Nuclear electromagnetic response: millions of hours of computing time, then Adelchi's PRC of 1997

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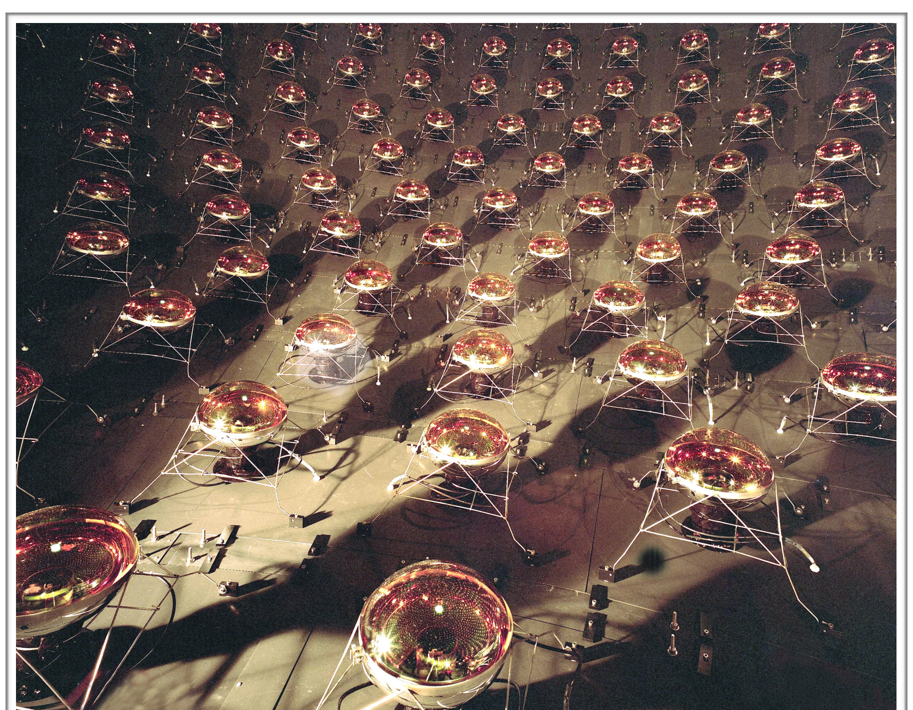
Introduction

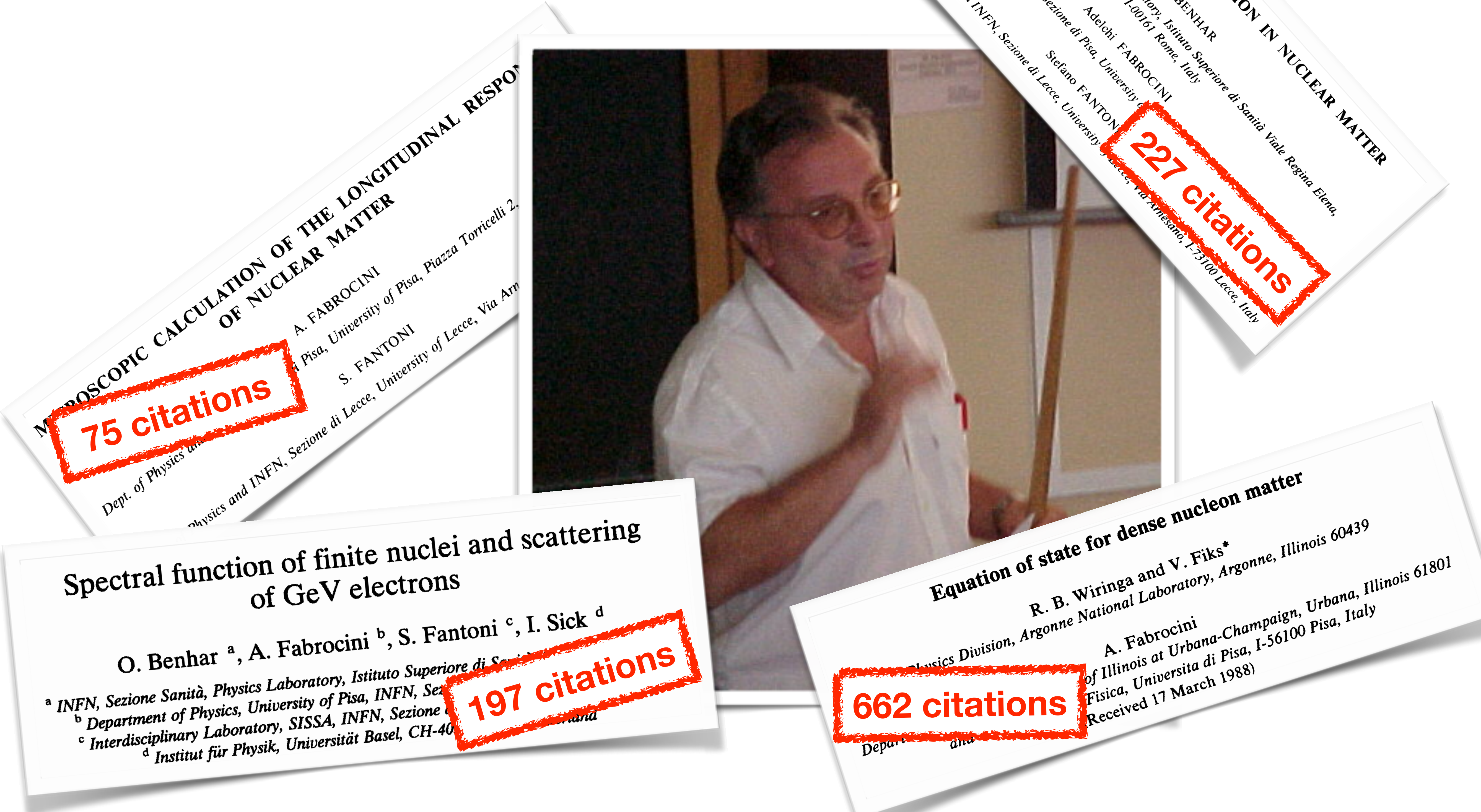
- Multi-messenger era for nuclear astrophysics
  - ✴ Gravitational waves have (just) been detected!
  - ✴ Supernovae neutrino will be detected by the current and next generation neutrino experiments
  - ✴ Nuclear dynamics determine the structure of neutron stars, neutrino abundances and neutrino propagation



- Ton-scale neutrino-oscillation and $0\nu\beta\beta$ experiments
  - ✴ Charge-parity (CP) violating phase and the mass hierarchy will be measured
  - ✴ Determine whether the neutrino is a Majorana or a Dirac particle
  - ✴ Need for including nuclear dynamics; mean-field models are inadequate to describe neutrino-nucleus interaction

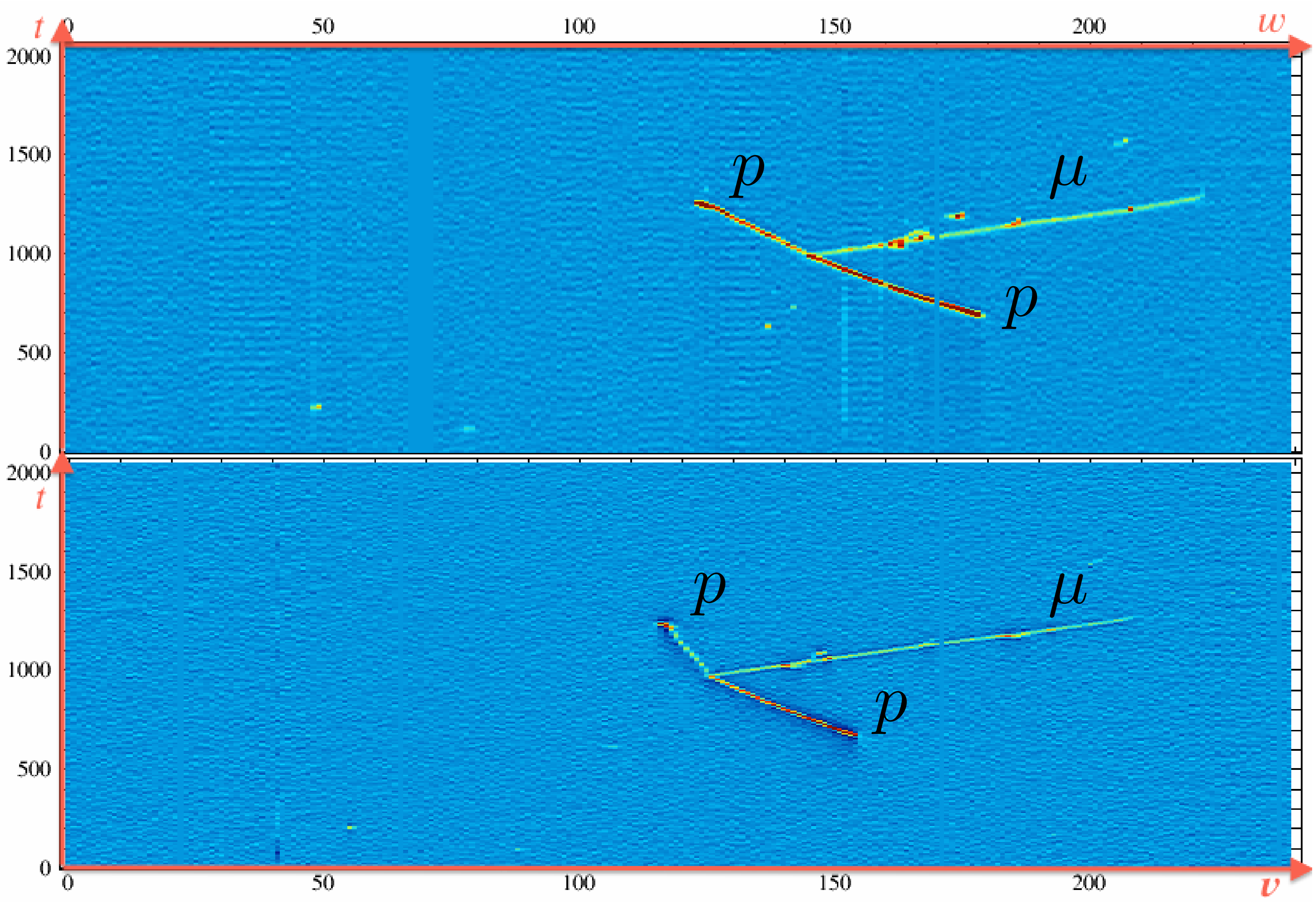
Introduction

- Adelchi's work was (**AND STILL IS**) central



Nuclear correlations

Recently, the liquid Argon detector ArgoNeuT was able to elucidate the role of nuclear correlations in neutrino-nucleus scattering events.

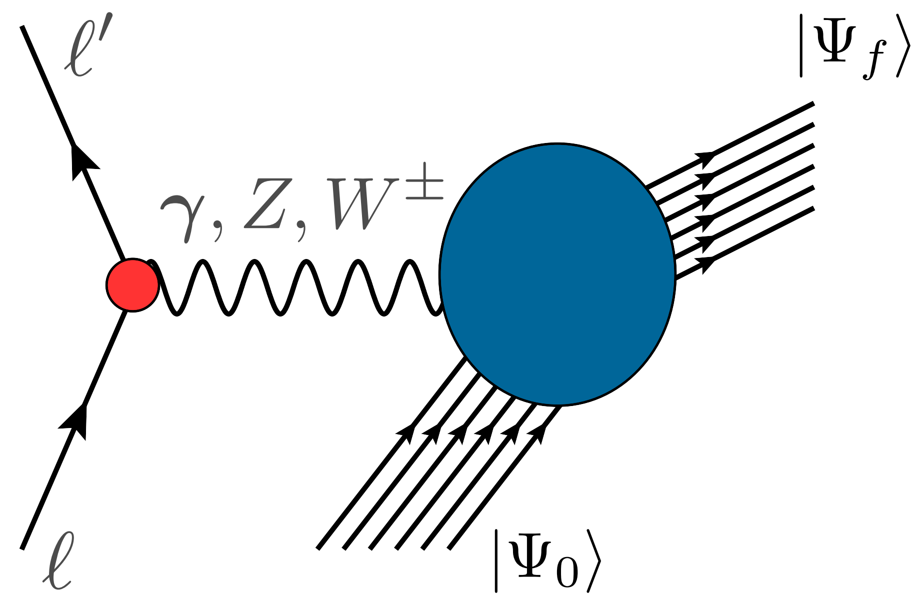


PRD 90, 012008 (2014)

Lepton-nucleus scattering

The inclusive cross section of the process in which a lepton scatters off a nucleus can be written in terms of five response functions

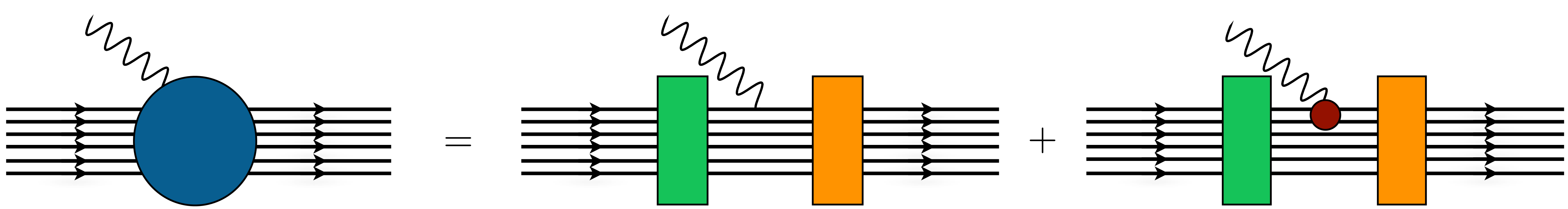
$$\frac{d\sigma}{dE_{\ell'}d\Omega_\ell} \propto \left[v_{00}R_{00} + v_{zz}R_{zz} - v_{0z}R_{0z} + v_{xx}R_{xx} \mp v_{xy}R_{xy}\right]$$



- The response functions contain all the information on target structure and dynamics

