Hadron Structure Approach to the Running Coupling at Low Energy

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IFPA, Universite de Liege

Workshop on Confinement Physics
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Outline

talk in collaboration with S. Liuti

- Strong Coupling Constant
  - Perturbative determination
  - Non-perturbative approaches
- Hadron Structure Phenomenology
  - Final State Interaction and Parton Distribution Functions
  - Parton-Hadron Duality
- Nonperturbative QCD coupling from Phenomenology PRELIMINARY

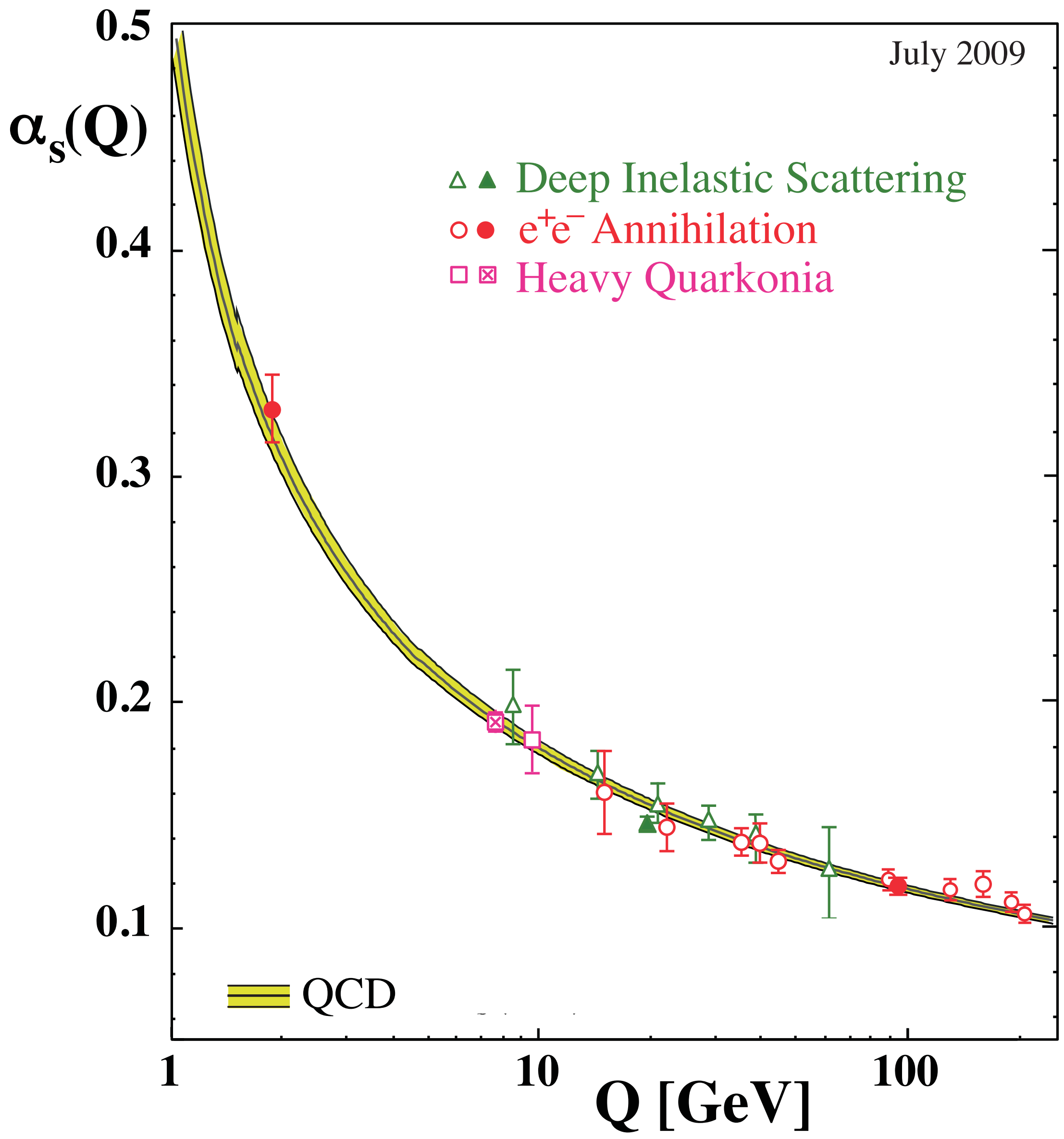
QCD Coupling Constant in pQCD

- QCD with massless quarks
  - ➡ no scale parameters
- RGE introduces a momentum scale Λ
  - ➡ interaction strength =1
- Renormalization scheme dependence of Λ
- World data average (2009)

$$\alpha_s(M_{Z^0}) = 0.1184 \pm 0.0007$$

that corresponds to

$$\Lambda^{(5)}_{\overline{MS}} = (213 \pm 9\ )\,\mathrm{MeV}$$



Bethke, Eur.Phys.J. C64

QCD Running Coupling Constant

$$\frac{d\,a(Q^2)}{d(\ln\ Q^2)} = \beta_{N^m LO}(\alpha) = \sum_{k=0}^{m} a^{k+2}\beta_k$$

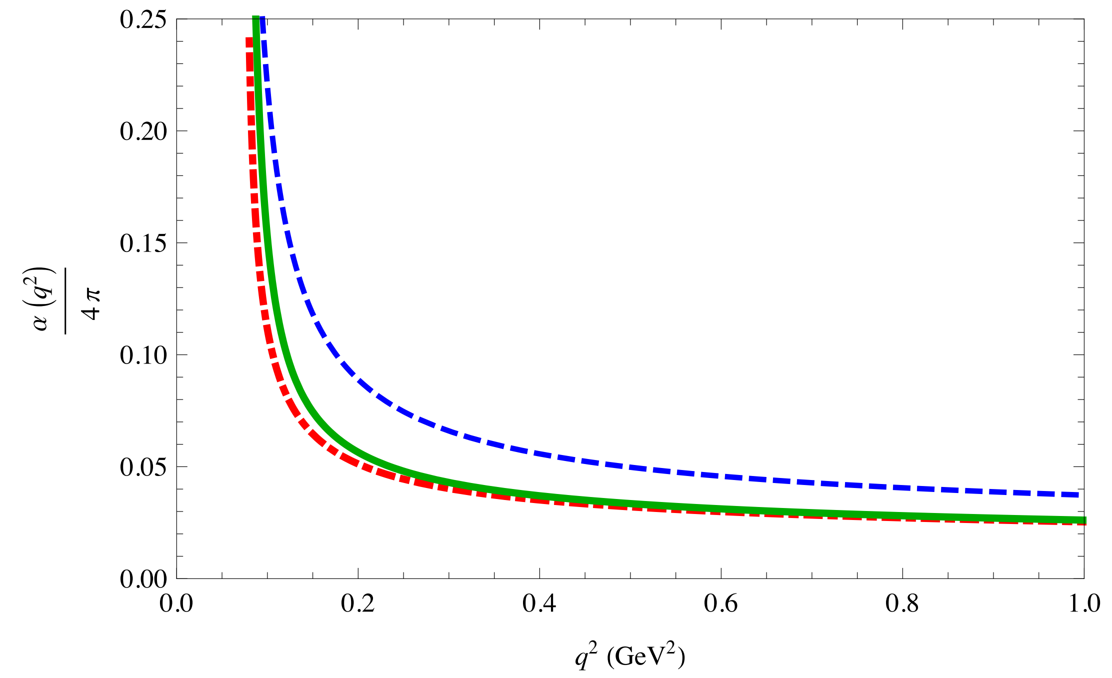
$\overline{\mathrm{MS}}$ scheme

$a=\alpha_s/4\pi$

LO exact perturbative solution Λ=250 MeV

NLO exact perturbative solution Λ=250 MeV

NNLO exact perturbative solution Λ=250 MeV



QCD predicts the shape of the running coupling constant, not its value

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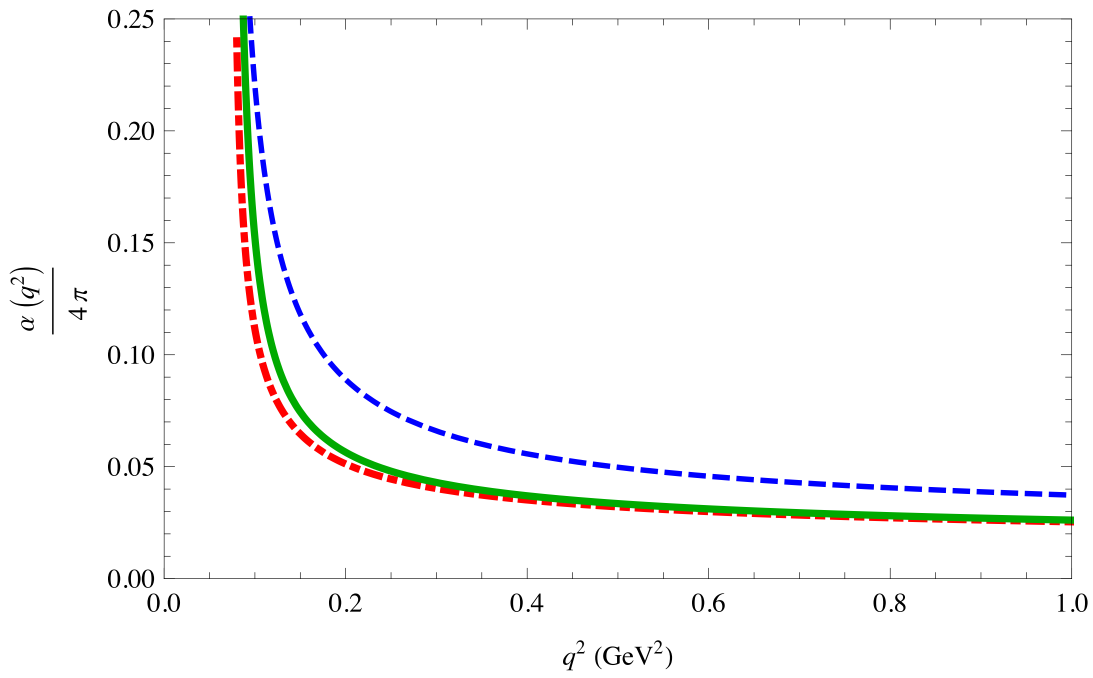


QCD predicts the shape of the running coupling constant, not its value

Intermediate energy?
Perturbative to nonperturbative transition?

Effective Charges

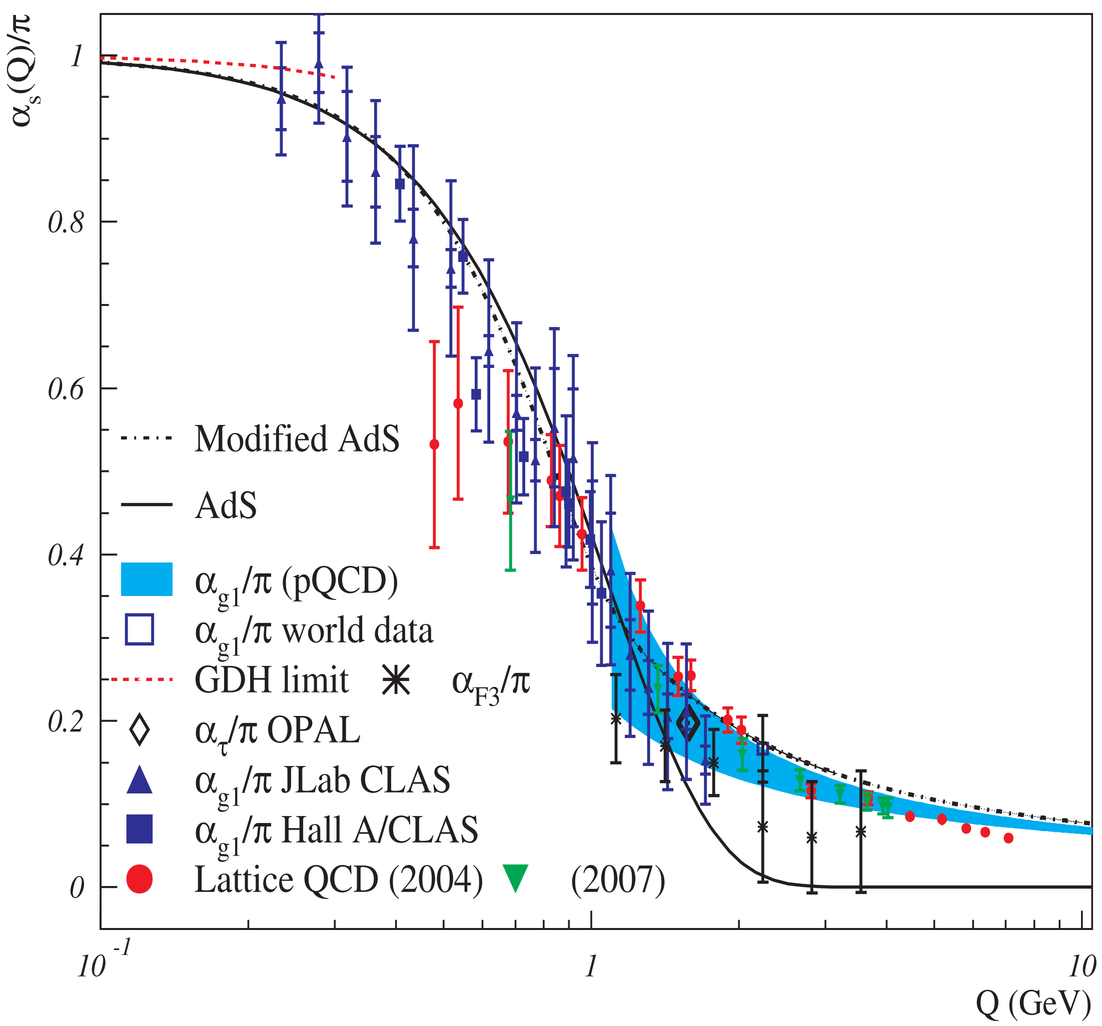
The non-perturbative approach:

- Importance of finite couplings
- Taming the Landau pole

The non-perturbative extraction:

- Effective couplings from phenomenology
- Dimensional transmutation (RG-improved)

  ➡ from RS dependence to Observable dependence (à la Grunberg)



[Brodsky et al., Phys.Rev.D81]
[Deur et al., Phys.Lett.B60]

Nonperturbative Gluon Propagator

Solving the Schwinger-Dyson eqs ...

