Hadron Structure and Lattice QCD

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Workshop on Confinement Physics
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Outline

- ***Hadron Structure from Lattice QCD***
  - ***Anatomy of a calculation***
  - ***Pion Form Factor***
- ***Nucleon Structure***
- ***Three-dimensional Imaging***
  - ***Generalized Parton Distributions***
  - ***Transverse-momentum-dependent distributions and transversity***
- ***Flavor-singlet Structure***
- ***"EMC effect"***
- ***Excited States and Transition Form Factors***
- ***Outlook***

Hadron Structure

How are

- charge and currents
- momentum
- spin and angular momentum

apportioned amongst the quarks and gluons that make up a hadron?

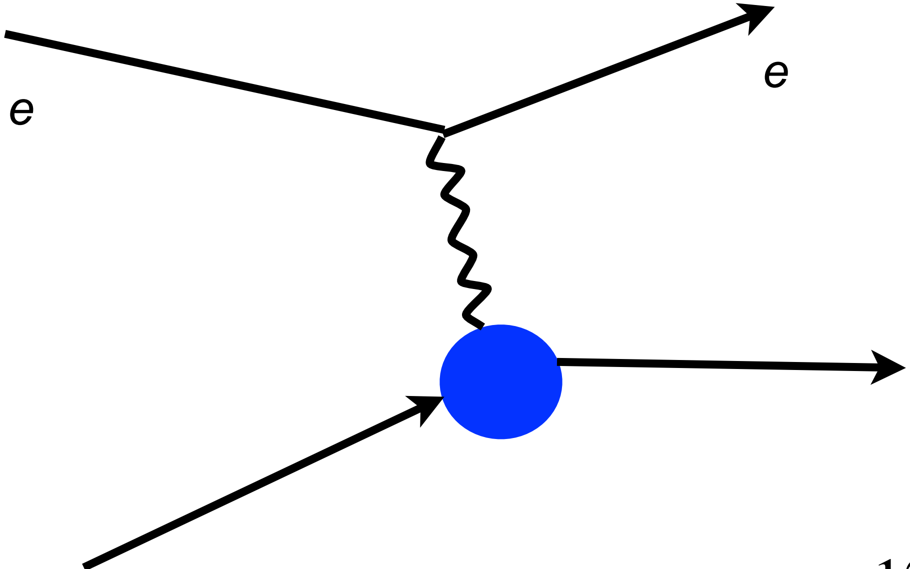
Encapsulated in

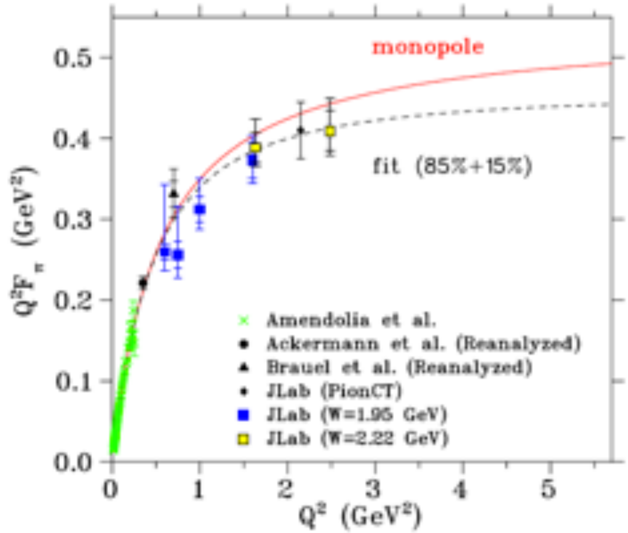
- electromagnetic form factors
- unpolarized structure functions and Transverse-momentum-dependent distributions (TMDs)
- polarized structure functions, Generalized Parton Distributions (GPDs), TMDs

Lattice QCD can either compute all of these or constrain them!

Technique: calculation of hadronic matrix elements.

Paradigm: Pion EM form factor





12 GeV

$$\langle \pi(\vec{p}_f) \mid V_\mu(0) \mid \pi(\vec{p}_i) \rangle = (p_i + p_f)_\mu F(Q^2)$$

where

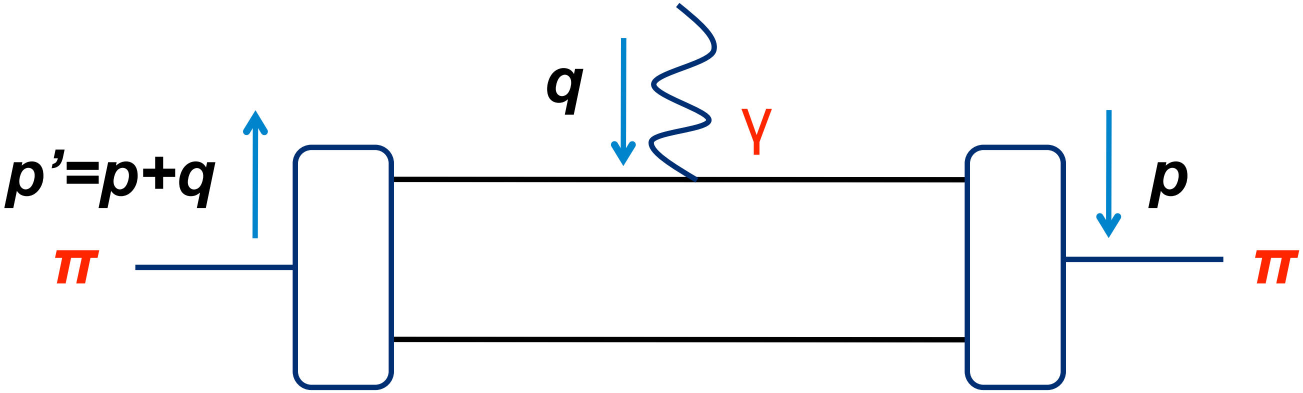
$$V_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d$$

$$-Q^2 = [E_\pi(\vec{p}_f) - E_\pi(\vec{p}_i)]^2 - (\vec{p}_f - \vec{p}_i)^2$$

Anatomy of a Matrix Element Calculation - I

Pion Interpolating Operator

$$\begin{aligned} \phi(x) &= \bar{d}(x)\gamma_5 u(x) \\ \phi^\dagger(x) &= -\bar{u}(x)\gamma_5 d(x) \\ V_\mu(x) &= e_u \bar{u}(x)\gamma_\mu u(x) + e_d \bar{d}(x)\gamma_\mu d(x). \end{aligned}$$

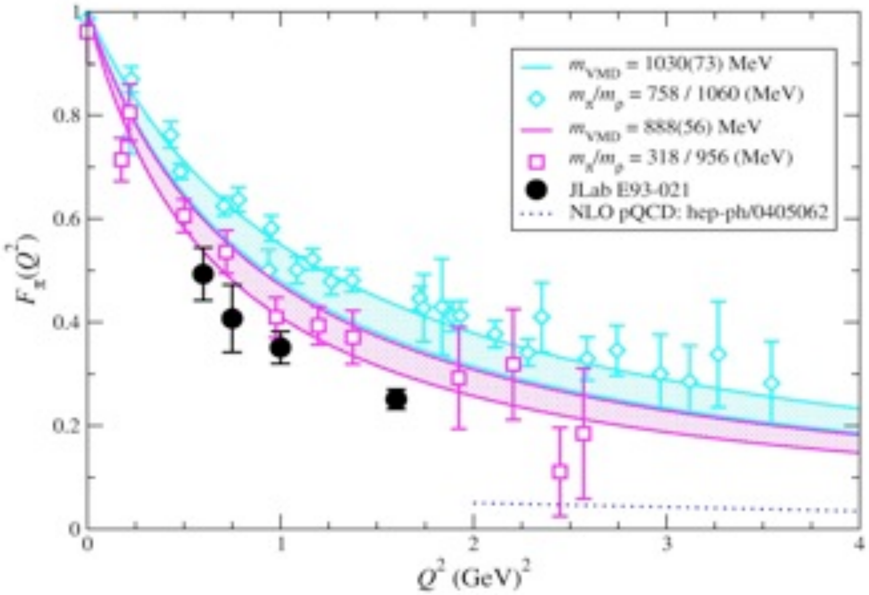


$$\Gamma_{\pi^+\mu\pi^+}(t_f, t; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0|\phi(\vec{x}, t_f) V_\mu(\vec{y}, t)\phi^\dagger(\vec{0}, 0)|0\rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}},$$

Resolution of unity – insert states

$$\langle 0 \mid \phi(0) \mid \pi, \vec{p}+\vec{q}\rangle\langle\pi, \vec{p}+\vec{q} \mid V_\mu(0) \mid \pi, \vec{p}\rangle\langle\pi, \vec{p} \mid \phi^\dagger \mid 0\rangle e^{-E(\vec{p}(t-t_i)} e^{-E(\vec{p}+\vec{q})(t_f-t)}$$

Pion Form Factor - I



LHPC, Bonnet et al,
Phys.Rev. D72 (2005) 054506

$$F(Q^2) = \frac{1}{1 + Q^2/{M_{\rm VMD}}^2}$$

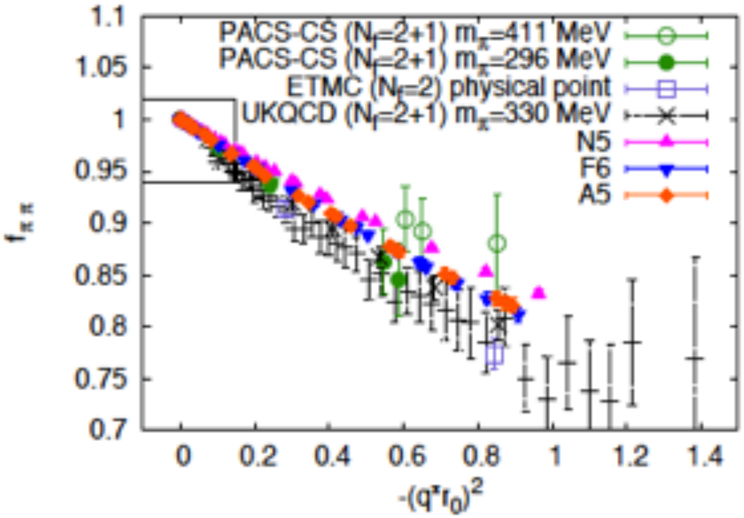
$$Q_{\rm max} \,|\simeq \frac{1}{a}$$

Quark distribution amplitudes

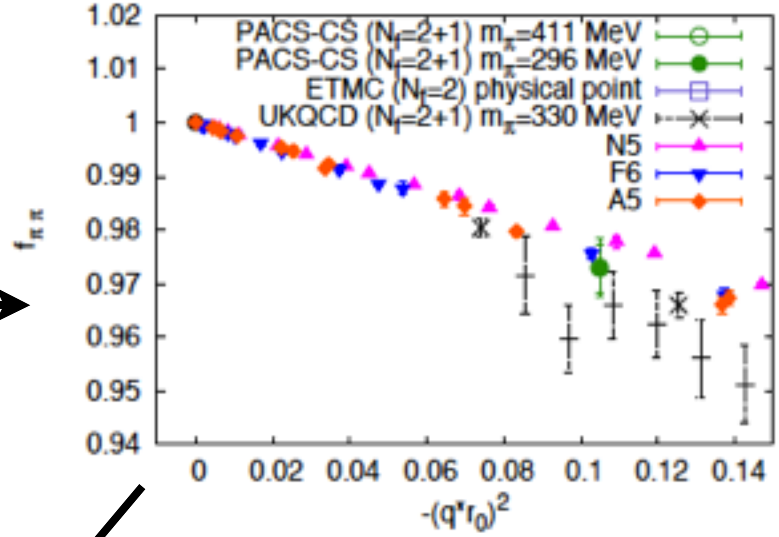
Charge radius

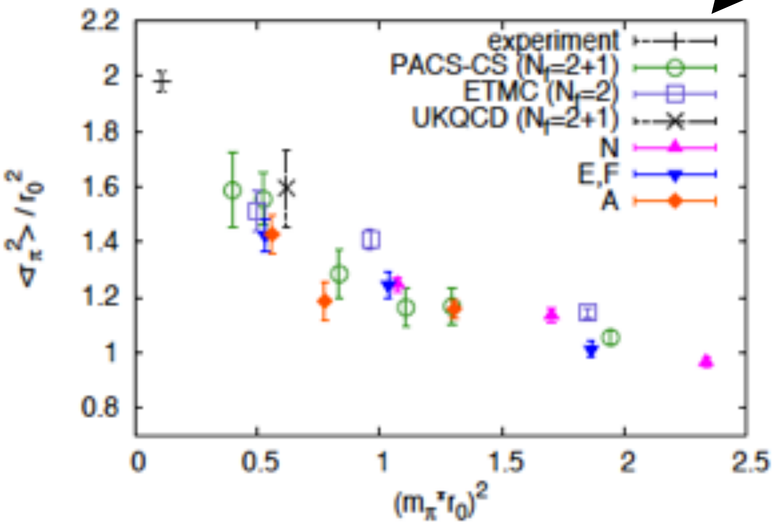
$$\langle r^2 \rangle = 6 \left. \frac{dF(q^2)}{dq^2} \right|_{q^2=0}$$

Pion Form Factor - II



Twisted boundary conditions





Brandt, Jutter, Wittig, arXiv:1109.0196

Nucleon EM Form Factors

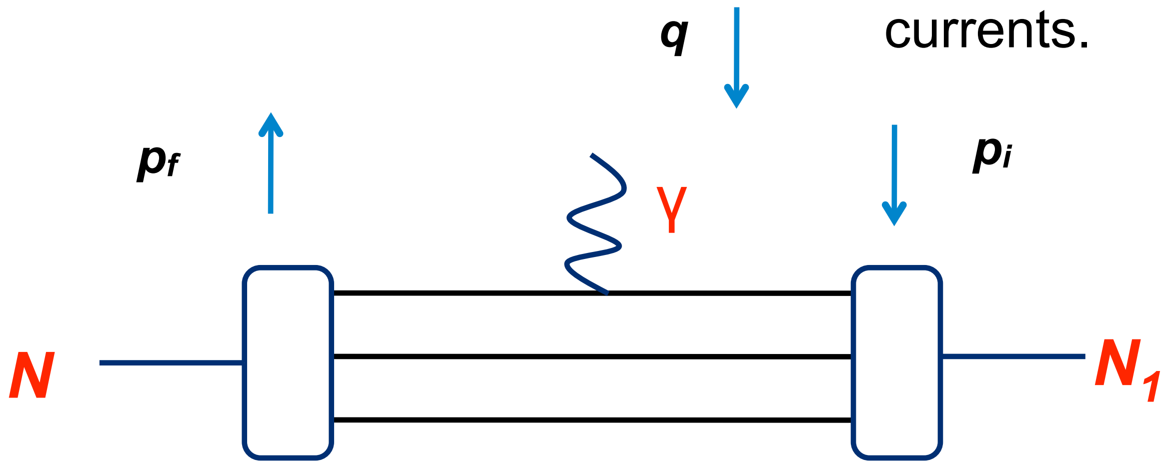
Two form factors

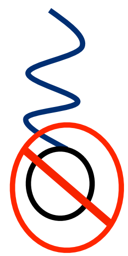
$$\langle p_f \mid V_\mu \mid p_i \rangle = \bar{u}(p_f) \left[\underbrace{\gamma_\mu F_1(q^2)}_{\text{Dirac}} + \underbrace{i q_\nu \frac{\sigma_{\mu\nu}}{2m_N} F_2(q^2)}_{\text{Pauli}} \right] u(p_i)$$

Related to familiar Sach's electromagnetic form factors through

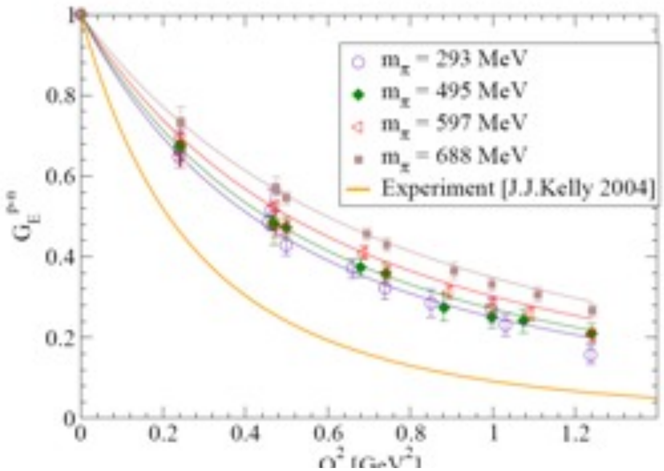
$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \frac{Q^2}{(2m_N)^2} F_2(Q^2) \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2) \end{aligned}$$

Isovector: difference between p and n or difference between u and d currents.





