Coherent electron cooling*

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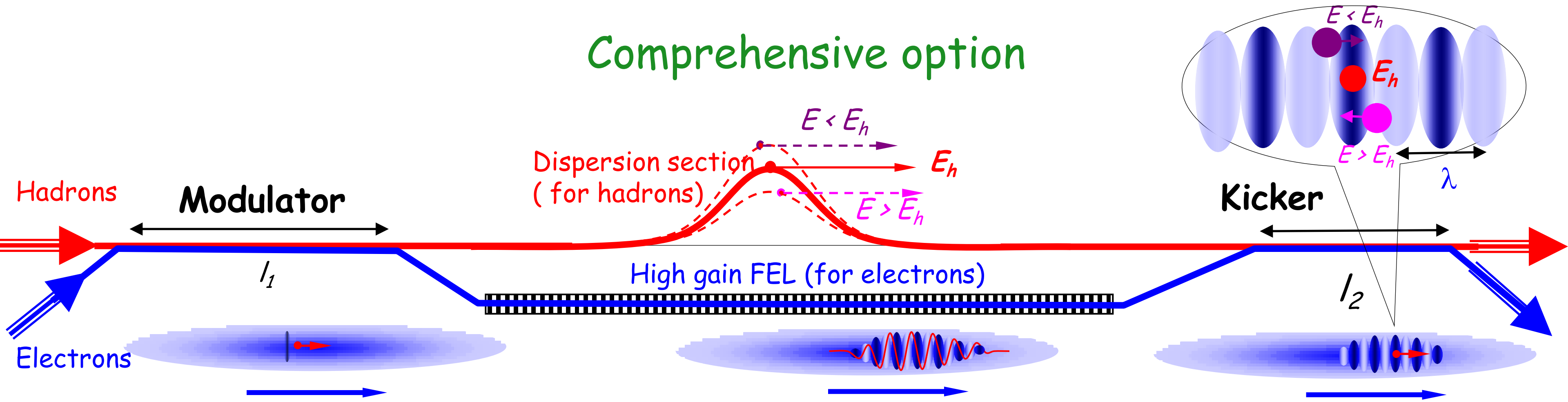
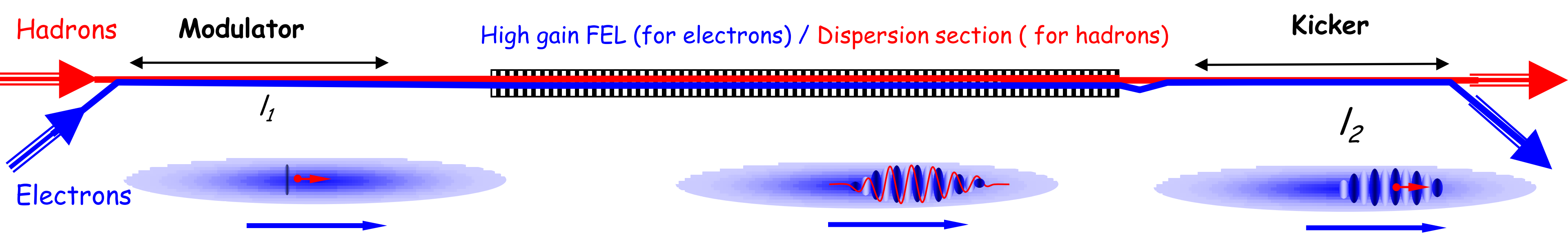
*Coherent Electron Cooling, Vladimir N. Litvinenko, Yaroslav S. Derbenev, Physical Review Letters **102**, 114801 (2009)
Original papers are in proceedings of FEL'07 and FEL'08 conferences

Coherent Electron Cooling

Comprehensive option



Economic option

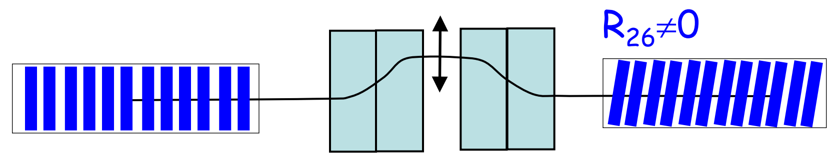


Transverse cooling

PAC09

- Transverse cooling can be obtained by using coupling with longitudinal motion via transverse dispersion
- Sharing of cooling decrements is similar to sum of decrements theorem for synchrotron radiation damping, i.e. decrement of longitudinal cooling can be split into appropriate portions to cool both transversely and longitudinally: $J_s+J_h+J_v=1$
- Vertical (better to say the second eigen mode) cooling is coming from transverse coupling

Non-achromatic chicane installed at the exit of the FEL before the kicker section turns the wave-fronts of the charged planes in electron beam



$$\delta(ct) = -R_{26}\cdot x$$

$$\Delta \mathbf{E} = -eZ^2\cdot E_o\cdot l_2\cdot \sin\left\{k\left(D\frac{\mathbf{E}-\mathbf{E}_o}{\mathbf{E}_o}+R_{16}x'-R_{26}x+R_{36}y'+R_{46}y\right)\right\};$$

$$\Delta x = -D_x\cdot eZ^2\cdot E_o\cdot L_2\cdot kR_{26}x+....$$

$$\zeta_\perp = J_\perp \zeta_{CeC};\quad \zeta_{//} = (1-2J_\perp)\zeta_{CeC};$$

$$\frac{d\varepsilon_x}{dt} = -\frac{\varepsilon_x}{\tau_{CeC\perp}};\quad \frac{d\sigma_\varepsilon^{\ 2}}{dt} = -\frac{\sigma_\varepsilon^{\ 2}}{\tau_{CeC//}}$$

$$\tau_{CeC\perp} = \frac{1}{2J_\perp\zeta_{CeC}};\quad \tau_{CeC\perp} = \frac{1}{2(1-2J_\perp)\zeta_{CeC}};$$

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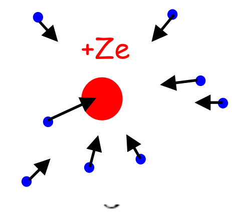
e-Density modulation caused by a hadron (co-moving frame)

Induces charge: $q = -Ze \cdot (1 - \cos \omega_p t)$



Analytical: for kappa-2 anisotropic electron plasma,

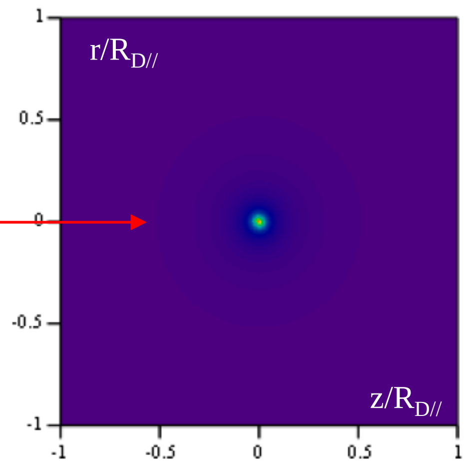
G. Wang and M. Blaskiewicz, Phys Rev E 78, 026413 (2008)

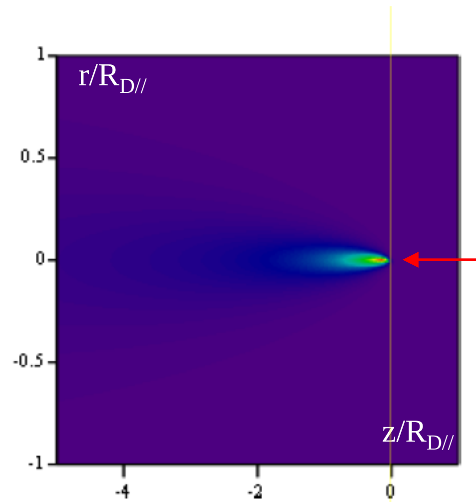
$$\tilde{n}(\vec{r},t) = \frac{Z n_o \omega_p^3}{\pi^2 \sigma_{vx}\sigma_{vy}\sigma_{vz}} \int_0^{\omega_p t} \tau \sin\tau \left(\tau^2 + \left(\frac{x - v_{hx}\tau/\omega_p}{r_{Dx}}\right)^2 + \left(\frac{y - v_{hy}\tau/\omega_p}{r_{Dy}}\right)^2 + \left(\frac{z - v_{hz}\tau/\omega_p}{r_{Dz}}\right)^2 \right)^{-2} d\tau$$

Density plots for a quarter of plasma oscillation

Ion rests in c.m.

(0,0) is the location of the ion





Ion moves in c.m. with

$$v_{hz} = 10\sigma_{vze}$$

(0,0) is the location of the ion

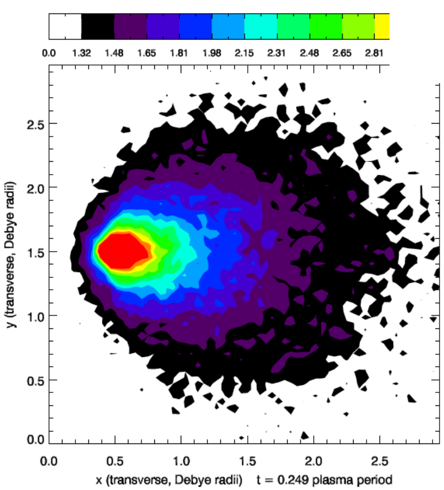


Figure 3: A transverse cross section of the wake behind a gold ion, with the color denoting density enhancement.

Numerical: VORPAL @ TechX)

Parameters of the problem

$$R_{D_\alpha} \propto \left(|v_\alpha| + \sigma_{v_\alpha}\right)/\omega_p; \quad \alpha = x,y,z$$

$$t = \tau/\omega_p; \ \vec{v} = \vec{\nu}\sigma_{v_z}; \ \vec{r} = \vec{\rho}\sigma_{v_z}/\omega_p; \ \omega_p = \sqrt{\frac{4\pi e^2 n_e}{m}} \qquad s = r_{D_z} = \frac{\sigma_{v_z}}{\omega_p}$$

$$R = \frac{\sigma_{v_\perp}}{\sigma_{v_z}}; \ T = \frac{v_{hx}}{\sigma_{v_z}}; \ L = \frac{v_{hz}}{\sigma_{v_z}}; \ \xi = \frac{Z}{4\pi n_e R^2 s^3};$$

$$A = \frac{a}{s}; \ X = \frac{x_{ho}}{a}; Y = \frac{y_{ho}}{a}.$$

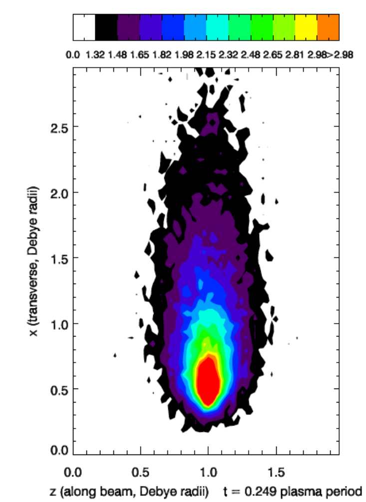
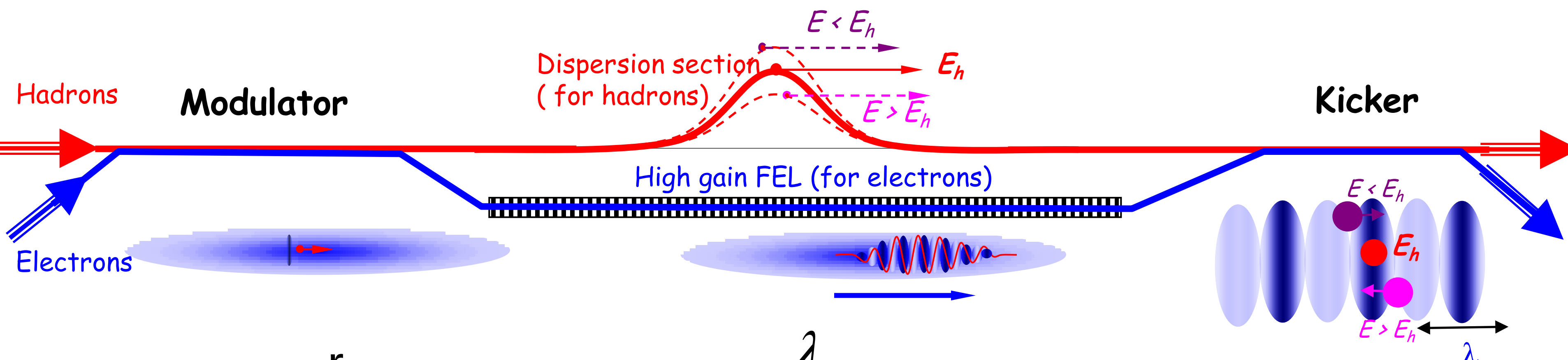