J. M. Cornwall, Phys. Rev. D26, 1453 (1982)
A. C. Aguilar and J. Papavassiliou, JHEP0612, 012 (2006)

$$\Delta^{-1}(Q^2) = Q^2 + m^2(Q^2)$$

$$m^2(Q^2) = m_0^2 \left[\ln\left(\frac{Q^2 + \rho m_0^2}{\Lambda^2}\right) \Big/ \ln\left(\frac{\rho m_0^2}{\Lambda^2}\right) \right]^{-1-\gamma}$$

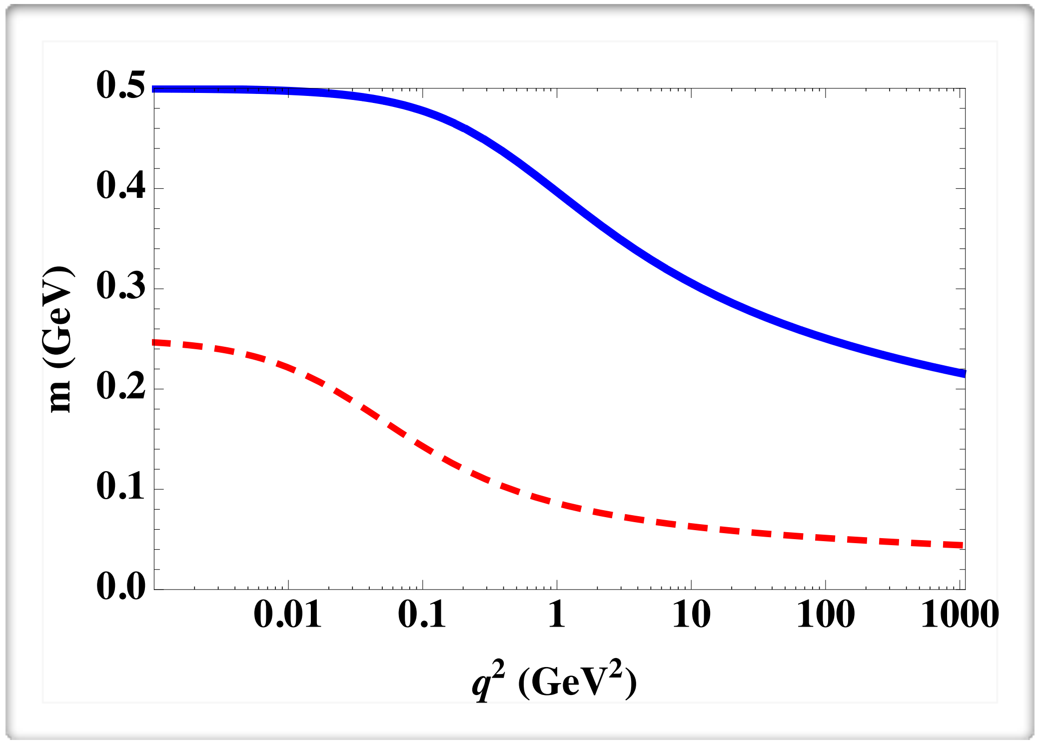
Gluon Mass as IR Regulator

- **effective gluon mass**

phenomenological estimates

$$m_0 \sim \Lambda - 2\Lambda$$

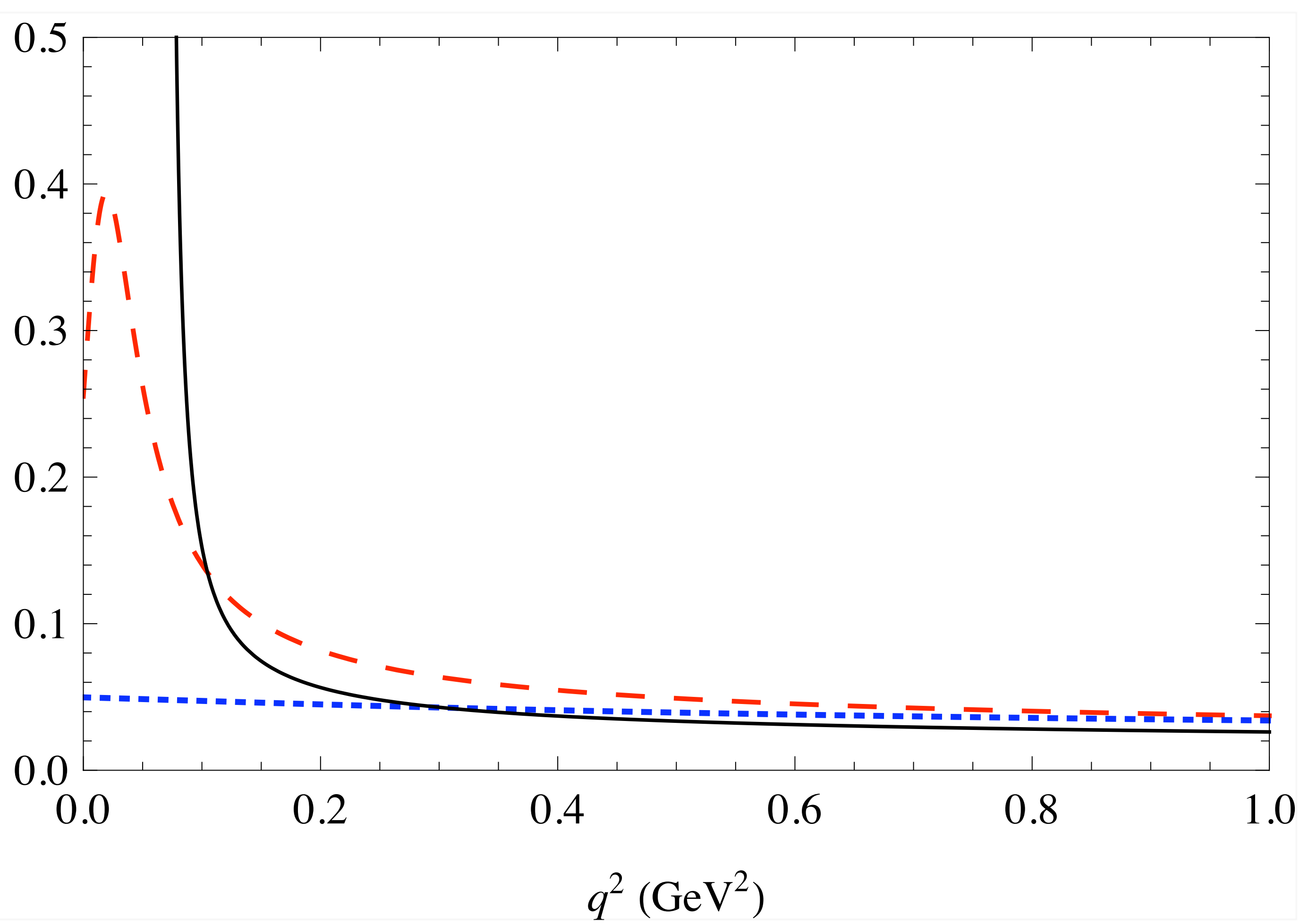
- **Solution free of Landau pole**
- **Freezes in the IR**



Low mass scenario
High mass scenario

NP Momentum-dependence of the Coupling Constant

$$\frac{\alpha_{\rm NP}(Q^2)}{4\pi} = \left[\beta_0 \ln\left(\frac{Q^2 + \rho m^2(Q^2)}{\Lambda^2}\right)\right]^{-1}$$



LO perturbative evolution

Λ=250 MeV ; $\overline{MS}$ scheme

Low mass scenario NP coupling constant

m_0=250 MeV ; Λ=250 MeV ; ρ=1.5

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[A.C., Vento & Scopetta, Eur.Phys.J.A47]

