Progress Report on F_L and Diffractive Physics Program to Measure Gluon Distributions in Nuclei

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BROOKHAVEN NATIONAL LABORATORY

The Big Questions

NSAC Long Range Plan ’07- Overarching Question:

What is the role of gluons and gluon self-interactions in nucleons and nuclei?

Studying gluons implies measurements of:

1. gluon momentum distributions $G(x,Q^2)$
2. gluon space time distribution

Incremental in ep, transformational in eA

Main Focus (Discovery Potential)

Establishment/Clarification of saturation and validity of CGC approach $\Rightarrow$ one of the fundamental outstanding problems in QCD

Gluon Distributions and Saturation

How to probe saturation? $G(x,Q^2)$ is not an observable!

Measurement
$\sigma(x, Q^2, A, t, W, \ldots)$

→ Structure Function F_2, F_L, F_2^D, F_L^D, Dipole $d\sigma/db$, ...

→ model dependent → $G(x,Q^2)$

Linear QCD Models
(DGLAP, BFKL)

Non-Linear QCD
Higher Twist,
saturation models,
CGC

⟺ comparison ⟺ $G(x,Q^2)$

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model dependent

$G(x,Q^2)$

comparison

Linear QCD Models
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Comparison (to constrain/reject models) requires

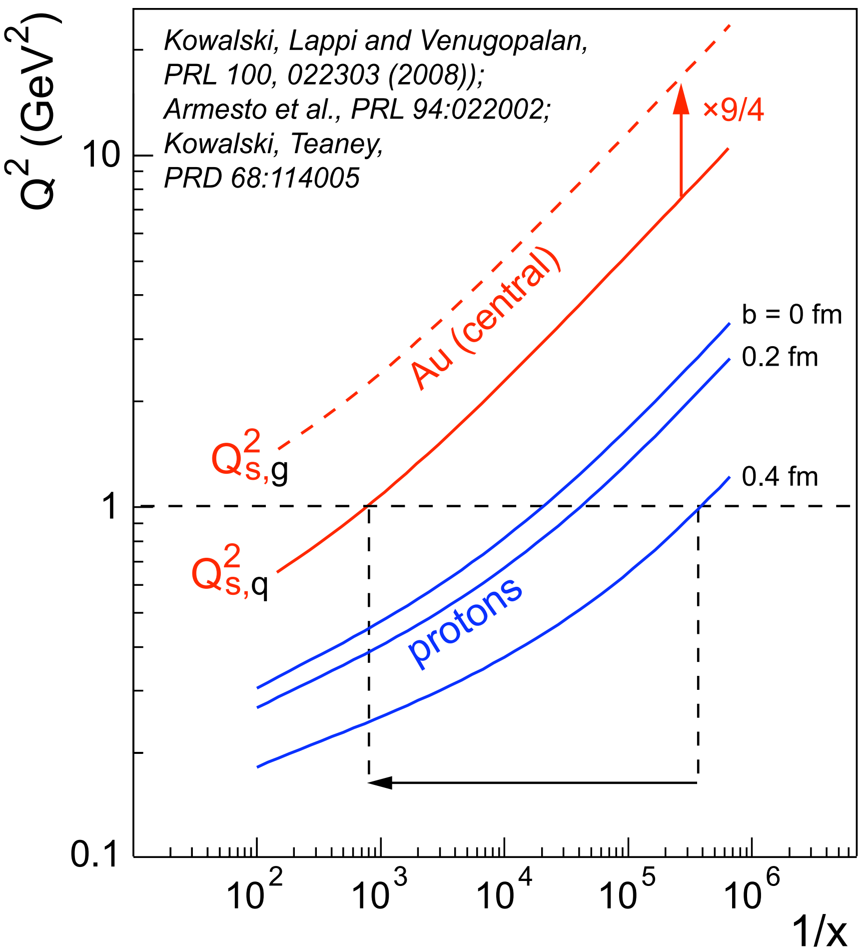
- “lever arm” in x, Q^2, A, ...
- complementary measurements (incl., semi-incl., excl., DIS & diffractive, varying probes, ...)

Measurements & Techniques

- Gluon Distribution $G(x,Q^2)$
  - Scaling violation in F_2: $\delta F_2/\delta \ln Q^2$
  - $F_L \sim xG(x,Q^2)$
  - 2+1 jet rates
  - Diffractive vector meson production ($[xG(x,Q^2)]^2$)
- Space-Time Distribution
  - Exclusive diffractive VM production (J/ψ, ϕ, ρ)
  - Deep Virtual Compton Scattering (nGPDs)
  - Structure functions for various mass numbers A and its impact parameter dependence

Ongoing studies (blue) | On To-Do List (green)

Saturation & Kinematic Range



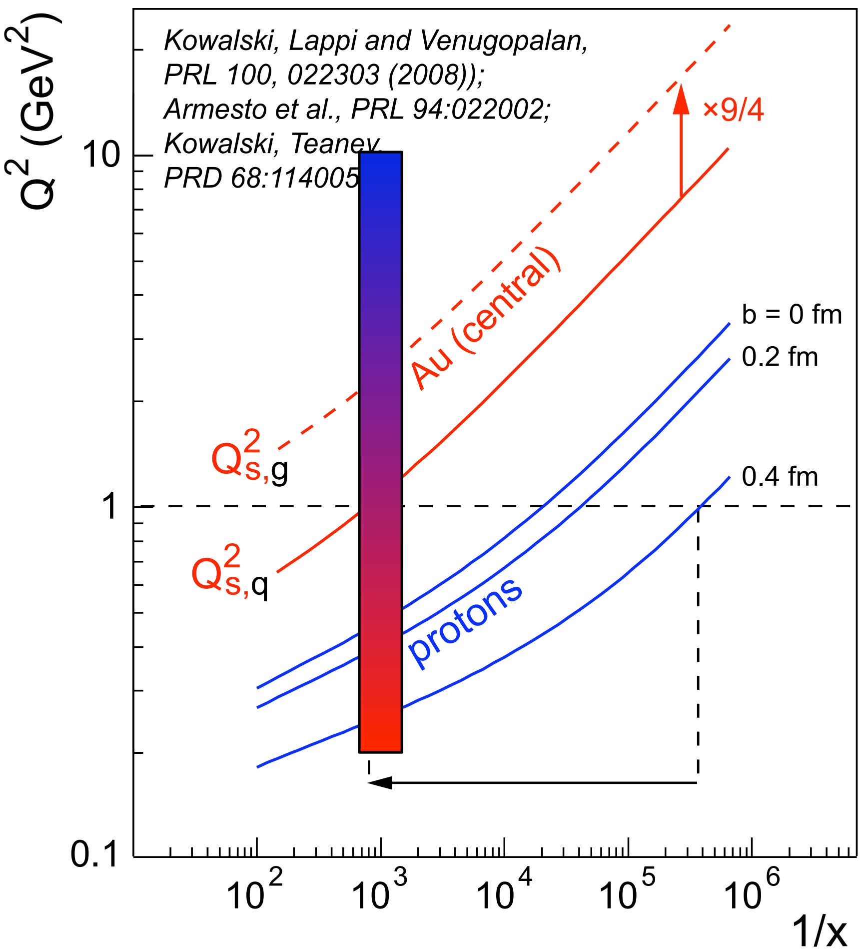
Nuclear Enhancement of Q_s

$$(Q_s^A)^2 \approx c Q_0^2 \left(\frac{A}{x} \right)^{1/3}$$

~6 for Au/U ⇒ at fix Q^2 translates into huge increase in x (~500)

pp, pA, AA: $Q_{s,g}$
DIS (ep, eA): $Q_{s,q}$

Saturation & Kinematic Range



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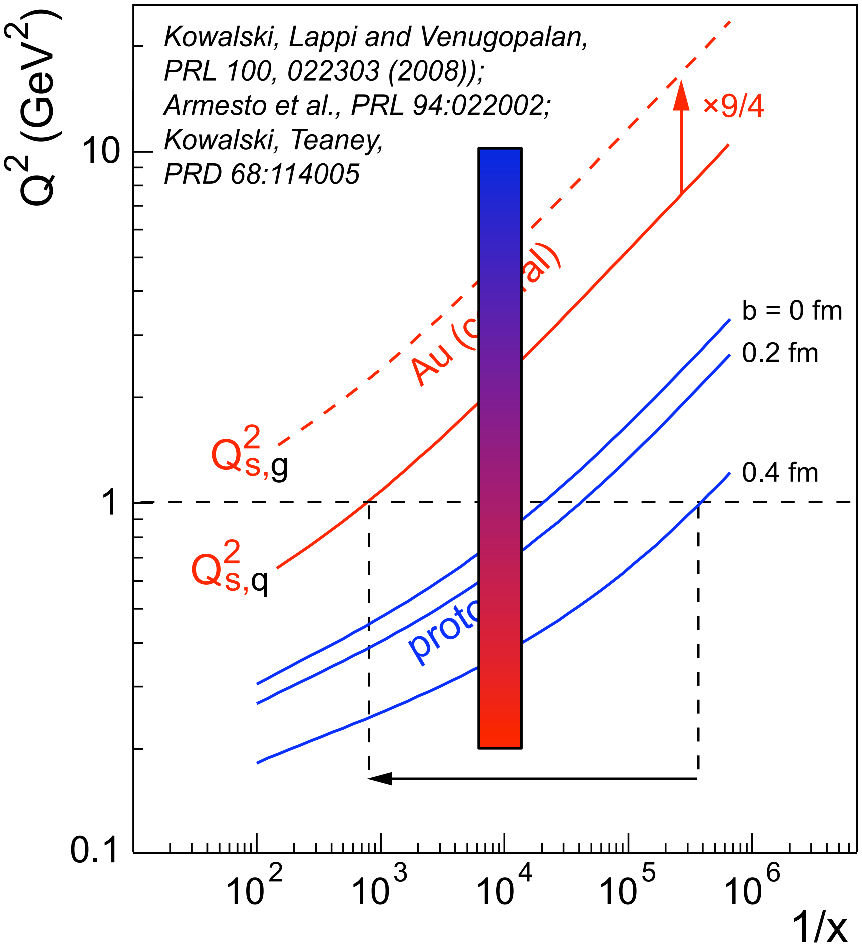
x, Q^2 kinematics:

$x = 10^{-3}$: $Q^2 = 0.2 \ldots 10\ \mathrm{GeV}^2$
$\sqrt{s} = 14 \ldots 100$ GeV

Saturation & Kinematic Range



$E_e + E_A$ (GeV)	√s (GeV)
4+100	40
10+100	63
20+100	89
30+100	110

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DIS (ep, eA): $Q_{s,q}$

x, Q^2 kinematics:

x = 10^{-3}: Q^2 = 0.2 ...10 GeV^2
√s = 14 ... 100 GeV

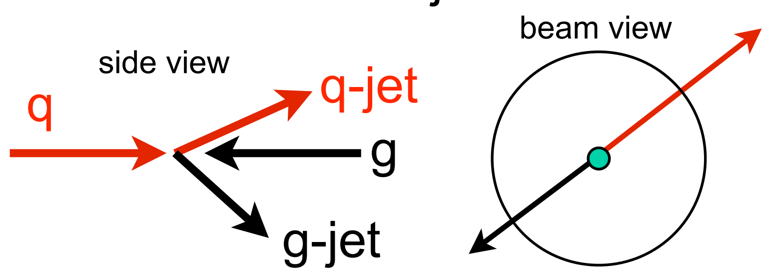
x = 10^{-4}: Q^2 = 0.2 ...10 GeV^2
√s = 45 ... 316 GeV

New Hints from RHIC: Saturation at $x=10^{-3}$?

Disappearance of angular correlations in Run 8 dAu data at forward rapidities (log x ~ 2.5 - 3)

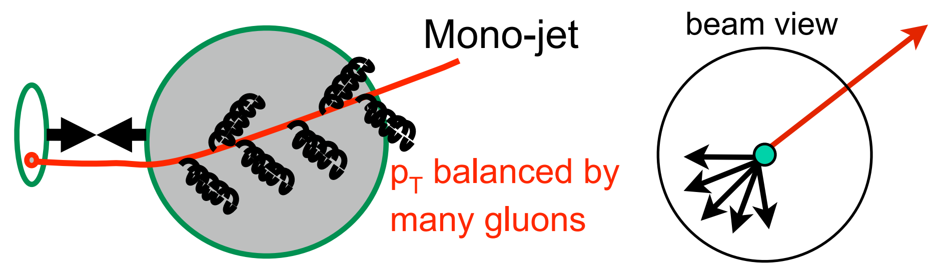
Low gluon density (pp):

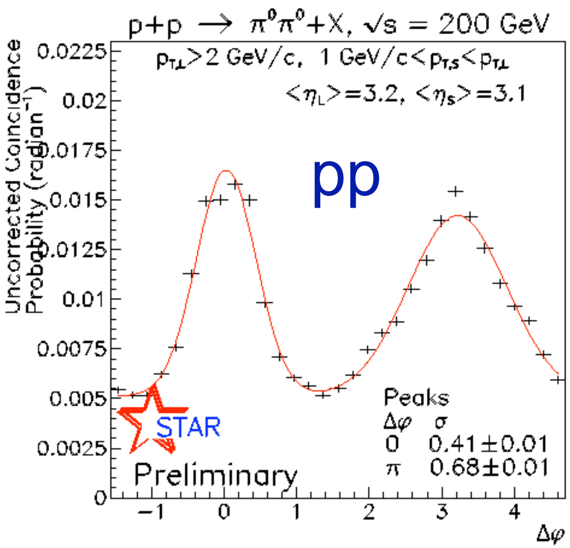
pQCD predicts 2→2 process
⇒ back-to-back di-jet