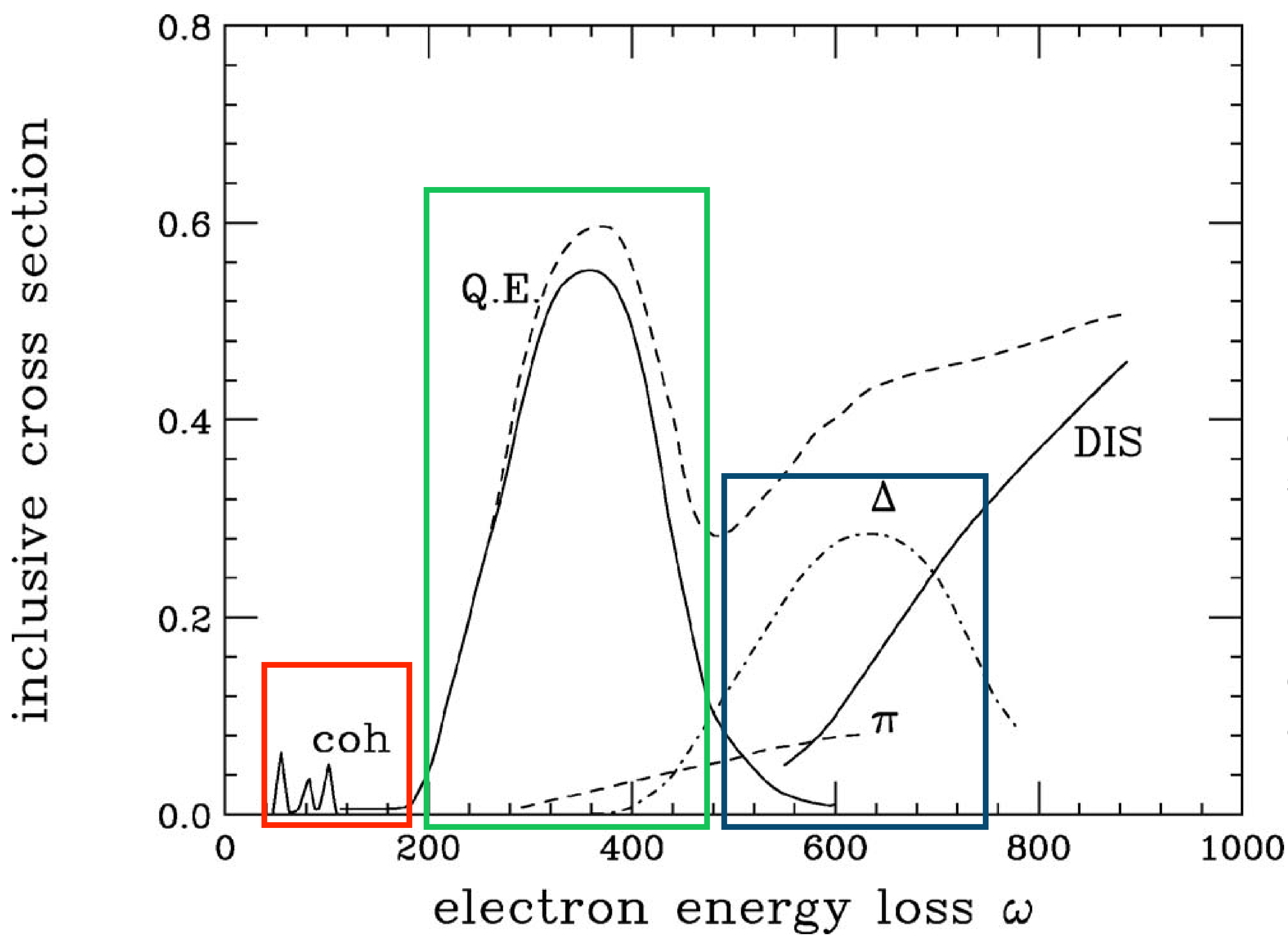
$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle\Psi_0|J^\dagger_\alpha(\mathbf{q})|\Psi_f\rangle\langle\Psi_f|J_\beta(\mathbf{q})|\Psi_0\rangle\delta(\omega - E_f + E_0)$$

- They account for initial state correlations, final state correlations and two-body currents



Electron-nucleus scattering

Schematic representation of the inclusive cross section as a function of the energy loss.



- Elastic scattering and inelastic excitation of discrete nuclear states.
- Broad peak due to quasi-elastic electron-nucleon scattering.
- Excitation of the nucleon to distinct resonances (like the Δ) and pion production.

Nuclear hamiltonian

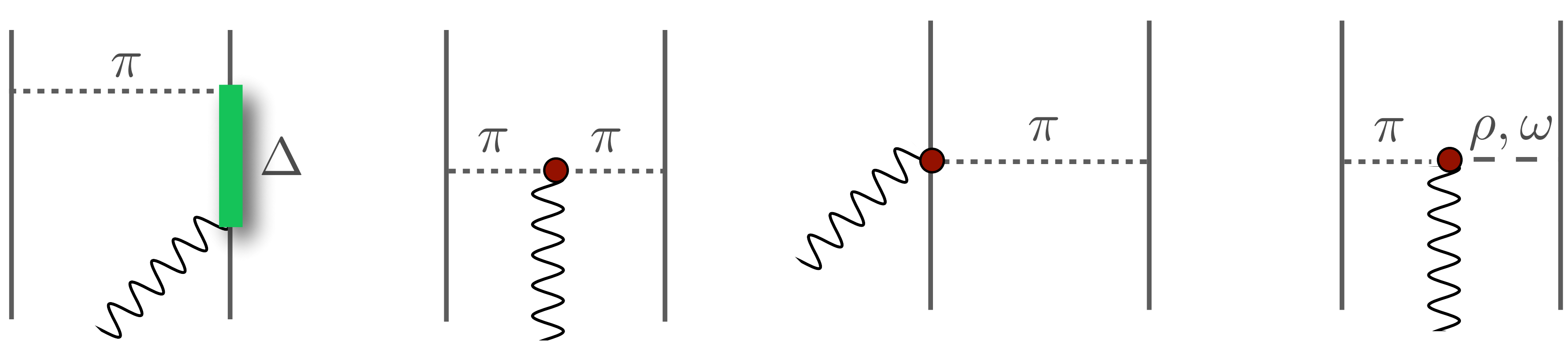
Ab initio approaches are based on a non-relativistic nuclear hamiltonian

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

- v_{ij} provides an accurate description of the NN scattering data and reduces to Yukawa's one-pion-exchange potential at large distances

- V_{ijk} effectively includes the lowest nucleon excitations

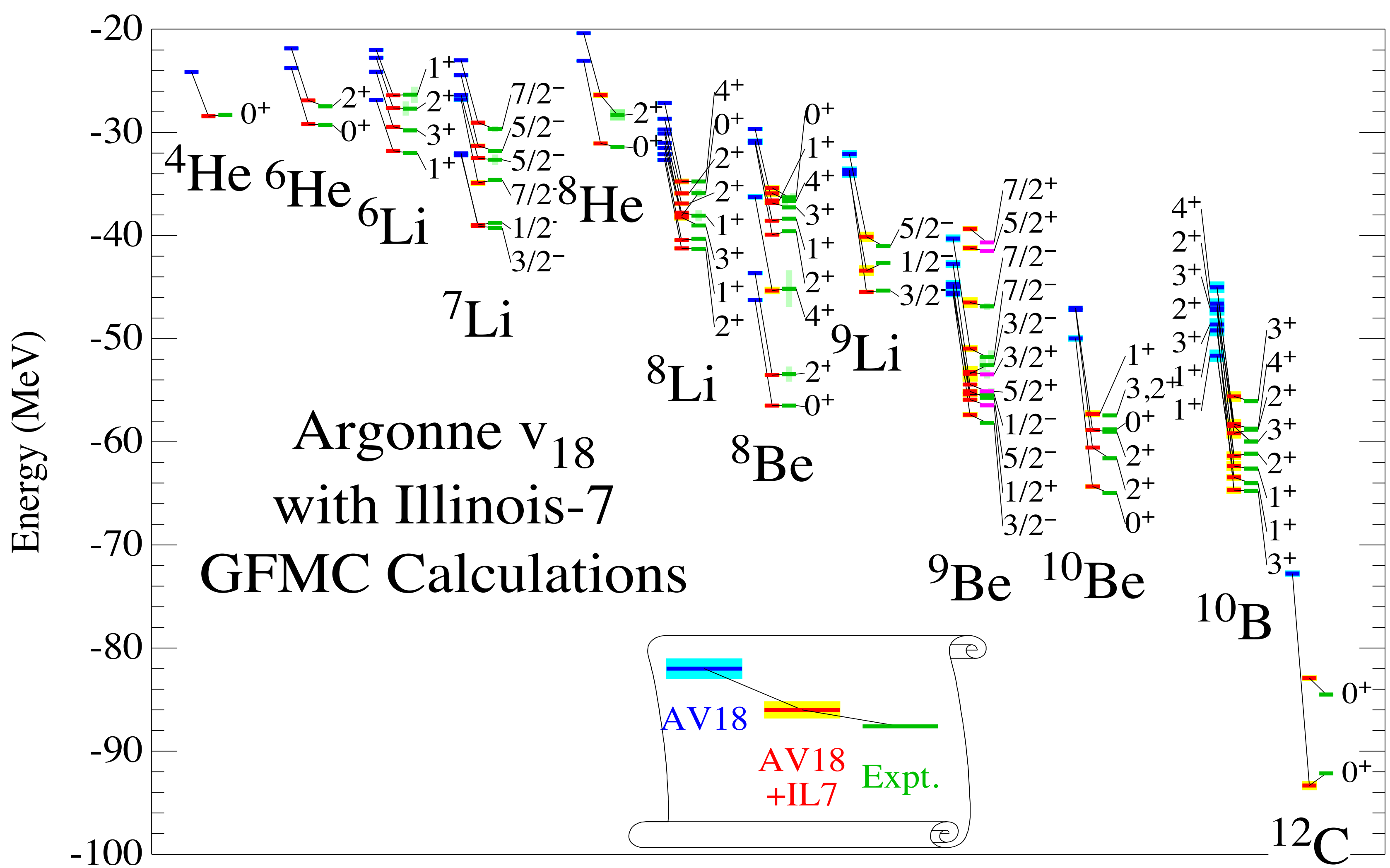
- Consistent two-body currents account for processes in which the vector boson couples to the meson exchanged between two nucleons or to the excitations of nuclear resonances



Quantum Monte Carlo

- Diffusion Monte Carlo methods use an imaginary-time projection technique to enhance the ground-state component of a starting trial wave function.

$$H|\Psi_0\rangle = E_0|\Psi_0\rangle \quad \longleftrightarrow \quad \lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = |\Psi_0\rangle \quad \longleftrightarrow \quad |\Psi_T\rangle = \mathcal{F}|\Phi\rangle$$



Moderate momentum transfer

- Both initial and final states are eigenstates of the nuclear Hamiltonian

$$H|\Psi_0\rangle = E_0|\Psi_0\rangle \qquad\qquad H|\Psi_f\rangle = E_f|\Psi_f\rangle$$

- For electron scattering on ^{12}C

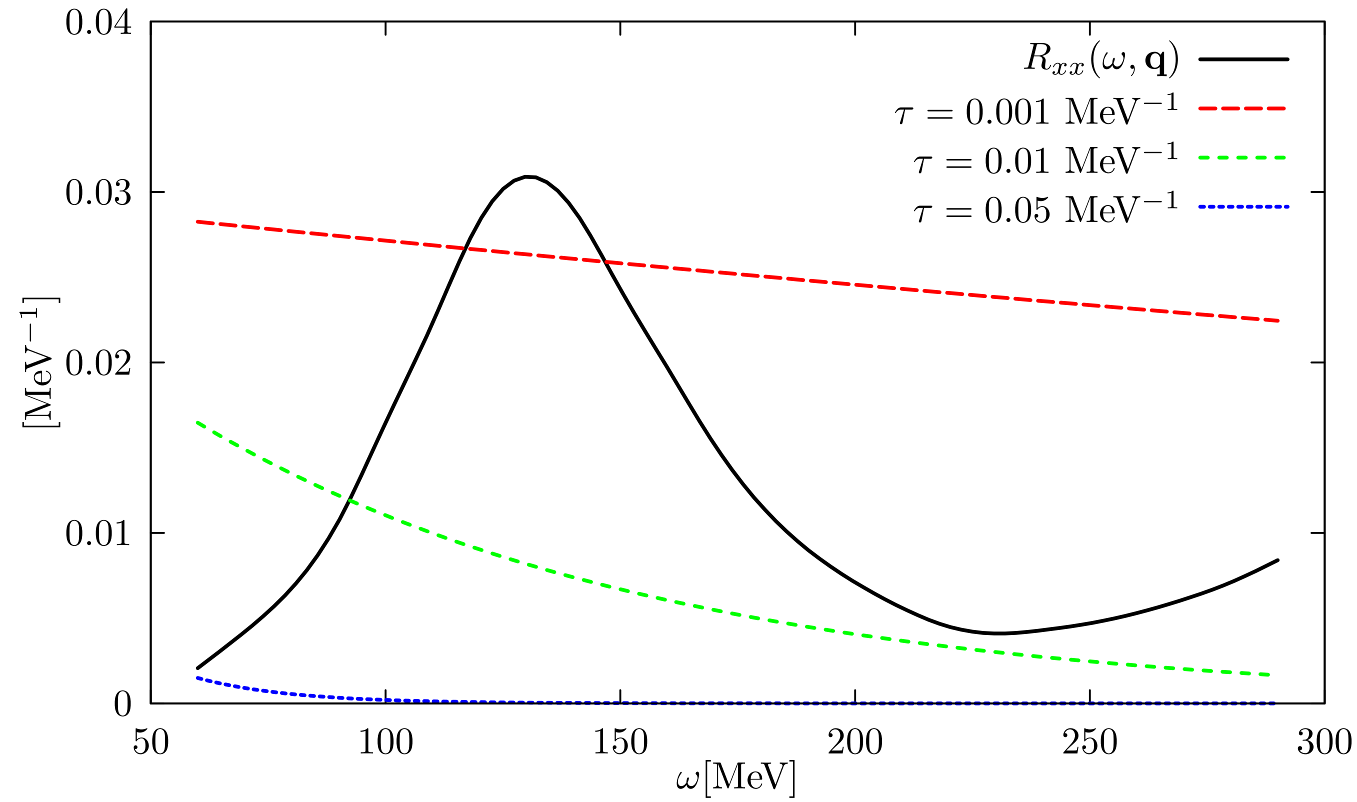
$$|^{12}C^*\rangle, |^{11}\mathrm{B}, p\rangle,\ |^{11}\mathrm{C}, n\rangle,\ |^{10}\mathrm{B}, pn\rangle,\ |^{10}\mathrm{B}, pp\rangle \ldots$$