Figure 4: A longitudinal cross section of the wake behind a gold ion, with the color denoting density enhancement.

Central Section of CeC

$$D = D_{free} + D_{chicane};\ D_{free} = \frac{L}{\gamma^2};\ \ D_{chicane} = l_{chicane}\cdot\theta^2$$



$$\lambda_{fel} = \lambda_w\left(1+\langle \vec{a}_w^2\rangle\right)/2\gamma_o^2 \qquad L_{Go} = \frac{\lambda_w}{4\pi\rho\sqrt{3}} \qquad L_G = L_{Go}(1+\Lambda)$$

Electron density modulation is amplified in the FEL and made into a train with duration of $N_c \sim L_{gain}/\lambda_w$ alternating hills (high density) and valleys (low density) with period of FEL wavelength λ. Maximum gain for the electron density of High Gain FEL is ~ 10^3.

$$v_{group} = (c+2v_{//})/3 = c\left(1-\frac{1+a_w^2}{3\gamma^2}\right) = c\left(1-\frac{1}{2\gamma^2}\right) + \frac{c}{3\gamma^2}\left(1-2a_w^2\right) = v_{hadrons} + \frac{c}{3\gamma^2}\left(1-2a_w^2\right)$$

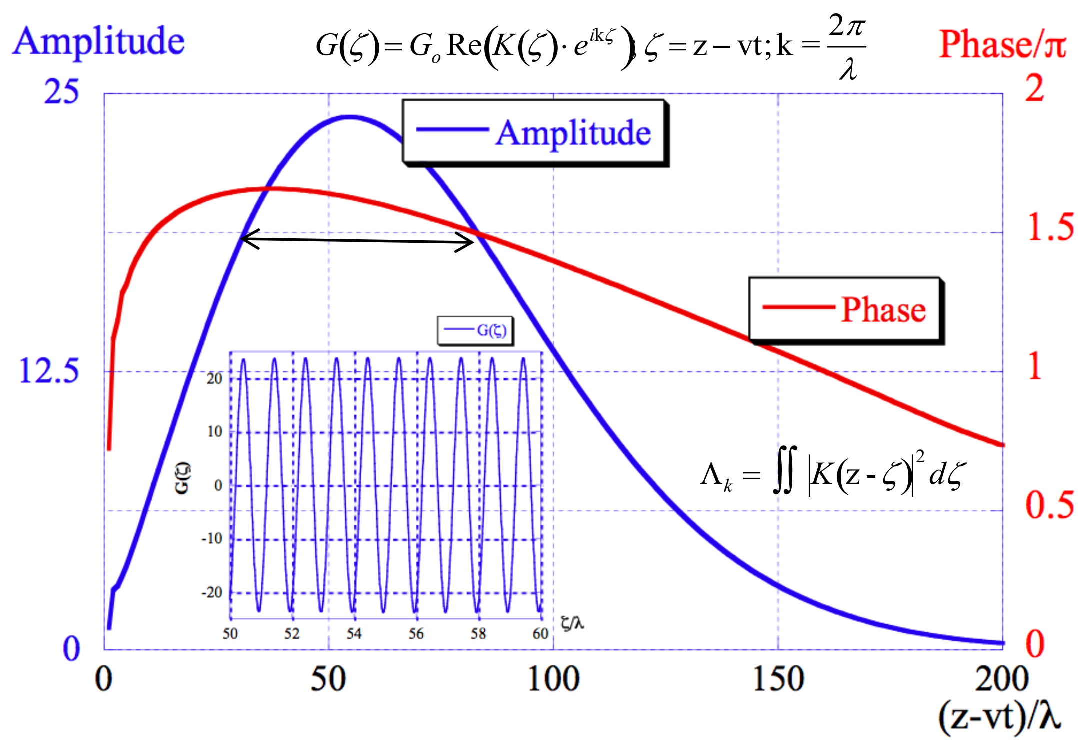
Economic option requires: $2a_w^2 < 1$!!!

3D FEL response calculated Genesis 1.3, confirmed by RON

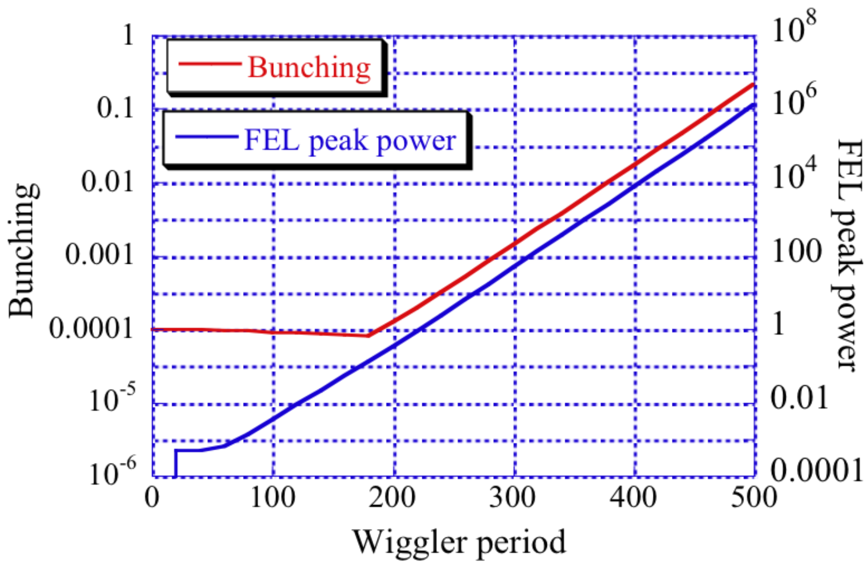
Main FEL parameters for eRHIC with 250 GeV protons

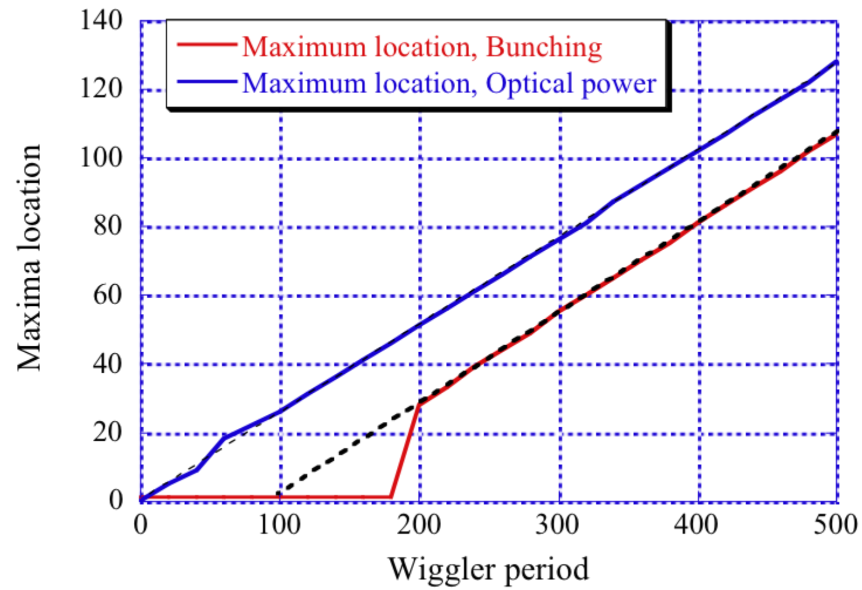
Energy, MeV	136.2	γ	266.45
Peak current, A	100	λ_o, nm	700
Bunchlength, psec	50	λ_w, cm	5
Emittance, norm	5 mm mrad	a_w	0.994
Energy spread	0.03%	Wiggler	Helical

The amplitude (**blue line**) and the phase (**red line**, in the units of π) of the FEL gain envelope after 7.5 gain-lengths (300 period). Total slippage in the FEL is 300λ, λ=0.5 μm. A clip shows the central part of the full gain function for the range of $\zeta=\{50\lambda, 60\lambda\}$.



Genesis: 3D FEL





Evolution of the maximum bunching in the e-beam and the FEL power simulated by Genesis.
The location of the maxima, both for the optical power and the bunching progresses with a lower speed compared with prediction by 1D theory,
i.e. electrons carry ~75% for the "information"

Evolution of the maxima locations in the e-beam bunching and the FEL power simulated by Genesis.
Gain length for the optical power is 1 m (20 periods) and for the amplitude/modulation is 2m (40 periods)

$$v_g \cong \frac{c + 3\langle v_z \rangle}{4} = c\left(1 - \frac{3}{8}\frac{1 + a_w^2}{\gamma_o^2}\right)$$

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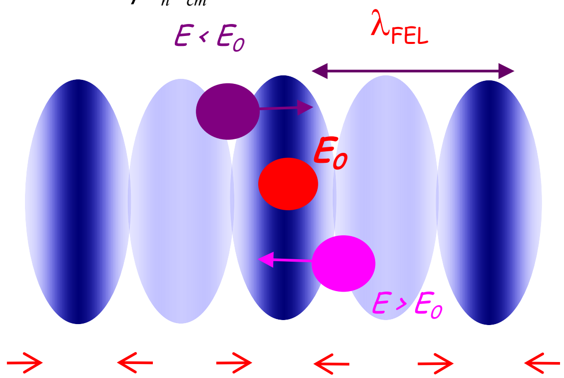
The Kicker

A hadron with central energy (E_o) phased with the hill where longitudinal electric field is zero, a hadron with higher energy ($E > E_o$) arrives earlier and is decelerated, while hadron with lower energy ($E < E_o$) arrives later and is accelerated by the collective field of electrons