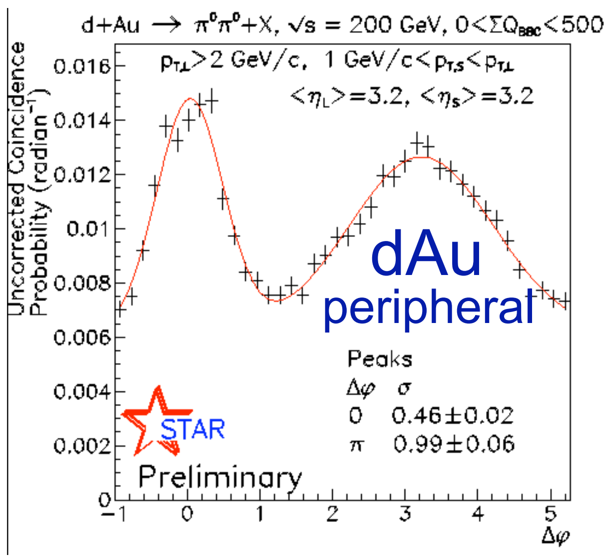


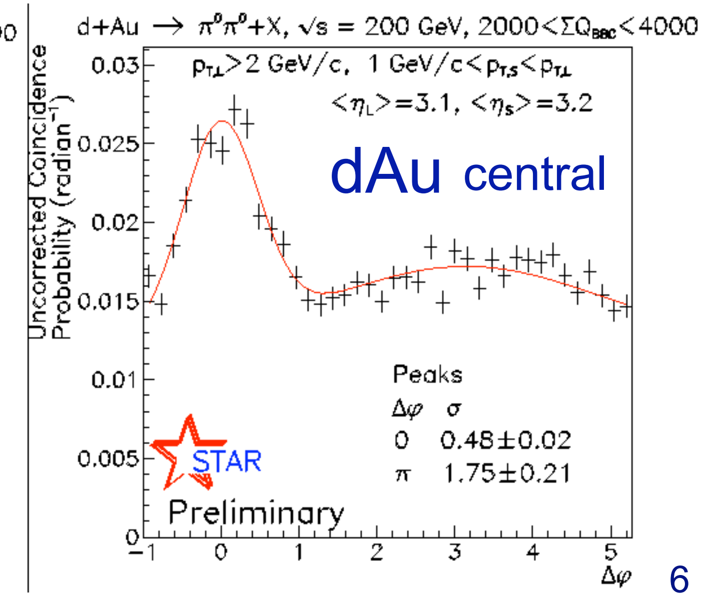
High gluon density (pA):

2→1 (2→many) process ⇒ mono-jet







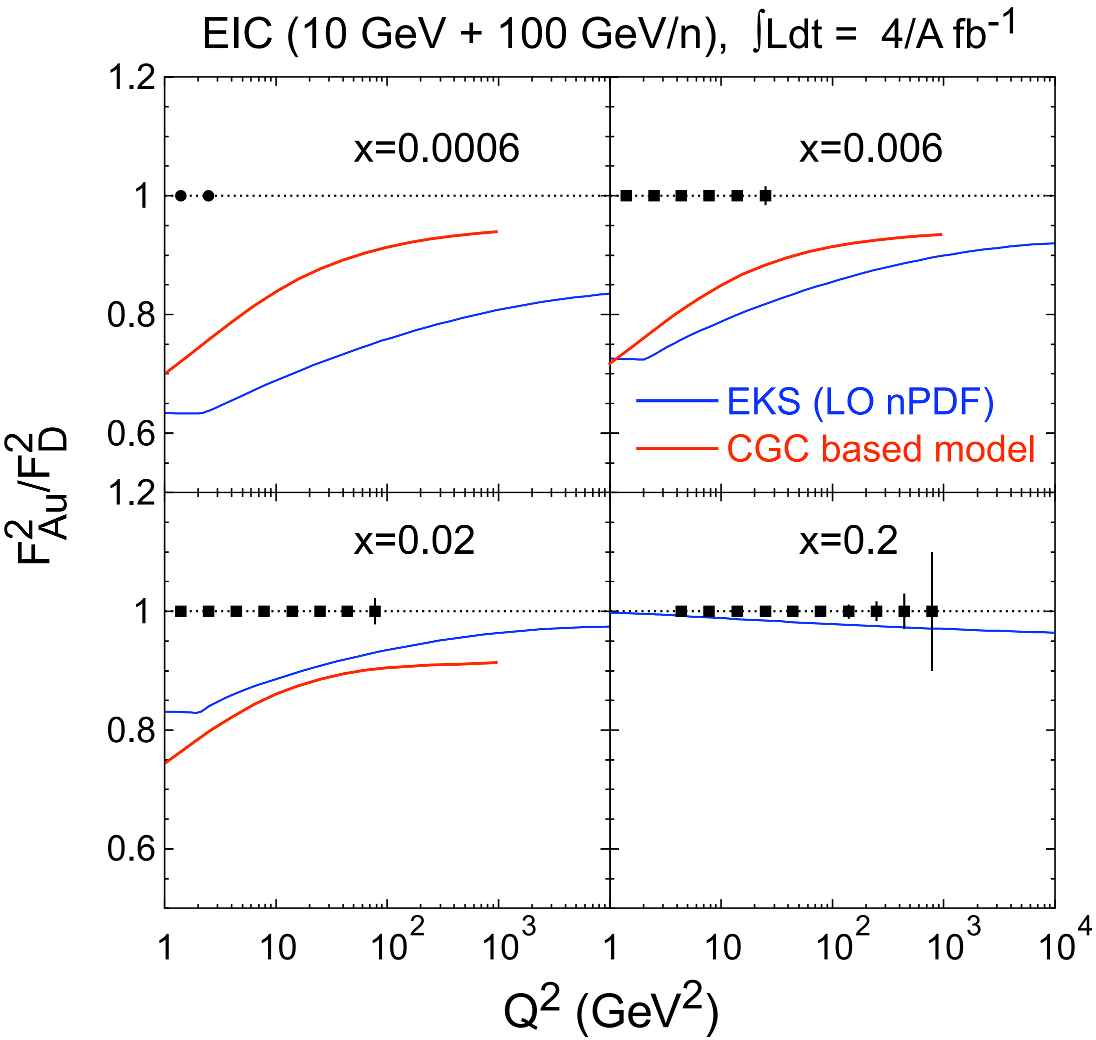


Measuring F_2 with the EIC

$$\frac{d^2\sigma^{ep\rightarrow eX}}{dxdQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left[\left(1-y+\frac{y^2}{2}\right)F_2(x,Q^2) - \frac{y^2}{2}F_L(x,Q^2)\right]$$

Inclusive DIS:

F_2 is day 1 measurement



Assumptions:

- 10 GeV + 100 GeV/n
  - $\sqrt{s}$ = 63 GeV
- Ldt = 4/A fb^{-1}
  - equiv. to L = 3.8 10^{33} cm^{-2} s^{-1}, T = 4 weeks, duty cycle: 50%
- Detector: 100% efficient
  - Q^2 up to kin. limit s·x
  - see talk by Elke
- Statistical errors only

Note: L ~ 1/A

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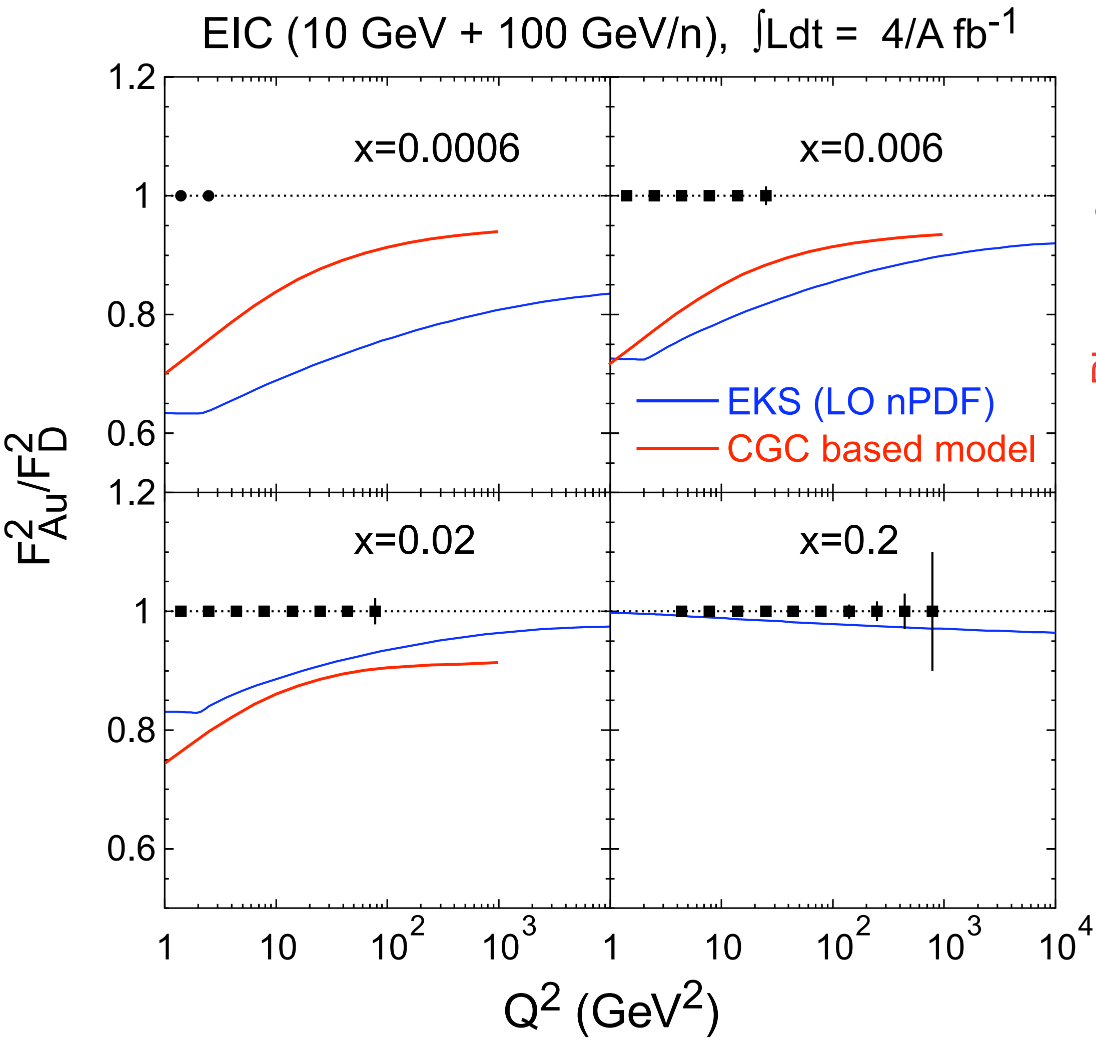

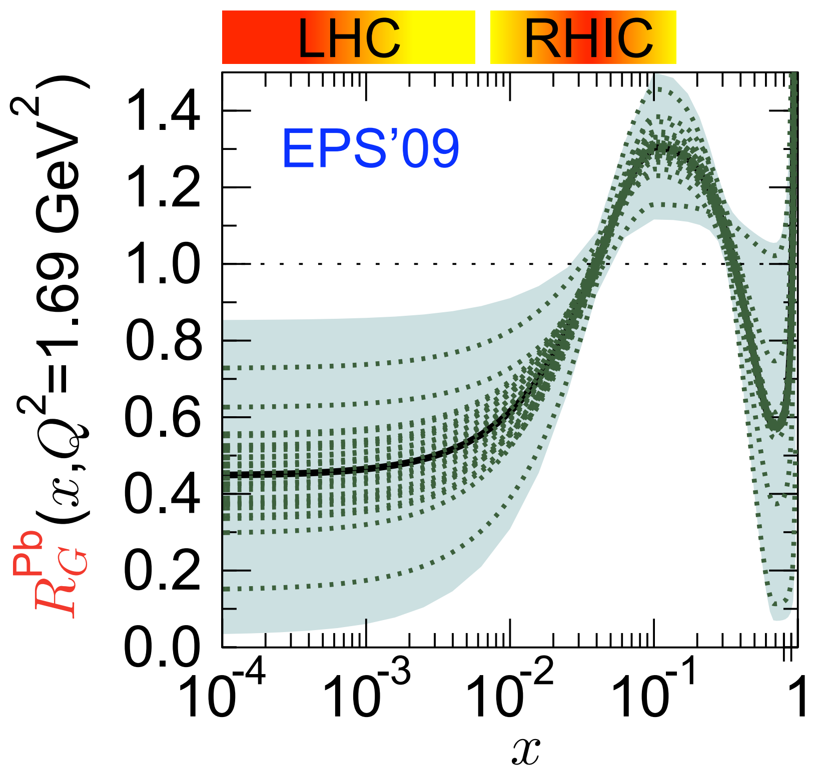

Initial state effects
in pA, AA relevant
for heavy ion program !