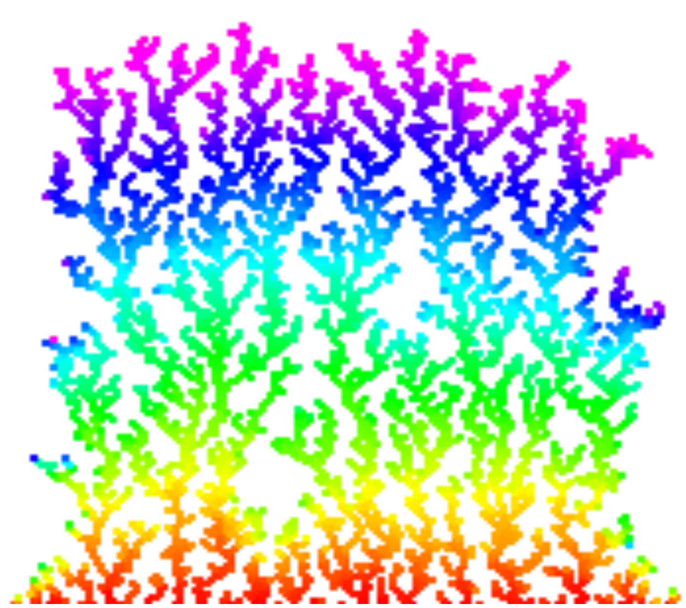
Euclidean response function

Valuable information on the energy dependence of the response functions can be inferred from the their Laplace transforms

$$E_{\alpha\beta}(\tau,\mathbf{q}) \equiv \int d\omega e^{-\omega\tau} R_{\alpha\beta}(\omega,\mathbf{q})$$

At finite imaginary time the contributions from large energy transfer are quickly suppressed





The system is first heated up by the transition operator.
Its cooling determines the Euclidean response of the system

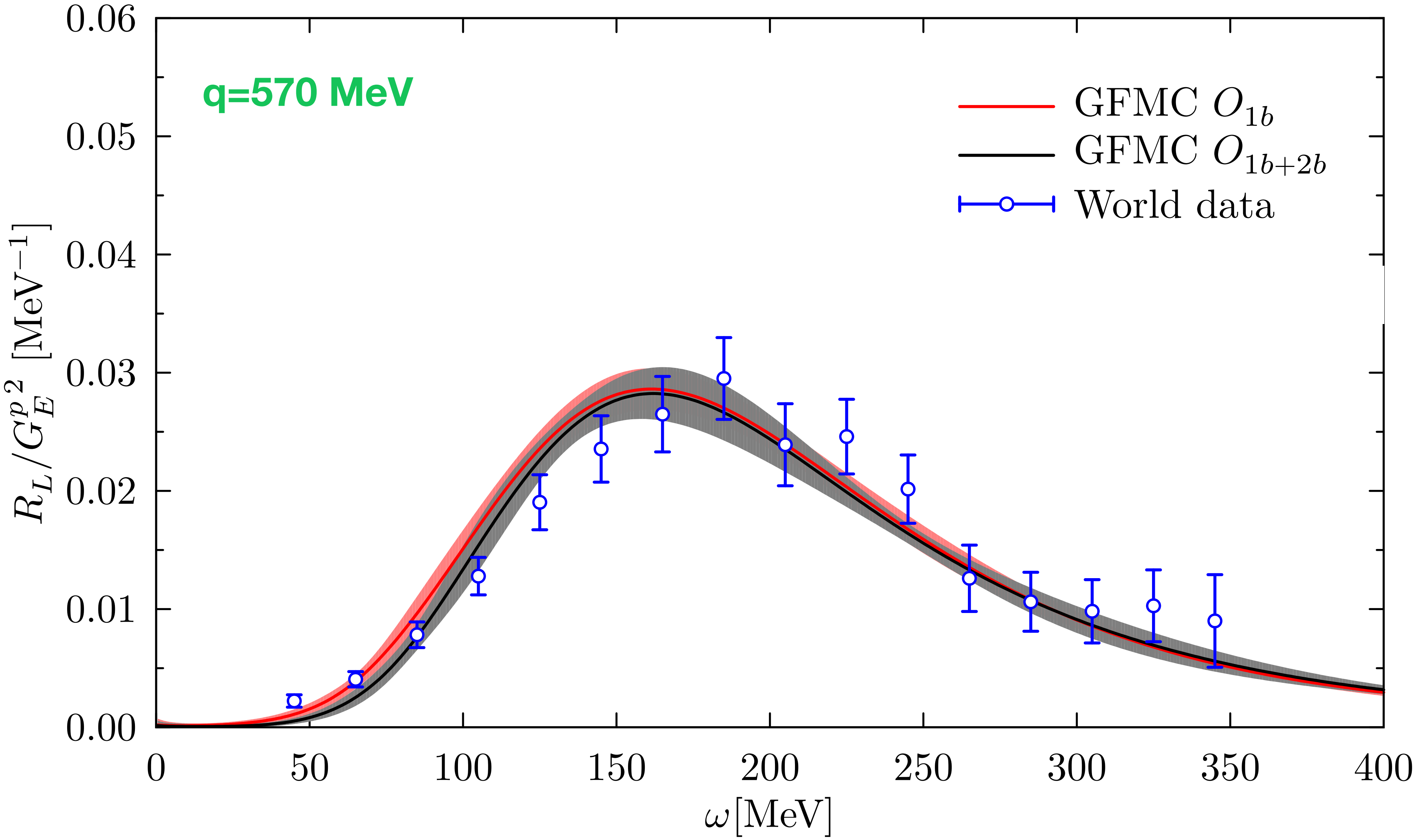
$$E_{\alpha\beta}(\tau,\mathbf{q}) = \langle\Psi_0|J^\dagger_\alpha(\mathbf{q})e^{-(H-E_0)\tau}J_\beta(\mathbf{q})|\Psi_0\rangle$$

Same technique used in Lattice QCD, condensed matter physics…

^{12}C electromagnetic response

- Very good agreement with the experimental data
- Small contribution from two-body currents to the transverse response functions.
- No quenching of the proton electric form factor!

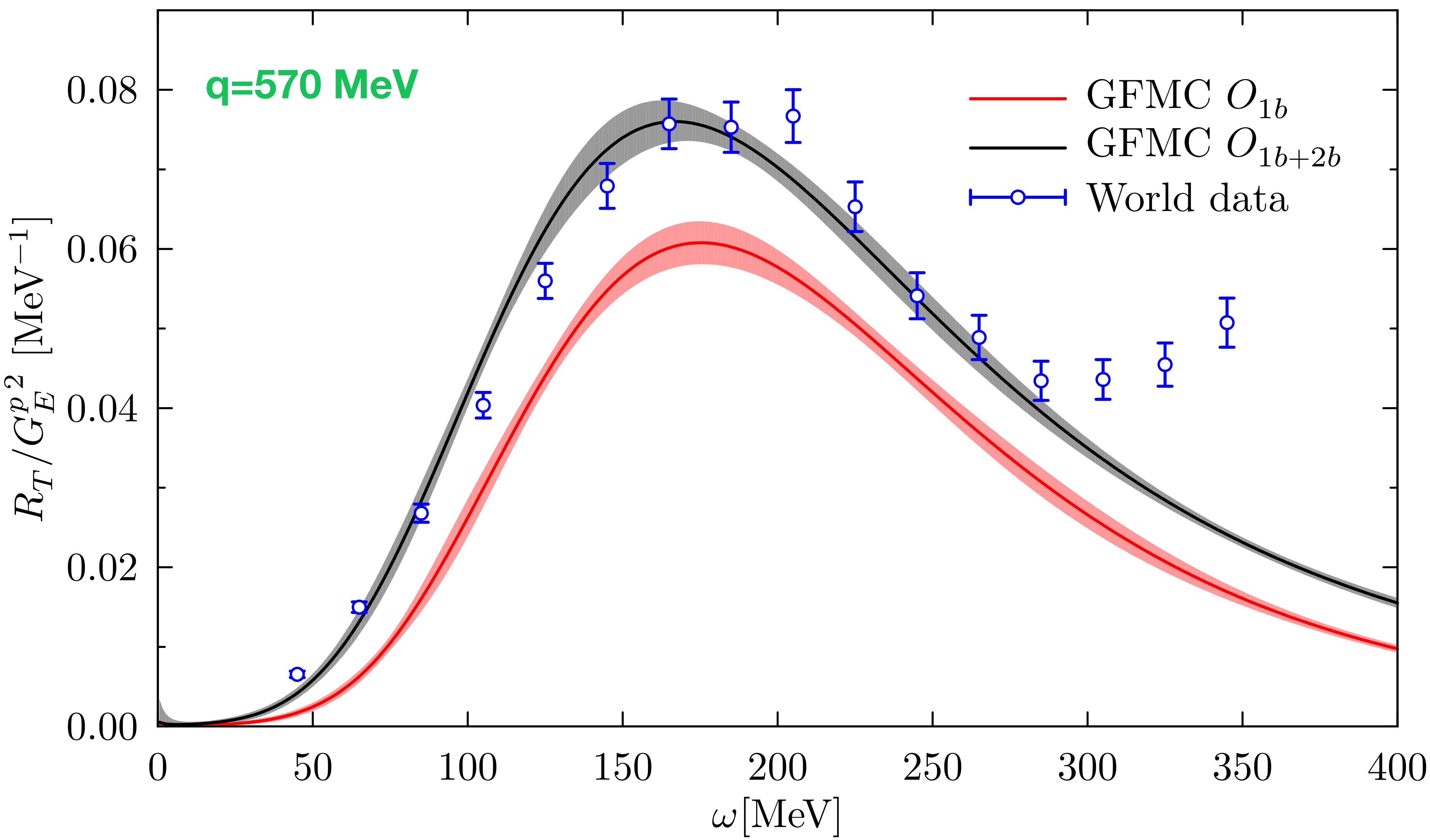
AL et al. ArXiv 1605.00248



^{12}C electromagnetic response

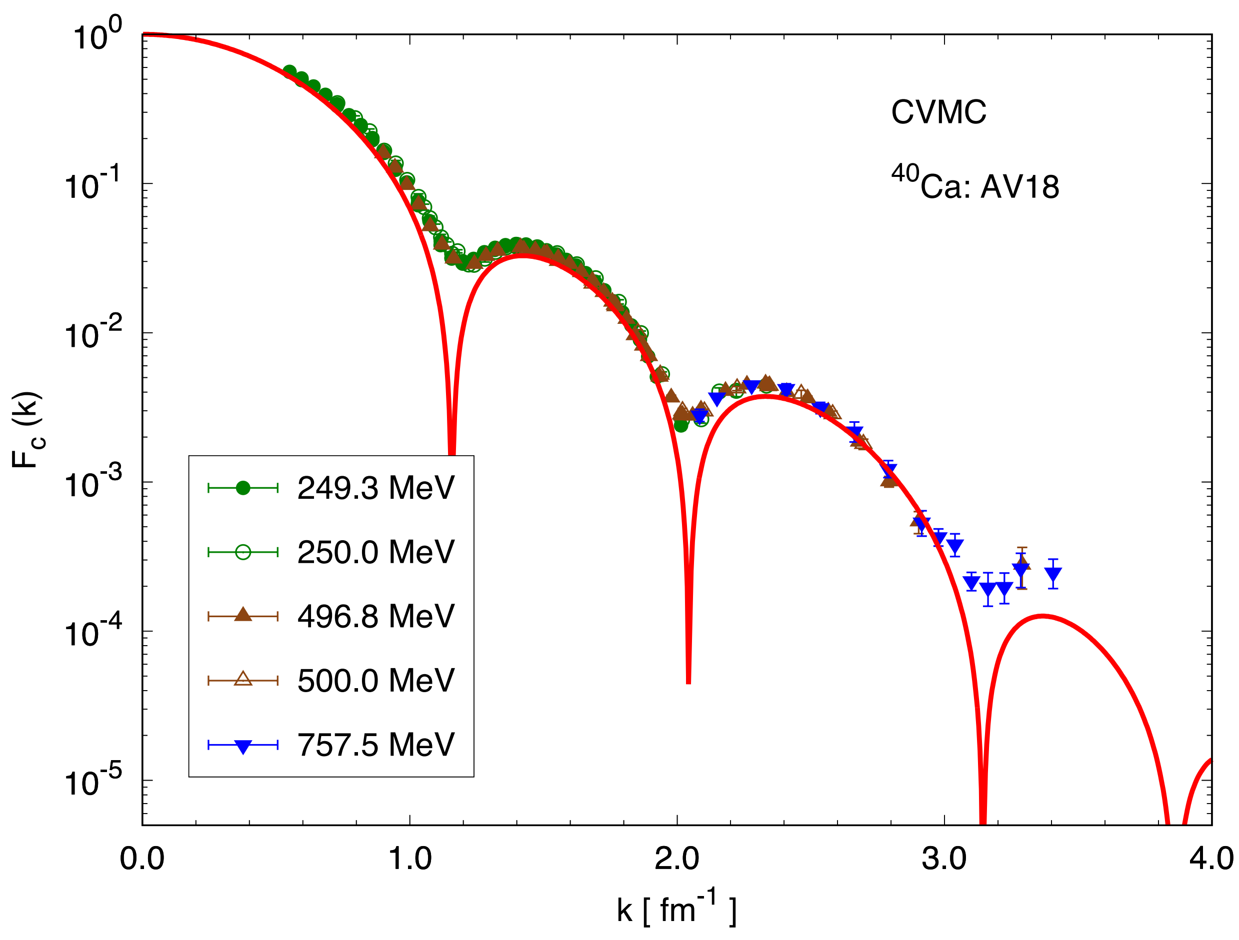
- Very good agreement with the experimental data.
- Sizable contribution from two-body currents to the transverse response functions.
- Very likely the solution of the axial mass puzzle !