Analytical estimation

$$\Delta\varphi = 4\pi\rho \Rightarrow \varphi = -\frac{8G\cdot Ze}{\pi\beta\varepsilon_n k_{cm}}\cdot\cos(k_{cm}z); \quad \vec{E} = -\vec{\nabla}\varphi = -\hat{z}\frac{8G\cdot Ze}{\pi\beta\varepsilon_n}\cdot\sin(k_{cm}z)$$



Periodical longitudinal electric field

$$\frac{d\mathbf{E}}{dz} = -eE_{peak}\cdot\sin\left\{kD\frac{\mathbf{E}-\mathbf{E}_o}{\mathbf{E}_o}\right\};$$

$$kD\sigma_\delta \sim 1$$

$$\sigma_\delta = \frac{\sigma_{\mathbf{E}}}{\mathbf{E}_o}$$

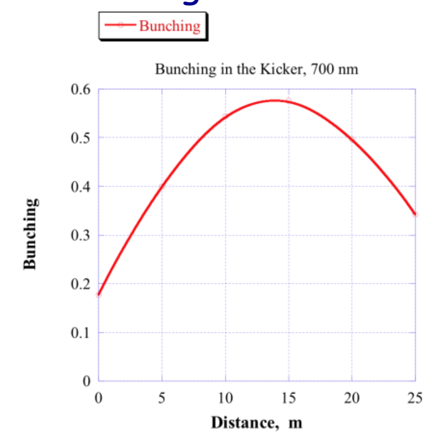
$$\zeta_{CEC} = -\frac{\Delta\mathbf{E}}{\mathbf{E}-\mathbf{E}_o} \approx \frac{e\cdot E_o\cdot l_2}{\gamma_o m_p c^2\cdot\sigma_\varepsilon}\cdot\frac{Z^2}{A}$$

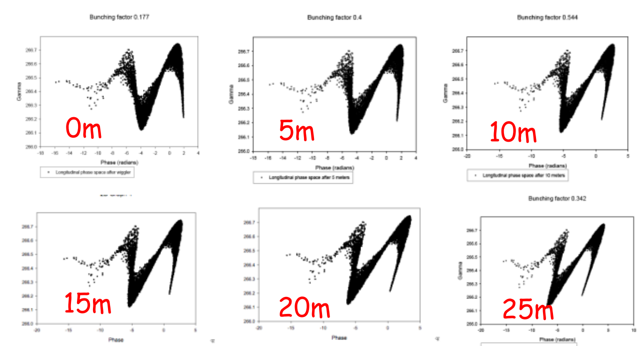
Simulations: only started

Step 1: use 3D FEL code out output + tracking
First simulation indicate that equations on the left significantly underestimate the kick, i.e. the density modulation continues to grow after beam leaves the FEL

Output from Genesis propagated for 25 m

©I.Ben Zvi





Step 2:
use VORPAL with input from Genesis, in preparation

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Analytical formula for damping decrement

$$\left\langle \zeta_{CeC} \right\rangle = \zeta \frac{\sigma_{\tau,e}}{\sigma_{\tau,h}} = \kappa \cdot 2G_o \cdot \frac{Z^2}{A} \cdot \frac{r_p \cdot \sigma_{\tau,e}}{\varepsilon_{\perp n}\left(\sigma_\delta \cdot \sigma_{\tau,h}\right)};\ \kappa \sim 1$$

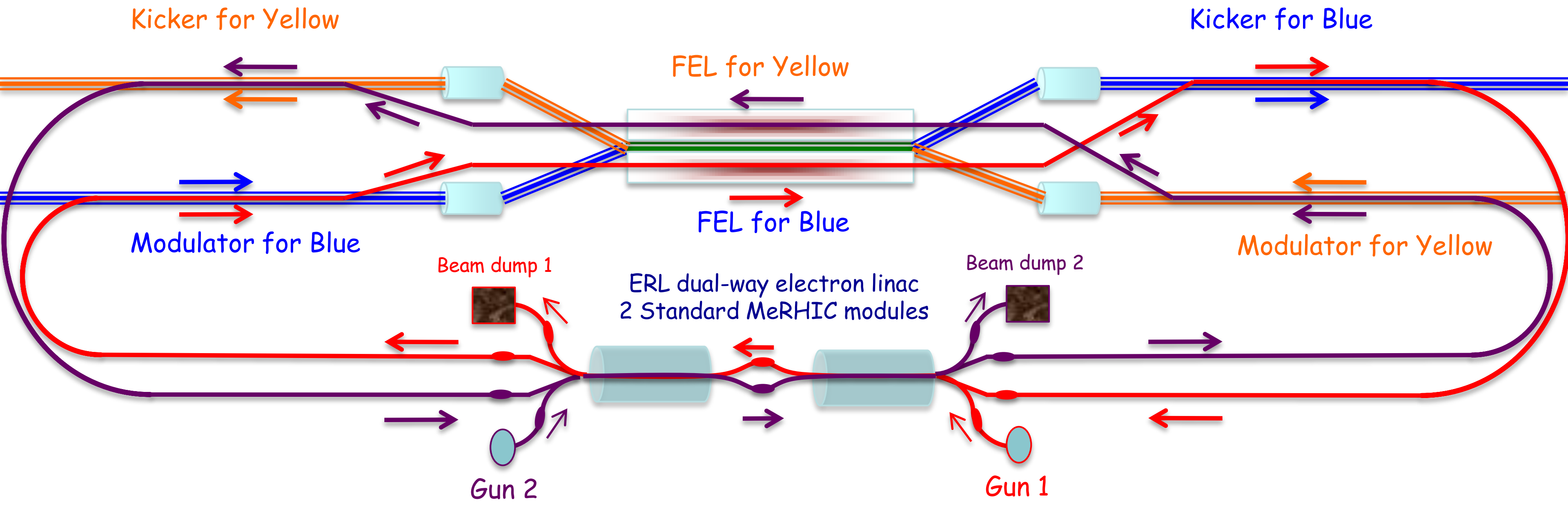
$$\left\langle \zeta_{CeC} \right\rangle \sim \frac{1}{\varepsilon_{long,h}\varepsilon_{trans,h}}$$

Note that damping decrement

a) **Does not depend on the energy of particles !**
b) **Improves as cooling goes on**

It makes it realistic to think about cooling intense proton beam in RHIC & LHC at 100s of GeV and 7 TeV energies
Even though LHC needs one more trick *(back up slides)*

Possible layout in RHIC IP of CeC driven by a single linac - **to boost polarized pp- luminosity**



E_p, GeV	γ	E_e, MeV
100	106.58	54.46
250	266.45	136.15
325	346.38	177.00

Example: CeC vs. IBS at RHIC

J.LeDuff, "Single and Multiple Touschek effects",
Proceedings of CERN Accelerator School,
Rhodes, Greece, 20 September - 1 October, 1993,
Editor: S.Turner, CERN 95-06, 22 November 1995, Vol. II, p. 573

$$\frac{\sigma_\varepsilon^2}{\tau_{IBS\,//}} = \frac{N r_c^2 c}{2^5 \pi \gamma^3 \varepsilon_x^{3/2} \sigma_s} \left\langle \frac{f(\chi_m)}{\beta_y \nu} \right\rangle; \quad \frac{\varepsilon_x}{\tau_{IBS\perp}} = \frac{N r_c^2 c}{2^5 \pi \gamma^3 \varepsilon_x^{3/2} \sigma_s} \left\langle \frac{H}{\beta_y^{1/2}} f(\chi_m) \right\rangle; \kappa = 1$$

$$f(\chi_m) = \int_{\chi_m}^{\infty} \frac{d\chi}{\chi} \ln\left(\frac{\chi}{\chi_m}\right) e^{-\chi}; \; \chi_m = \frac{r_c m^2 c^4}{b_{max} \sigma_E^2}; b_{max} \cong n^{-1/3}; \; r_c = \frac{e^2}{mc^2}; \quad (e -> Ze; m -> Am)$$

IBS in RHIC for 250 GeV, $N_p = 2\cdot 10^{11}$ were scaled from the data below
Reference value was provided by A.Fedotov using Beta-cool code © Dubna

$$X = \frac{\varepsilon_x}{\varepsilon_{xo}}; \; S = \left(\frac{\sigma_s}{\sigma_{so}}\right)^2 = \left(\frac{\sigma_E}{\sigma_{sE}}\right)^2;$$

$$\frac{dX}{dt} = \frac{1}{\tau_{IBS\perp}} \frac{1}{X^{3/2} S^{1/2}} - \frac{\xi_\perp}{\tau_{CeC}} \frac{1}{S};$$

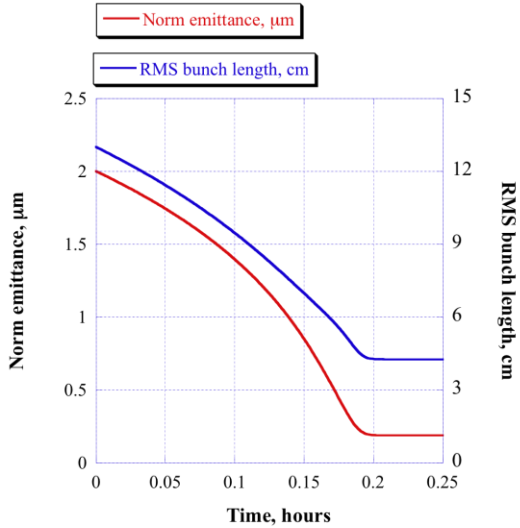
$$\frac{dS}{dt} = \frac{1}{\tau_{IBS\,//}} \frac{1}{X^{3/2} S^{1/2}} - \frac{1 - 2\xi_\perp}{\tau_{CeC}} \frac{1}{X};$$

$$\varepsilon_{xn0} = 2\,\mu m; \; \sigma_{s0} = 13\ cm; \; \sigma_{\delta 0} = 4\cdot 10^{-4}$$
$$\tau_{IBS\perp} = 4.6\ hrs; \; \tau_{IBS\,//} = 1.6\ hrs;$$

Stationary solution:

$$X = \frac{\tau_{CeC}}{\sqrt{\tau_{IBS\,//}\tau_{IBS\perp}}} \frac{1}{\sqrt{\xi_\perp(1-2\xi_\perp)}}; \quad S = \frac{\tau_{CeC}}{\tau_{IBS\,//}} \cdot \sqrt{\frac{\tau_{IBS\perp}}{\tau_{IBS\,//}}} \cdot \sqrt{\frac{\xi_\perp}{(1-2\xi_\perp)^3}}$$

$$\varepsilon_{xn} = 0.2\,\mu m; \; \sigma_s = 4.9\ \text{cm}$$



This may allow

a) RHIC pp - keep the luminosity at beam-beam limit all the time
b) RHIC pp – reduce bunch length to few cm (from present 1 m)
   1. to reduce hourglass effect
   2. To concentrate event in short vertexes of the detectors
c) eRHIC - reduce polarized beam current down to 50 mA while keeping the same luminosity
d) eRHIC - increase electron beam energy to 20 GeV
e) Both - increase luminosity by reducing β* to 5-10 cm from present 0.5m