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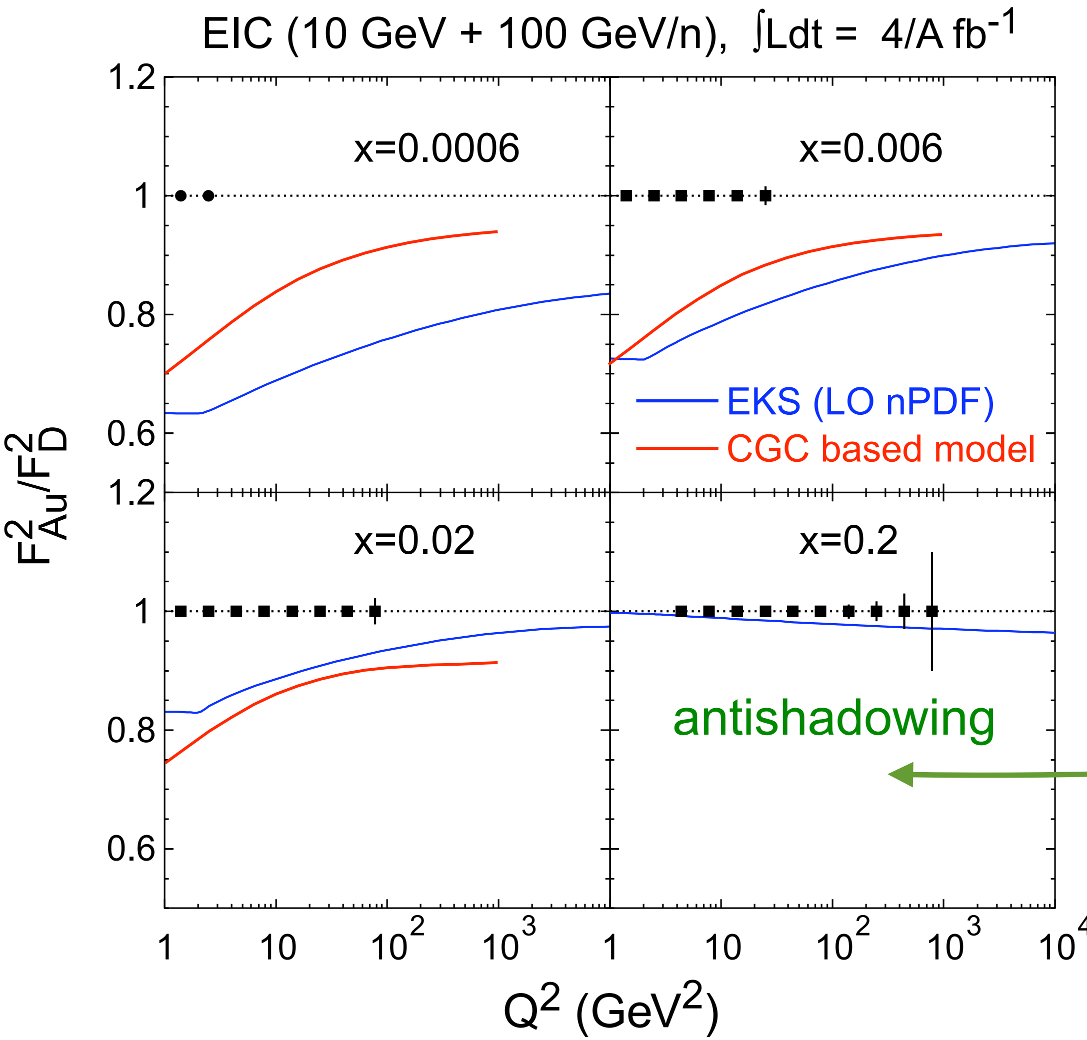

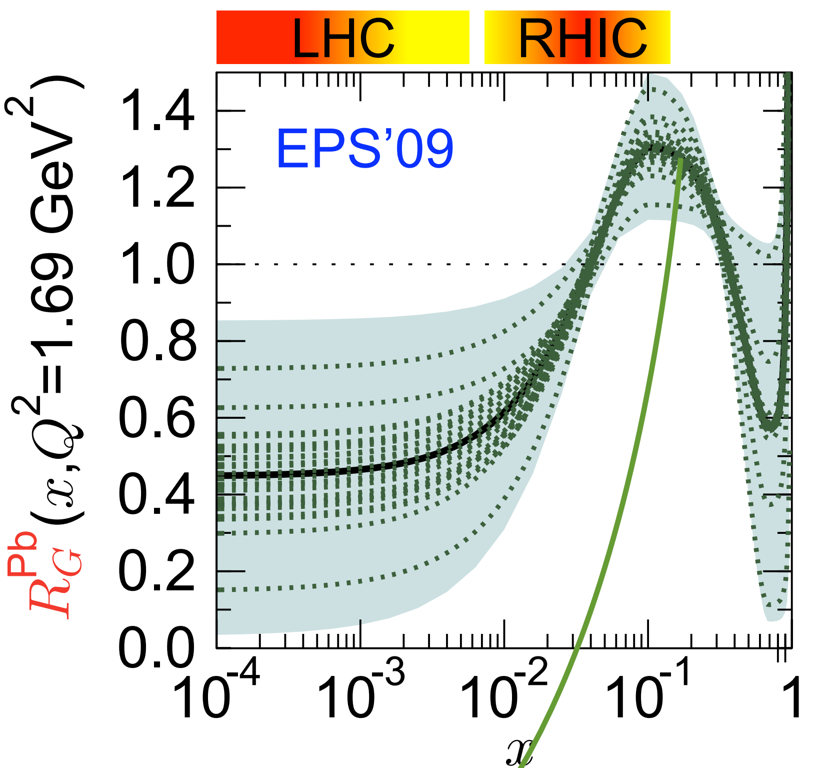

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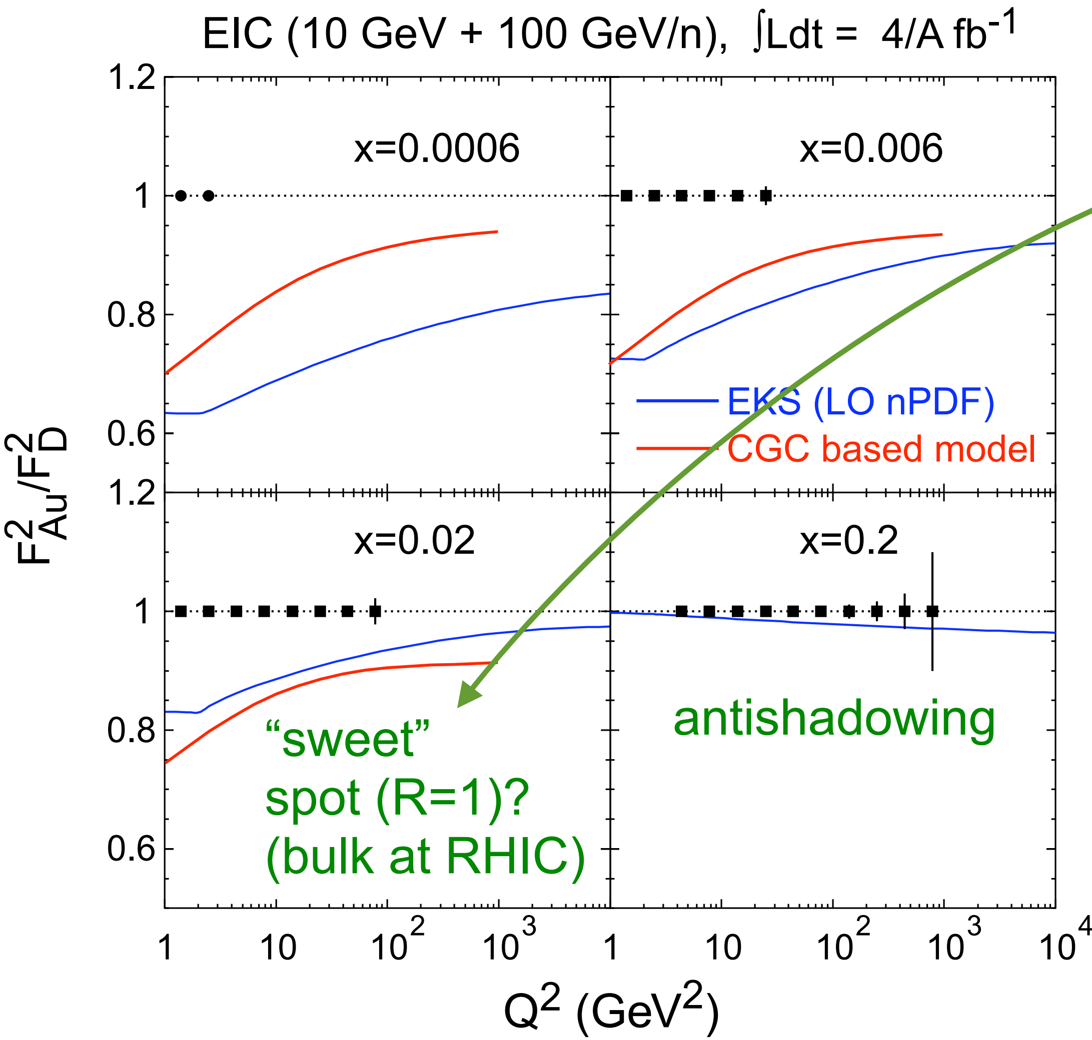
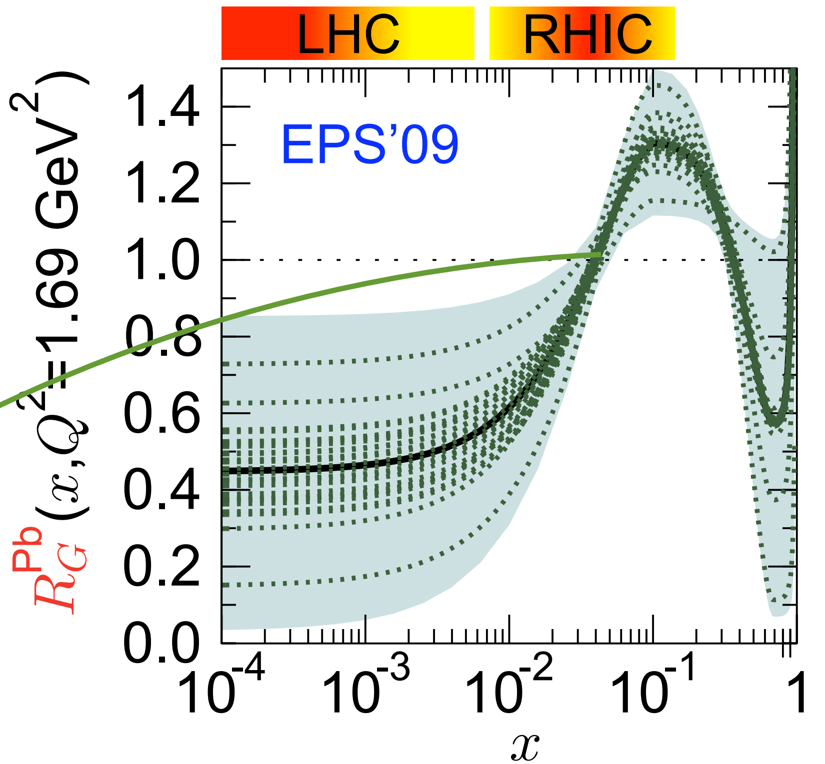
$$\frac{d^2\sigma^{ep\to eX}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left[\left(1-y+\frac{y^2}{2}\right)F_2(x,Q^2)-\frac{y^2}{2}F_L(x,Q^2)\right]$$





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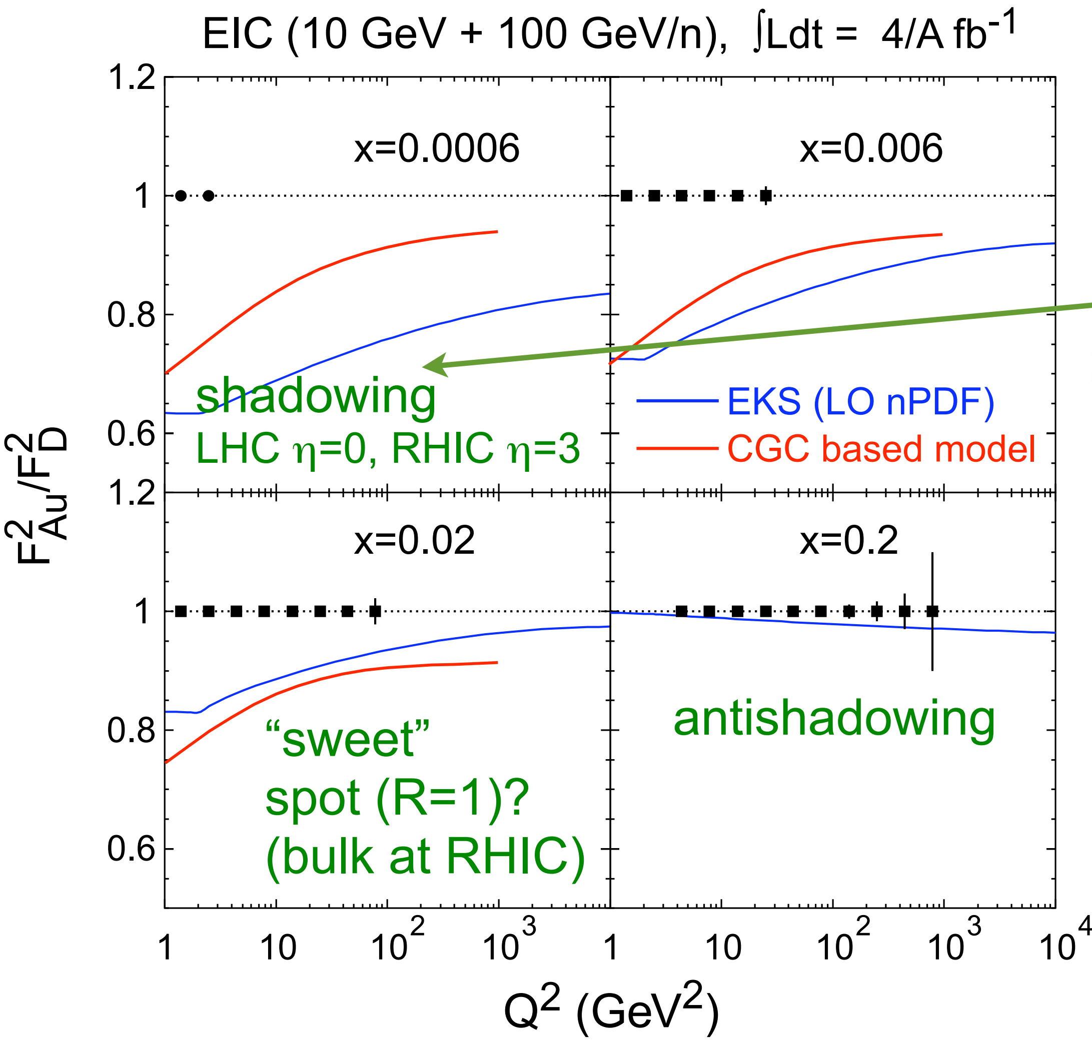
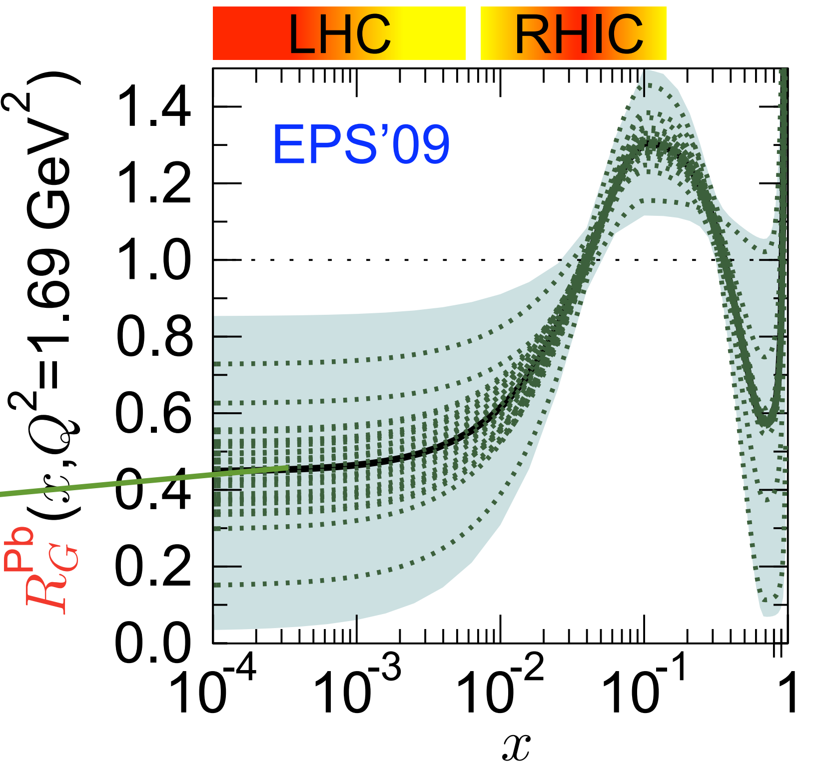




Initial state effects in pA, AA relevant for heavy ion program !