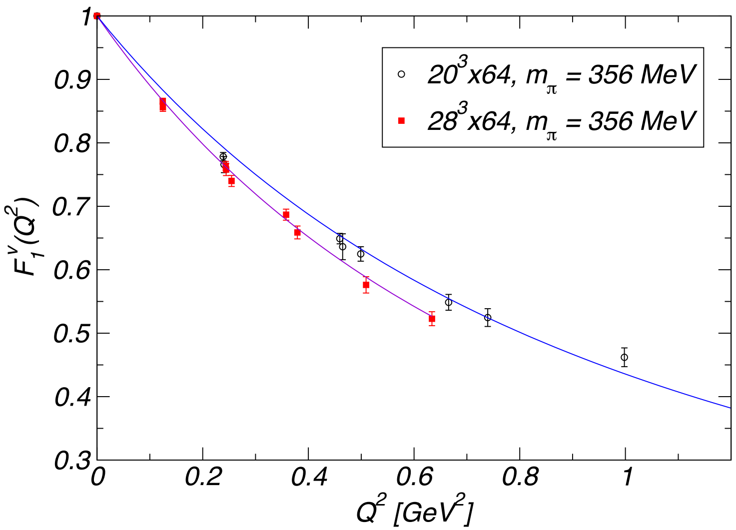
Isovector Form Factor

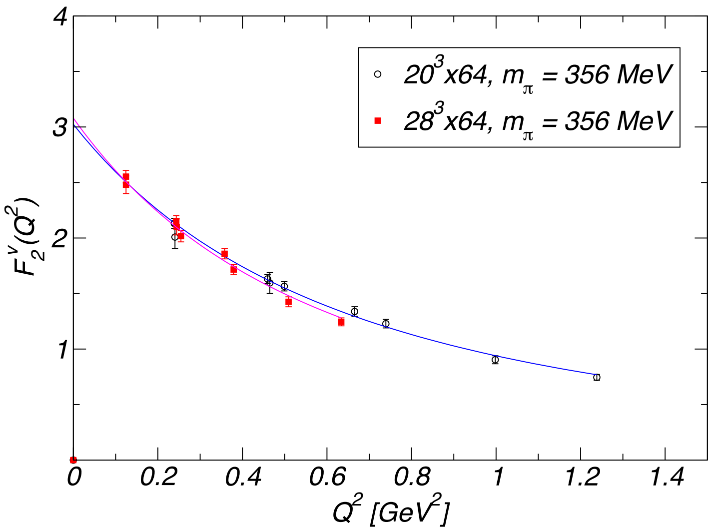


DWF valence/Asqtad sea

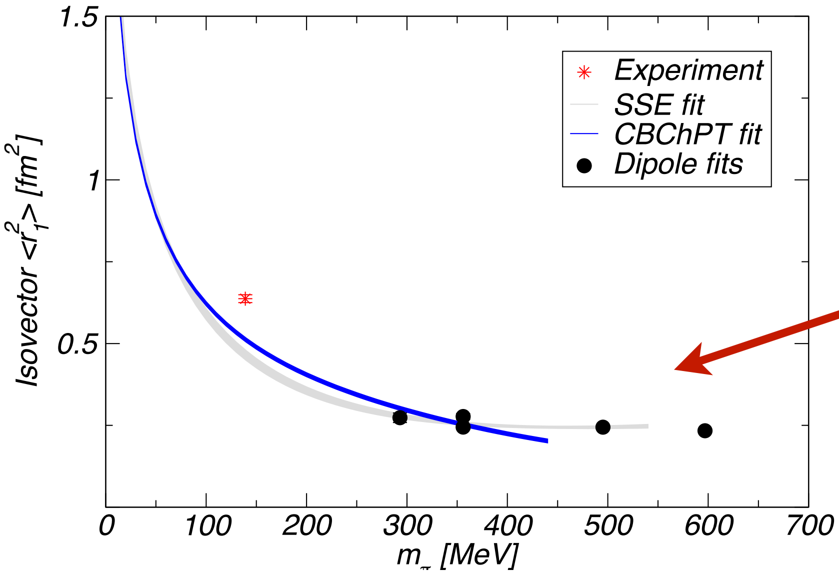
J.D.Bratt et al (LHPC), arXiv:0810.1933

Data well described by dipole form - but example of notable finite-volume effect:





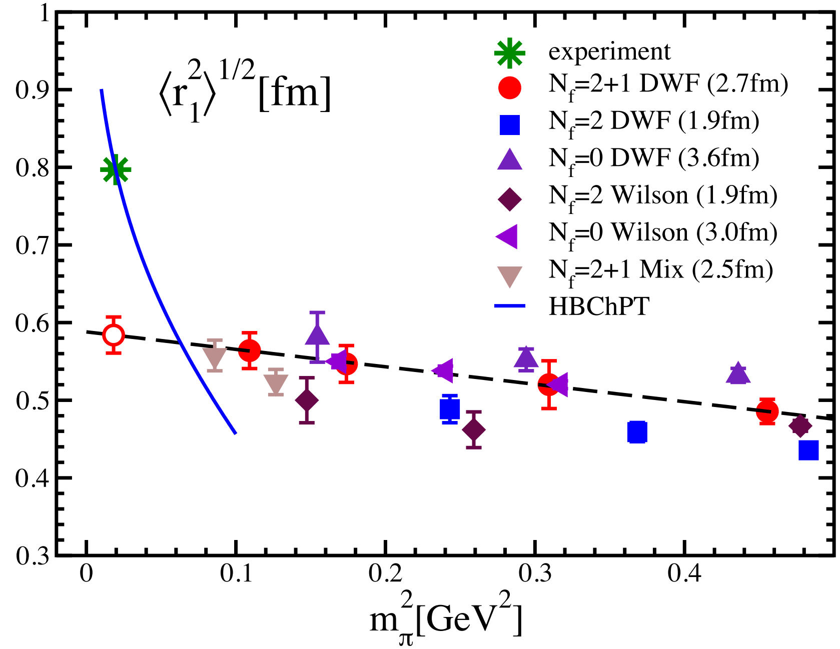
Nucleon Form Factors - III



LHPC, arXiv:1001.3620

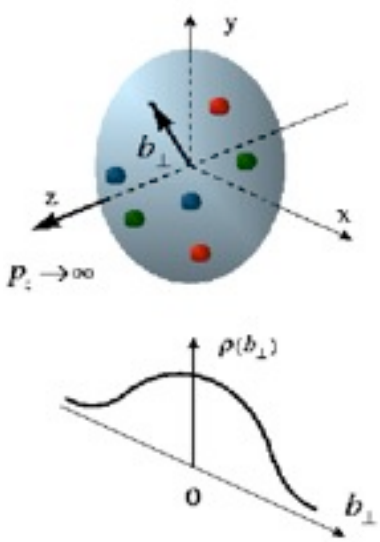
Dipole fits at each pion mass

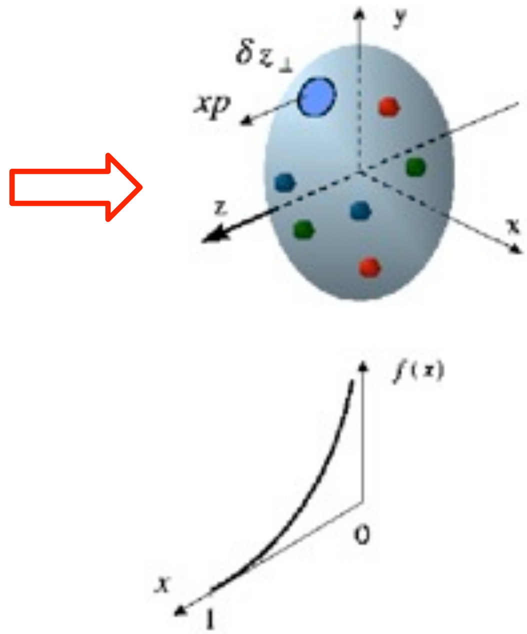
RBC/UKQCD, arXiv:0904.2039



Large extrapolation to Chiral limit

Different Regimes in Different Experiments





Form Factors
transverse quark distribution in Coordinate space

Structure Functions
longitudinal quark distribution in momentum space

Moments of Structure Functions

- **Describe distribution of longitudinal momentum and spin in proton**
- **Matrix elements of light-cone correlation functions**

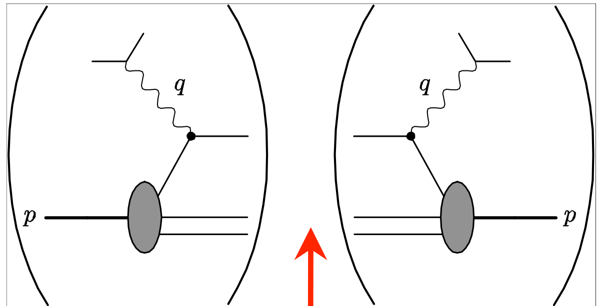
$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}\left(-\frac{\lambda}{2}n\right) nP e^{-ig\int_{\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi\left(\frac{\lambda}{2}n\right)$$

- **Expand *O(x)* around light-cone**

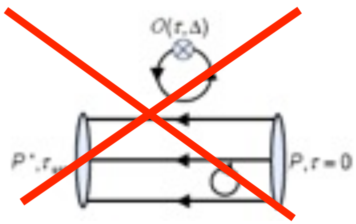
$$O_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \bar{\psi}_q \gamma_5 \gamma^{\{\mu_1} iD^{\mu_2}\ldots D^{\mu_n\}}\psi_q$$

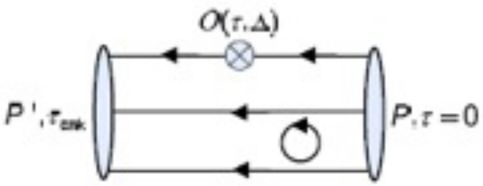
- **Diagonal matrix element**

$$\langle P|O_q^{\{\mu_1\ldots\mu_n\}}|P\rangle \simeq \int dx\, x^{n-1} q(x)$$



Dominated by lightest state

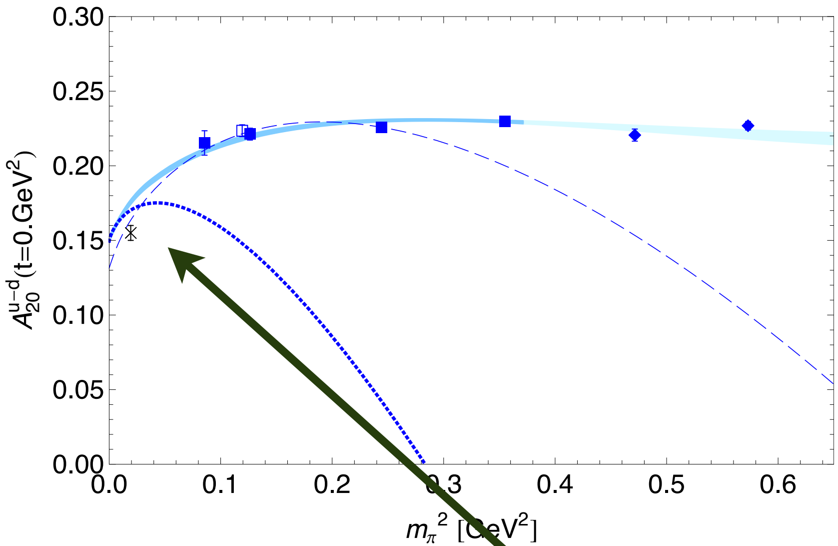




Iso-vector Momentum Fraction

Isovector momentum fraction

$$\langle x \rangle_{u-d}$$



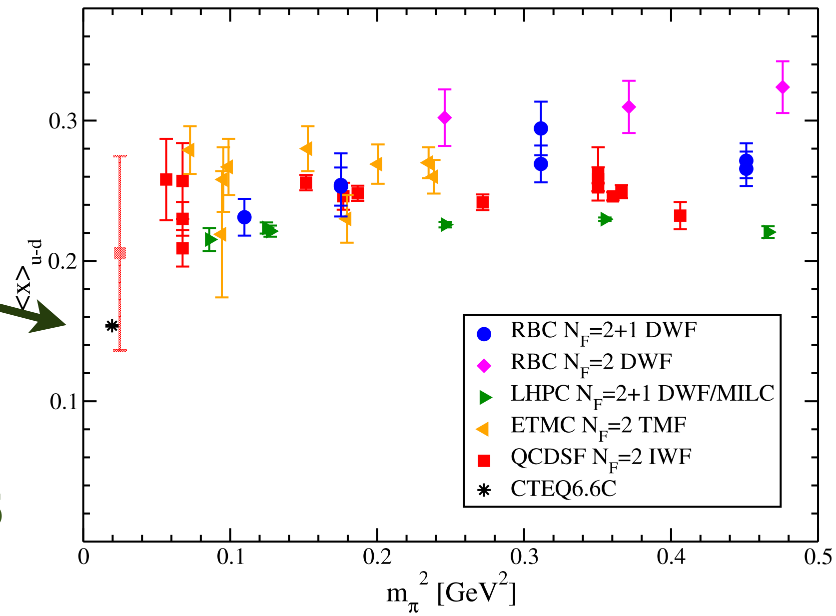
Covariant BChPT

LHPC,arXiv:1001.3620

HBChPT

HB limit of BChPT

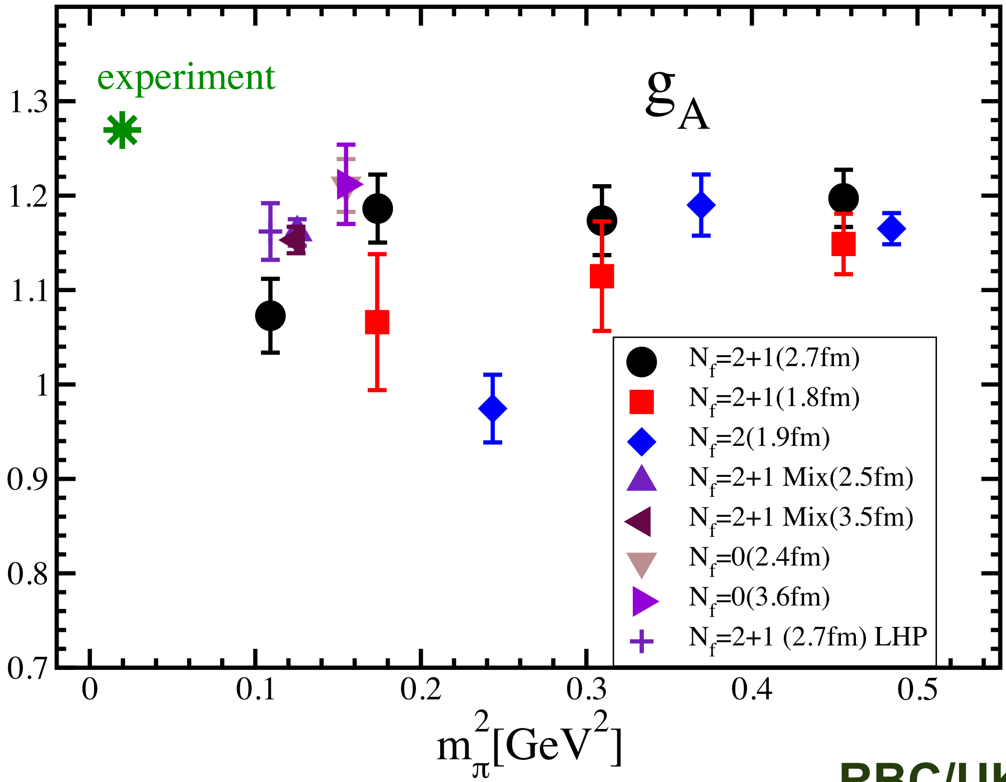
CTEQ6



Excited-state contributions?