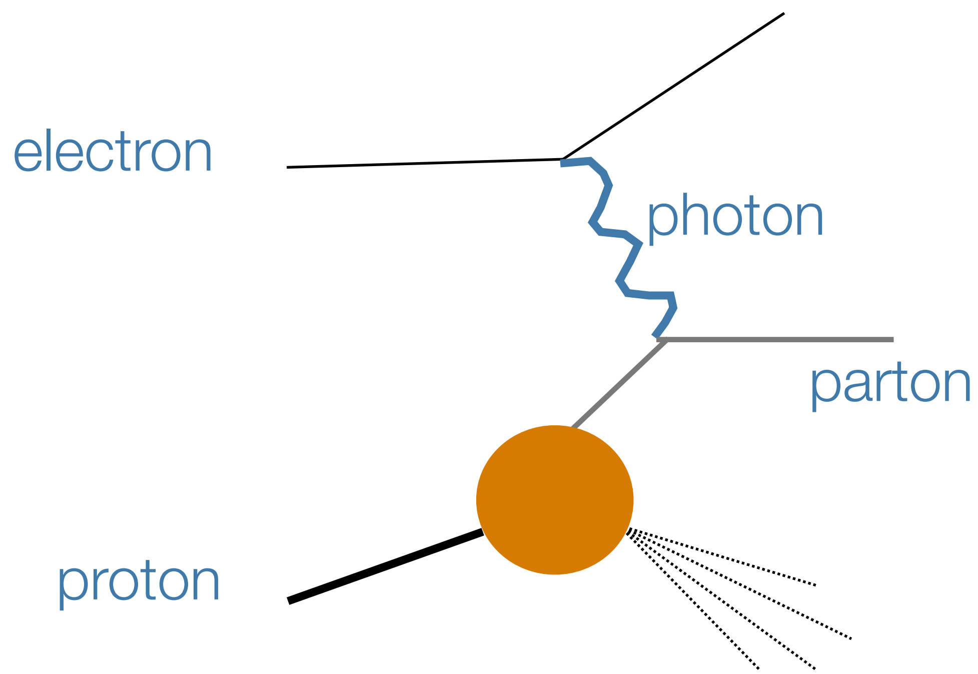
Hadron Structure Phenomenology

Final State Interaction and Parton Distribution Functions

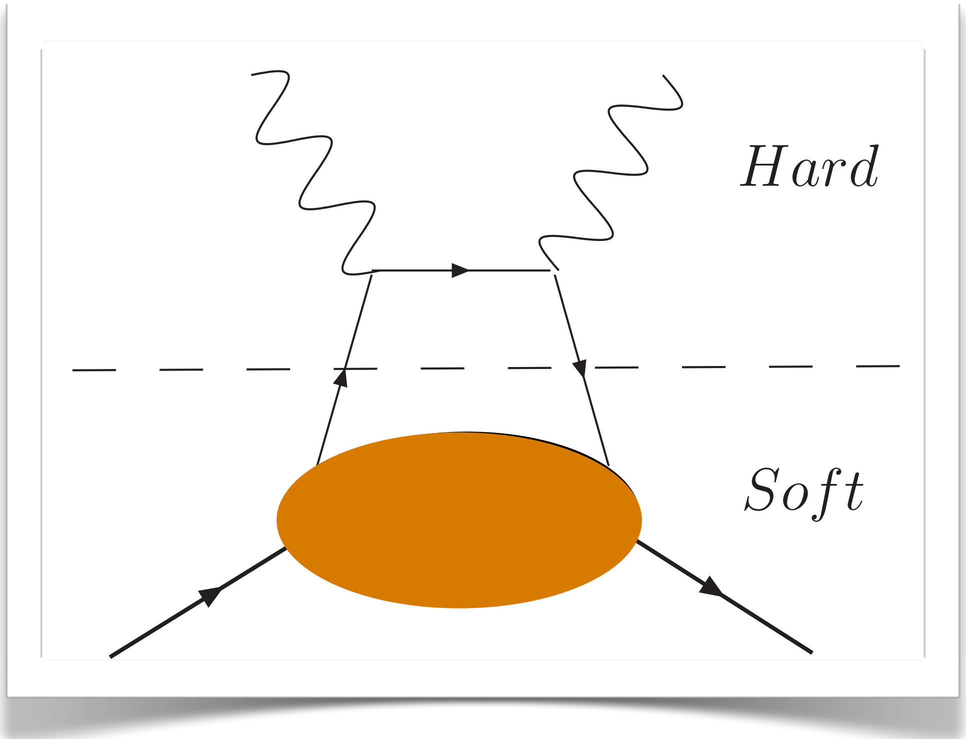
Hard Probes and Factorization

Small size configuration ⇒ Hard Probes ⇒ Hard processes

Deep Inelastic Scattering



Hadronic tensor ⇒



Parton Model
High energy photon Q^2
Fast-moving proton
Bjorken scaling

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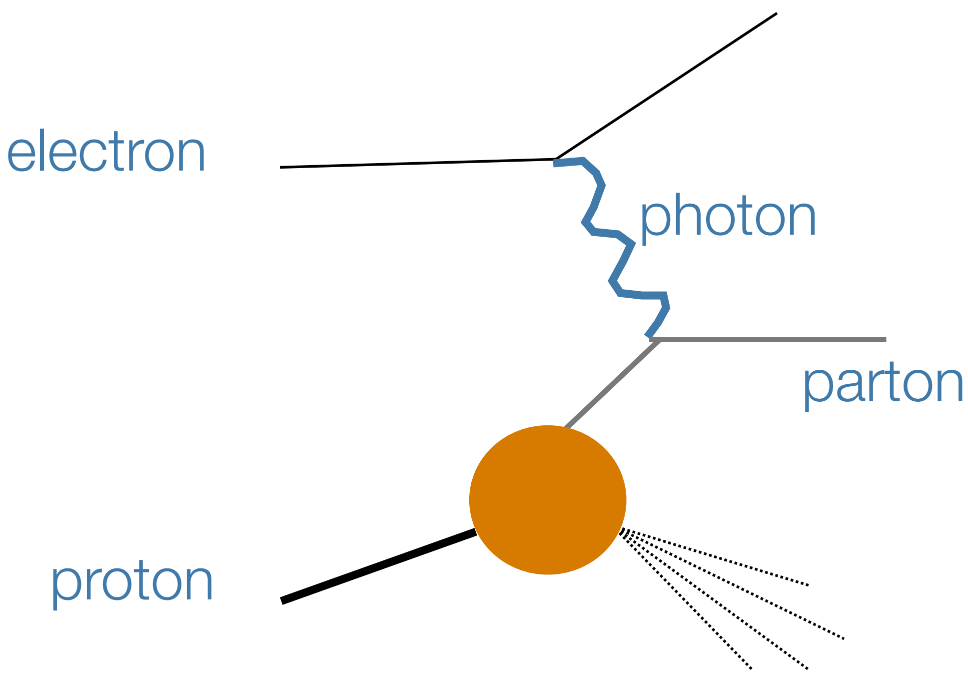

Factorization
& factorization scale

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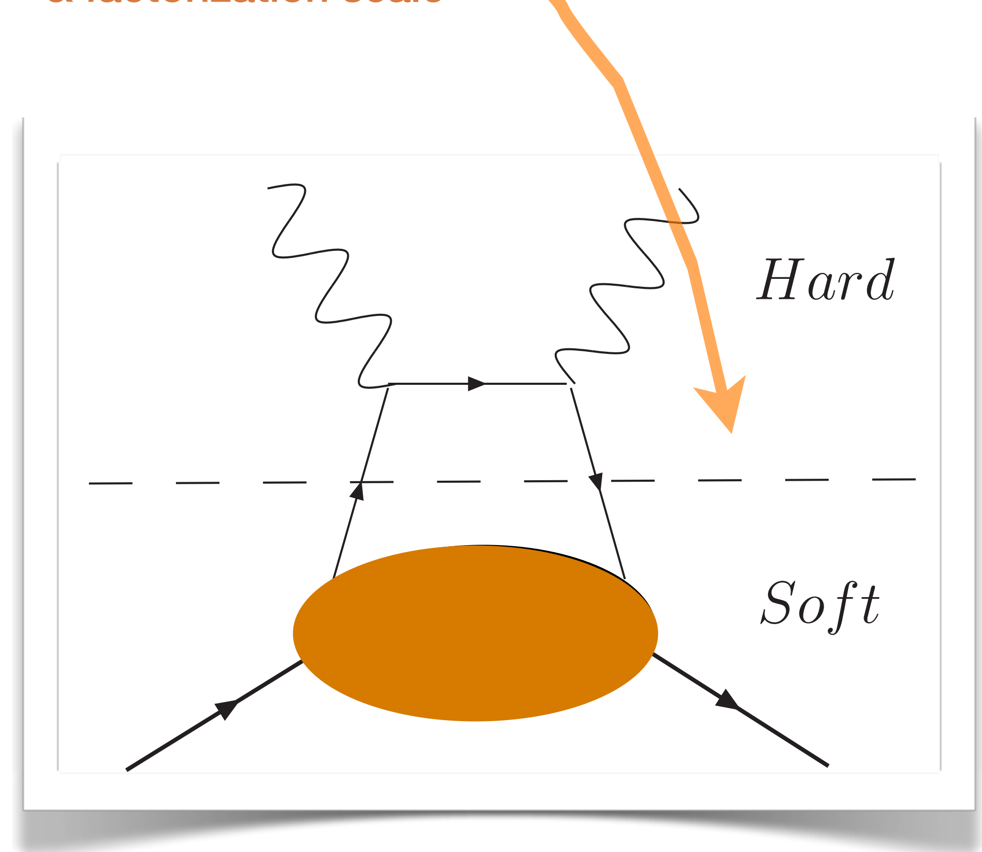

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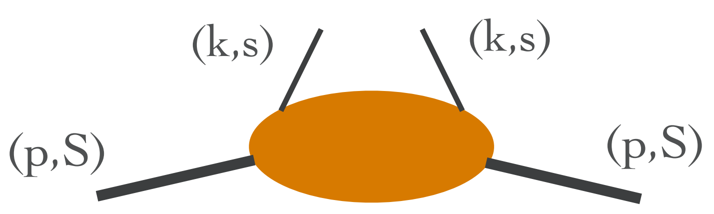
Transverse Momentum Dependent PDFs



Hadronic matrix elements to $f(x, k_T)$

$\Downarrow$

Number of independent structure functions

$\updownarrow$

Number of Lorentz scalars +hermiticity+parity invariance+Time-reversal invariance

- Relaxing Time-reversal invariance $\Rightarrow$ naive T-odd functions

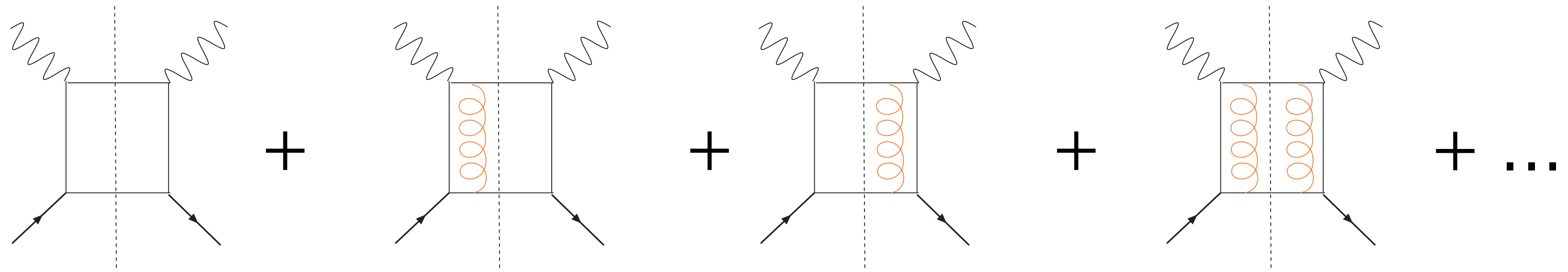
Sivers & Boer-Mulders functions

Sivers, Phys.Rev.D41
Boer & Mulders, Phys.Rev.D57

- Existence of Final State Interactions (FSI) at leading-order

Brodsky, Hwang & Schmidt, Phys.Lett.B530

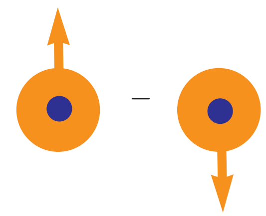
- Importance of the gauge link



T-odd TMDs

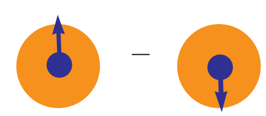
The Sivers function $f_{1T}^{\perp Q}(x, k_T)$

$\Rightarrow$ Distribution of **unpolarized quarks** inside a **transversely polarized proton**



The Boer-Mulders functions $h_1^{\perp Q}(x, k_T)$

$\Rightarrow$ Distribution of **transversely polarized quarks** inside a **unpolarized proton**



- Matrix element of low twist operator

$$f_{1T}^{\perp q}(x, k_T) = -\frac{M}{2k_x}\int \frac{d\xi^- d^2\vec{\xi}_T}{(2\pi)^3}\, e^{-i(xp^+\xi^- - \vec{k}_T\cdot\vec{\xi}_T)} \times \frac{1}{2}\sum_{S_y=-1,1} S_y \langle PS_y|\overline{\psi}_q(\xi^-, \vec{\xi}_T)\mathcal{L}^\dagger_{\vec{\xi}_T}(\infty, \xi^-)\gamma^+ \mathcal{L}_0(\infty, 0)\psi_q(0,0)|PS_y\rangle + \text{h.c.}$$

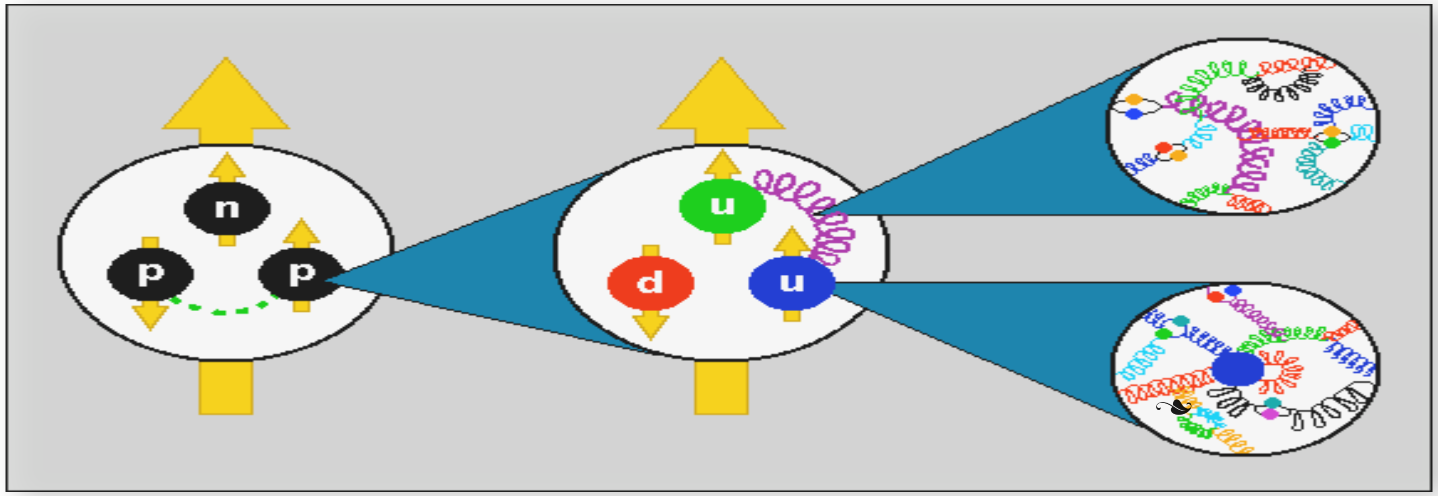
- Importance of gauge link

$$\mathcal{L}_{\vec{\xi}_T}(\infty, \xi^-) = \mathcal{P}\exp\left(-ig\int_{\xi^-}^{\infty} A^+(\eta^-, \vec{\xi}_T)\, d\eta^-\right)$$

- holds in covariant gauges
- process dependent

Hadronic Models

Hadron ⇔ Constituent quarks ⇔ Current quarks



Nonperturbative vs. Perturbative QCD

Models of Hadron Structure

Renormalization Group Eqs.