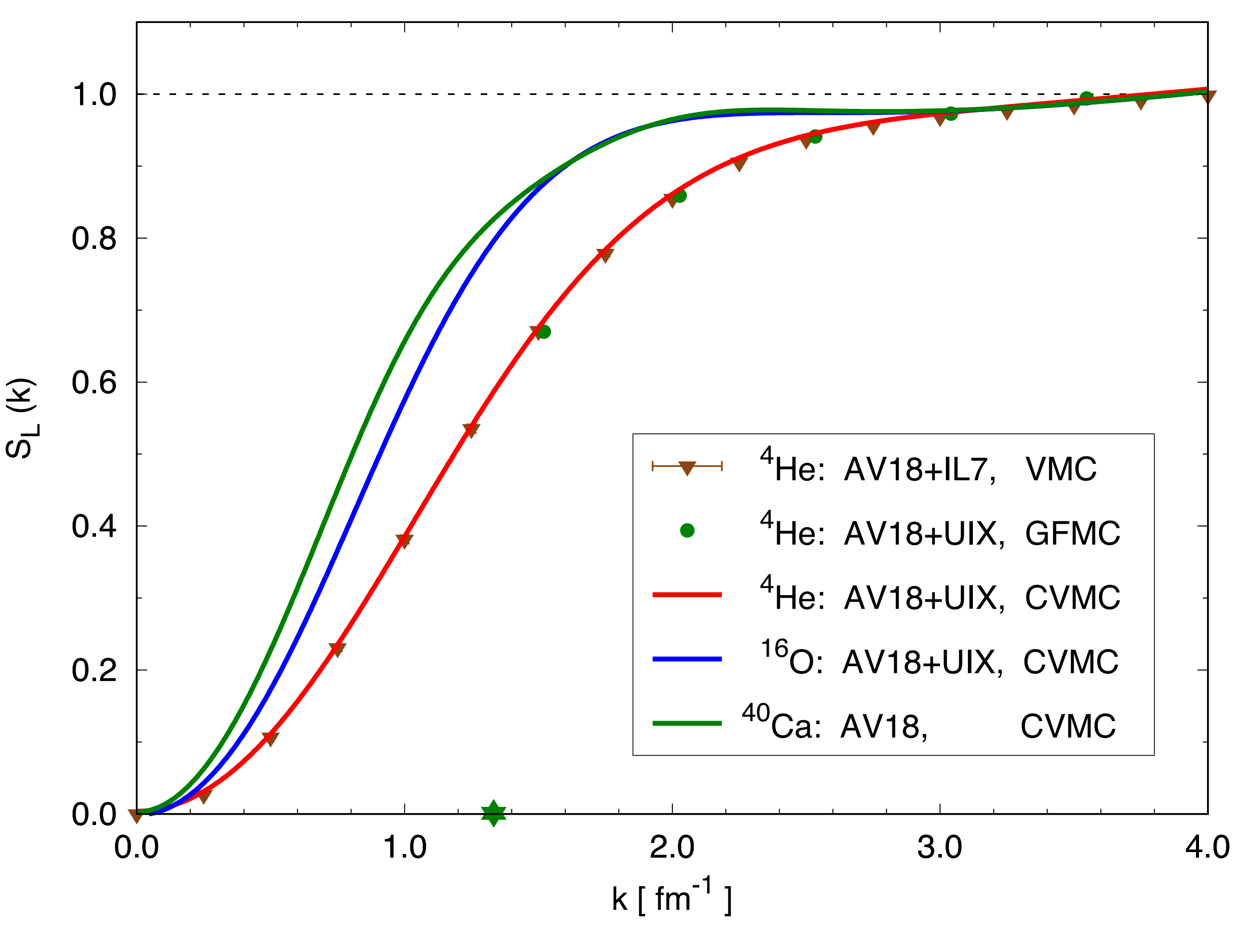
AL et al. ArXiv 1605.00248



Using supercomputers

- With Diego we are moving towards larger nuclei like ^{16}O and ^{40}Ca
- Since we are lazier than (and not as smart as) Adelchi, we use quantum Monte Carlo!





- The agreement with the experimental data is remarkably good!

Using supercomputers

- We do indeed generate very nice plots, but **millions of hours of computing time burned**!

Large momentum transfer

- Same initial state but final state factorizes

$$H|\Psi_0\rangle = E_0|\Psi_0\rangle \qquad\qquad |\Psi_f\rangle = |\mathbf{p}\rangle \otimes |\Psi_{\tilde{f}}\rangle_{A-1}$$

- Sum of individual cross sections weighted by the spectral function

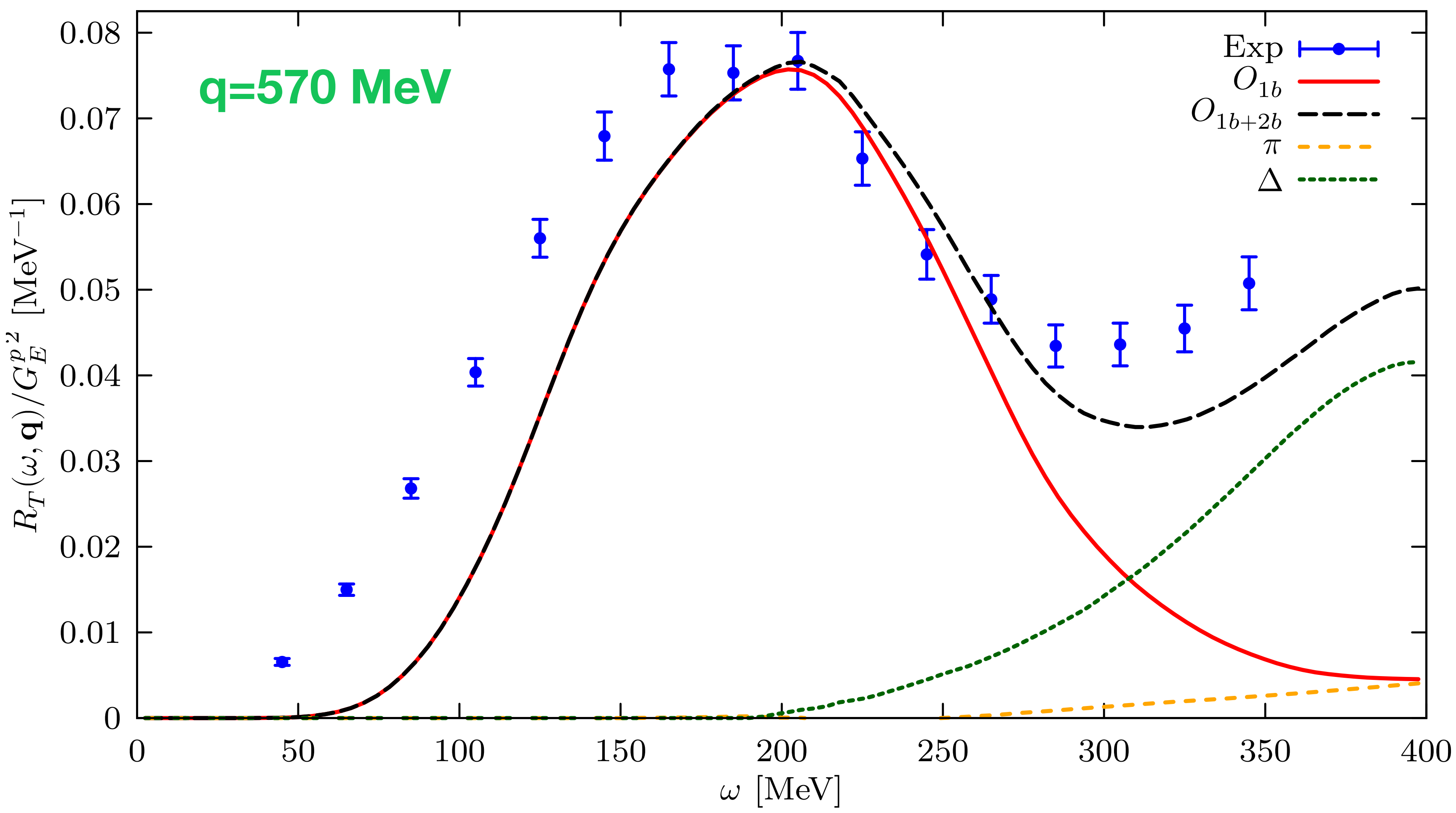
$$\frac{d\sigma_{IA}}{d\Omega_{e'} dE_{e'}} = \int d^3p\, dE\, P(\mathbf{p}, E) \left[Z \frac{d\sigma_{ep}}{d\Omega_{e'} dE_{e'}} + (A - Z) \frac{d\sigma_{en}}{d\Omega_{e'} dE_{e'}} \right]$$

Spectral function approach

With Noemi, to use relativistic MEC and realistic description of the nuclear ground state we have extended the factorization scheme to account for two-nucleon emission amplitude

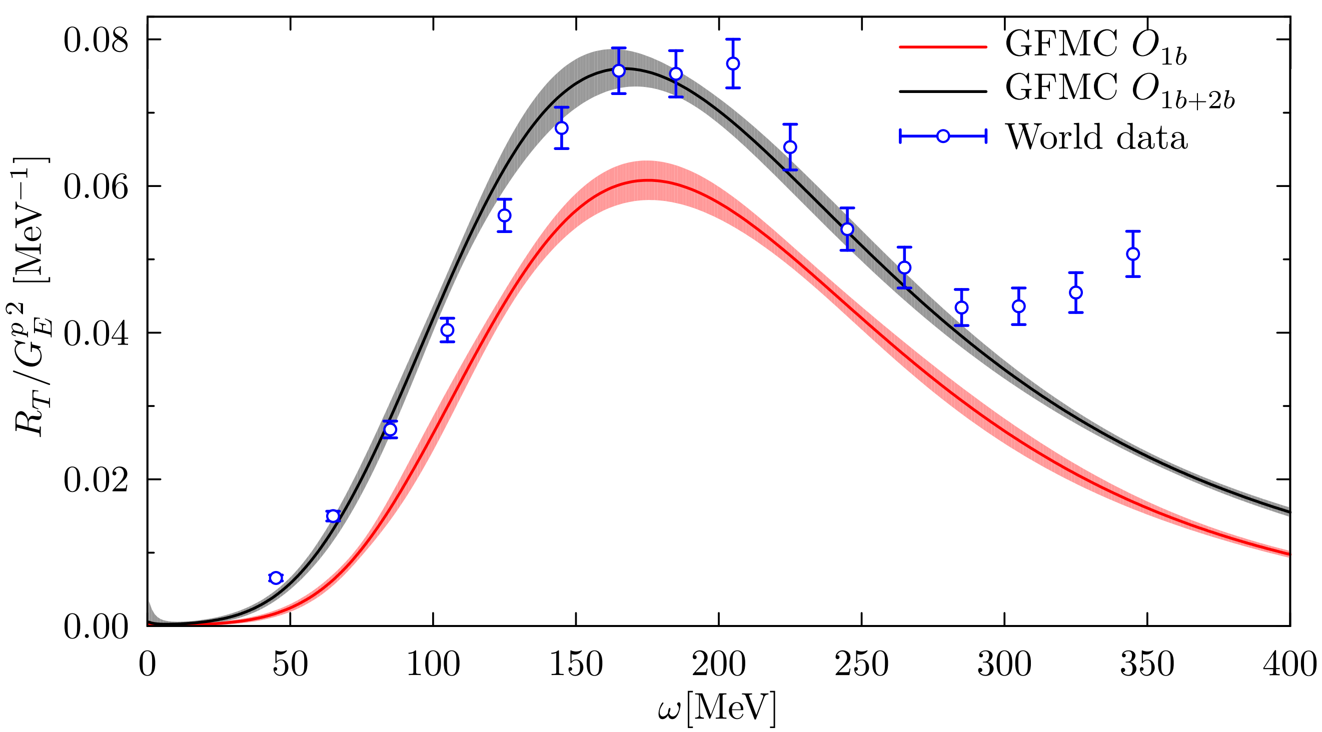
$$|\Psi_f\rangle \rightarrow |\mathbf{p}\mathbf{p}'\rangle \otimes |\Psi_{\tilde{f}}\rangle_{A-2}$$

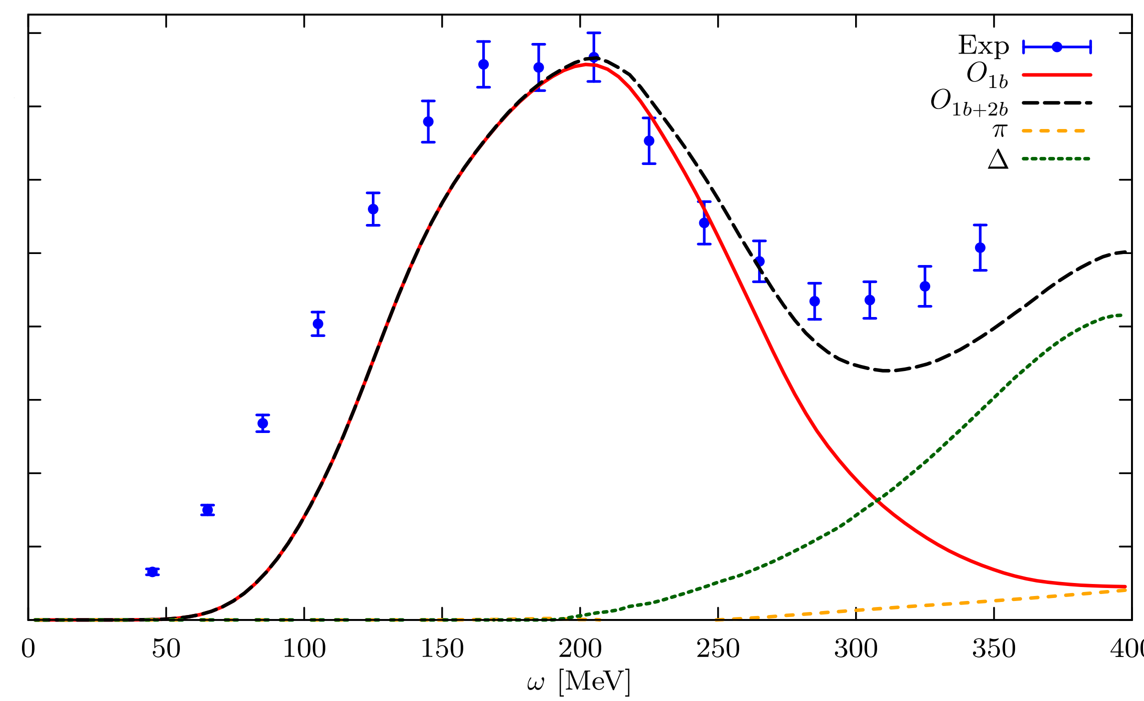
^{12}C calculations indicate a sizable enhancement of the electromagnetic transverse response