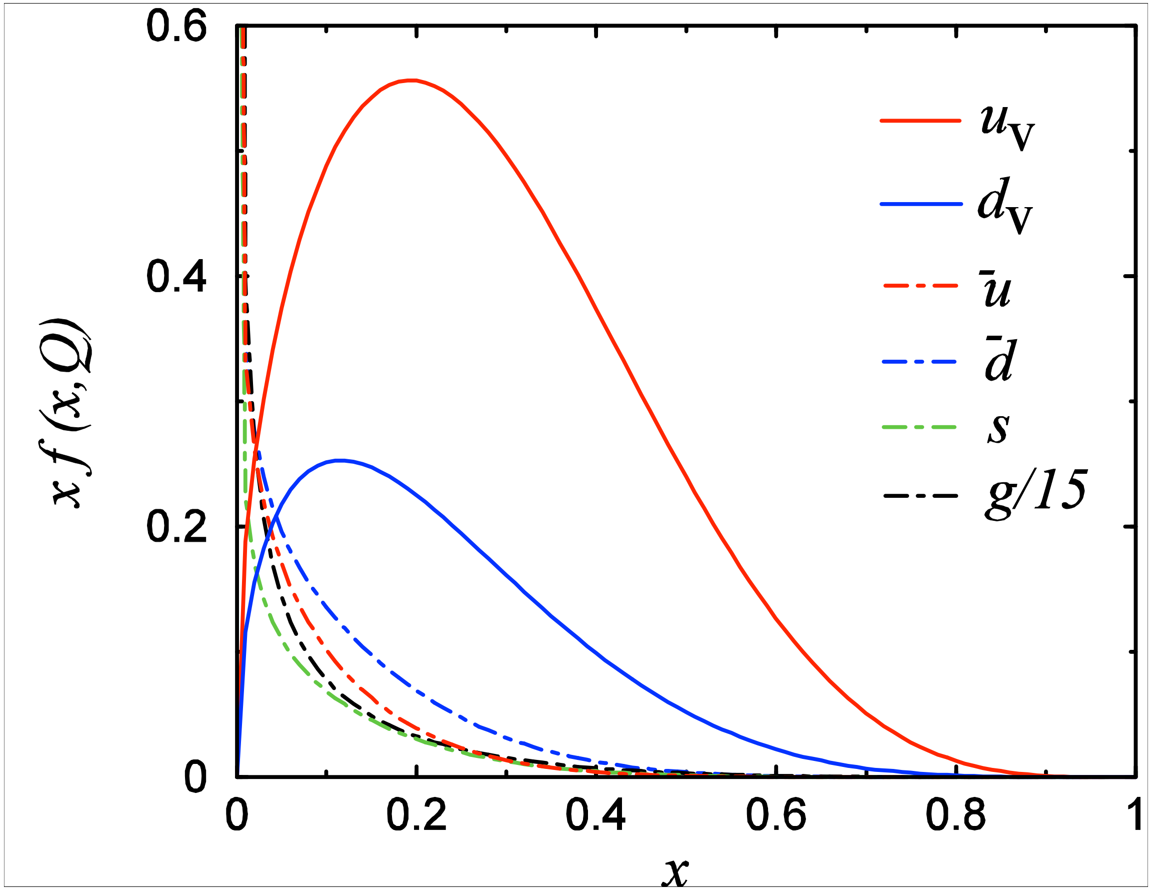
Dru Renner, arXiv:1002.0925

Nucleon Axial-Vector Charge



RBC/UKQCD, 2+ 1 flavor DWF

Moments of Parton Distributions



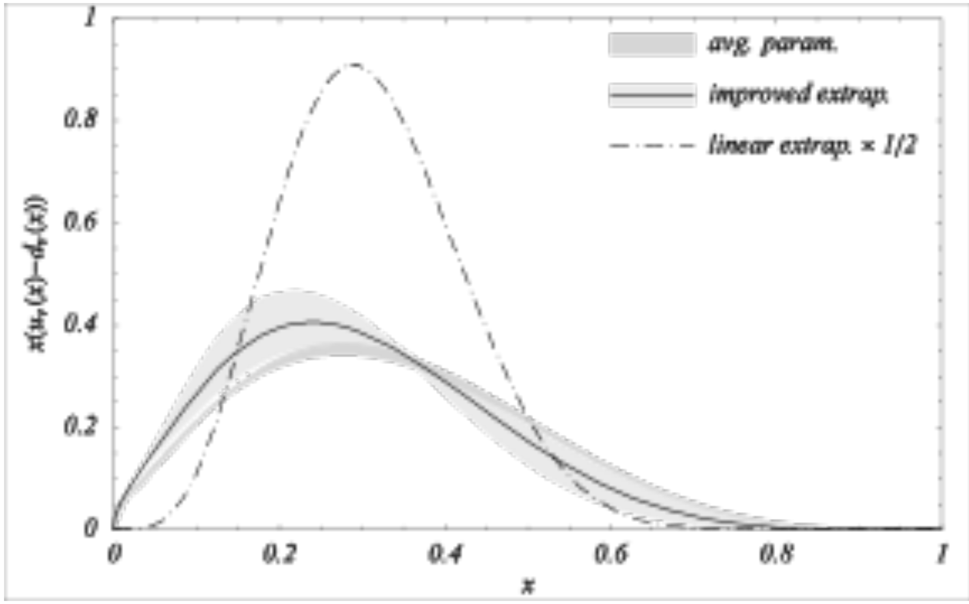
Distributions at 5 GeV

Can we recover distribution?

- **Calculations give moments of distributions**
- **High moments of distributions (>4) - hypercubic symmetry, *mix with lower moments.***
- **Can we recover shape from knowledge of, say, first three moments?**

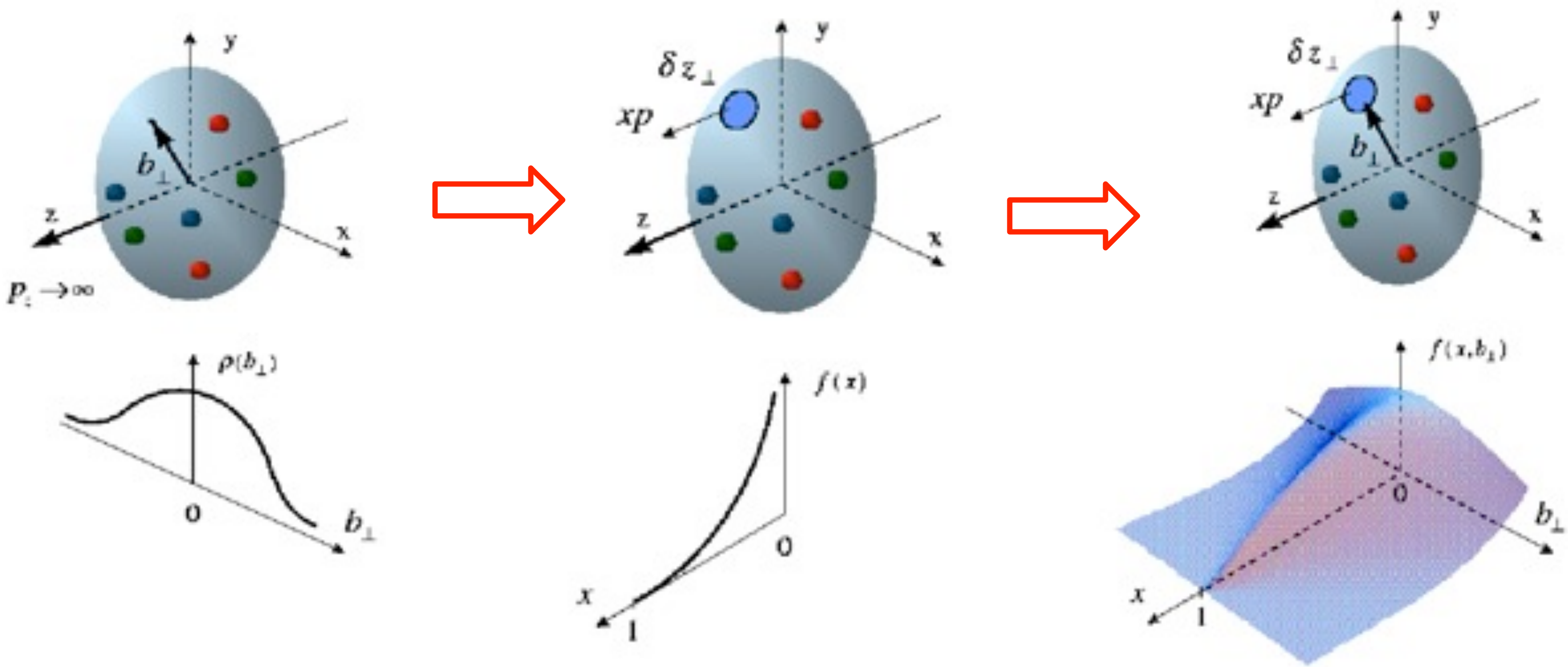
Detmold, Melnitchouk, Thomas

Need to assume parametrization



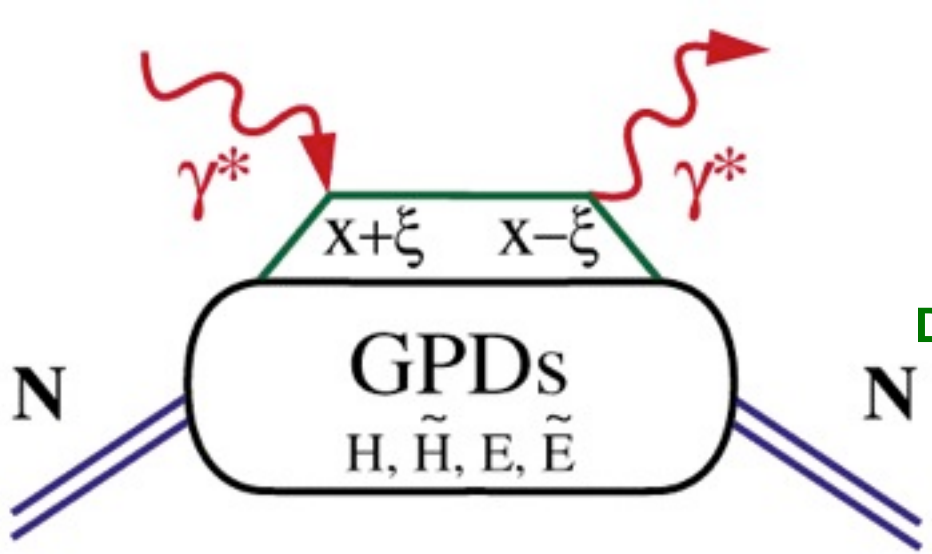
$$x(u_v(x) - d_v(x)) = a\,x^b(1-x)^c(1+\varepsilon\sqrt{x}+\gamma\,x)$$

Different Regimes in Different Experiments



Form Factors	Structure Functions	GPDs
transverse quark distribution in Coordinate space	longitudinal quark distribution in momentum space	Fully-correlated quark distribution in both coordinate and momentum space

Generalized Parton Distributions (GPDs)



D. Muller *et al* (1994), X. Ji & A. Radyushkin (1996)

- Matrix elements of light-cone correlation functions

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}\left(-\frac{\lambda}{2}n\right) n P e^{-ig\int_{\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi\left(\frac{\lambda}{2}n\right)$$

- Expand *O(x)* around light-cone

$$O_q^{\{\mu_1\mu_2\cdots\mu_n\}} = \bar{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots D^{\mu_n\}} \psi_q$$

LHPC, QCDSF, 2003

- *Off-forward* matrix element

Co-efficient of ξ^i

$$\langle P'|O_q^{\{\mu_1\cdots\mu_n\}}|P\rangle \simeq \int dx\, x^{n-1}[H(x,\xi,t), E(x,\xi,t)]$$

$$\longrightarrow A_{ni}(t), B_{ni}(t), C_n(t), \tilde{A}_{ni}(t), \tilde{B}_{ni}(t), \tilde{C}_n(t)$$

GPDs and Orbital Angular Momentum

- **Form factors of energy momentum tensor - *quark and gluon angular momentum***

"$\bar{q}\gamma_\mu D_\nu q$"

$$\frac{1}{2} = \sum_q J^q + J^g$$

X.D. Ji, PRL 78, 610 (1997)

$$= \frac{1}{2}\left\{\sum_q \left(A_{20}^q(t=0) + B_{20}^q(t=0)\right) + A_{20}^g(t=0) + B_{20}^g(t=0)\right\}$$

$$\sum_q \left(\frac{1}{2}\Delta\Sigma^q + L^q\right)$$

Decomposition

- **Gauge-invariant**
- **Renormalization-scale dependent**
- **Handle on Quark orbital angular momentum**

