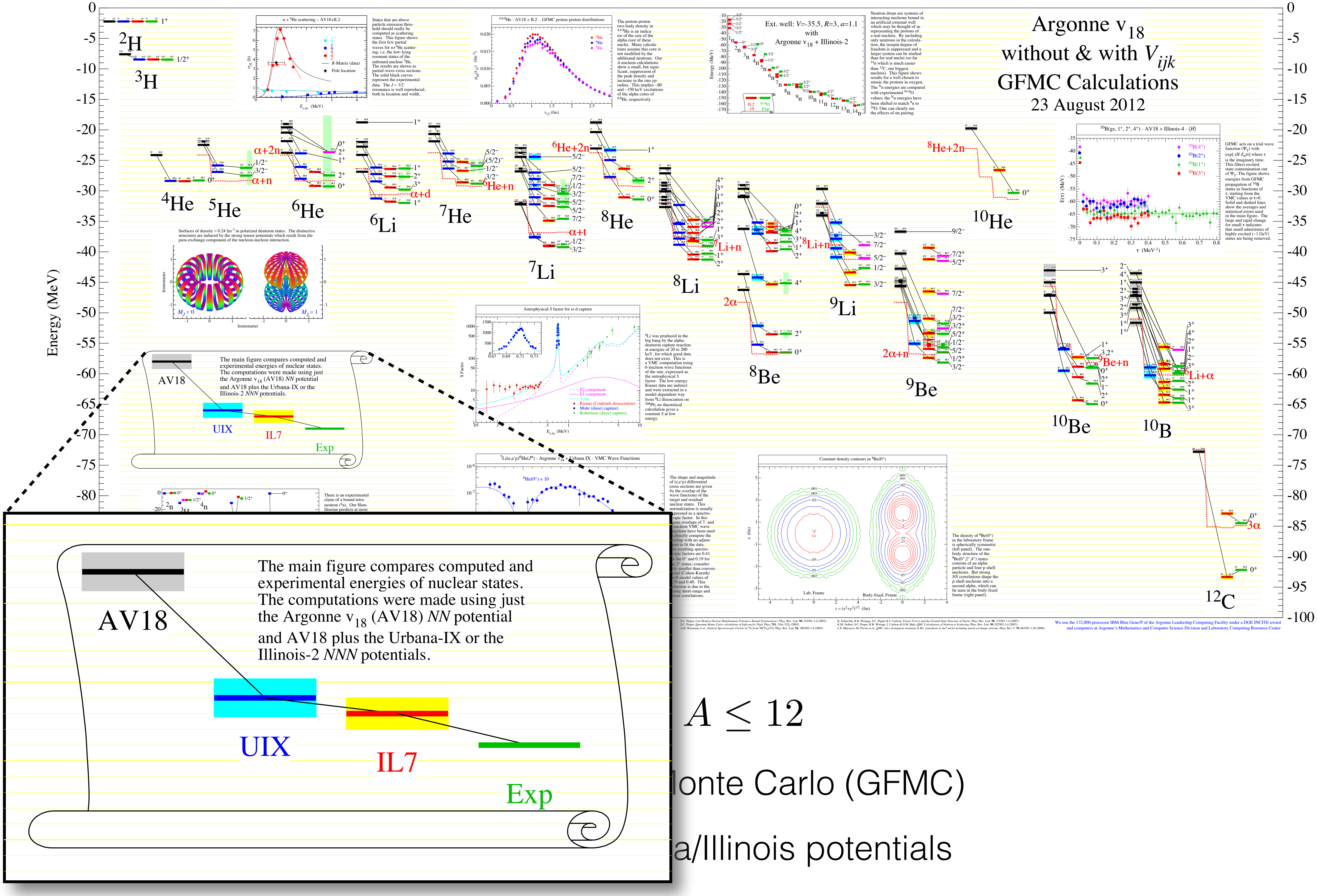


nuclei $A \leq 12$

Green’s function Monte Carlo (GFMC)

Argonne + Urbana/Illinois potentials

Introduction: non-strange sector



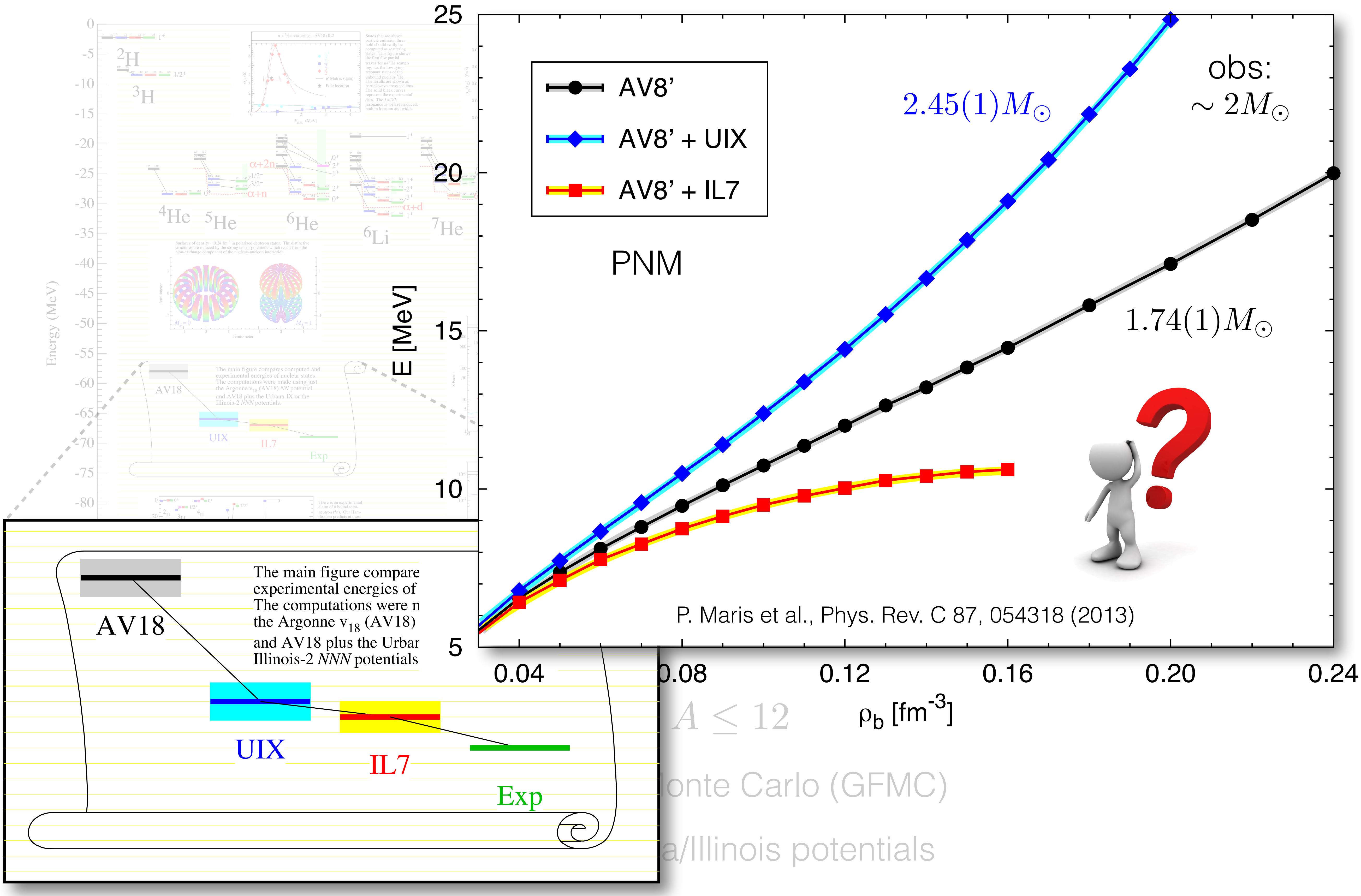
The main figure compares computed and experimental energies of nuclear states. The computations were made using just the Argonne v_{18} (AV18) *NN* potential and AV18 plus the Urbana-IX or the Illinois-2 *NNN* potentials.

$A \le 12$

onte Carlo (GFMC)

a/Illinois potentials

Introduction: non-strange sector



P. Maris et al., Phys. Rev. C 87, 054318 (2013)

$A \leq 12$

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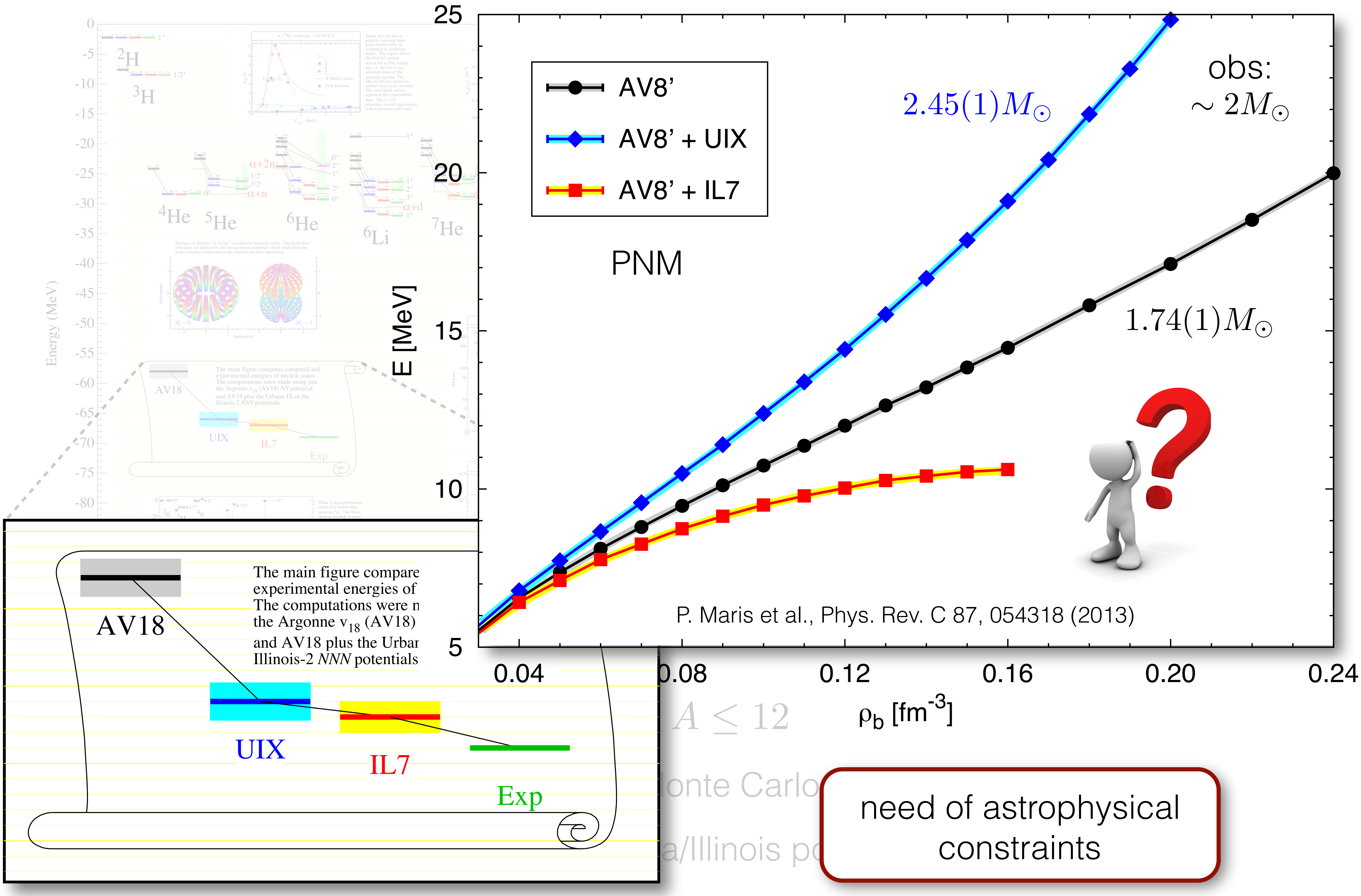
Introduction: non-strange sector



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$A \leq 12$

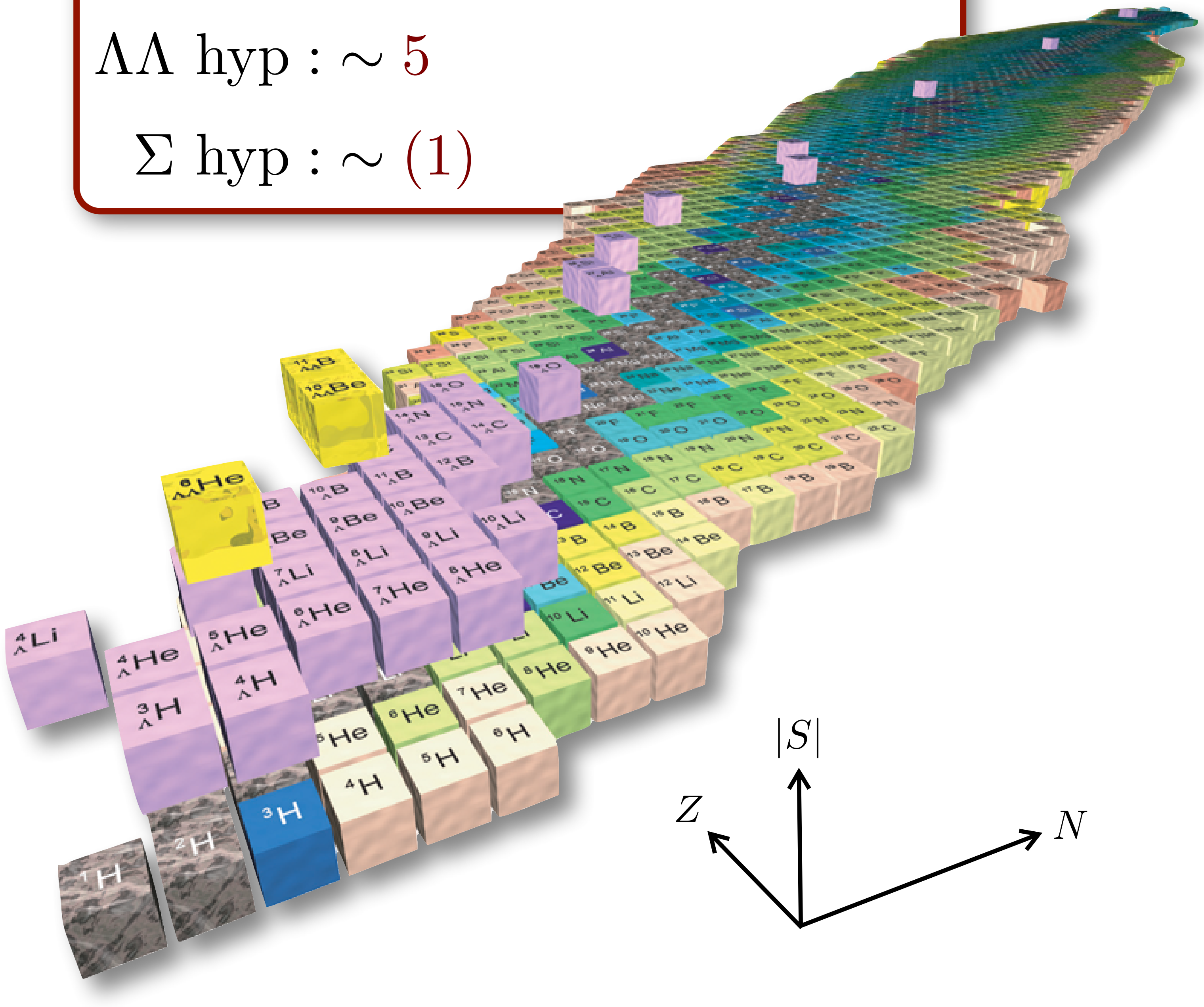
onte Carlo

a/Illinois po

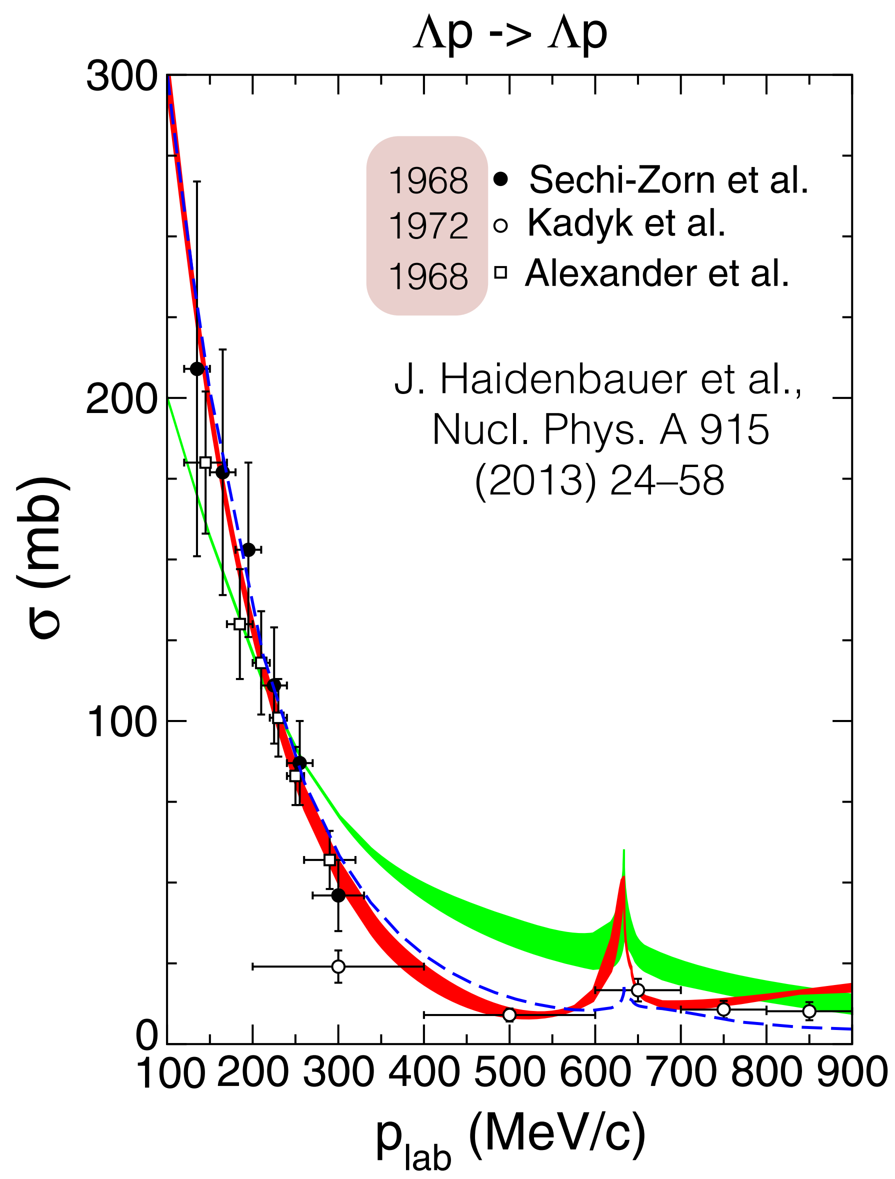
need of astrophysical constraints

Introduction: strange sector

binding energies:	scattering data:
nuc : ~ 3340	$NN : \sim 4300$
Λ hyp : ~ 41	$YN : \sim 52$
$\Lambda\Lambda$ hyp : ~ 5	
Σ hyp : $\sim (1)$	



- ✓ lack of experimental data



J. Haidenbauer et al., Nucl. Phys. A 915 (2013) 24–58

Introduction: strange sector

binding energies:	scattering data:
$\mathrm{nuc} : \sim 3340$	$NN : \sim 4300$
$\Lambda\ \mathrm{hyp} : \sim 41$	$YN : \sim 52$
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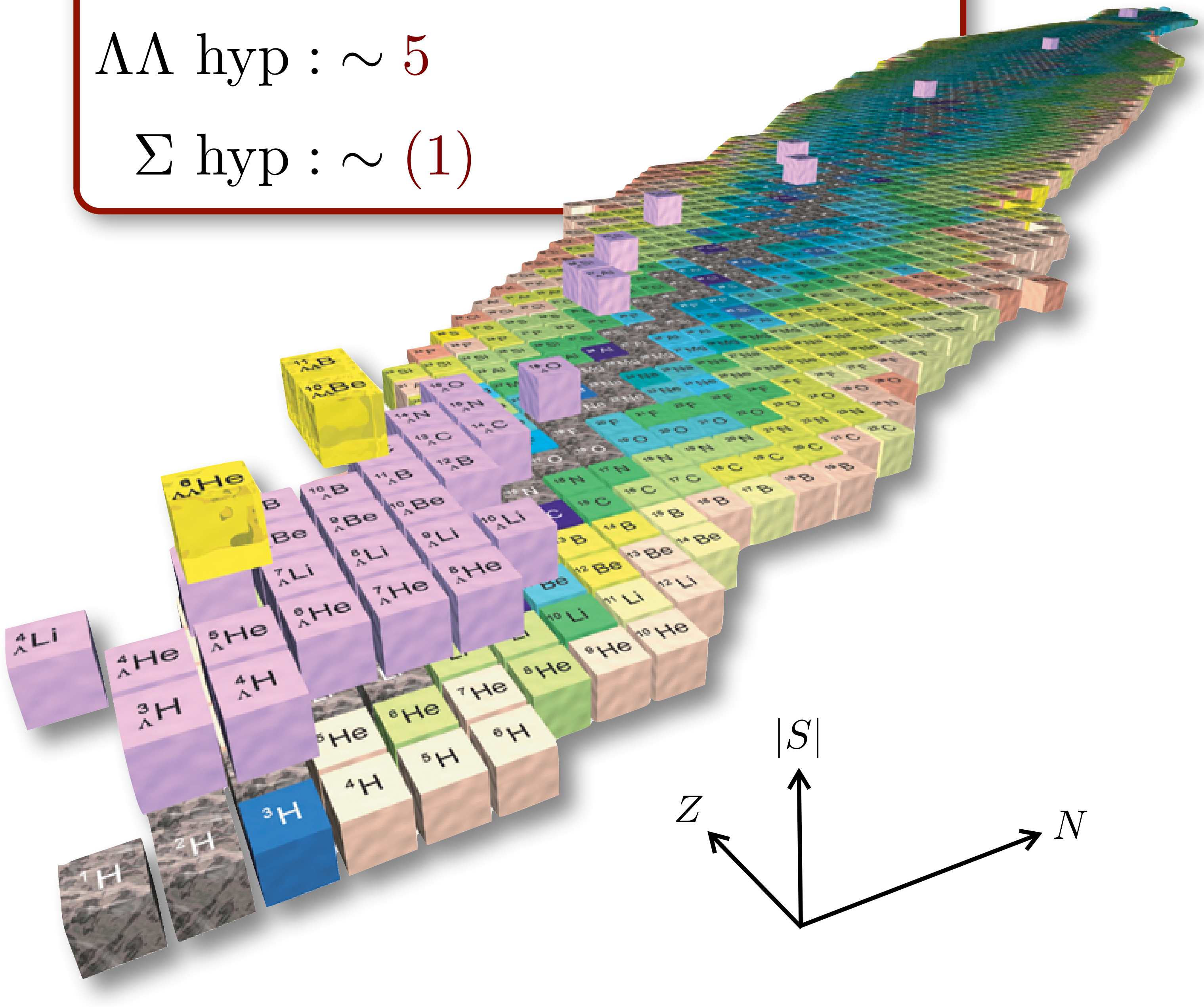
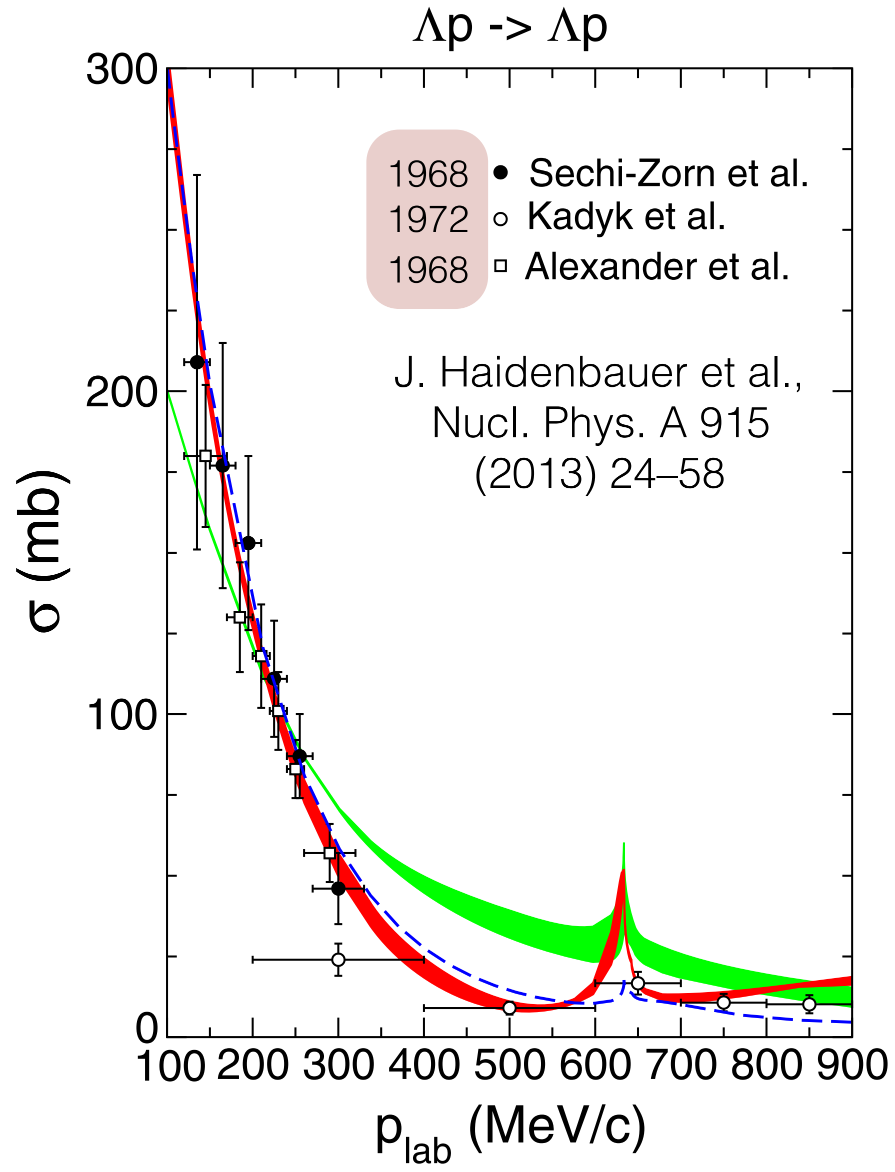


- ✓ lack of experimental data
- ✓ constraints from astrophysics?

NS observations: $\sim 2M_\odot$



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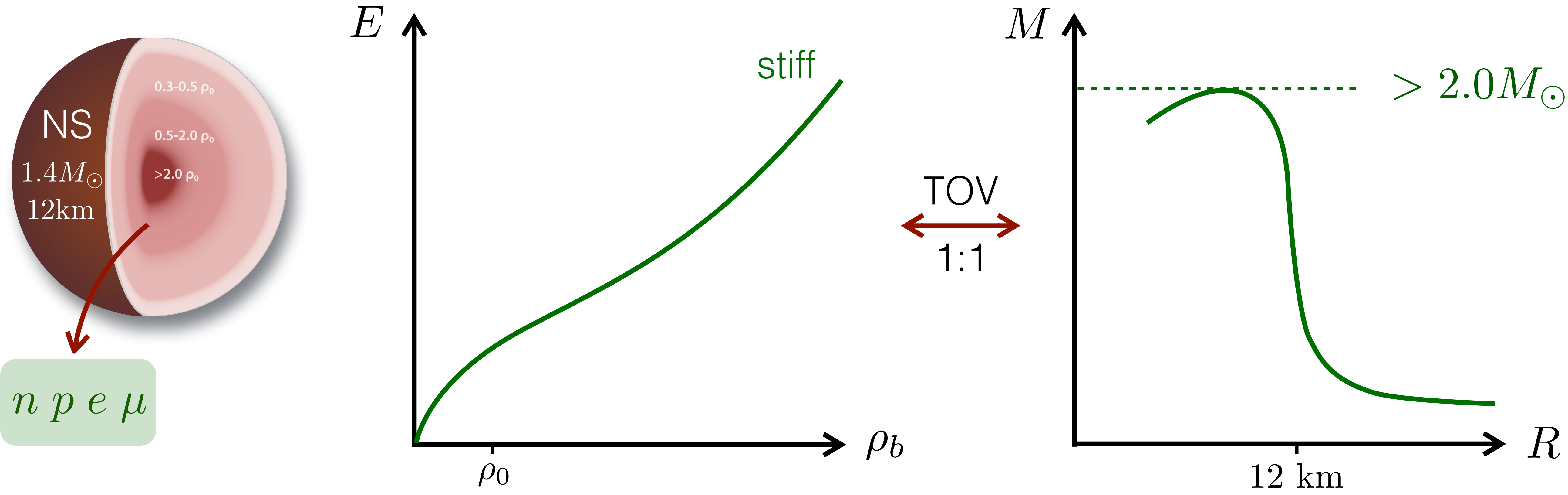
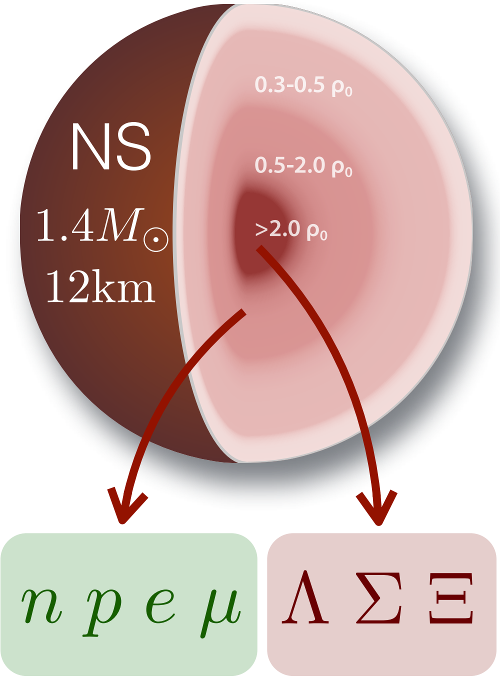
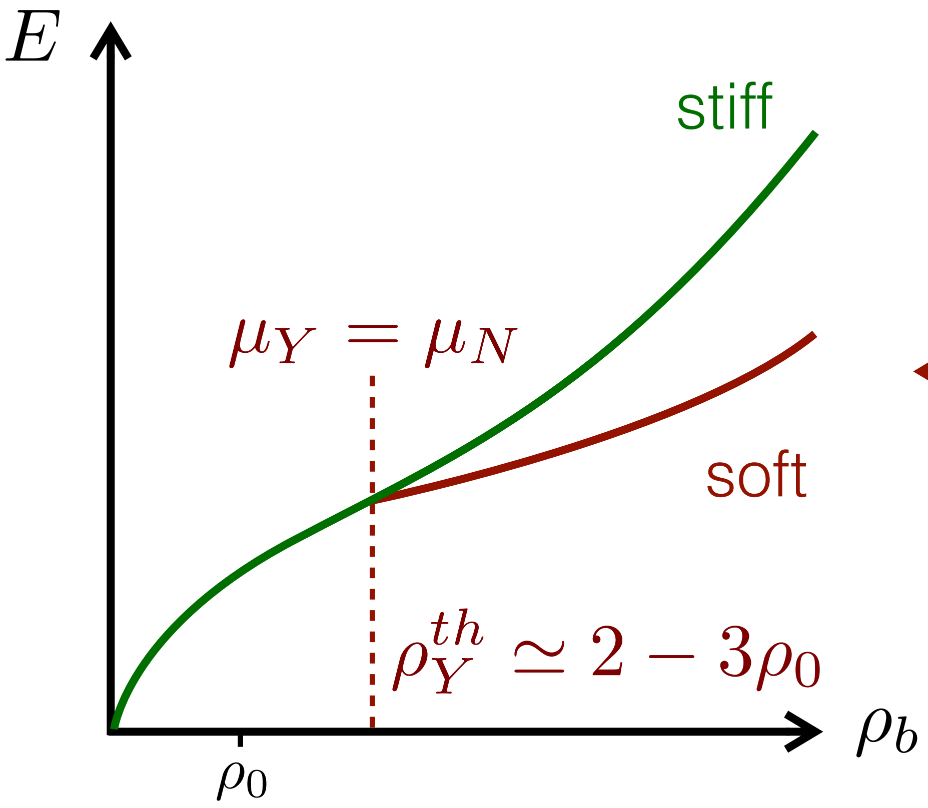
Introduction: the hyperon puzzle



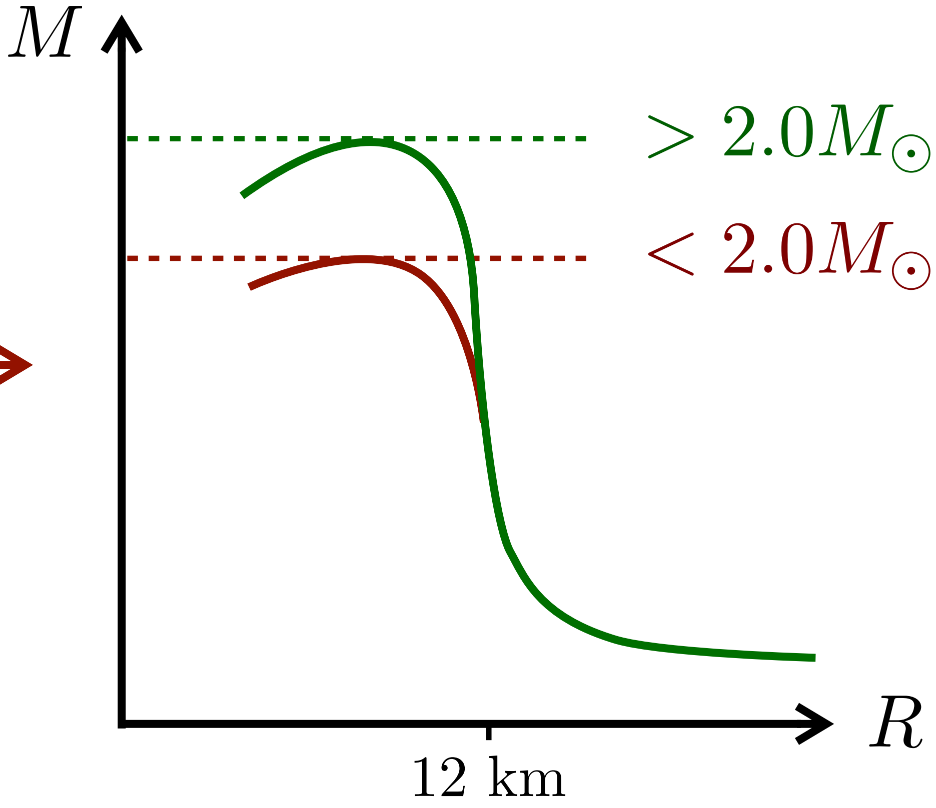
Introduction: the hyperon puzzle





TOV

1:1



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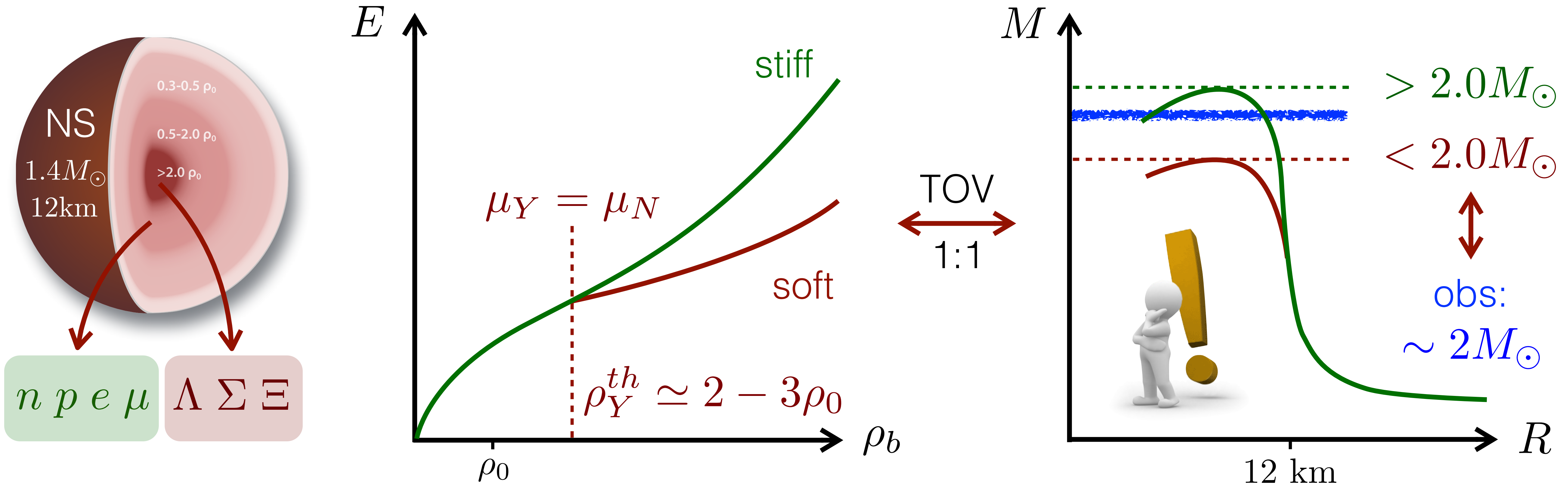