Spectral function vs GFMC

As pointed out by Noemi, we have some problems here





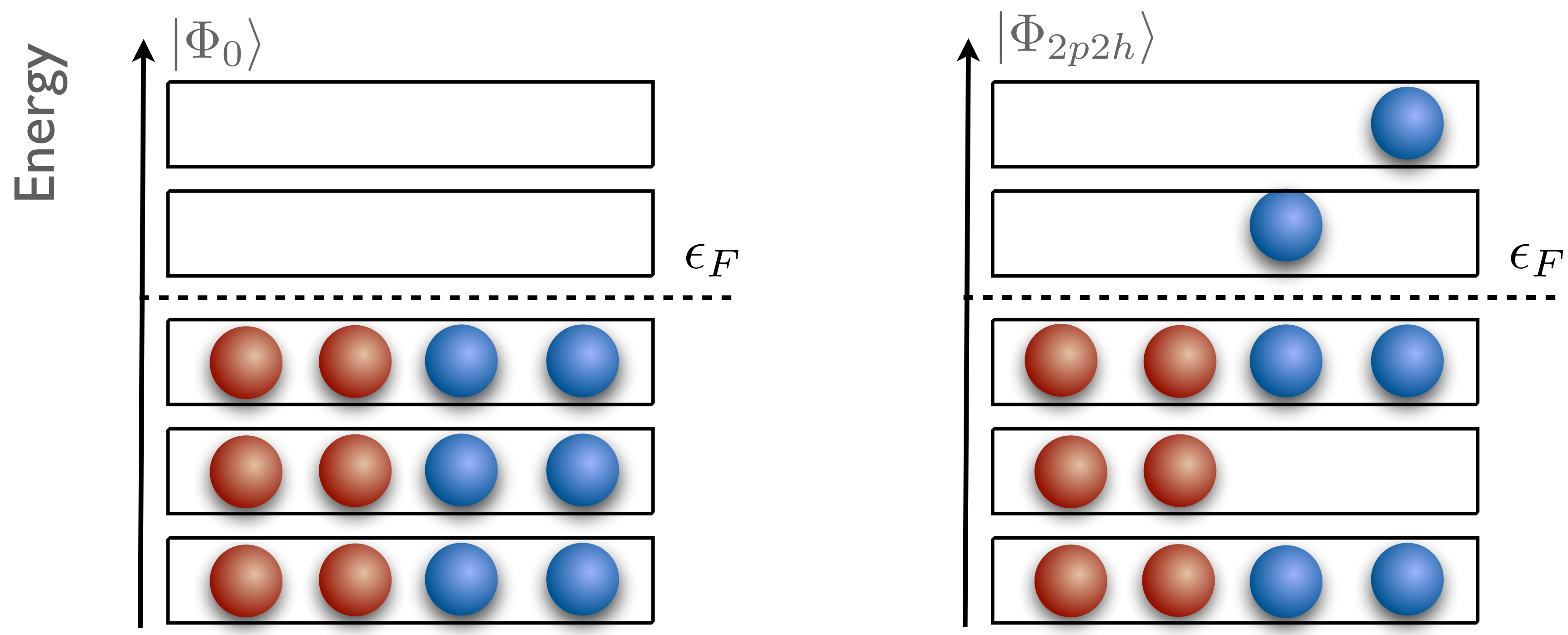
This discrepancy can be ascribed to

- Differences in the two-nucleon currents employed in the two cases
- The non relativistic nature of the GFMC calculations
- Interference between amplitudes involving the one- and two-body currents and 1p1h final states

1p-1h final states

- One-particle one-hole, two-particle two-hole states are definition dependent

$$|\Psi_0\rangle \approx \mathcal{F}|\Phi_0\rangle = \sum_{n=1}^{N>>1} |\Phi_{np,nh}\rangle$$



- Only n-particle n-holes correlated states are asymptotic states of the hamiltonian, hence observables, in principle.

- Two-particle two-hole correlated state correspond to a linear combination of n-particle n-hole mean field states

Adelchi's 1997 PRC

- The enhancement in the quasi elastic peak is surprising, but NOT NEW

PHYSICAL REVIEW C VOLUME 55, NUMBER 1 JANUARY 1997

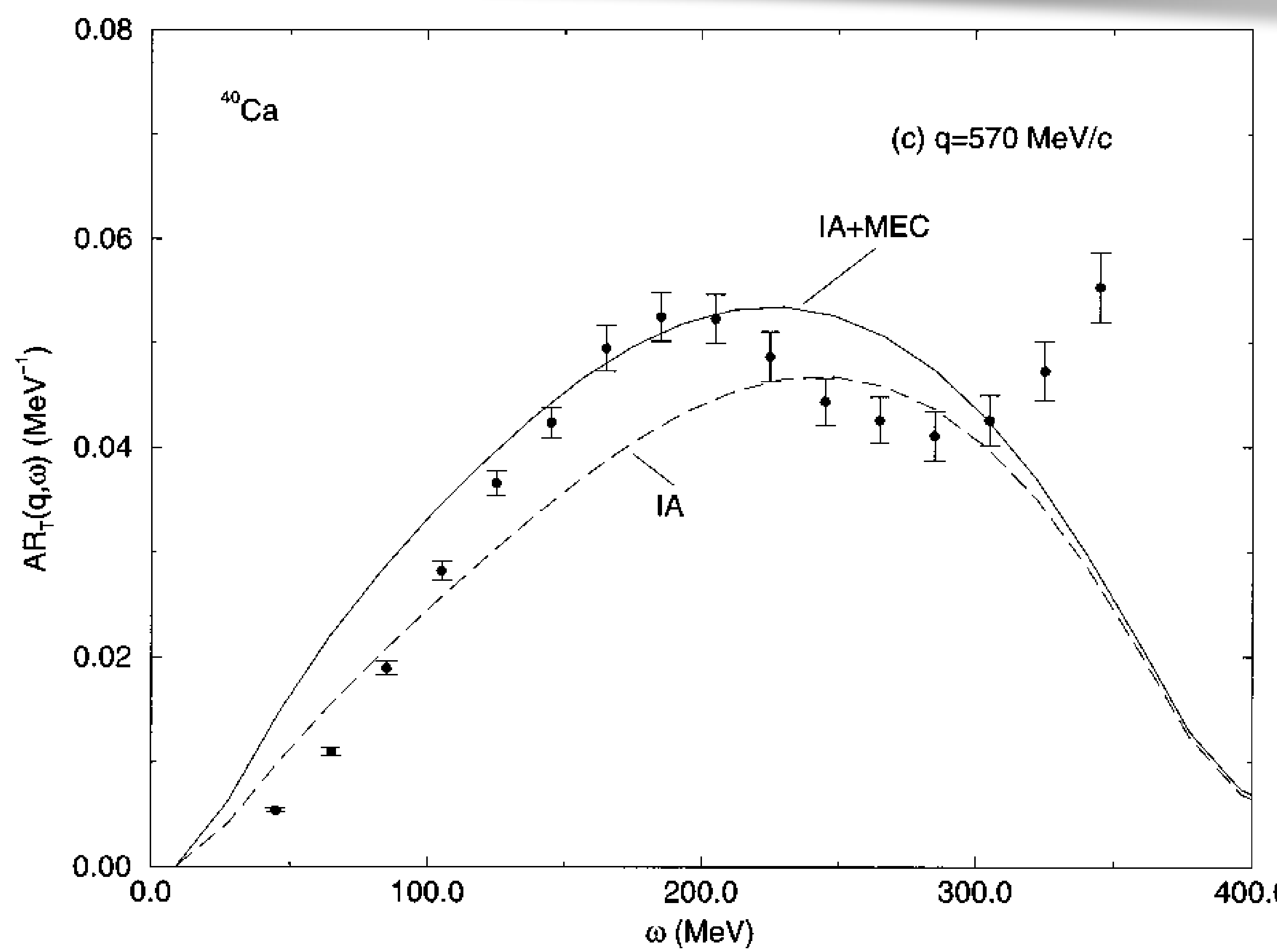
Inclusive transverse response of nuclear matter

Adelchi Fabrocini

- Back in 1997 Adelchi found a significant enhancement of the transverse response function from two-body current .

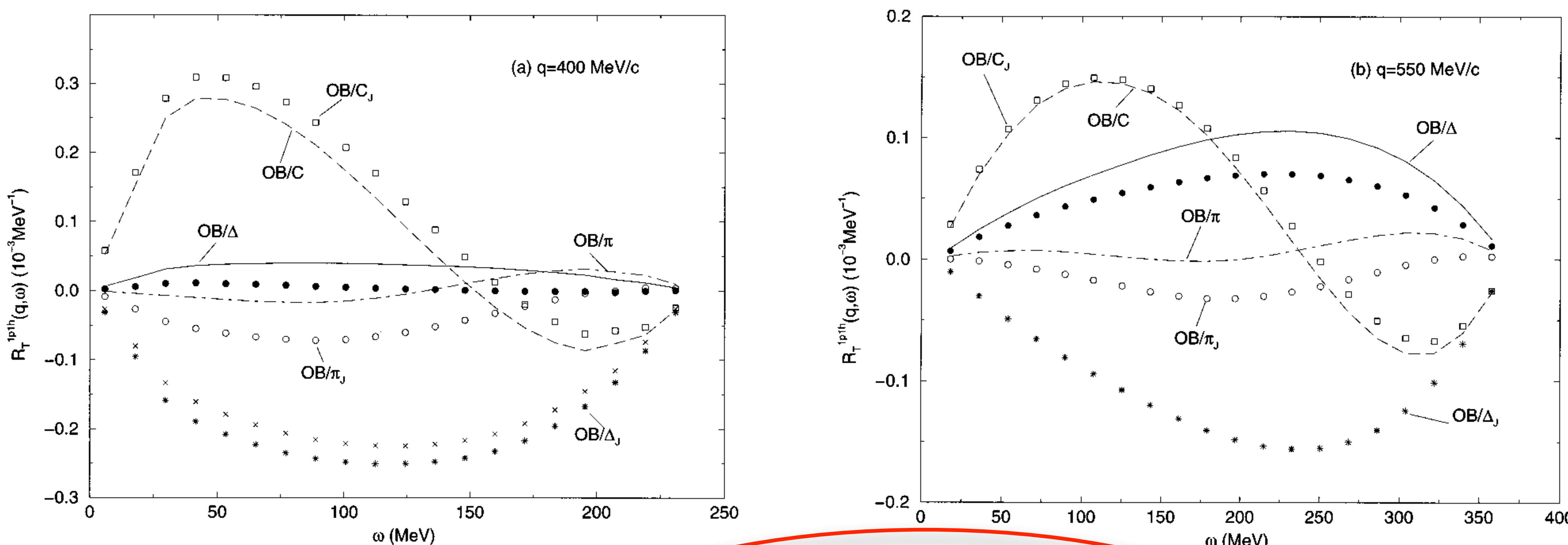
- This enhancement, in the quasielastic peak region, is due to correlated one-particle one-hole final states.

$$R_T^{1p1h}(q,\omega)=\frac{1}{A}\sum_{ph}|\langle 0|\mathbf{j}(\mathbf{q})|\mathbf{ph}\rangle|^2\delta(\omega-e_p+e_h)$$



Adelchi’s 1997 PRC

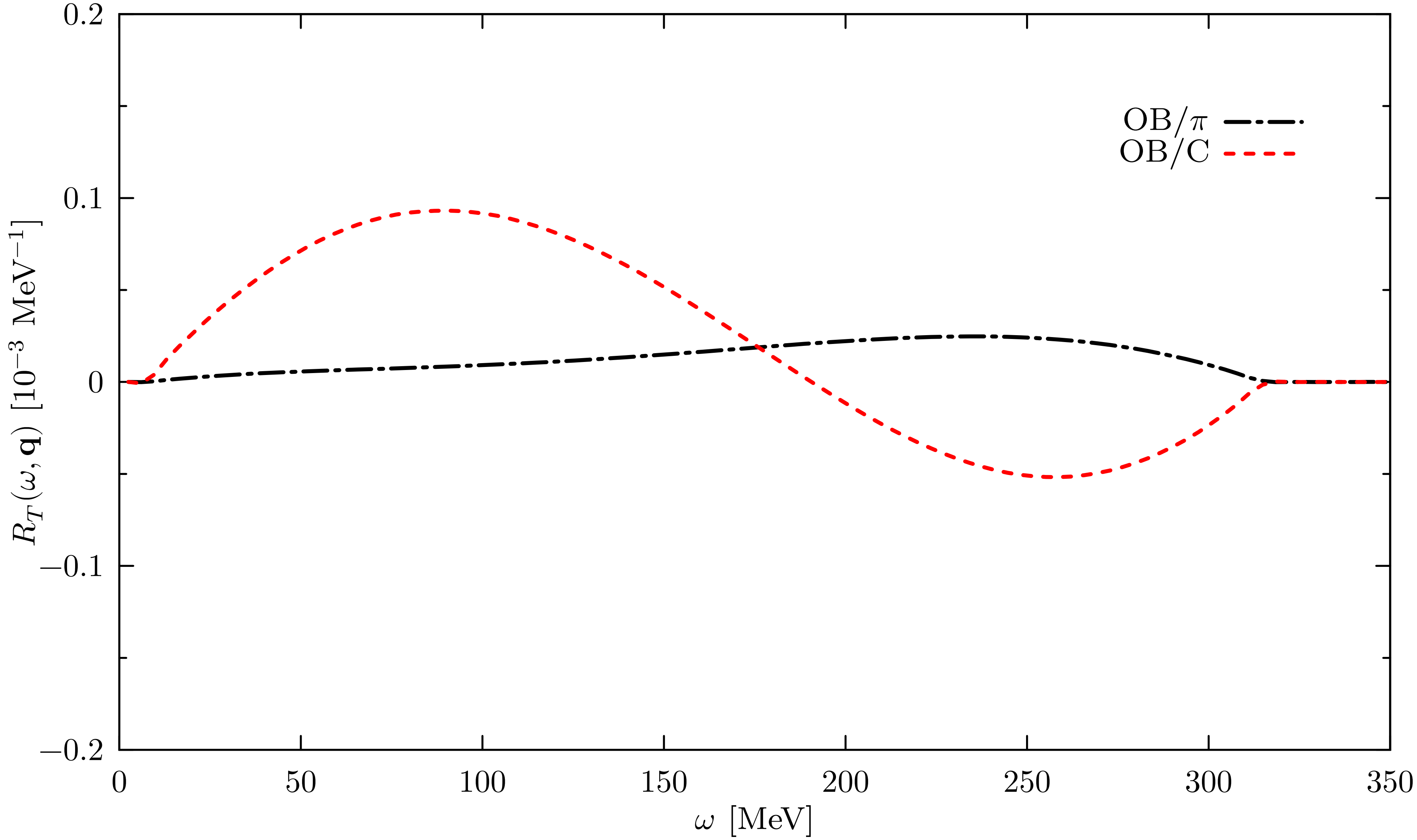
- Adelchi showed that the tensor-isospin component of the correlation is the leading factor responsible for such a behavior.