Mathur et al., *Phys.Rev. D62 (2000) 114504*

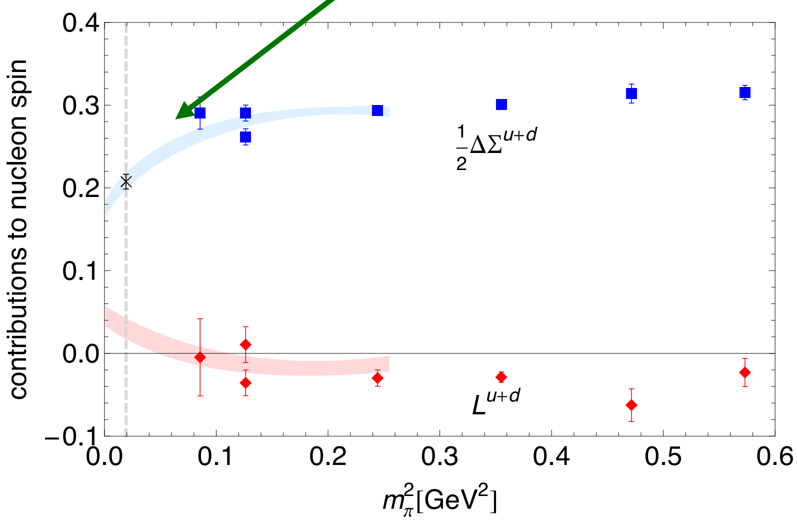
Origin of Nucleon Spin

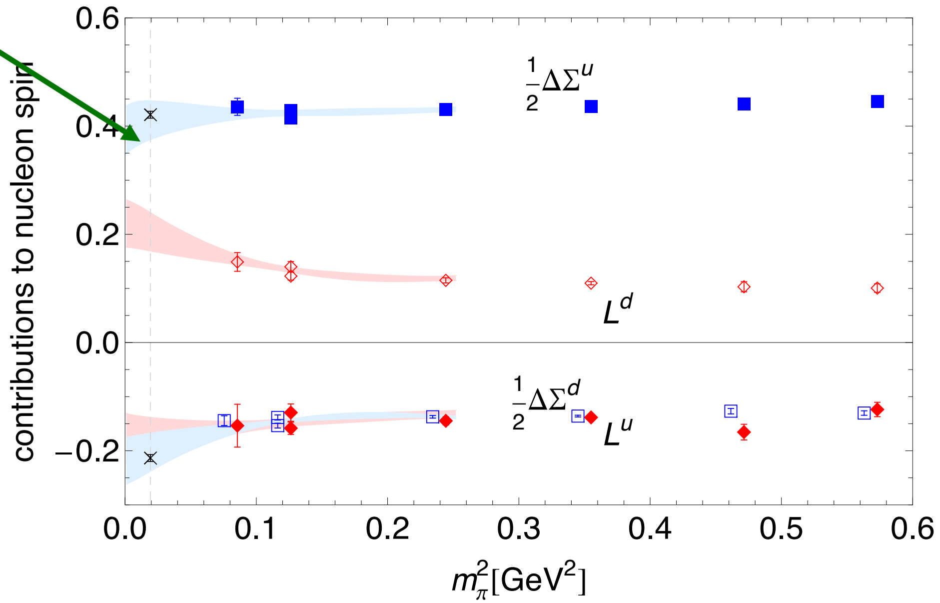
$$J^q = 1/2\left(A^q_{20}(t=0) + B^q_{20}(t-0)\right)$$
$$\Delta\Sigma^q/2 = \tilde{A}^q_{10}(t=0)/2$$
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^{u+d} + L^{u+d} + J^g$$

- **Total orbital angular momentum carried by quarks small**
- **Orbital angular momentum carried by individual quark flavours substantial.**

LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008); arXiv.1001.3620

HERMES, PRD75 (2007)

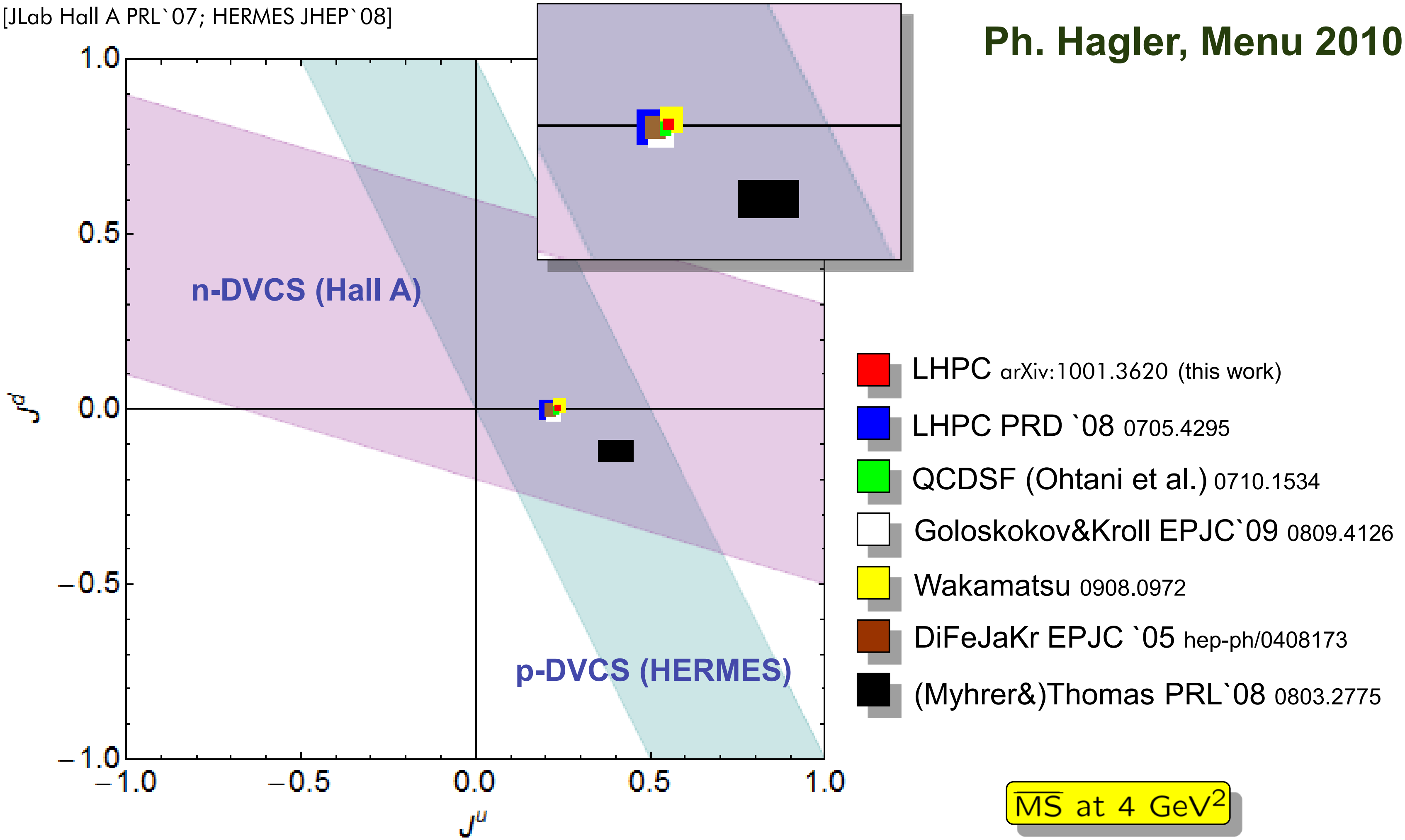




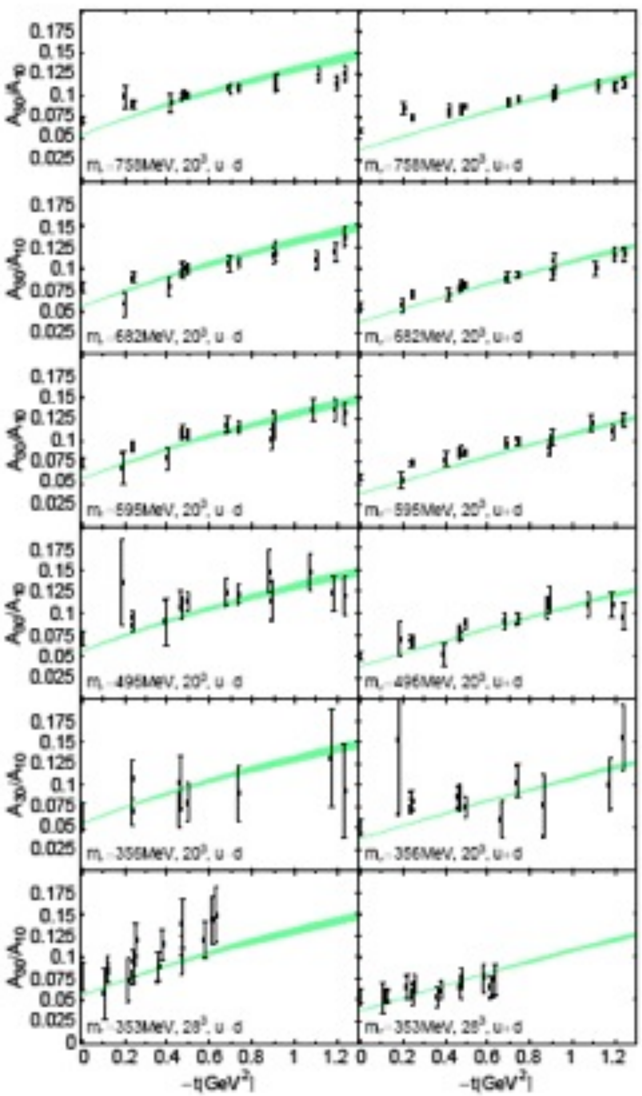
Disconnected contributions neglected.

Origin of Nucleon Spin - II

Ph. Hagler, Menu 2010



Parametrizations of GPDs



Provide phenomenological guidance for GPD's

- ***CTEQ, Nucleon Form Factors, Regge***

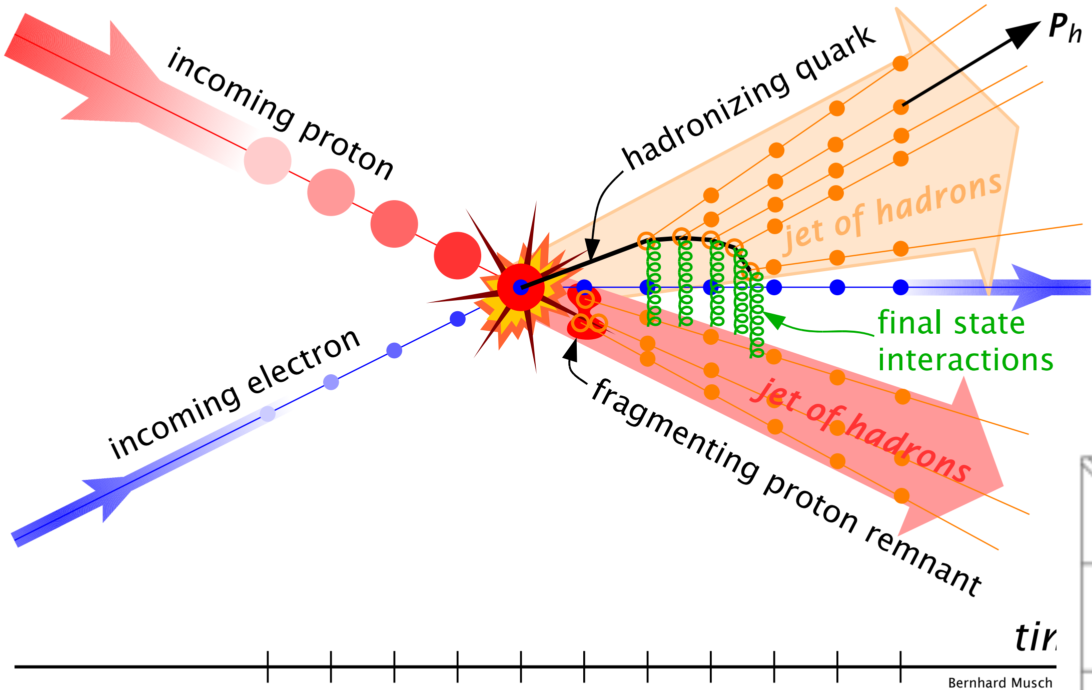
Comparison with *Diehl et al, hep-ph/0408173*

Important Role for LQCD

Transverse momentum distributions (TMDs)

from experiment, e.g., **SIDIS** (semi-inclusive deep inelastic scattering)

HERMES, COMPASS, JLab 6 GeV, JLab 12 GeV , ... , EIC



Cf: measured in Drell-Yan, eg at RHIC-spin

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Boer-Mulders ($h_1^\perp$)

Sivers ($f_{1T}^\perp$) ← time-reversal odd

final state interactions!
explain large asymmetries otherwise forbidden!
signature of QCD!

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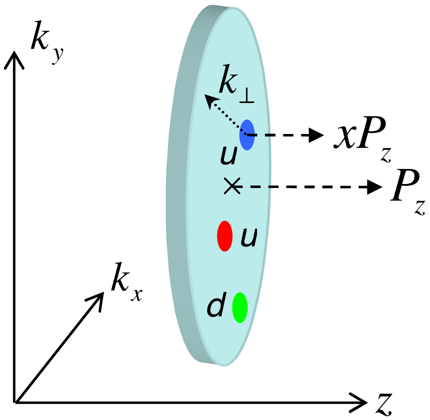
TMDs in Lattice QCD



B. Musch, PhD Thesis; Haegler, Musch, Negele, Schafer arXiv: 0908.1283

Introduce Momentum-space correlators

$$\Phi_\Gamma = \int d(n \cdot k) \int \frac{d^4 l}{2(2\pi)^4} e^{-ik \cdot l} \tilde{\Phi}_\Gamma(l; P, S)$$

$$= \int d(n \cdot k) \int \frac{d^4 l}{2(2\pi)^4} e^{-ik \cdot l} \langle P, S \mid \bar{q}(l) \Gamma \mathcal{U} q(0) \mid P, S \rangle$$