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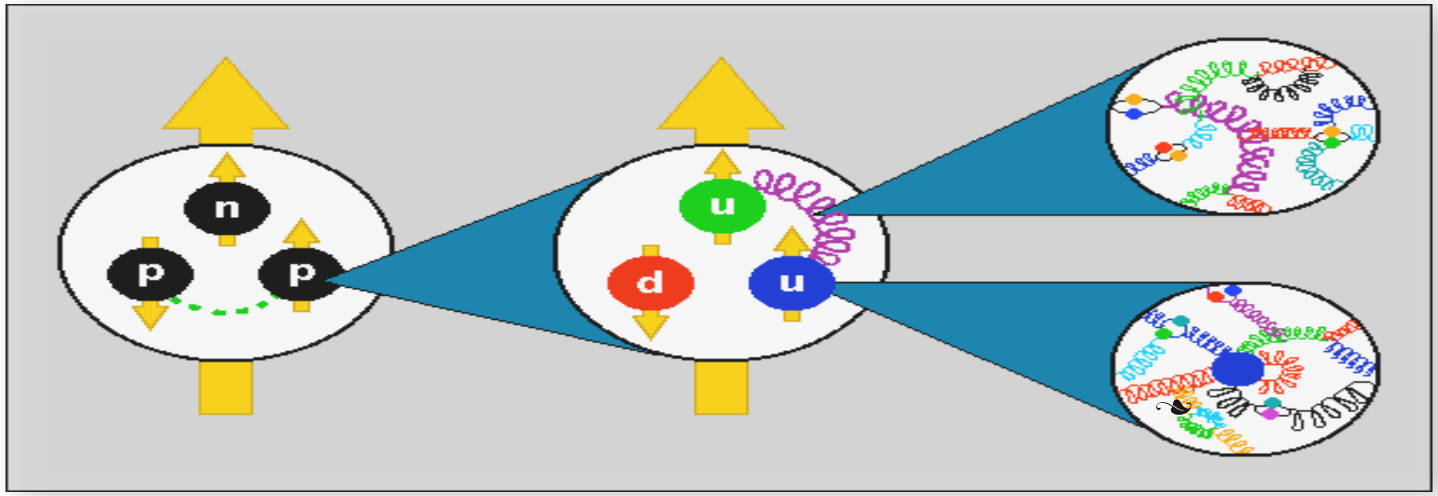

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Observable

- calculated in hadronic model
- at scale μ_0
- switch on QCD evolution

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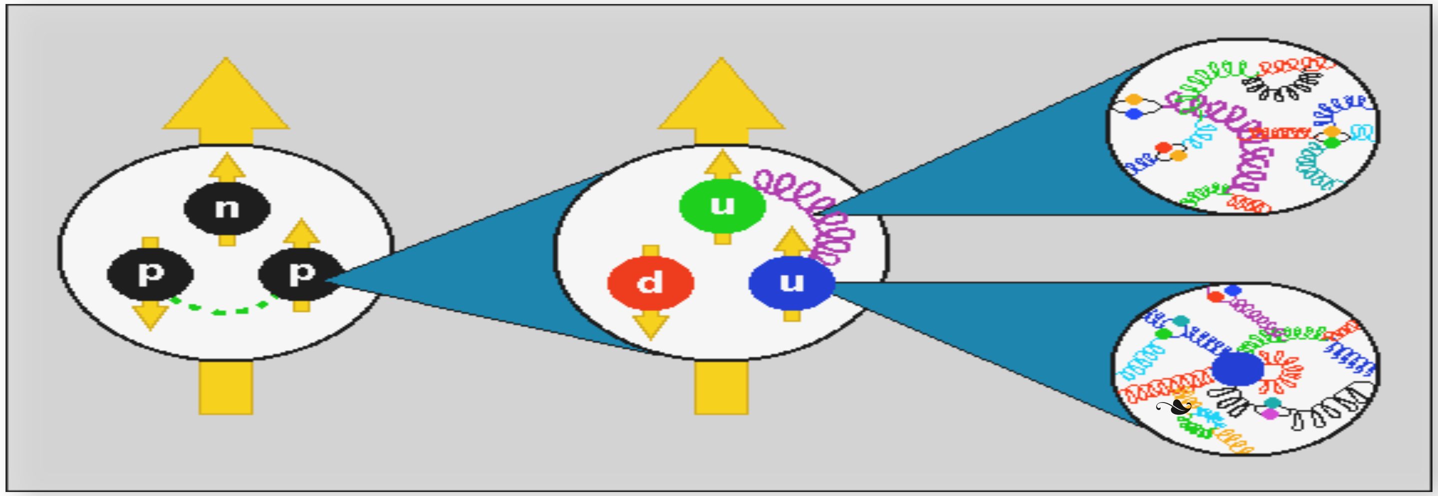

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?

Hadronic Scale from collinear PDFs, e.g. CTEQ, GRV,...

We use RGE and one *first principle* based assumption.
Then we set scenarios ...

Parisi & Petronzio, Phys. Lett. B 62 (1976) 331
Traini et al, Nucl. Phys. A 614, 472 (1997)

Hadronic Scale from collinear PDFs, e.g. CTEQ, GRV,...

**We use RGE and one *first principle* based assumption.
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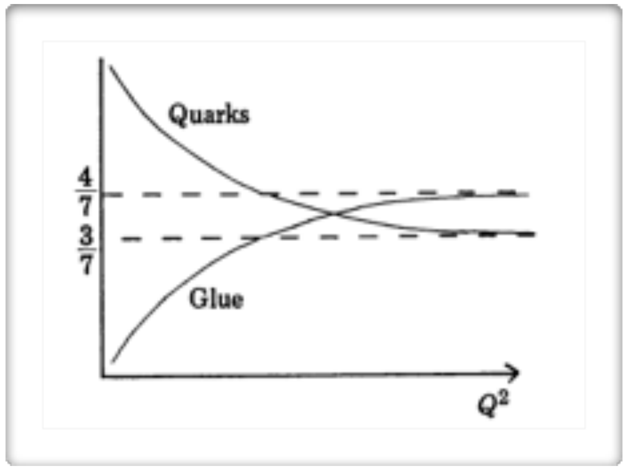
Say there exists a scale at which there is no sea and no gluon, then

$$\langle (u_v + d_v)\,(\mu_0^2) \rangle_{n=2} = 1$$

QCD evolution introduces gluons and sea quarks:

$$\langle (u_v + d_v)\,(Q^2 = 10\,\mathrm{GeV}^2) \rangle_{n=2} = 0.36$$

DATA= PDFs parameterization



R.G.Roberts
"The Structure of the Proton"

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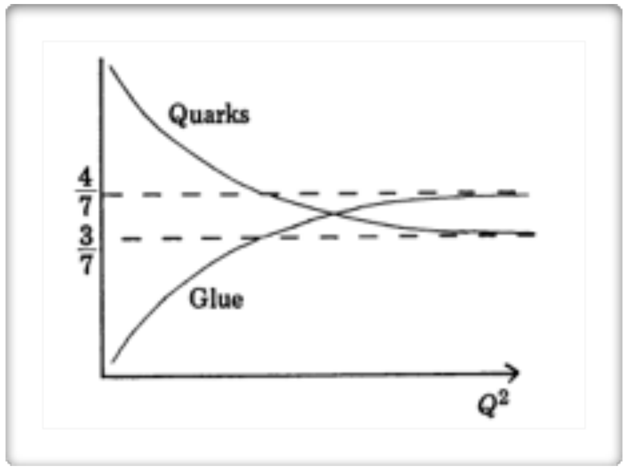

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DATA= PDFs parameterization

Evolve in energy until 2nd moment=1
Find μ_0^2~0.1GeV^2 +$\Delta\mu_0^2$

Parisi & Petronzio, Phys. Lett. B 62 (1976) 331
Traini et al, Nucl. Phys. A 614, 472 (1997)

Perturbative vs. NP 'evolution':Fixing the hadronic scale

2nd moment of f_1

$$\langle q_v(Q^2)\rangle_n = \langle q_v(\mu_0^2)\rangle_n \left(\frac{\alpha(Q^2)}{\alpha(\mu_0^2)}\right)^{d^n_{NS}}$$

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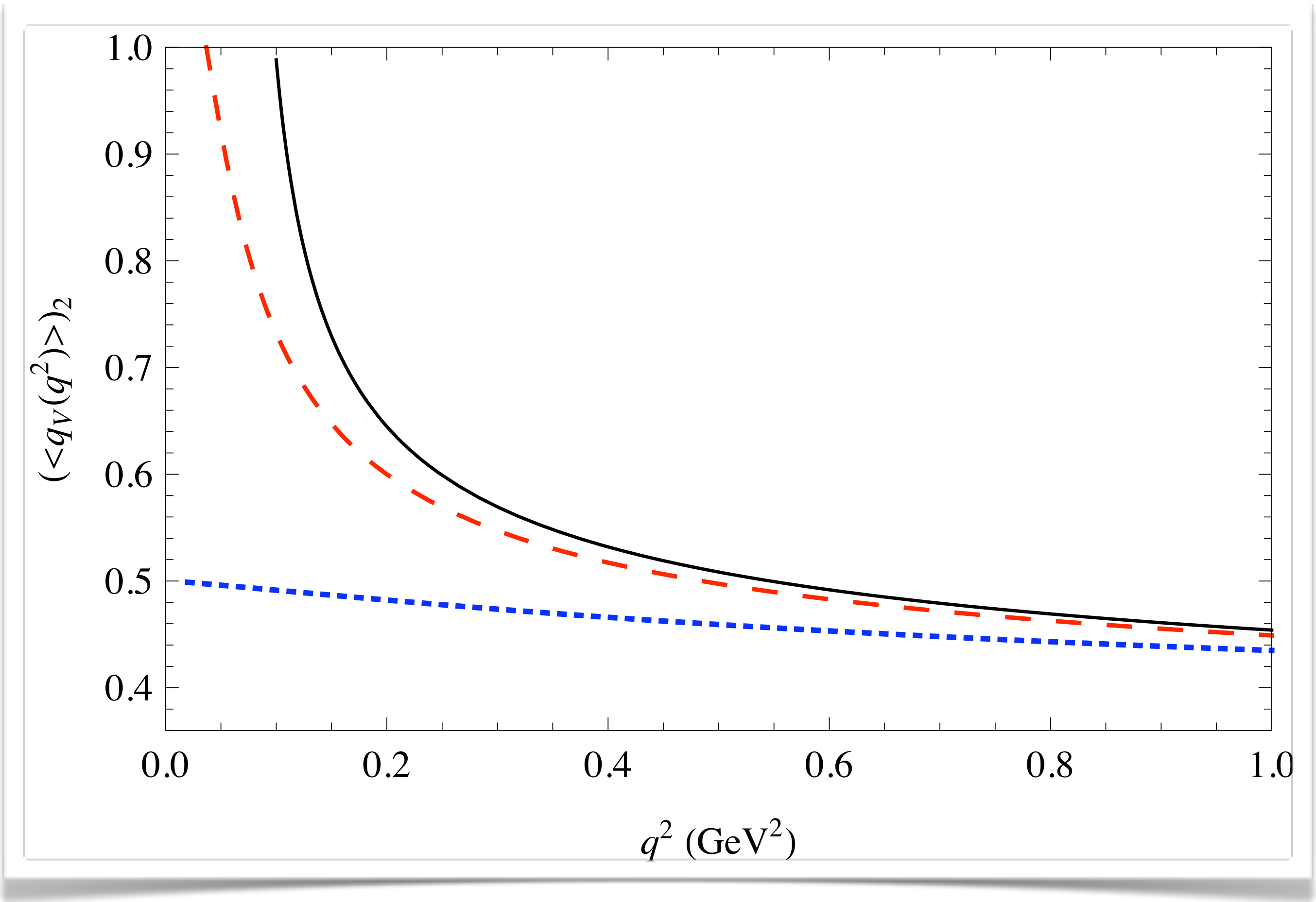

[A.C., Vento & Scopetta, Eur.Phys.J.A47]