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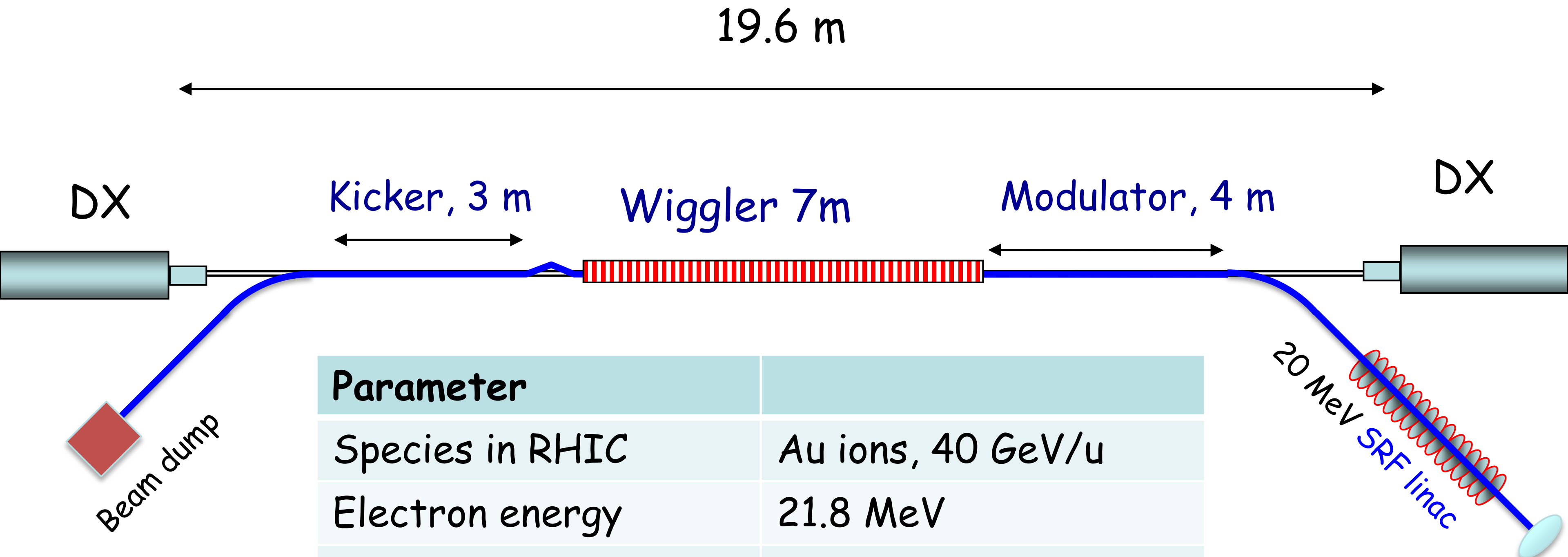
Possible layout for Coherent Electron Cooling proof-of-principle experiment in RHIC IR



Parameter	
Species in RHIC	Au ions, 40 GeV/u
Electron energy	21.8 MeV
Charge per bunch	1 nC
Train	5 bunches
Rep-rate	78.3 kHz
e-beam current	0.39 mA
e-beam power	8.5 kW

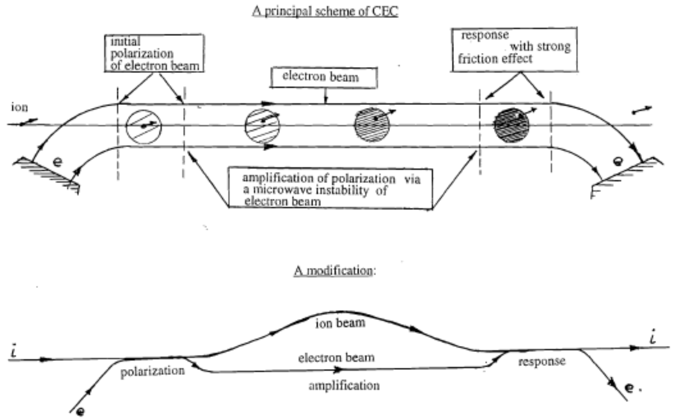
Content

- A bit of history
- Principles of Coherent Electron Cooling (CeC)
- Analytical estimations, Simulations
- Proof of Principle test using R&D ERL
- Conclusions

History

possibility of coherent electron cooling was discussed qualitatively by Yaroslav Derbenev about 28 years ago

- Y.S. Derbenev, Proceedings of the 7th National Accelerator Conference, V. 1, p. 269, (Dubna, Oct. 1980)
- Coherent electron cooling, Ya. S. Derbenev, Randall Laboratory of Physics, University of Michigan, MI, USA, UM HE 91-28, August 7, 1991
- Ya.S.Derbenev, Electron-stochastic cooling, DESY , Hamburg, Germany, 1995 ……….



UM HE 91-28

August 7, 1991

COHERENT ELECTRON COOLING

1. Physics of the method in general

Ya. S. Derbenev

Randall Laboratory of Physics, University of Michigan

Ann Arbor, Michigan 48109-1120 USA

Conclusion

The method considered above combines principles of electron and stochastic cooling and microwave amplification. Such an unification promises to frequently increase the cooling rate and stacking of high-temperature, intensive heavy particle beams. Certainly, for the whole understanding of new possibilities thorough theoretical study is required of all principle properties and other factors of the method.

Q: What's new in today's presentation?

- It is a new CeC is the scheme and the first with complete analytical and quantitative evaluation
- The spirit of amplifying the interaction remains the same as in 80's. but the underlying physics of interaction is different and also specific
- ERLs and FEL did advanced in last 30 years – hence, the practicality of the scheme
- Now we can analytically estimate and numerically calculate CeC cooling decrements for a wide variety of cases

[1] *Coherent Electron Cooling*, V.N. Litvinenko, Y.S. Derbenev, Physical Review Letters 102, 114801 (Feb 2009)

[2] *Free Electron Lasers and High-energy Electron Cooling*, V.N. Litvinenko, Y.S. Derbenev, Proc. FEL'07, P. 268-275 (Sep 2007)

[3] *Use of an Electron Beam for Stochastic Cooling*, Y.S. Derbenev, COOL'07 (2007)

[4] *FEL-based Coherent Electron Cooling for High-energy Hadron Colliders*, V.N. Litvinenko, Y.S. Derbenev, Proc. EPAC'09, WEPP016 (2008)

[5] *The Dynamics of Ion Shielding in an Anisotropic Electron Plasma*, G. Wang and M. Blaskiewicz, Phys Rev E 78, 026413 (2008)

[6] *Progress with FEL-base coherent electron cooling*, V.N.Litvinenko, I. Ben Zvi, M. Blaskiewicz, Y.Hao, D.Kayran, E.Pozdeyev, G. Wang, G.I. Bell, D.L. Bruhwiler, A. Sobol, O.A. Shevchenko, N.A. Vinokurov, Y.S. Derbenev, S. Reiche, FEL'08, THDAU05, (2008)

[7] *High Gain FEL Amplification of Charge Modulation Caused by a Hadron*, V.N.Litvinenko, J. Bengtsson, I. Ben Zvi, Y.Hao, D.Kayran, E.Pozdeyev, G. Wang, S. Reiche, O.A. Shevchenko, N.A. Vinokurov, FEL'08, MOPPH026 (2008)

[8] *VORPAL Simulations Relevant to Coherent Electron Cooling*, G.I. Bell, D.L. Bruhwiler, A.V. Sobol, I. Ben-Zvi, V.N. Litvinenko, Y. Derbenev, EPAC'08, (2008)

[9] *Simulation of Coherent Electron Cooling for High-Intensity Hadron Colliders*, D.L. Bruhwiler, G.I. Bell, A.V. Sobol, I. Ben-Zvi, V.N. Litvinenko, Y.S. Derbenev, Proc. HB2008 (2008)

[10] *Analytical Studies of Coherent Electron Cooling*, G. Wang, M. Blaskiewicz, V. N. Litvinenko, this conference

[11] *Simulating Electron-Ion Dynamics in Relativistic Electron Coolers*, D.L. Bruhwiler, Invited talk, this conference

[12] *Integrated modeling of the modulator, amplifier and kicker in a Coherent Electron Cooling system*, G.I. Bell, D.L. Bruhwiler, A.V. Sobol, V.N. Litvinenko, E. Pozdeyev and I. Ben-Zvi, this conference

……….

Thesis: G. Wang, SBU (def. 2008), S. Webb, SBU (since 2008)….

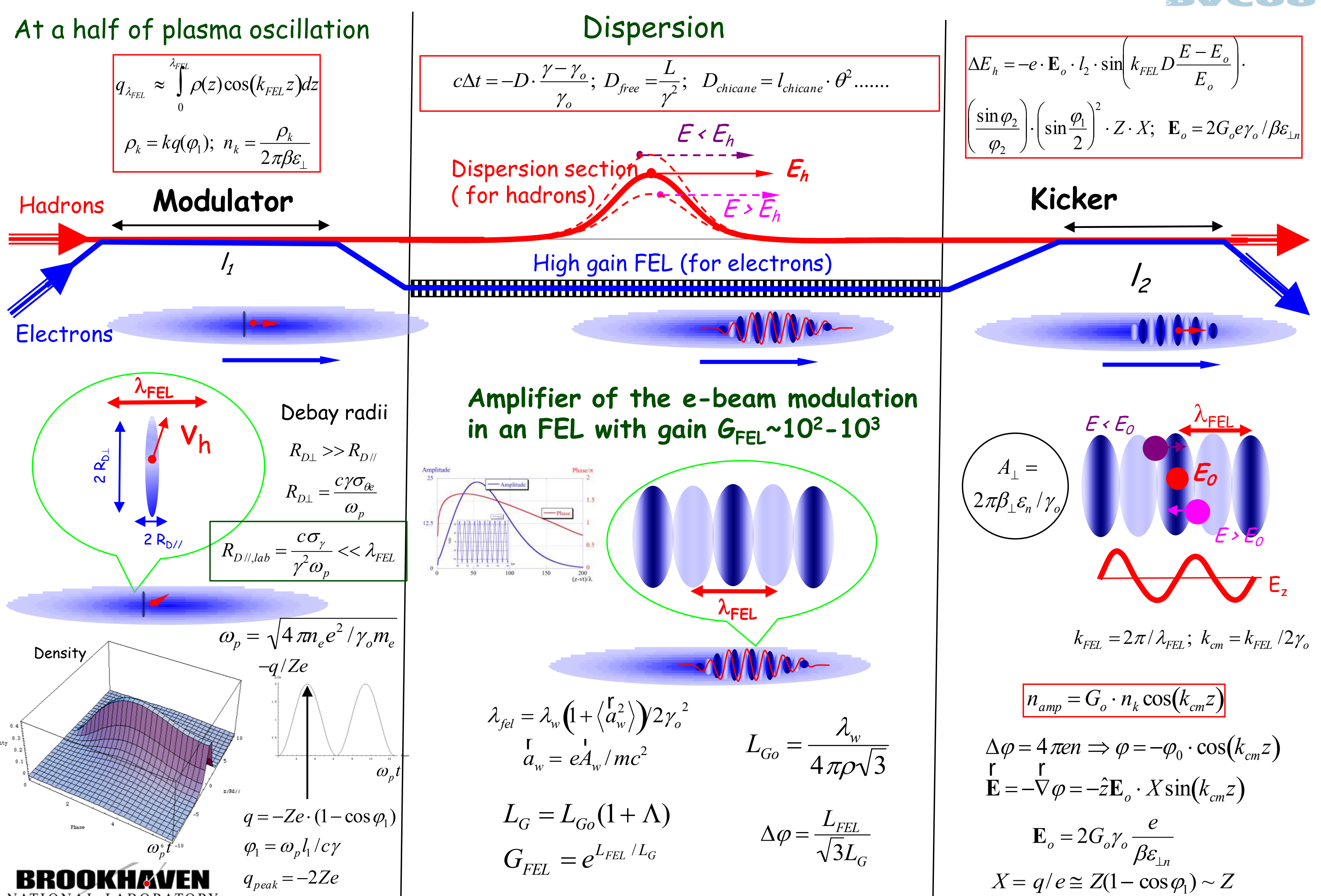
Examples of hadron beams cooling

Machine	Species	Energy GeV/n	Trad. Stochastic Cooling, hrs	Synchrotron radiation, hrs	Trad. Electron cooling hrs	Coherent Electron Cooling, hrs 1D/3D
RHIC PoP	*Au*	*40*	-	-	~ 1	*0.02/0.06*
eRHIC	Au	130	~1	20,961 ∞	~ 1	0.015/0.05
eRHIC	**p**	**325**	**~100**	**40,246 ∞**	**> 30**	**0.1/0.3**
LHC	p	7,000	~ 1,000	13/26	∞ ∞	0.3/<1