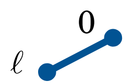
continuum

$$\mathcal{U} \equiv \mathcal{P} \exp\left(-ig \int_0^\ell d\xi^\mu A_\mu(\xi)\right)$$

along path from 0 to ℓ

Choice of path - retain gauge invariance

SIDIS: path runs to infinity

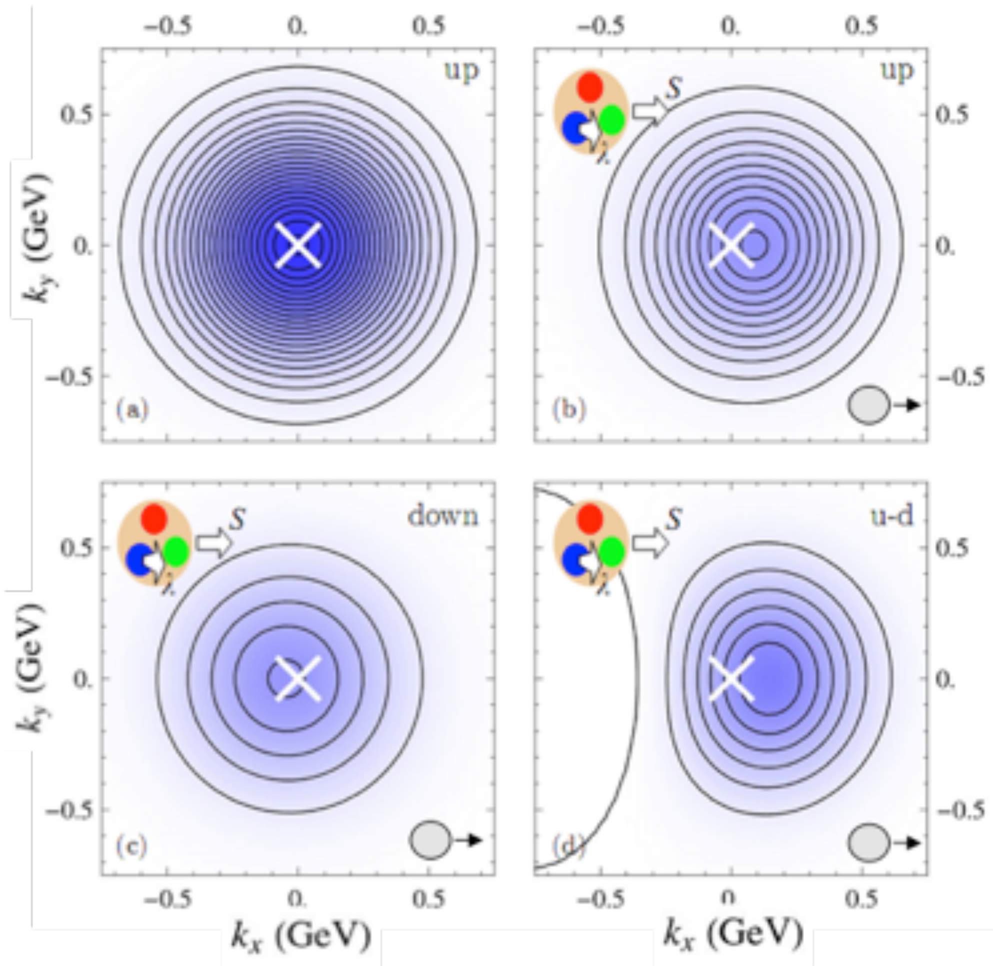


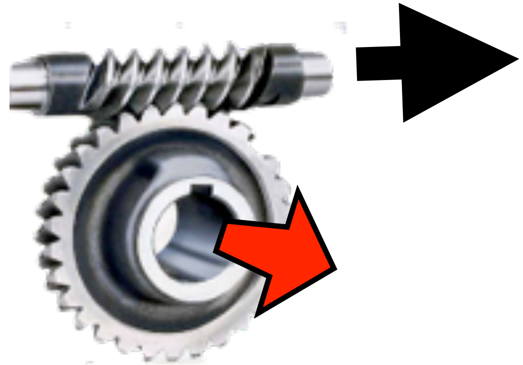
Lattice: equal time slice

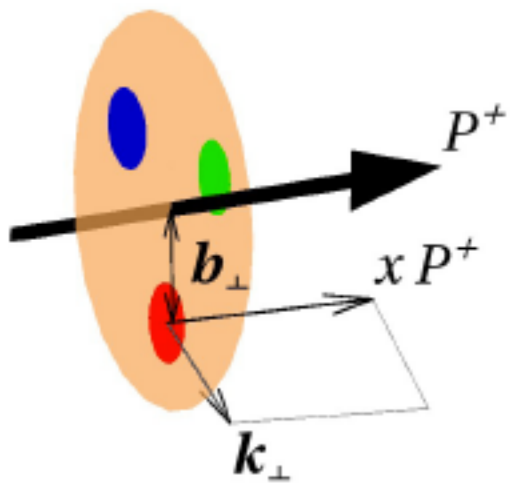
Slide: A. Bacchetta

Worm gears on the lattice

P. Hägler, B. U. Musch, J. W. Negele, and A. Schäfer, Europhys. Lett. 88 (2009) 61001



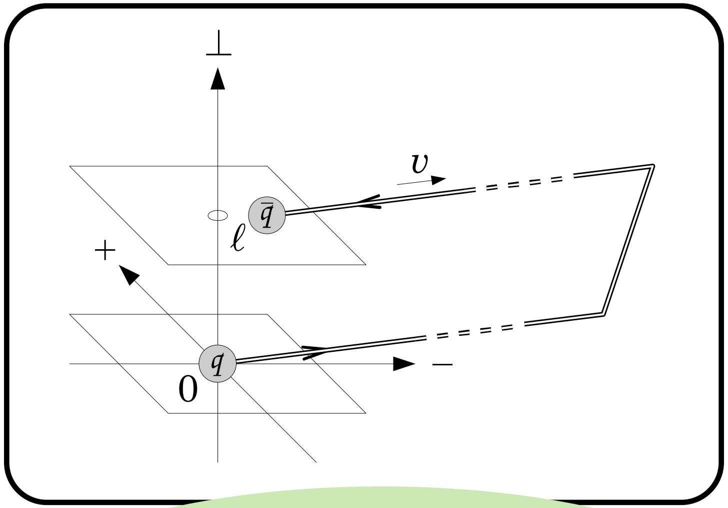


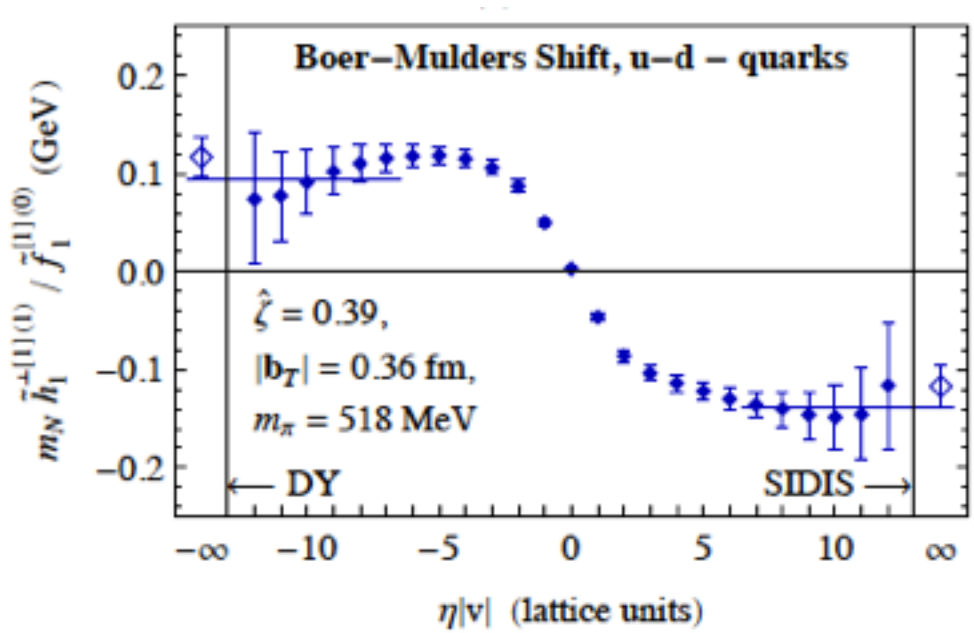


Transverse momentum distributions (TMDs)

Lattice QCD, including effect of final state interactions

Transversely-polarized quarks, unpolarized nucleon





new calculations with extended operator

B.Musch et al.,arXiv:1111.4249

preliminary results
m_π=800 MeV,
MILC lattices,
LHPC propagators

T-odd TMDs accessible...

Boer-Mulders function :
odd signal (SIDIS vs. Drell-Yan)
still far from light cone
⇒ **Proposal to study pion TMDs**

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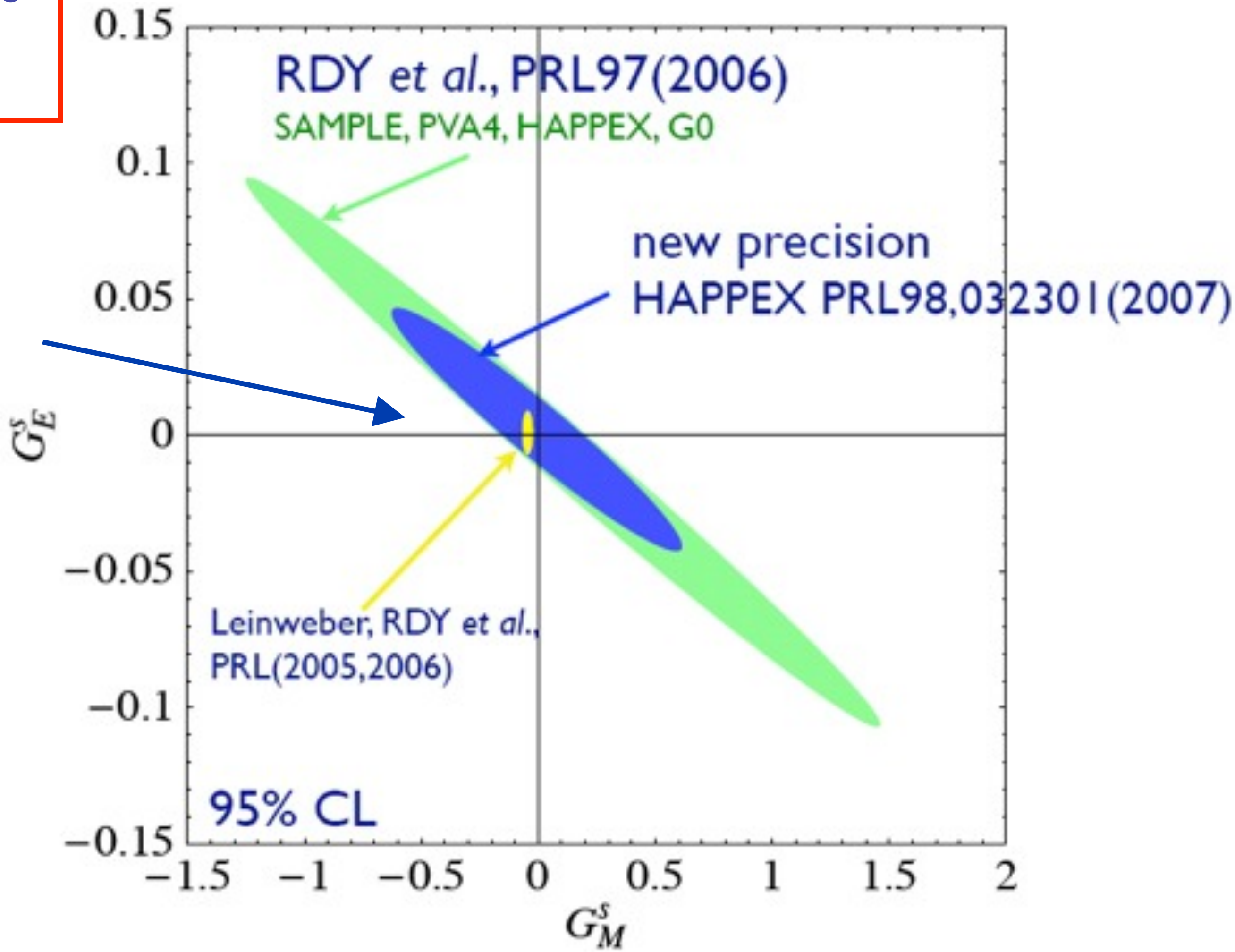
Flavor-Singlet Hadron Structure

Flavor-singlet: Disconnected Contributions

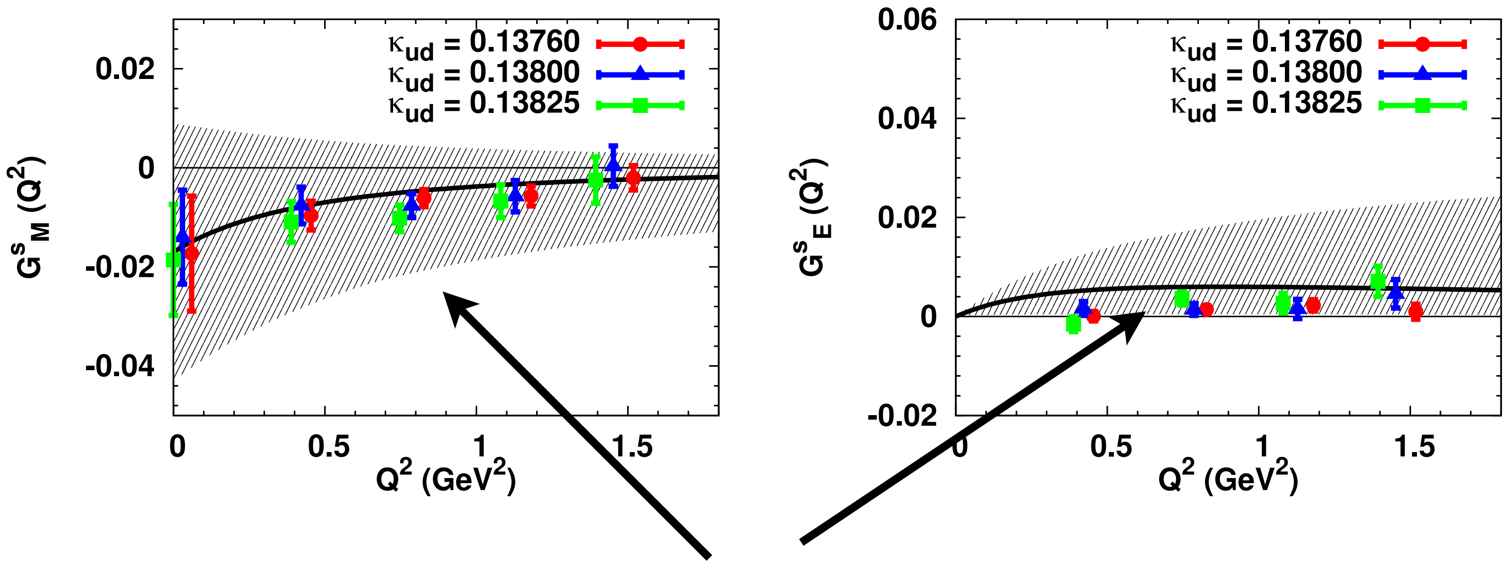
Strangeness contribution to electric and magnetic form factors.