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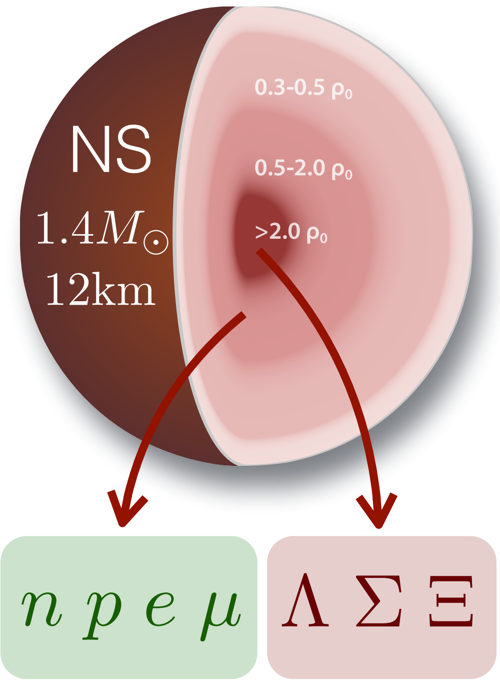

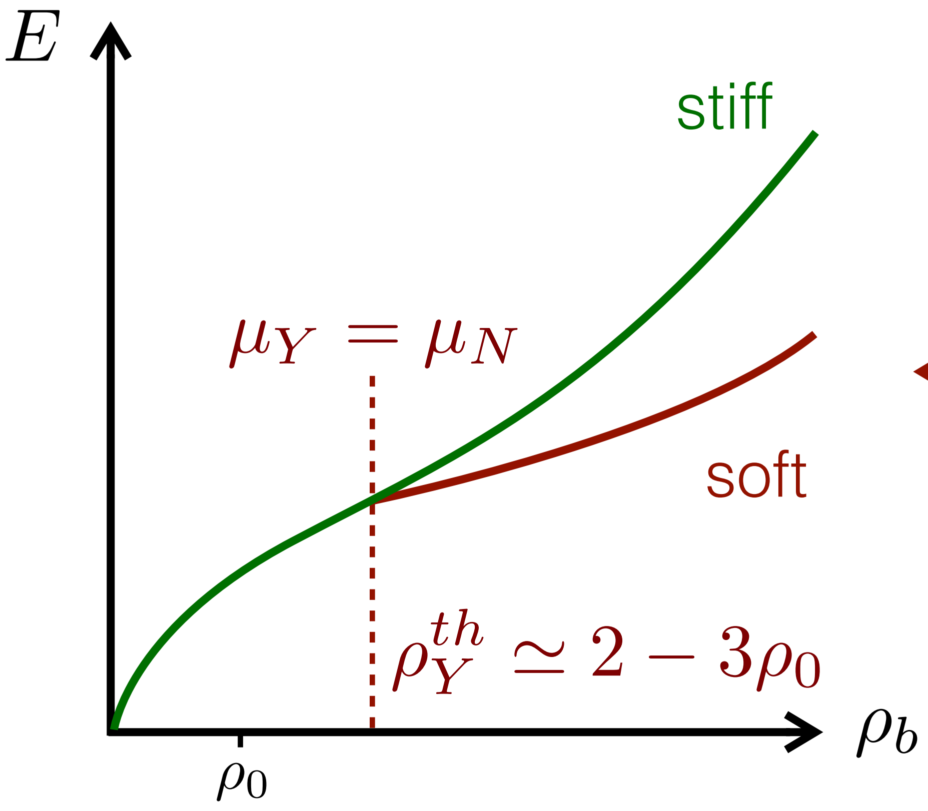

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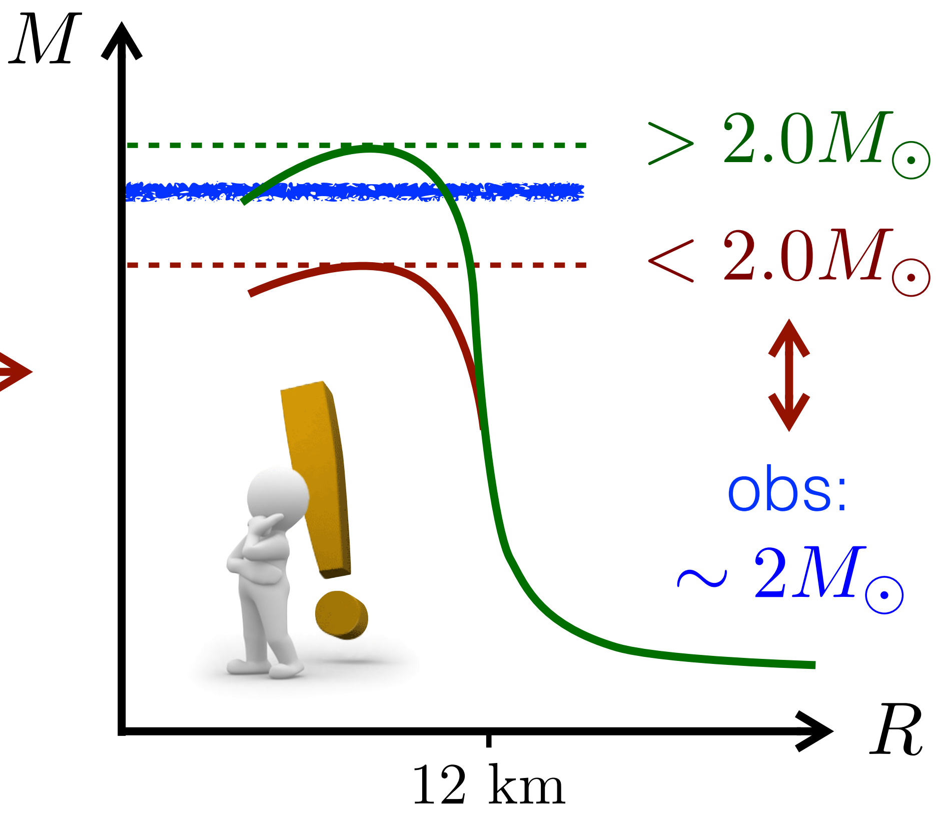
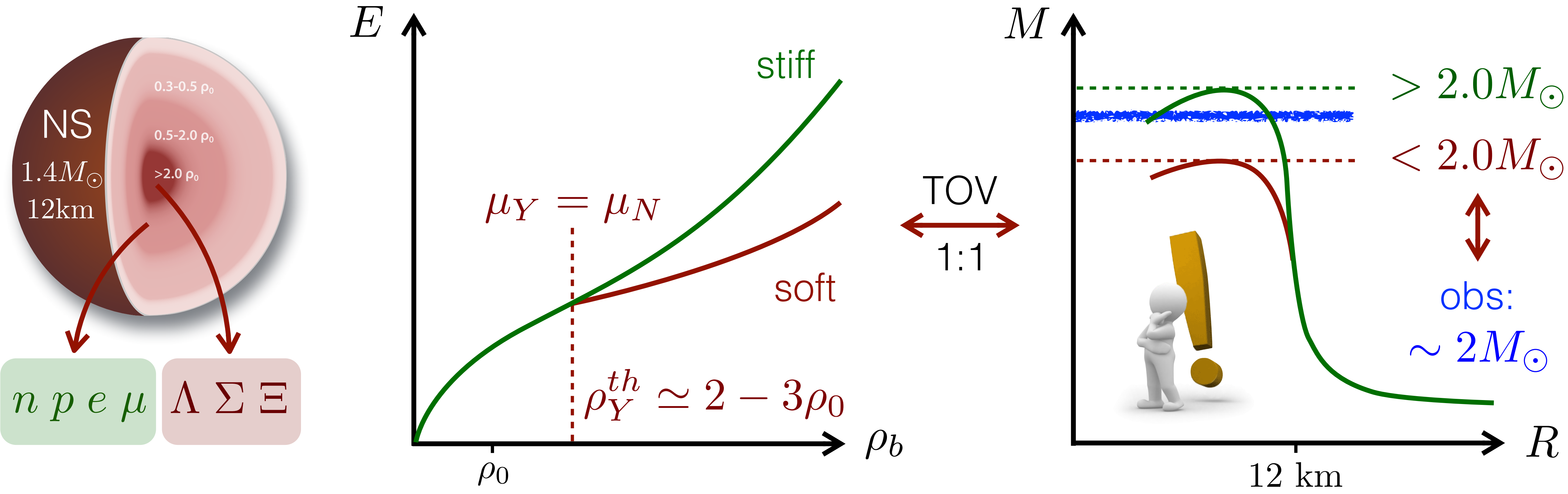


Hyperon puzzle

- ✓ Indication for the appearance of hyperons in NS core
- ✓ Apparent inconsistency between theoretical calculations and observations

Introduction: the hyperon puzzle



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Quantum Monte Carlo → YN interaction

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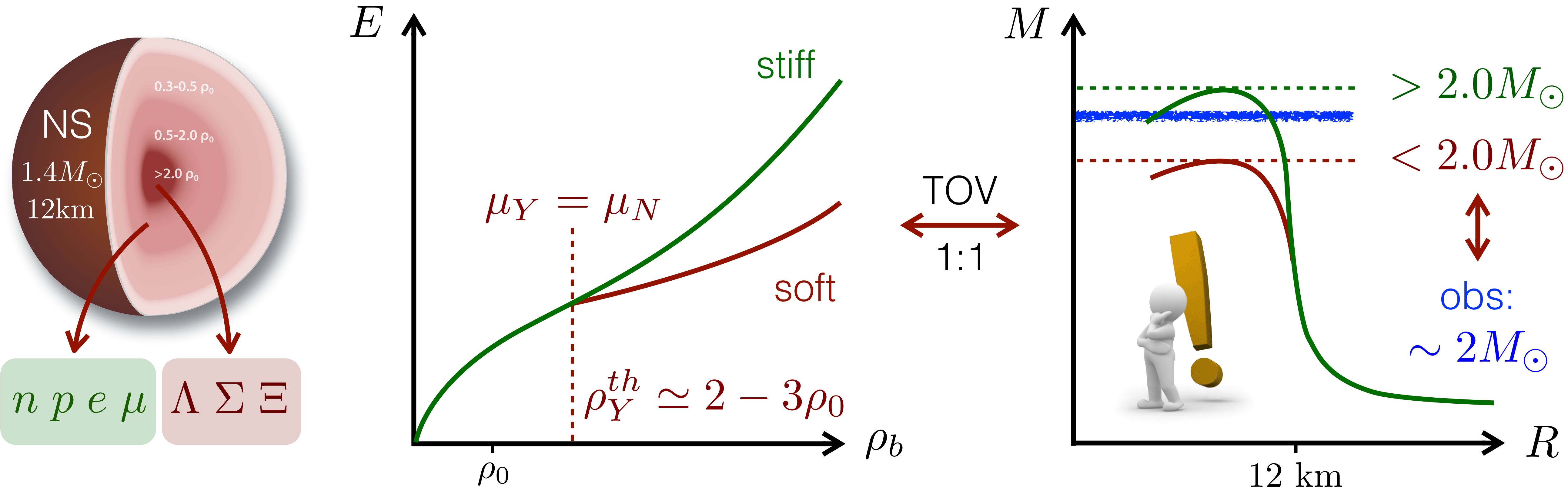

Hyperon puzzle

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Quantum Monte Carlo → YN interaction

light- to medium-heavy hypernuclei

AFDMC for strange systems

- ✓ AFDMC method

$$-\frac{\partial}{\partial\tau}|\psi(\tau)\rangle = (H - E_0)|\psi(\tau)\rangle \qquad \tau = it/\hbar$$

imaginary time

AFDMC for strange systems

- ✓ AFDMC method

$$-\frac{\partial}{\partial\tau}|\psi(\tau)\rangle = (H - E_0)|\psi(\tau)\rangle$$

$\tau = it/\hbar$ imaginary time

$$\downarrow$$

$$|\psi(\tau)\rangle = \mathrm{e}^{-(H-E_0)\tau}|\psi(0)\rangle$$

$$|\psi(0)\rangle = |\psi_T\rangle = \sum_{n=0}^{\infty} c_n|\varphi_n\rangle$$

AFDMC for strange systems

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$$|\psi(\tau)\rangle = \mathrm{e}^{-(H-E_0)\tau}|\psi(0)\rangle \qquad |\psi(0)\rangle = |\psi_T\rangle = \sum_{n=0}^{\infty} c_n|\varphi_n\rangle$$

$$= \sum_{n=0}^{\infty} \mathrm{e}^{-(E_n-E_0)\tau} c_n|\varphi_n\rangle \xrightarrow{\tau\to\infty} \boxed{c_0|\varphi_0\rangle}$$

projection

AFDMC for strange systems

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$$-\frac{\partial}{\partial\tau}|\psi(\tau)\rangle = (H - E_0)|\psi(\tau)\rangle \qquad \tau = it/\hbar$$

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projection

$$\Big\downarrow$$

$$E = \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} \xrightarrow{\tau\to\infty} \boxed{E_0}$$

ground state

AFDMC for strange systems

✓ AFDMC algorithm

- imaginary time projection $\longrightarrow$ exact ground state
- single particle wf + HS transformation $\longrightarrow$ large number of particles
- stochastic method $\longrightarrow$ error estimate: $\sigma \sim 1/\sqrt{\mathcal{N}}$

AFDMC for strange systems

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✓ AFDMC Hamiltonians

- nucleon-nucleon phenomenological interaction: Argonne & Urbana

$$H = \boxed{\sum_i \frac{p_i^2}{2m_N} + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}}$$

2B: NN scattering + deuteron

3B: nuclei + nuclear matter

AFDMC for strange systems

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- nucleon-nucleon phenomenological interaction: Argonne & Urbana
- hyperon-nucleon phenomenological interaction: Argonne & Urbana like

$$H = \sum_i \frac{p_i^2}{2m_N} + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

$$+ \sum_\lambda \frac{p_\lambda^2}{2m_\Lambda} + \sum_{\lambda,i} v_{\lambda i} + \sum_{\lambda,i<j} v_{\lambda ij}$$

2B: Λp scattering + $A = 4$ CSB*

3B:

AFDMC for strange systems

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3B: no unique fit

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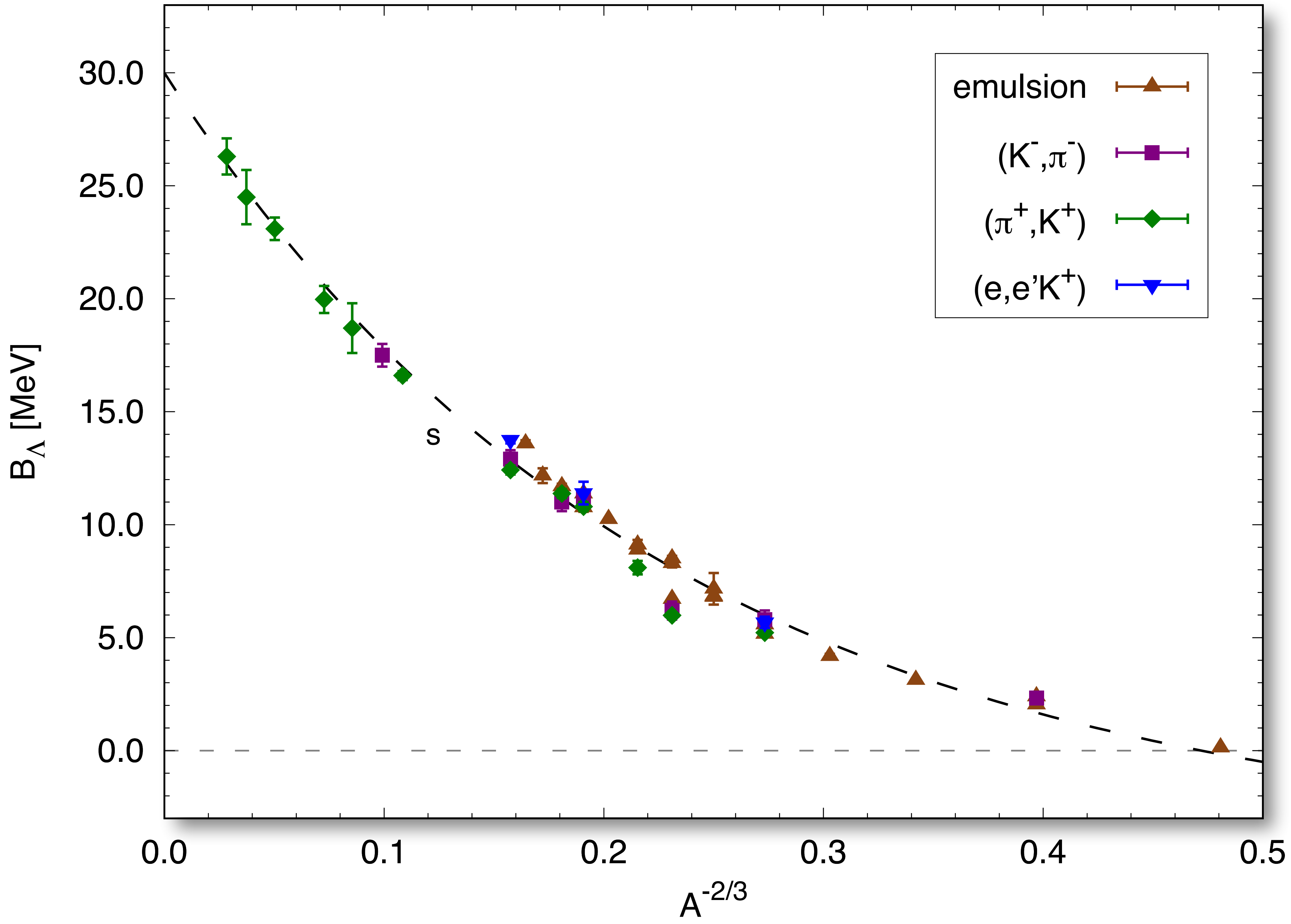
$$+ \sum_\lambda \frac{p_\lambda^2}{2m_\Lambda} + \sum_{\lambda,i} v_{\lambda i} + \boxed{\sum_{\lambda,i<j} v_{\lambda ij}}$$

use QMC to fit hyp. exp. data

$$B_\Lambda = E(^{A-1}Z) - E(^{A}_{\Lambda}Z)$$

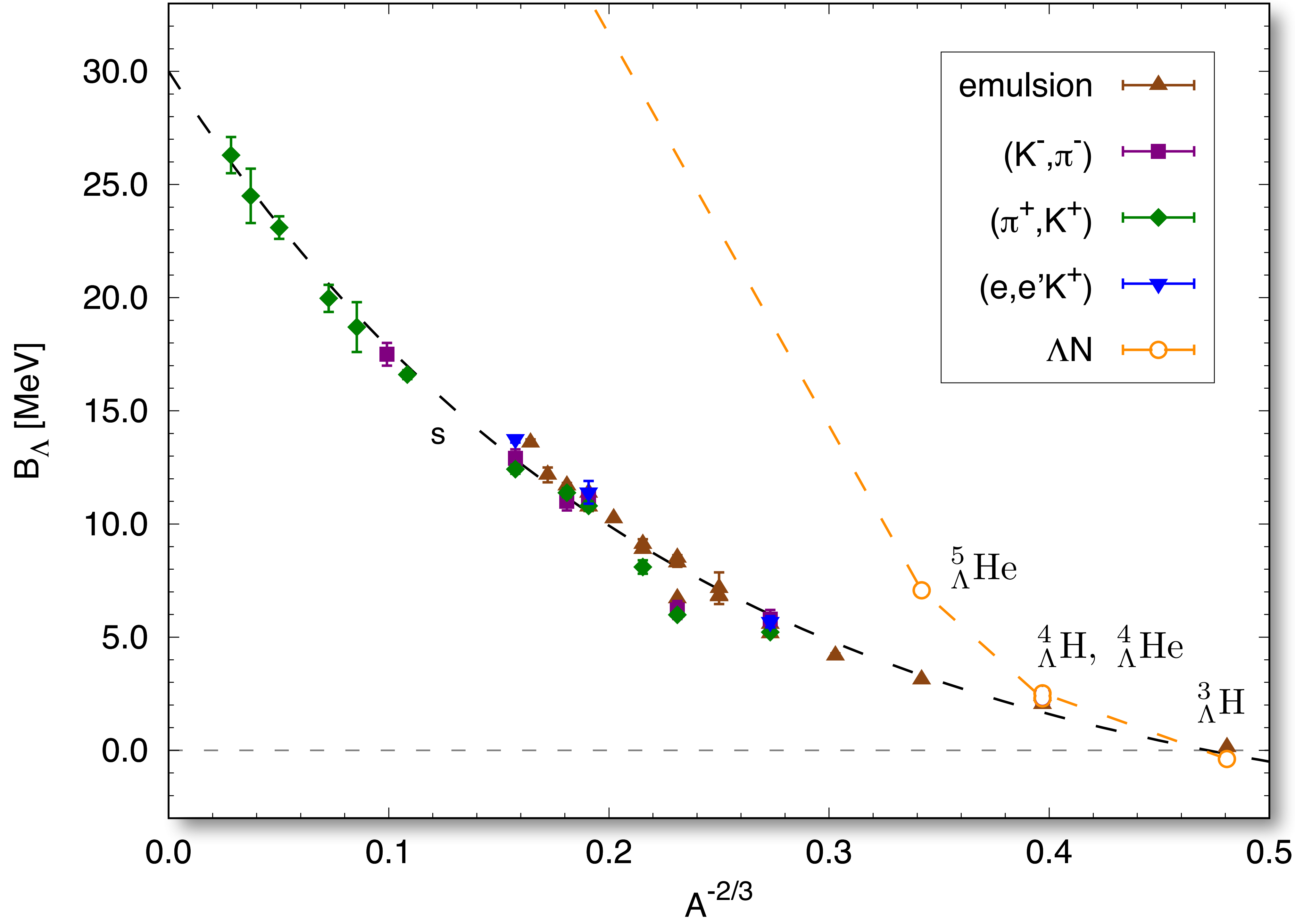
3B: no unique fit

Hyperons in nuclei



D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)

Hyperons in nuclei



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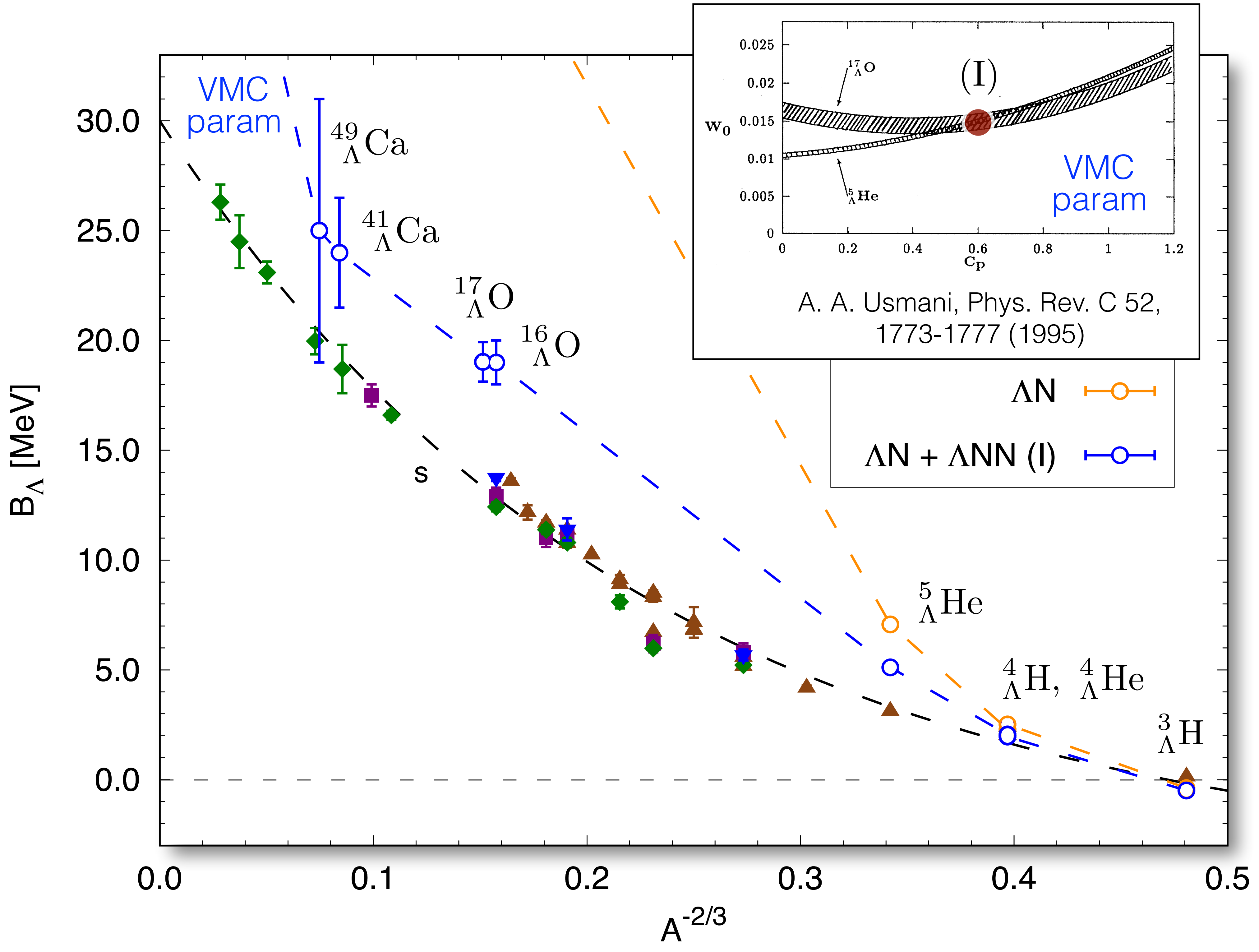

A. A. Usmani, Phys. Rev. C 52, 1773-1777 (1995)

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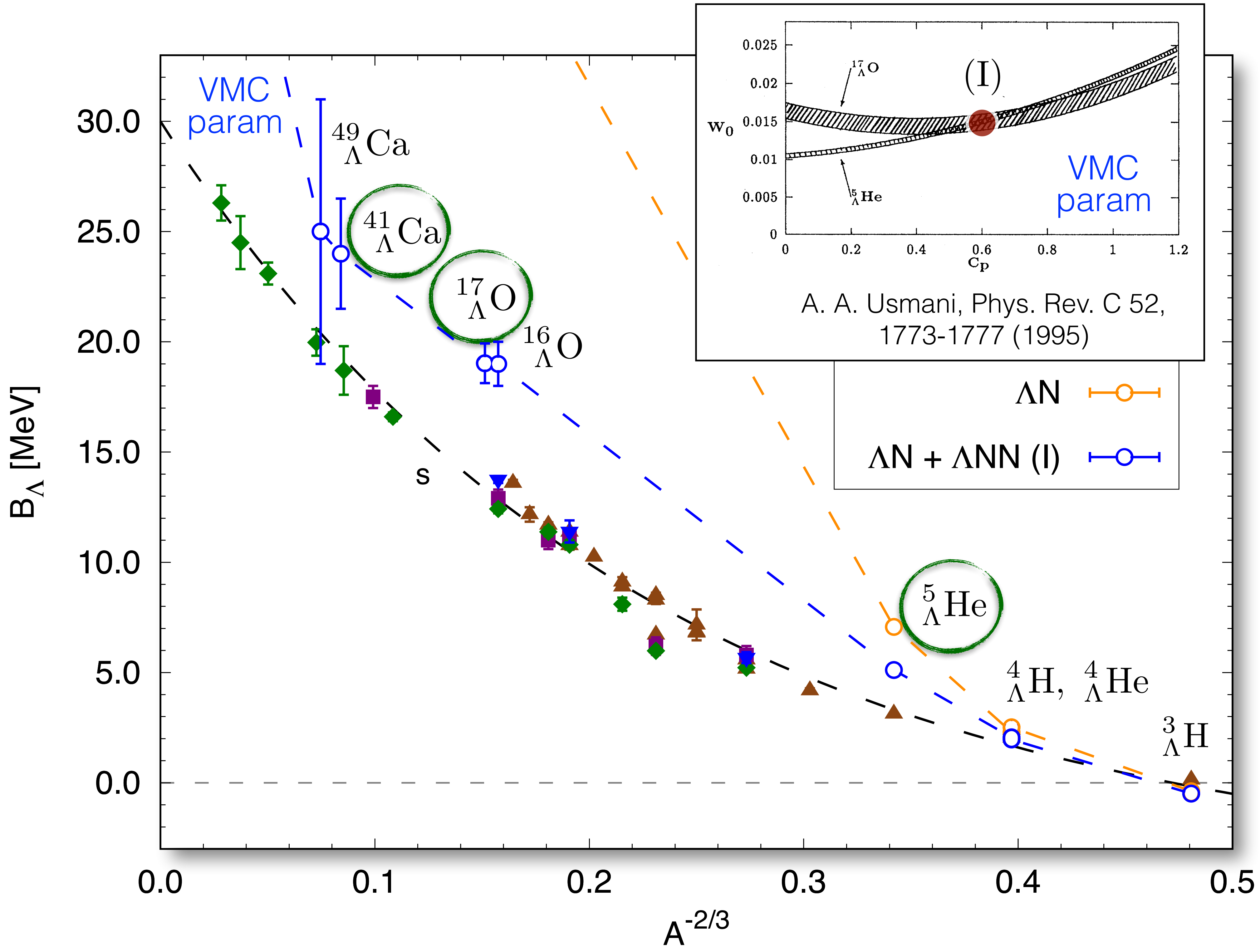

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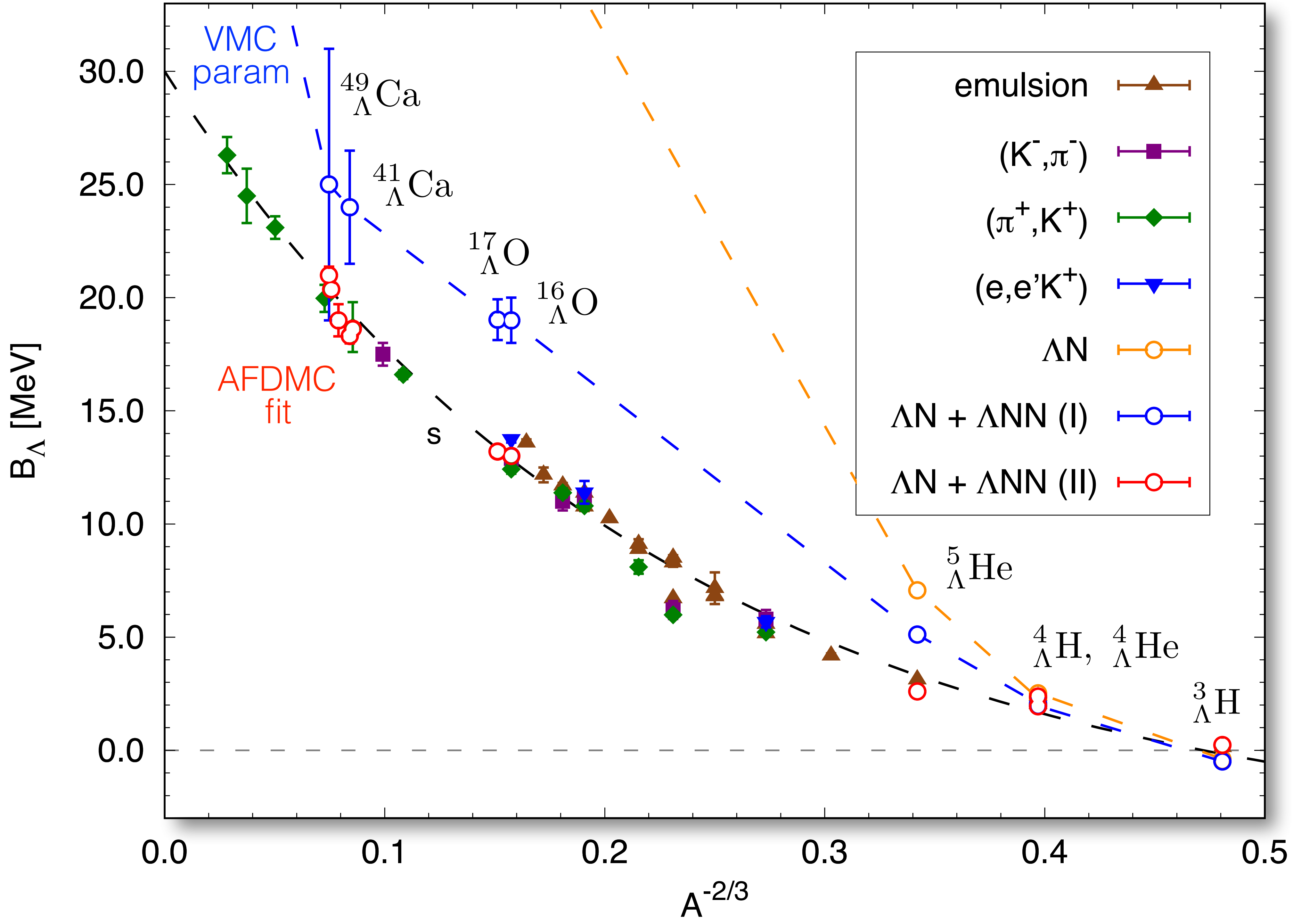

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F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)

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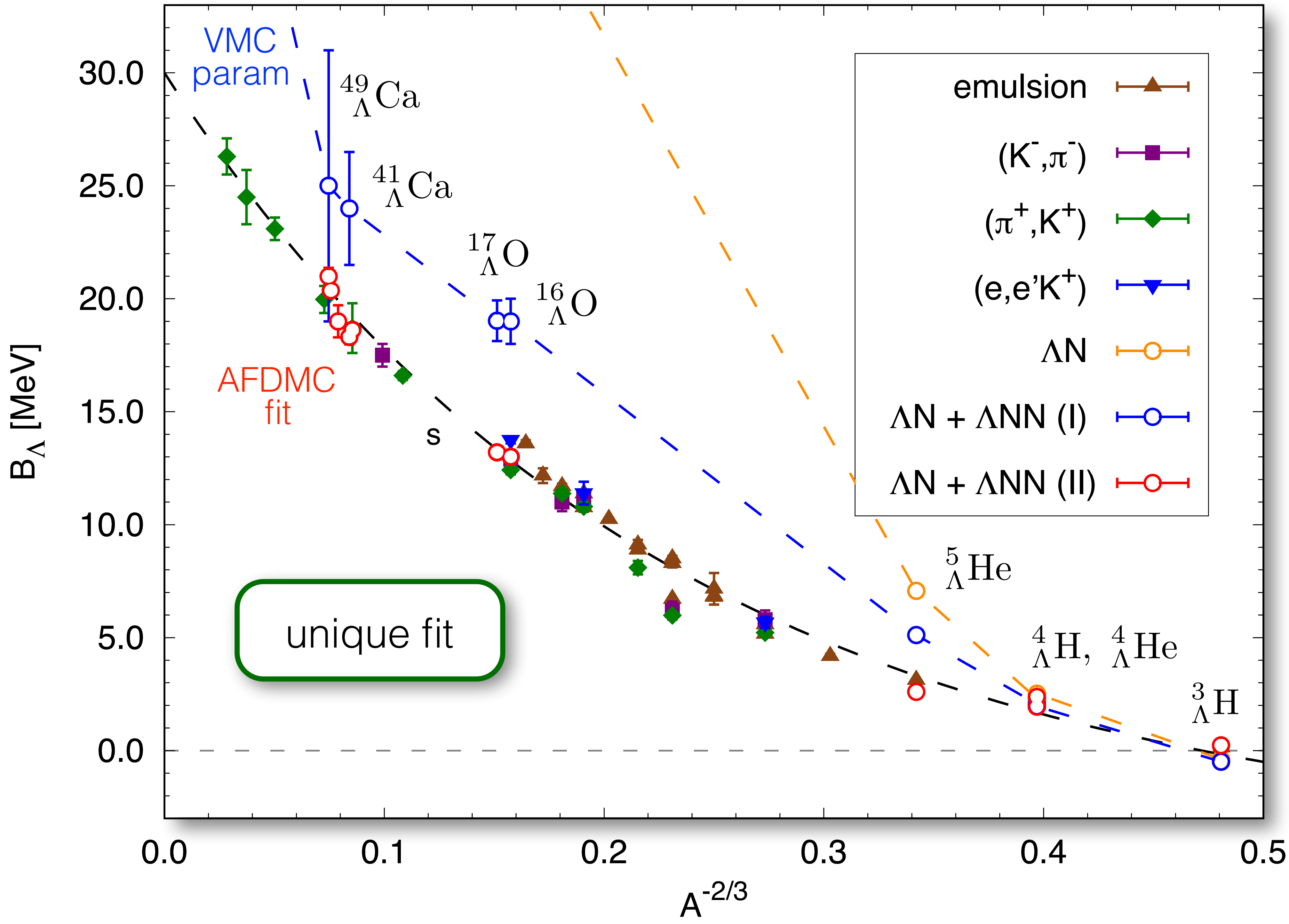