Potential increases in luminosities:

RHIC polarized pp ~ 2-4 fold, eRHIC ~ 5-10 fold, LHC ~ 2 fold

Start from longitudinal cooling , ultra-relativistic case (γ>>1)



Analytical formula for damping decrement

PAC09

- 1/2 of plasma oscillation in the modulator creates a pancake of electrons with the charge -2Ze
- electron clamp is well within $\Delta z \sim \lambda_{FEL}/2\pi$
- gain in SASE FEL is $G \sim 10^2$-10^3
- electron beam is wider than $2\gamma_o \lambda_{FEL}$ - it is 1D field
- Length of the kicker is ~ β-function

$$\zeta = -\frac{\Delta E_i}{E - E_o} = A \cdot \frac{L_2}{\beta} \cdot \chi \cdot \frac{\sin\varphi_3}{\varphi_3} \cdot \frac{\sin\varphi_2}{\varphi_2} \cdot \left(\sin\frac{\varphi_1}{2}\right)^2$$

$$A = 2G_o \frac{Z^2}{A} \cdot \frac{r_p}{\varepsilon_{\perp n}\sigma_\delta}; \quad \chi = k_{FEL} D \cdot \sigma_\delta;$$

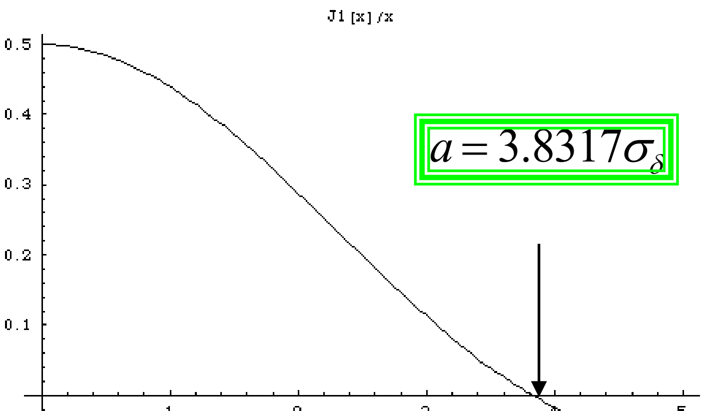
$$\varphi_3 = k_{FEL} D\delta; \quad \delta = \frac{E - E_o}{E_o}$$

$$\delta = a \cdot \sin\Omega_s t$$

$$\langle \delta^2 \rangle' = -\left\langle 2A \cdot a^2 \cdot \cos^2\Omega_s t \cdot \sin\left(\frac{a}{\sigma_\delta} \cdot \chi \cdot \sin\Omega_s t\right)\right\rangle$$

$$= -2A \cdot \langle \delta^2 \rangle \cdot J_1\left(\chi \cdot \frac{a}{\sigma_\delta}\right)$$

$$\frac{L_2}{\beta} \cdot \chi \cdot \mathrm{sinc}(\varphi_3) \cdot \mathrm{sinc}\varphi_2 \cdot \left(\sin\frac{\varphi_1}{2}\right)^2 \sim 1$$



$$a = 3.8317\sigma_\delta$$

Beam-Average decrement

$$\int \frac{2J_1(x)}{x} e^{-x^2/2} dx = 0.889$$

•Electron bunches are usually much shorter and cooling time for the entire bunch is proportional to the bunch-lengths ratios

$$\chi = 1$$

$$\langle \zeta_{CeC} \rangle = \zeta \frac{\sigma_{z,e}}{\sigma_{z,h}} = 2\frac{G_o \sigma_{z,e}}{\sigma_\delta \sigma_{z,h}} \frac{Z^2}{A} \frac{r_p}{\varepsilon_{\perp n}} \cdot \kappa; \qquad \kappa \sim 1$$

ε_1

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Office of Science U.S. DEPARTMENT OF ENERGY

Effects of the surrounding particles

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Each charged particle causes generation of an electric field wave-packet proportional to its charge and synchronized with its initial position in the bunch

$$\mathbf{E}_{total}(\zeta) = \mathrm{E}_o \cdot \mathrm{Im}\left(X \cdot \sum_{i,hadrons} K(\zeta - \zeta_i) e^{ik(\zeta - \zeta_i)} - \sum_{j,electrons} K(\zeta - \zeta_j) e^{ik(\zeta - \zeta_j)} \right)$$

$$\mathbf{E}_o = 2G_o \cdot \gamma_o \cdot \frac{e}{\beta \varepsilon_{\perp n}}$$

$$X = q/e \cong Z(1 - \cos\varphi_1) \sim Z$$

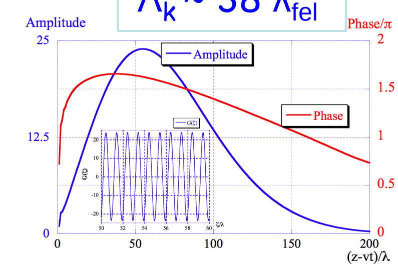
Evolution of the RMS value resembles stochastic cooling!
Best cooling rate achievable is ~ $1/N_{eff}$, N_{eff} is effective number of hadrons in coherent sample ($\Lambda_k = N_c \lambda$)

$\Lambda_k \sim 38\ \lambda_{fel}$

$$\left\langle \delta^2 \right\rangle' = -2\xi \left\langle \delta^2 \right\rangle + D$$

$$\Lambda_k = \iint \left| K(z - \zeta) \right|^2 d\zeta$$

$$N_{eff} \cong N_h \frac{\Lambda_k}{\sqrt{4\pi}\sigma_{z,h}} + \frac{N_e}{X^2} \frac{\Lambda_k}{\sqrt{4\pi}\sigma_{z,e}}$$



$$\xi = -g \left\langle \delta_i \,\mathrm{Im}\left(K(\Delta\zeta_i) e^{ik\Delta\zeta_i} \right) \right\rangle / \left\langle \delta^2 \right\rangle; \; D = g^2 N_{eff}/2;$$

$$g = G_o \frac{Z^2}{A} \frac{r_p}{\varepsilon_{\perp n}} \left\{ 2 f(\varphi_2)(1 - \cos\varphi_1) \frac{l_2}{\beta} \cdot \right\},$$

$$\xi_{CeC}(\max) = \frac{\Delta}{2\sigma_\gamma} = \frac{2}{N_{eff}} (kD\sigma_\varepsilon) \propto \frac{1}{N_{eff}}$$

Fortunately, the bandwidth of FELs $\Delta f \sim 10^{13}$-10^{15} Hz is so large that this limitation does not play any practical role in most HE cases

Conclusions

- Coherent electron cooling has potential of cooling high intensity TeV scale proton and ion beams with reasonable (under an hour) cooling time
- Electron accelerator of choice for such cooler is energy recovery linac (ERL)
- ERL seems to be capable of providing required beam quality for such coolers
- Majority of the technical limitation and requirements on the beam and magnets stability are well within limit of current technology, even though satisfying all of them in nontrivial fit
- We plan a proof of principle experiment of coherent electron cooling with Au ions in RHIC at ~ 40 GeV/n and existing R&D ERL as part of eRHIC R&D

Conclusions

- Coherent electron cooling is very promising method for significant luminosity increases in hadron colliders from RHIC to LHC
- Initial studies did not find any phenomena, which challenges the concept of CeC
- Our CeC estimations passed a number of tests
- At the same time, we found a number of new and interesting details to pursue further
- Future studies will refine the model and improve the quality of predictions
- We plan to test validity of the concept experimentally in Proof-of-Principle experiment using BNL's R&D ERL installed in one of available IPs at RHIC

Supported by the Office on Nuclear Physics, US DoE

Coherent electron cooling, ultra-relativistic case ($\gamma >> 1$)

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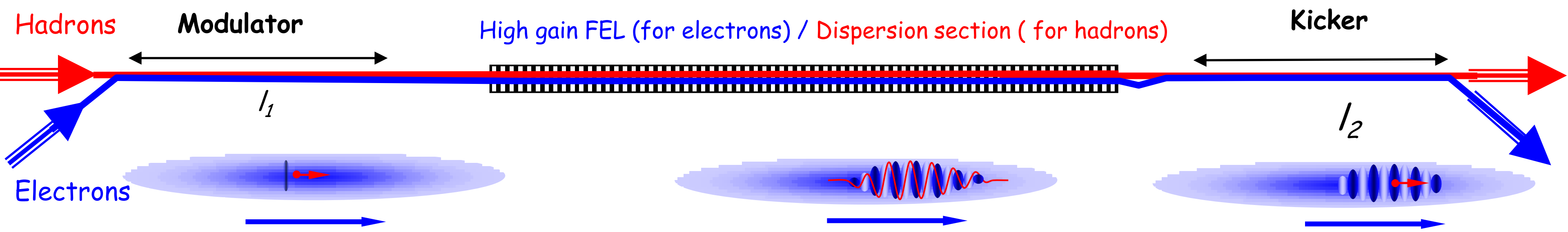
Economic option



Electrons

Modulator: region 1
a quarter to a half of plasma oscillation

Amplifier of the e-beam modulation via High Gain FEL and
Longitudinal dispersion for hadrons

Kicker: region 2

Electron density modulation is amplified in the FEL and made into a train with duration of $N_c \sim L_{gain}/\lambda_w$ alternating hills (high density) and valleys (low density) with period of FEL wavelength λ. Maximum gain for the electron density of HG FEL is ~ 10^3.

$$v_{group} = (c + 2v_{//})/3 = c\left(1 - \frac{1 + a_w^2}{3\gamma^2}\right) = c\left(1 - \frac{1}{2\gamma^2}\right) + \frac{c}{3\gamma^2}\left(1 - 2a_w^2\right) = v_{hadrons} + \frac{c}{3\gamma^2}\left(1 - 2a_w^2\right)$$

Economic option requires: $2a_w^2 < 1$!!!