The introduction of tensor-isospin-dependent correlations drastically changes this picture. The Δ contribution is largely modified, as it becomes positive and increasing with the momentum transfer. As a result, MEC’s produce an extra strength (10–20 %) in the QE peak region. This is in agreement with exact GFMC calculations in light nuclei.

Adelchi's 1997 PRC

- We are including this reaction mechanism within the spectral function formalism



- This could potentially be seen in (e,e',p) experiments, including the one of Omar on ^{40}Ar!

Conclusions

- Adelchi made key contributions in the study of the structure and of the response functions of strongly interacting nuclear systems.

  ✴ He is one of the three (crazy) people who embarked in the calculation of the nuclear matter spectral function within CBF

  ✴ He computed the spin longitudinal and transverse responses of infinite nuclear matter, using the full FHNC/SOC machinery

  ✴ He performed a full calculation of the electromagnetic longitudinal and transverse response functions, including meson-exchange currents

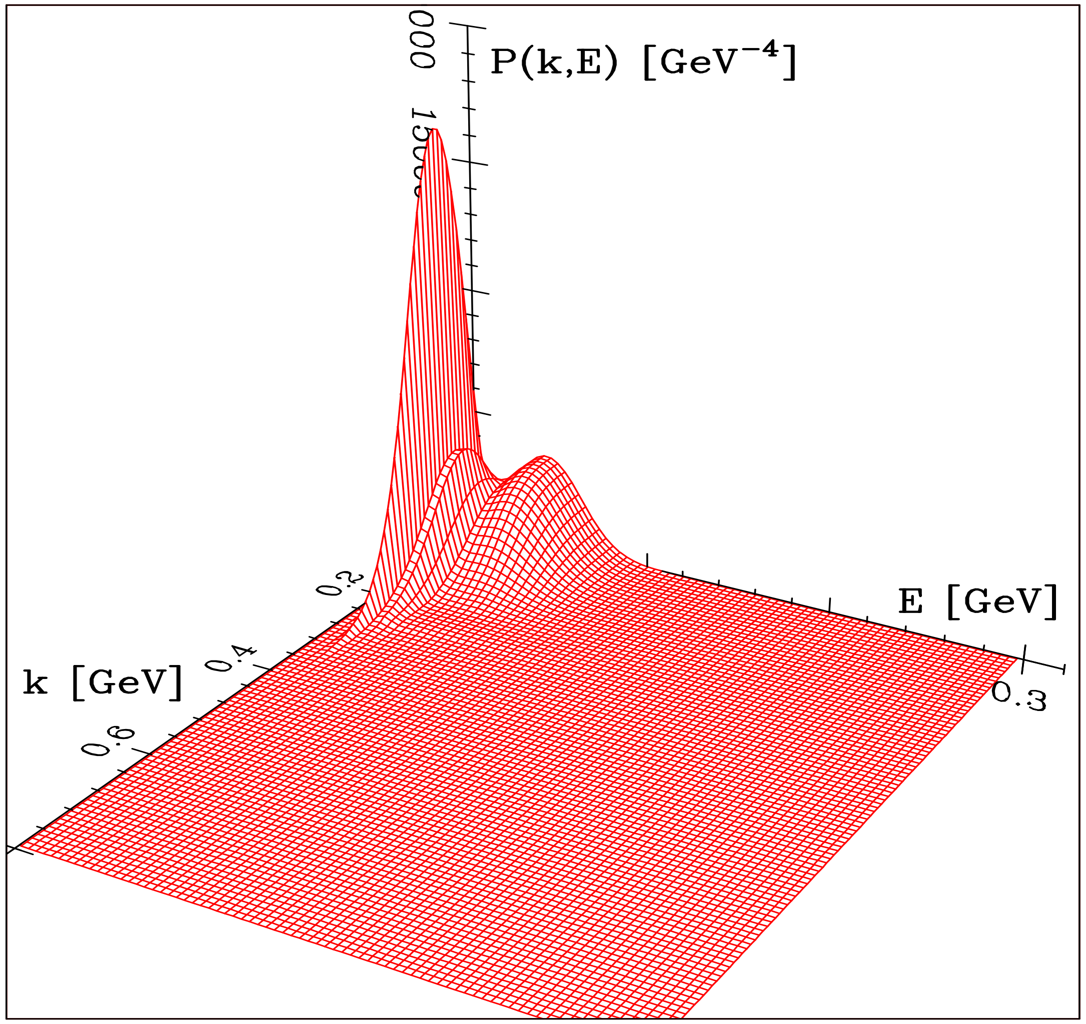
- I really enjoy Adelchi's papers and I would like to point out a recommendation

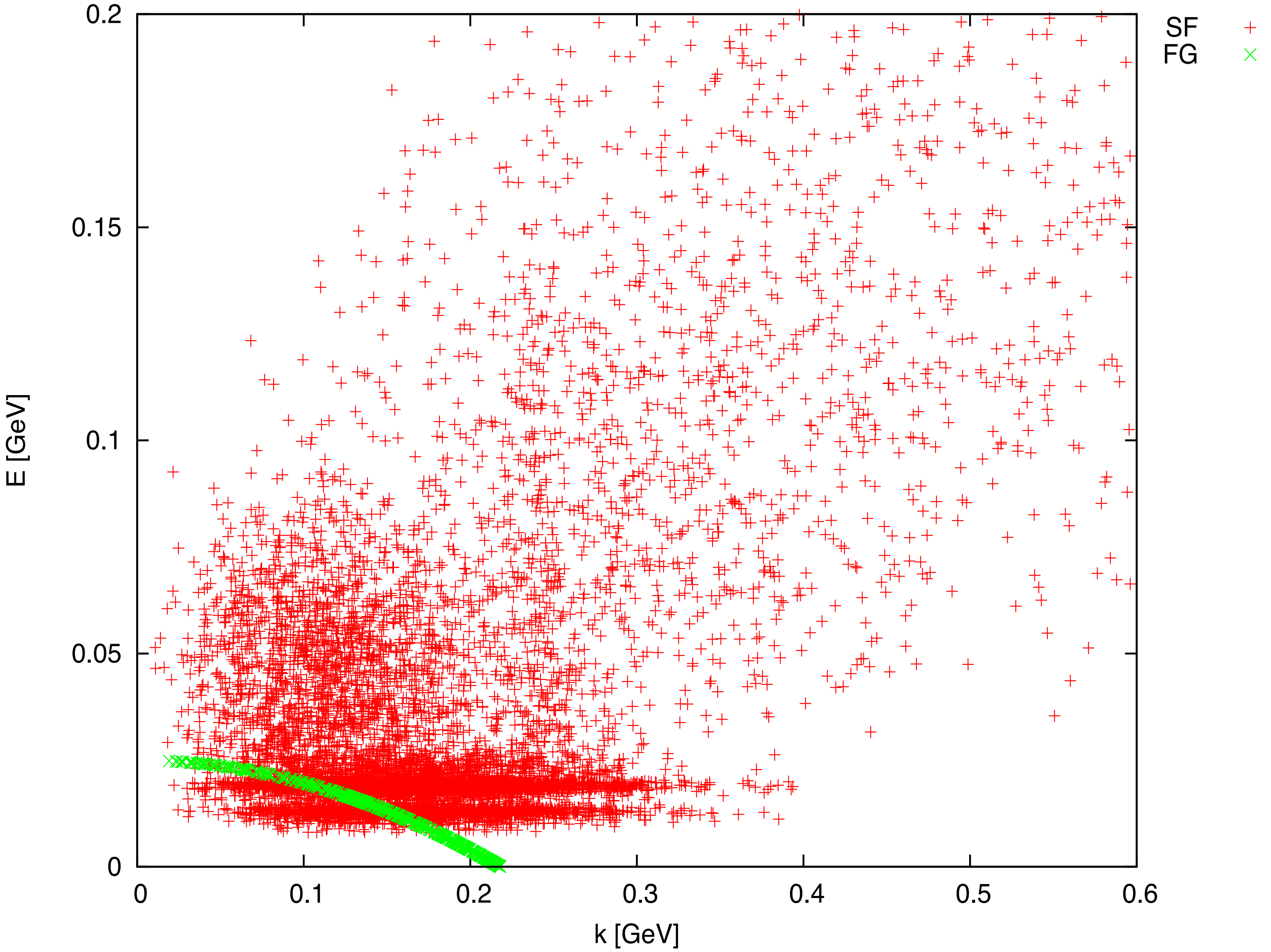
> We have tried to minimize the possibility of programming errors by computing most of the terms with two independently written codes, and carefully comparing results to search for and repair discrepancies.

Thank you

IA: Spectral function approach

The spectral function and the factorization of the nuclear transition matrix elements allows to combine a fully relativistic description of the electromagnetic interaction with an accurate treatment of nuclear dynamics





Constraining the spectral function with QMC

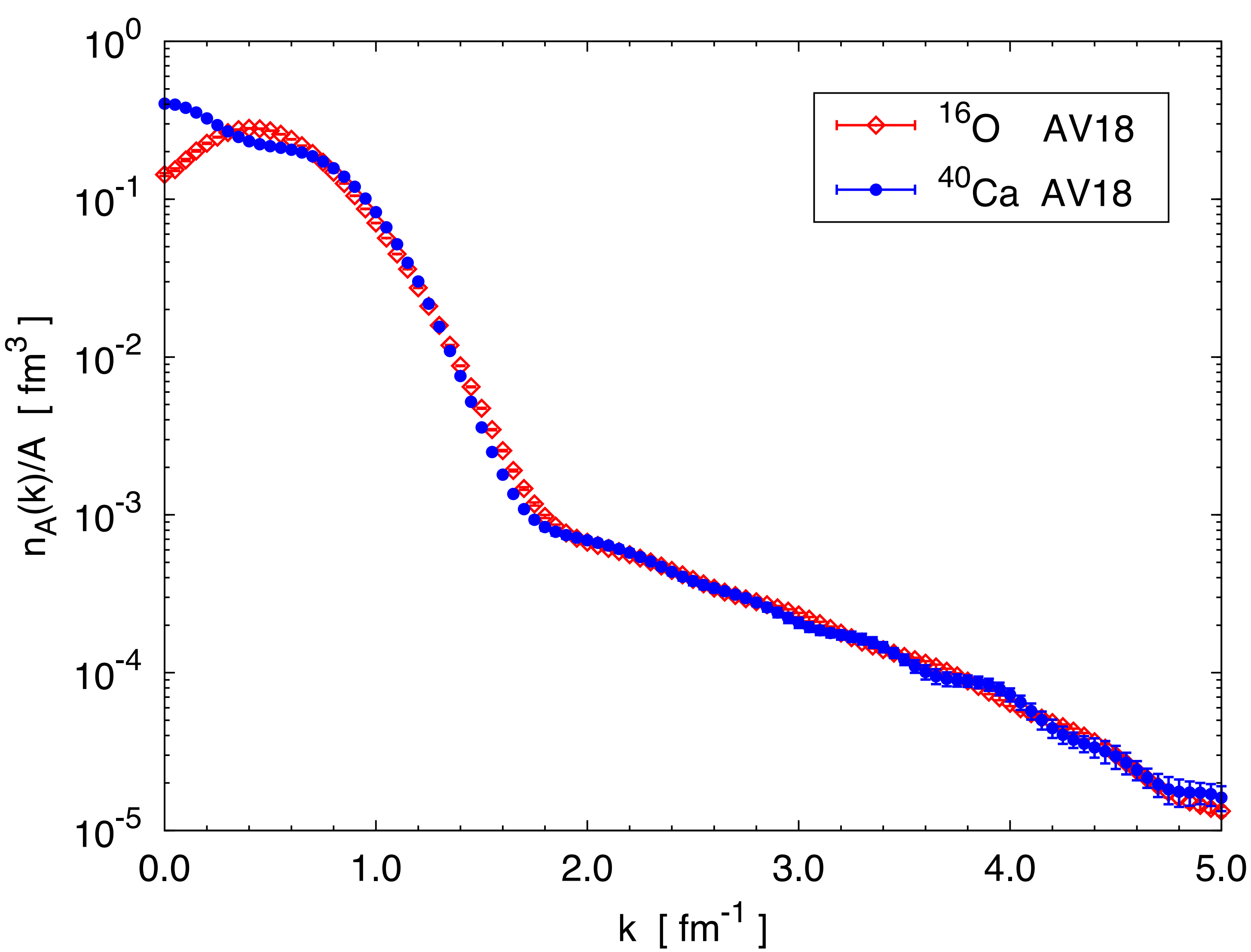
The sum rule of the spectral function corresponds to the momentum distribution

$$\int dEP(\mathbf{k}, E) = n(\mathbf{k})$$

- Within Quantum Monte Carlo, we have already computed the momentum distribution of nuclei as large as ^{16}O and ^{40}Ca.