Amalgam of Lattice QCD and Phenomenology by Leinweber *et al.*



Ab initio calculation

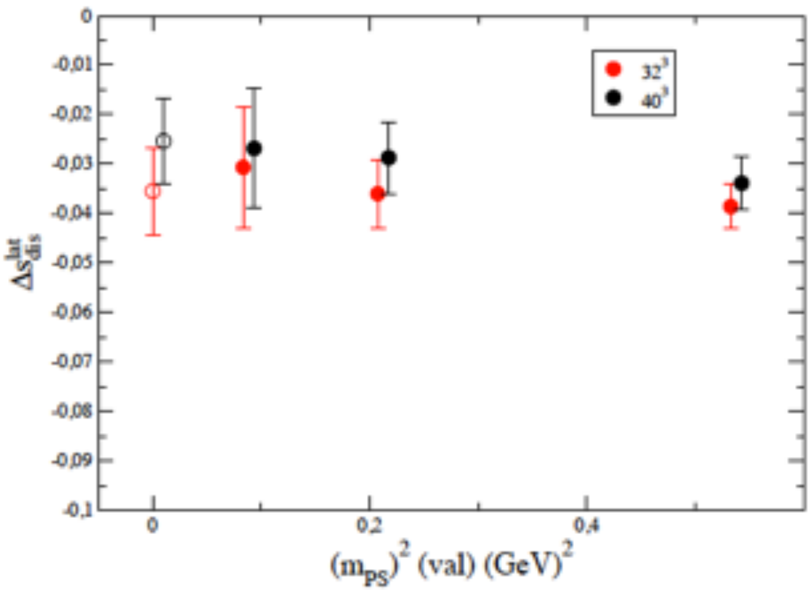
Doi et al. (ChQCD Collaboration),
arXiv:0910.2687, PRD79:094502,2009



Uncertainties: statistical, Q^2 dependence, chiral extrapolation

$$G^s_M(0) = -0.017(25)(07)$$

Strange-quark contribution to hadron spin



QCDSF, arXiv:1112.3354

$$\Delta s^{\overline{MS}}(\sqrt{7.4}\ \text{GeV}) = -0.020(10)(4)$$

Small, negative contribution

In general, Quark and gluons mix under renormalization

$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} q^S \\ g \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi} \begin{pmatrix} P_{qq} & 2n_f P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q^S \\ g \end{pmatrix}$$

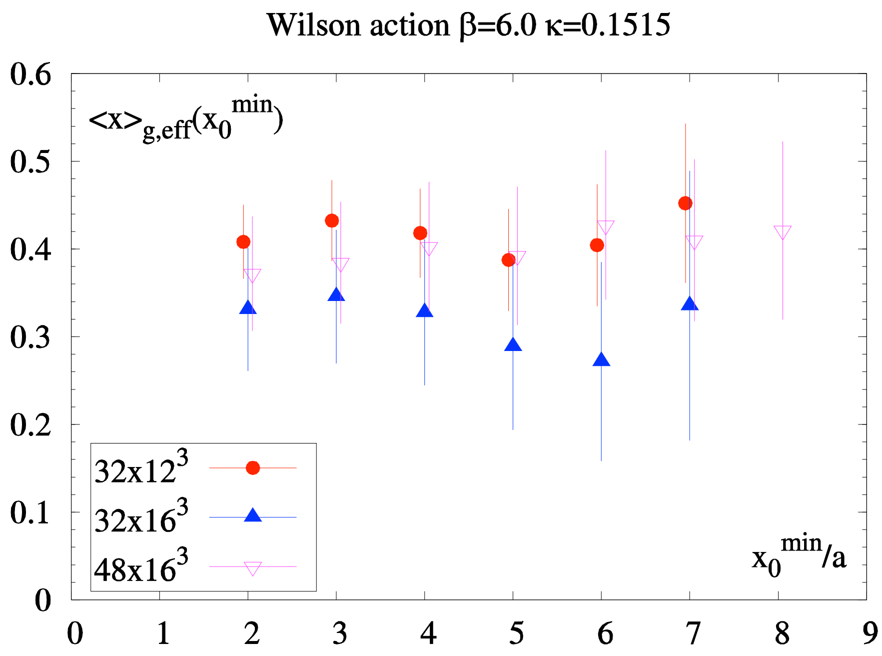
The local operators mix as follows:

$$O^{qS}_{\mu_1\cdots\mu_N} = \frac{1}{2^N} \overline{\psi}\, \gamma_{[\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_N]} (1 \pm \gamma_5) \psi$$

$$O^{gS}_{\mu_1\cdots\mu_N} = \sum_\rho \mathrm{Tr}\left[F_{[\mu_1\rho} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_{N-1}} F_{\rho\mu_N]} \right]$$

Gluon Momentum Fraction in Pion

- **Flavour-singlet:** mixing of quark and gluon contributions
- **Notoriously difficult, but essential**
- **Improved operator $E^2 - B^2$: 40x increase in signal**
- **Normalize operator by ratio of entropy at finite T**



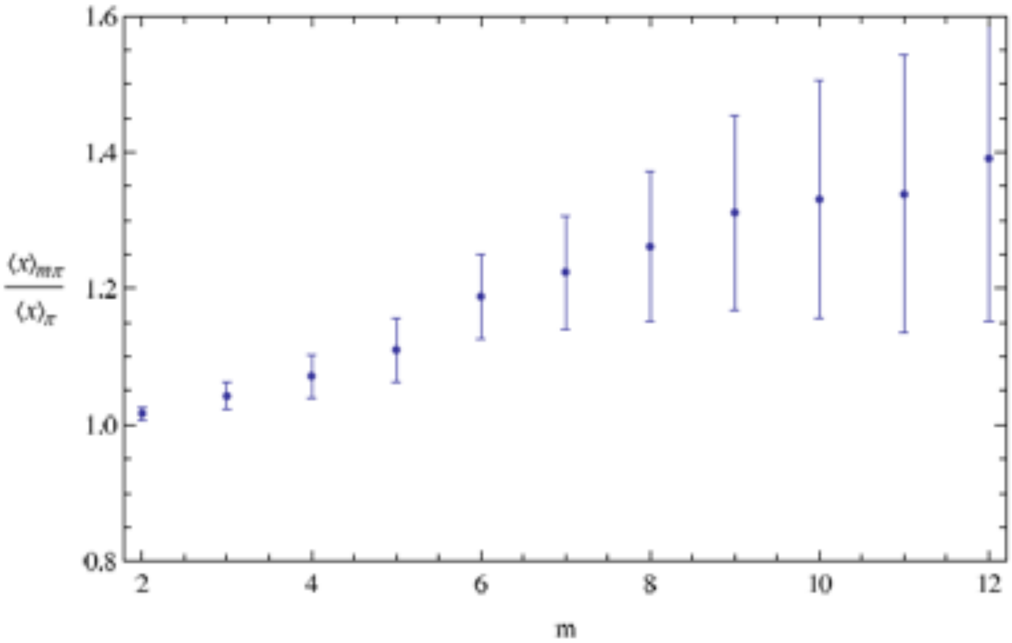
H. Meyer, J. Negele, PRD (2008)

$$\langle x\rangle_{\text{glue}}(\mu = 2\ \text{GeV}) = 0.37 \pm 8 \pm 12$$

Momentum sum rule: $\langle x\rangle_{\text{glue}} + \langle x\rangle_{\text{quarks}} = 0.99 \pm 8 \pm 12$

Medium modification of structure

- How is the structure of a hadron modified "in medium" - EMC effect?
- First attempt - momentum fraction carried by quarks in Bose-condensed pion gas.



W Detmold, H-W Lin, arXiv:1112.5682

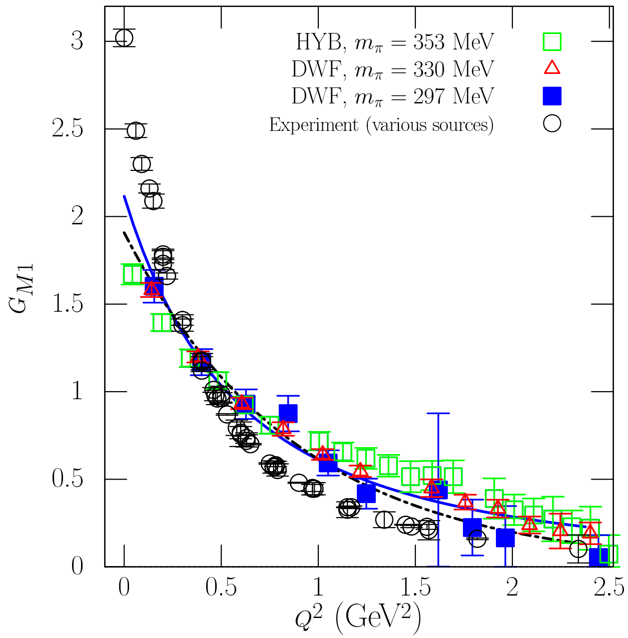
Proof of concept

Transition Form Factors

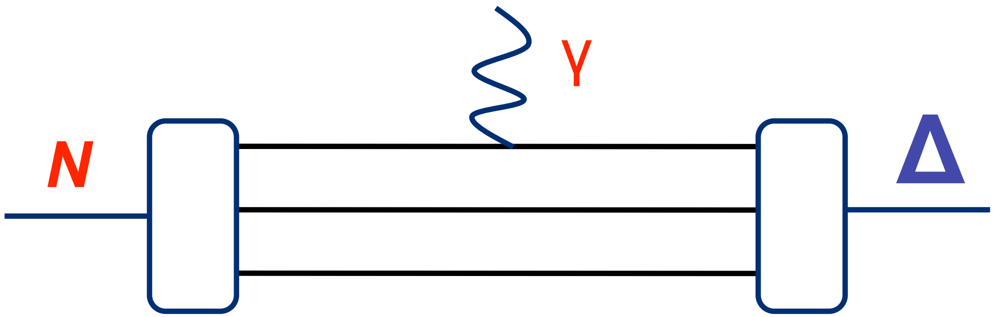
Form factors of excited states, and transition form factors to excited states, provide additional insight into nature of QCD. Precise electro-production data

Program of computations looking at Δ form factor, and Nγ → Δ transition form factors
N.B. Δ → Nπ is p-wave decay, suppressed at zero momentum.

Admits *three* multipoles: magnetic dipole, electric quadrupole and Coulomb quadrupole: G_{M1}, G_{E2}, G_{C2}



Alexandrou et al, DWF + DWF valence/Asqtad sea

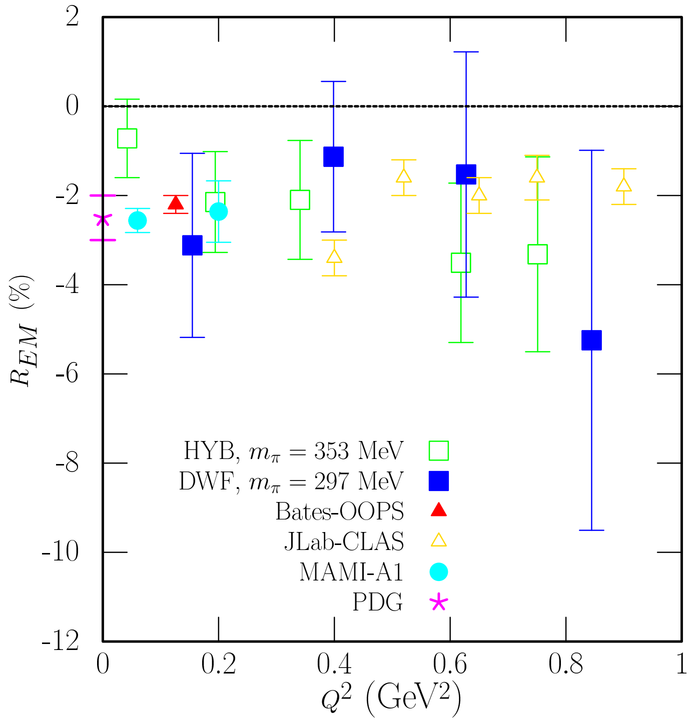


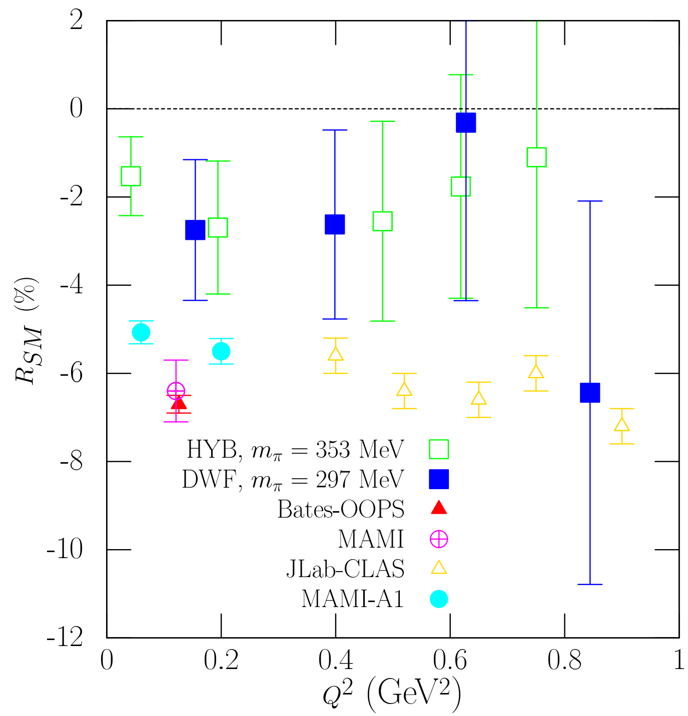
Free of disconnected contributions

N-Δ Transition Form Factor

$$R_{EM} = -\frac{G_{E2}(Q^2)}{G_{M1}(Q^2)}$$

$$R_{SM} = -\frac{|\vec{q}|}{2m_\Delta}\frac{G_{C2}(Q^2)}{G_{M1}(Q^2)}$$





Non-zero values: sphericity in either N or Δ - zero quadrupole moment for spin-1/2 system

Delta is unstable - Luscher, Lellouch

Form factors at High Q^2

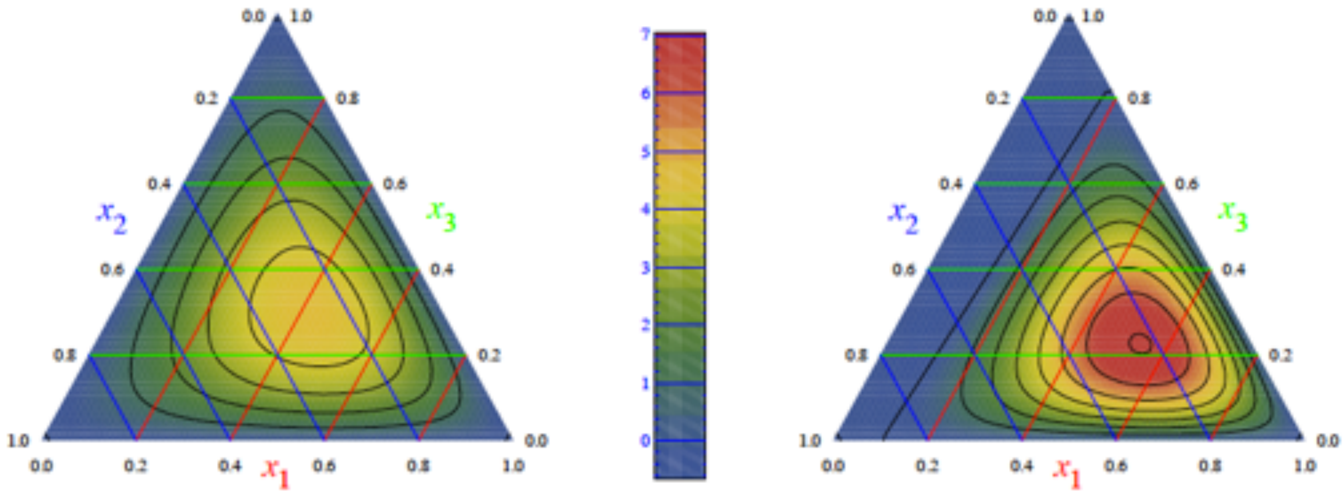
- For exclusive processes at sufficiently high Q^2, can describe processes in terms of quark distribution amplitudes, e.g. for N(*)

$$|N,\uparrow\rangle = f_N \int \frac{[dx]\varphi(x_i)}{2\sqrt{24x_1x_2x_3}} \{|u^\uparrow(x_1)u^\downarrow(x_2)d^\uparrow(x_3)\rangle - |u^\uparrow(x_1)d^\downarrow(x_2)u^\uparrow(x_3)\rangle\}.$$