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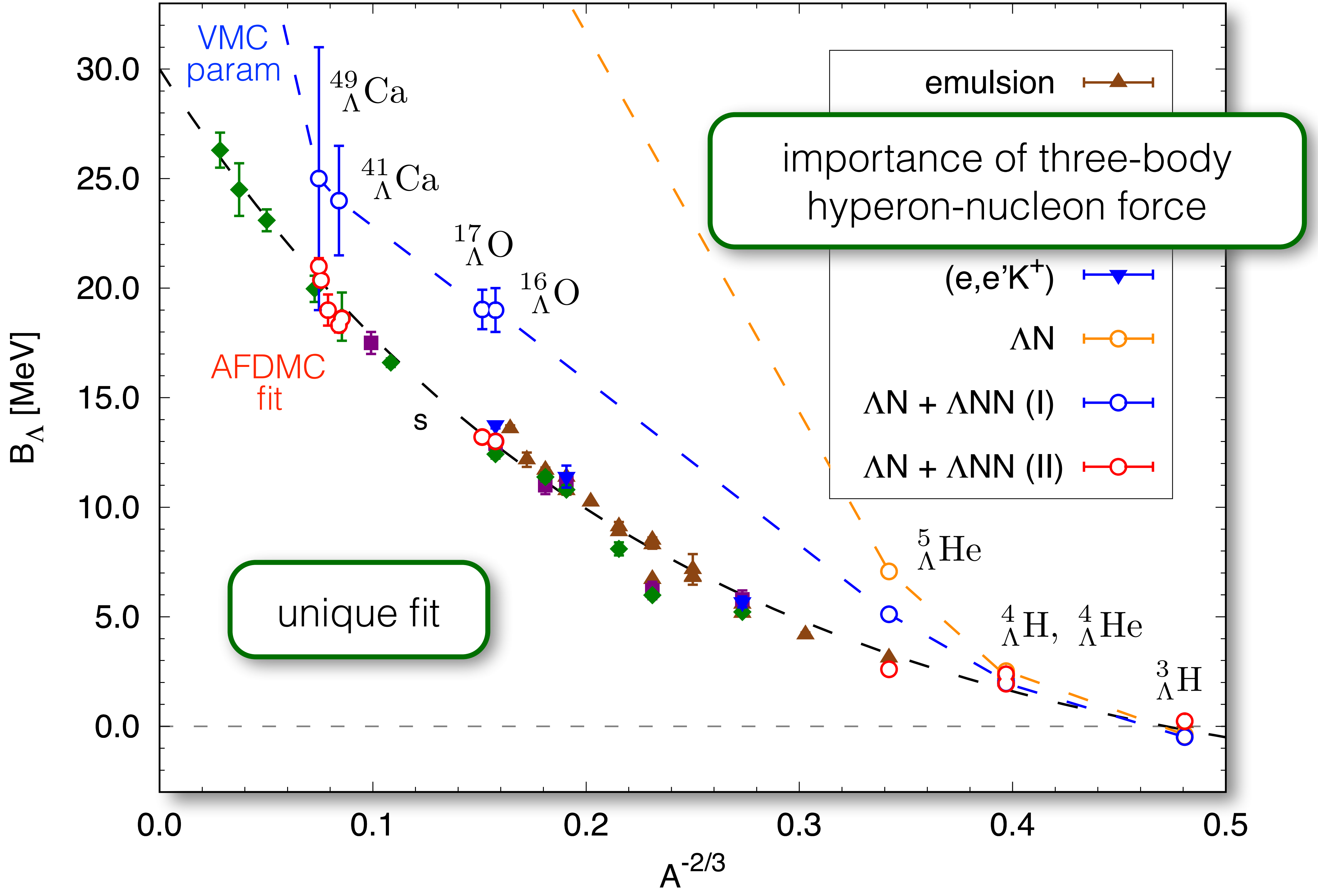

importance of three-body hyperon-nucleon force

unique fit

D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)

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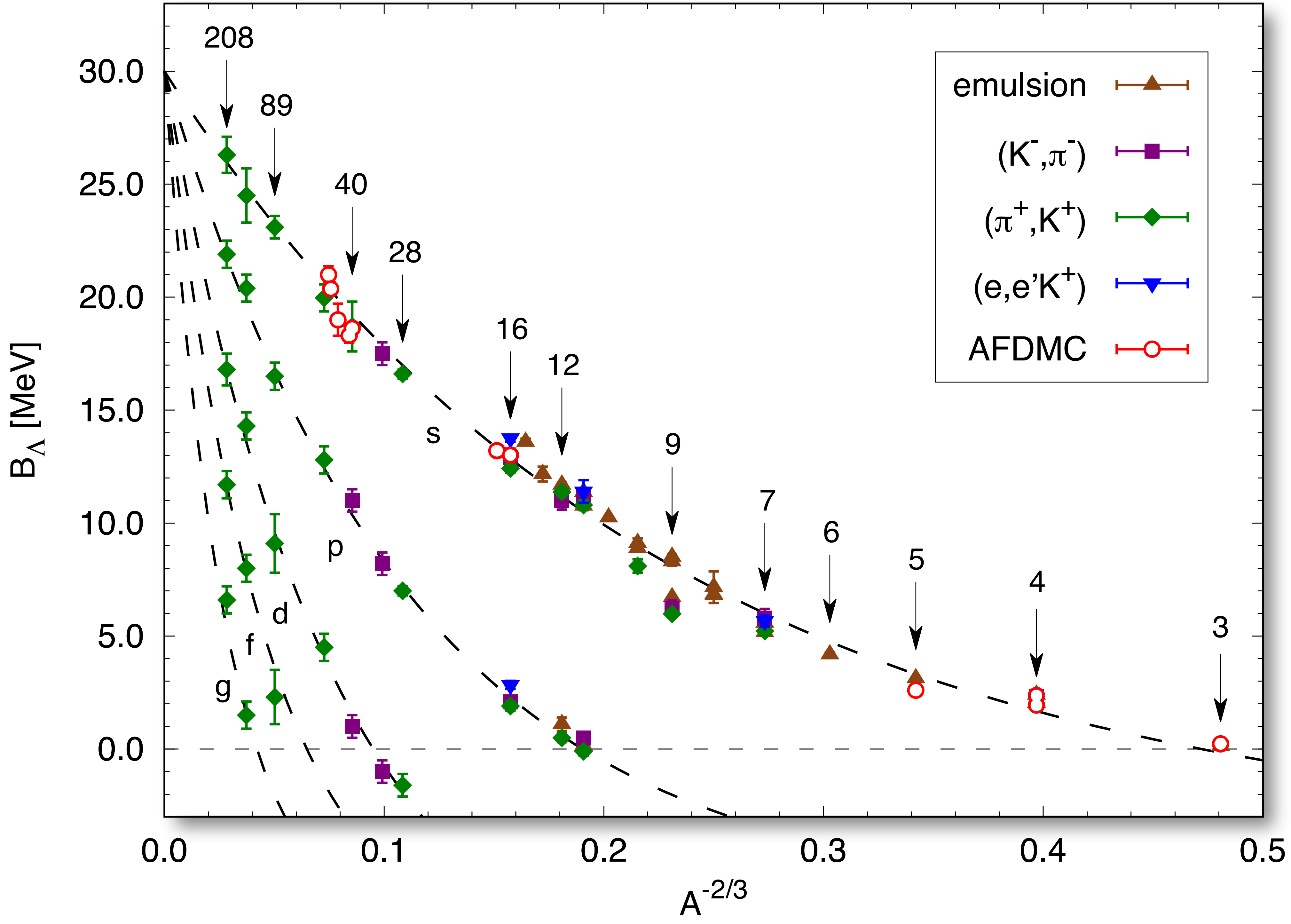

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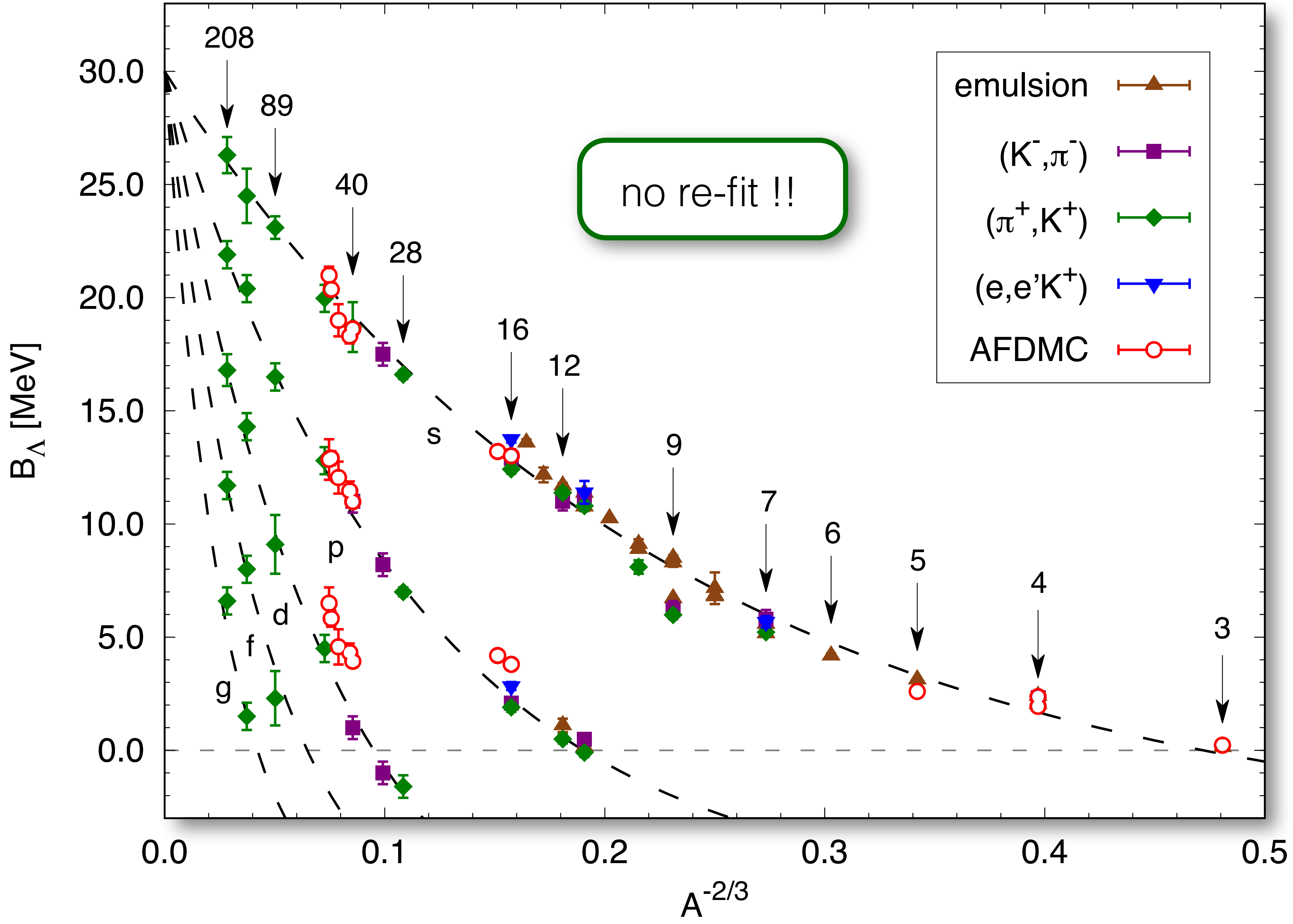

no re-fit !!

D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)

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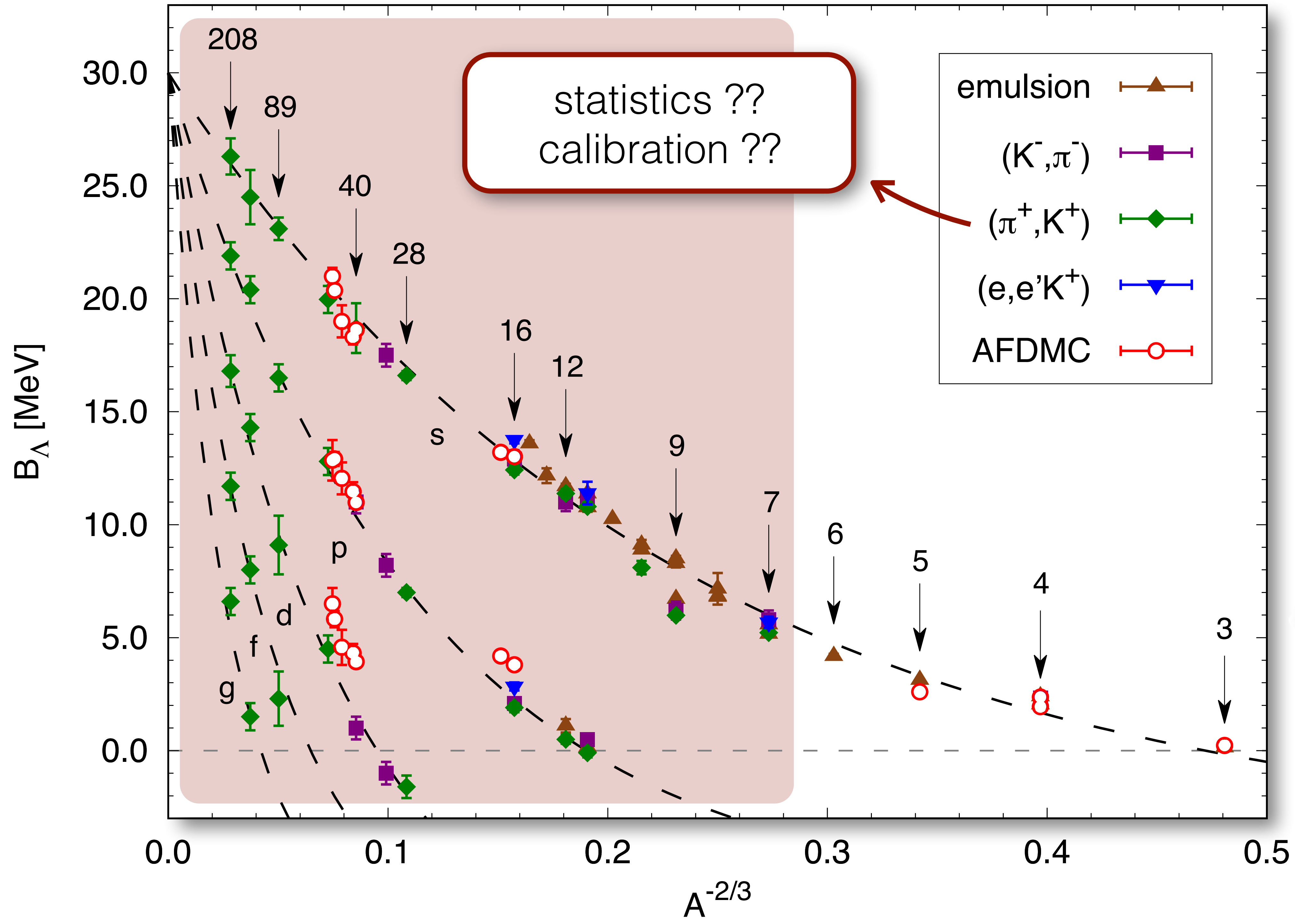

statistics ??
calibration ??

D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)
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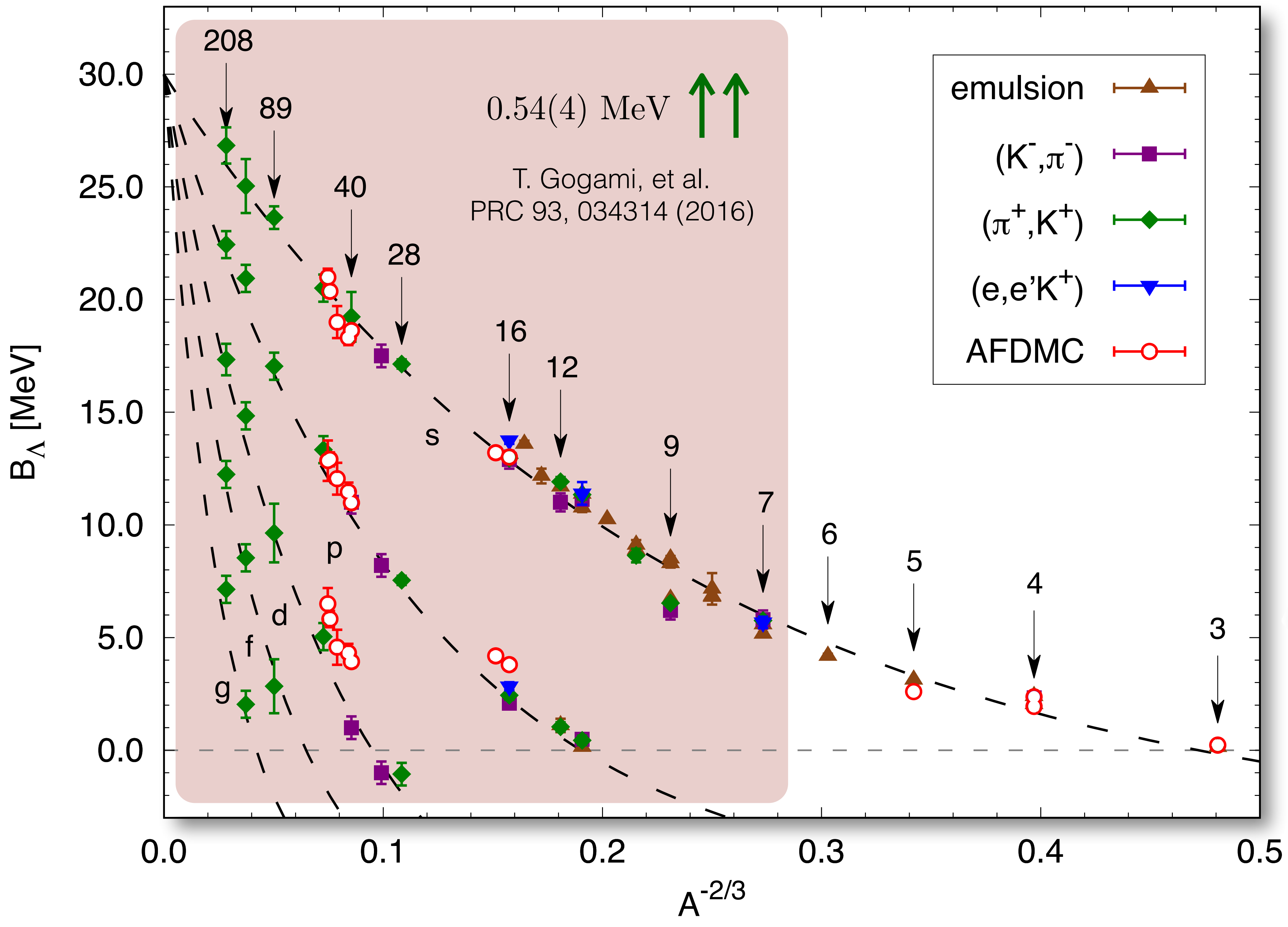

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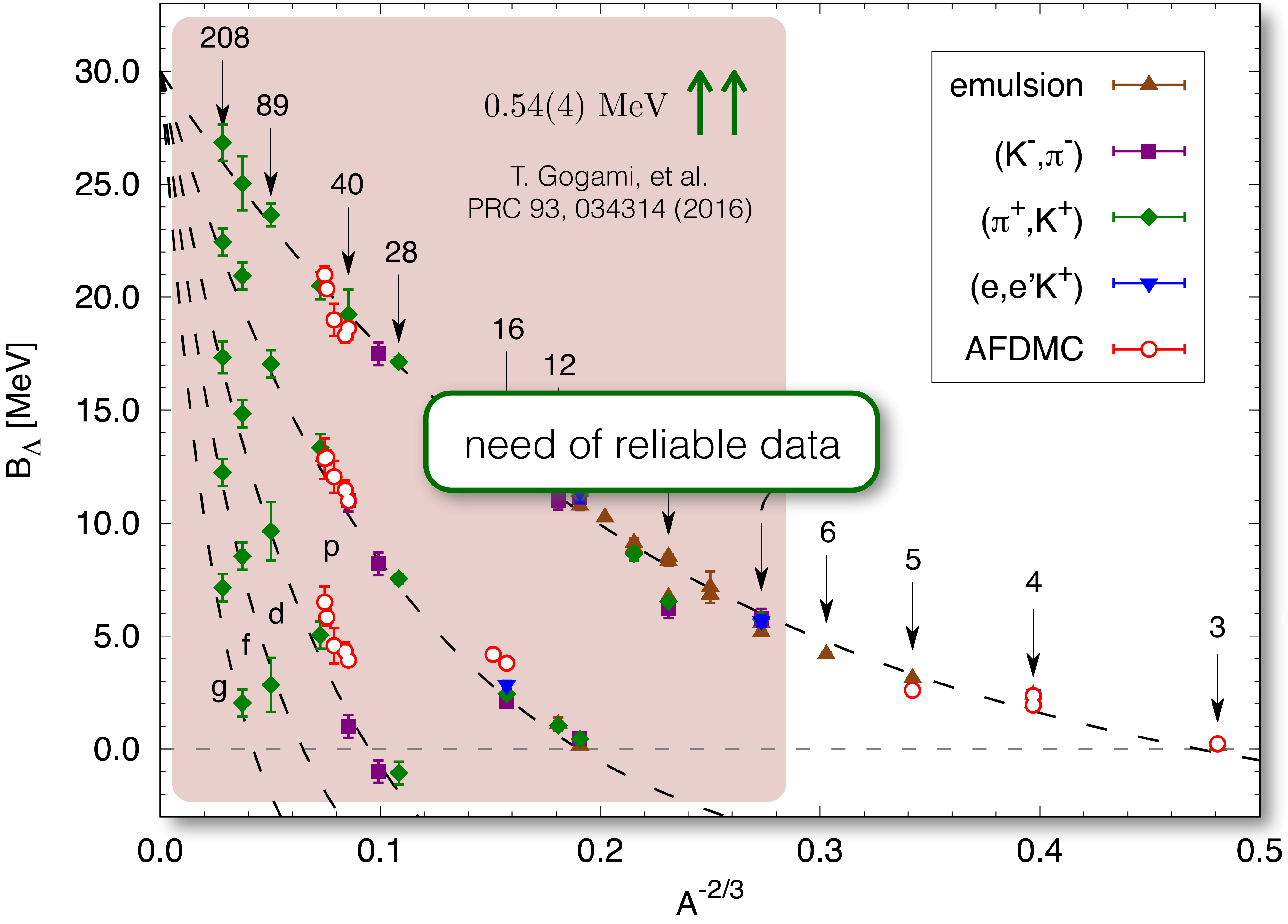
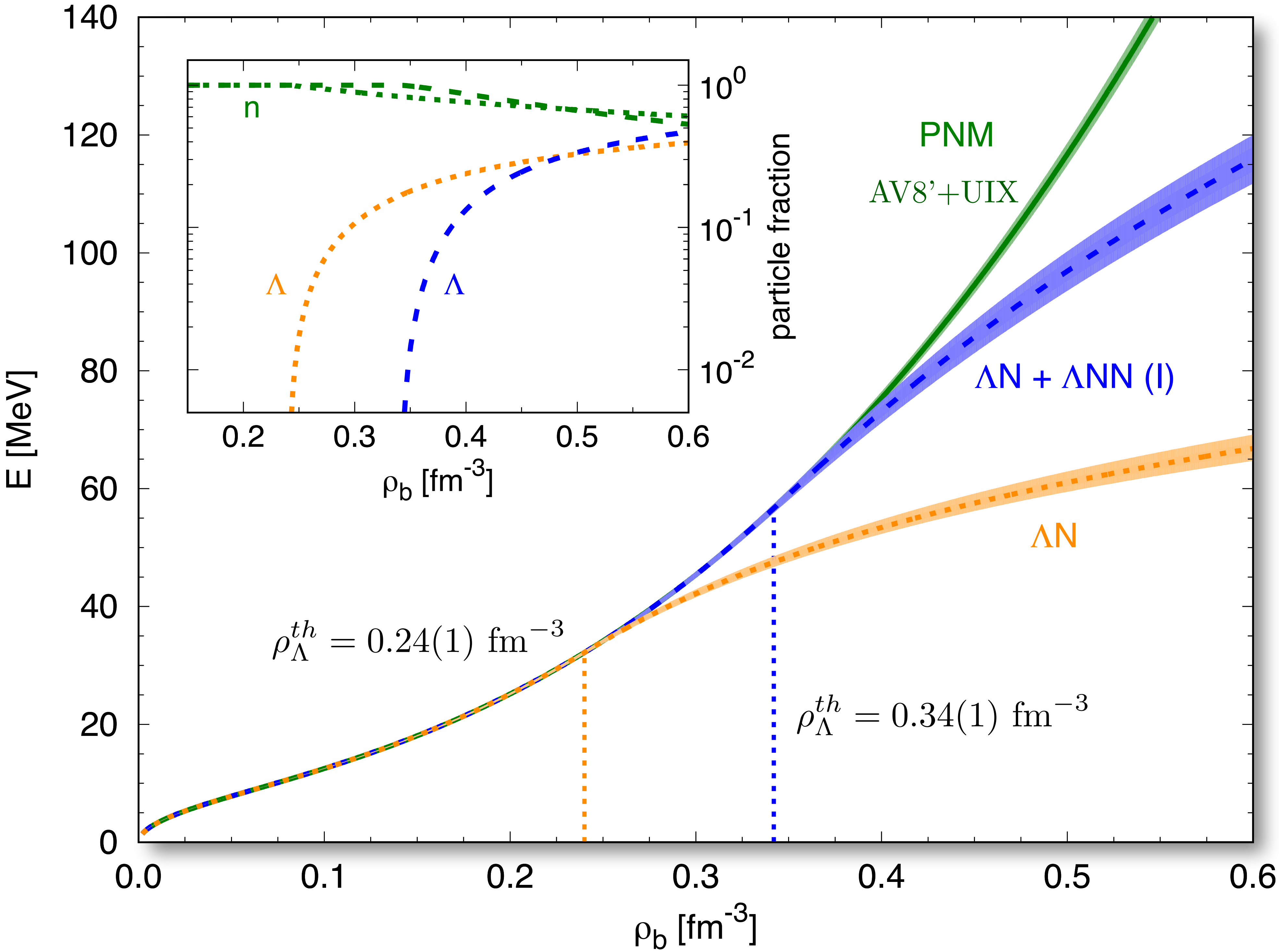


T. Gogami, et al.
PRC 93, 034314 (2016)

need of reliable data

D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)
F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)

Hyperons in neutron stars



D. L., A. Lovato, S. Gandolfi, F. Pederiva, Phys. Rev. Lett. 114, 092301 (2015)

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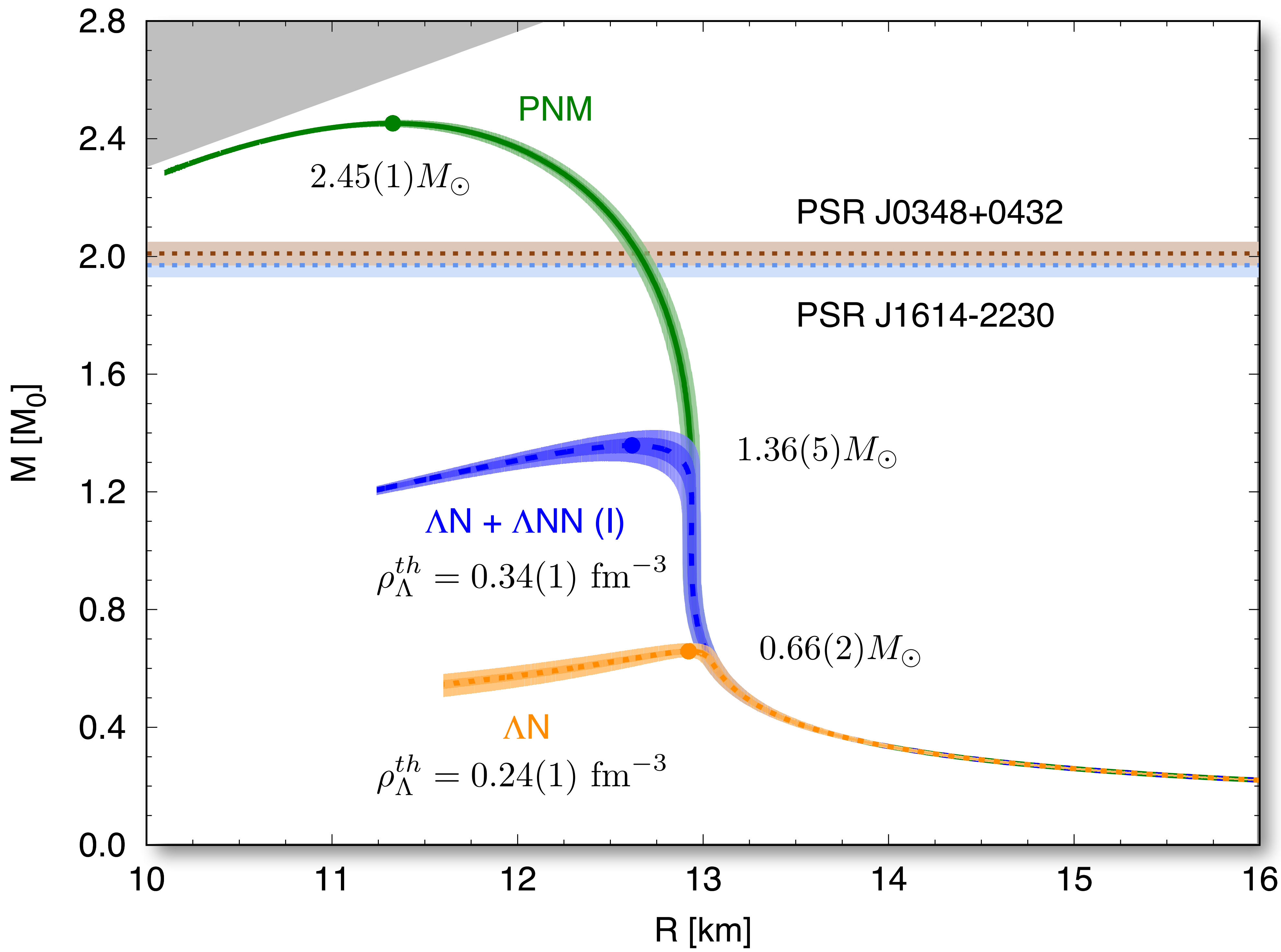