- The energy weighted sum rules of the spectral function can also be computed within cluster variational Monte Carlo

$$\int dEEP(\mathbf{k}, E) = \langle \Psi_0 | a^\dagger_{\mathbf{k}} [H, a_{\mathbf{k}}] | \Psi_0 \rangle$$



Work led by Diego Lonardoni

Path forward

The results we obtained are very nice, but limited and not completely satisfactory

- The continuity equation only constraints the longitudinal components of the current
- The transverse component and the axial terms are phenomenological (the coupling constant is fitted on the tritium beta-decay)
- Two- and three- body forces not fully consistent

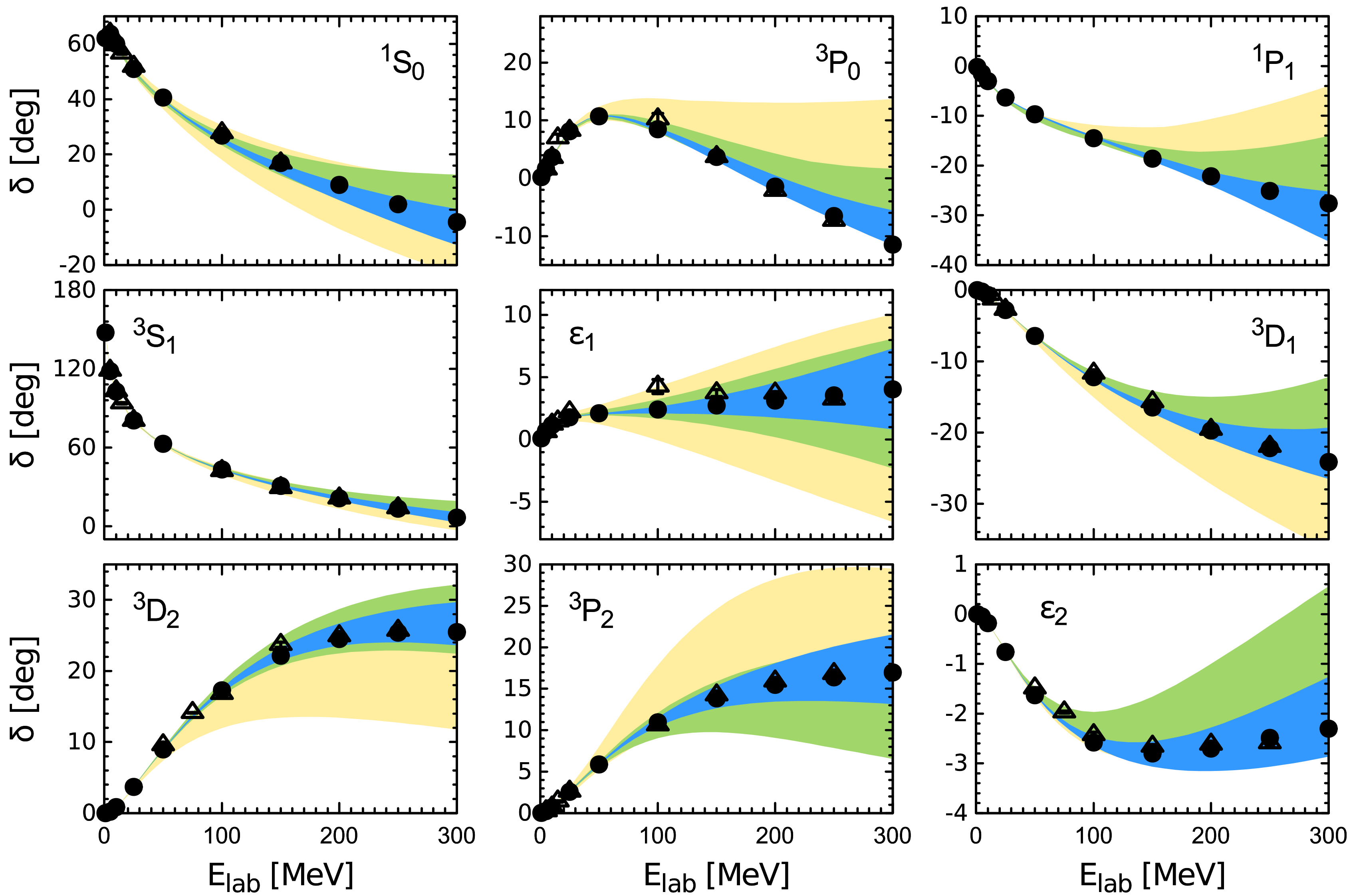
Within this framework, the theoretical error arising from modeling the nuclear dynamics cannot be properly assessed!

Chiral effective field theory (χEFT) has witnessed much progress during the two decades since the pioneering papers by Weinberg (1990, 1991, 1992)

In χEFT, the symmetries of quantum chromodynamics (QCD), in particular its approximate chiral symmetry, are employed to systematically constrain classes of Lagrangians describing the interactions of baryons with pions as well as the interactions of these hadrons with electroweak fields

Chiral EFT

χEFT provides a framework to derive consistent many-body forces and currents and the tools to rigorously estimate their uncertainties, along with a systematic prescription for reducing them.



Epelbaum et al., arXiv:1412.0142

QMC allows to propagate the theoretical uncertainty arising from the nuclear interaction to the response functions

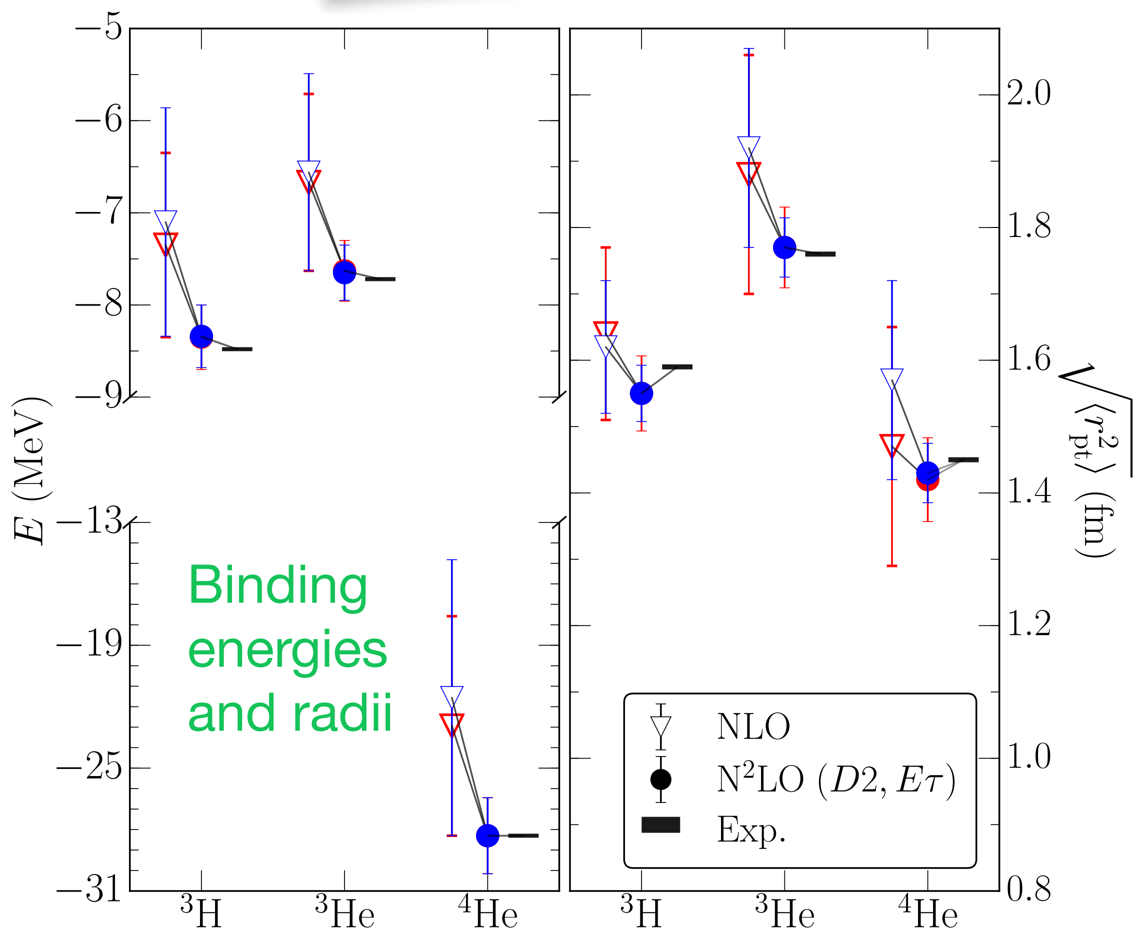
Chiral EFT

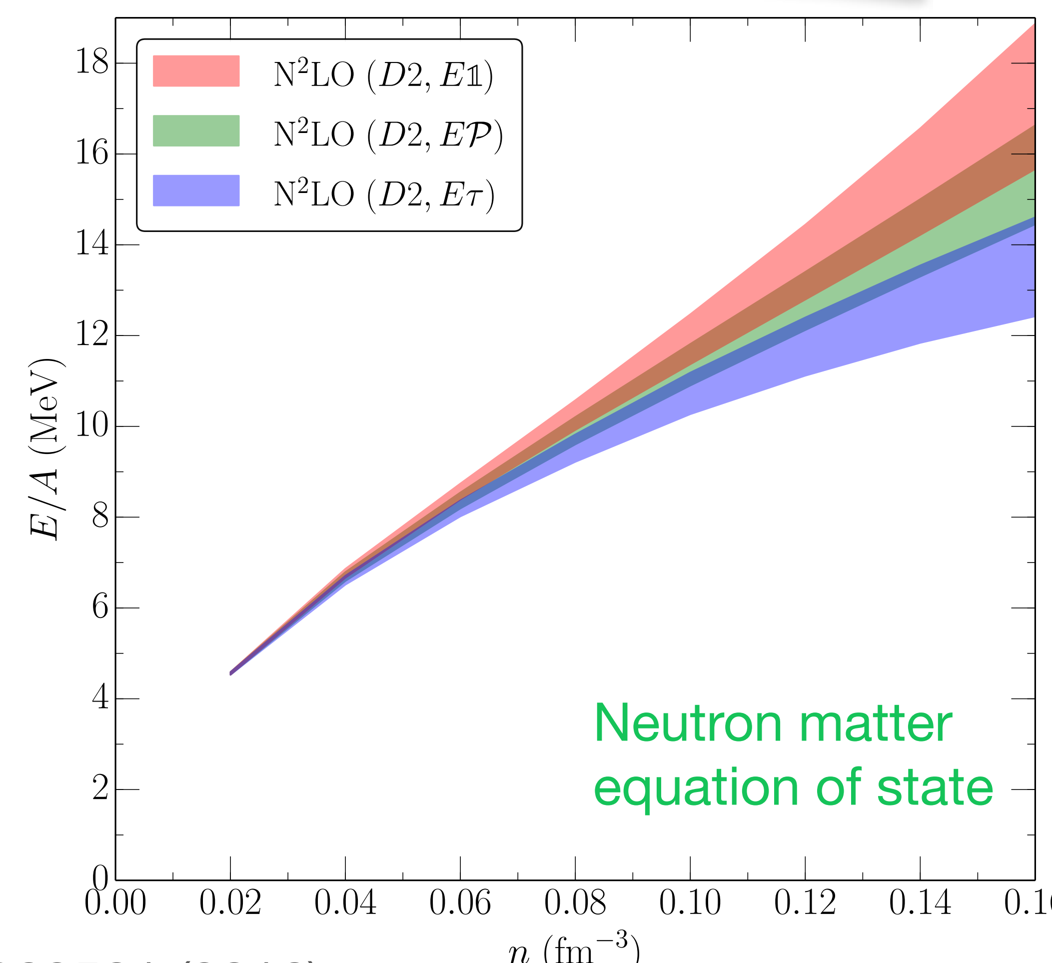
Recently chiral nuclear interactions, including the Δ degrees of freedom have been developed

PHYSICAL REVIEW C **91**, 024003 (2015)

Minimally nonlocal nucleon-nucleon potentials with chiral two-pion exchange including Δ resonances

M. Piarulli,[1] L. Girlanda,[2,3] R. Schiavilla,[1,4] R. Navarro Pérez,[5] J. E. Amaro,[5] and E. Ruiz Arriola[5]





J. Lynn et al. PRL 116, 062501 (2016)

Maximum entropy algorithm

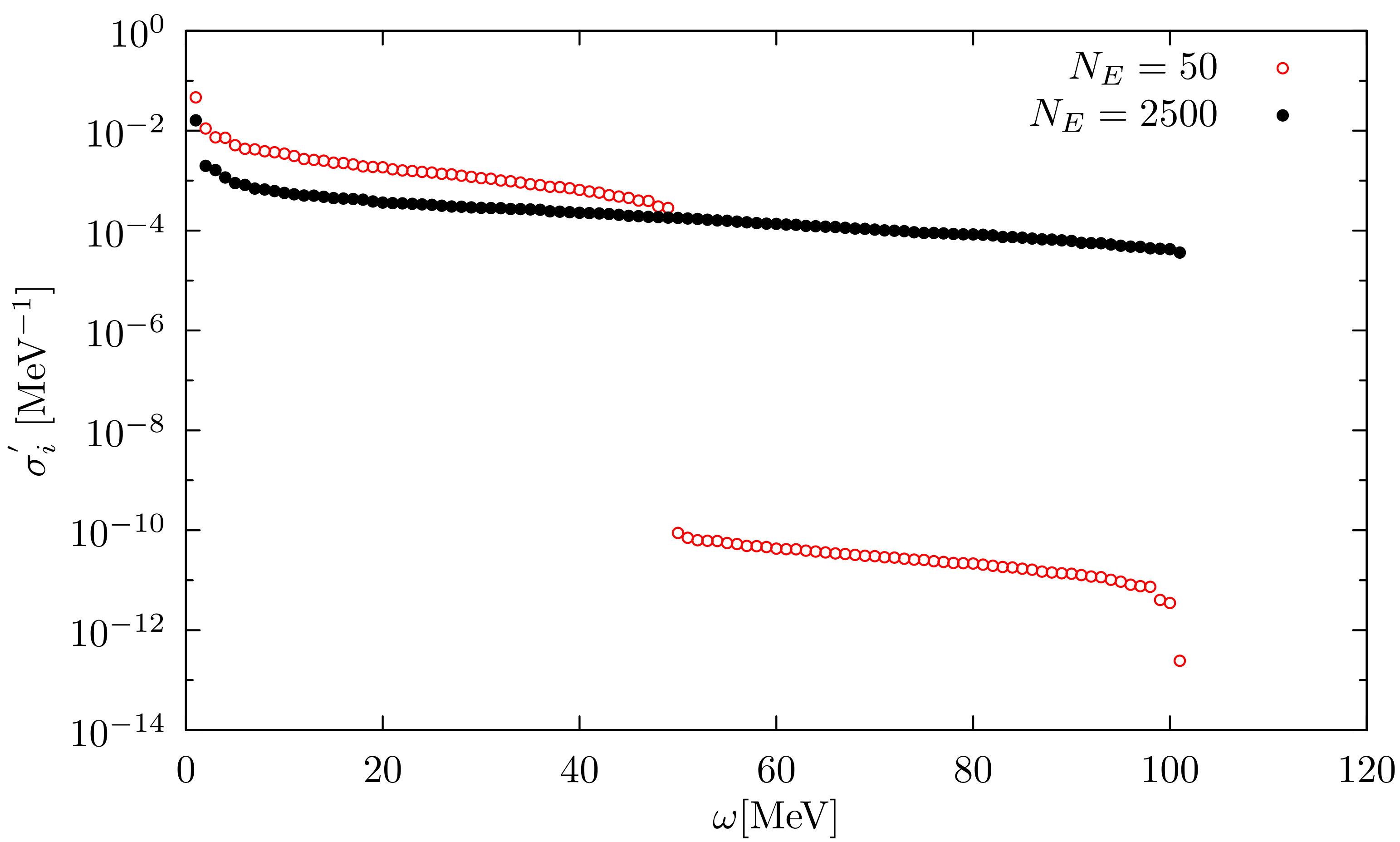
We estimate the mean and the covariance matrix from N_E Euclidean responses

$$\bar{E}(\tau_i) = \frac{1}{N}\sum_n E^n(\tau_i) \qquad C(\tau_i,\tau_j) = \frac{1}{N(N-1)}\sum_n (\bar{E}^n(\tau_i) - E^n(\tau_i))(\bar{E}^n(\tau_j) - E^n(\tau_j))$$

- The covariance matrix in general is NOT diagonal, and it is convenient to diagonalize it

$$(\mathbf{U}^{-1}\mathbf{C}\mathbf{U})_{ij} = \sigma_i'^2\delta_{ij}$$

- If N is not sufficiently large, the spectrum of the covariance eigenvalues becomes pathological.