Sivers & Boer-Mulders functions

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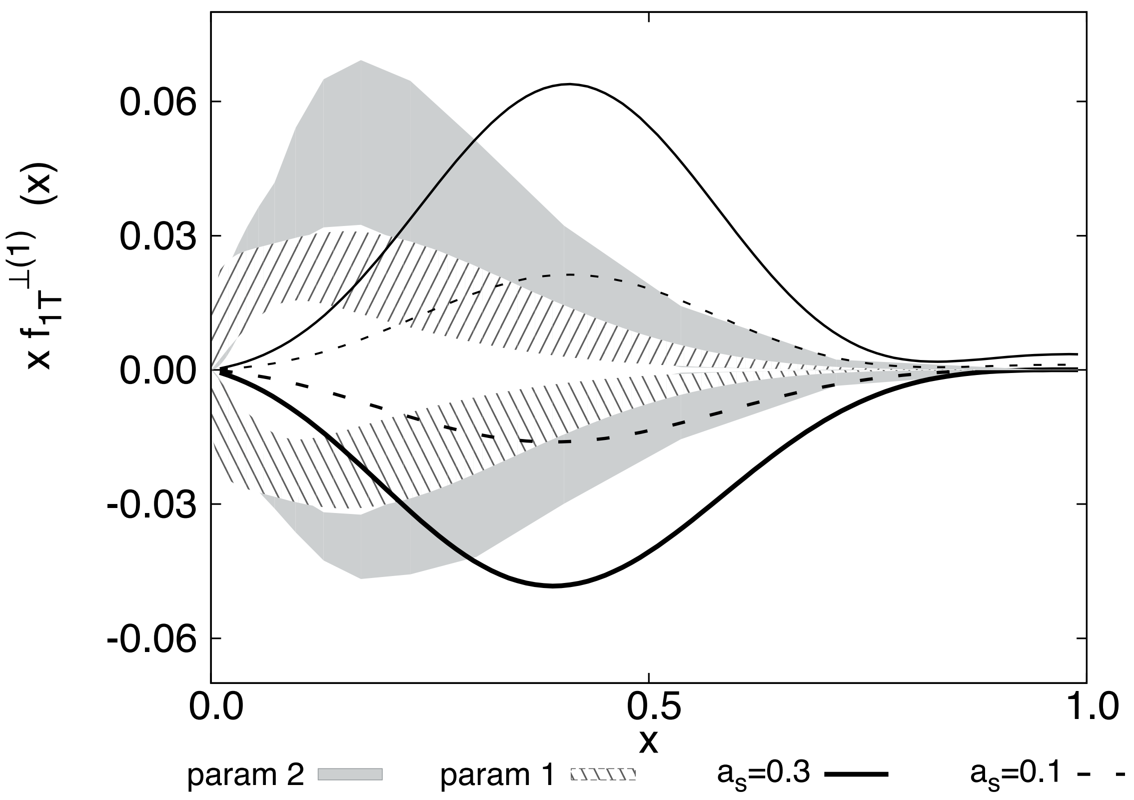
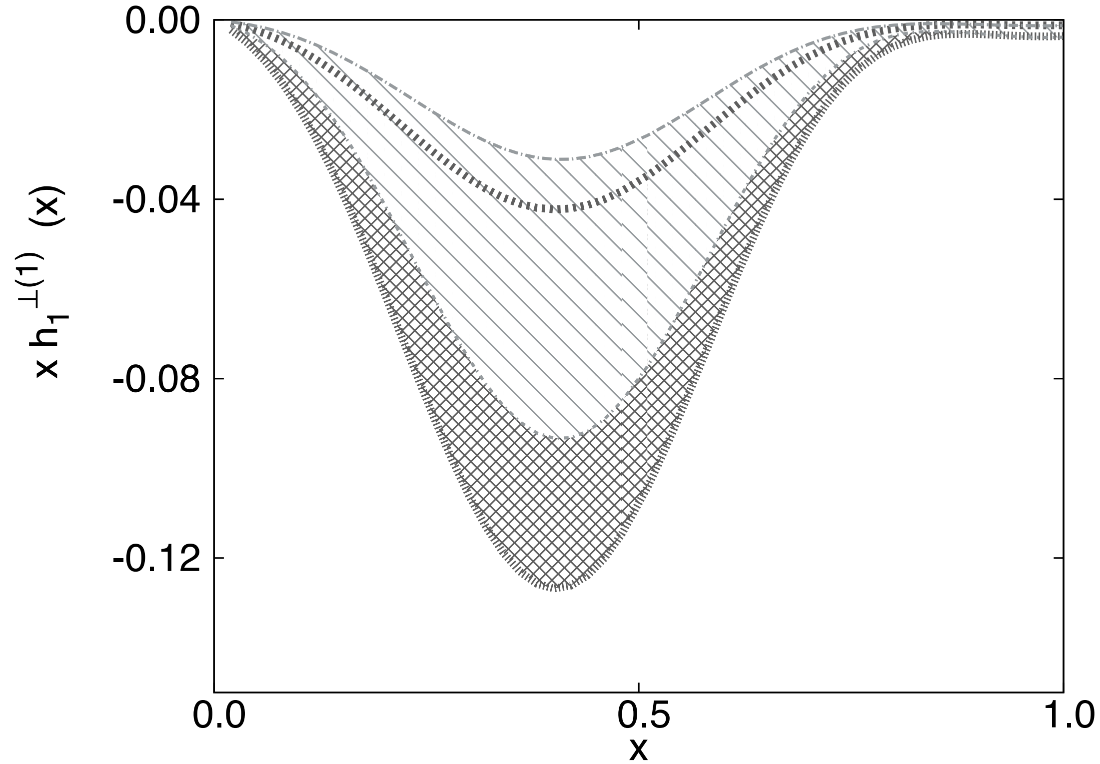


$$0.1 < \frac{\alpha_s(\mu_0^2)}{4\pi} < 0.3$$

dashed $\frac{\alpha_s(\mu_0^2)}{4\pi} = 0.1$

solid $\frac{\alpha_s(\mu_0^2)}{4\pi} = 0.3$

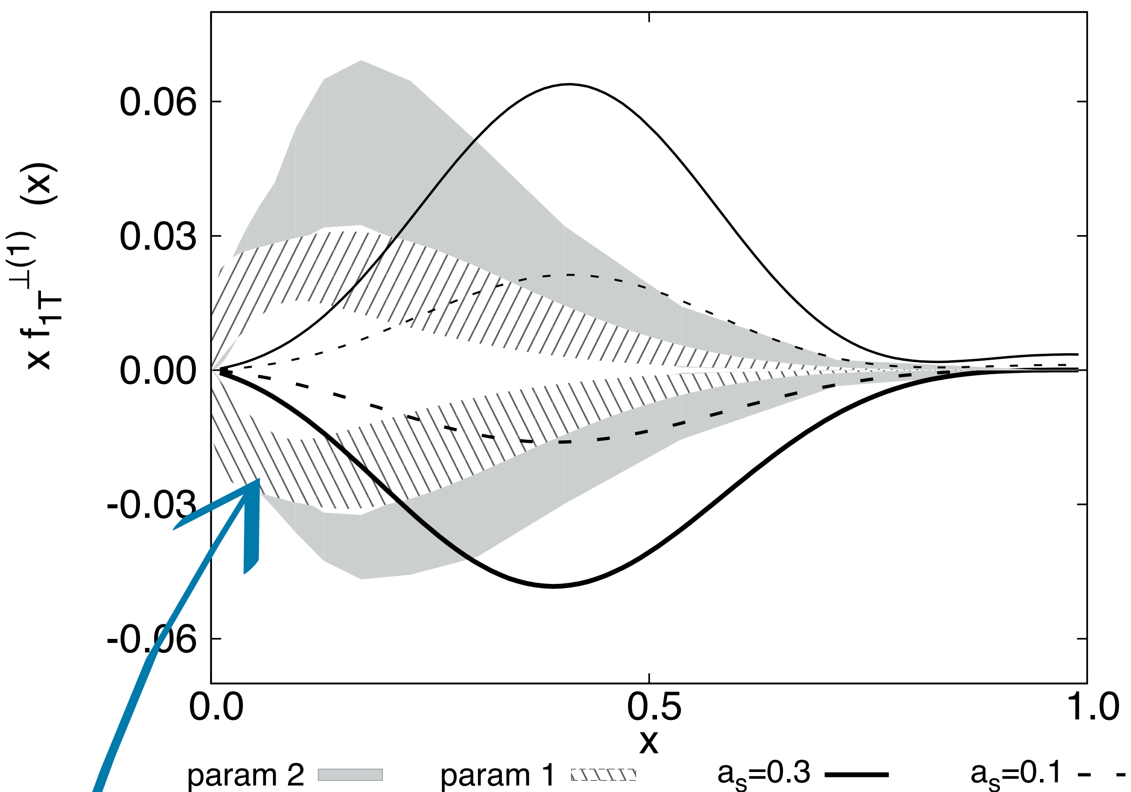


Bag Model:

rescaling/defining error of $f_{1T}^{\perp}$ & $h_1^{\perp}$

Sivers & Boer-Mulders functions

[A.C., Scopetta & Vento, Eur.Phys.J. A47]



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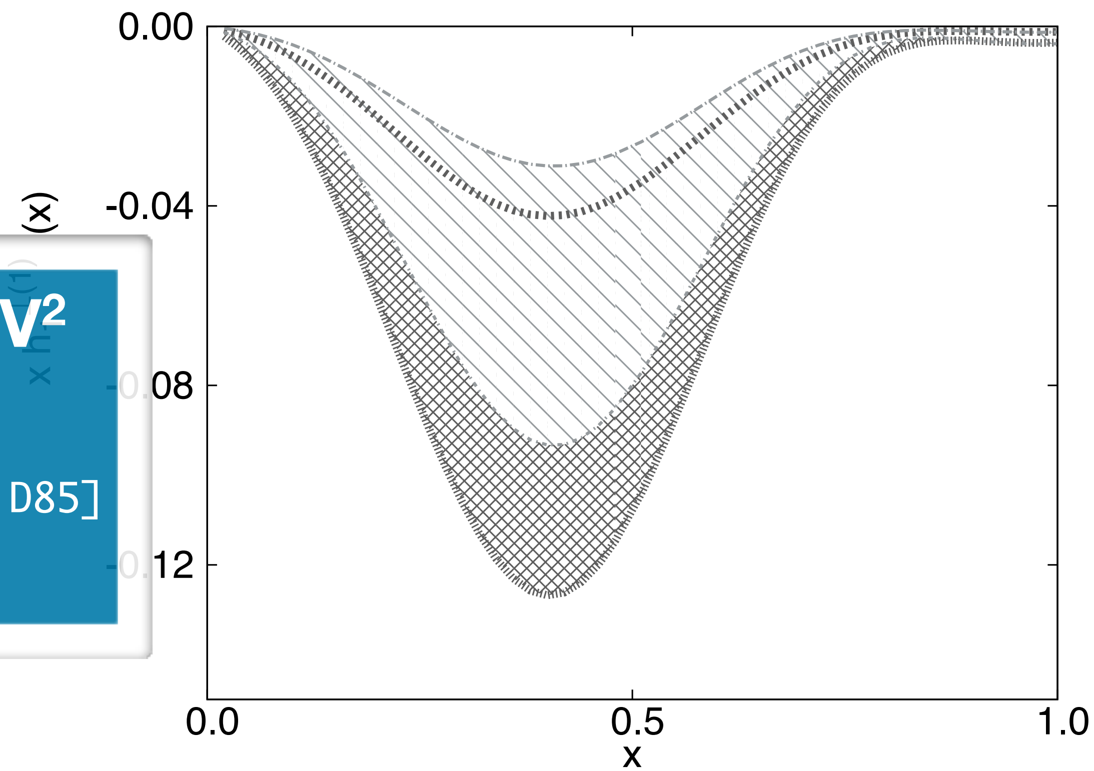


Phenomenological extractions at Q^2=2.5GeV^2

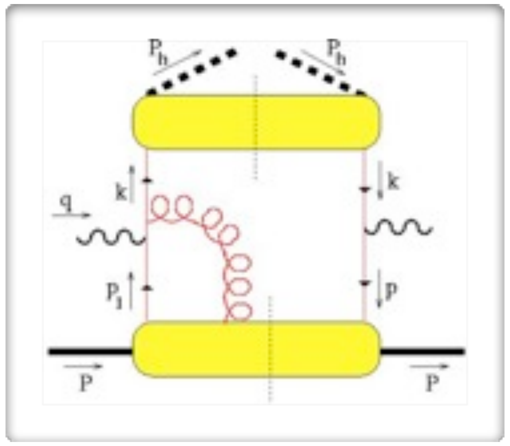
➡ **Need for QCD formalism for T-odd TMDs**

[Aybat, Collins, Qiu & Rogers, Phys.Rev. D85]

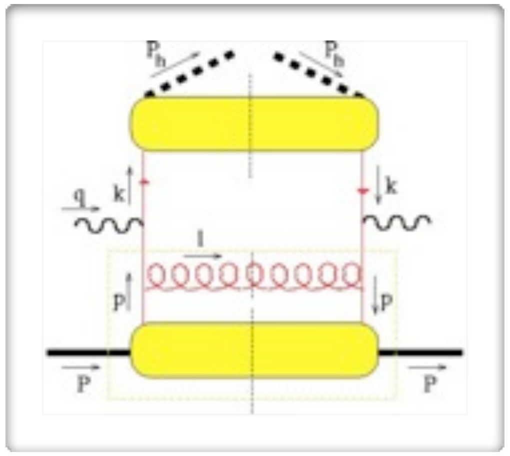
➡ **Additional source of error**

Work in progress for T-odd TMDs

- Ambiguity Sivers function and Qiu-Sterman function
  - Model dependent definition of the FSI and of the proton



- TMD evolution: Coupled CSS and RGE -> two scales ![Aybat et al., PRD85]
  - Definition of momentum regions [Bacchetta et al., JHEP 0808]
  - Redefinition of both scale for model calculations (in collaboration with T. Rogers)
- Correspondance effective coupling from the soft blob with pQCD
  - [Brodsky et al., Phys.Rev.D81] *À la Grunberg?* [Phys. Rev. D29]
  - Commensurate Scale Relations [Brodsky & Lu, Phys. Rev. D251]