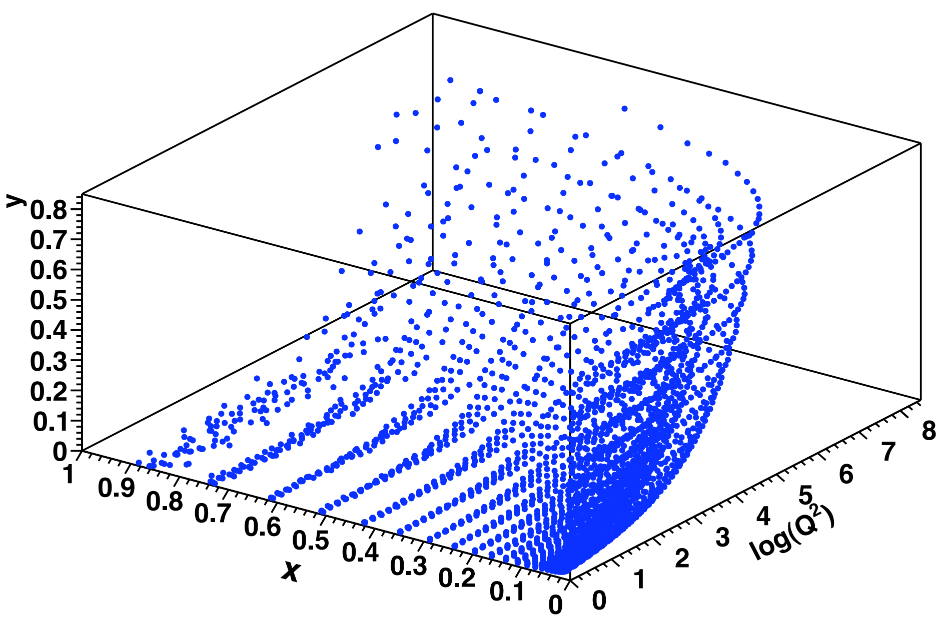
Measuring F_L with the EIC

$F_L \sim \alpha_s\, G(x,Q^2)$: the most "direct" way to $G(x,Q^2)$

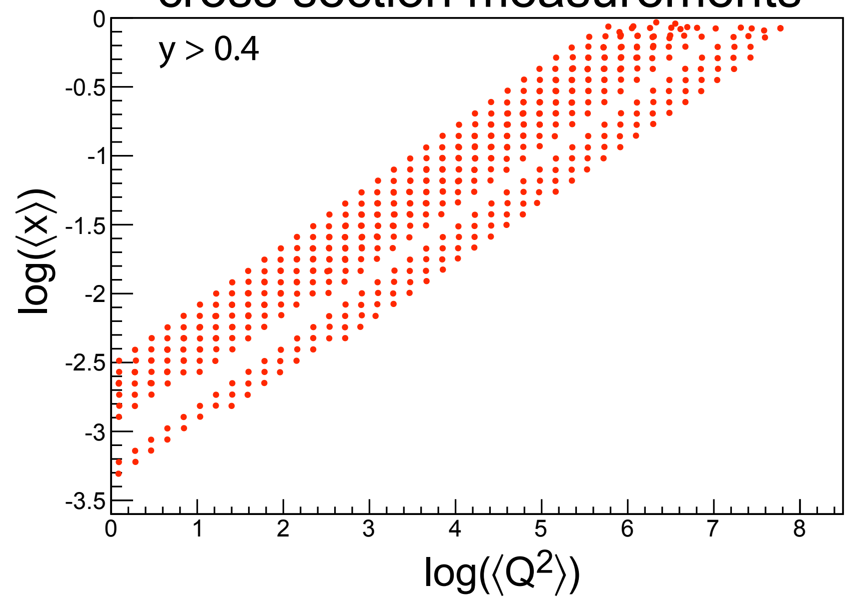
F_L runs at various $\sqrt{s}$
$\Rightarrow$ longer program

$$\frac{d^2\sigma^{ep\to eX}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left[\left(1 - y + \frac{y^2}{2}\right)F_2(x,Q^2) - \frac{y^2}{2}F_L(x,Q^2)\right]$$

In order to extract F_L one needs at least two measurements of the inclusive cross section with "wide" span in inelasticity parameter **y** ($Q^2 = sxy$)



Coverage in x and Q^2 for inclusive cross section measurements

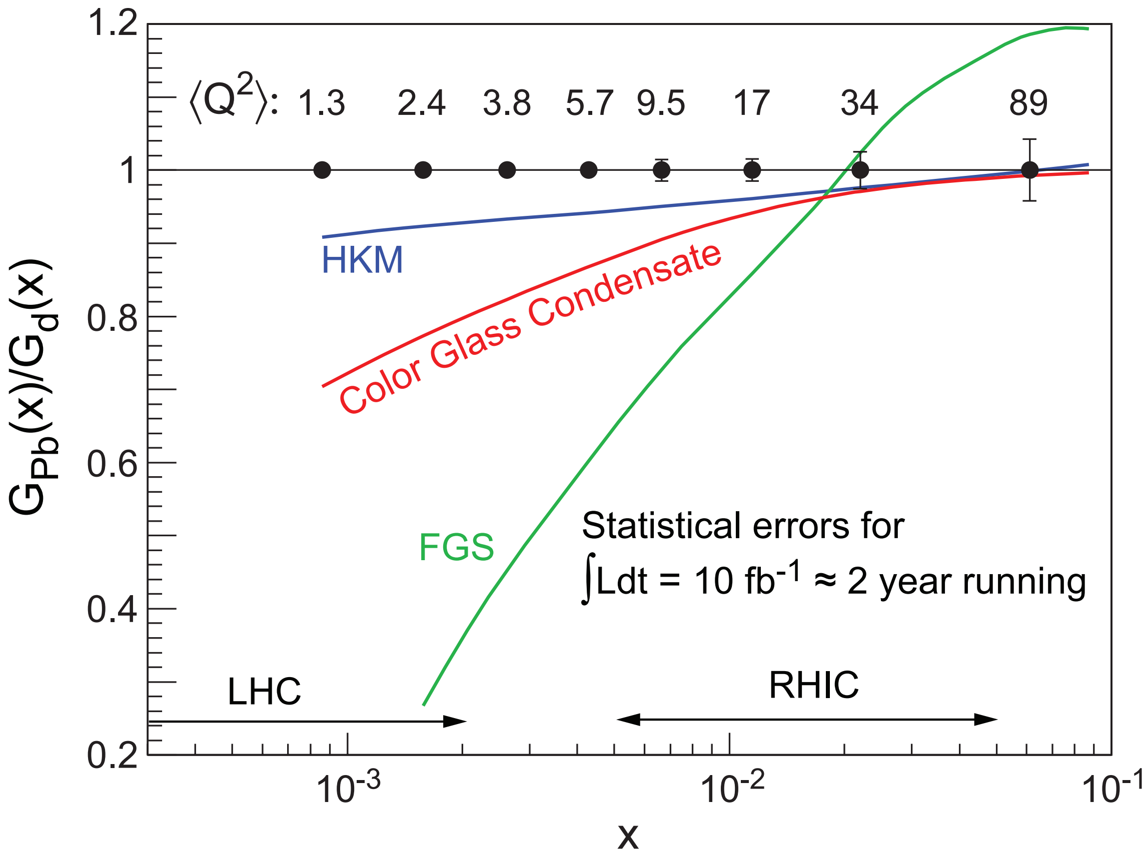


Plots for 4 GeV electrons on 50-250 GeV protons

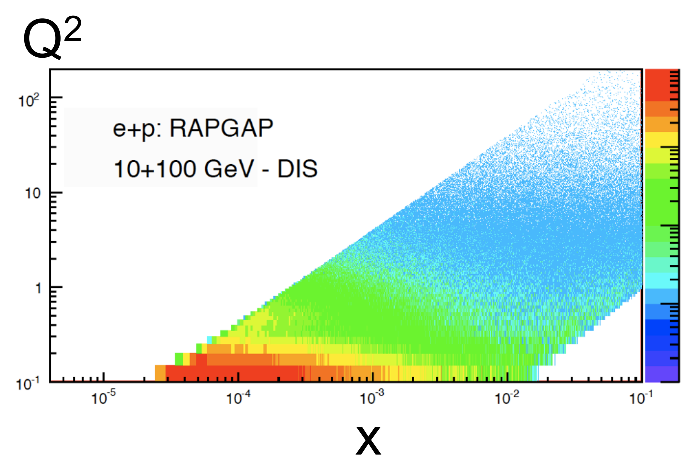
Measuring F_L with the EIC

Assumptions:

- $\int \mathcal{L}dt$ = 4/A fb^{-1} (10+100) GeV
  = 4/A fb^{-1} (10+50) GeV
  = 2/A fb^{-1} (5+50) GeV
- Detector: 100% efficient
  - Q^2 up to kin. limit s·x
- Statistical errors only



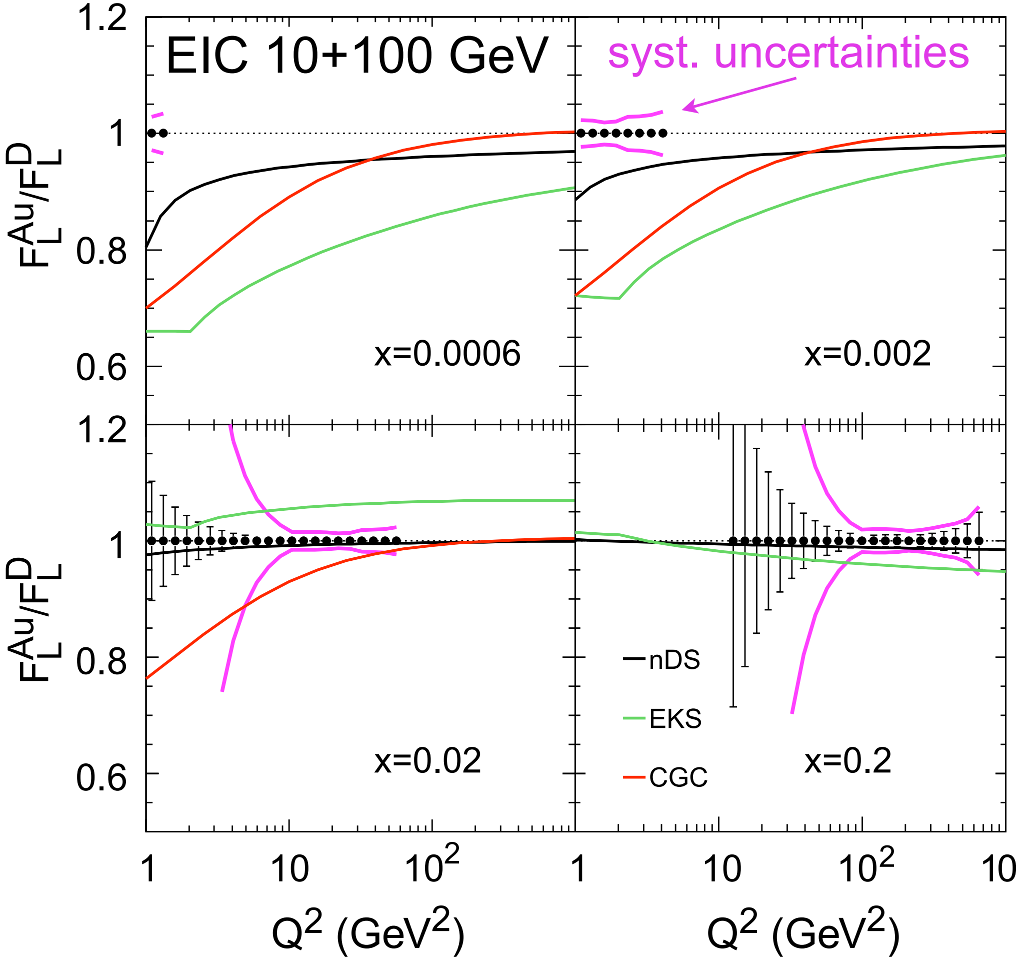
⟨Q^2⟩ reflects kinematic limits



Measuring F_L: Uncertainties

First attempt to get a feeling for systematic uncertainties

1% energy-to-energy normalization (can we do better?)