- Can compute low moments of quark distribution amplitudes

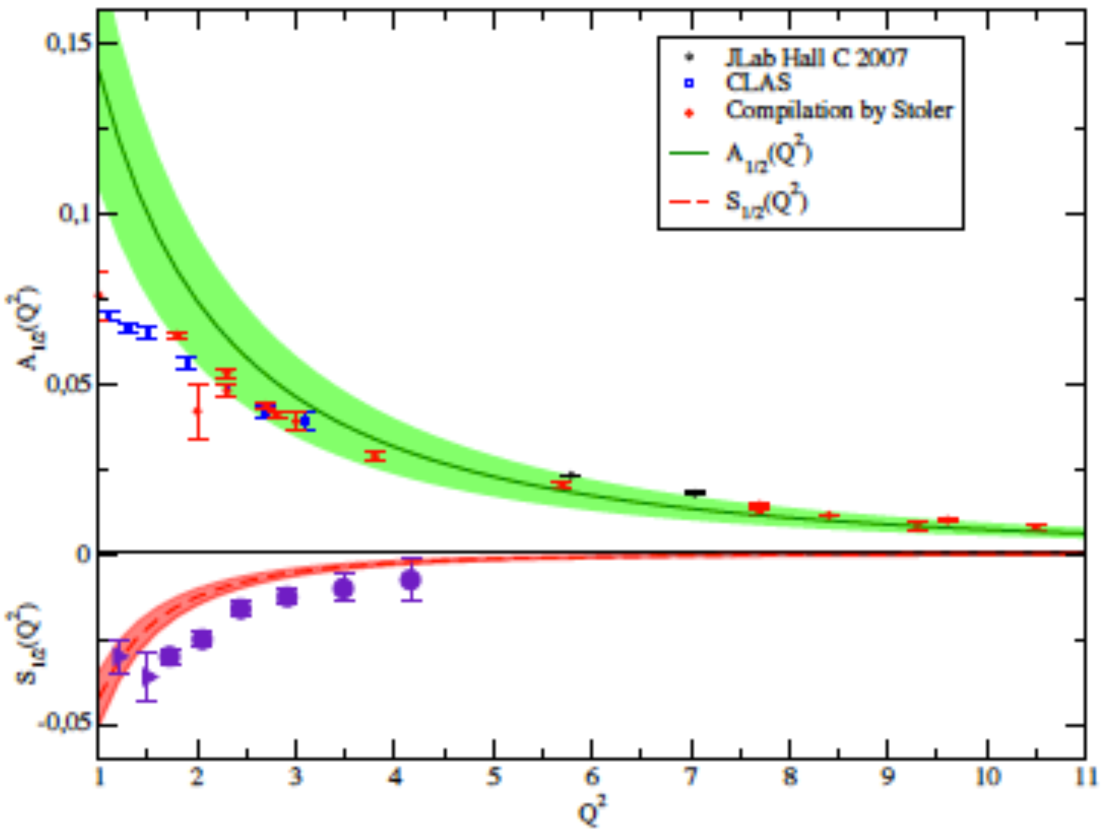
$$\varphi^{lmn} = \int [dx] x_1^l x_2^m x_3^n \varphi(x_1,x_2,x_3).$$

QCDSF, arXiv:1112.0473



N　　　　*N*(1535)*

Form factors at High Q^2



V.Braun, arXiv:1008.5228

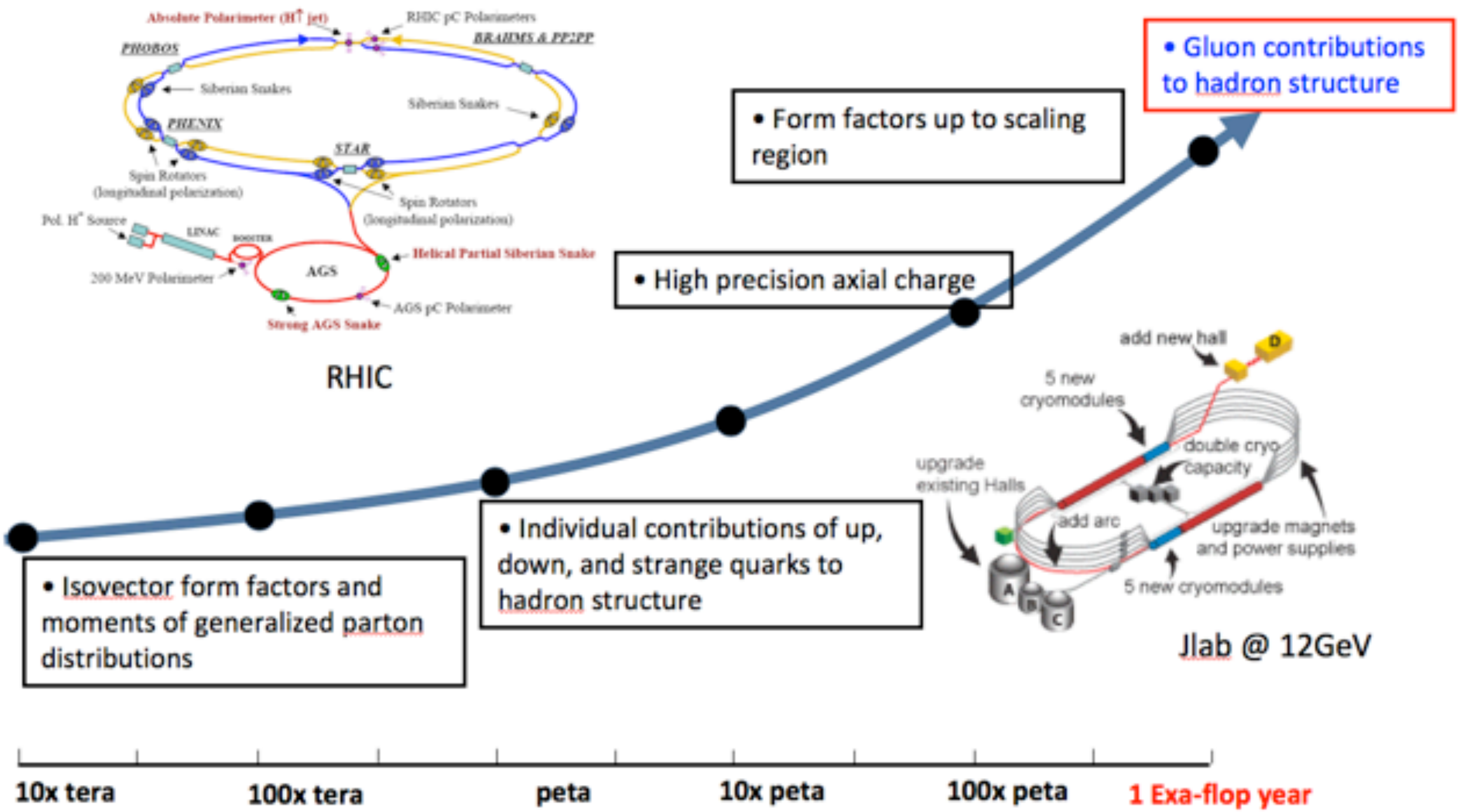
Helicity amplitudes from DA

Summary

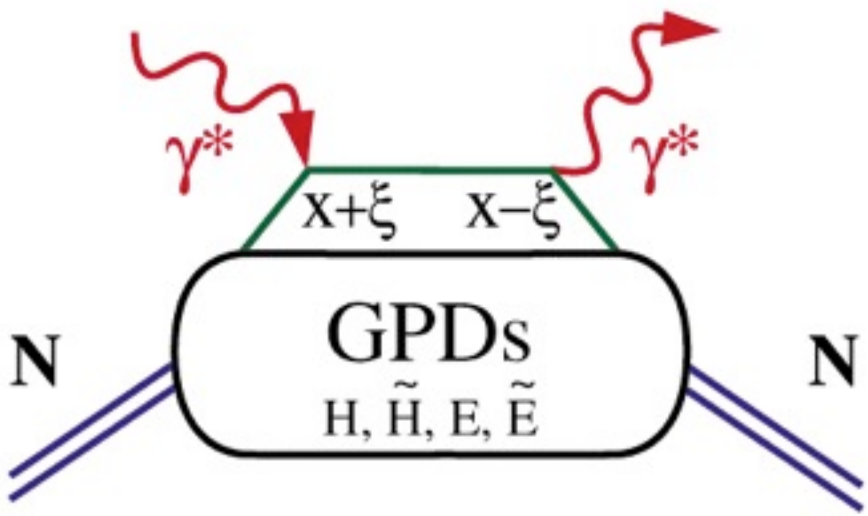
- GPDs and TMDs are Drawing a three-dimensional picture of the Proton
- Control over systematic uncertainties
  - Finite-pion mass
  - Finite volume
  - Excited states
- Role of sea quarks and of gluons now being addressed
- New questions
  - Can we go beyond moments?
  - How is hadron structure "modified" in medium?
  - Formalism for properties of unstable hadrons?
  - Form factors at high Q^2

Lattice QCD Roadmap

Workshop on Extreme Computing, Jan. 2009

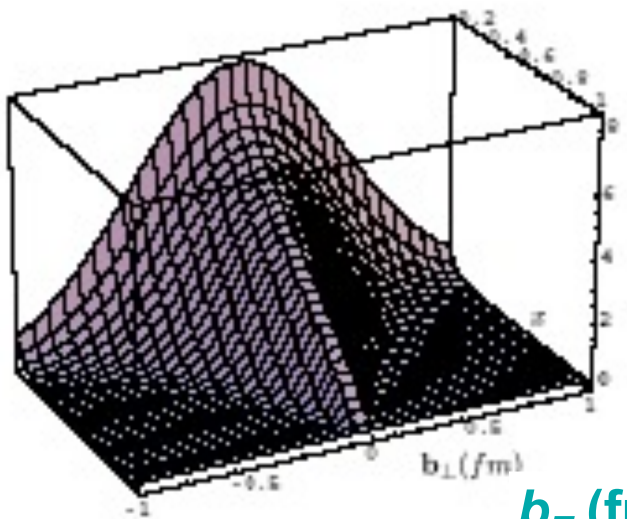


Transverse Distribution - I



• **Lattice QCD can compute moments of GPDs and PDFs, and the *t*-dependence**

$$A^q_{n0}(-\vec{\Delta}^2_\perp) = \int d^2b_\perp \, e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \int_{-1}^{1} dx \, x^{n-1} q(x, \vec{b}_\perp)$$



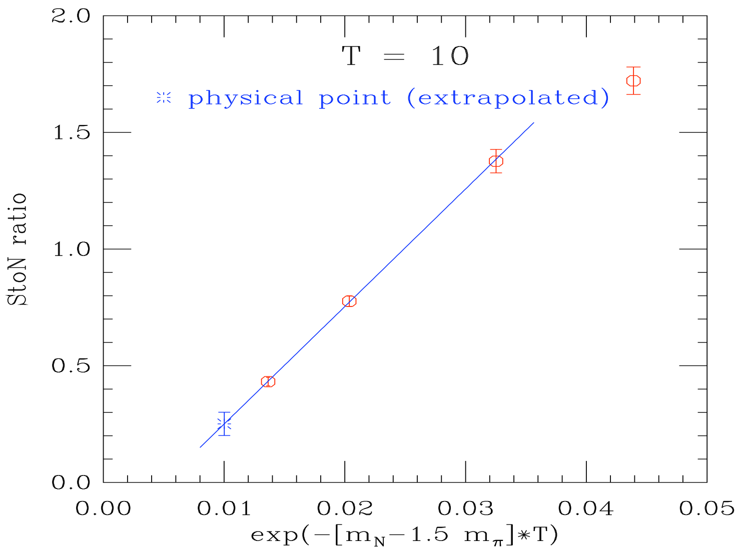
x

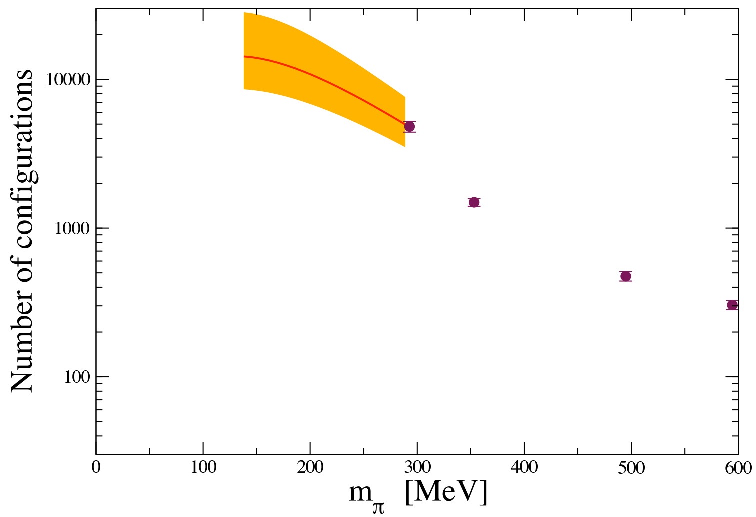
b_T (fm)

Compare to phenomenological models

Decrease slope : decreasing transverse size as x ! 1
Burkardt

Statistics for Hadron Structure





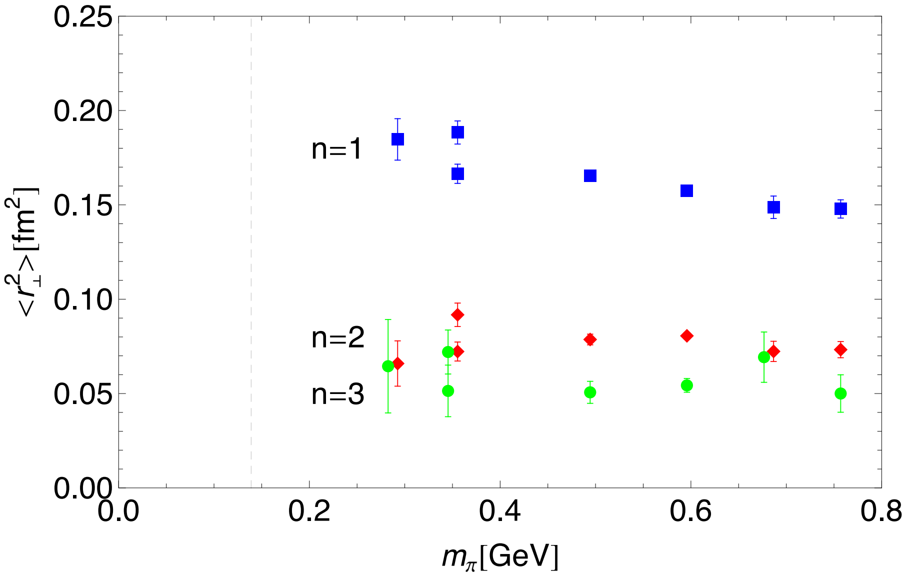
Increasing statistics in approach to physical quark mass: ***more severe for baryons than mesons***