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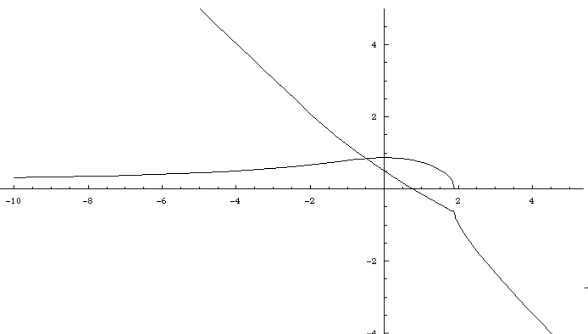
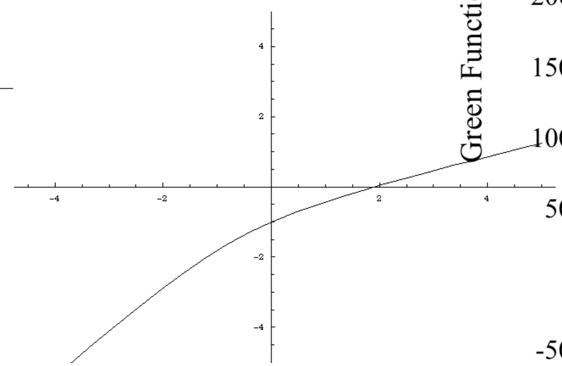
Response - 1D FEL after 10 gain lengths

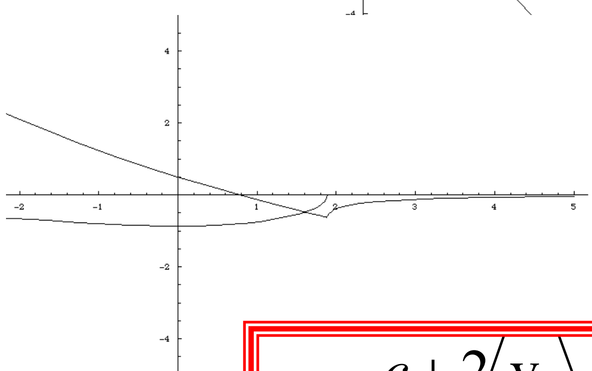
$$\left\{\left\{x \to \frac{1}{3}\left(-2 i c - \frac{2^{1/3} c^2}{\left(27 i - 2 i c^3 + 3\sqrt{3}\sqrt{-27+4c^3}\right)^{1/3}} + \frac{\left(27 i - 2 i c^3 + 3\sqrt{3}\sqrt{-27+4c^3}\right)^{1/3}}{2^{1/3}}\right)\right\},\right.$$

$$\left\{x \to -\frac{2 i c}{3} + \frac{\left(1 + i\sqrt{3}\right) c^2}{3\, 2^{2/3}\left(27 i - 2 i c^3 + 3\sqrt{3}\sqrt{-27+4c^3}\right)^{1/3}} - \frac{\left(1 - i\sqrt{3}\right)\left(27 i - 2 i c^3 + 3\sqrt{3}\sqrt{-27+4c^3}\right)^{1/3}}{6\, 2^{1/3}}\right\},$$

$$\left.\left\{x \to -\frac{2 i c}{3} + \frac{\left(1 - i\sqrt{3}\right) c^2}{3\, 2^{2/3}\left(27 i - 2 i c^3 + 3\sqrt{3}\sqrt{-27+4c^3}\right)^{1/3}} - \frac{\left(1 + i\sqrt{3}\right)\left(27 i - 2 i c^3 + 3\sqrt{3}\sqrt{-27+4c^3}\right)^{1/3}}{6\, 2^{1/3}}\right\}\right\}$$

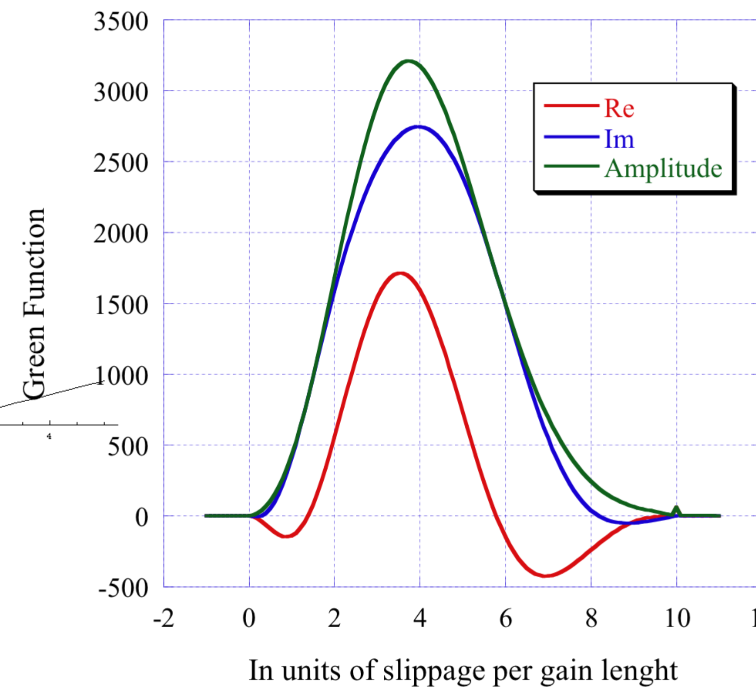






$$v_g = \frac{c + 2\langle v_z\rangle}{3} = c\left(1 - \frac{1 + a_w^2}{3\gamma_o^2}\right)$$

Green-function envelope (Abs, Re and Im)



Maximum located at 3.744 slippage units,
(i.e. just a bit further that expected 3 and 1/3)
The Green function (with oscillations) had effective RMS length of 1.48 slippage units.

FEL's Green Function

1D - analytical approach $G(\tau;z) = \mathrm{Re}\left(\tilde{G}_z(\tau)e^{i\omega_o\tau}\right)$

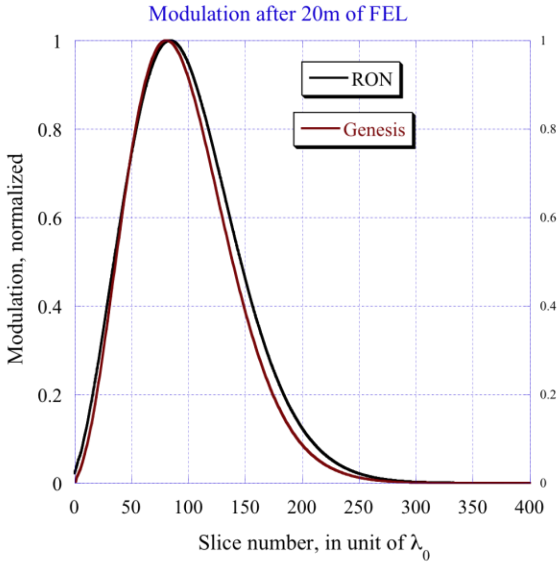
3D - 3D FEL codes RON and Genesis 1.3

FEL parameters for Genesis 1.3 and RON simulations

FEL gain length: 1 m (power), 2m (amplitude)

Main FEL parameters for eRHIC with 250 GeV protons

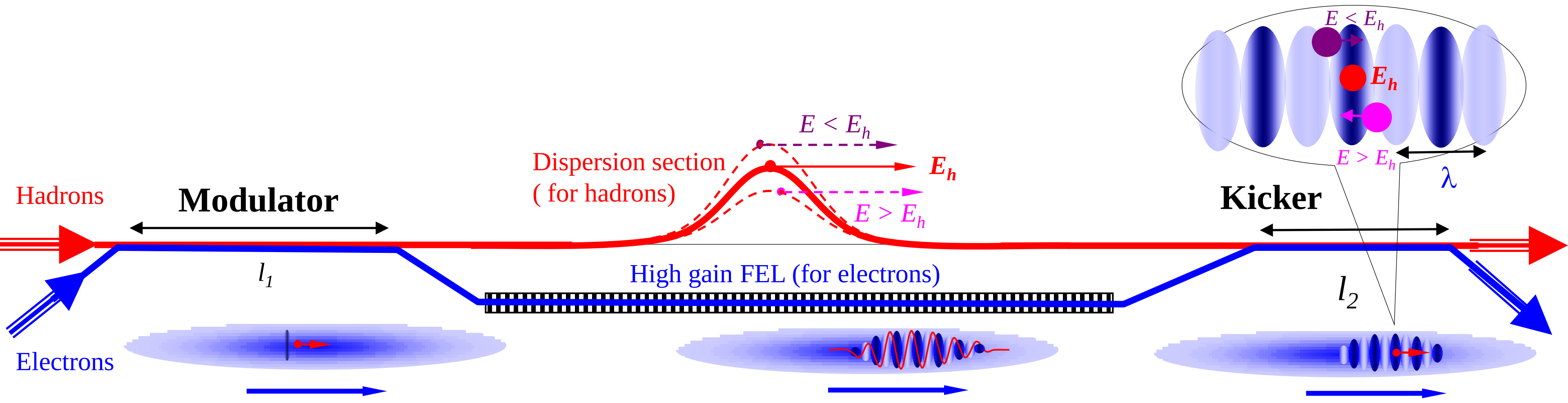
Energy, MeV	136.2	γ	266.45
Peak current, A	100	λ_o, nm	700
Bunchlength, psec	50	λ_w, cm	5
Emittance, norm	5 mm mrad	a_w	0.994
Energy spread	0.03%	Wiggler	Helical



CeC: FEL response

$$f_{input}(\vec{r}_\perp,\vec{p},t) = f_{o\ input}(\vec{r}_\perp,\vec{p}) + \delta f(\vec{r}_\perp,\vec{p},t)$$

$$f_{exit}(\vec{r}_\perp,\vec{p},t) = f_{o\ exit}(\vec{r}_\perp,\vec{p}) + \int K(\vec{r}_\perp,\vec{p},\vec{r}_{\perp 1},\vec{p}_1,t-t_1)\cdot \delta f(\vec{r}_1,\vec{p}_1,t_1)\cdot d\vec{r}_{\perp 1} d\vec{p}_1 dt_1$$



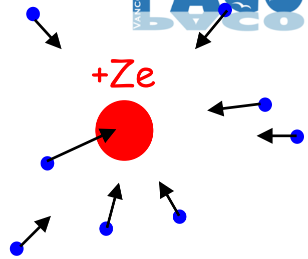
1D FEL response

$$\rho_{exit}(t;z) = \rho_o + \int G(\tau;z)\cdot \delta\rho(t-\tau;0)\cdot d\tau$$

$$G(\tau;z) = \mathrm{Re}\left(\tilde{G}_z(\tau)e^{i\omega_o\tau}\right) \qquad \omega_0 = \frac{2\pi c}{\lambda_o};$$