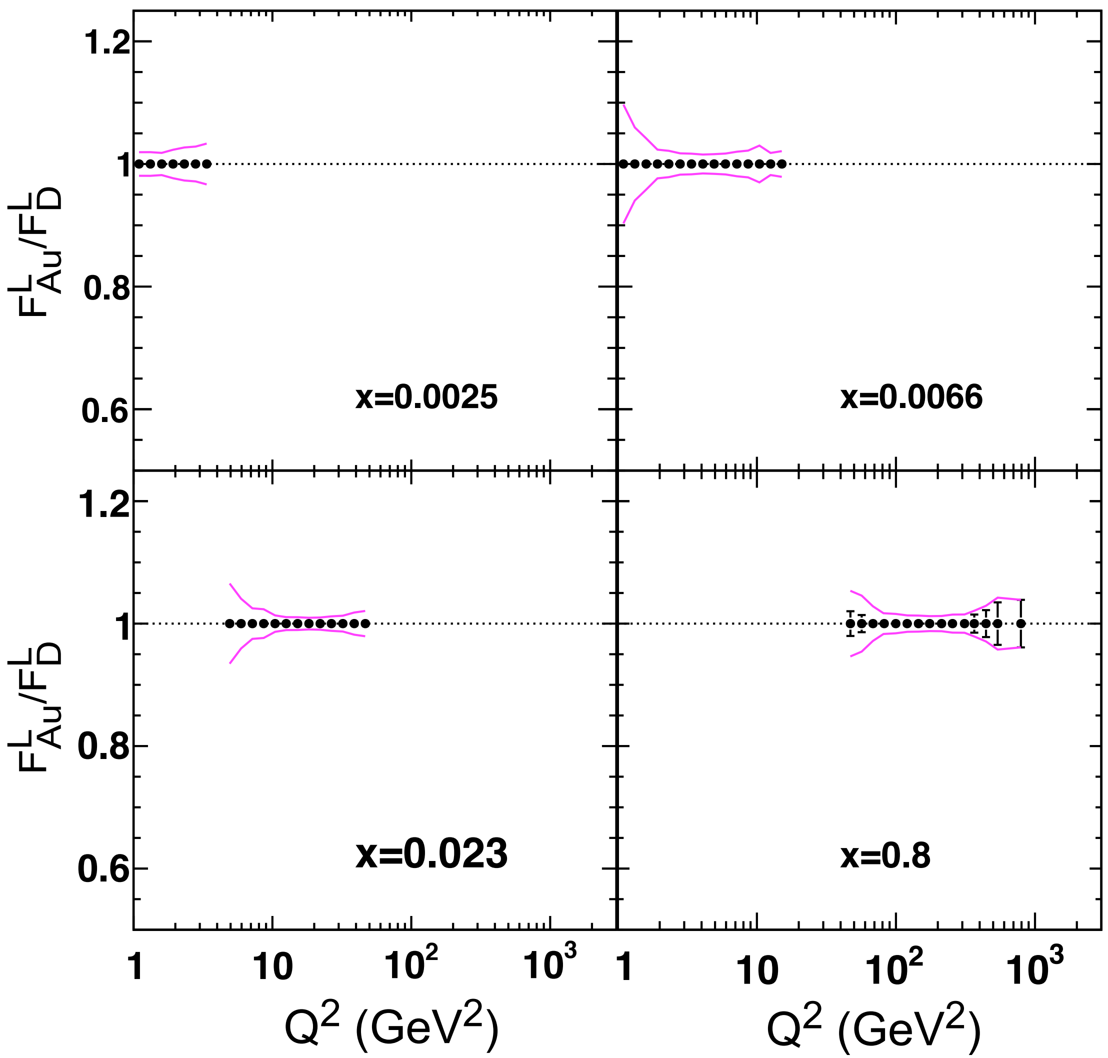
Conclusion from this study:

- Dominated by sys. uncertainties
- It makes little sense to collect more statistics when dominated by systematical errors
- Depending on x and Q^2 might be able take a hit in luminosity

  ⇒ need more detailed studies (detector simulations)

F_L for Staged EIC: E_e = 4 GeV

F_L for electron energy fixed at 4 GeV and proton energies: 50, 70, 100, 250 GeV (4fb^{-1} each)



The magenta lines shows the statistical and systematic error (1% uncertainty in normalization) added in quadrature.

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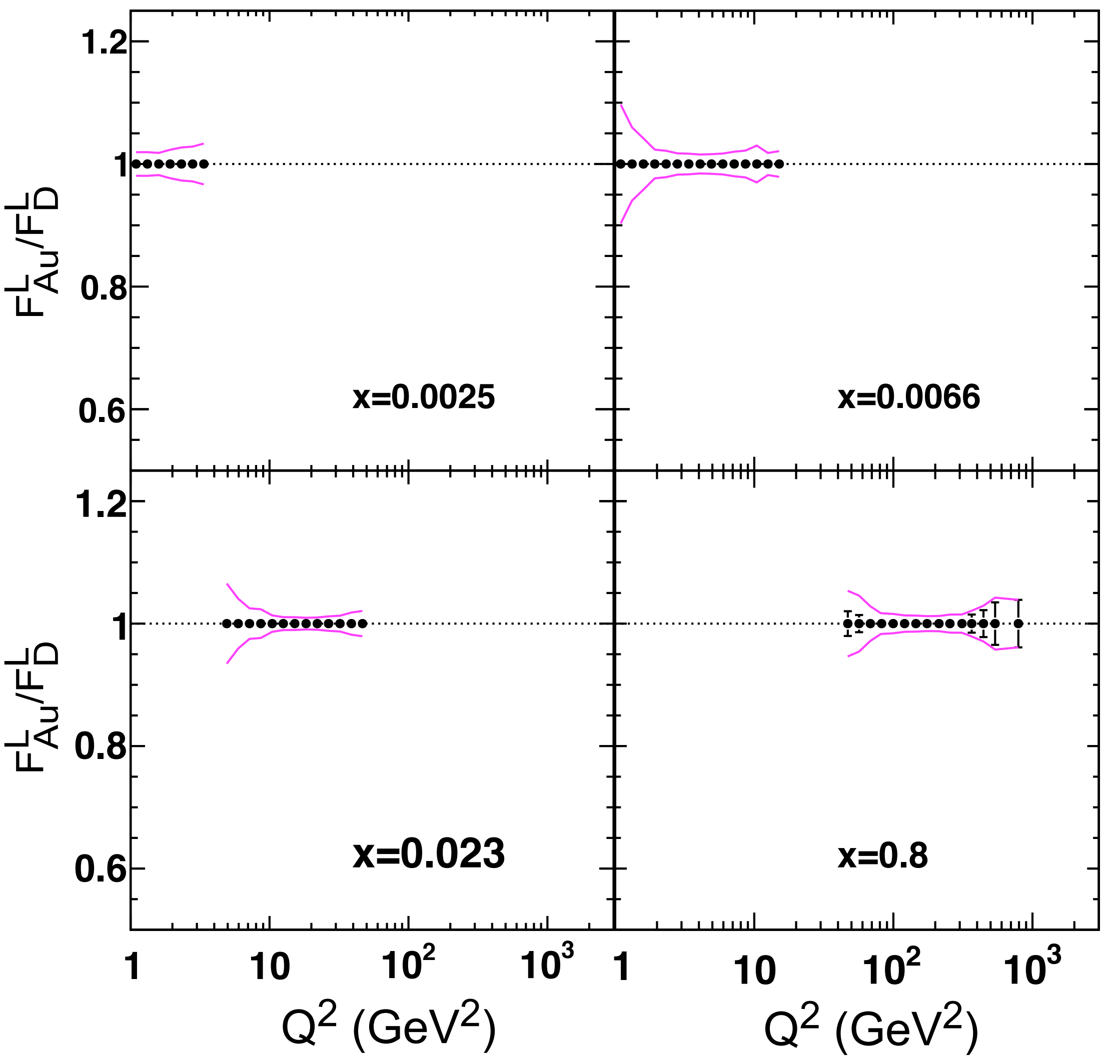

The magenta lines shows the statistical and systematic error (1% uncertainty in normalization) added in quadrature.

Again, the extraction of F_L is dominated by systematic uncertainties

Extracting $G(x,Q^2)$ from Diffractive Events

General Assumption:

Diffractive processes are the most sensitive means to probe $G(x,Q^2)$ and saturation since $\sigma \propto G(x,Q^2)^{\mathbf{2}}$

Note: quadratic dependence not for all processes

Caveats:

- Theoretical
  - How to extract G from σ ?
  - At what scale (Q^2) and what *x* are we probing G ?
- Experimental
  - Detecting diffractive eA events ?
    - testing breakup of nuclei versus rapidity gap
  - Separating coherent from incoherent processes
    - How to detect breakup of nuclei ?
  - How to measure *t* ?

Extracting $G(x,Q^2)$ from Diffractive Events

Smoking Gun (?): exclusive diffractive vector meson production

pQCD:
$$\frac{d\sigma_L^{\gamma*A\rightarrow VA}}{dt}\Big|_{t=0} \propto \frac{\alpha_S^2(Q^2)}{Q^6}\left[xG_A(x,Q^2)\right]^2$$

Brodsky et al.
Frankfurt,Koepf,Strikman

but: only valid at large Q^2 ($Q^2 \gg M_V^2$)

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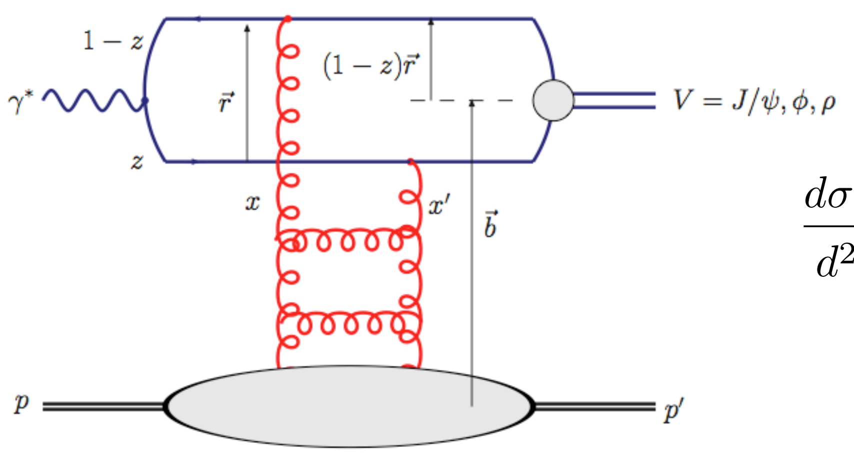
Dipole model:

Kowalski,Motyka,Watt

$$\frac{d\sigma_{T,L}^{\gamma^* p \to pV}}{dt} = \frac{1}{16\pi} \left| \int dr(2\pi r) \int_0^1 \frac{dz}{4\pi} \int db(2\pi b) (\Psi_V^* \Psi)_{T,L} J_0(b\Delta) J_0([1-z]r\Delta) \frac{d\sigma_{q\bar{q}}}{d^2\vec{b}} \right|^2$$



$$\frac{d\sigma_{q\bar{q}}}{d^2\vec{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_S(\mu^2) xg(x, \mu^2) T(b)\right)\right].$$

Glauber-Mueller

Modeling Diffractive VM Production

Implemented various dipole models (b-Sat, b-CGC) in a single program (xdvmp) for ep and recently for eA.

Various VM wave functions are implemented. The implementation of the pQCD model is underway.

(Dipole: H. Kowalski, L. Motyka, G. Watt, PhysRev D74, 074016, arXiv:hep-ph/0606272v2; Henri Kowalski , Derek Teaney, PRD68:114005, hep-ph/0304189; H. Kowalski, T. Lappi, R. Venugopalan, PRL100:022303, arXiv:0705.3047 [hep-ph]
pQCD: S. Brodsky et al., Phys.Rev.D50:3134,1994, e-Print: hep-ph/9402283; L. Frankfurt et al., Phys. Rev. D 54, 3194 - 3215 (1996); L. Frankfurt et al., Phys.Rev.D57:512,1998, hep-ph/9702216)

The dipole model describes VM (J/ψ, ϕ, ρ) production at HERA very well.

Both will be used to test sensitivity to different $G(x,Q^2)$ and can be used in detector simulations.