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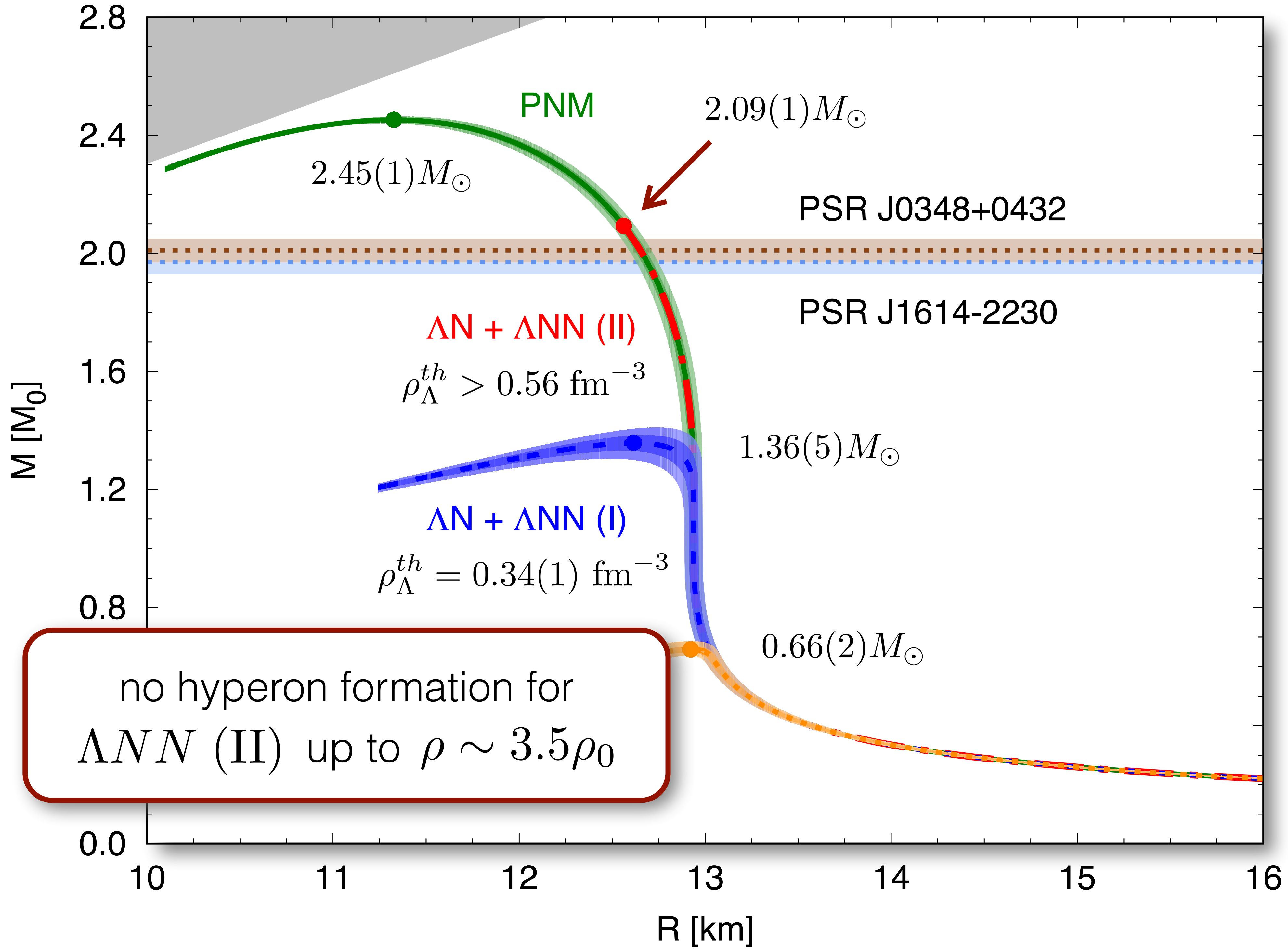

no hyperon formation for $\Lambda NN\ (\mathrm{II})$ up to $\rho \sim 3.5\rho_0$

D. L., A. Lovato, S. Gandolfi, F. Pederiva, Phys. Rev. Lett. 114, 092301 (2015)

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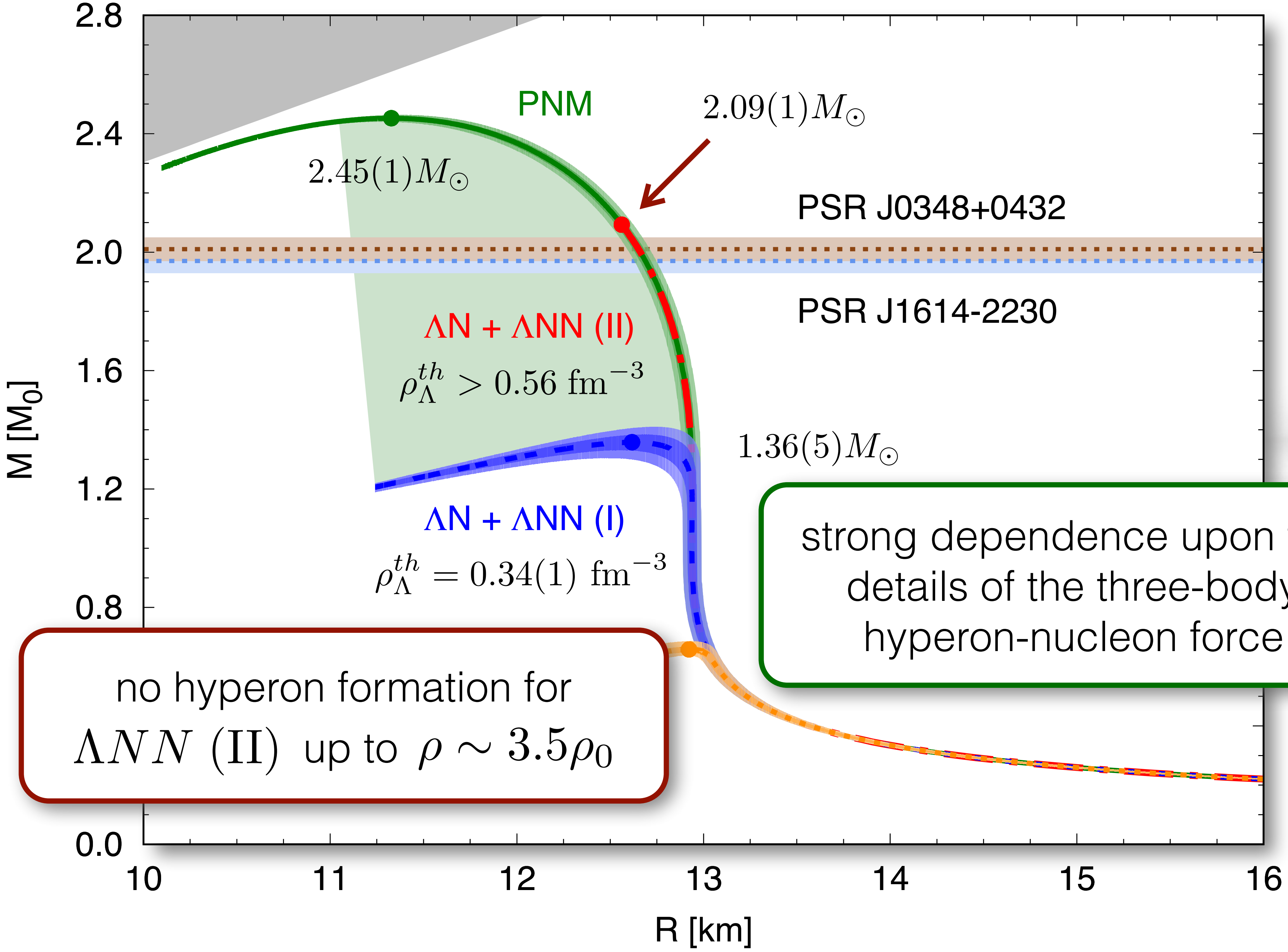

strong dependence upon the details of the three-body hyperon-nucleon force

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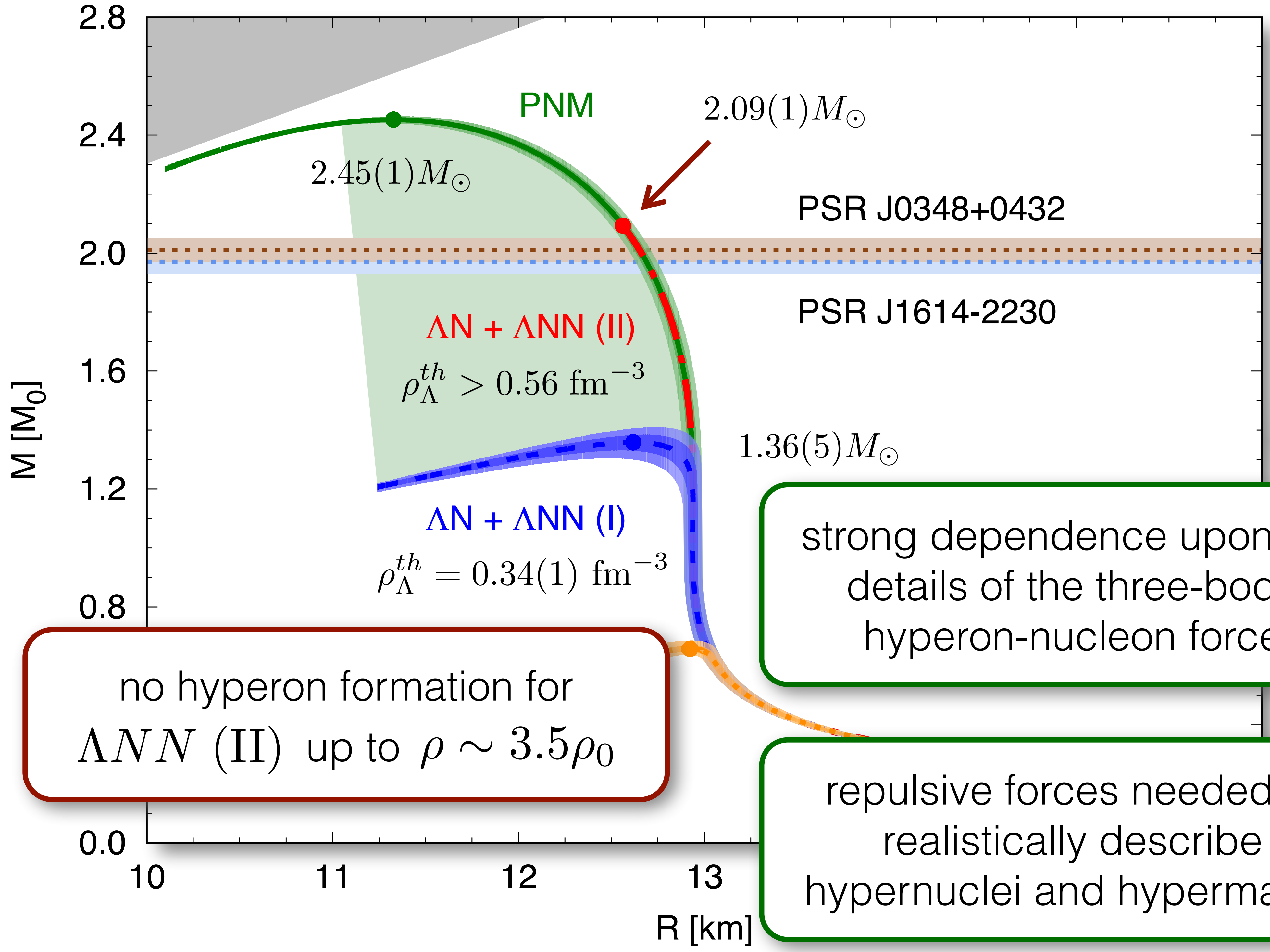

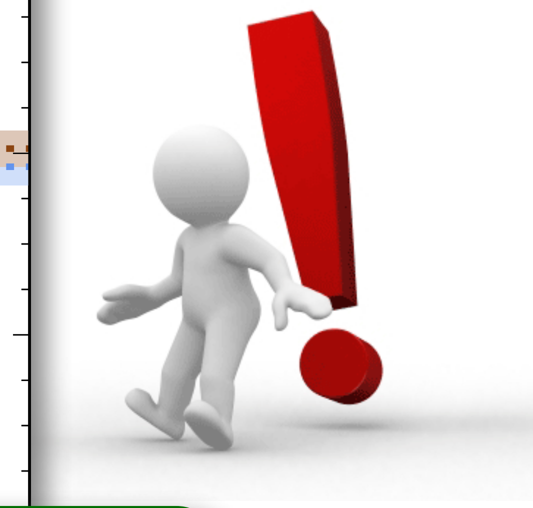

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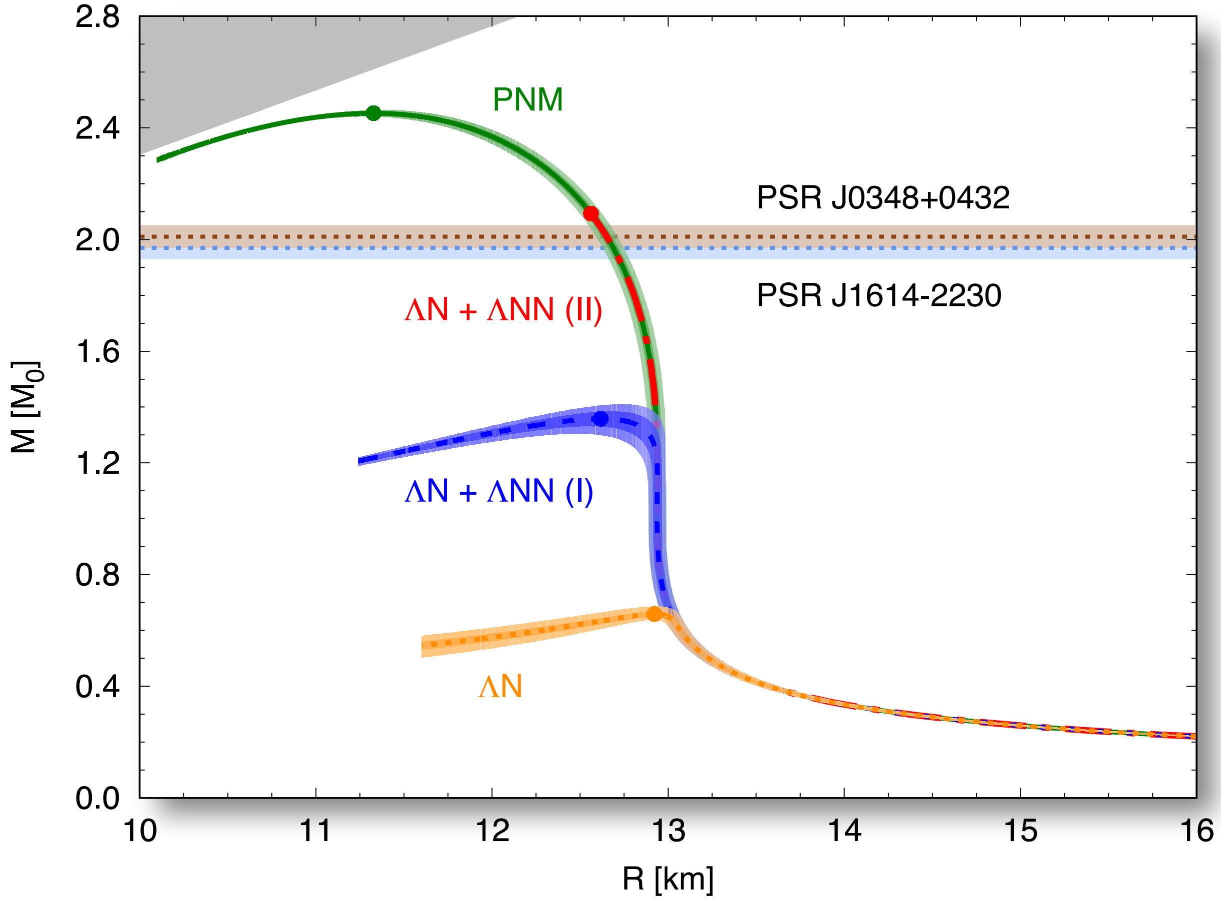
repulsive forces needed to realistically describe hypernuclei and hypermatter

D. L., A. Lovato, S. Gandolfi, F. Pederiva, Phys. Rev. Lett. 114, 092301 (2015)

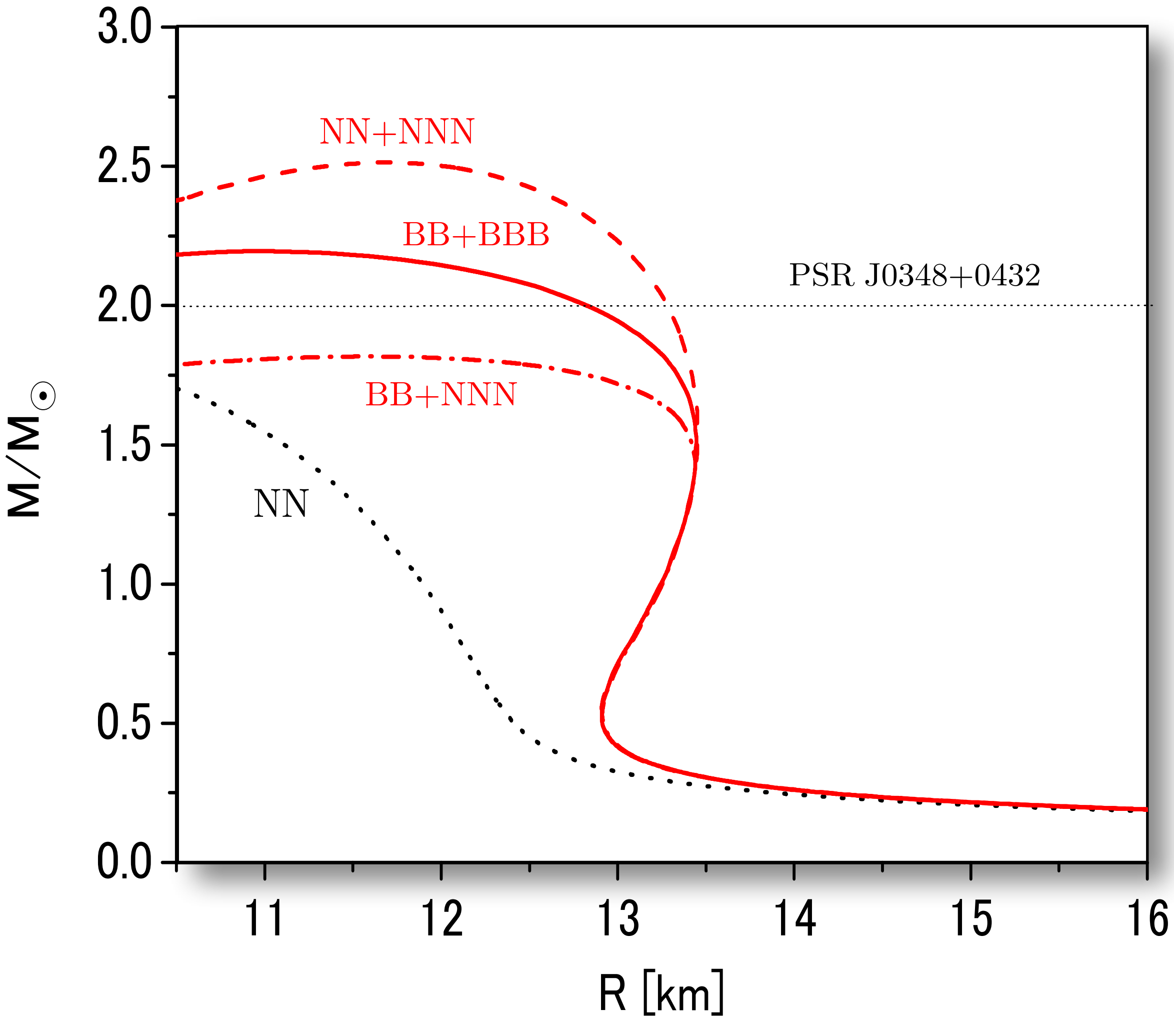
Hyperons in neutron stars

AFDMC: Argonne + Urbana



Phys. Rev. Lett. 114, 092301 (2015)

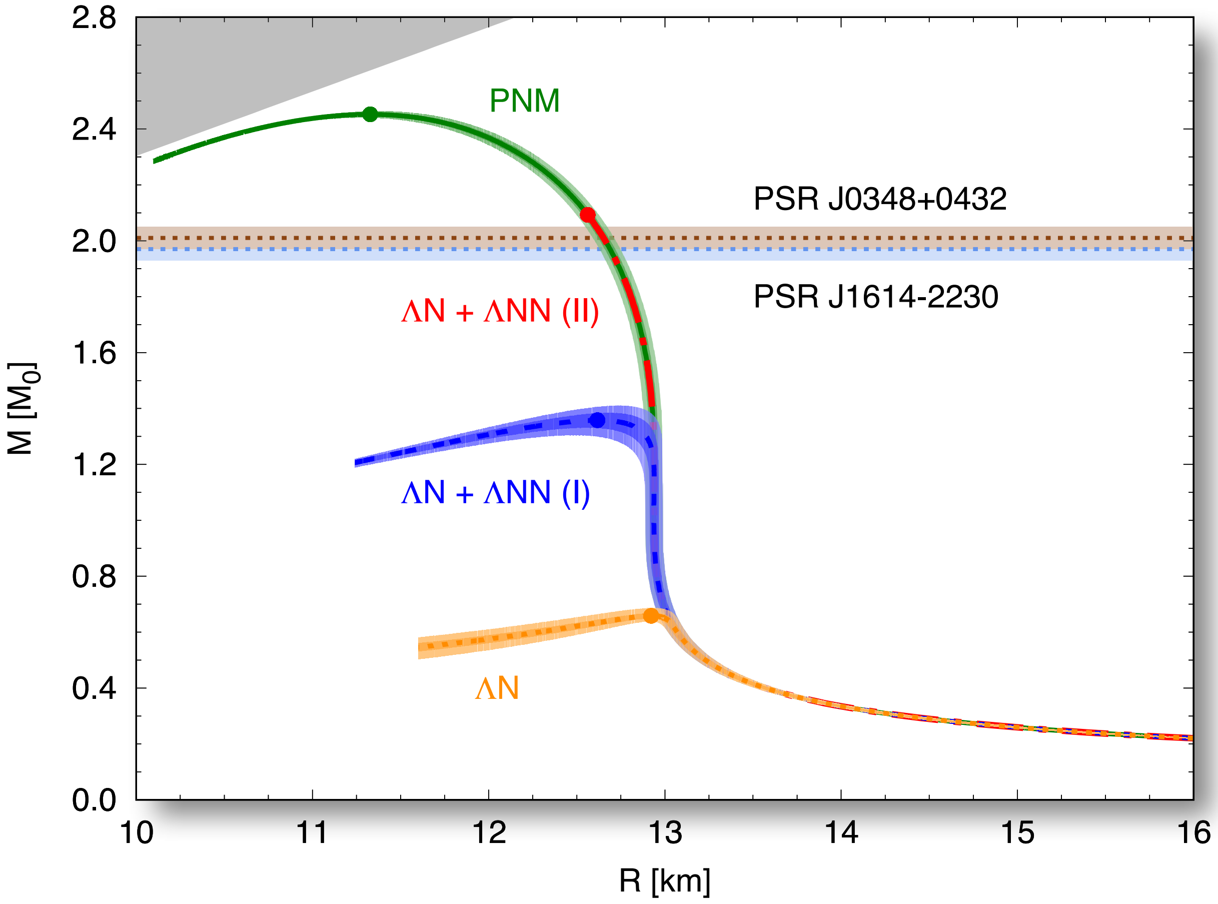
G-Matrix: ESC08 + MPa



Phys. Rev. C 90, 045805 (2014)

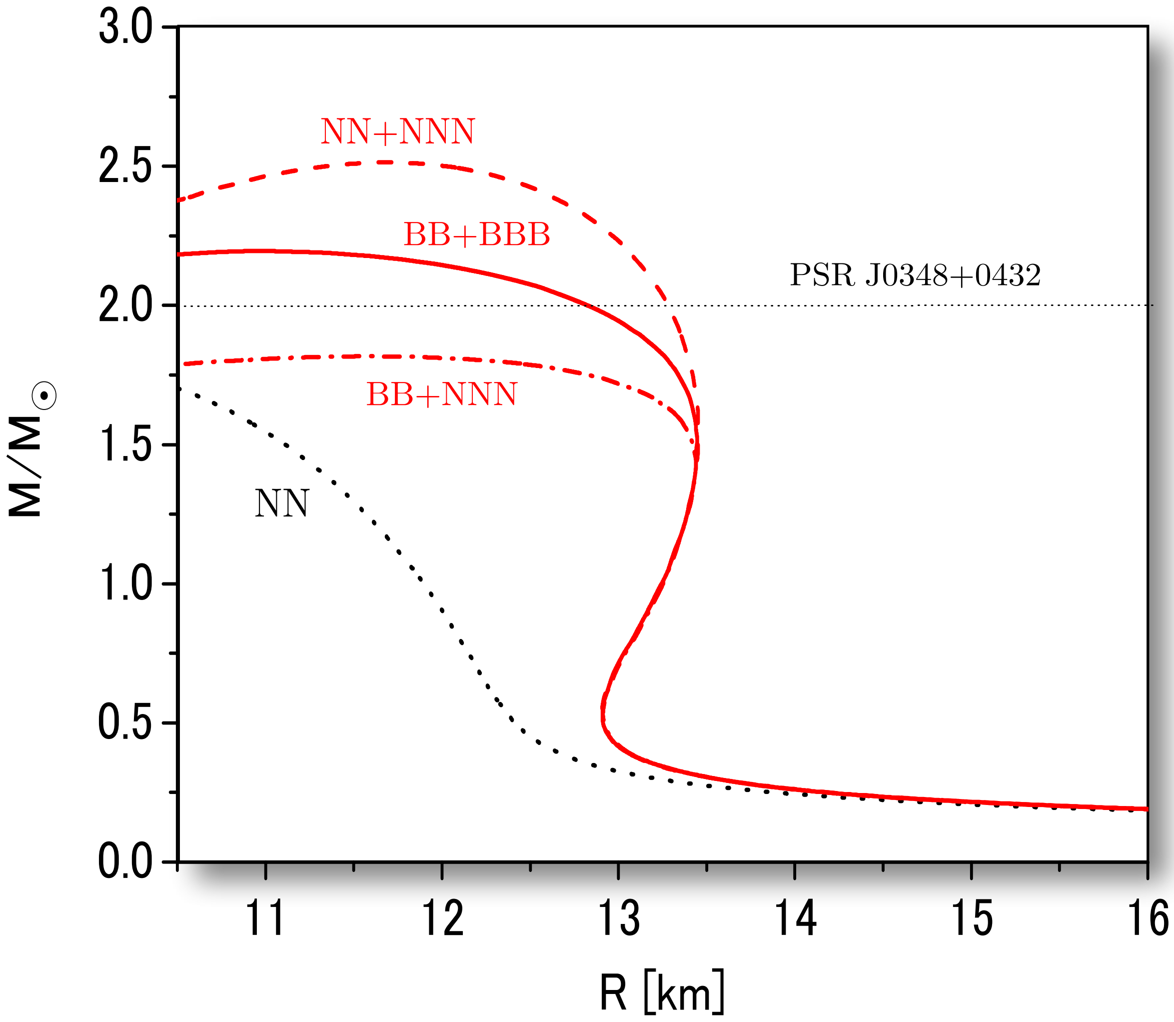
Hyperons in neutron stars

AFDMC: Argonne + Urbana



Phys. Rev. Lett. 114, 092301 (2015)

G-Matrix: ESC08 + MPa



Phys. Rev. C 90, 045805 (2014)

2010

~2013

hyperons in NS? **yes!**

good EOS

hyperons in NS? **no!**

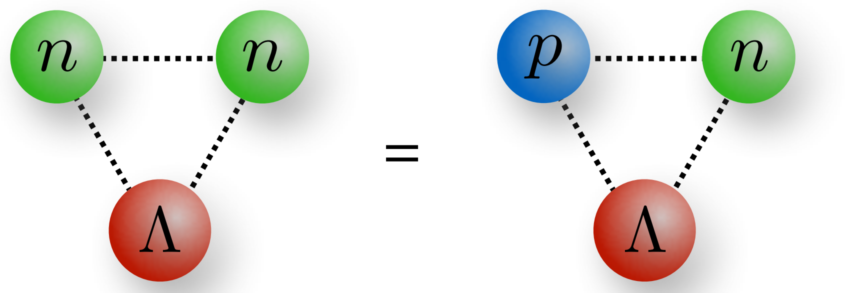
too soft EOS

hyperons in NS? **maybe…**

need of repulsion

Hyperons in nuclei

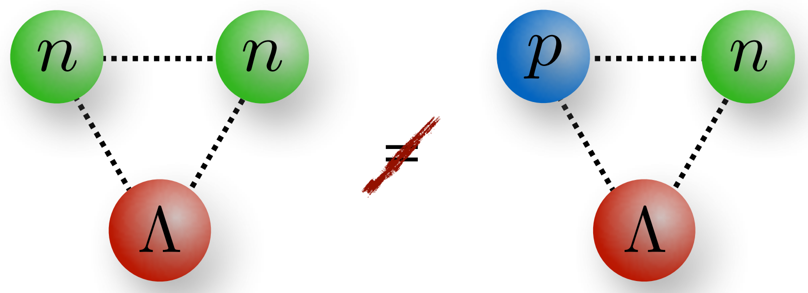
3-body interaction → fit on symmetric hypernuclei



ΛNN force: no dependence on singlet or triplet nucleon isospin state

Hyperons in nuclei

3-body interaction → fit on symmetric hypernuclei



ΛNN force: no dependence on singlet or triplet nucleon isospin state

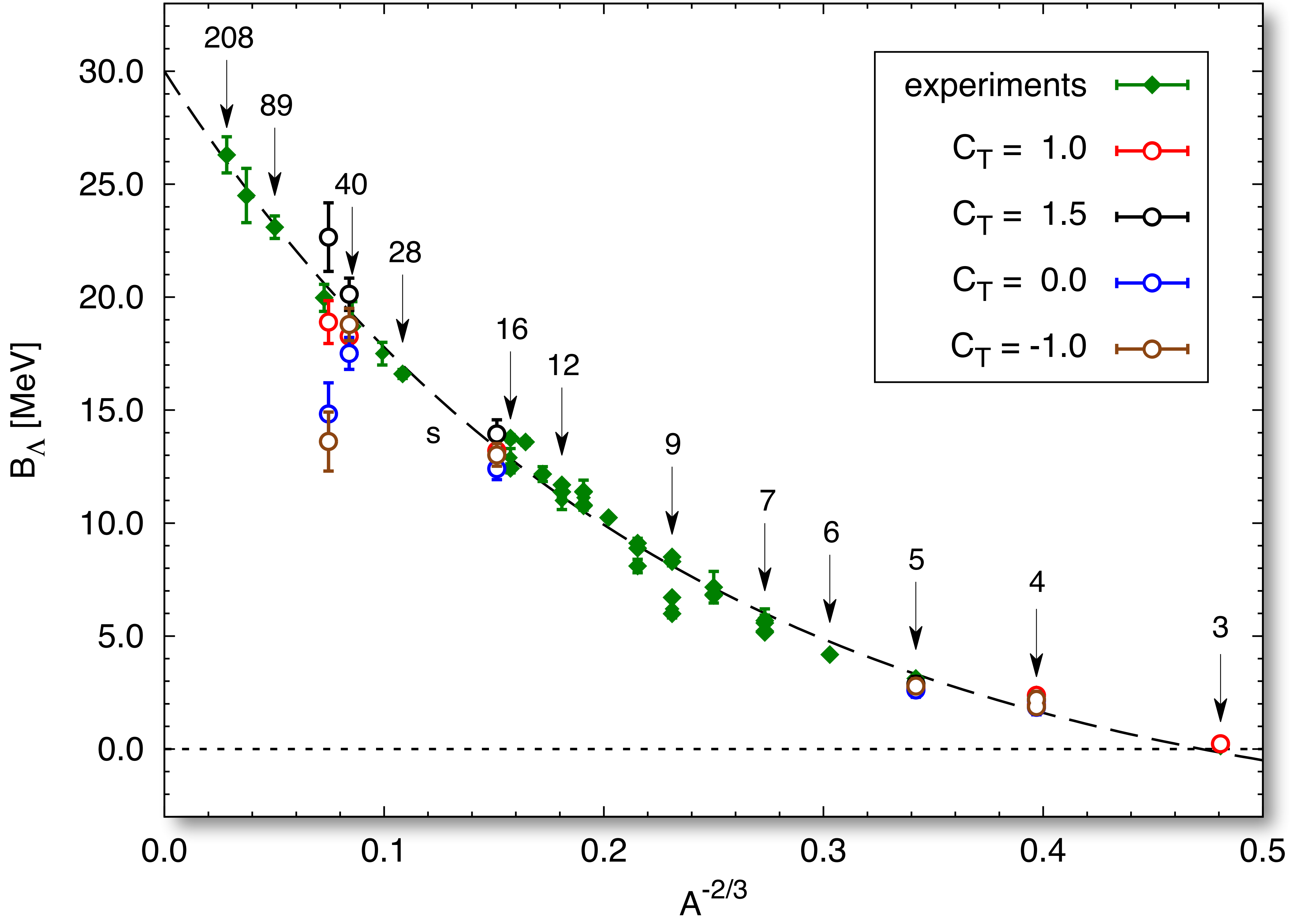
$$\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j = -3\,\mathcal{P}^{T=0} + \mathcal{P}^{T=1} \longrightarrow -3\,\mathcal{P}^{T=0} + C_T\,\mathcal{P}^{T=1}$$

isospin projectors

control parameter:
strength and sign of the nucleon isospin triplet channel

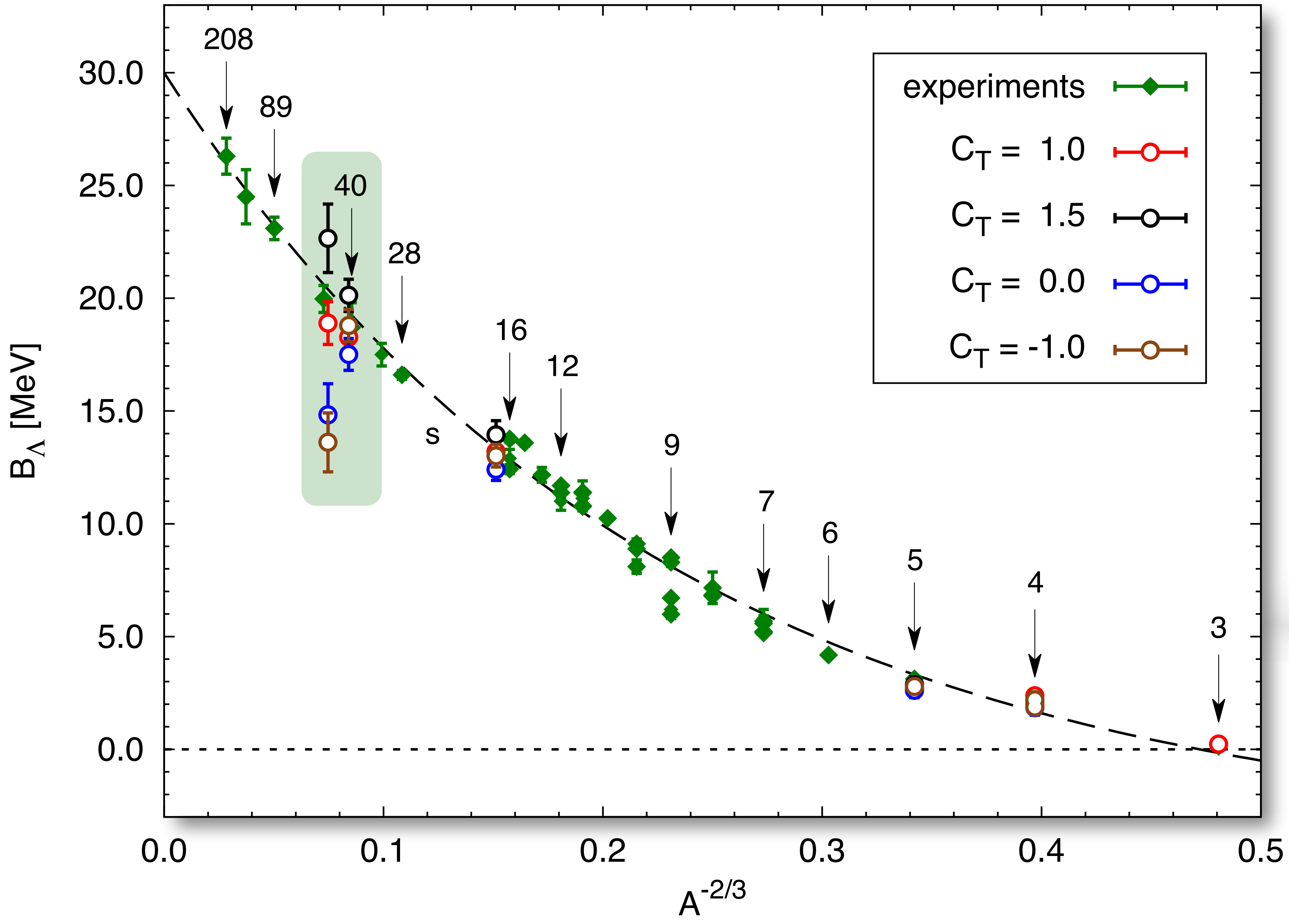
sensitivity study:
light- & medium-heavy hypernuclei

Hyperons in nuclei



F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)

Hyperons in nuclei



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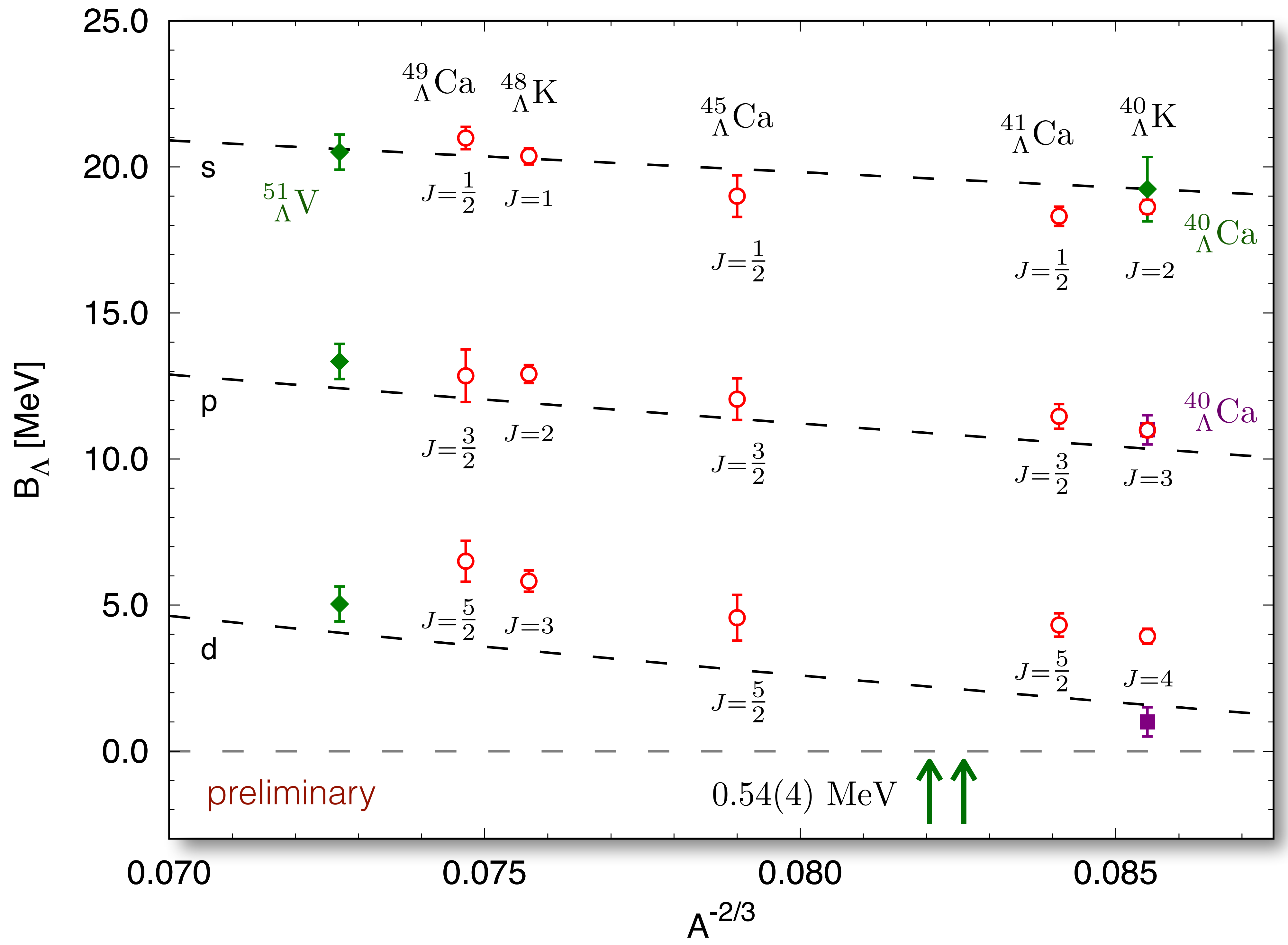
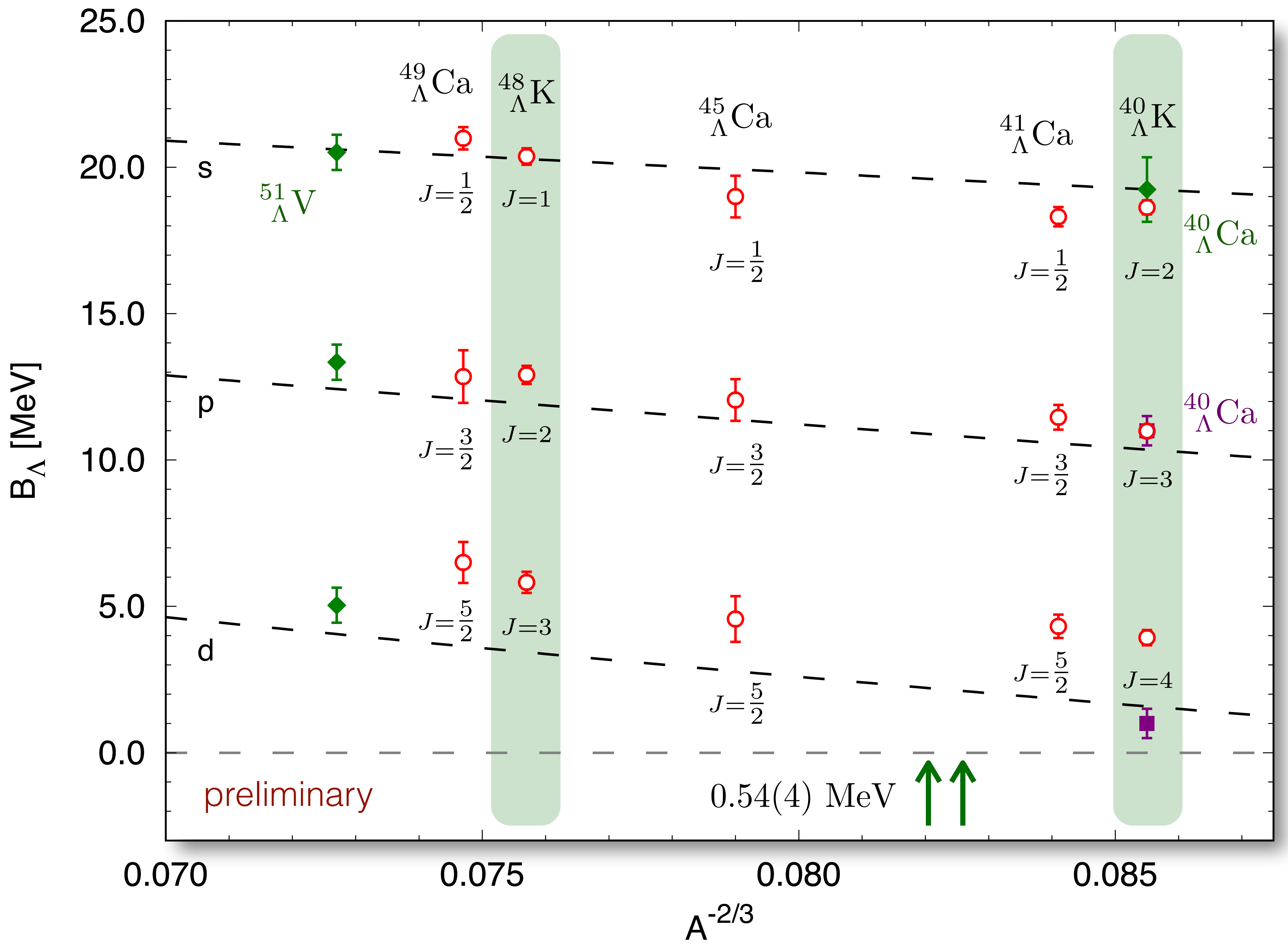