Hadron Structure Phenomenology

Parton-Hadron Duality

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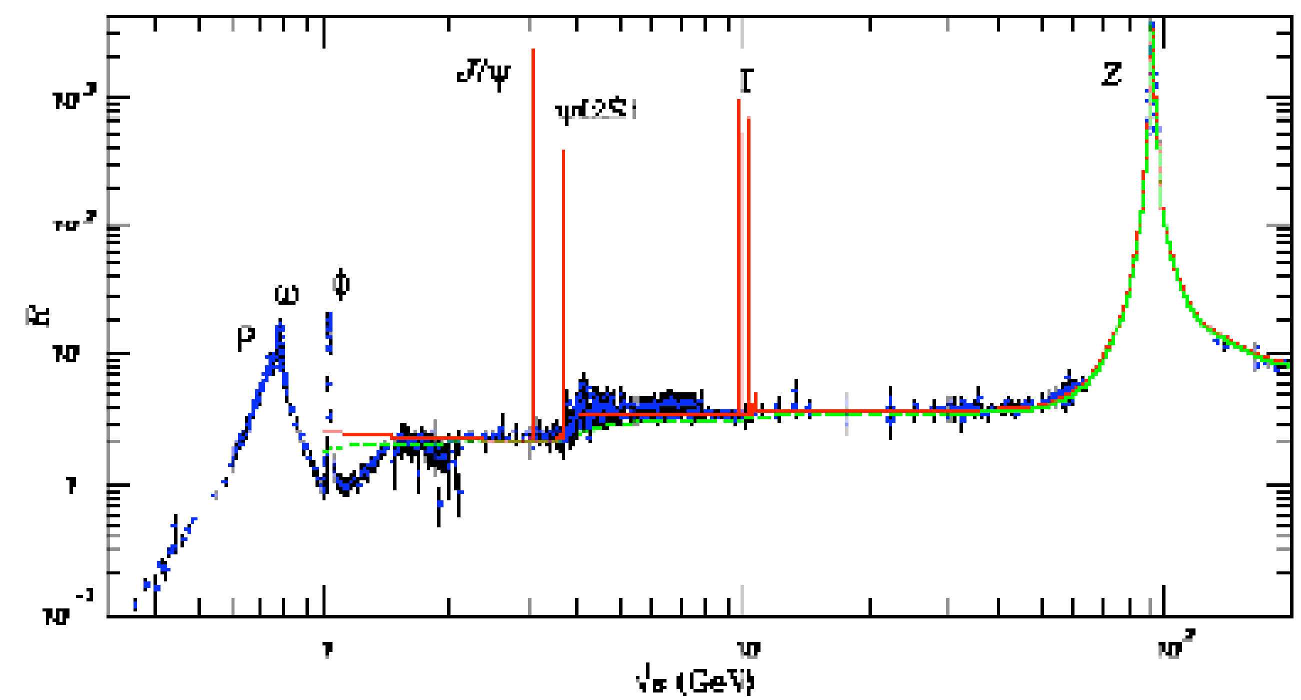
[Poggio, Quinn & Weinberg, Phys Rev D13]

$$e^+ - e^- \to \text{hadrons} \equiv \sum_q (e^+e^- \to q\bar{q}) \Rightarrow \sigma_{hadrons} \equiv \sum_q \hat{\sigma}_q$$



Complementarity between Parton and Hadron descriptions of observable

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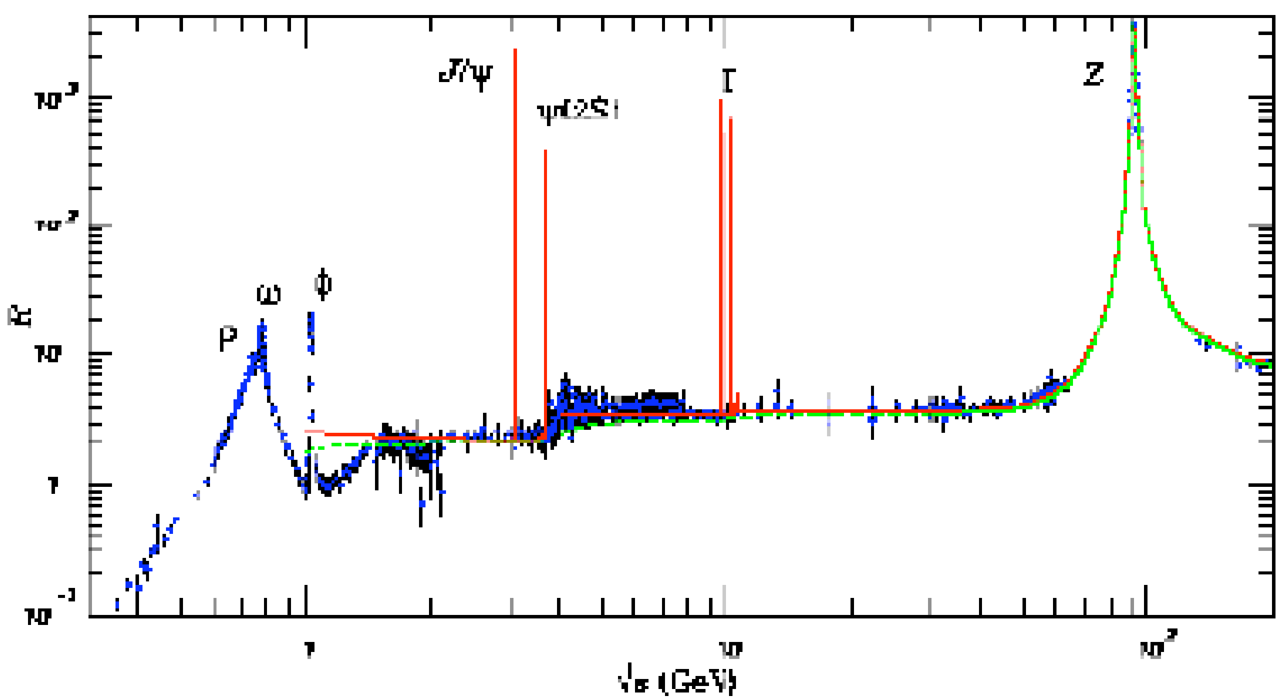

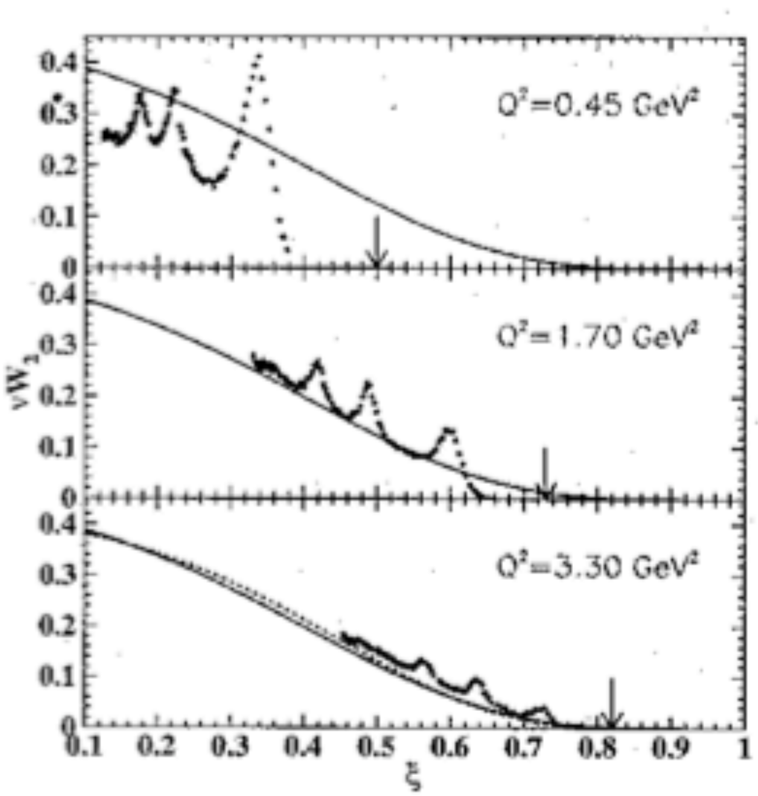

Structure functions

Resonance region ⇔ Scaling region

x_{Bj}>0.5, Q^2 multi-GeV region ⇒ $W^2 \leq 5 GeV^2$

[Bloom & Gilman, Phys.Rev.Lett.25]

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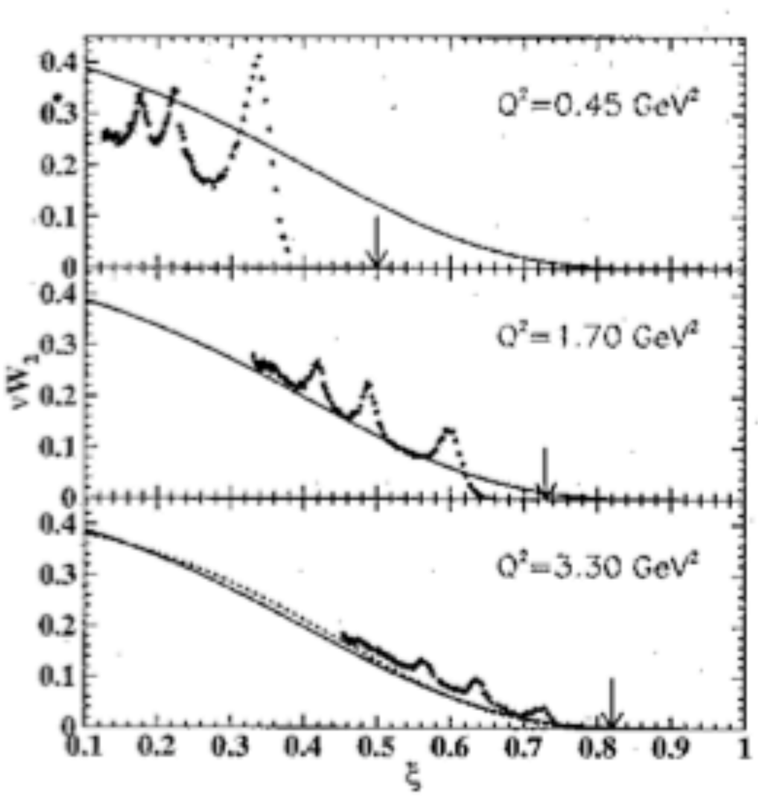
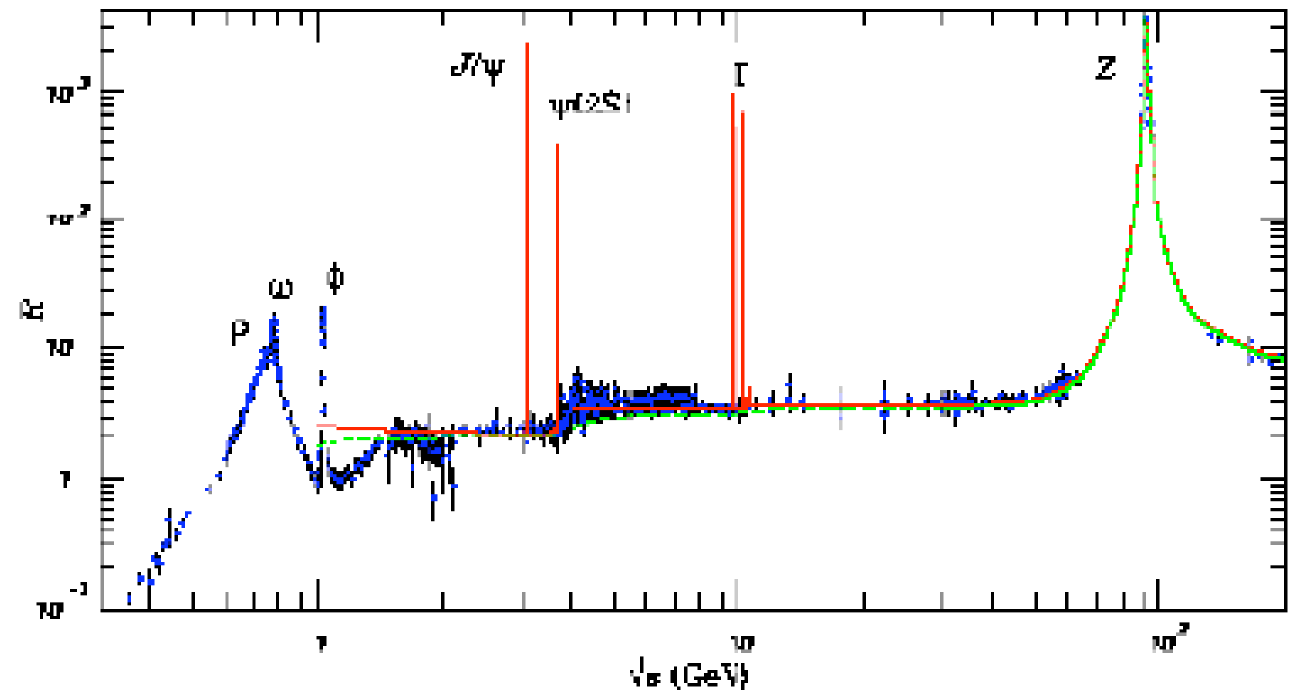
Talk by W. Melnitchouk





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Two Complementary Approaches to Structure Functions

$$I^{res}(Q^2) = \int_{x_m}^{x_M} F_2^{\rm Res}(x, Q^2)\, dx$$

$$I^{DIS}(Q^2) = \int_{x_m}^{x_M} F_2^{\rm DIS}(x, Q^2)\, dx$$

experiment

perturbative QCD

$x_M \div x_m \Leftrightarrow W^2_m \div W^2_M \Rightarrow 1 \div 4$ GeV^2

- **Nonperturbative models analysis**
- **Perturbative analysis**

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[Bianchi, Fantoni & Liuti, PRD69]

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Start with NLO PDF and then ...