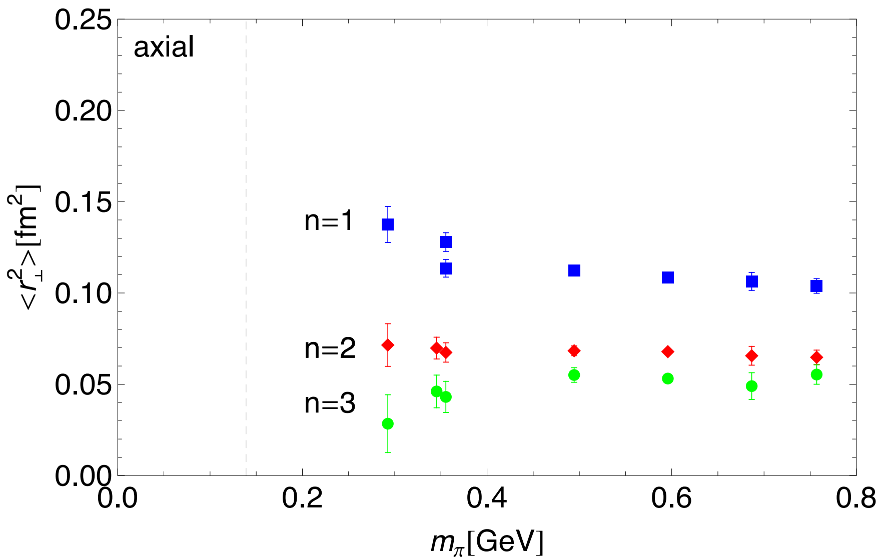
Transverse Distribution - II

Lattice results consistent with narrowing of transverse size with increasing x

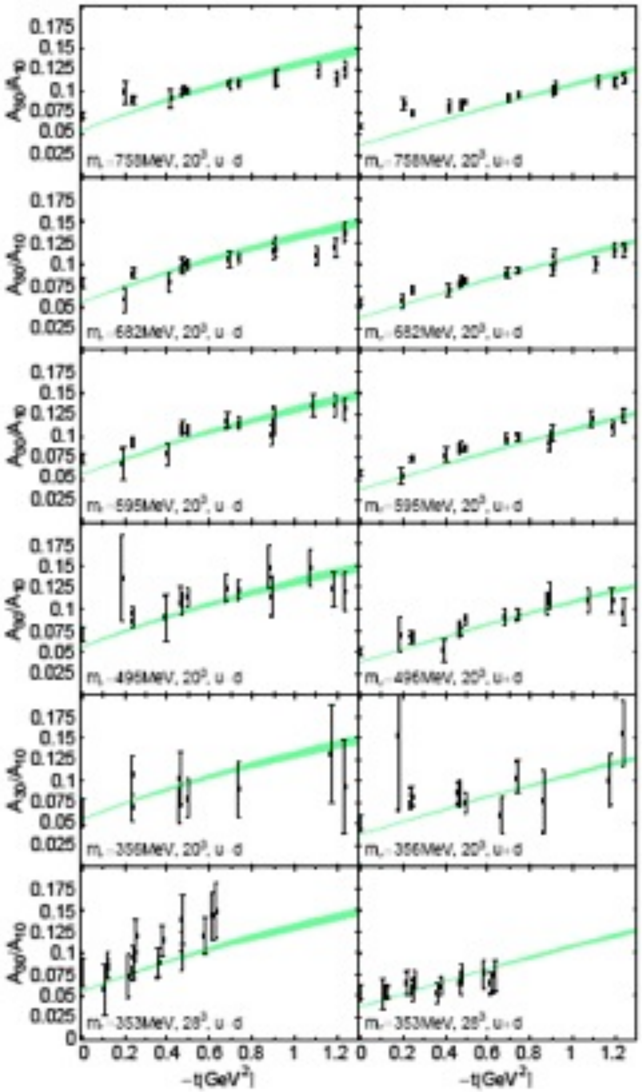
LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008)

Flattening of GFFs with increasing n





Parametrizations of GPDs



Provide phenomenological guidance for GPD's

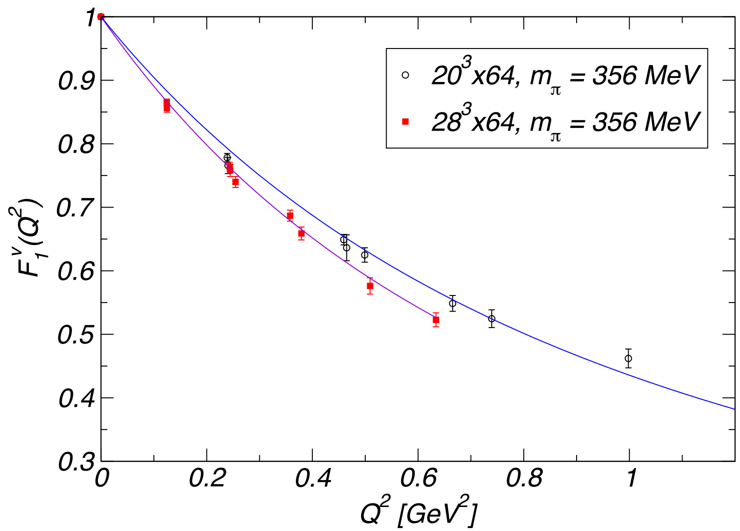
– *CTEQ, Nucleon Form Factors, Regge*

Comparison with *Diehl et al,* hep-ph/0408173

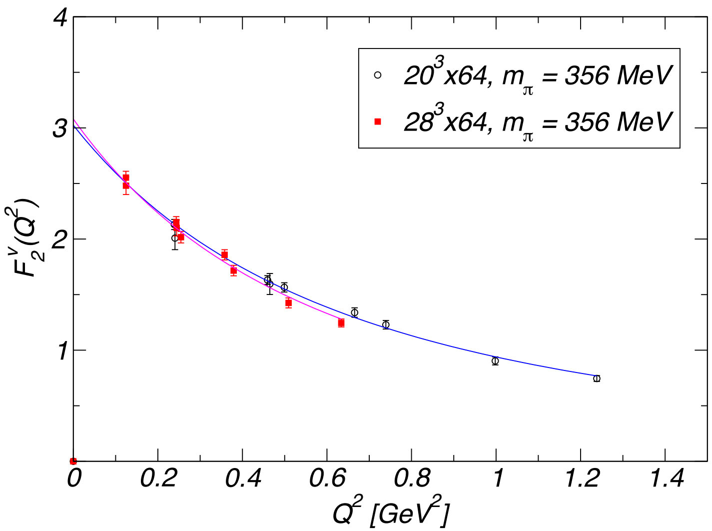
Important Role for LQCD

Nucleon Form Factors - II

LHPC, arXiv:1001:3620



T. Hemmert, HS2



Data well described by dipole form - but example of notable finite-volume effect

LHP Collaboration

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J.W. Negele,[1] K. Orginos,[7] A.V. Pochinsky,[1] M. Procura,[1,4] D.G. Richards,[2] W. Schroers,[8,9,*] and S.N. Syritsyn[1]
(LHPC)

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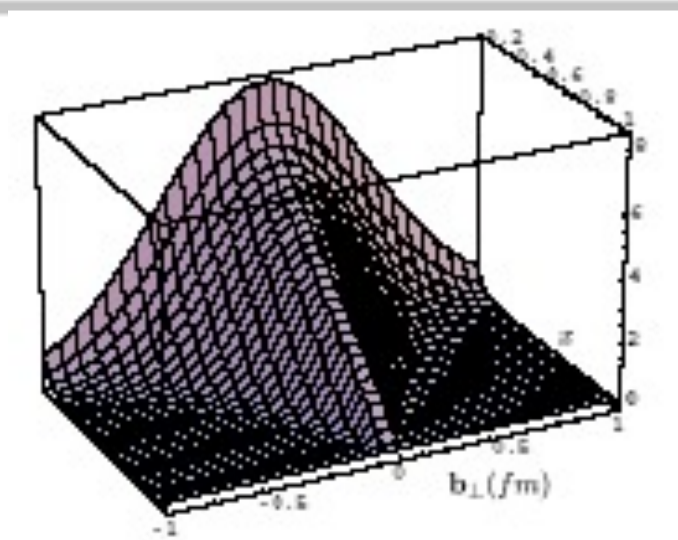
National Taiwan University, Taipei 10617, Taiwan, R.O.C.
(Dated: June 29, 2010)

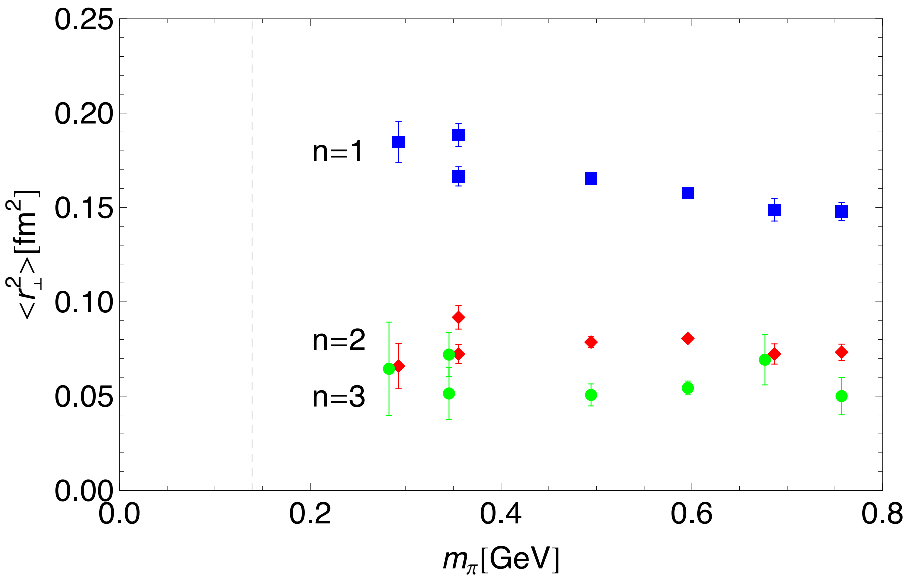
Transverse Structure

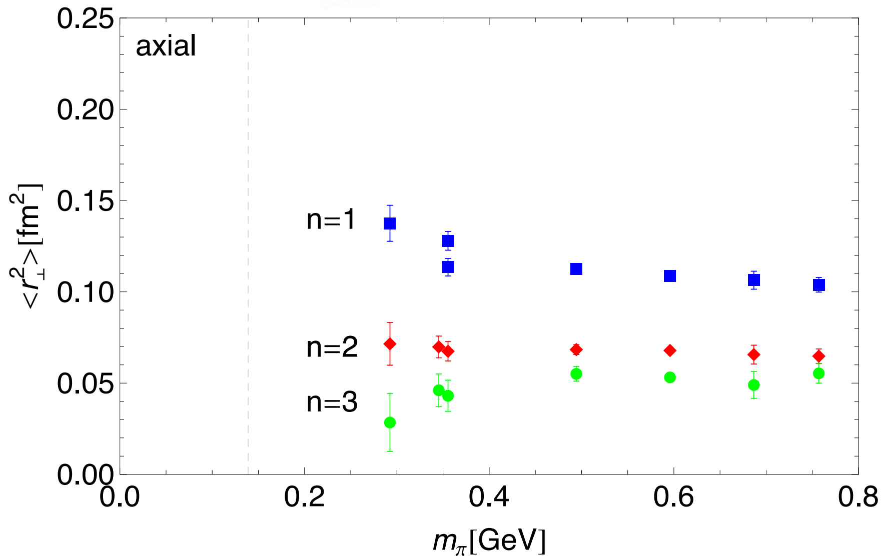
Lattice results consistent with narrowing of transverse size with increasing x *Burkardt*



Flattening of GFFs with increasing n

$$A^q_{n0}(-\vec{\Delta}^2_\perp) = \int d^2 b_\perp \; e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \int_{-1}^{1} dx \; x^{n-1} q(x, \vec{b}_\perp)$$



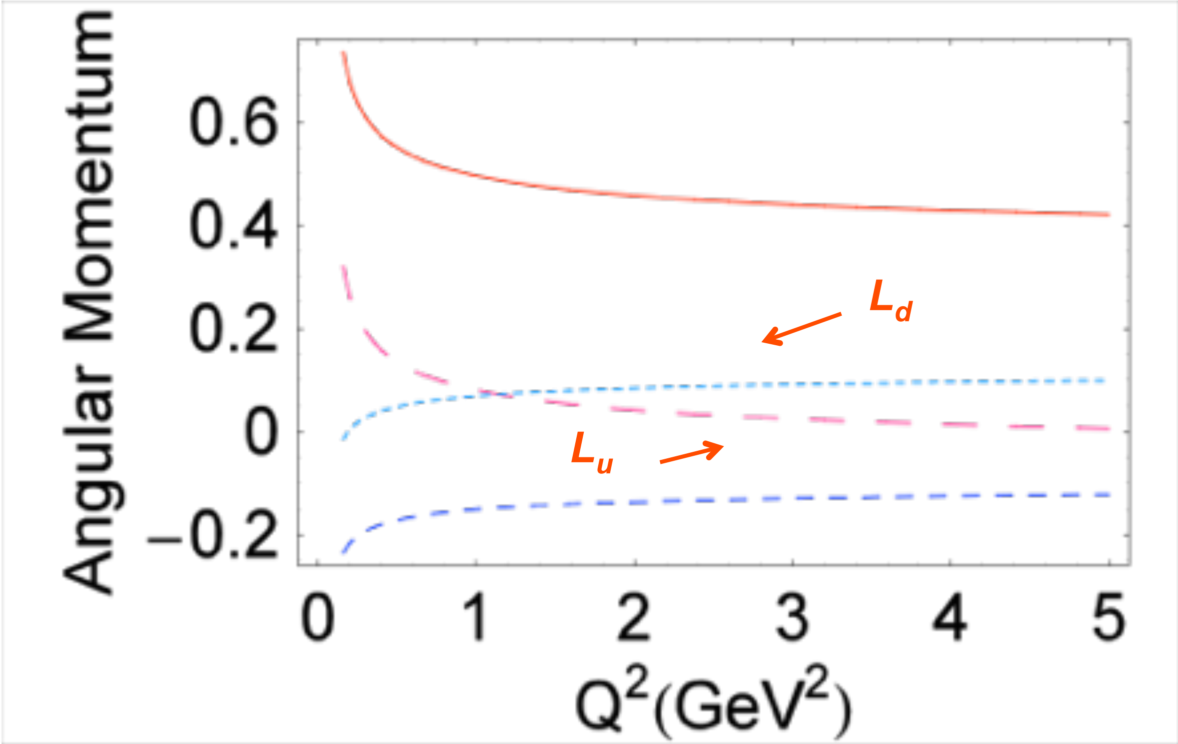


Origin of nucleon spin - III

"Missing spin": orbital angular momentum of quarks and anti-quarks

L_u +ve, L_d small, -ve at model scale

Myhrer & Thomas, Phys.Lett.B663:302 (2008)



A.Thomas, Phys. Rev. Lett. 101:102003 (2008)

Transverse Spin in Nucleon

Measuring generalized form factors corresponding to tensor current gives provides information on transverse spin of nucleon

$$\langle P'\Lambda'|\mathcal{O}_T^{\mu\nu}|P\Lambda\rangle = \overline{u}(P',\Lambda')\Big\{\sigma^{\mu\nu}\gamma_5\Big(A_{T10}(t) - \frac{t}{2m^2}\widetilde{A}_{T10}(t)\Big) + \frac{\epsilon^{\mu\nu\alpha\beta}\Delta_\alpha\gamma_\beta}{2m}\overline{B}_{T10}(t) - \frac{\Delta^{[\mu}\sigma^{\nu]\alpha}\gamma_5\Delta_\alpha}{2m^2}\widetilde{A}_{T10}(t)\Big\}u(P,\Lambda)\ ,$$

QCDSF/UKQCD, PRL, 0612021

$$\mathcal{O}_T^{\mu\nu} = \bar{q}\sigma_{\mu\nu}\gamma_5 q$$

Lowest moment $B_{T10}(t)$