Hyperons in nuclei

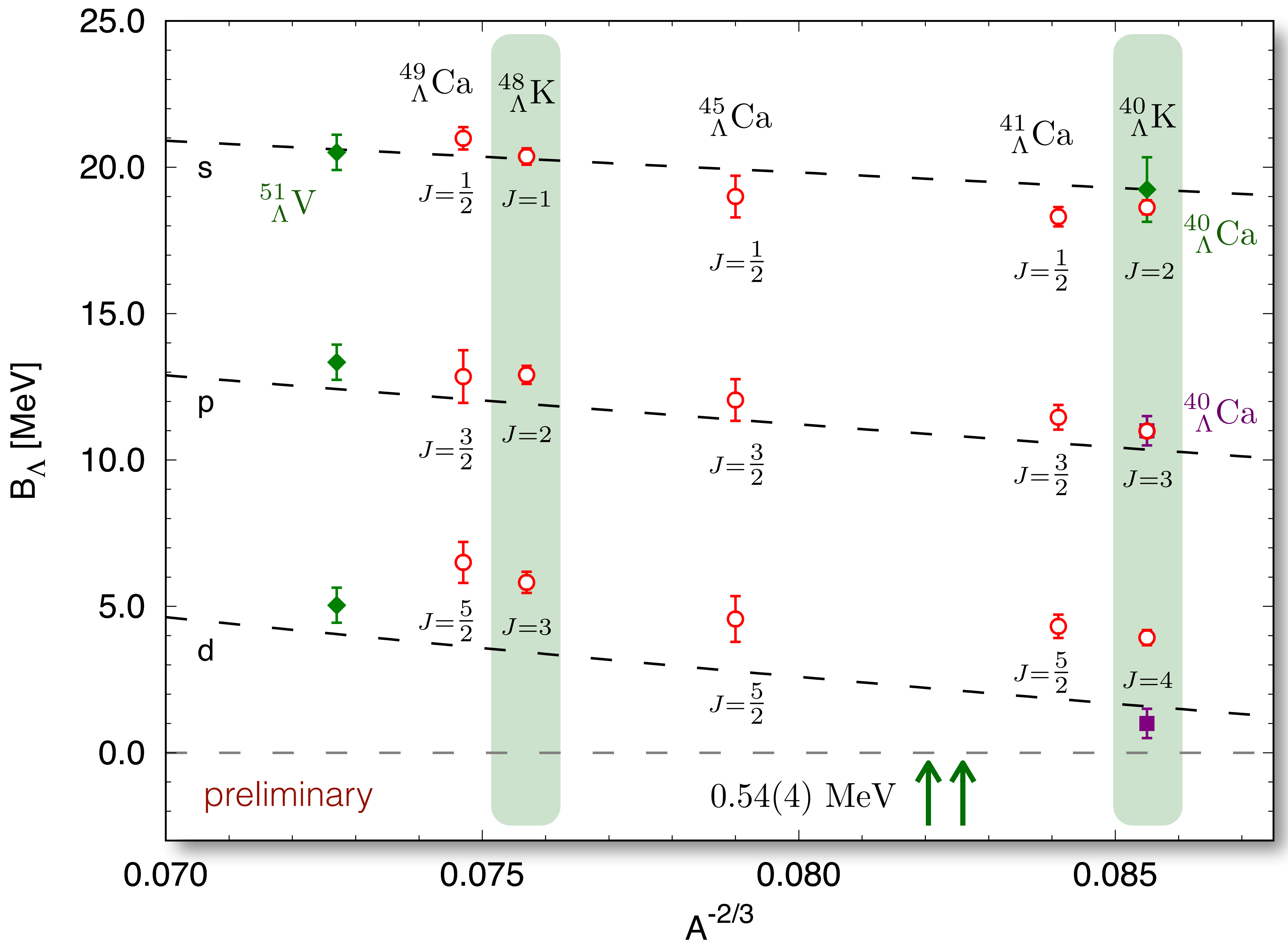


Jefferson Lab

$^{48}\mathrm{Ca}(e,e'K^+)^{48}_{\Lambda}\mathrm{K}$

$^{40}\mathrm{Ca}(e,e'K^+)^{40}_{\Lambda}\mathrm{K}$

Hyperons in nuclei



$$\delta = \frac{N-Z}{A}$$

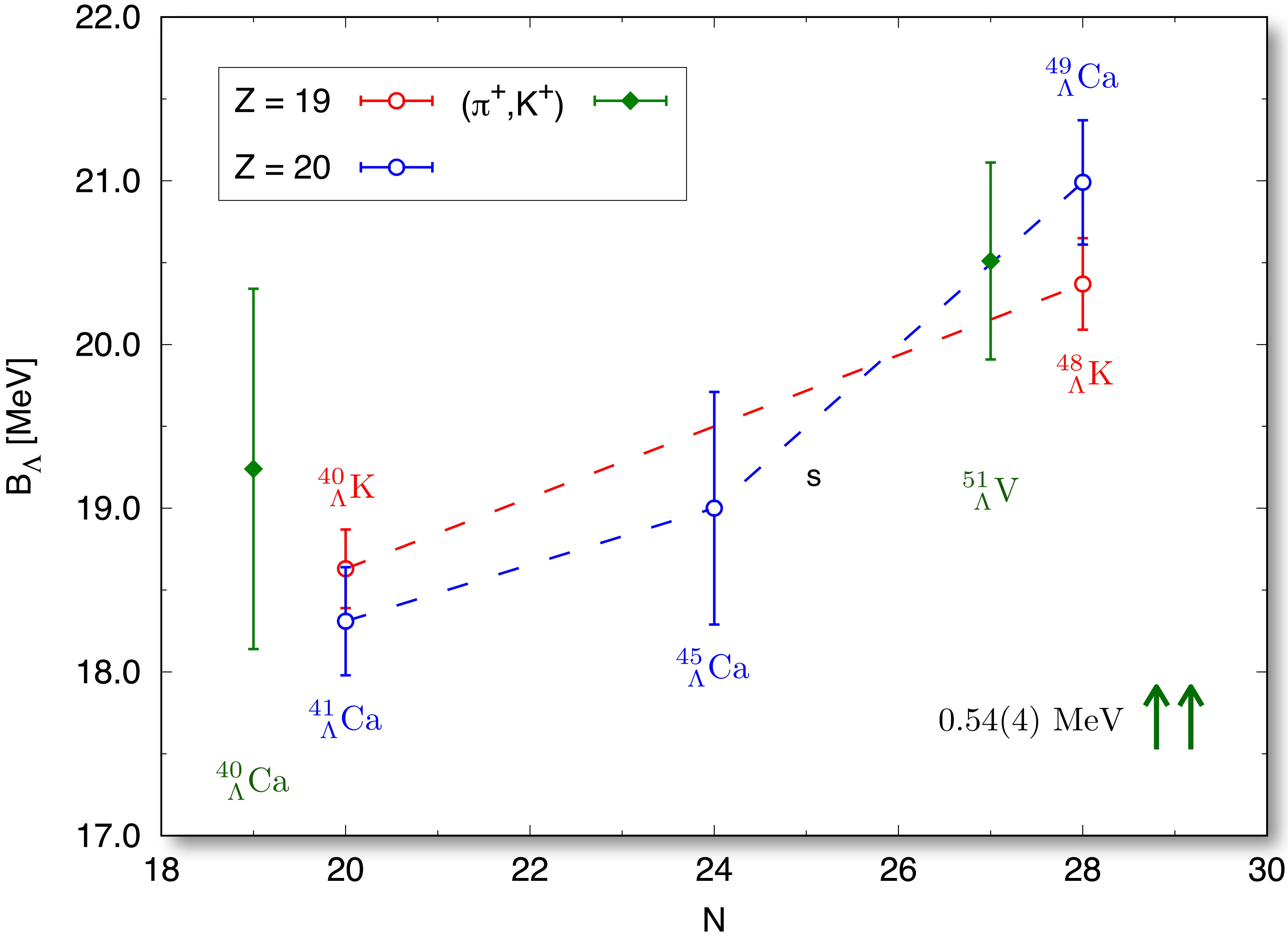
$$^{48}\mathrm{Ca}(e,e'K^{+})^{48}_{\Lambda}\mathrm{K}$$

$$^{48}_{\Lambda}\mathrm{K} : \delta = 0.188$$

$$^{40}\mathrm{Ca}(e,e'K^{+})^{40}_{\Lambda}\mathrm{K}$$

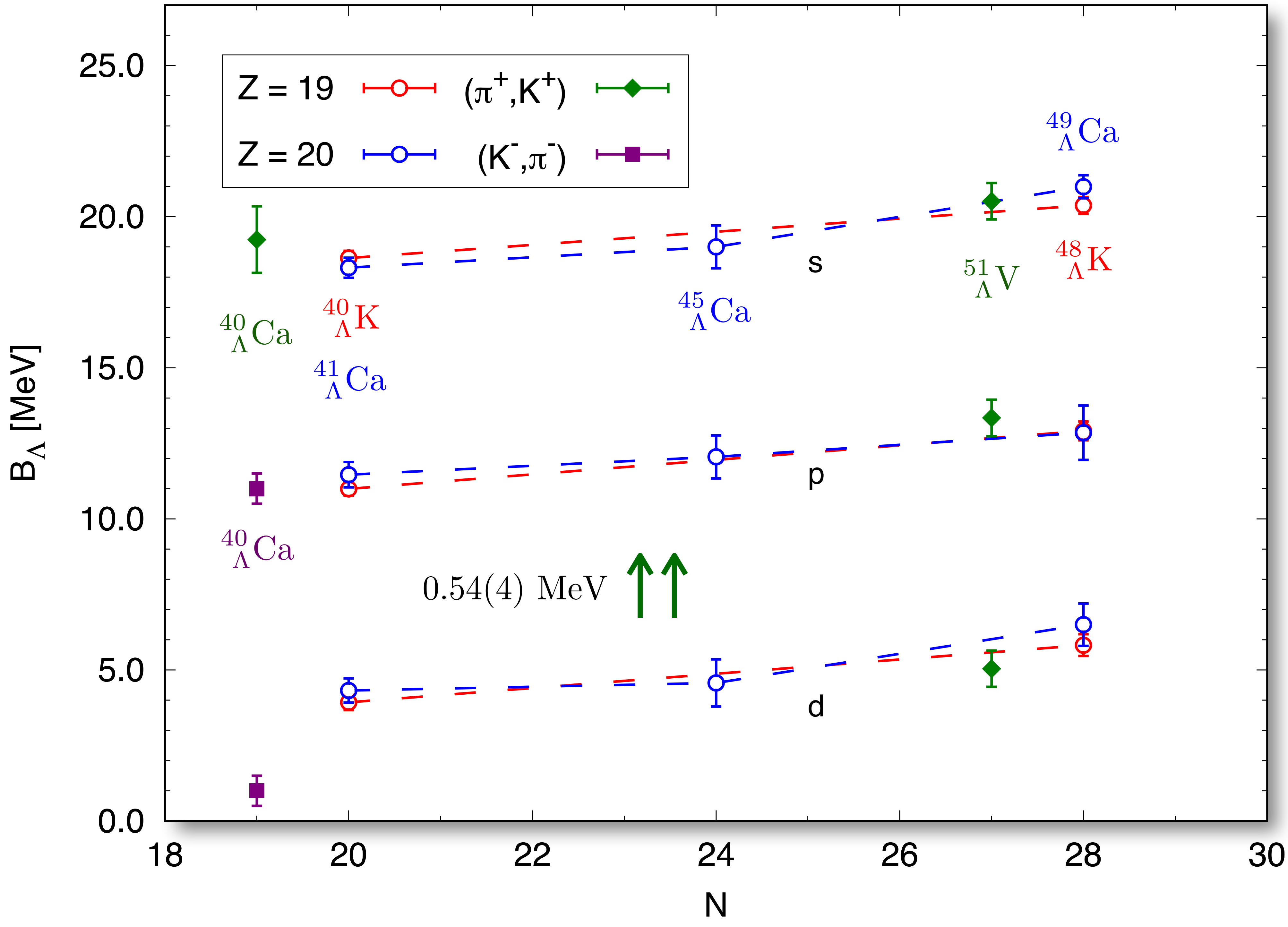
$$^{40}_{\Lambda}\mathrm{K} : \delta = 0.025$$

Hyperons in nuclei



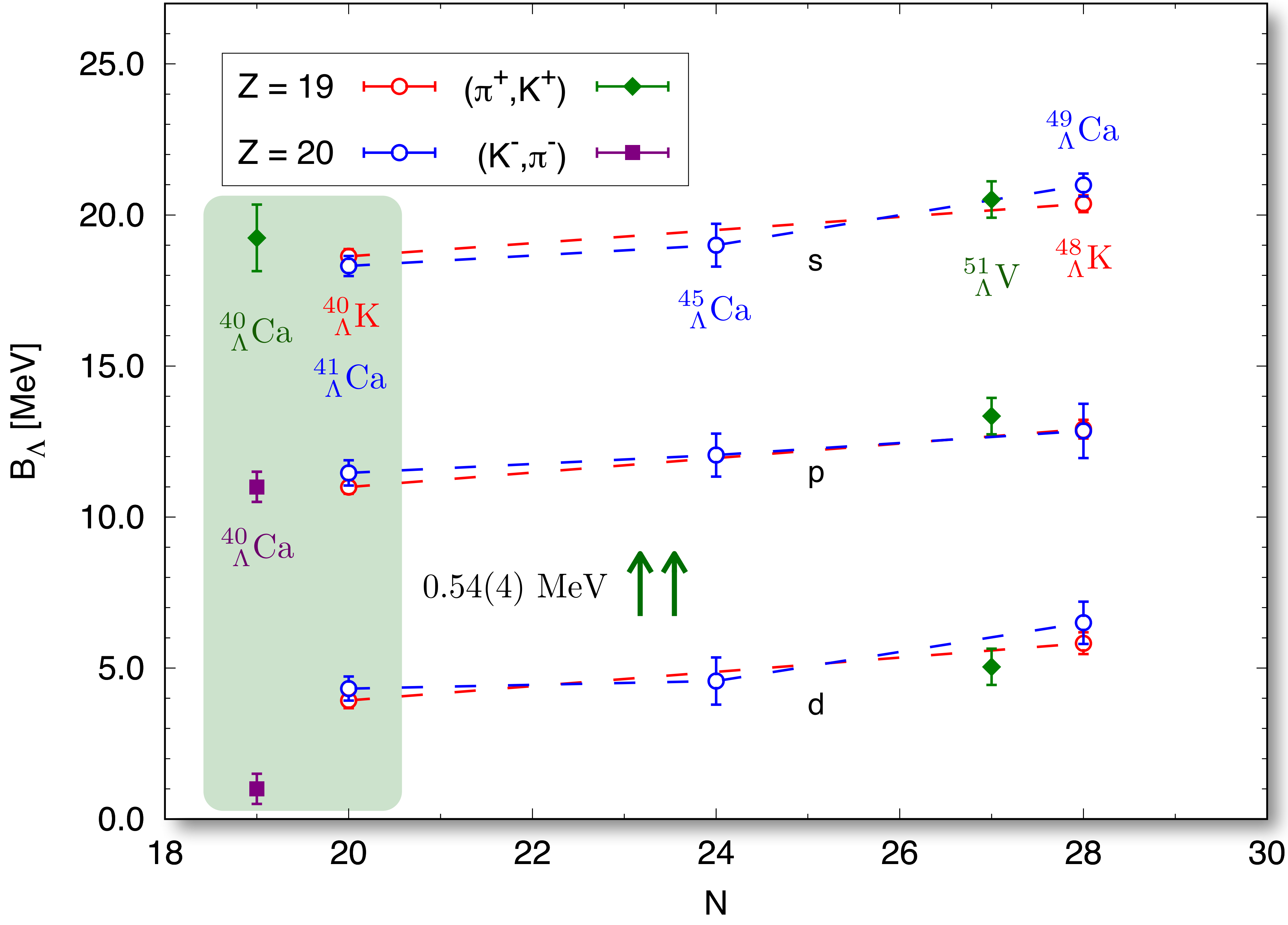
preliminary

Hyperons in nuclei



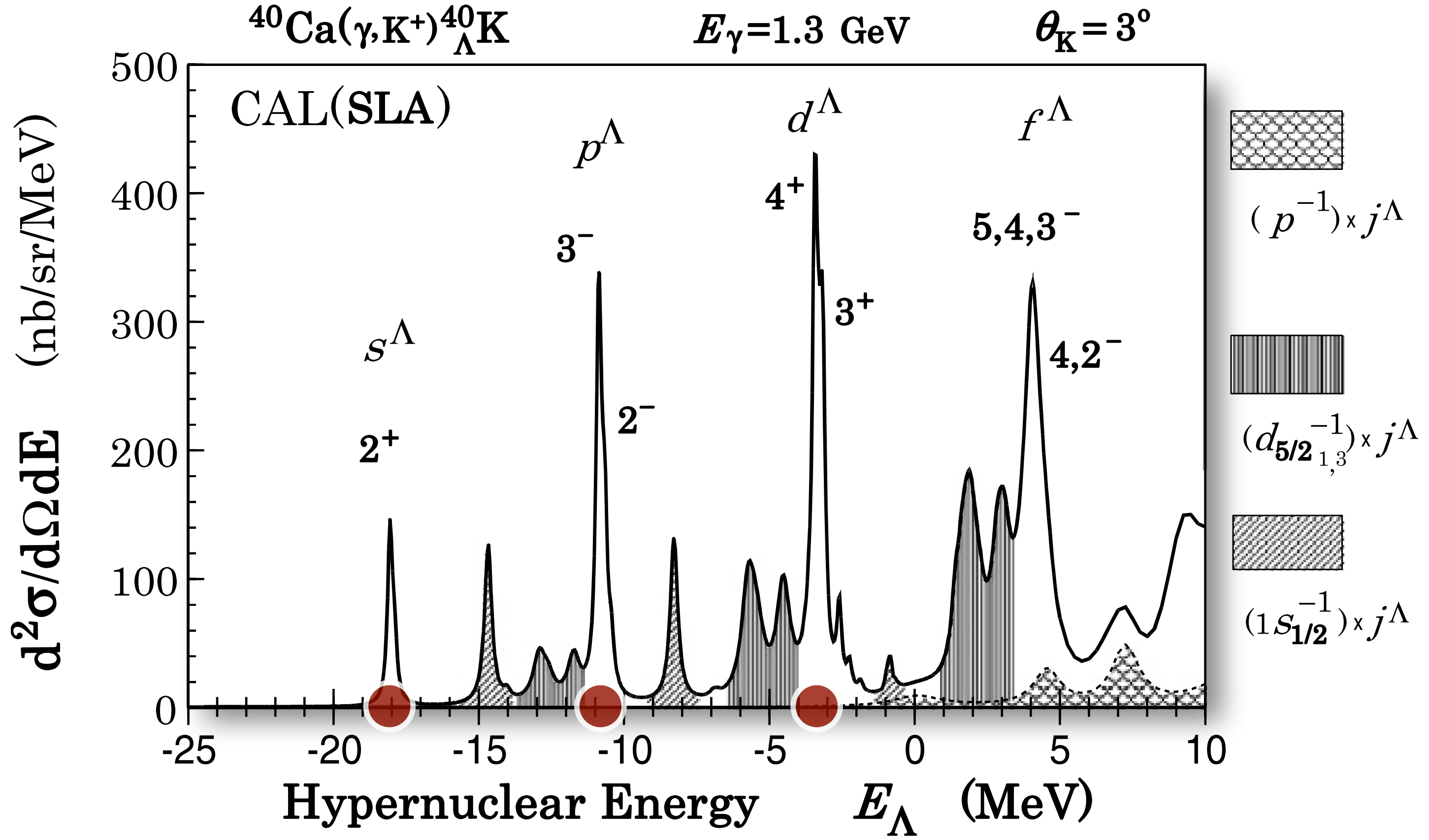
preliminary

Hyperons in nuclei



preliminary

Hyperons in nuclei



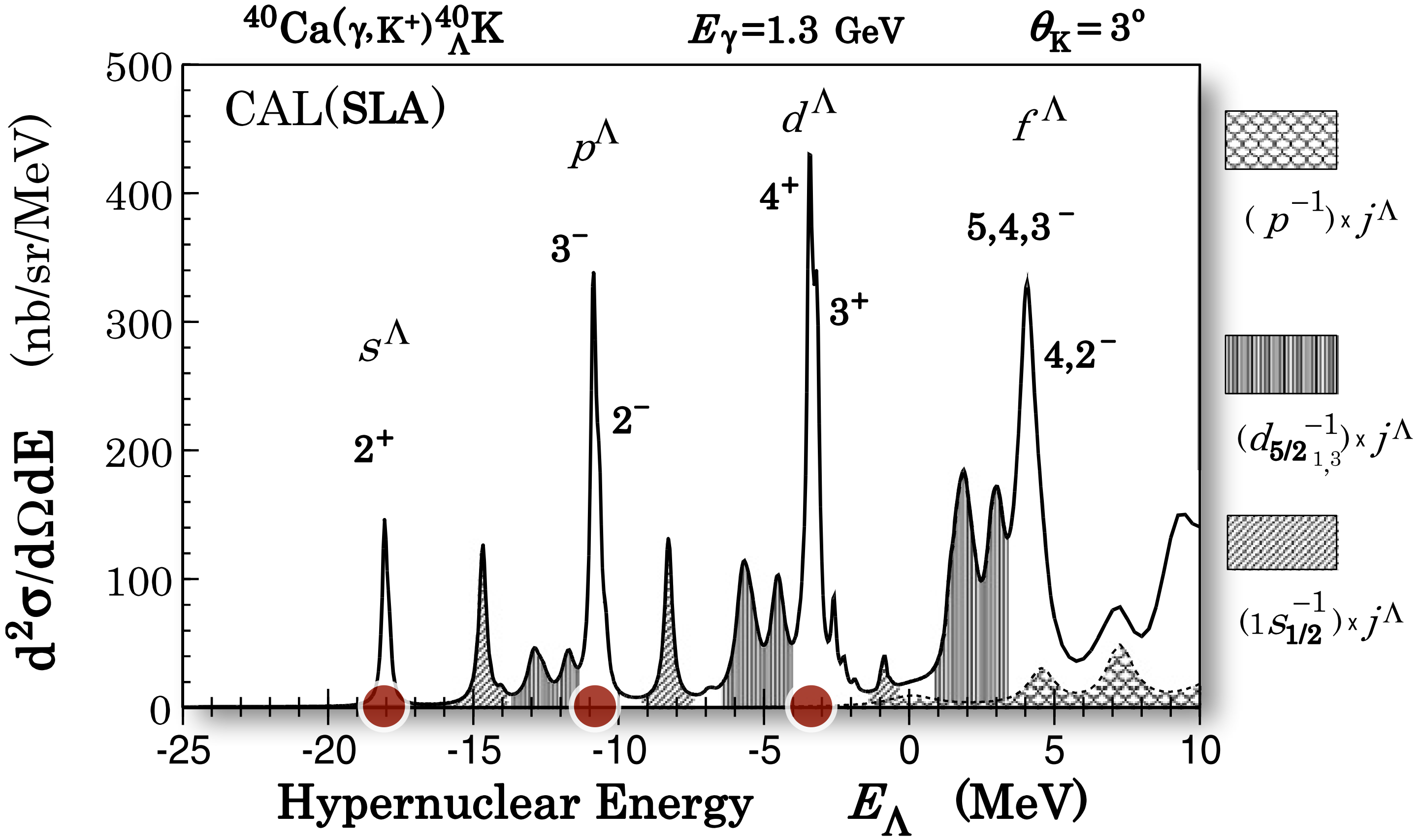
P. Bydžovský, M. Sotona, T. Motoba, K. Itonaga,
K. Ogawa, O. Hashimoto,
Nucl. Phys. A 881 (2012) 199-217

$$B^s_\Lambda \simeq 18.0 \text{ MeV}$$

$$B^p_\Lambda \simeq 10.7 \text{ MeV}$$

$$B^d_\Lambda \simeq 3.3 \text{ MeV}$$

Hyperons in nuclei



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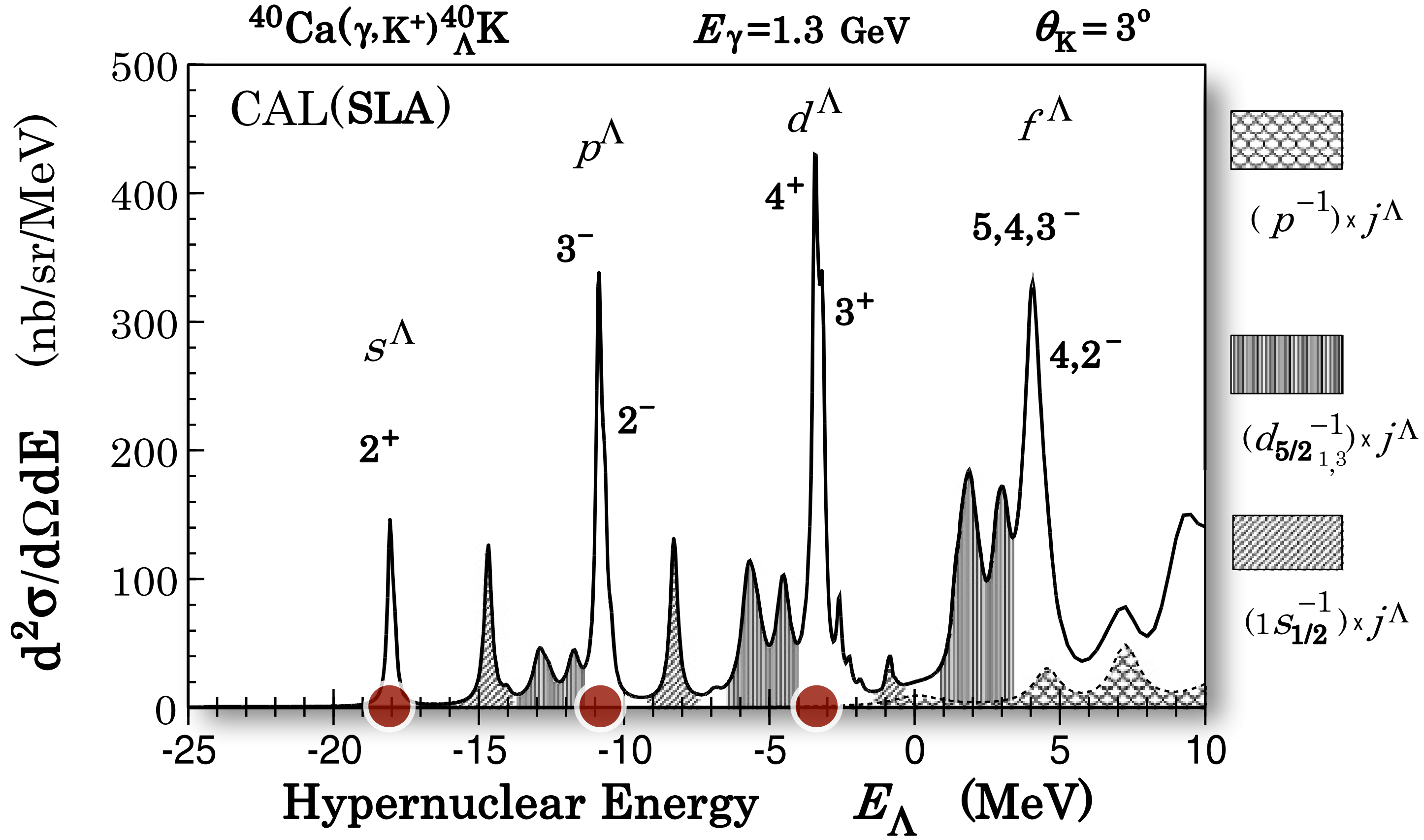
hypernucleus	s-wave	p-wave	d-wave
${}^{40}_{\Lambda}$K AFDMC	18.63(24)	10.99(22)	3.93(26)
${}^{41}_{\Lambda}$Ca AFDMC	18.31(33)	11.46(42)	4.32(40)
${}^{40}_{\Lambda}$Ca (π^+, K^+)	18.7(1.1)	–	–
${}^{40}_{\Lambda}$Ca (K^-, π^-)	–	11.0(5)	1.0(5)

Hyperons in nuclei



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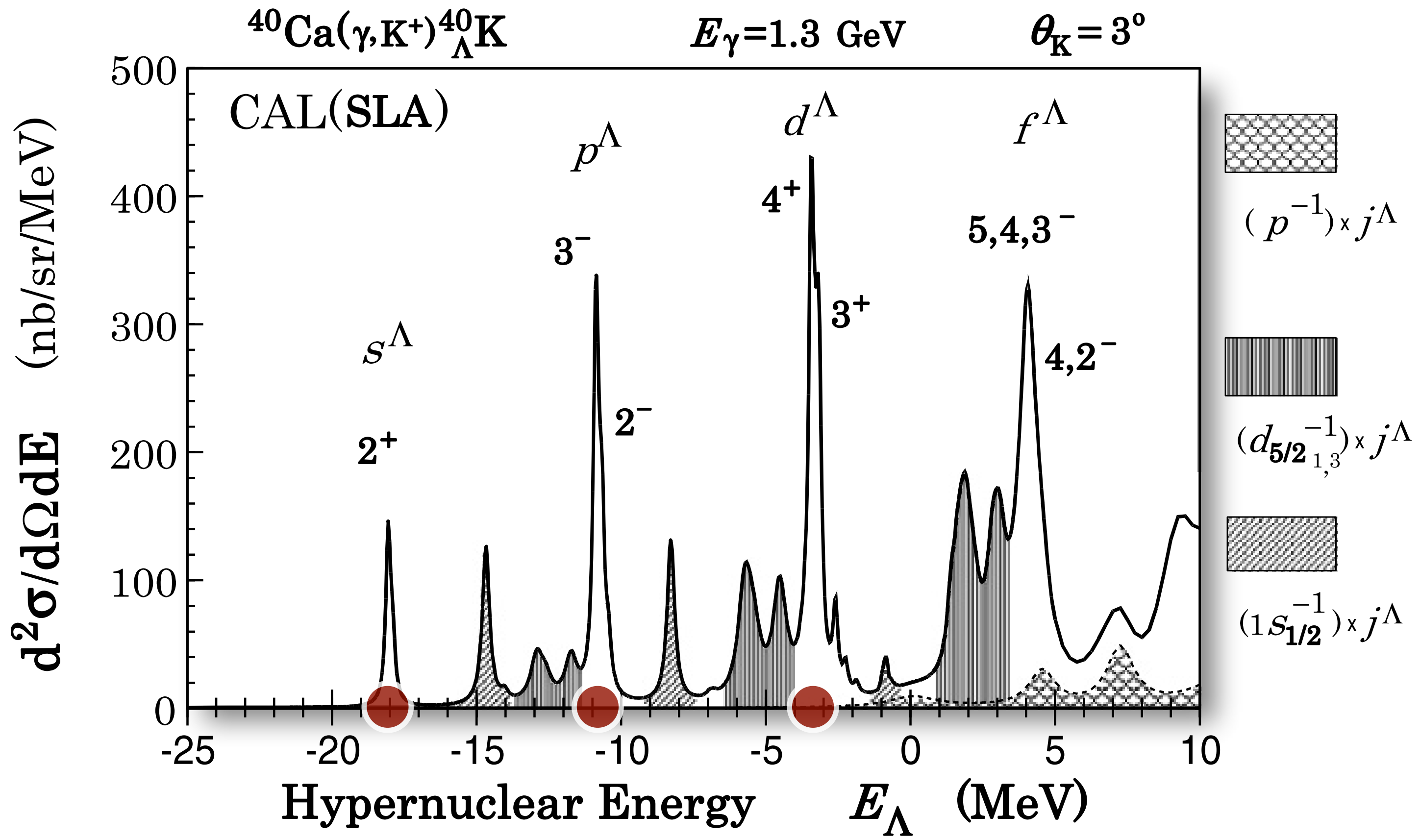
Hyperons in nuclei



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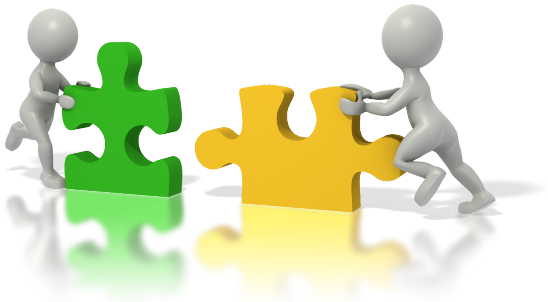
$^{40}_{\Lambda}$Ca (K^-, π^-)	–		

need of medium-heavy
neutron-rich hypernuclei

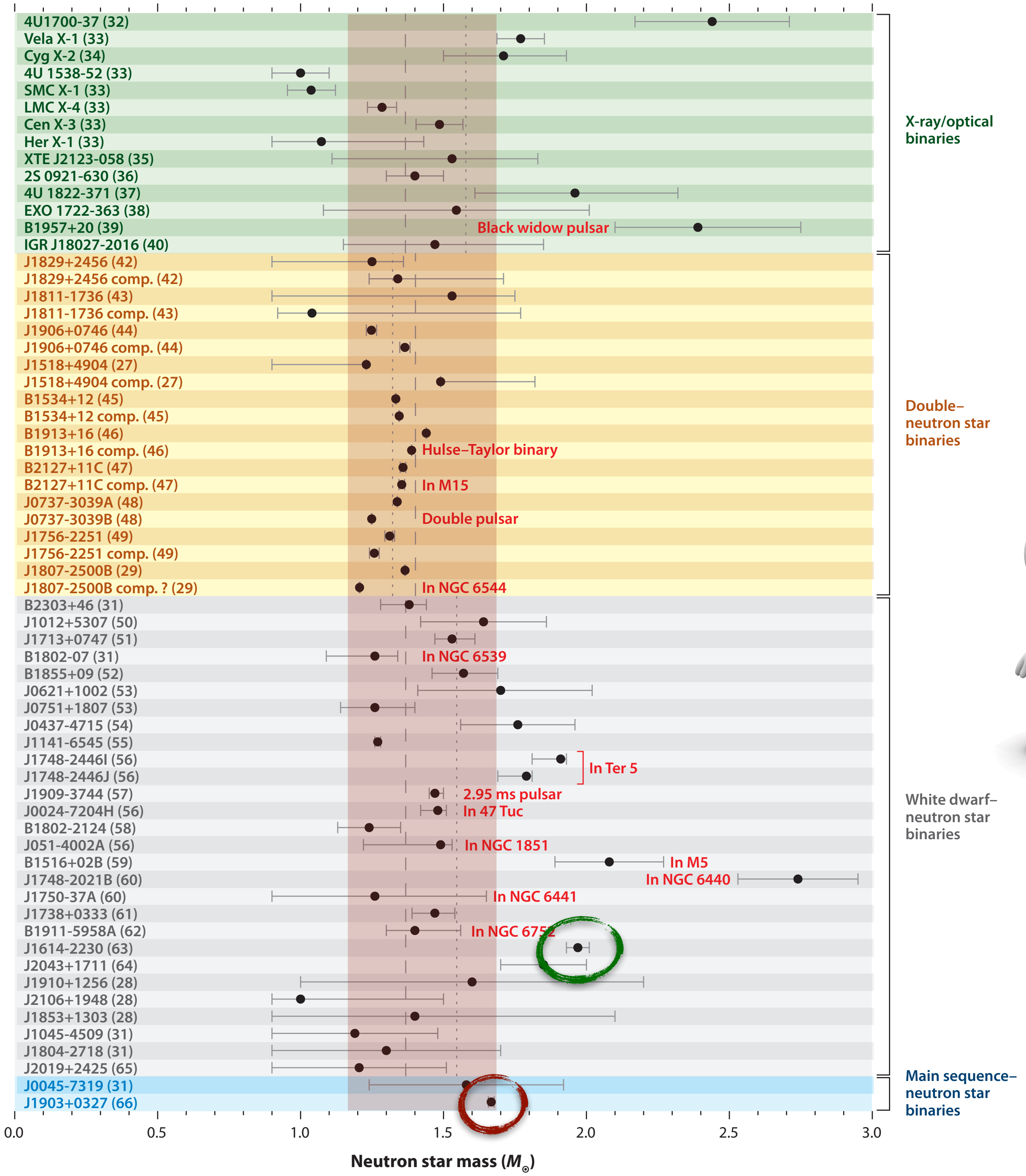
Conclusions

- ✓ The extrapolation from finite size to infinte nuclear systems can be non trivial: need for astrophysical constraints and/or inputs from medium-heavy systems
- ✓ An accurate description of the physics of strange nuclear systems seems to demand for more repulsion (why…?)
- ✓ The presence of hyperons in the core of neutron stars cannot be ruled out based on current information on hyperon-nucleon forces
- ✓ Accurate experimental information is needed, in particular for medium-heavy neutron-rich hypernuclei (but also scattering information)
- ✓ Theoretical efforts: extend the progresses reached in AFDMC calculations for nuclei and nuclear matter to the strange sector



Thank you!!