- Target Mass Corrections (TMC)
- Large-x Resummation (LxR)
- Higher-order in pQCD
- Higher-Twists

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[A. Accardi, J. -W. Qiu, JHEP 0807]
[A. De Rujula,et al., Phys. Lett. B64]

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- Higher-order in pQCD
- Higher-Twists → **?**

[A. Accardi, J. -W. Qiu, JHEP 0807]
[A. De Rujula,et al., Phys. Lett. B64]

Large-x Resummation

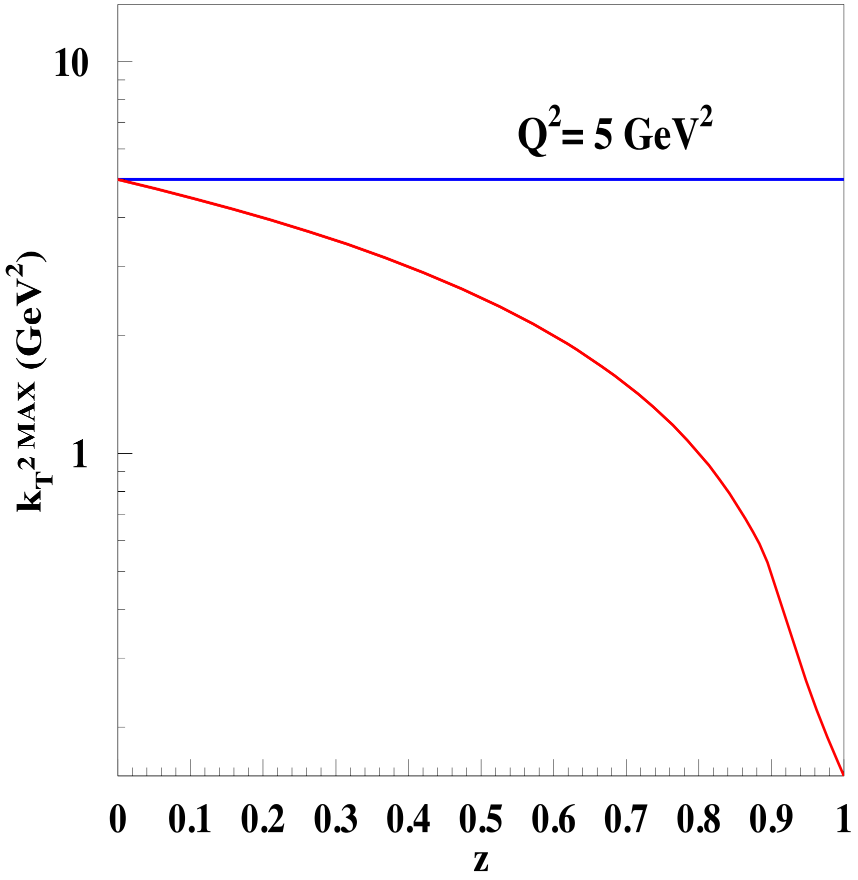
Text Book, e.g. *Cornerstones of QCD*, M. Pennington.

- Large invariants: $\Lambda^2 \ll W^2 \ll Q^2$
- Argument for α_s is ω^2, mass square of final state of γ^* parton collision

$$\omega^2 = \frac{Q^2}{z}(1-z)$$

Without LxR, upper limit $=Q^2$

$$q(x,Q^2) = \int_x^1 \frac{dz}{z} \int_{\mu^2}^{Q^2 \frac{1-z}{4z}} dk_T^2\, \alpha_S(k_T^2)\, P_{qq}(z)\, q\left(\frac{x}{z}, k_T^2\right)$$



The structure functions become

$$F_2^{NS}(x,Q^2) = \sum_q \int_x^1 dz\, \frac{\alpha_s\left(\frac{Q^2(1-z)}{4z}\right)}{2\pi}\, C_{NS}(z)\, q_{NS}\left(\frac{x}{z}, Q^2\right)$$

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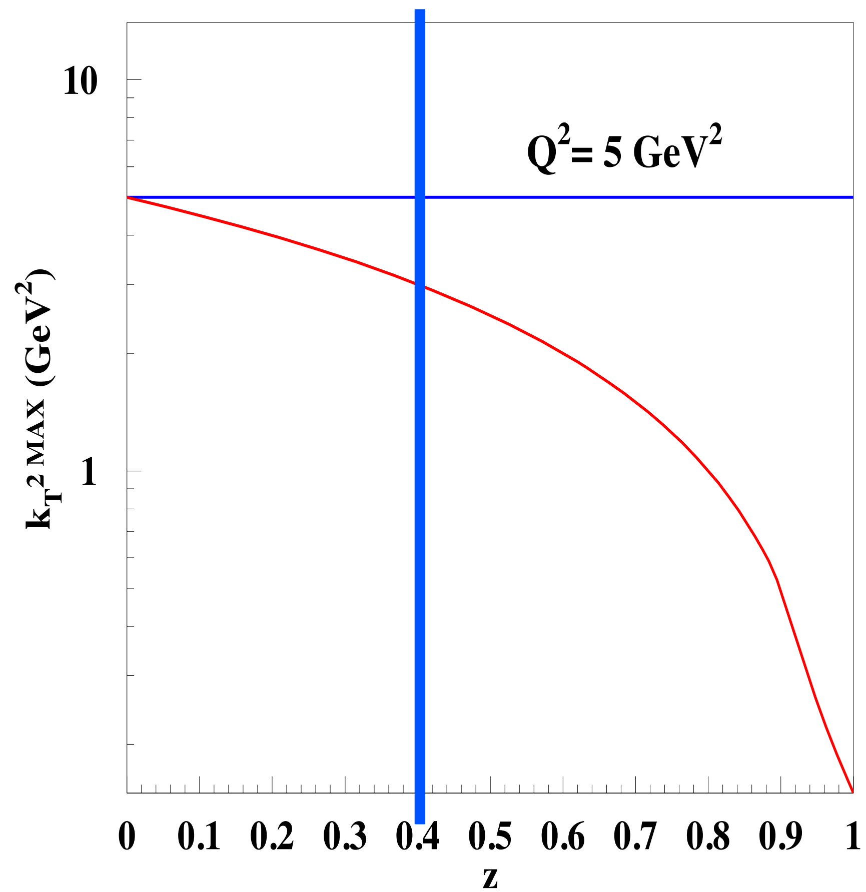

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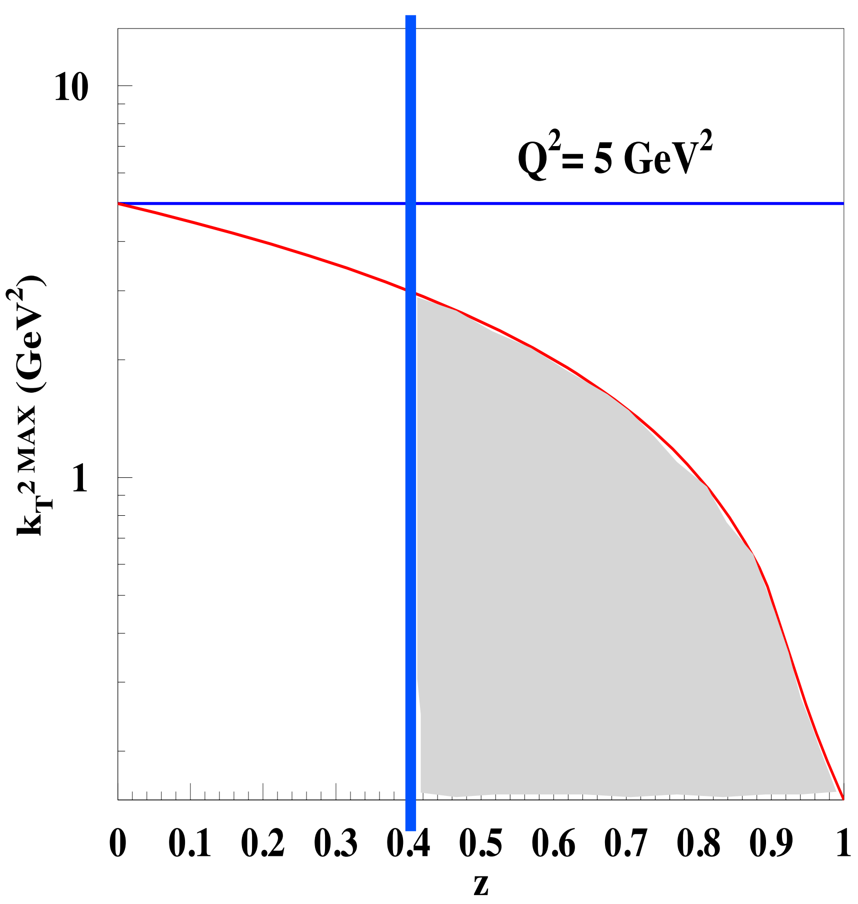
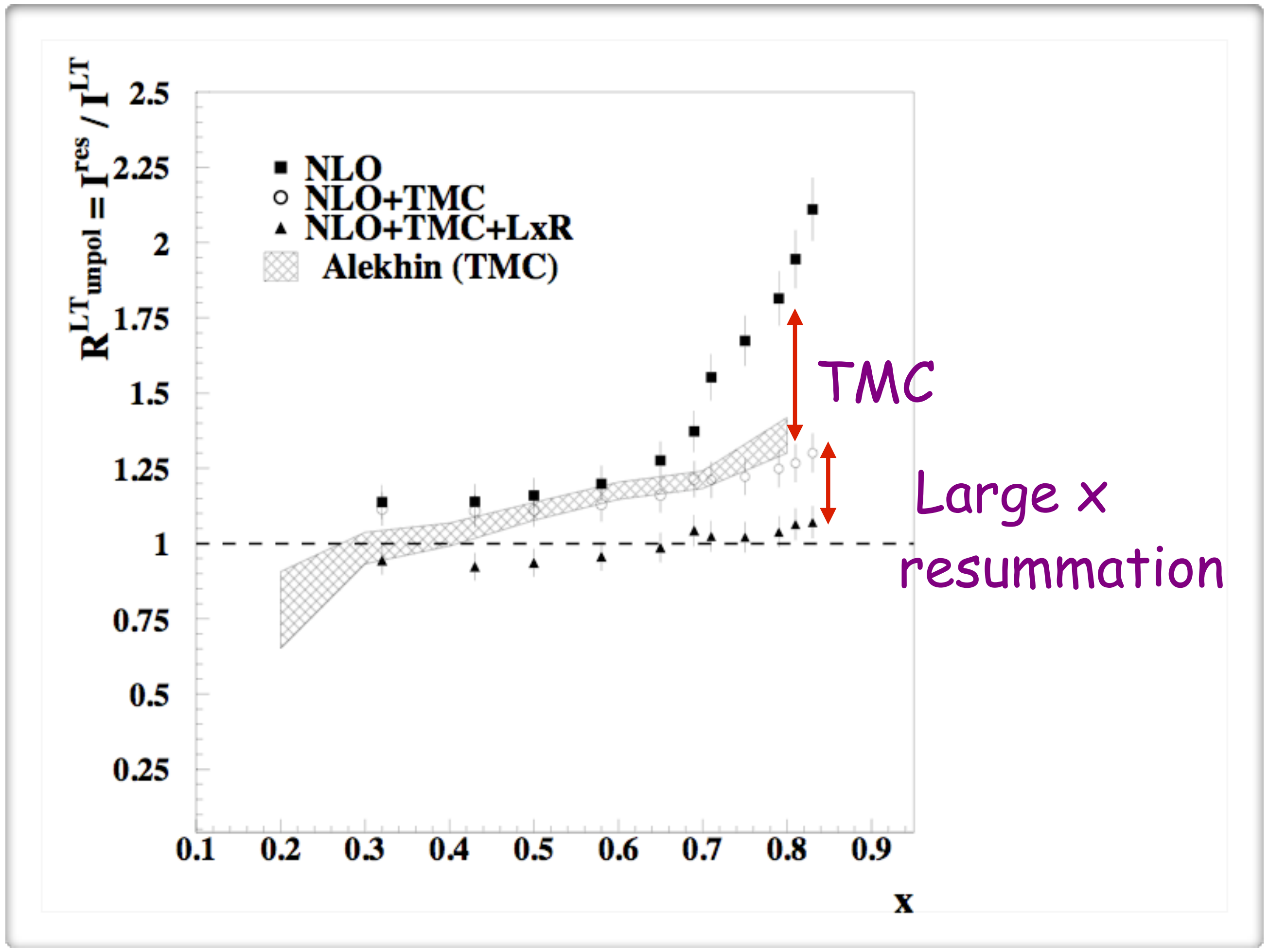


Size of Nonperturbative Contributions

[Niculescu et al., PRD60]

[Bianchi, Fantoni & Liuti, PRD69]



$$R \equiv \frac{I^{\rm res}(Q^2)}{I^{\rm DIS}(Q^2)}$$ $\Leftrightarrow$ Duality fulfilled if R=1