Modulator

Dimensionless equations of motion



$$\frac{\partial f_e}{\partial t} + \frac{\partial f_e}{\partial \vec{v}}\cdot\frac{e\vec{E}}{m} + \frac{\partial f_e}{\partial \vec{r}}\cdot\vec{v} = 0; \quad \vec{r}_h(t) \cong \vec{r}_o + \vec{v}_h t;$$

$$\left(\vec{\nabla}\cdot\vec{E}\right) = 4\pi e n_e\left(\frac{Z}{n_e}\delta\left(\vec{r} - \vec{r}_h(t)\right) - \int f_e d\vec{v}^3\right).$$

$$\frac{\partial f_e}{\partial \tau} + \frac{\partial f_e}{\partial \vec{v}}\cdot\vec{g} + \frac{\partial f_e}{\partial \vec{\rho}}\cdot\vec{v} = 0; \quad \vec{g} = \frac{e\vec{E}}{m\omega_p{}^2 s};$$

$$\left(\vec{\nabla}_n\cdot\vec{g}\right) = \frac{Z}{s^3 n_e}\delta\left(\vec{\rho} - \vec{\rho}_i(t)\right) - \int f_e d\vec{v}^3; \quad \vec{\nabla}_n \equiv \partial_{\vec{\rho}}.$$

$$t = \tau/\omega_p; \quad \vec{v} = \vec{v}\sigma_{v_z}; \quad \vec{r} = \vec{\rho}\sigma_{v_z}/\omega_p; \quad \omega_p^2 = \frac{4\pi e^2 n_e}{m}$$

$$s = r_{D_z} = \sigma_{v_z}/\omega_p$$

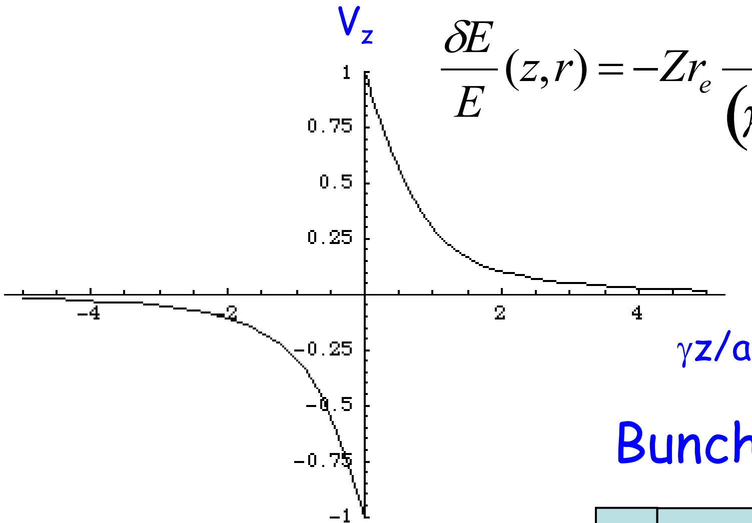
Parameters of the problem

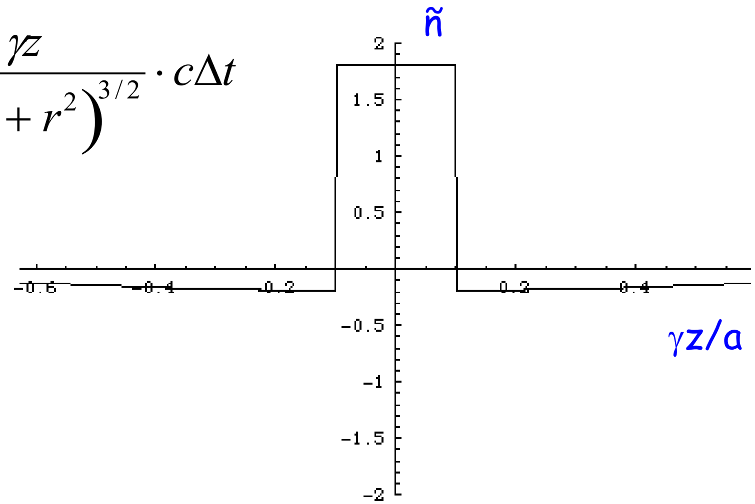
$$R = \frac{\sigma_{v_\perp}}{\sigma_{v_z}}; \quad T = \frac{v_{hx}}{\sigma_{v_z}}; \quad L = \frac{v_{hz}}{\sigma_{v_z}}; \quad \xi = \frac{Z}{4\pi n_e R^2 s^3};$$

$$A = \frac{a}{s}; \quad X = \frac{x_{ho}}{a}; Y = \frac{y_{ho}}{a}.$$

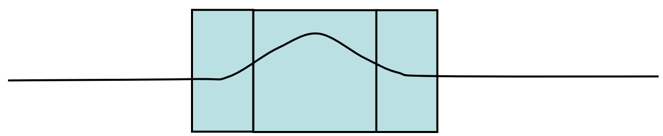
Velocity map & buncher (γ>1000)

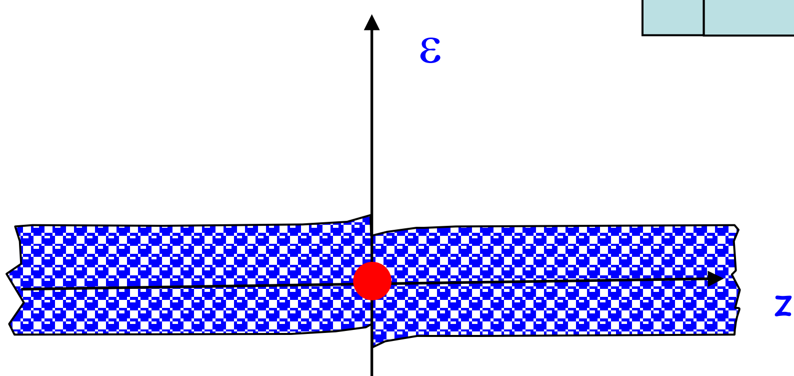
$$\frac{\delta E}{E}(z,r) = -Z r_e \frac{\gamma z}{\left(\gamma^2 z^2 + r^2\right)^{3/2}} \cdot c\Delta t$$

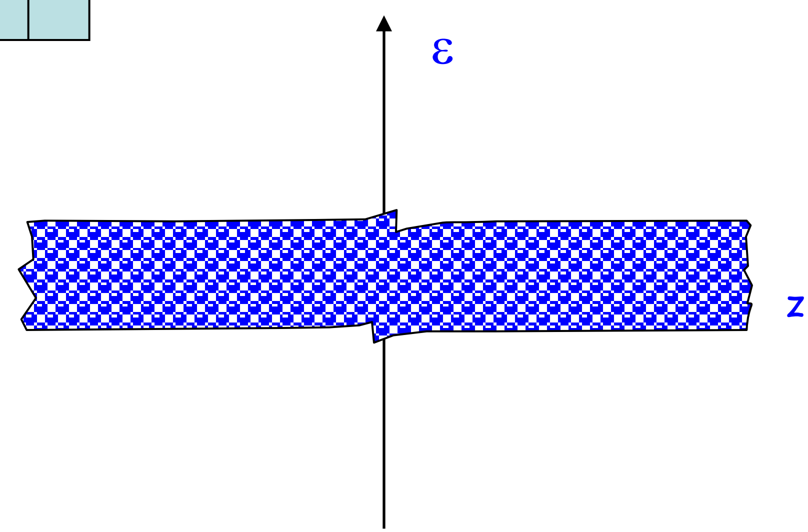




Buncher

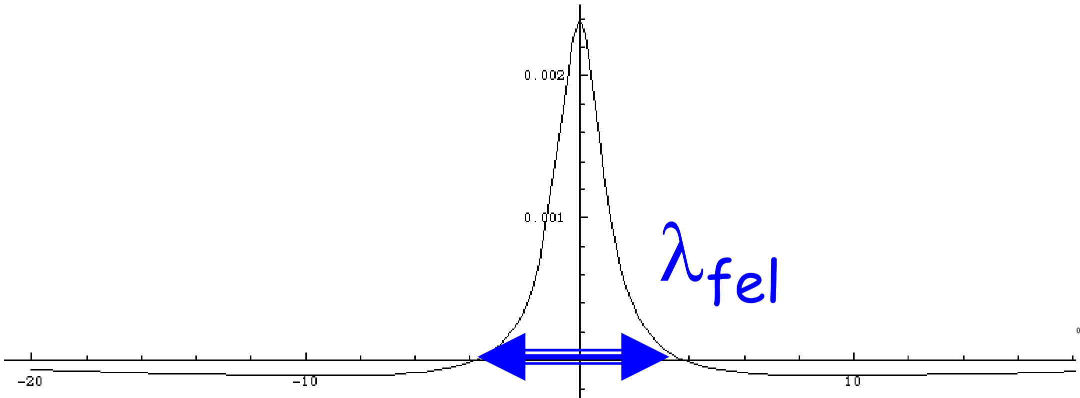


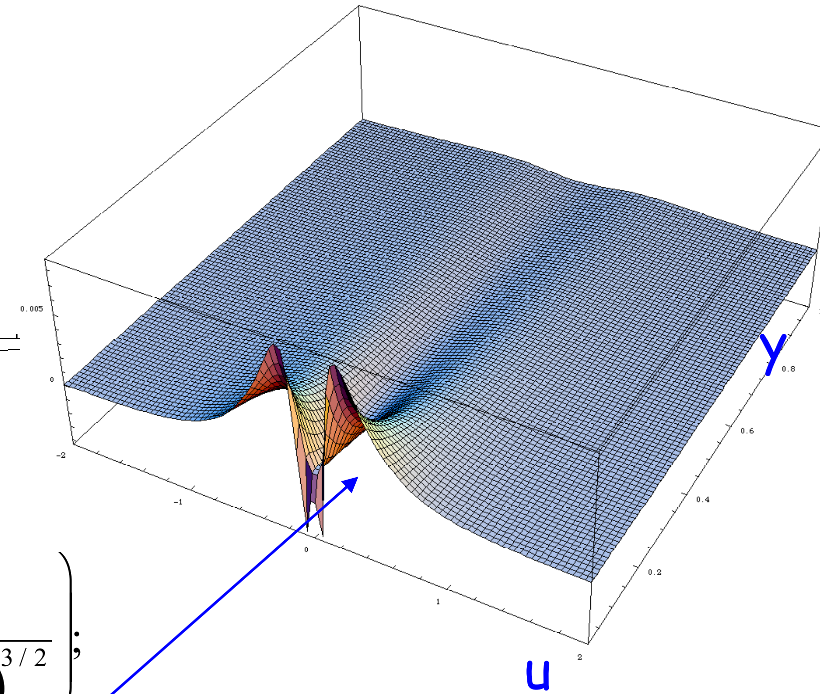




$$\left\langle \frac{\delta E}{E} \right\rangle \cong -2Z \frac{r_e}{a^2} \cdot \frac{L_{pol}}{\gamma} \cdot \left(\frac{z}{|z|} - \frac{z}{\sqrt{a^2/\gamma^2 + z^2}} \right)$$

Exact calculations: solving Vlasov equation





$$f_o(r,\vec{p}_\perp,z,\gamma)=\frac{\theta(r-a)}{a^2/2}\cdot\frac{\theta(z-l_z)}{l_z}\cdot\frac{1}{\sqrt{2\pi}\sigma_\gamma}e^{-\frac{(\gamma-\gamma_o)^2}{2\sigma_\gamma^2}}\cdot g(\vec{p}_\perp)$$

$$\frac{\delta\gamma}{\gamma_o}=\frac{\delta\gamma_i}{\gamma_o}-A\frac{\gamma_o z_i}{\left(r_i^2+\gamma_o^2 z_i^2\right)^{3/2}};\quad z=z_i+D\left(\frac{\delta\gamma_i}{\gamma_o}-A\frac{\gamma_o z_i}{\left(r_i^2+\gamma_o^2 z_i^2\right)^{3/2}}\right);$$

$$l_z\rho(z)=\Phi(s)=\frac{1}{\kappa^2\sqrt{2\pi}}\int_0^{\kappa^2}dy\int_{-L/2}^{L/2}\left[\exp\left\{-\frac{1}{2}\left(s-u\left(1-\frac{G}{\left(y+u^2\right)^{3/2}}\right)\right)^2\right\}-\exp\left\{-\frac{(s-u)^2}{2}\right\}\right]du;$$

$$G=Z\frac{r_e L_{\mathrm{mod}}|D|}{\left(\gamma_o\sigma_{p_1}|D|\right)^3};\quad \kappa=\frac{a}{\gamma_o\sigma_{p_1}|D|};\quad L=\frac{l_z}{\sigma_{p_1}|D|}$$

$$u=\frac{x_1}{\sigma_{p_1}|D|};\quad s=\frac{z}{\sigma_{p_1}|D|};\quad y=\frac{r^2}{\left(\gamma_o\sigma_{p_1}|D|\right)^2}$$

For 7 TeV p in LHC CeC case: simple "gut-feeling" estimate gave 22.9 boost in the induced charge by a buncher, while exact calculations gave 21.7.