Backup: the hyperon puzzle



J. M. Lattimer, Annu. Rev. Nucl. Part. Sci. 2012. 62:485–515

< 2010:

$$M_{\max} = 1.67(2) M_\odot$$

D. J. Champion et al.
Science 320, 1309 (2008)

2010:

$$M_{\max} = 1.97(4) M_\odot$$

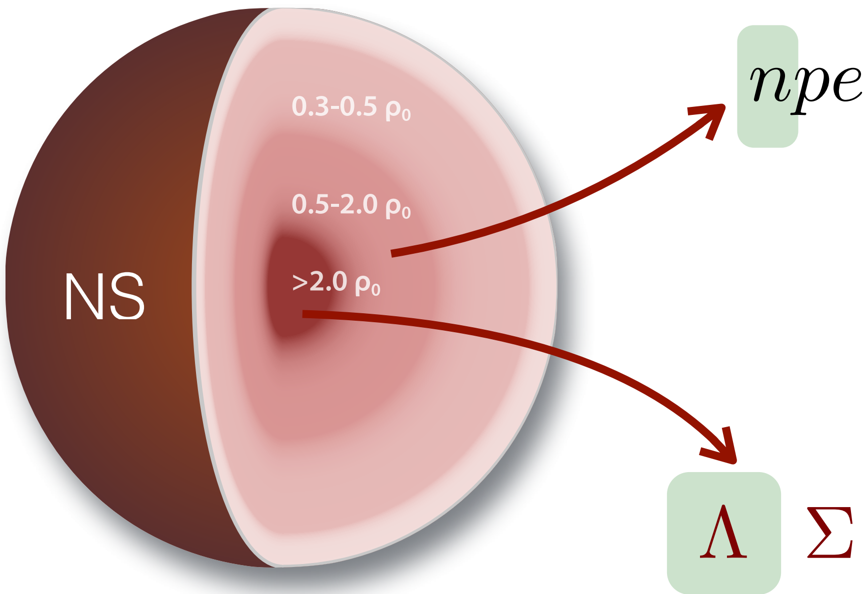
P. B. Demorest et al.
Nature 467, 1081 (2010)

2013:

$$M_{\max} = 2.01(4) M_\odot$$

J. Antoniadis et al.
Science 340, 1233232 (2013)

Backup: the hyperon puzzle



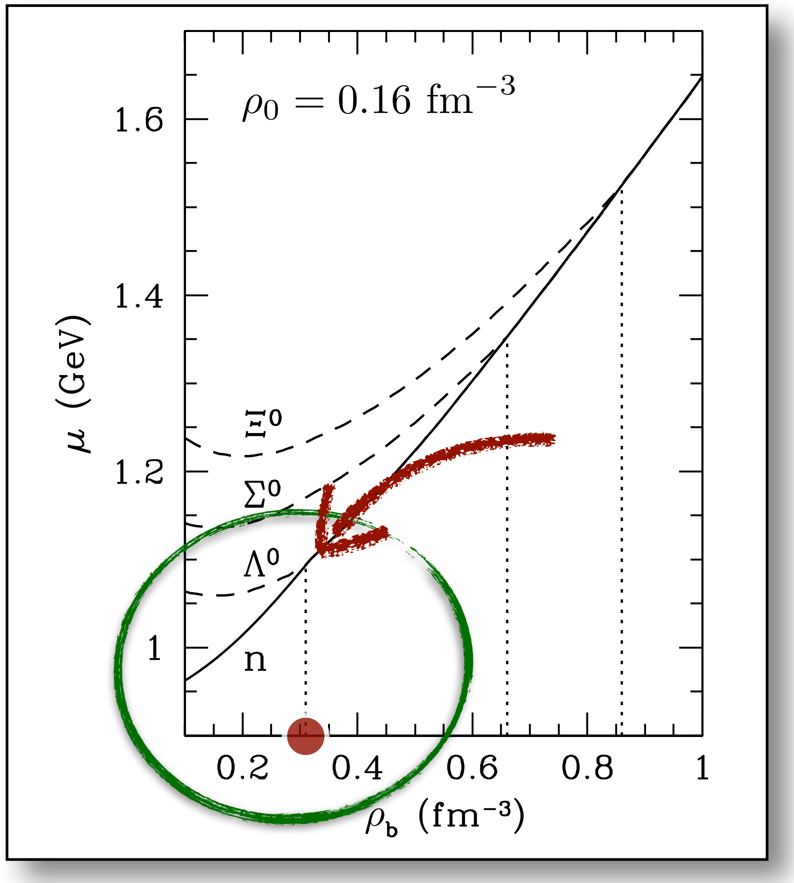
npe

$\Lambda \quad \Sigma \quad \Xi$

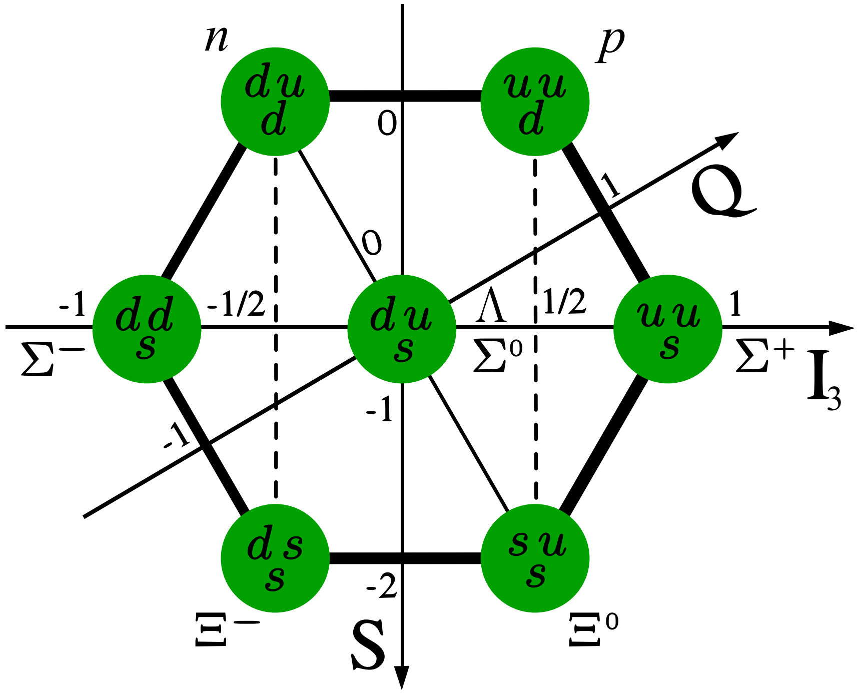
$\pi_c \; K_c \; q_p$?

$R \sim 12$ km

$M \sim 1.4\, M_\odot$



P. Haensel, A. Y. Potekhin, D. G. Yakovlev
Neutron Stars 1, Springer 2007



940 MeV

1116 MeV

1200 MeV

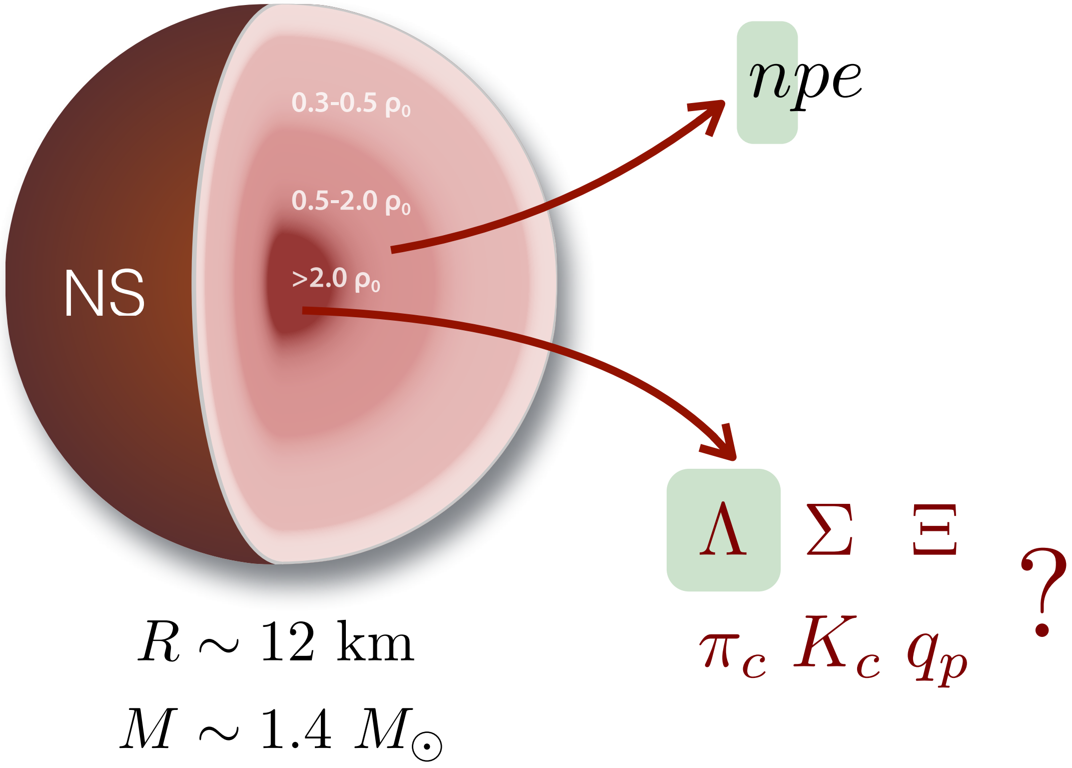
1300 MeV

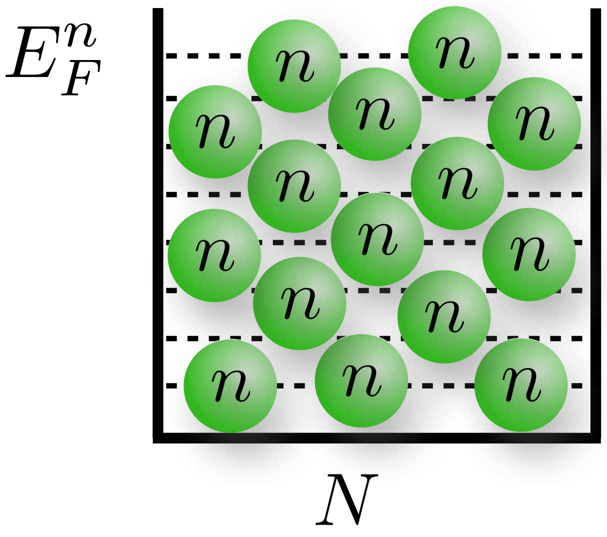
$$Q = -1 \;:\; \mu_{b^-} = \mu_n + \mu_e$$

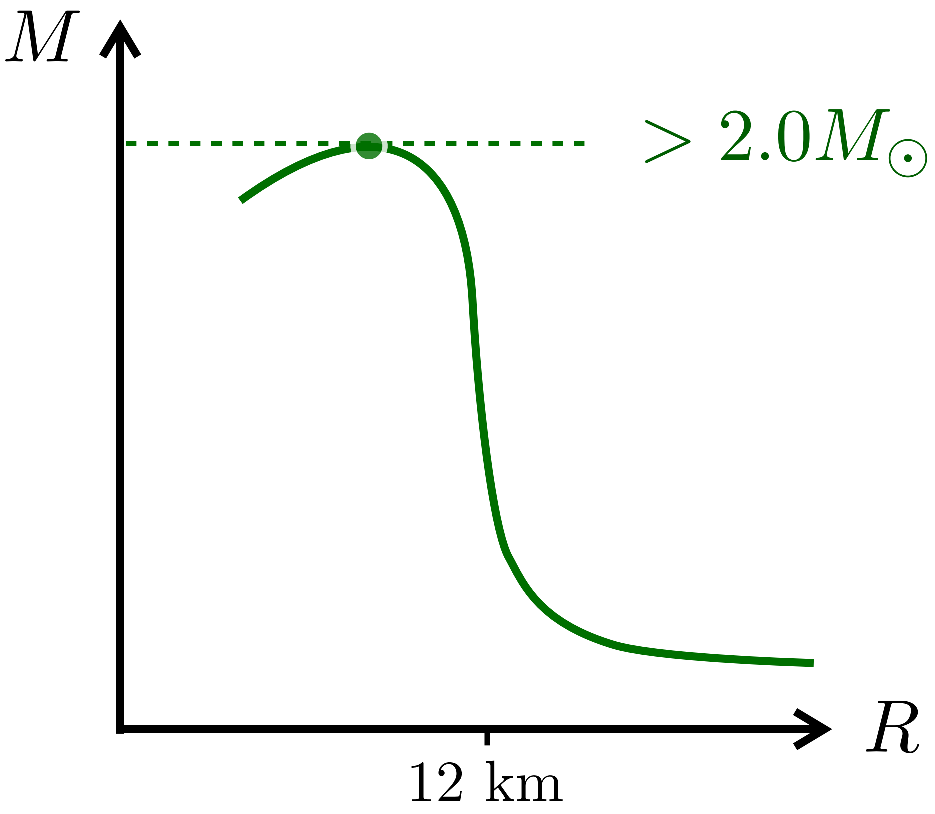
$$Q = \;\;\, 0 \;:\; \mu_{b^0} = \mu_n$$

$$Q = +1 \;:\; \mu_{b^+} = \mu_n - \mu_e$$

Backup: the hyperon puzzle



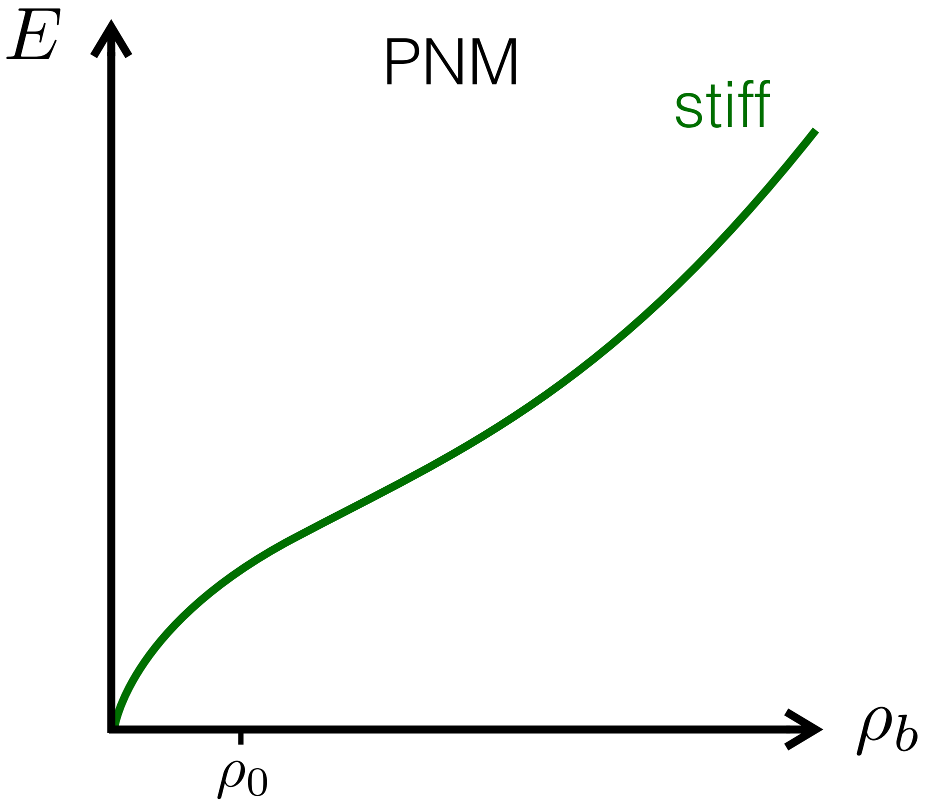




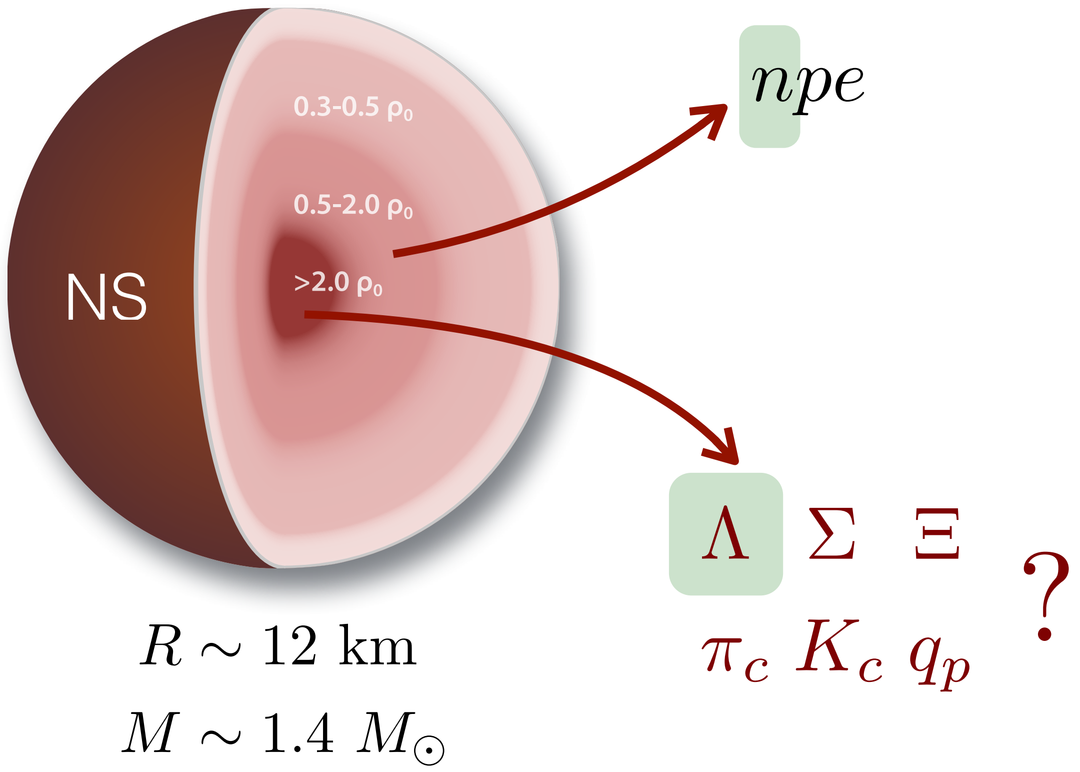
TOV

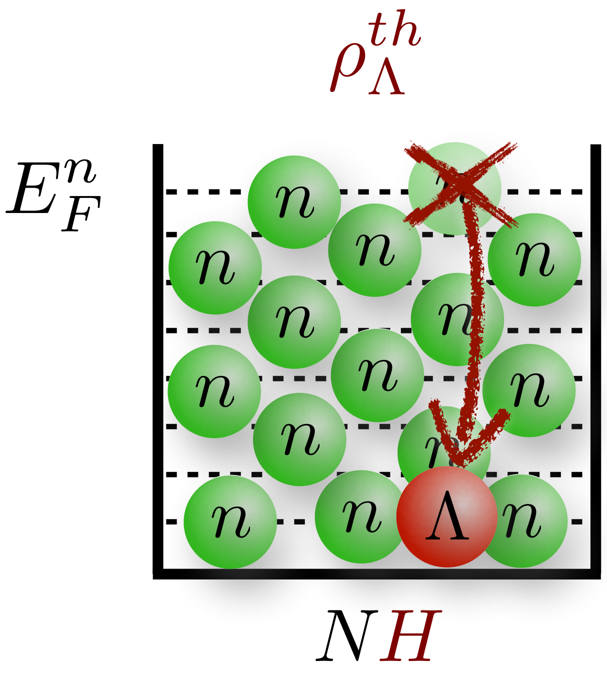
↔

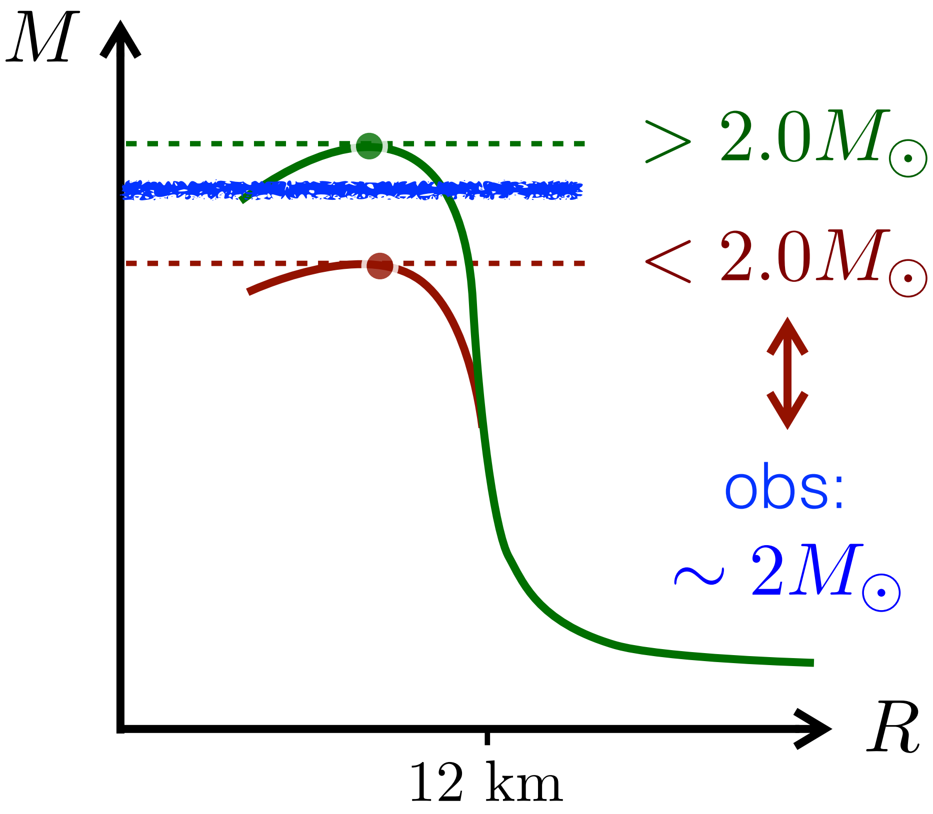
1:1



Backup: the hyperon puzzle

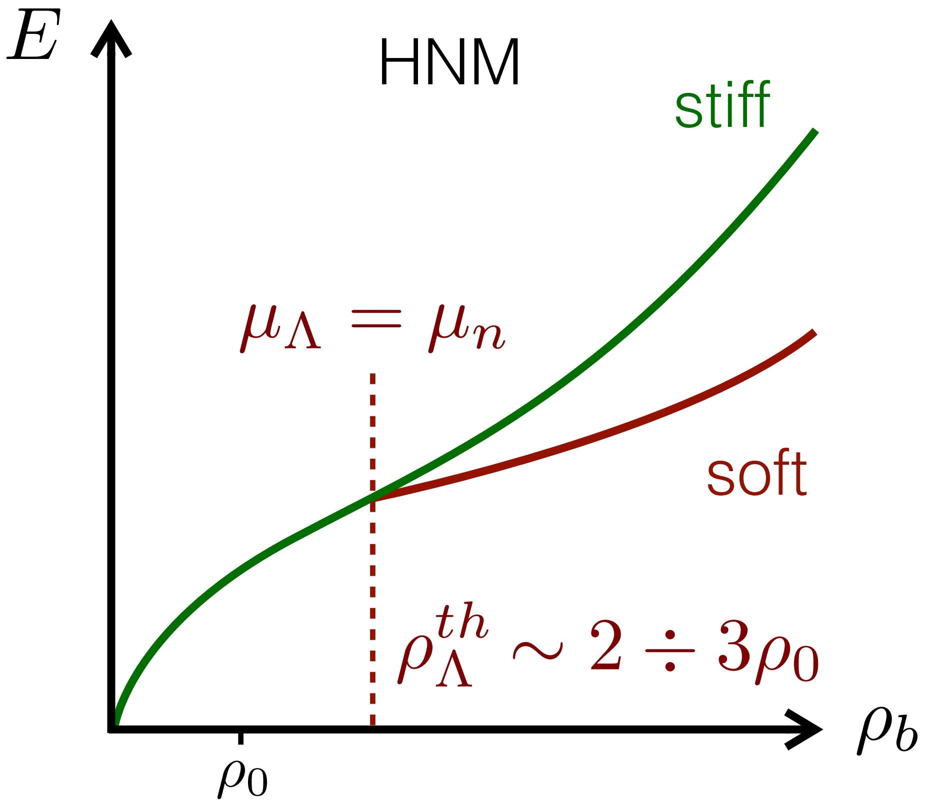






TOV

1:1



Backup: the hyperon puzzle

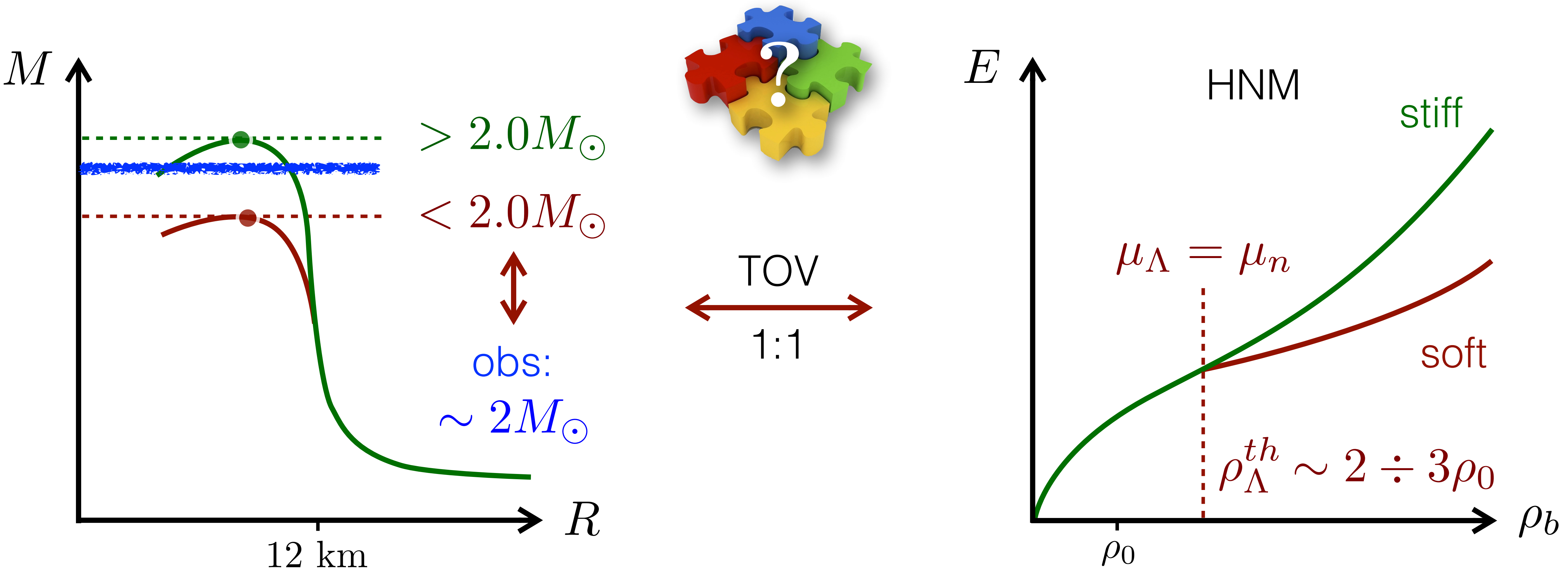
Hyperon puzzle

- ✓ Theoretical indication for hyperons in NS core: softening of the EOS
- ✓ Observation of massive NS: stiff EOS
- ✓ Magnitude of the softening: strongly model dependent

Problems

- ✓ Interactions poorly known
- ✓ Non trivial many-body problem: very dense system, strong interactions

QMC → HN interaction



Backup: terrestrial experiments

- ✓ Charge conserving reactions

$$ {}^{A}Z\,(K^-,\pi^-)\,{}^{A}_{\Lambda}Z $$

$$ {}^{A}Z\,(\pi^+,K^+)\,{}^{A}_{\Lambda}Z $$

- ✓ Single charge exchange reactions (SCX)

$$ {}^{A}Z\,(K^-,\pi^0)\,{}^{A}_{\Lambda}[Z-1] $$

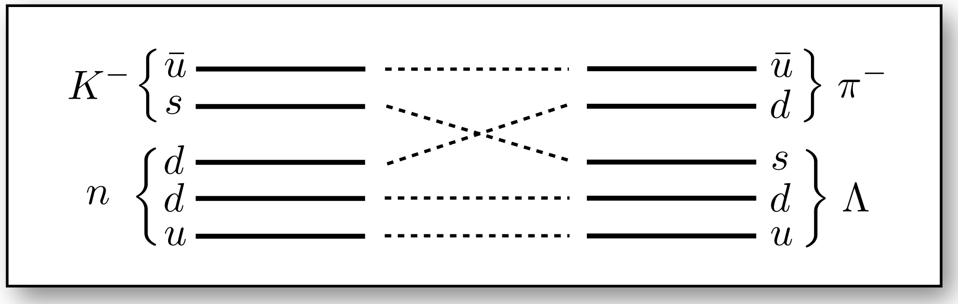
$$ {}^{A}Z\,(\pi^-,K^0)\,{}^{A}_{\Lambda}[Z-1] $$

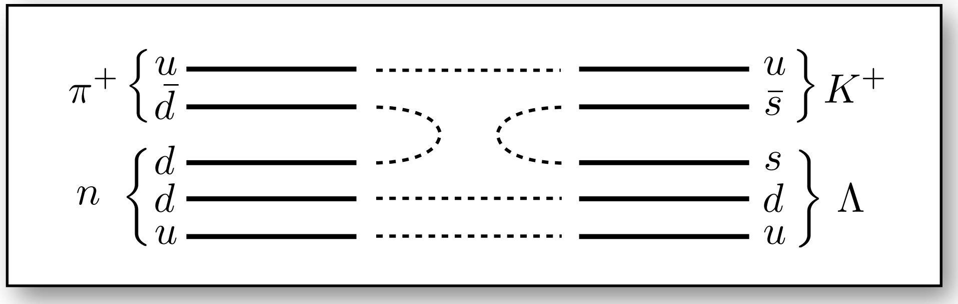
$$ {}^{A}Z\,(e,e'K^+)\,{}^{A}_{\Lambda}[Z-1] $$

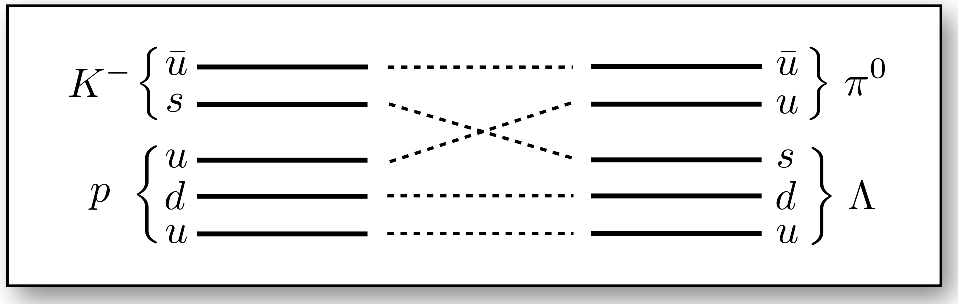
- ✓ Double charge exchange reactions (DCX)

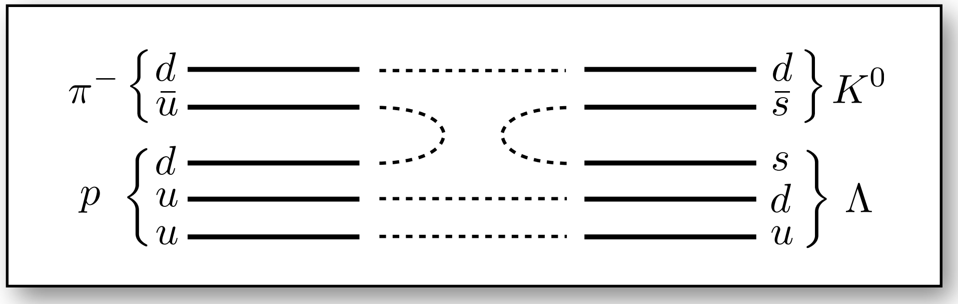
$$ {}^{A}Z\,(\pi^-,K^+)\,{}^{A+1}_{\Lambda}[Z-2] $$

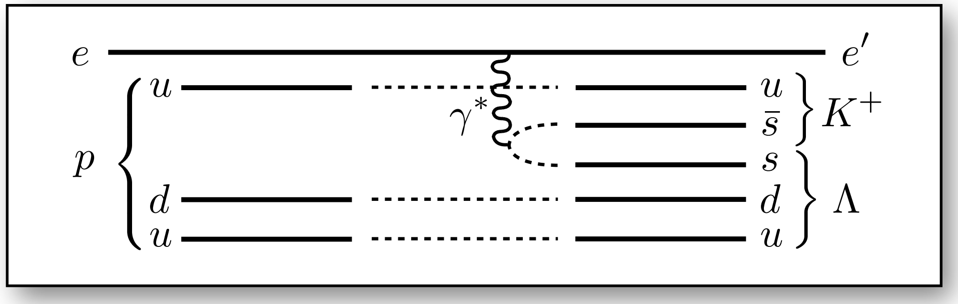
$$ {}^{A}Z\,(K^-,\pi^+)\,{}^{A+1}_{\Lambda}[Z-2] $$











Backup: terrestrial experiments

$^{89}\mathrm{Y}\left(\pi^{+}, K^{+}\right){}^{89}_{\Lambda}\mathrm{Y}$

SKS spectrometer

KEK 12-GeV Proton Synchrotron

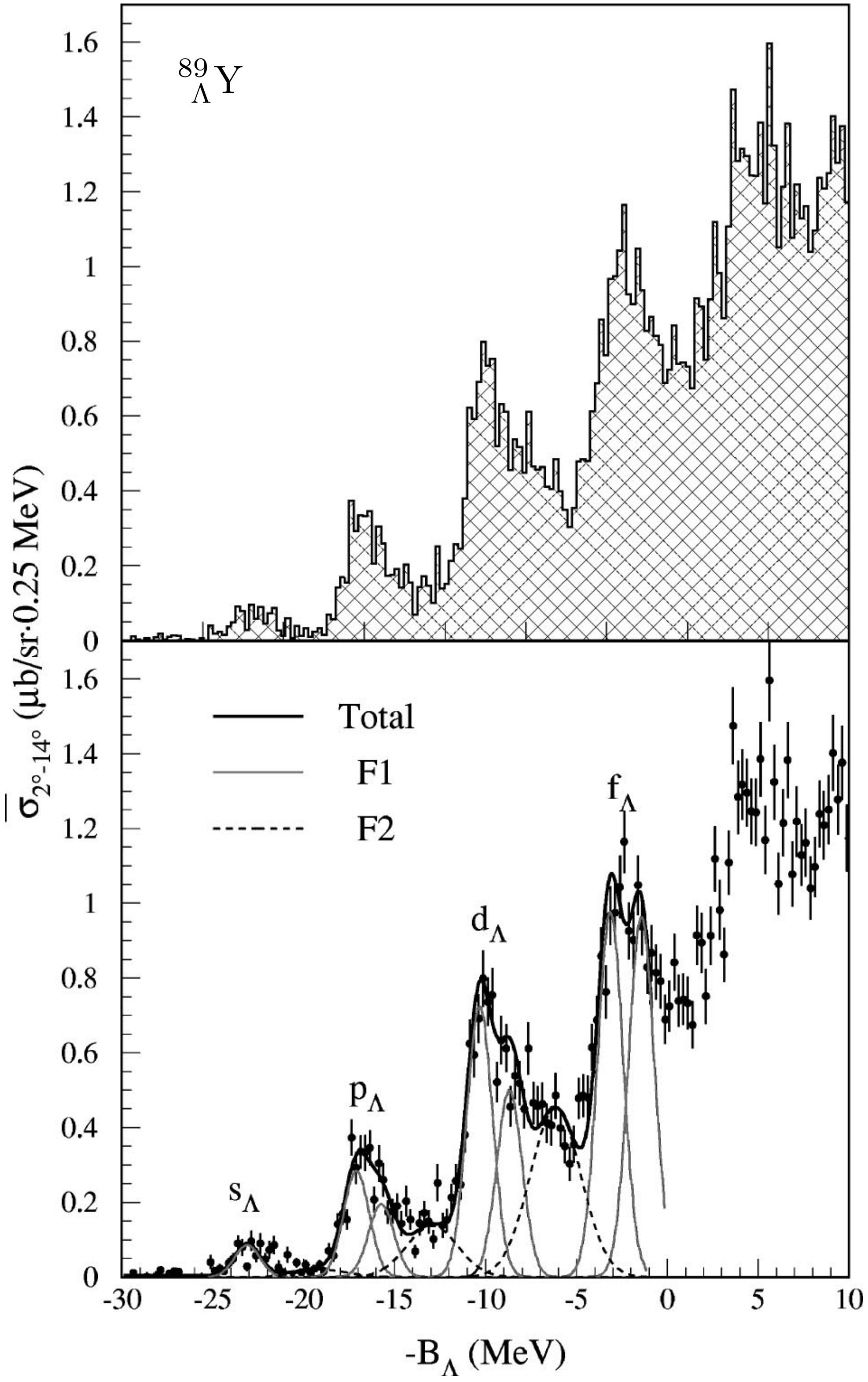
Japan

$$M_{HY} = \sqrt{\left(E_{\pi} + M_{A} - E_{K}\right)^{2} - \left(p_{\pi}^{2} + p_{K}^{2} - 2 p_{\pi}\, p_{K} \cos\theta\right)}$$

$$B_{\Lambda} = M_{A-1} + M_{\Lambda} - M_{HY}$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{A}{\rho_{x} \cdot N_{\mathcal{A}}} \cdot \frac{1}{N_{beam} \cdot f_{beam}} \cdot \frac{N_{K}}{\varepsilon_{\exp} \cdot d\Omega}$$

$$\bar{\sigma}_{2^{\circ}-14^{\circ}} = \int_{\theta=2^{\circ}}^{\theta=14^{\circ}} \left(\frac{d\sigma}{d\Omega}\right) d\Omega \Big/ \int_{\theta=2^{\circ}}^{\theta=14^{\circ}} d\Omega$$



H. Hotchi et al., Phys. Rev. C 64, 044302 (2001)

Backup: hyperon-nucleon interaction

hyperon-nucleon interaction ?