First Lessons Learned for EIC

Cross-section for production of final state VM:

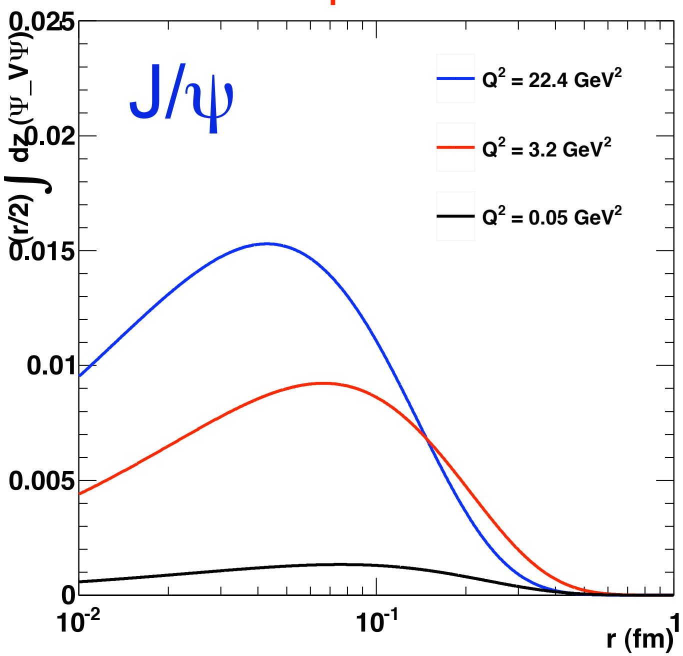
$$\frac{\mathrm{d}\sigma^{\gamma^* p\rightarrow Ep}_{T,L}}{\mathrm{d}t} = \frac{1}{16\pi}\left|\mathcal{A}^{\gamma^* p\rightarrow Ep}_{T,L}\right|^2 = \frac{1}{16\pi}\left|\int \mathrm{d}^2\boldsymbol{r}\int_0^1\frac{\mathrm{d}z}{4\pi}\int \mathrm{d}^2\boldsymbol{b}\,(\Psi^*_E\Psi)_{T,L}\,\mathrm{e}^{-\mathrm{i}[\boldsymbol{b}-(1-z)\boldsymbol{r}]\cdot\boldsymbol{\Delta}}\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\boldsymbol{b}}\right|^2$$

Amplitude

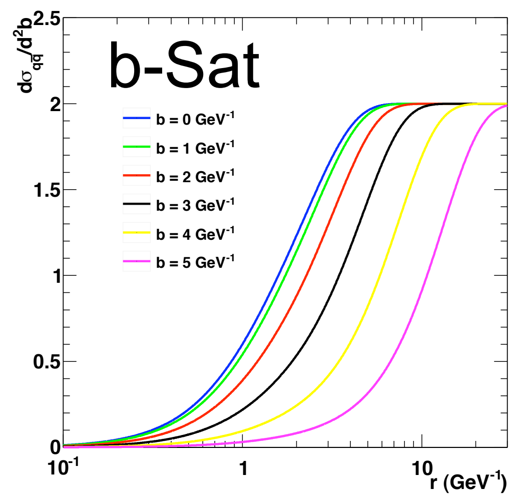
Overlap between photon and VM wave function

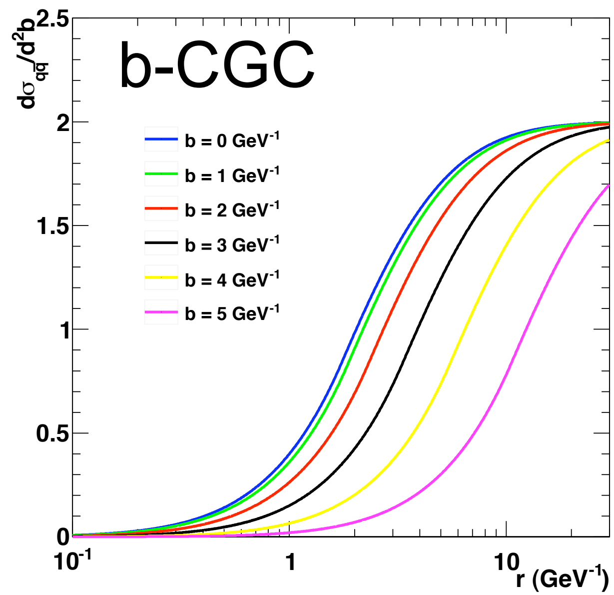
Dipole Cross-Section

Overlap Function



Dipole Cross-Section





$$\frac{d\sigma_{q\bar{q}}}{d^2\vec{b}} = 2\left[1-\exp\left(-\frac{\pi^2}{2N_c}r^2\alpha_S(\mu^2)xg(x,\mu^2)T(b)\right)\right].$$

First Lessons Learned for EIC

Cro

Overlap function (and thus σ) vanishes for large dipole radii where saturation kicks in (Q ~ 1/r)

The J/ψ seems too small to probe saturation physics

ϕ looks better, ρ is ideal

Problem is that the wave functions for ρ, ϕ are less known (can - in principle - be solved)

Measuring VM other than J/ψ → $\ell^+\ell^-$ requires **particle ID capabilities** (K, π) of the detector over a wide p_T range

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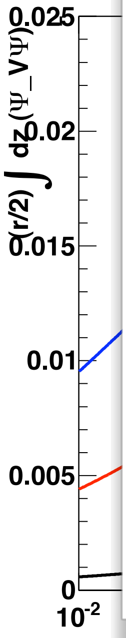
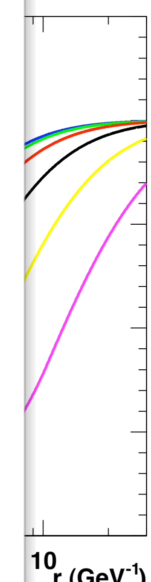


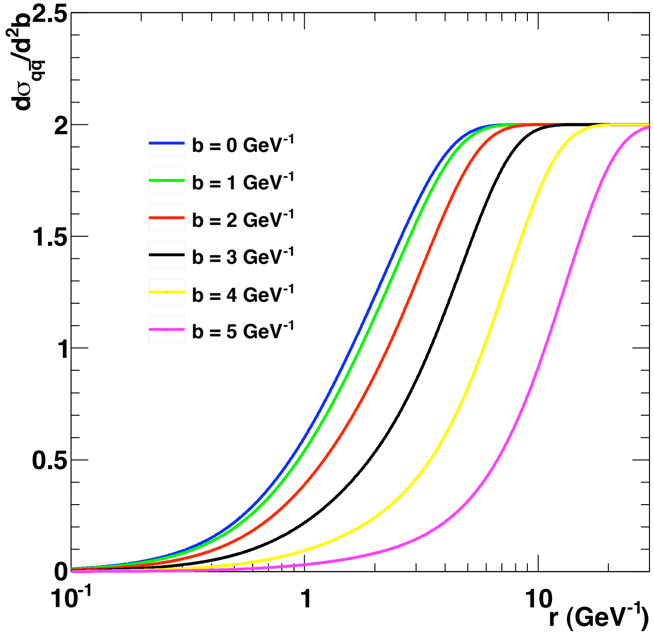


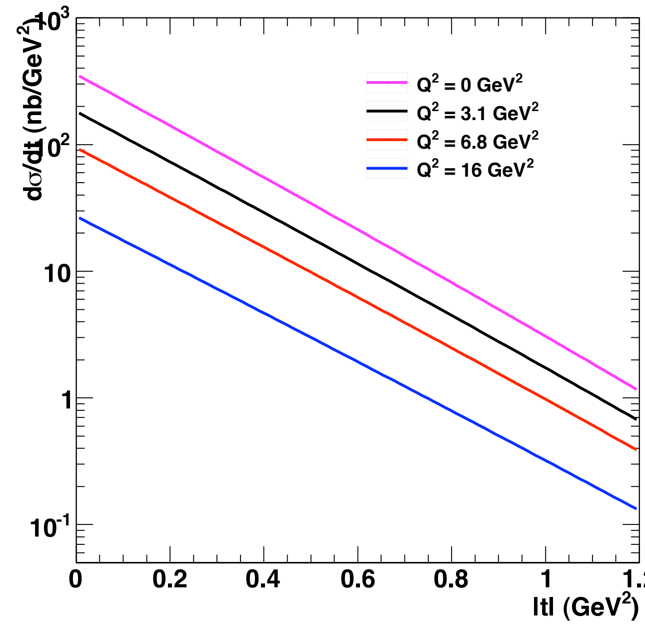
$$\frac{d\sigma_{q\bar{q}}}{d^2\vec{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_S(\mu^2) x g(x,\mu^2) T(b)\right)\right].$$

From ep to eA ...

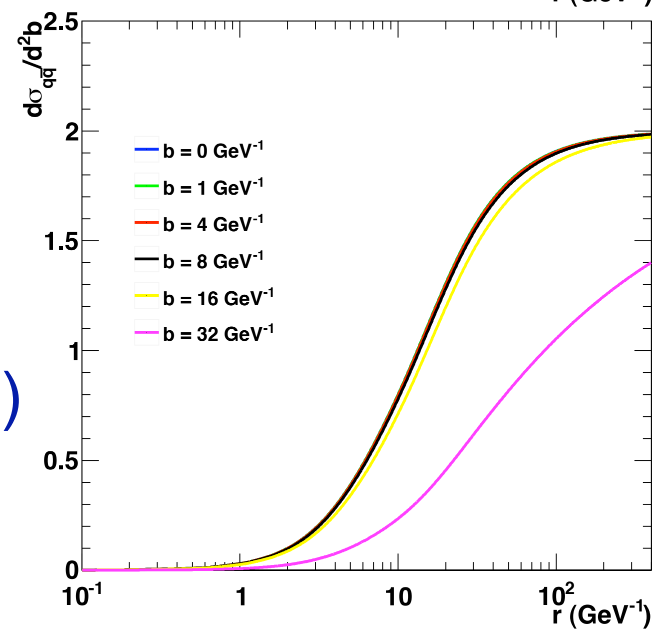
(All xdvmp)

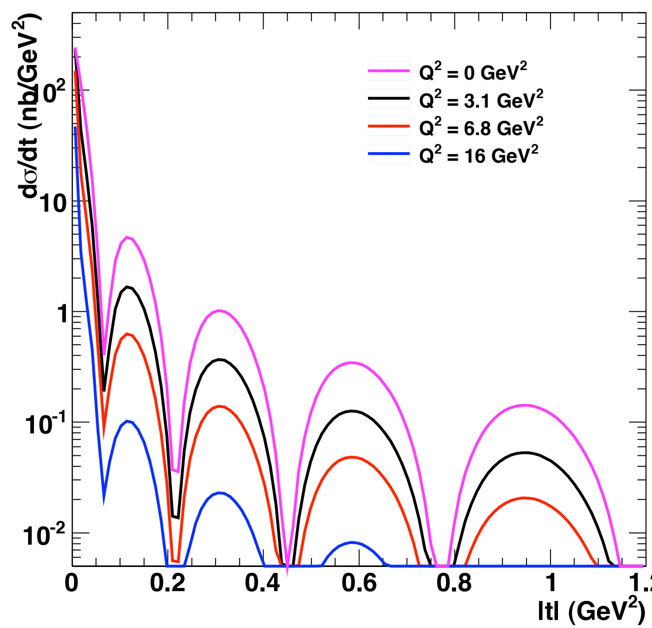
ep →





eA →

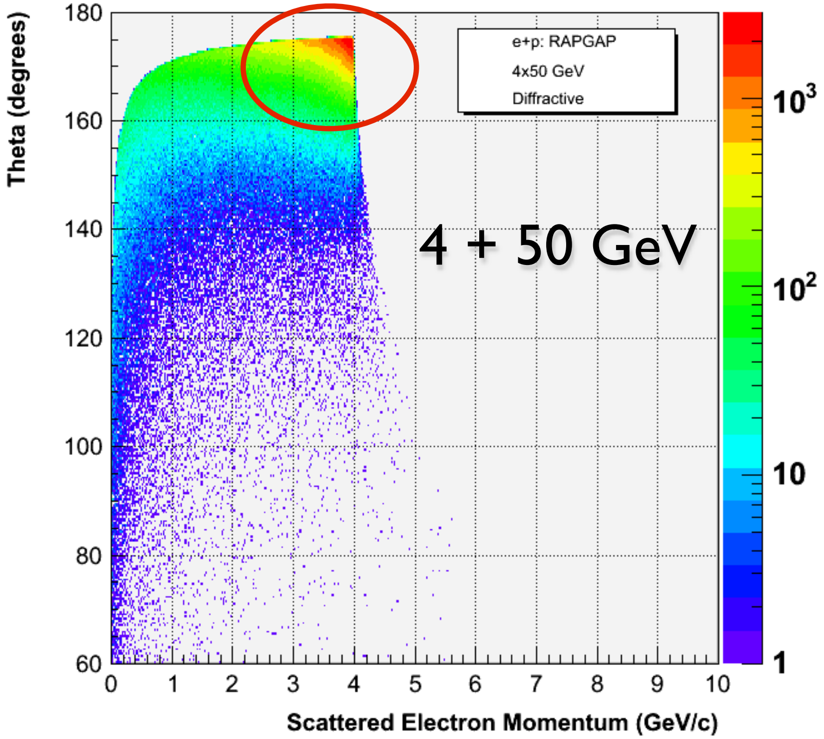


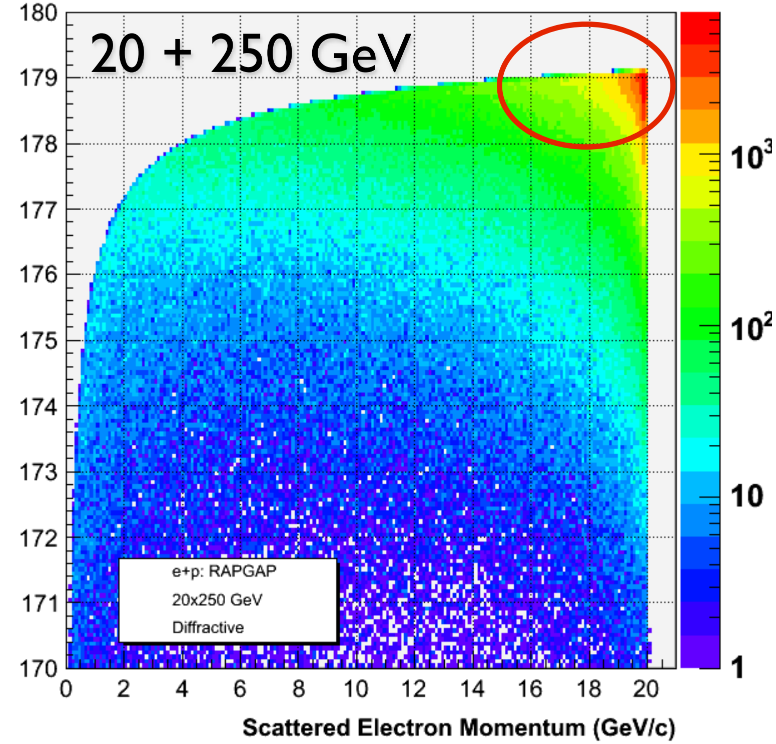


Note: eA is less b dependent than ep (which is good)

Diffractive Physics is Experimentally Hard

Scattered Electron θ(p)





Stringent constraints on detector:

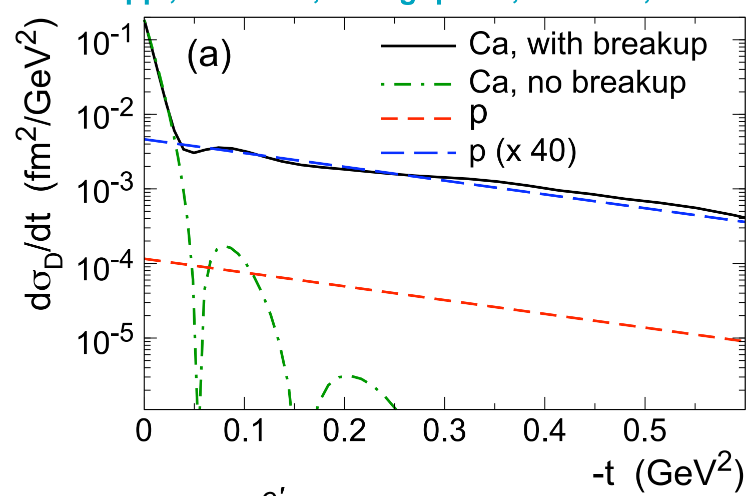
Need to measure electrons (PID + p) down to very low angles (up to 1° off the beam line)

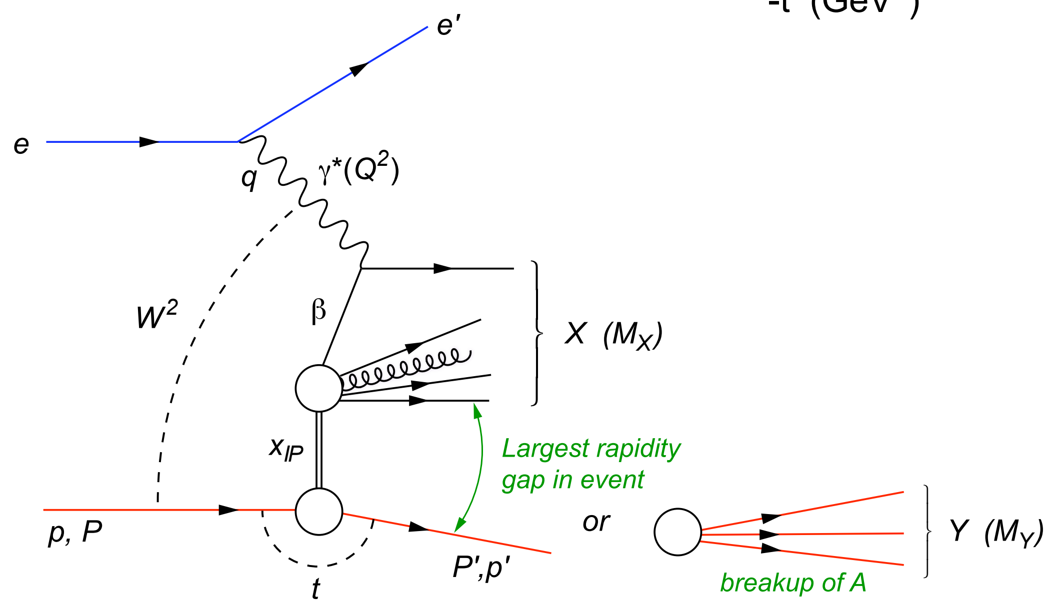
⇒ need dipole magnet(s) to bend e in "sane" region

Identifying Diffractive Events

- Beam angular divergence limits smallest outgoing p(A) angle that can be measured
- Cannot measure coherent diffraction in heavy ions (small *t*) using forward spectrometry (Roman Pots)
  - separate ion only if $p_T > p_{T,min}$
  - possible for p and light ions
- Can determine *t* in exclusive production from e, e', X

Lappi, Kowalski, Venugopalan, PRL 100, 022303





species (A)	p_T^{min} (GeV/c)
d (2)	0.02
Si (28)	0.22
Cu (64)	0.51
In (115)	0.92
Au (197)	1.58
U (238)	1.90

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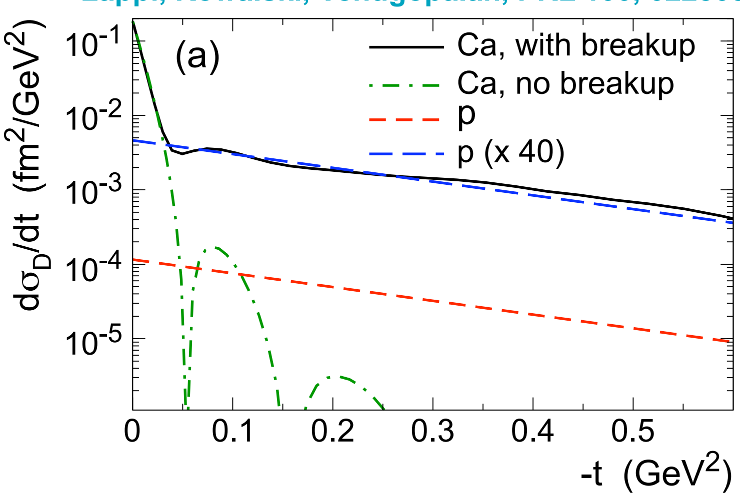
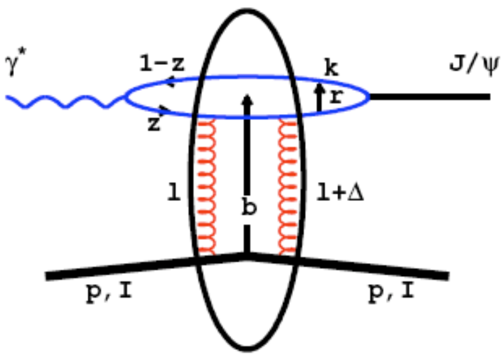


- Need to rely on rapidity gap method
  - simulations look good
  - high efficiency, high purity possible with gap *alone*
    - ~1% contamination,
    - ~80% efficiency
  - depends critically on hermeticity of detector
  - improve further by veto on breakup of nuclei (DIS)
- Very critical:
  - Mandatory to detect nuclear fragments from breakup
  - n: Zero-Degree Calorimeter
  - p, A_{frag}: Forward Spectrometer
  - New idea: Use U instead of Au (fission)

New: Probing Gluonic Structure of Nuclei

Basic Idea: Studying diffractive exclusive J/ψ production at $Q^2=0$ (photo-production)

(H. Kowalski & A. Caldwell)



Ideal probe

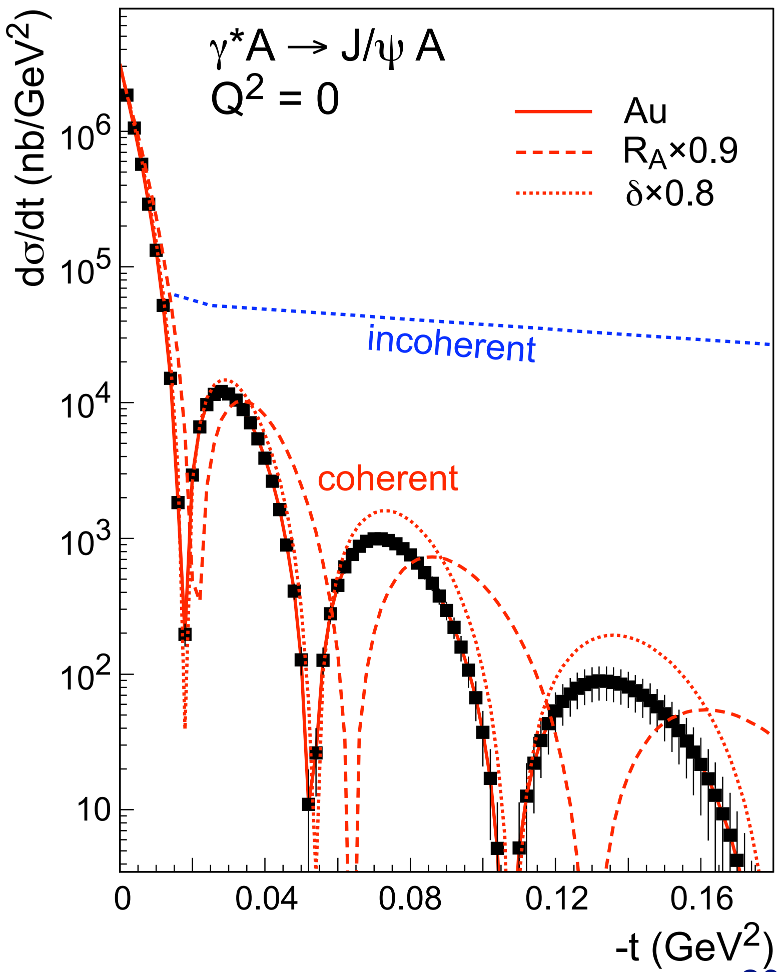
- large photo-production cross sections
- t can be derived from e, e', and J/ψ 4-momentum
  - no measurement of ion momentum necessary
  - beam electron p_T < 1 MeV (0.2 with cooling MeV) for E < 5 GeV
  - scattered electron can be detected in the forward detector (beam optic needs to be studied)
- J/ψ has small width well separated from background
- J/ψ dipole interacts only by 2g exchange at low x
  - process is well understood in QCD

Probing Gluonic Structure of Nuclear Forces

Simplified assumption for proof of principle:

- Random and uncorrelated distribution of nucleons within the nucleus
- Shape of the nucleus given by the Woods-Saxon distribution $\rho(b_T)$
- Average (sum) over all configurations
- Fourier transform the average $\Rightarrow d\sigma_A/dt$

Promising method to measure **gluon form factor** F_g in nuclei



Crucial:

- Need to suppress background by factor 100
- Dynamics of nuclear disintegration?
  - studies underway (QMD?)
  - easier with Uranium?

Summary

Study of measurements of $G(x,Q^2)$ in progress

- F_2 (existing studies but not updated yet)
- F_L (EIC & staged EIC)
- 2+1 jets (EIC & staged EIC) [not shown, see add. material]
- Diffractive VM production - no error evaluation yet but all we need is there

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Next Steps

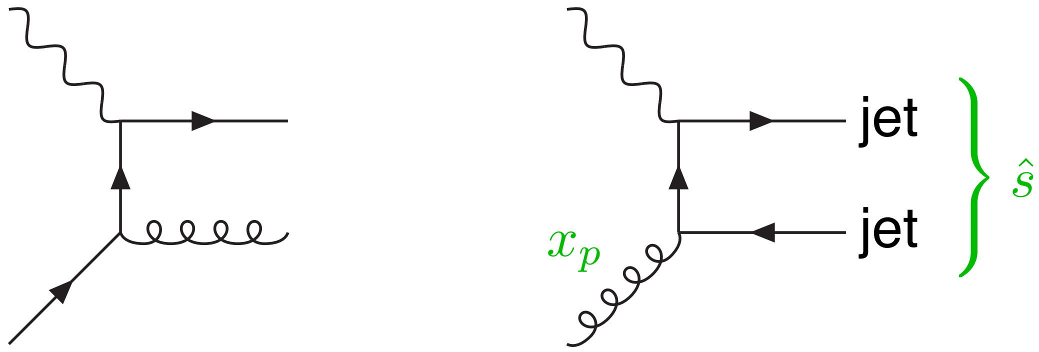
- Further investigate nuclear breakup
- Simulation on diffractive VM
- Run measurements through detector simulation (see Elke's talk)

Additional Slides

Gluon Distribution from Jet Analysis at EIC

Jets: window to partons, DIS is a clean environment

"2+1 jets" becomes more interesting



Main formula:

$$\frac{d^2\sigma^{2+1}}{dx_p dQ^2} = \alpha_s \left[a\, g(x_p, Q^2) + b\, q(x_p, Q^2) \right]$$

Technique:

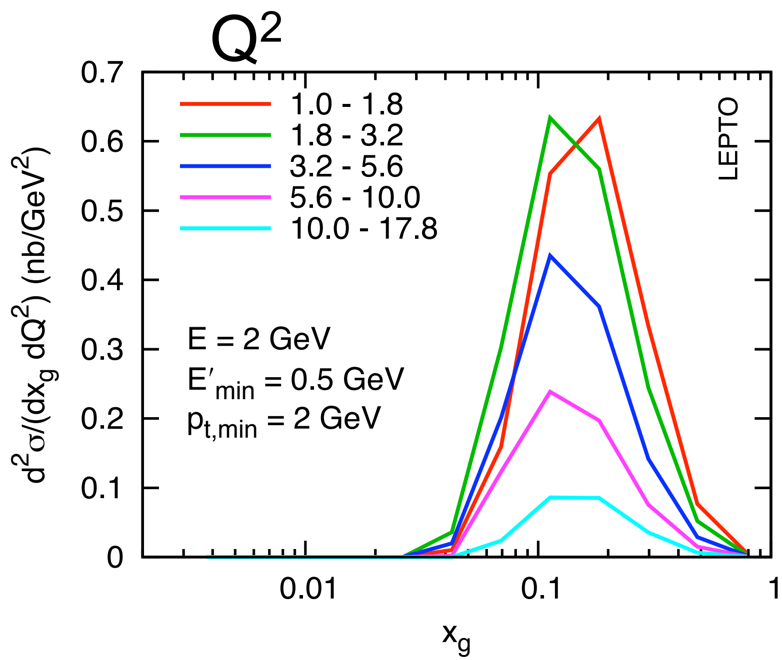
1. a and bq: matrix elements & quark piece from Monte Carlo
2. $x_p = x \left(1 + \frac{\hat{s}}{Q^2} \right)$
3. Extract the gluon distrib: $g_{\text{extr.}} = \frac{1}{a_{\text{MC}}} \left(\sigma_{\text{meas.}} - b_{\text{MC}} q \right)$