Comprehensive studies

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Analytical, Numerical and Computer Tools to:

1. find reaction *(distortion of the distribution function of electrons)* on a presence of moving hadron inside an electron beam

$$\frac{\partial f_e}{\partial t}+\frac{\partial f_e}{\partial \vec{v}}\cdot\frac{e\vec{E}}{m}+\frac{\partial f_e}{\partial \vec{r}}\cdot\vec{v}=0;\quad \vec{r}_h(t)\cong\vec{r}_o+\vec{v}_h t;$$

$$\left(\vec{\nabla}\cdot\vec{E}\right)=4\pi e n_e\left(\frac{Z}{n_e}\delta\left(\vec{r}-\vec{r}_h(t)\right)-\int f_e d\vec{v}^3\right).$$

$$f \Rightarrow f_o+\delta f$$

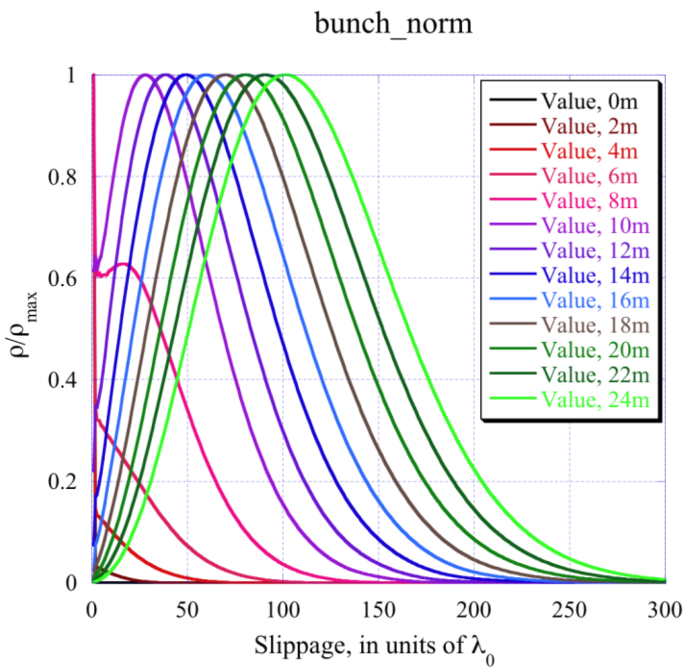
2a. Find how an arbitrary δf is amplified in high-gain FEL

$$f_{exit}(\vec{r}_\perp,\vec{p},t)=f_{o\ exit}(\vec{r}_\perp,\vec{p})+\int K\left(\vec{r}_\perp,\vec{p},\vec{r}_{\perp 1},\vec{p}_1,t-t_1\right)\cdot\delta f(\vec{r}_1,\vec{p}_1,t_1)\cdot d\vec{r}_{\perp 1}d\vec{p}_1 dt_1$$

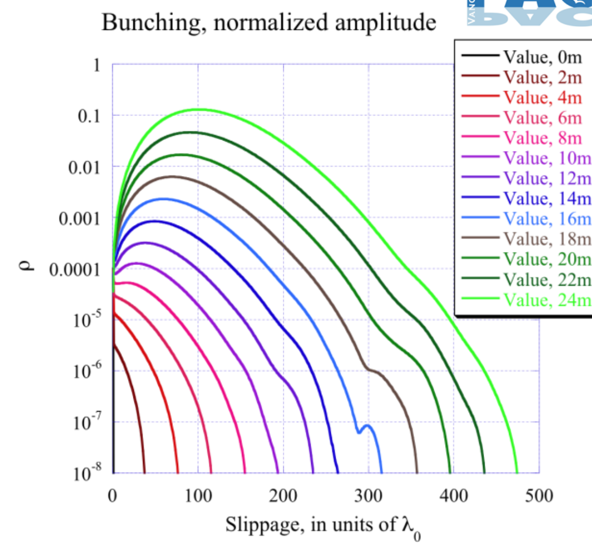
2b. Design cost effective lattice for hadrons + coupling

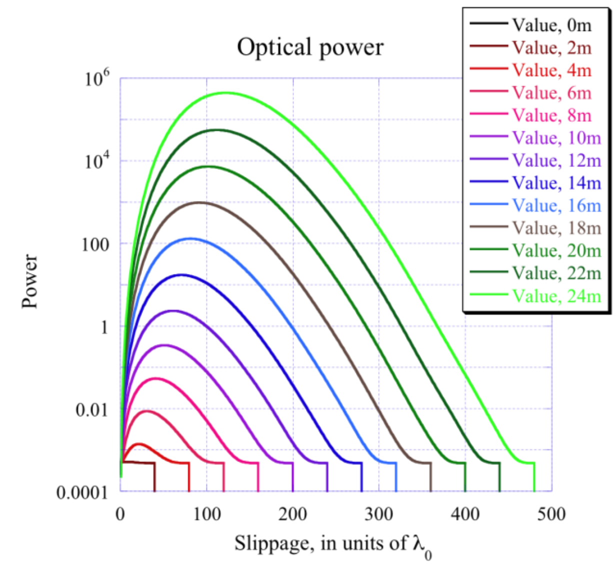
3. Find how the amplified reaction of the e-beam acts on the hadron (including coupling to transverse motion)

Genesis: 3D FEL



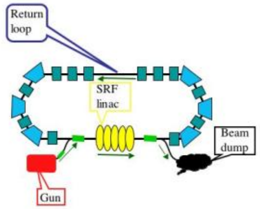
Evolution of the normalized bunching envelope





The Green function (with oscillations) after 10 gain-lengths had also smaller effective RMS length [1] of 0.96 slippage units (i.e. about 38 optical wavelengths, or 27 microns

Evolution of the bunching and optical power envelopes (vertical scale is logarithmic)



PoP test using BNL R&D ERL: Au ions in RHIC with 40 GeV/n, L_{cooler} = 14 m

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N per bunch	$1\ 10^9$	Z, A	79, 197
Energy Au, GeV/n	40	γ	42.63
RMS bunch length, nsec	3.2	Relative energy spread	0.037%
Emittance norm, μm	2.5	$\beta_\perp$, m*	8
Energy e^-, MeV	21.79	Peak current, A	60
Charge per bunch, nC	5 (or 4 x 1.4)	Bunch length, RMS, psec	83
Emittance norm, μm	5 (4)	Relative energy spread	0.15%
$\beta_\perp$, m	5	L_1 (lab frame) ,m	4
ω_{pe}, CM, Hz	$5.03\ 10^9$	Number of plasma oscillations	0.256
$\lambda_{D\perp}$, μm	611	$\lambda_{D\vert\vert}$, μm	3.3
λ_{FEL}, μm	**18**	**λ_w, cm**	**5**
a_w	**0.555**	**L_{Go}, m**	**0.67**
Amplitude gain =150, L_w , m	**6.75 (7)**	**L_{G3D}, m**	**1.35**
L_2 (lab frame) ,m	**3**	**Cooling time, local, minimum**	**0.05 minutes**
N_{turns}, Ñ, 5% BW	$8\ 10^6 > 6\ 10^4$	**Cooling time, beam, min**	**2.6 minutes**

325 GeV polarized protons in RHIC, L_{cooler} fits in IR

N per bunch	2 10^{11}	Z, A	1, 1
Energy Au, GeV/n	250	γ	266.45
RMS bunch length, nsec	1	Relative energy spread	0.04%
Emittance norm, μm	2.5	$\beta_{\perp}$, m	10
Energy e^-, MeV	136.16	Peak current, A	100
Charge per bunch, nC	5	Bunch length, nsec	0.2
Emittance norm, μm	3	Relative energy spread	0.04%
$\beta_{\perp}$, m	10	L_1 (lab frame) ,m	30
ω_{pe}, CM, Hz	4.19 10^9	Number of plasma oscillations	0.25
$\lambda_{D\perp}$, μm	1004	$\lambda_{D\|\|}$, μm	0.17
λ_{FEL}, μm	**0.5**	**λ_w, cm**	**5**
a_w	**0.648**	**L_{Go}, m**	**0.87**
Amplitude gain =100, L_w , m	**13 (-> 15)**	**L_{G3D}, m**	**1.22**
L_2 (lab frame) ,m	10	**Cooling time, local, min**	**1.96**
$N_{min\ turns}$ or $\tilde{N}$ in 10% BW	6.7 10^6 > 5.9 10^6	**Cooling time, beam, min**	**49.2**

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Not optimized!

Au ions in RHIC with 100 GeV/n, L_{cooler} ~ 20 m

N per bunch	$2\ 10^9$	Z, A	79, 197
Energy Au, GeV/n	100	γ	106.58
RMS bunch length, nsec	1	Relative energy spread	0.1%
Emittance norm, μm	2.5	$\beta_\perp$, m	5
Energy e^-, MeV	54.5	Peak current, A	50
Charge per bunch, nC	5	Bunch length, nsec	0.1
Emittance norm, μm	3	Relative energy spread	0.1%
$\beta_\perp$, m	10	L_1 (lab frame) ,m	8.5
ω_{pe}, CM, Hz	$5.9\ 10^9$	Number of plasma oscillations	0.25
$\lambda_{D\perp}$, μm	78	$\lambda_{D\|\|}$, μm	0.75
λ_{FEL}, μm	3	λ_w, cm	5
a_w	0.603	L_{Go}, m	0.5
Amplitude gain =200, L_w , m	8.11 (-> 9)	L_{G3D}, m	0.77
L_2 (lab frame) ,m	5	Cooling time, local, minimum	0.08 minutes
$N_{min\ turns}$ or $\tilde{N}$ in 5% BW	$6\ 10^5$> $2\ 10^5$	Cooling time, beam, min	1.93 minutes

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