Maximum entropy algorithm

- The likelihood is defined in terms of the covariance matrix

$$\chi^2 = \sum_{i,j=1}^{N_\tau} \left(\overline{E}_i - E_i\right) \left(C^{-1}\right)_{ij} \left(\overline{E}_j - E_j\right) \qquad E_i = \sum_{j=1}^{N_\omega} K_{ij}\, R_j$$

- We rotate both the data and the kernel in the diagonal representation of the covariance matrix

$$\mathbf{K}' = \mathbf{U}^{-1}\mathbf{K} \qquad \bar{\mathbf{E}}' = \mathbf{U}^{-1}\bar{\mathbf{E}} \qquad \leftrightarrow \qquad (\mathbf{U}^{-1}\mathbf{C}\mathbf{U})_{ij} = {\sigma'_i}^2\delta_{ij}$$

- The likelihood can be written in terms of the statistically independent measurements and the rotated kernel

$$\chi^2 = \frac{1}{N_\tau}\sum_i \frac{(\sum_j K'_{ij}R_j - \bar{E}'_i)^2}{{\sigma'_i}^2}$$

Maximum entropy algorithm

Maximum entropy approach can be justified on the basis of Bayesian inference. The best solution will be the one that maximizes the conditional probability

$$Pr[R|\bar{E}] = \frac{Pr[\bar{E}|R]\, Pr[R]}{Pr[\bar{E}]}$$

- The evidence is merely a normalization constant

$$Pr[\bar{E}] = \int \mathcal{D}R\, Pr[\bar{E}|R]\, Pr[R]$$

- When the number of measurements becomes large, the asymptotic limit of the likelihood function is

$$Pr[\bar{E}|R] = \frac{1}{Z_1} e^{-L[R]} = \frac{1}{Z_1} e^{-\frac{1}{2}\chi^2[R]} \qquad \chi^2 = \frac{1}{N_\tau} \sum_i \frac{(\sum_j K'_{ij} R_j - \bar{E}'_i)^2}{\sigma_i'^2}$$

Limiting ourselves to the minimization of the χ^2, we implicitly make the assumption that the prior probability is important or unknown.

Maximum entropy algorithm

Since the response function is nonnegative and normalizable, it can be interpreted as a probability distribution function.

The principle of maximum entropy states that the values of a probability function are to be assigned by maximizing the entropy expression

$$S[R] \equiv -\int d\omega (R(\omega) - D(\omega) - R(\omega)\ln[R(\omega)/D(\omega)]) \quad \longleftrightarrow \quad D(\omega)\text{: Default model}$$

The prior probability then reads

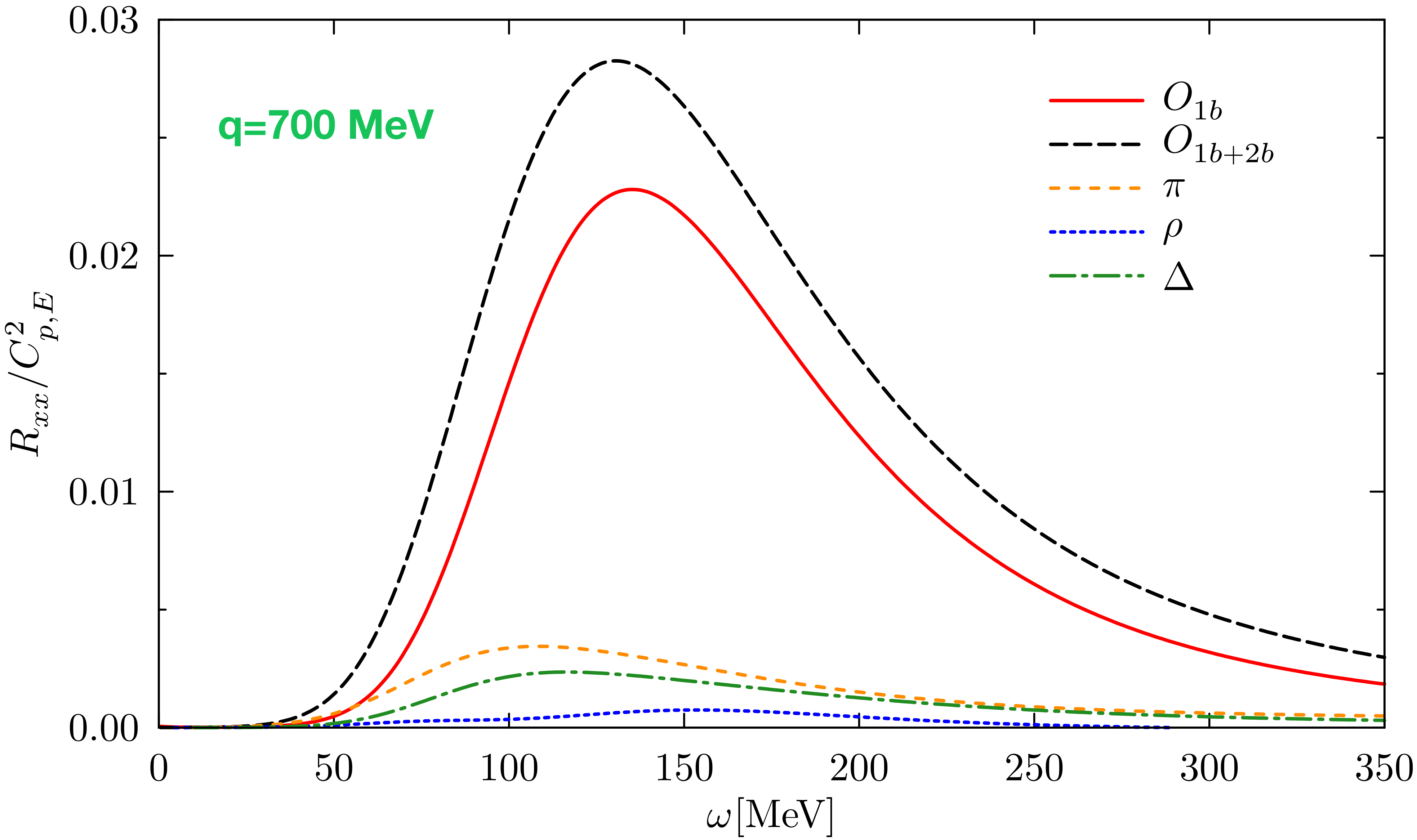
$$Pr[R] = \frac{1}{Z_2} e^{\alpha S[R]}$$

and the posterior probability can be rewritten as

$$Pr[R|\bar{E}] = \frac{e^{-Q[R]}}{Z_1 Z_2 Pr[\bar{E}]} \quad \longleftrightarrow \quad Q[R] \equiv \frac{1}{2}\chi^2[R] - \underbrace{\alpha}_{\text{Regularization parameter}} S[R]$$

^{4}He electromagnetic response

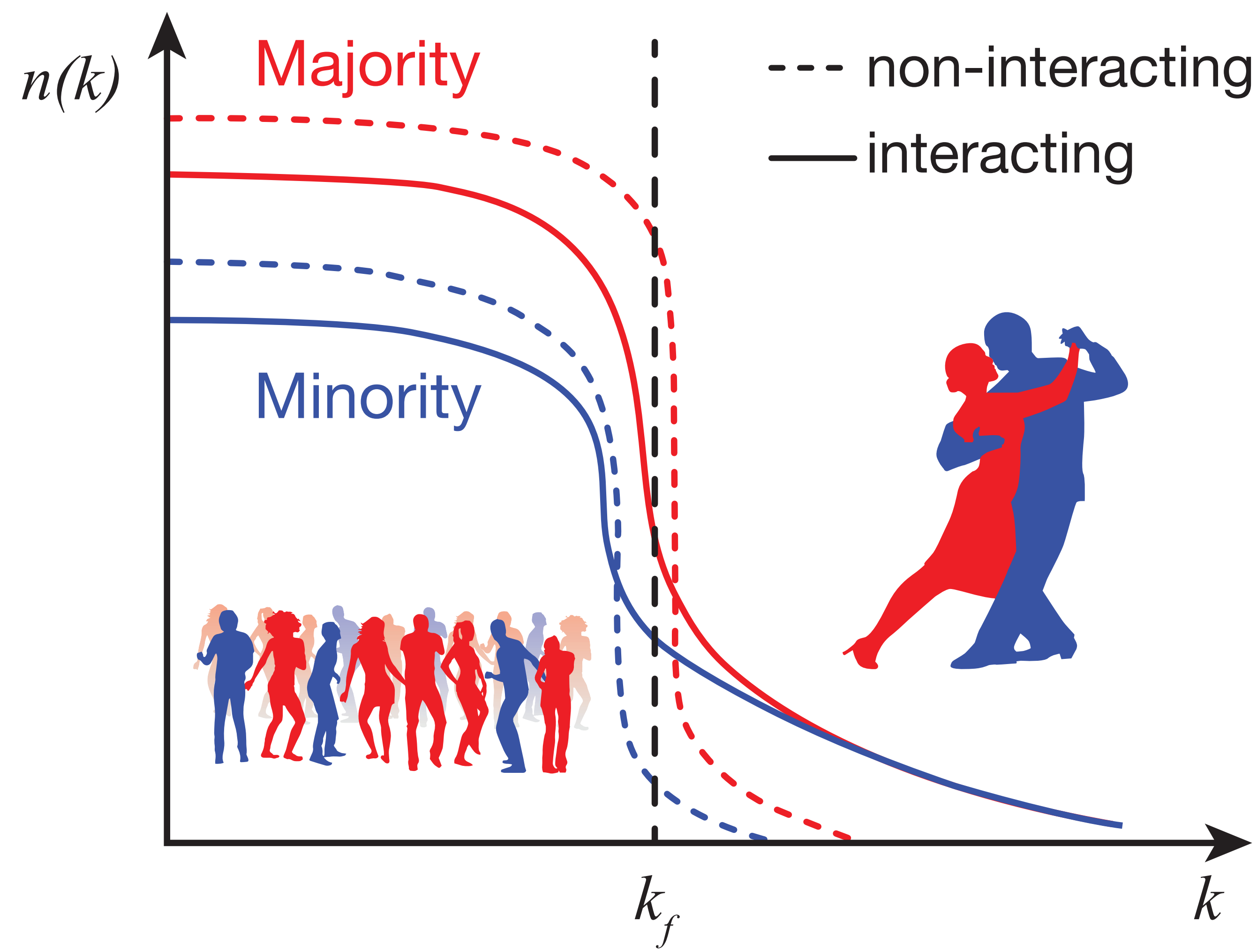
The enhancement is driven by process involving one-pion exchange and the excitation of the Delta degrees of freedom

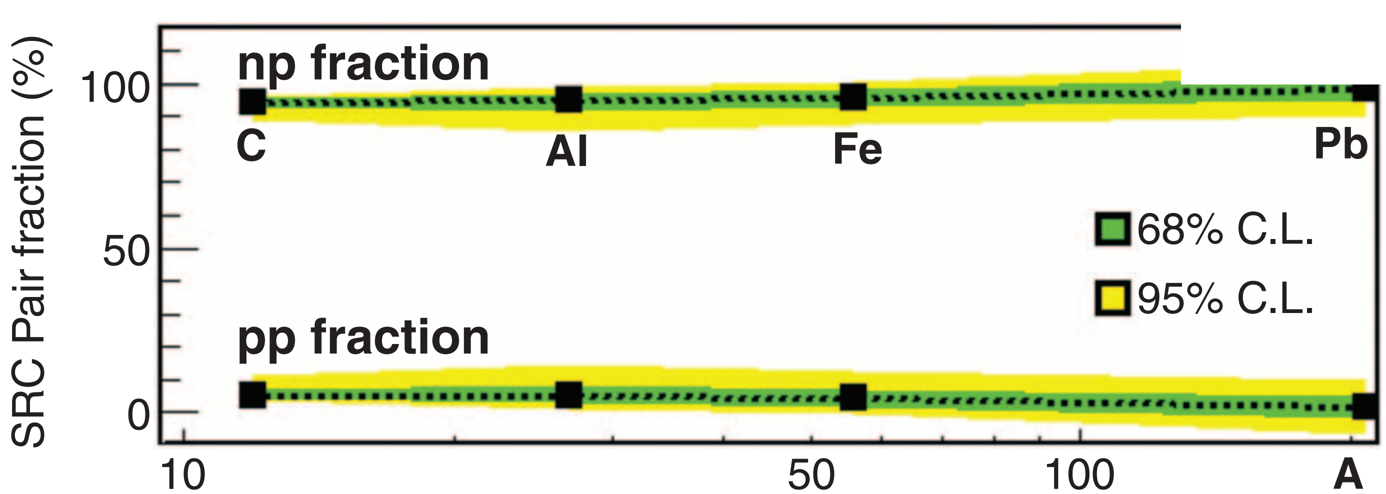


Nuclear correlations

- Nuclear interaction creates short-range correlated pairs of unlike fermions with large relative momentum and pushes fermions from low momenta to high momenta creating a "high-momentum tail."

- Like in a dance party with a majority of girls, where boy-girl interactions will make the average boy dance more than the average girl





- Even in neutron-rich nuclei, protons have a greater probability than neutrons to have momentum larger than the Fermi momentum.

Science 346, 614 (2014)