- ✓ one boson exchange model Nijmegen & Jülich

  Th. A. Rijken, M. M. Nagels, Y. Yamamoto,
  Few-Body Syst. (2013) 54, 801

  J. Haidenbauer, Ulf-G. Meißner,
  Phys. Rev. C 72, 044005 (2005)

- ✓ χ-EFT (NLO)

  J. Haidenbauer, S. Petschauer, N. Kaiser,
  U. -G. Meißner, A. Nogga, W. Weise,
  Nucl. Phys. A 915 (2013) 24–58

- ✓ effective - mean field models
  - cluster approach

    E. Hiyama,Y. Yamamoto,
    Prog. Theor. Phys. (2012) 128 (1) 105

  - Skyrme-Hartree-Fock

    H.-J. Schulze, E. Hiyama
    Phys. Rev. C 90, 047301 (2014)

- ✓ phenom. pion exchange model Argonne-Urbana like

  A. A. Usmani, F. C. Khanna, J. Phys. G: Nucl. Part. Phys. 35 (2008) 025105

  → good for QMC

Backup: hyperon-nucleon interaction

- ✓ 2-body interaction: AV18 & Usmani

$$NN \quad \left\{ \begin{array}{l} v_{ij} = \sum_{p=1,18} v_p(r_{ij})\, \mathcal{O}^p_{ij} \\[2ex] \mathcal{O}^{p=1,8}_{ij} = \left\{1, \sigma_{ij}, S_{ij}, \boldsymbol{L}_{ij}\cdot\boldsymbol{S}_{ij}\right\} \otimes \left\{1, \tau_{ij}\right\} \end{array}\right.$$

NN scattering

deuteron

$$\Lambda N \quad \left\{ \begin{array}{l} v_{\lambda i} = \sum_{p=1,4} v_p(r_{\lambda i})\, \mathcal{O}^p_{\lambda i} \\[2ex] \mathcal{O}^{p=1,4}_{\lambda i} = \left\{1, \sigma_{\lambda i}\right\} \otimes \left\{1, \tau^z_i\right\} \end{array}\right.$$

Λp scattering

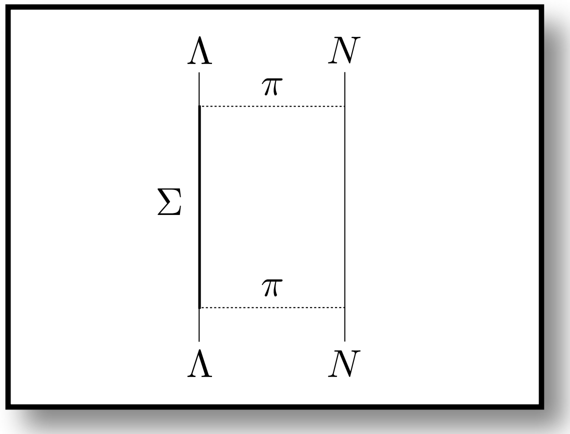
$A = 4$ CSB

Note: ~~$\Lambda\pi\Lambda$ vertex~~

forbidden

$\Lambda\pi\Sigma$ vertex

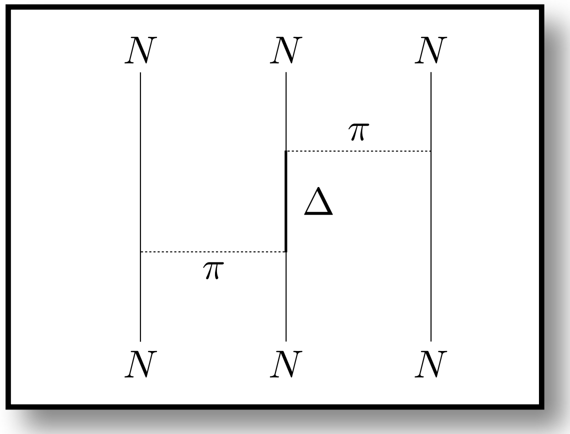
2π exchange

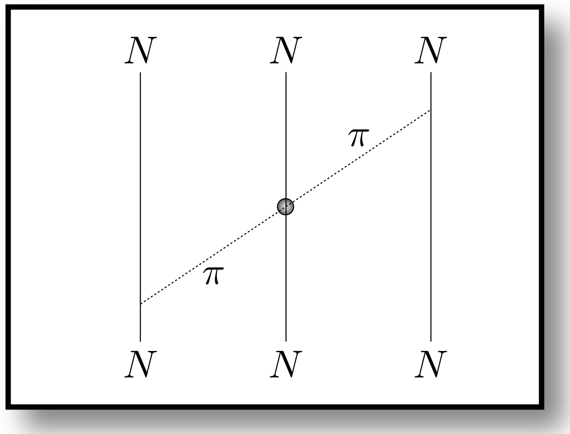


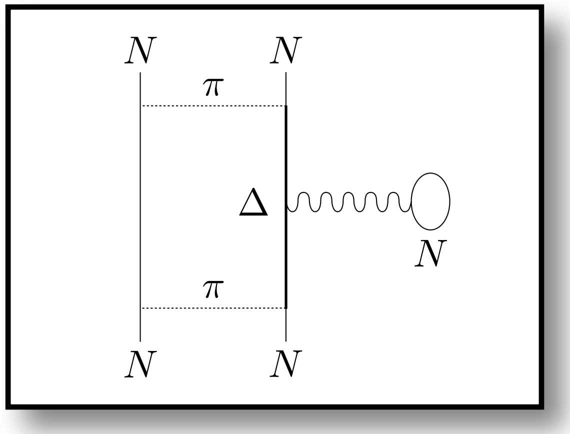
Backup: hyperon-nucleon interaction

✓ 3-body interaction: Urbana IX & Usmani

NNN





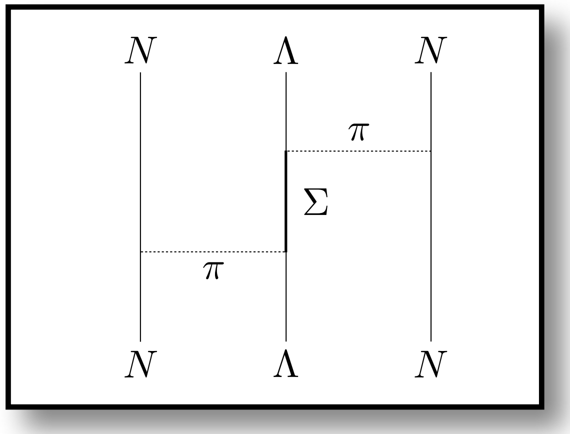


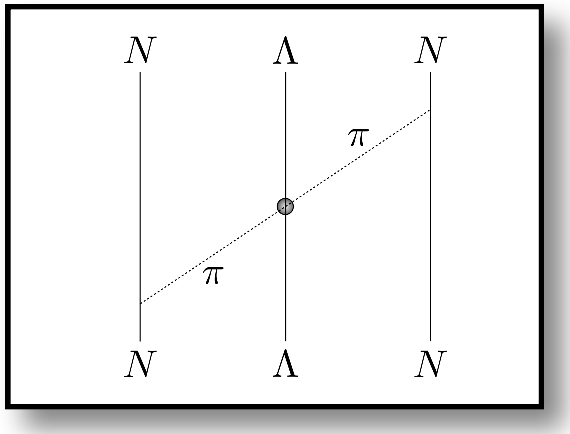
nuclei

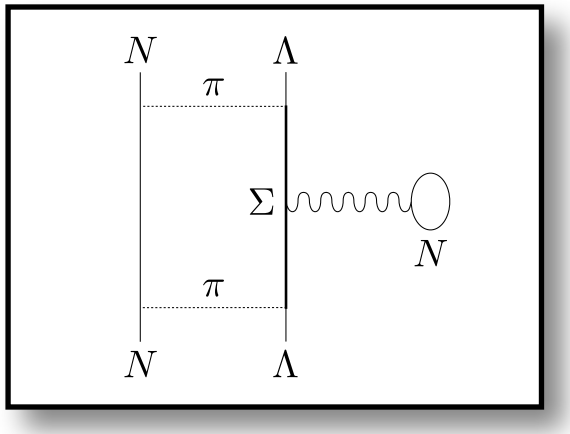
nuclear matter

$$v_{ijk} = A_{2\pi}^{P}\,\mathcal{O}_{ijk}^{2\pi,P} + A_{2\pi}^{S}\,\mathcal{O}_{ijk}^{2\pi,S} + A_R\,\mathcal{O}_{ijk}^{R}$$

ΛNN







VMC calc.

no unique fit

$$v_{\lambda ij} = C_P\,\mathcal{O}_{\lambda ij}^{2\pi,P} + C_S\,\mathcal{O}_{\lambda ij}^{2\pi,S} + W_D\,\mathcal{O}_{\lambda ij}^{R}$$

Backup: hyperon-nucleon interaction

- ✓ 2-body interaction

$$v_{\lambda i} = v_0(r_{\lambda i}) + \frac{1}{4} v_\sigma T_\pi^2(r_{\lambda i})\, \boldsymbol{\sigma}_\lambda \cdot \boldsymbol{\sigma}_i$$

charge symmetric

$$v_{\lambda i}^{CSB} = C_\tau \, T_\pi^2 \, (r_{\lambda i}) \, \tau_i^z$$

charge symmetry breaking
(spin independent)

A. R. Bodmer, Q. N. Usmani, Phys.Rev.C 31, 1400 (1985)

- ✓ 3-body interaction

$$v_{\lambda ij} = v_{\lambda ij}^{2\pi,P} + v_{\lambda ij}^{2\pi,S} + v_{\lambda ij}^{D}$$

$$\begin{cases} v_{\lambda ij}^{2\pi,P} = -\dfrac{C_P}{6} \Big\{ X_{i\lambda} \, , X_{\lambda j} \Big\} \, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\ v_{\lambda ij}^{2\pi,S} = C_S \, Z\,(r_{\lambda i})\, Z\,(r_{\lambda j}) \;\, \boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{r}}_{i\lambda} \, \boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{j\lambda} \, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\ v_{\lambda ij}^{D} = W_D \, T_\pi^2 \,(r_{\lambda i})\, T_\pi^2\,(r_{\lambda j}) \Big[1 + \dfrac{1}{6} \boldsymbol{\sigma}_\lambda \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \Big] \end{cases}$$

use QMC to fit on
hyp. exp. data

Backup: hyperon-nucleon interaction

$$v_0(r) = v_c(r) - \bar{v}\, T_\pi^2(r)$$

$$v_c(r) = W_c \left(1 + \mathrm{e}^{\frac{r-\bar{r}}{a}}\right)^{-1}$$

$$\bar{v} = (v_s + 3v_t)/4 \qquad v_\sigma = v_s - v_t$$

$$Y_\pi(r) = \frac{\mathrm{e}^{-\mu_\pi r}}{\mu_\pi r}\xi_Y(r)$$

$$T_\pi(r) = \left[1 + \frac{3}{\mu_\pi r} + \frac{3}{(\mu_\pi r)^2}\right]\frac{\mathrm{e}^{-\mu_\pi r}}{\mu_\pi r}\xi_T(r)$$

$$\mu_\pi = \frac{m_\pi}{\hbar} = \frac{1}{\hbar}\frac{m_{\pi^0} + 2\, m_{\pi^\pm}}{3}$$

$$\xi_Y(r) = \xi_T^{1/2}(r) = 1 - \mathrm{e}^{-cr^2}$$

$$Z_\pi(r) = \frac{\mu_\pi r}{3}\Big[Y_\pi(r) - T_\pi(r)\Big]$$

$$X_{\lambda i} = Y_\pi(r_{\lambda i})\,\boldsymbol{\sigma}_\lambda \cdot \boldsymbol{\sigma}_i + T_\pi(r_{\lambda i})\, S_{\lambda i}$$

$$S_{\lambda i} = 3\,(\boldsymbol{\sigma}_\lambda \cdot \hat{\boldsymbol{r}}_{\lambda i})\,(\boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{r}}_{\lambda i}) - \boldsymbol{\sigma}_\lambda \cdot \boldsymbol{\sigma}_i$$

Constant	Value	Unit
W_c	2137	MeV
$\bar{r}$	0.5	fm
a	0.2	fm
v_s	6.33, 6.28	MeV
v_t	6.09, 6.04	MeV
$\bar{v}$	6.15(5)	MeV
v_σ	0.24	MeV
c	2.0	fm^{-2}
C_τ	-0.050(5)	MeV
C_P	$0.5 \div 2.5$	MeV
C_S	$\simeq 1.5$	MeV
W_D	$0.002 \div 0.058$	MeV

Backup: AFDMC

- ✓ AFDMC propagation

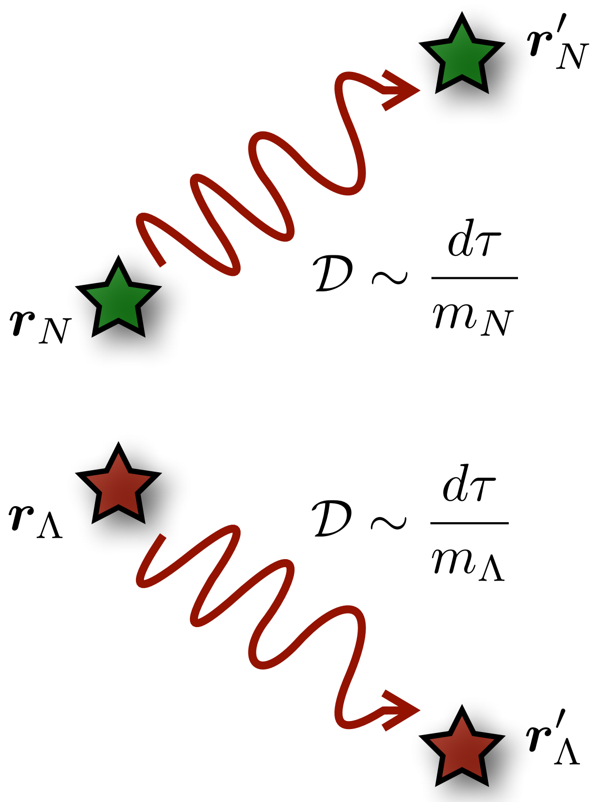
$$\langle SR|\psi\left(\tau+d\tau\right)\rangle = \int dR' dS' \langle SR|\, \mathrm{e}^{-(H-E_0)d\tau}\, |R'S'\rangle \langle S'R'|\psi_T(\tau)\rangle$$

final walkers

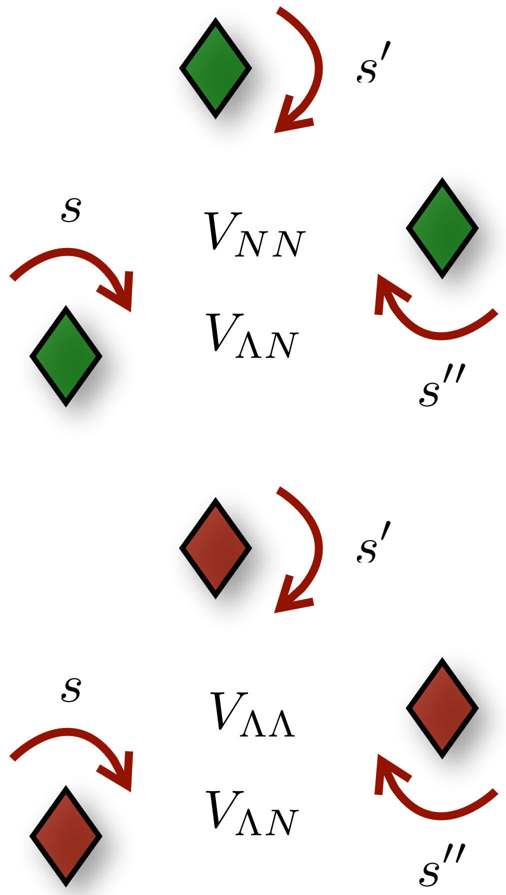
propagator

initial walkers

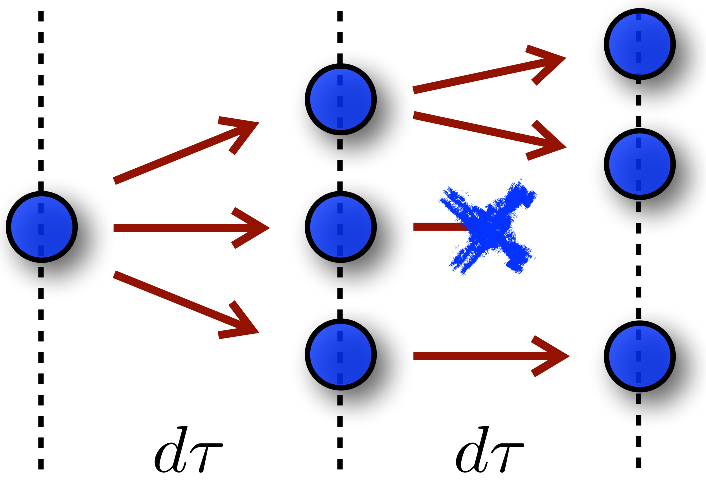
diffusion (DMC): $d\tau$



rotation (AF): $\sqrt{d\tau}$



branching: $d\tau$



Backup: AFDMC

- ✓ AFDMC wave function: single particle representation

$$\psi_T(R,S) = \psi_T^N(R_N,S_N)$$

$$\begin{cases} \psi_T^\kappa(R_\kappa,S_\kappa) = \displaystyle\prod_{i<j} f_c^{\kappa\kappa}(r_{ij})\,\Phi_\kappa(R_\kappa,S_\kappa) & \kappa = N \\ \Phi_\kappa(R_\kappa,S_\kappa) = \mathcal{A}\left[\displaystyle\prod_{i=1}^{\mathcal{N}_\kappa}\varphi_\epsilon^\kappa(\boldsymbol{r}_i,s_i)\right] = \det\left\{\varphi_\epsilon^\kappa(\boldsymbol{r}_i,s_i)\right\} & \end{cases}$$

s.p. orbitals

plane waves

$$s_i = \begin{pmatrix} a_i \\ b_i \\ c_i \\ d_i \end{pmatrix}_i = a_i|p\uparrow\rangle_i + b_i|p\downarrow\rangle_i + c_i|n\uparrow\rangle_i + d_i|n\downarrow\rangle_i$$

Backup: AFDMC

- ✓ AFDMC wave function: single particle representation

$$\psi_T(R,S) = \prod_{\lambda i} f_c^{\Lambda N}(r_{\lambda i})\,\psi_T^N(R_N,S_N)\,\psi_T^\Lambda(R_\Lambda,S_\Lambda)$$

$$\begin{cases} \psi_T^\kappa(R_\kappa,S_\kappa) = \displaystyle\prod_{i<j} f_c^{\kappa\kappa}(r_{ij})\,\Phi_\kappa(R_\kappa,S_\kappa) & \qquad \kappa = N,\Lambda \\ \Phi_\kappa(R_\kappa,S_\kappa) = \mathcal{A}\left[\displaystyle\prod_{i=1}^{\mathcal{N}_\kappa} \varphi_\epsilon^\kappa(\boldsymbol{r}_i,s_i)\right] = \det\Big\{\varphi_\epsilon^\kappa(\boldsymbol{r}_i,s_i)\Big\} \end{cases}$$

s.p. orbitals

plane waves

$$s_i = \begin{pmatrix} a_i \\ b_i \\ c_i \\ d_i \end{pmatrix}_i = a_i|p\uparrow\rangle_i + b_i|p\downarrow\rangle_i + c_i|n\uparrow\rangle_i + d_i|n\downarrow\rangle_i$$

$$s_\lambda = \begin{pmatrix} u_\lambda \\ v_\lambda \end{pmatrix}_\lambda = u_\lambda|\Lambda\uparrow\rangle_\lambda + v_\lambda|\Lambda\downarrow\rangle_\lambda$$

Backup: AFDMC

- ✓ diffusion Monte Carlo

$$-\frac{\partial}{\partial\tau}|\psi(\tau)\rangle = (H - E_0)|\psi(\tau)\rangle \qquad \tau = it/\hbar$$

imaginary time

$$\downarrow$$

$$|\psi(\tau)\rangle = \mathrm{e}^{-(H-E_0)\tau}|\psi(0)\rangle \qquad |\psi(0)\rangle = |\psi_T\rangle = \sum_{n=0}^{\infty} c_n|\varphi_n\rangle$$

$$= \sum_{n=0}^{\infty} \mathrm{e}^{-(E_n-E_0)\tau} c_n|\varphi_n\rangle \xrightarrow{\tau\to\infty} \boxed{c_0|\varphi_0\rangle}$$

projection

$$\downarrow$$

$$E = \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} \xrightarrow{\tau\to\infty} \boxed{E_0}$$

ground
state

Backup: AFDMC

- ✓ diffusion Monte Carlo

imaginary time evolution: $\tau = \mathcal{M} d\tau \qquad d\tau \ll 1$

$$\langle SR|\psi\left(\tau + d\tau\right)\rangle = \int dR'\, dS' \langle SR|\, \mathrm{e}^{-(H-E_0)d\tau}\, |R'S'\rangle \langle S'R'|\psi_T(\tau)\rangle$$

final walkers

propagator

initial walkers

$$\left\{\boldsymbol{r}^*, s^*\right\}_w \longleftarrow \left\{\boldsymbol{r}, s\right\}_w$$

Backup: AFDMC

- ✓ diffusion Monte Carlo

imaginary time evolution: $\tau = \mathcal{M} d\tau \qquad d\tau \ll 1$

$$\langle SR|\psi\left(\tau + d\tau\right)\rangle = \int dR' dS' \langle SR|\, \mathrm{e}^{-(H-E_0)d\tau}\, |R'S'\rangle \langle S'R'|\psi_T(\tau)\rangle$$

final walkers

propagator

initial walkers

$$\{\boldsymbol{r}_0, s_0\}_w \xleftarrow{\infty \leftarrow \tau \qquad \infty \leftarrow \mathcal{M}} \{\boldsymbol{r}, s\}_w$$

propagator: $H = T$ $\longrightarrow$ diffusion in coordinate space

$+ V(\boldsymbol{r})$ $\longrightarrow$ branching of configurations

$+ V(s)$ $\longrightarrow$ problem !!

Backup: AFDMC

- ✓ auxiliary field

$$\mathcal{P} \sim \mathrm{e}^{-\frac{1}{2}\gamma d\tau \mathcal{O}^2} \quad \longrightarrow \quad \mathrm{e}^{-\frac{1}{2}\gamma d\tau \mathcal{O}^2} \bigotimes_i |S\rangle_i \neq \bigotimes_i |\tilde{S}\rangle_i$$

many body $\quad |S\rangle : \quad 2^A \dfrac{A!}{(A-Z)!Z!}$ components $\quad$ GFMC: $A \leq 12$

single particle $\quad |S\rangle = \bigotimes_i |S\rangle_i : \quad 4A$ components $\quad$ AFDMC: $A \sim 90$

Idea: Hubbard-Stratonovich transformation

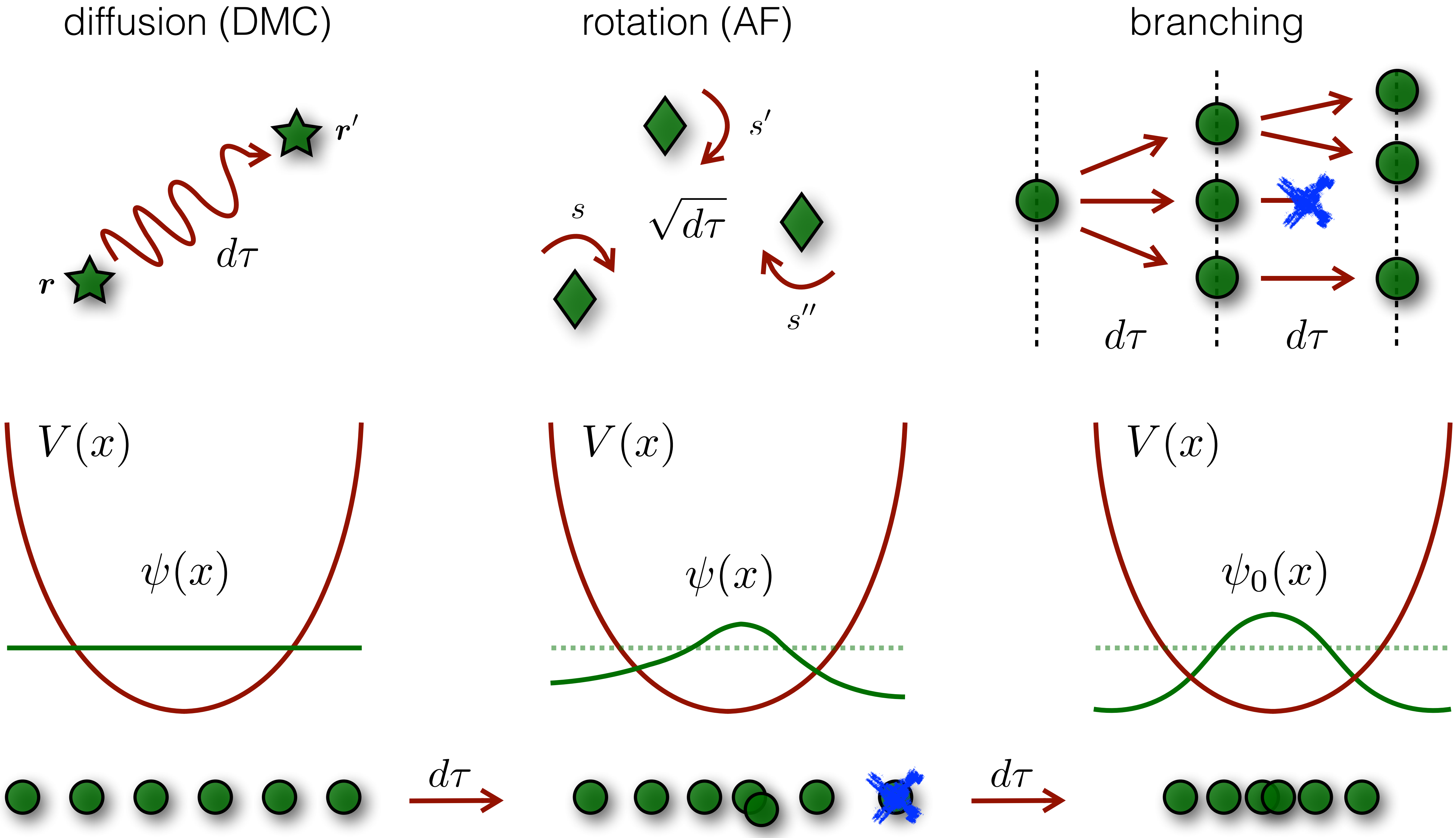
$$\mathrm{e}^{-\frac{1}{2}\gamma d\tau \mathcal{O}^2} = \frac{1}{\sqrt{2\pi}} \int dx\; \mathrm{e}^{-\frac{x^2}{2} + \sqrt{-\gamma d\tau}\, x \mathcal{O}}$$

auxiliary field

rotation over spin-isospin configurations

Backup: AFDMC

- ✓ auxiliary field diffusion Monte Carlo



Backup: strangeness in QMC calculations

$$
\begin{aligned}
V_{NN}^{SD} + V_{\Lambda N}^{SD} &= \frac{1}{2}\sum_{n=1}^{3\mathcal{N}_N} \lambda_n^{[\sigma]} \left(\mathcal{O}_n^{[\sigma]}\right)^2 && A_{i\alpha,j\beta}^{[\sigma]} \\
&+ \frac{1}{2}\sum_{n=1}^{3\mathcal{N}_N}\sum_{\alpha=1}^{3} \lambda_n^{[\sigma\tau]} \left(\mathcal{O}_{n\alpha}^{[\sigma\tau]}\right)^2 && A_{i\alpha,j\beta}^{[\sigma\tau]} \\
&+ \frac{1}{2}\sum_{n=1}^{\mathcal{N}_N}\sum_{\alpha=1}^{3} \lambda_n^{[\tau]} \left(\mathcal{O}_{n\alpha}^{[\tau]}\right)^2 && A_{ij}^{[\tau]} \\
&+ \frac{1}{2}\sum_{n=1}^{\mathcal{N}_\Lambda}\sum_{\alpha=1}^{3} \lambda_n^{[\sigma\Lambda]} \left(\mathcal{O}_{n\alpha}^{[\sigma\Lambda]}\right)^2 && C_{\lambda\mu}^{[\sigma]} \\
&+ \frac{1}{2}\sum_{n=1}^{\mathcal{N}_N\mathcal{N}_\Lambda}\sum_{\alpha=1}^{3} B_n^{[\sigma]} \left(\mathcal{O}_{n\alpha}^{[\sigma\Lambda N]}\right)^2 \\
&+ \sum_{i=1}^{\mathcal{N}_N} B_i^{[\tau]}\,\tau_i^z
\end{aligned}
$$

diagonalization:

λ_n eigenvalues

ψ_n eigenvectors

$\mathcal{O}_n = \sigma_n\,\psi_n$

direct calculation

Backup: strangeness in QMC calculations

computing time

- 5000 configurations, 3 time steps: nucleus & hypernucleus
- 10 nodes @ Edison (NERSC)
- 2 socket 12-core Intel "Ivy Bridge" processor @ 2.4 GHz