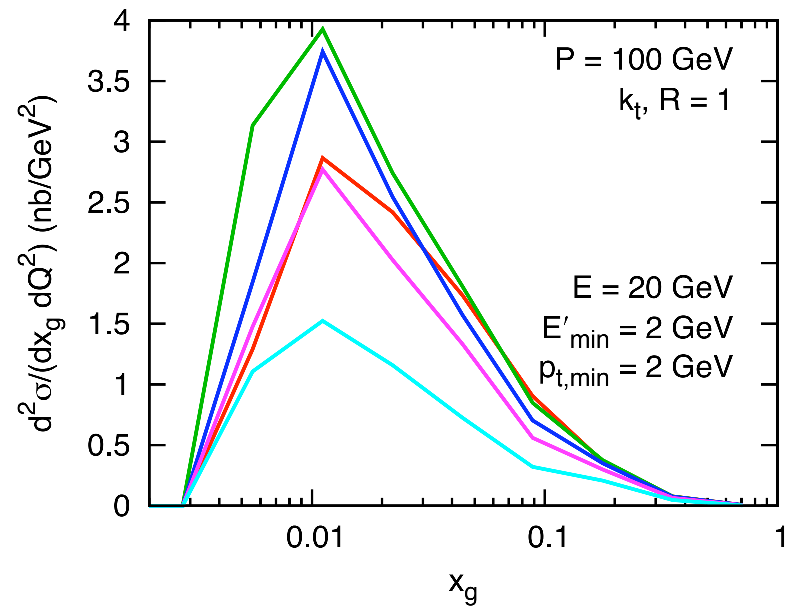
Results from Jets

Experimental cuts:

- Outgoing electron energy: E'_{min}
- Minimal jet p_T : $p_{T,min}$
- Azimuthal separation between the 2 jets: $\Delta\phi > \pi - \varepsilon$ (in the Breit frame — ensures that the 2 jets come from the hard scattering)
- Clustering: k_T algorithm with R=1 (large but OK in DIS)

Cross-section for gluon-initiated dijet events (obtained with LEPTO)





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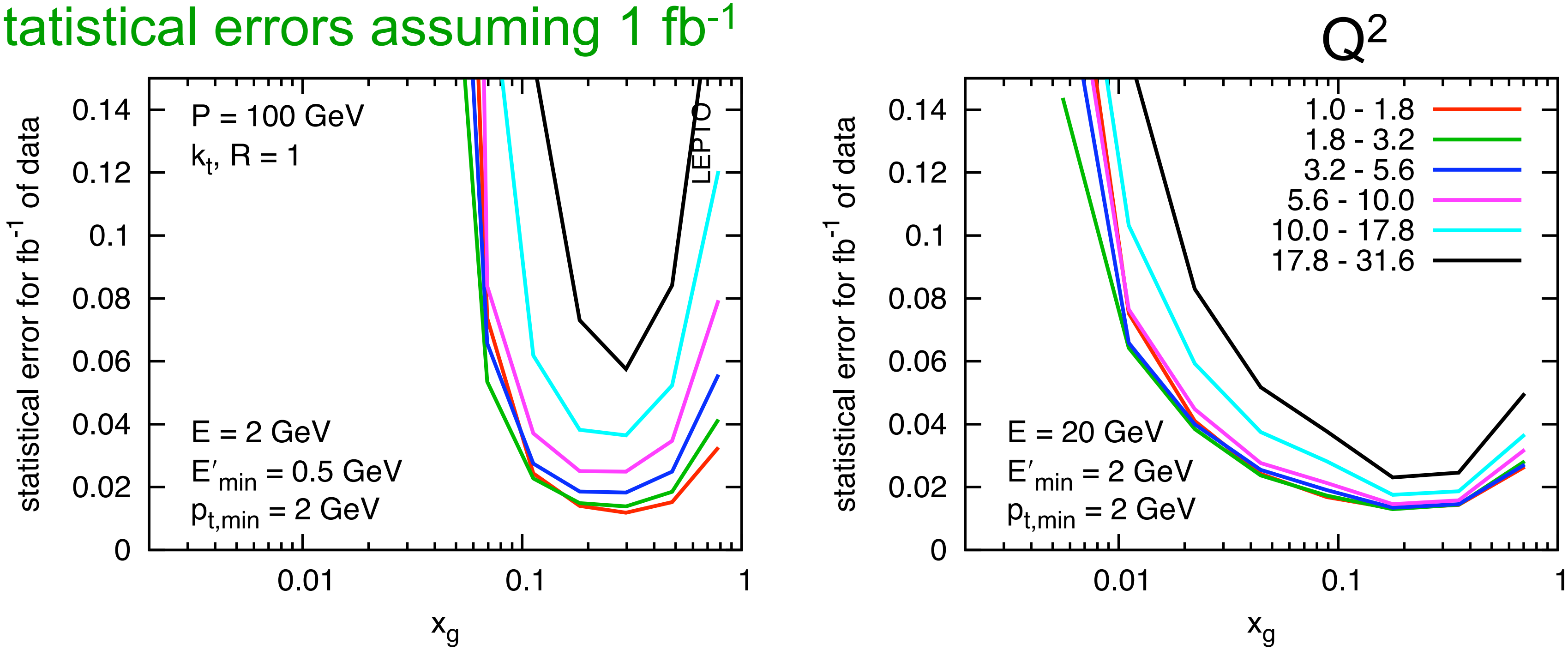
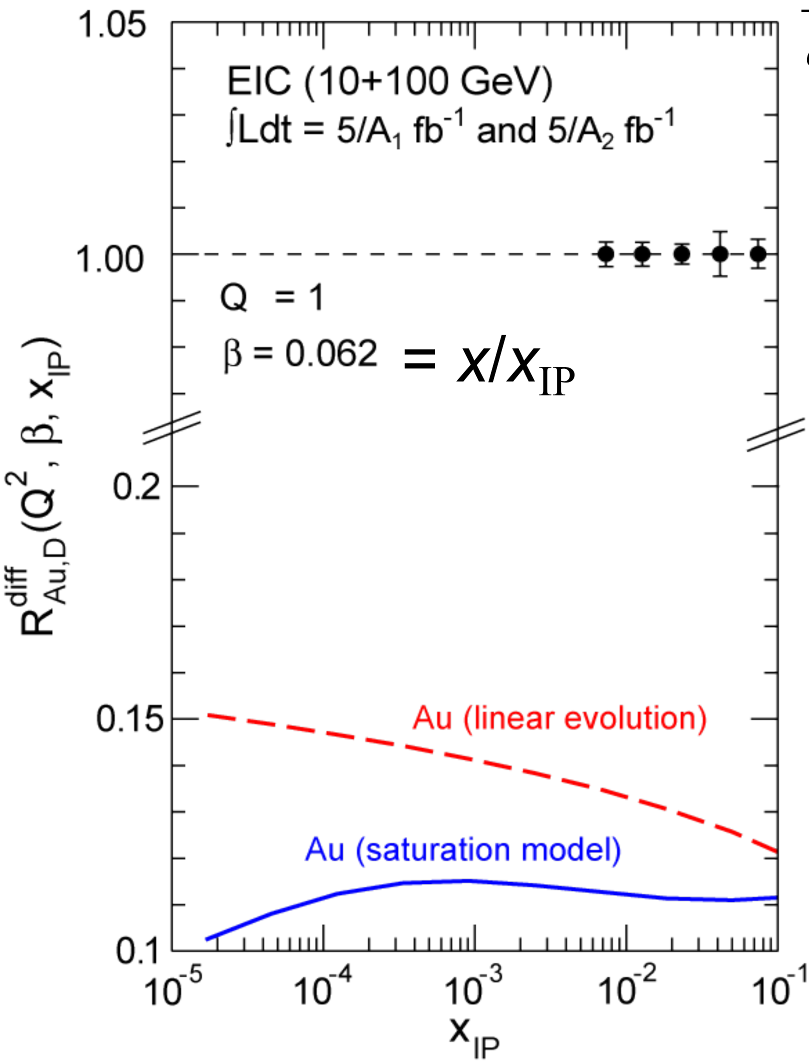
Statistical errors assuming 1 fb^{-1}



Diffractive Structure Function F_2^D at EIC



$$\frac{d^4\sigma^{eh\to eXh}}{dx\,dQ^2\,d\beta\,dt} = \frac{4\pi\alpha_{e.m.}^2}{\beta^2 Q^4}\left[\left(1 - y + \frac{y^2}{2}\right) F_2^D - \frac{y^2}{2} F_L^D\right]$$

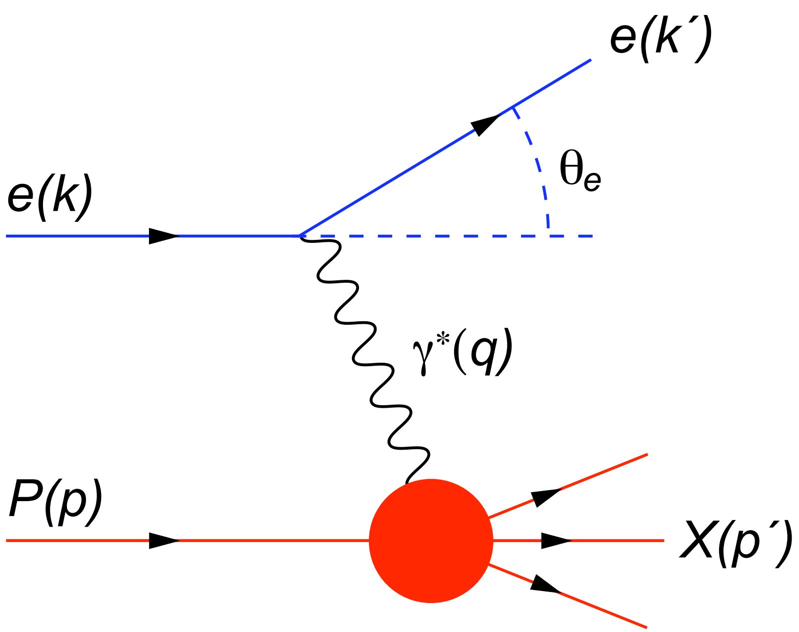
x_{IP} = momentum fraction of the pomeron w.r.t the hadron

⇒ Distinguish between linear evolution and saturation models

⇒ Insight into the nature of pomeron

Curves: Kugeratski, Goncalves, Navarra, EPJ C46, 413

Deep Inelastic Scattering (DIS)



Resolution power ("Virtuality"):

$$Q^2 = -q^2 = -(k - k')^2$$

$$Q^2 = 4E_e E'_e \sin^2\left(\frac{\theta'_e}{2}\right)$$

Inelasticity:

$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

p fraction of struck quark

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

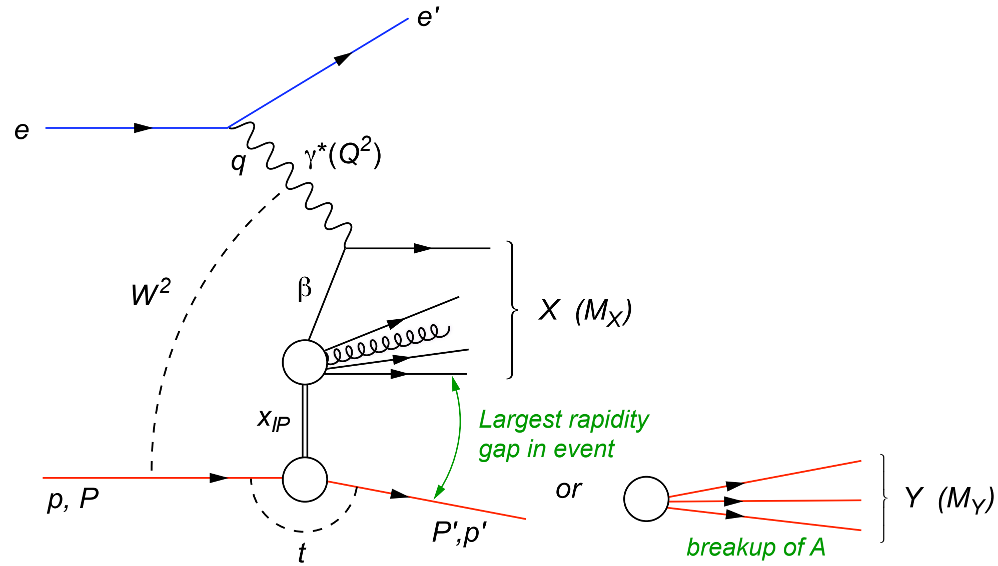
$$\frac{d^2\sigma^{ep\rightarrow eX}}{dxdQ^2} = \frac{4\pi\alpha^2_{e.m.}}{xQ^4}\left[\left(1 - y + \frac{y^2}{2}\right)F_2(x,Q^2) - \frac{y^2}{2}F_L(x,Q^2)\right]$$

quark+anti-quark momentum distributions (F_2)

gluon momentum distribution (F_L)

Hard Diffraction in DIS at Small x

Cyrille Marquet



$$\beta = \frac{Q^2}{2(p-p').(k-k')} = \frac{Q^2}{M_X^2 - t + Q^2}$$

is the momentum fraction of the struck parton w.r.t. the Pomeron

$x_{IP} = x/\beta$

momentum fraction of the exchanged object (Pomeron) w.r.t. the hadron

The measured cross-section:

$$\frac{d^4\sigma^{eh\to eXh}}{dx dQ^2 d\beta dt} = \frac{4\pi\alpha_{em}^2}{\beta^2 Q^4}\left[\left(1-y+\frac{y^2}{2}\right)F_2^{D,4}(x,Q^2,\beta,t) - \frac{y^2}{2}F_L^{D,4}(x,Q^2,\beta,t)\right]$$

The dipole picture:

Here inclusive DIS

$$\sigma_{tot}^{\gamma^* p\to X} = 2\int d^2r\, dz \sum_\lambda |\psi_\lambda(r,z,Q^2)|^2 \int d^2b\, T_{q\bar{q}}(r,x,b)$$

overlap of $\gamma^* \to q\bar{q}$ splitting functions

dipole-hadron cross-section

$T_{q\bar{q}}$ = dipole scattering amplitude