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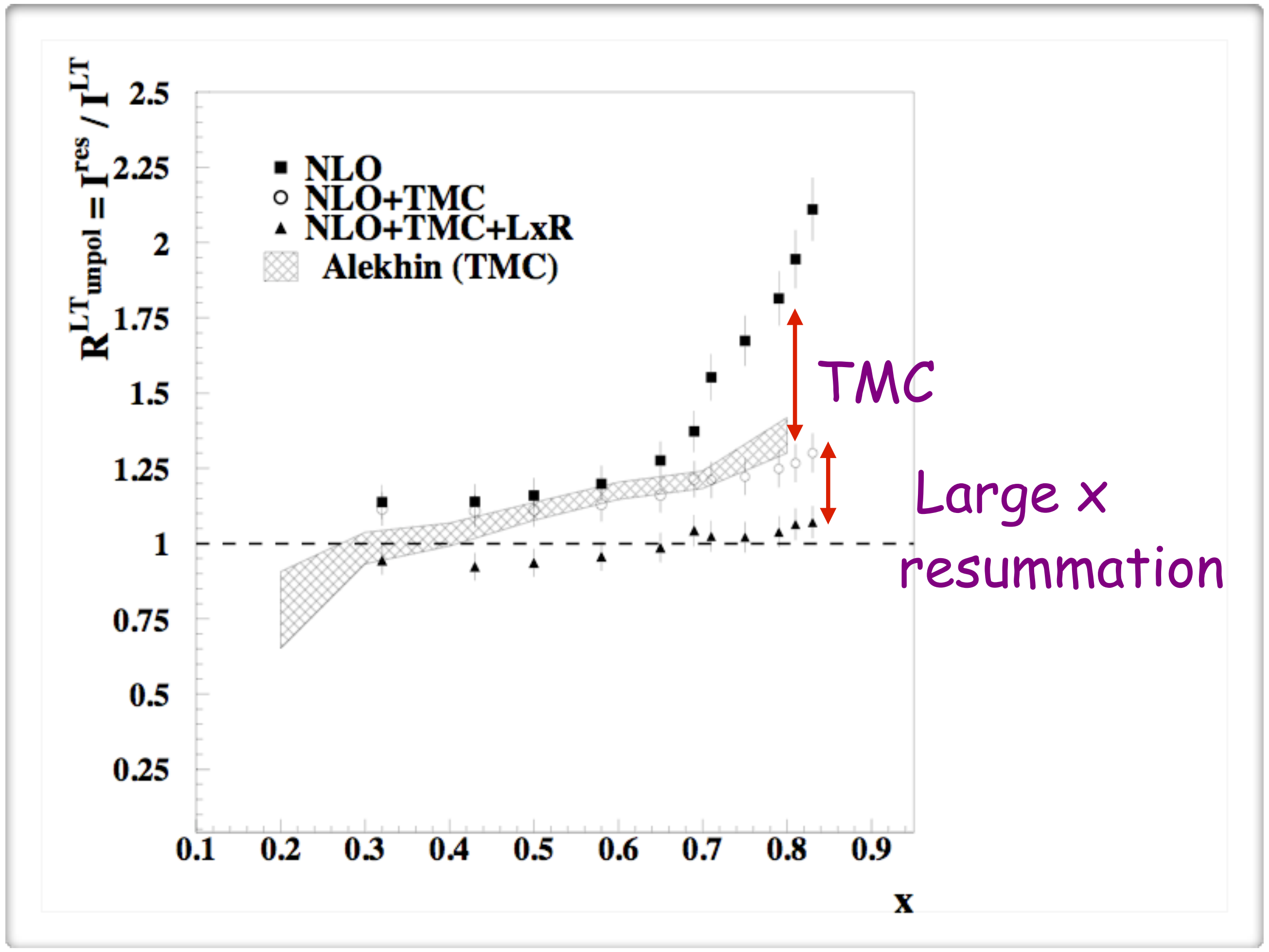

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⇐ LxR sensitive to α_s

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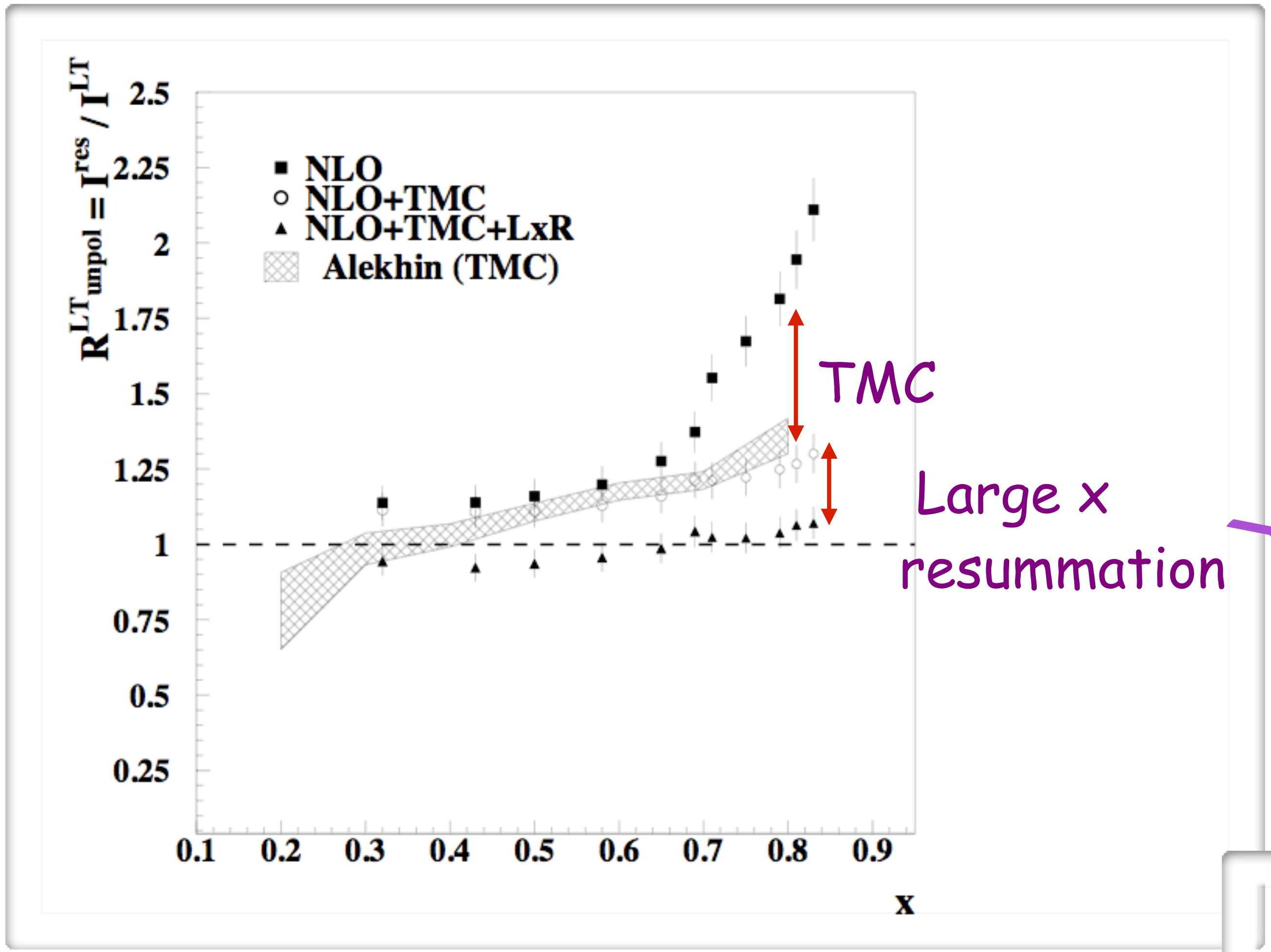

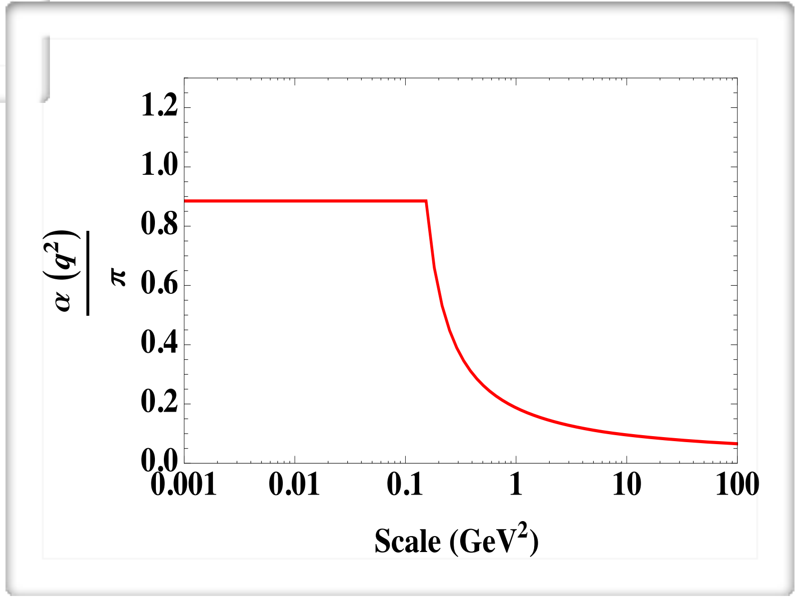

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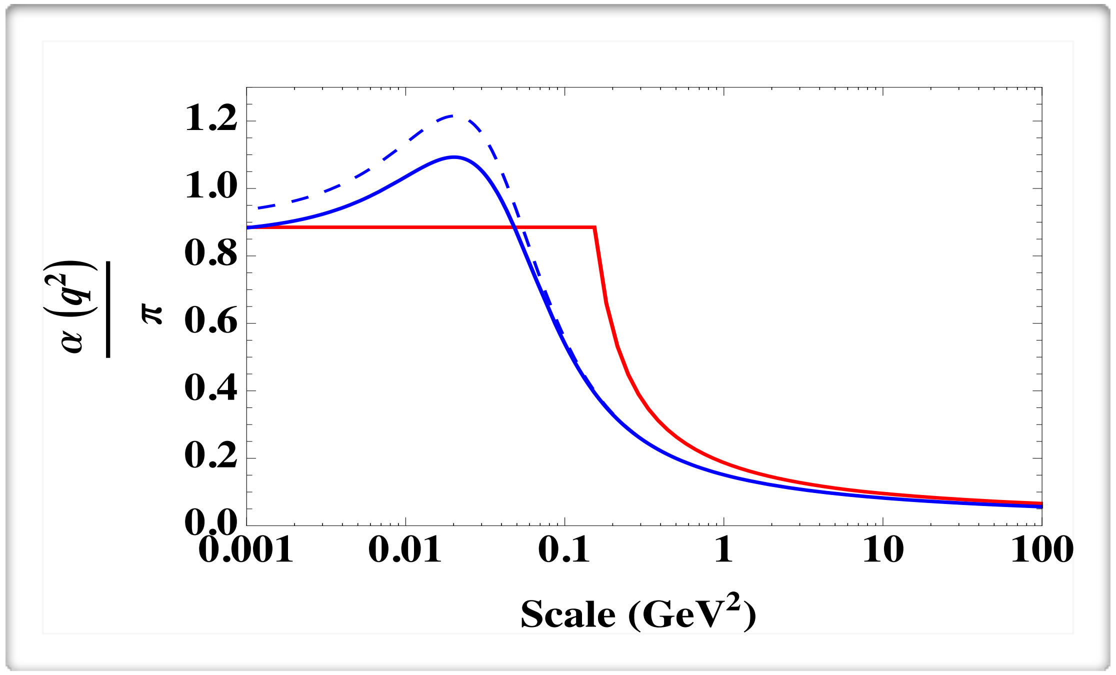
$\Leftarrow$ LxR sensitive to α_s

Nonperturbative Coupling Constant & LxR

How we go further : Nonperturbative Coupling Constant from DSE



Cornwall α_s^{NP}

3-4 free parameters

(up to physical constrains)

- Nonperturbative effects gathered in effective coupling α_s^{NP}
- Use of NP running coupling that scales to LO pQCD result
- Include in LxR
- Parameterization of the realization of duality

- Understand Higher-Twists ?
- Go to NNLO ?

Work in progress with S. Liuti

Nonperturbative QCD coupling from Phenomenology

Joint analysis: Chen, Courtoy, Deur, Liuti & Vento

Work in progress about α_s at low energy

- **Nonperturbative to perturbartive transition**
  - Final States Interactions and pQCD
  - Errorbands to measurements (even if error on "model dependence" is immeasurable)
- **Perturbative to nonperturbartive transition**
  - Realization of duality & parametrization via α_s^{NP}
  - New data for F_2 in the resonance region at JLab
- **How to relate the coupling constant?**
  - Commensurate Scale Relations? [Brodsky & Lu, Phys. Rev. D251]
  - RG-improved perturbation theory? [Grunberg, Phys. Rev. D29]

Extraction of α_s at low energy

- Polarized scattering from both proton and neutron

Deur et al. Phys.Lett. B650 (2007) 244-248

Natale, PoS QCD-TNT09 (2009) 031

Bjorken Sum Rule from JLab & GDH Sum Rule at Q^2=0 GeV^2

- Deep Inelastic Scattering (DIS) at large Bjorken-x & parton-hadron duality

Liuti, [arXiv:1101.5303 [hep-ph]].

- Semi-Inclusive DIS & Extraction of T-odd TMDs from SSAs

A.C., Vento & Scopetta, Eur. Phys. J. A47, 49 (2011)

Joint analysis: Chen, Courtoy, Deur, Liuti & Vento