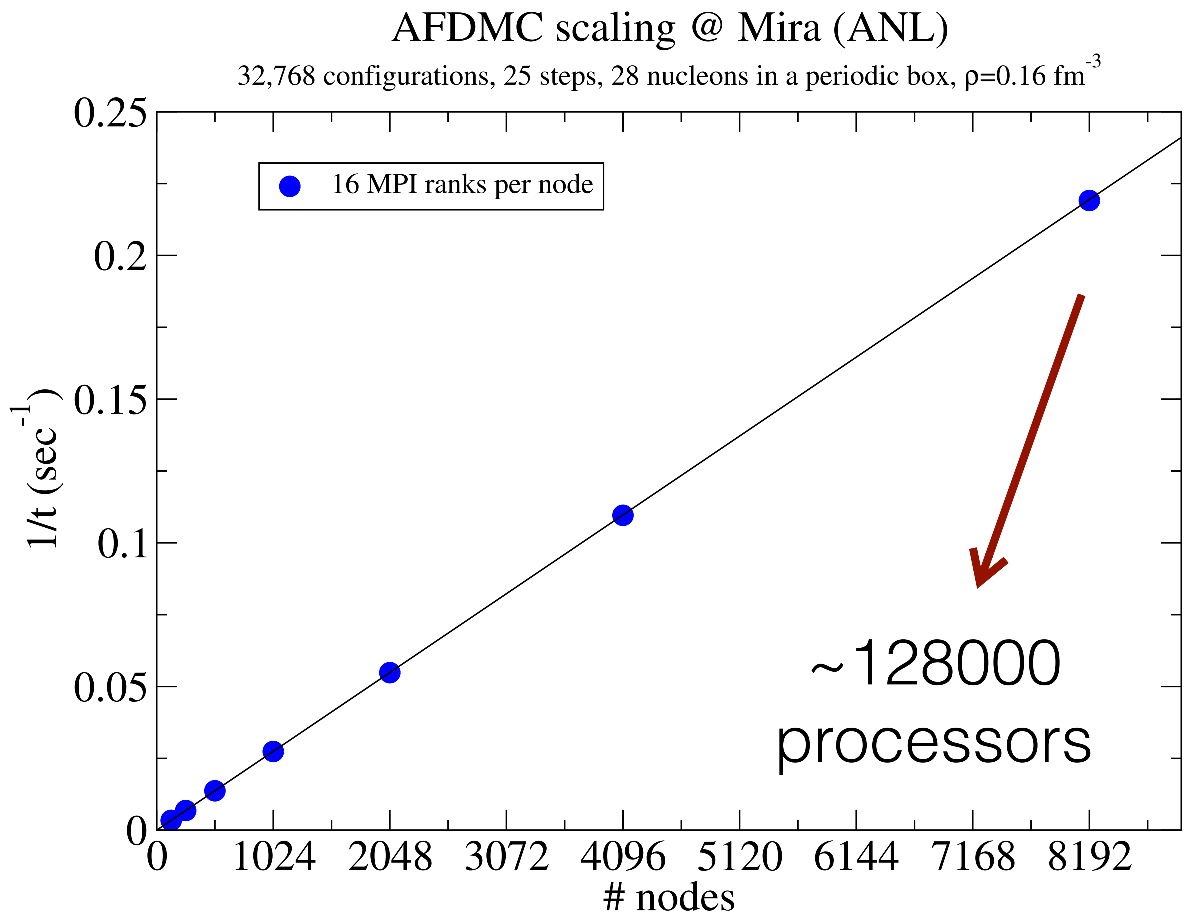
→ 240 processors

system	CPU time	B_Λ error
${}^{41}_{\Lambda}\mathrm{Ca}$ - ${}^{40}\mathrm{Ca}$	$\sim 30\,\mathrm{k\ hrs}$	~ 0.75 MeV
${}^{49}_{\Lambda}\mathrm{Ca}$ - ${}^{48}\mathrm{Ca}$	$\sim 55\,\mathrm{k\ hrs}$	~ 0.75 MeV
${}^{91}_{\Lambda}\mathrm{Zr}$ - ${}^{90}\mathrm{Zr}$	$\sim 350\,\mathrm{k\ hrs}$	~ 0.75 MeV
${}^{209}_{\Lambda}\mathrm{Pb}$ - ${}^{208}\mathrm{Pb}$	$\sim 4.2\,\mathrm{M\ hrs}$	~ 0.75 MeV
AFDMC	$\sim A^3$	$\sigma \sim 1/\sqrt{\mathcal{N}}$

↓

calculation accessible

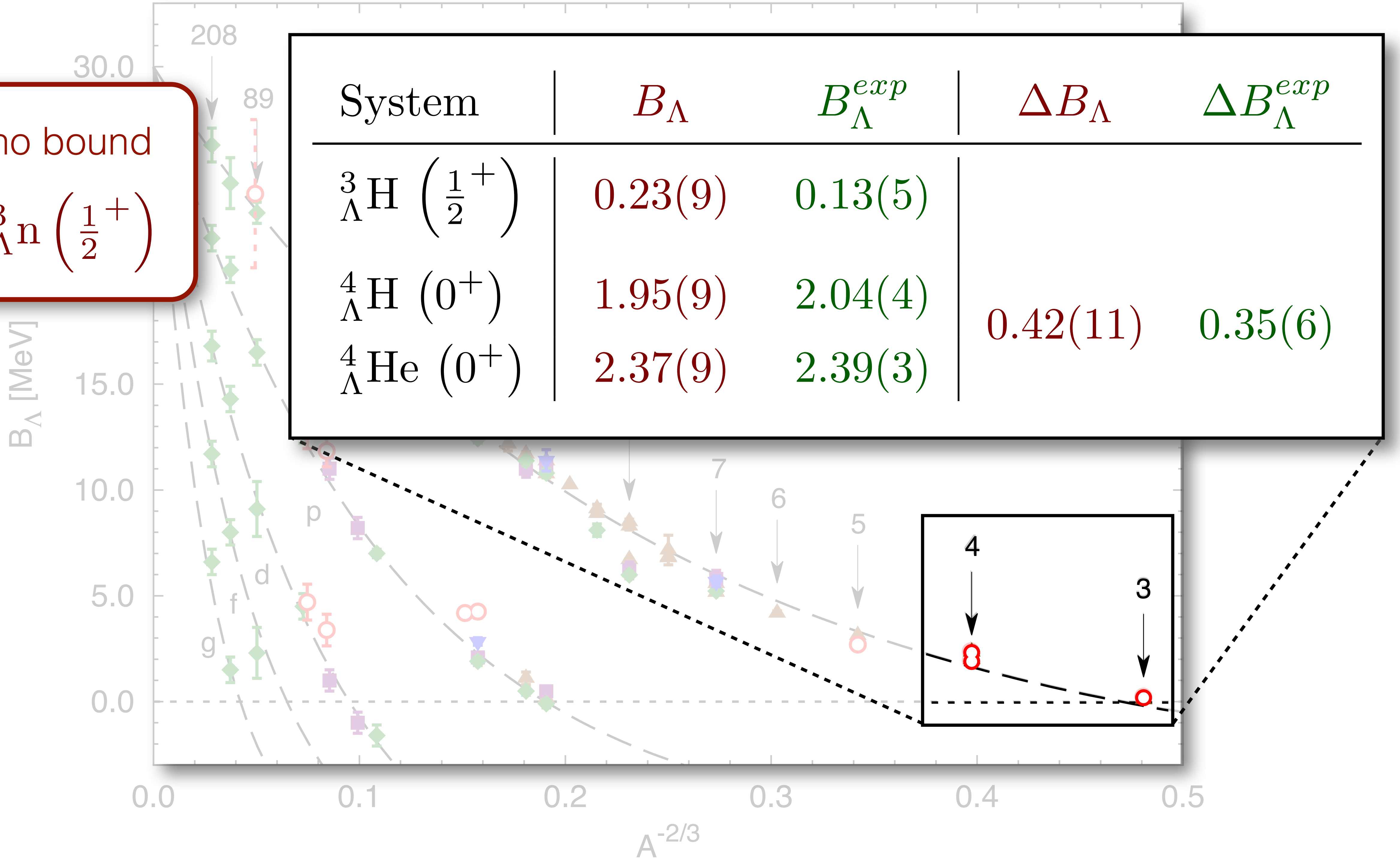
B_Λ in all waves, $A \pm 1$



~128000 processors

S. Gandolfi, unpublished

Backup: strangeness in nuclei



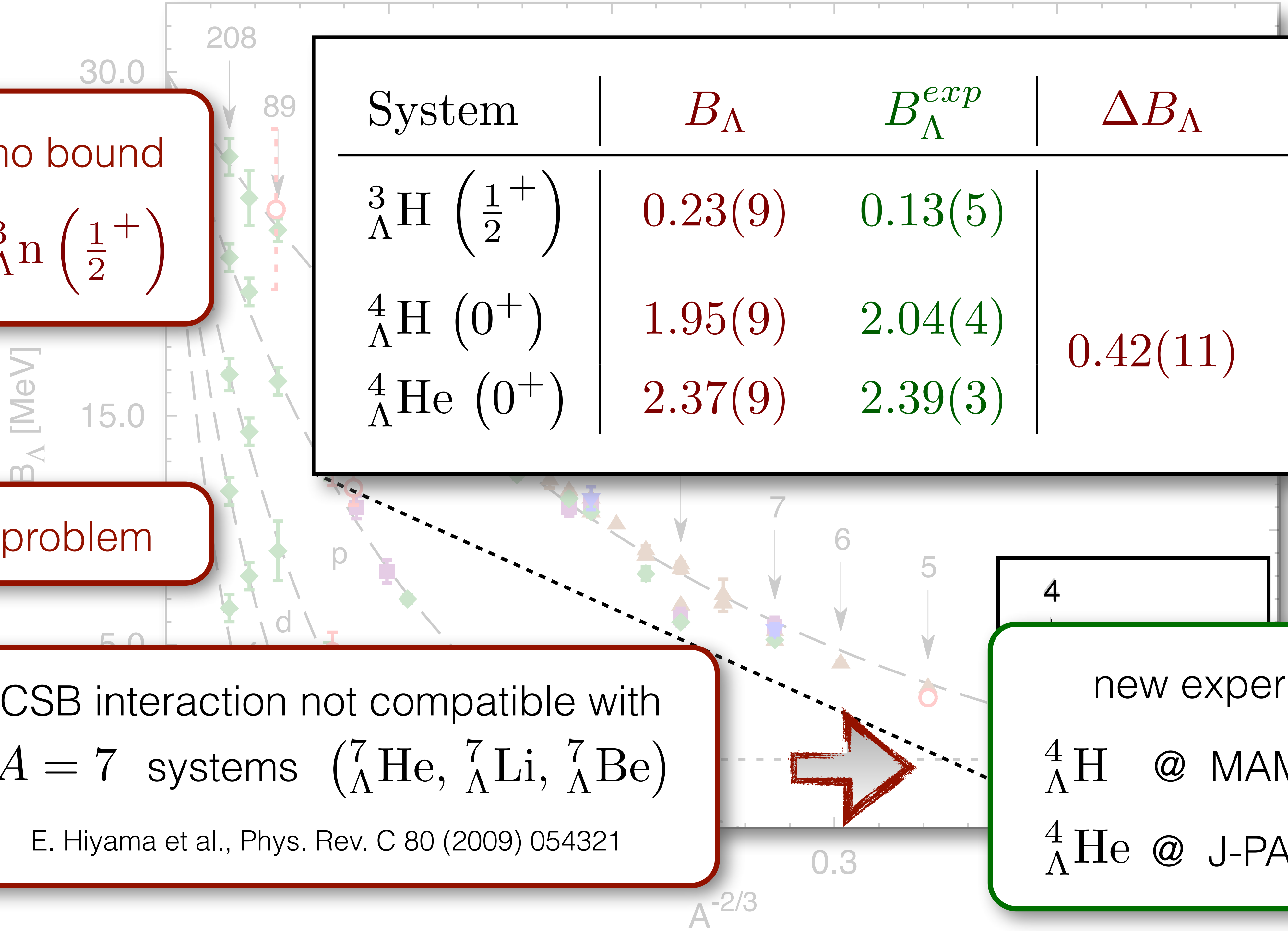
no bound

${}^{3}_{\Lambda}\mathrm{n}\left(\frac{1}{2}^{+}\right)$

System	B_Λ	B_Λ^{exp}	ΔB_Λ	ΔB_Λ^{exp}
${}^{3}_{\Lambda}\mathrm{H}\left(\frac{1}{2}^{+}\right)$	0.23(9)	0.13(5)		
${}^{4}_{\Lambda}\mathrm{H}\ (0^{+})$	1.95(9)	2.04(4)	0.42(11)	0.35(6)
${}^{4}_{\Lambda}\mathrm{He}\ (0^{+})$	2.37(9)	2.39(3)		

D. L., A. Lovato, S. Gandolfi, F. Pederiva, arXiv:1508.04722 (2015)

Backup: strangeness in nuclei



no bound

$^3_\Lambda n\left(\frac{1}{2}^+\right)$

problem

System	B_Λ	B_Λ^{exp}	ΔB_Λ	ΔB_Λ^{exp}
$^3_\Lambda\mathrm{H}\left(\frac{1}{2}^+\right)$	0.23(9)	0.13(5)		
$^4_\Lambda\mathrm{H}\,(0^+)$	1.95(9)	2.04(4)	0.42(11)	0.35(6)
$^4_\Lambda\mathrm{He}\,(0^+)$	2.37(9)	2.39(3)		

CSB interaction not compatible with

$A = 7$ systems $\left(^7_\Lambda\mathrm{He},\, ^7_\Lambda\mathrm{Li},\, ^7_\Lambda\mathrm{Be}\right)$

E. Hiyama et al., Phys. Rev. C 80 (2009) 054321

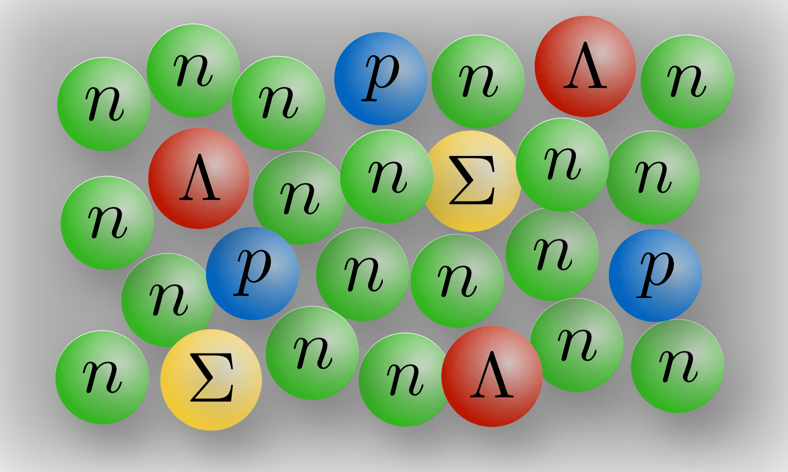
new experiments:

$^4_\Lambda\mathrm{H}$ @ MAMI, JLab

$^4_\Lambda\mathrm{He}$ @ J-PARC

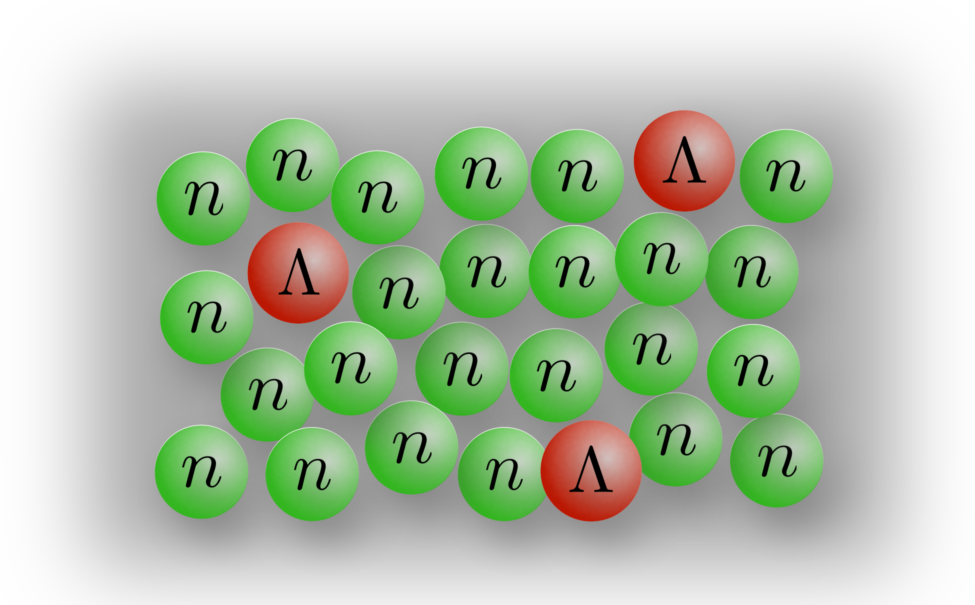
D. L., A. Lovato, S. Gandolfi, F. Pederiva, arXiv:1508.04722 (2015)

Backup: strangeness in neutron stars



hyper-nuclear matter

Backup: strangeness in neutron stars



lambda-neutron matter

PNM → hyperon fraction → energy per particle

equilibrium condition: chemical potentials

$$\mu_\Lambda(\rho_b, x_\Lambda) = \mu_n(\rho_b, x_\Lambda)$$

$$\text{EOS} \left\{ \begin{array}{lcl} E_{\mathrm{HNM}} & \equiv & E_{\mathrm{HNM}}(\rho_b) \\ \mathcal{E}_{\mathrm{HNM}} & \equiv & \mathcal{E}_{\mathrm{HNM}}(\rho_b) \\ P_{\mathrm{HNM}} & \equiv & P_{\mathrm{HNM}}(\rho_b) \end{array} \right. \Rightarrow \quad \text{TOV} \left\{ \begin{array}{l} M(R) \\ M_{\max} \end{array} \right.$$

$$E_{\mathrm{HNM}} \equiv E_{\mathrm{HNM}}(\rho_b, x_\Lambda) \quad \longleftrightarrow$$

AFDMC calculations
neutrons + lambdas

Backup: strangeness in neutron stars

neutrons
+
lambdas

$$\begin{cases} \rho_b = \rho_n + \rho_\Lambda \\ x_\Lambda = \dfrac{\rho_\Lambda}{\rho_b} \end{cases} \qquad \begin{cases} \rho_n = (1 - x_\Lambda)\rho_b \\ \rho_\Lambda = x_\Lambda \rho_b \end{cases}$$

$$E_{\mathrm{HNM}}(\rho_b, x_\Lambda) = \Big[E_{\mathrm{PNM}}((1 - x_\Lambda)\rho_b) + m_n\Big](1 - x_\Lambda) + \Big[E_\Lambda^F(x_\Lambda \rho_b) + m_\Lambda\Big] x_\Lambda + f(\rho_b, x_\Lambda)$$

Problem1: limitation in x_Λ due to simulation box

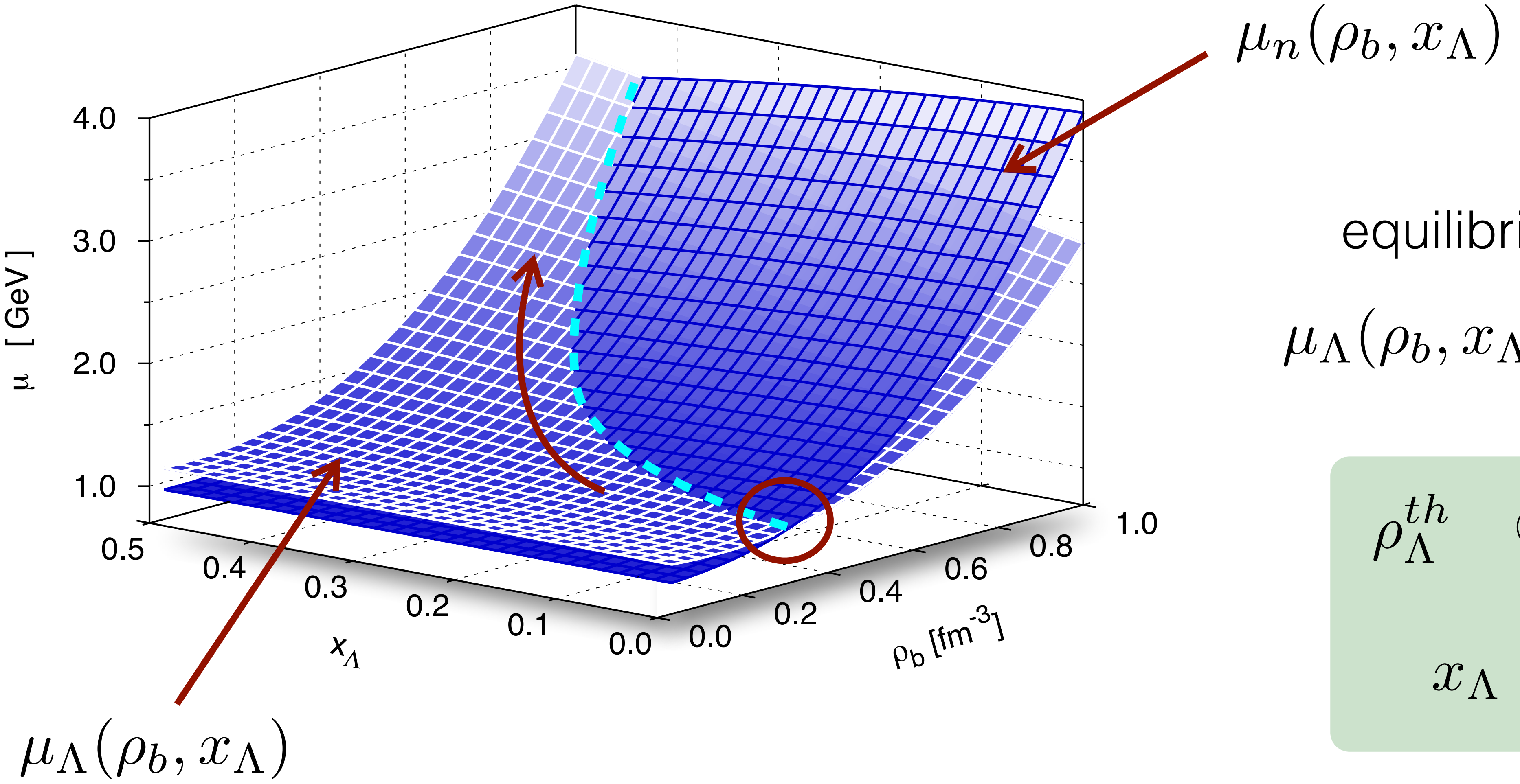
Problem2: finite size effects

Problem3: fitting procedure

$f(\rho_b, x_\Lambda)$ cluster expansion $\dfrac{\rho_\Lambda \rho_n}{\rho_b}$, $\dfrac{\rho_\Lambda \rho_n \rho_n}{\rho_b}$, $\cancel{\dfrac{\rho_\Lambda \rho_\Lambda \rho_n}{\rho_b}}$, $\cancel{\dfrac{\rho_\Lambda \rho_n \rho_n \rho_n}{\rho_b}}$

Backup: strangeness in neutron stars

$$\begin{cases} \mu_n(\rho_b, x_\Lambda) = E_{\rm PNM}(\rho_n) + \rho_n \dfrac{\partial E_{\rm PNM}}{\partial \rho_n} + m_n + f(\rho_b, x_\Lambda) + \rho_b \dfrac{\partial f}{\partial \rho_n} \\ \mu_\Lambda(\rho_b, x_\Lambda) = E_\Lambda^F(\rho_\Lambda) + \rho_\Lambda \dfrac{\partial E_\Lambda^F}{\partial \rho_\Lambda} + m_\Lambda + f(\rho_b, x_\Lambda) + \rho_b \dfrac{\partial f}{\partial \rho_\Lambda} \end{cases}$$



$\mu_n(\rho_b, x_\Lambda)$

$\mu_\Lambda(\rho_b, x_\Lambda)$

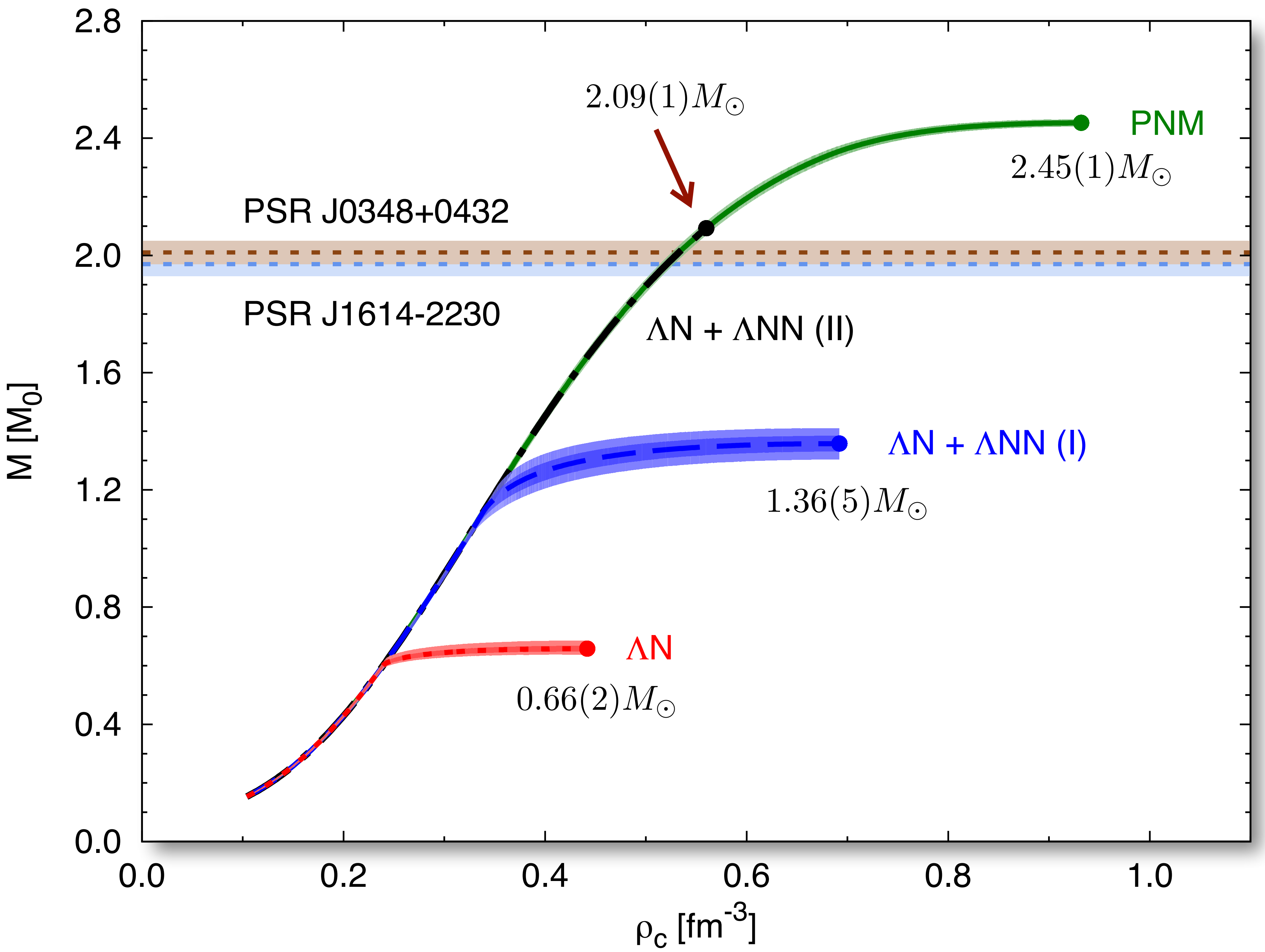
equilibrium condition:

$$\mu_\Lambda(\rho_b, x_\Lambda) = \mu_n(\rho_b, x_\Lambda)$$

$$\rho_\Lambda^{th} \quad @ \quad x_\Lambda \to 0$$

$$x_\Lambda \equiv x_\Lambda(\rho_b)$$

Backup: strangeness in neutron stars

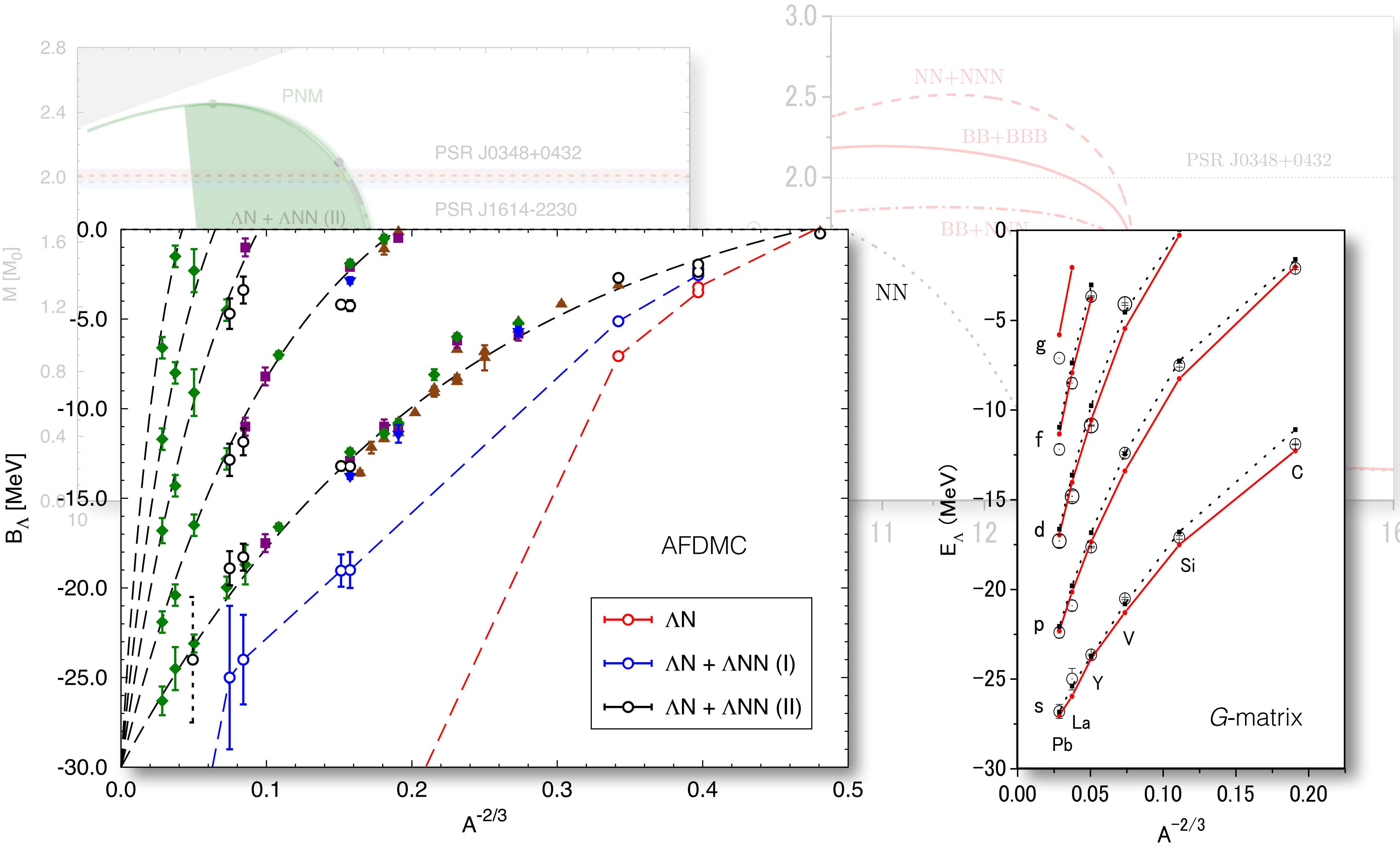


D. L., A. Lovato, S. Gandolfi, F. Pederiva, Phys. Rev. Lett. 114, 092301 (2015)

Backup: strangeness in neutron stars

Phys. Rev. Lett. 114, 092301 (2015)

Phys. Rev. C 90, 045805 (2014)



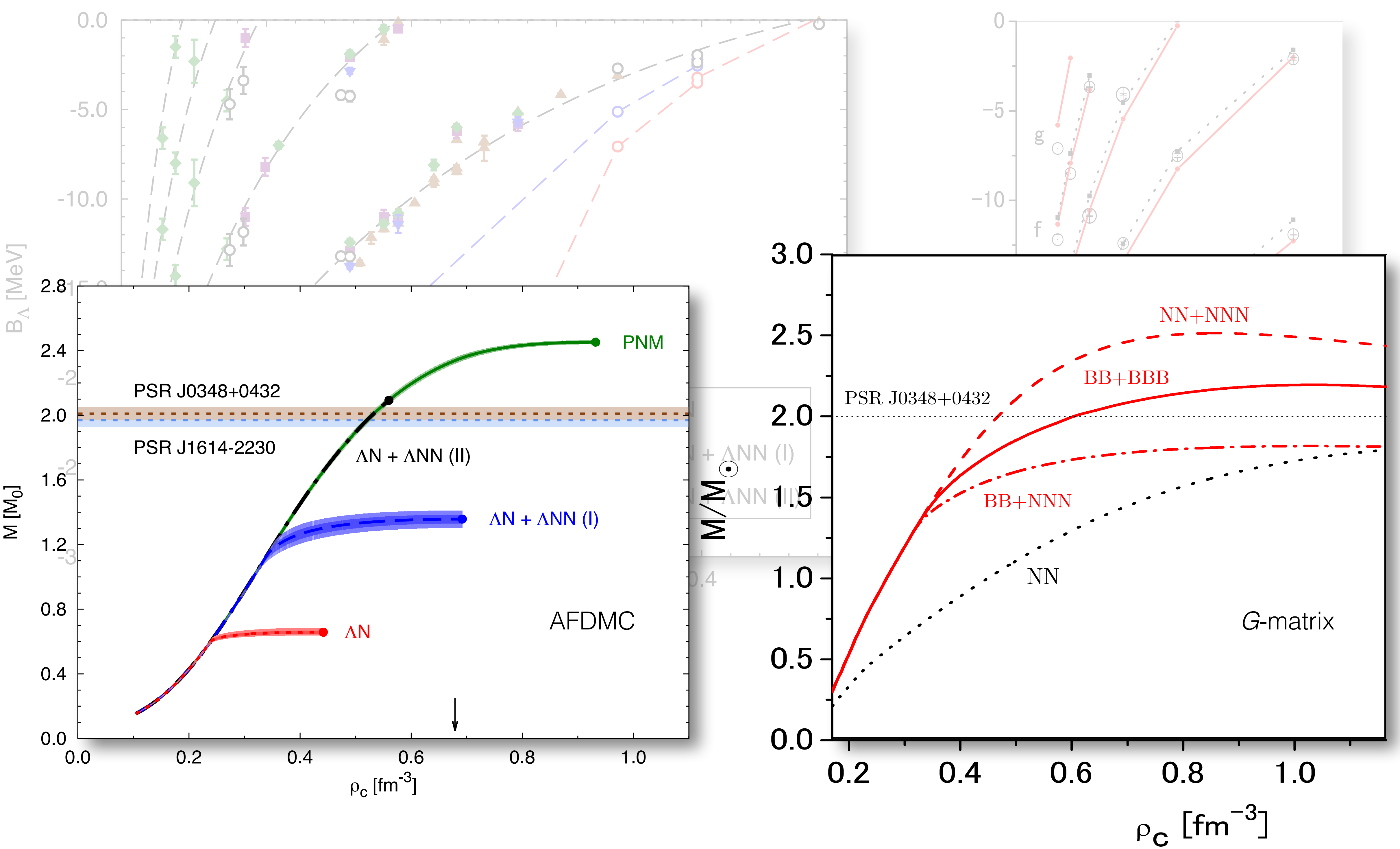
Phys. Rev. C 89, 014314 (2014), arXiv:1506.04042 (2015)

Phys. Rev. C 90, 045805 (2014)

Backup: strangeness in neutron stars

Phys. Rev. C 89, 014314 (2014), arXiv:1506.04042 (2015)

Phys. Rev. C 90, 045805 (2014)



Phys. Rev. Lett. 114, 092301 (2015)

Phys. Rev. C 90, 045805 (2014)

Backup: strangeness in nuclei

- ✓ 3-body interaction → fit on symmetric hypernuclei

$$v_{\lambda ij} = v_{\lambda ij}^{2\pi,P} + v_{\lambda ij}^{2\pi,S} + v_{\lambda ij}^{D}$$

isospin projectors

$$\begin{cases} v_{\lambda ij}^{2\pi,P} = -\dfrac{C_P}{6}\Big\{X_{i\lambda}\,,X_{\lambda j}\Big\}\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \\ v_{\lambda ij}^{2\pi,S} = C_S\,Z\left(r_{\lambda i}\right)Z\left(r_{\lambda j}\right)\,\boldsymbol{\sigma}_i\cdot\hat{\boldsymbol{r}}_{i\lambda}\,\boldsymbol{\sigma}_j\cdot\hat{\boldsymbol{r}}_{j\lambda}\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \\ v_{\lambda ij}^{D} = W_D\,T_\pi^2\left(r_{\lambda i}\right)T_\pi^2\left(r_{\lambda j}\right)\Big[1+\dfrac{1}{6}\boldsymbol{\sigma}_\lambda\cdot\left(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j\right)\Big] \end{cases}$$

$$\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j = -3\,\mathcal{P}^{T=0}\boxed{+\,\mathcal{P}^{T=1}} \longrightarrow -3\,\mathcal{P}^{T=0} + C_T\,\mathcal{P}^{T=1}$$

control parameter:
strength and sign of the nucleon
isospin triplet channel

sensitivity study:
light- & medium-heavy hypernuclei

Backup: nuclei

nucleon-nucleon interaction

nucleus	AV4′	AV6′	AV7′	AV4′+UIX$_c$	exp
^{4}He (0^+)	-32.83(5)	-27.09(3)	-25.7(2)	-26.63(2)	-28.295
^{15}O $(\frac{1}{2}^-)$	–	–	–	-99.43(2)	-111.955
^{16}O (0^+)	-180.1(4)	-115.6(3)	-90.6(4)	-119.9(2)	-127.619
^{39}K $(\frac{3}{2}^+)$	–	–	–	-360.8(2)	-333.724
^{40}Ca (0^+)	-597(3)	-322(2)	-209(1)	-383.3(3)	-342.051
^{44}Ca (0^+)	–	–	–	-397.8(5)	-380.960
^{47}K $(\frac{1}{2}^+)$	–	–	preliminary –	-386.3(2)	-400.199
^{48}Ca (0^+)	-645(3)	–	–	-413.2(3)	-416.001

S. Gandolfi, A. Lovato, J. Carlson, K. E. Schmidt, Phys. Rev. C 90, 061306(R) (2014)
F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)