Confinement and the Infra-Red

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Outline

- Constructing charges in gauge theory.
- Tests: perturbation theory and the lattice.
- Failure of the Bloch-Nordsieck & Lee-Nauenberg approaches to the IR problem.
- Renormalisation scheme dependence

Charges in Gauge Theories

- In QED and QCD matter fields ψ cannot be identified with observables – not gauge invariant

$$\psi \to U^{-1}\psi$$

- 'Dress' matter field with the gauge field, Dirac 1958

$$\Psi := h^{-1}[A]\,\psi.$$

- Gauge invariance implies

$$h^{-1}[A^U] = h^{-1}[A]U.$$

- What are the dressings? ML, D.McMullan, Phys. Rep. C 97

A First Guess

- Stringy state: link by Polyakov line (with some path)?

- Problem: in QED string state has electric potential energy:

$$V(x-y) \sim e^2 |\boldsymbol{x}-\boldsymbol{y}| \delta^2(0)$$

  Confining potential with a divergent coefficient!
- Infinitely excited state.

Haagensen, Johnson, 97

Refining the Guess

$$\bar{\psi}(\boldsymbol{y}, t) \exp\left(-ie \int_\Gamma dr_i A_i(\boldsymbol{r}, t)\right) \psi(\boldsymbol{x}, t)$$

gauge invariant but Γ dependent?

Remove by decomposition: $A_i \to A_i^T + A_i^L$, $\partial_i A_i^T = 0$, i.e., $A_i^L = \partial_i \partial_j A_j / \nabla^2$.

$$\exp\left(-ie \int_\Gamma dx_i A_i^T\right) \bar{\psi}(y) \exp\left(ie \frac{\partial_j A_j(y)}{\nabla^2}\right) \exp\left(-ie \frac{\partial_k A_k(x)}{\nabla^2}\right) \psi(x)$$

Factorised Γ dependence in gauge invariant way.

Charges in QED

$$\Psi = \exp\left[-\,ie\,\frac{\partial_i A_i}{\nabla^2} \right] \psi$$ is the static electron.

- Locally gauge invariant. Dirac 1958
- Commutator $[E_i^a(x), A_j^b(y)]_{\mathrm{e}t} = i\delta^{ab}\delta(\boldsymbol{x}-\boldsymbol{y})$ $\Rightarrow$ Coulomb field:

$$E_j\,\Psi|\,0\,\rangle = -\frac{e}{4\pi}\frac{r_j}{r^2}\,\Psi|\,0\,\rangle$$

Gauge Invariant Dressing in QCD

- The minimal static dressing in QED:

$$h^{-1} = \exp(-ie\chi)\,, \quad \text{with } \chi = \partial_i A_i/\nabla^2$$

- Transform QCD into Coulomb (arbitrary order in g).

In QCD we write

$$\exp(-ie\chi) \Rightarrow \exp(g\chi^a T^a) \equiv h^{-1}$$

with $g\chi^a T^a = (g\chi_1^a + g^2\chi_2^a + g^3\chi_3^a + \cdots)T^a$

The dressing gauge argument $\Rightarrow$

$$\chi_1^a = \frac{\partial_j A_j^a}{\nabla^2}\,; \quad \chi_2^a = f^{abc}\frac{\partial_j}{\nabla^2}\left(\chi_1^b A_j^c + \frac{1}{2}(\partial_j \chi_1^b)\chi_1^c\right)$$

Definition of Colour Charge

$$Q^a = \int d^3x \, (J_0^a(x) - f_{bc}^a E_i^b(x) A_i^c(x)) \, .$$

On gauge invariant states, non-abelian Gauss' law $\Rightarrow$

$$Q^a = \frac{1}{g} \int d^3x \, \partial_i E_i^a(x) \, .$$

Under a gauge transformation $E_i^a T^a \to U^{-1} E_i^a T^a U$, and hence

$$Q^a T^a \to \frac{1}{g} \int d^3x \, \partial_i (U^{-1} E_i^a T^a U) \, .$$

Can write this as the surface integral

$$\frac{1}{g} \lim_{R \to \infty} \int_{S^2} d\underline{s} \cdot U^{-1} \underline{E} U \, .$$

Definition of Colour Charge

Colour charge transforms on gauge invariant states as

$$Q^a T^a \to \frac{1}{g} \lim_{R\to\infty} \int_{S^2} d\underline{s} \cdot U^{-1} \underline{E} U \,.$$

Hence the colour charge will be gauge invariant if

- $U \to U_\infty$ so that
$$Q^a T^a \to U_\infty^{-1} Q^a T^a U_\infty$$
- where U_∞ lies in the centre of SU(3).
- continuity $\Rightarrow$ constant gauge transformations at spatial infinity

Perturbation Theory

- Leading order Coulombic:

$$V(r) = -\frac{g^2 C_F}{4\pi r}$$

- NLO:

$$V(r) = -\frac{g^2 C_F}{4\pi r}\left[1 + \frac{g^2}{4\pi}\frac{C_A}{2\pi}\left(4 - \frac{1}{3}\right)\log(\mu r)\right]$$

  Compare with the one-loop beta function

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left[4 - \frac{1}{3}\right]$$

- The dominant antiscreening contribution comes from longitudinal glue (minimal dressing) and the screening part from gauge invariant glue (an additional dressing)

Overlaps

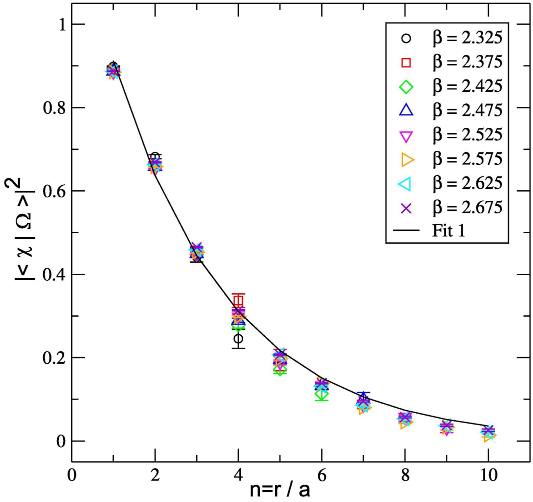
Trial quark anti-quark state, separation r. At large T

$$\langle \text{trial} | e^{-HT} | \text{trial} \rangle = |\langle \text{trial}|\Omega\rangle|^2 \, e^{-V(r)T}$$

- Measure overlap $|\langle \text{trial}|\Omega\rangle|^2$ with ground state $|\Omega\rangle$
- SU(2) Yang-Mills, 20^4 lattices, Wilson and improved actions.

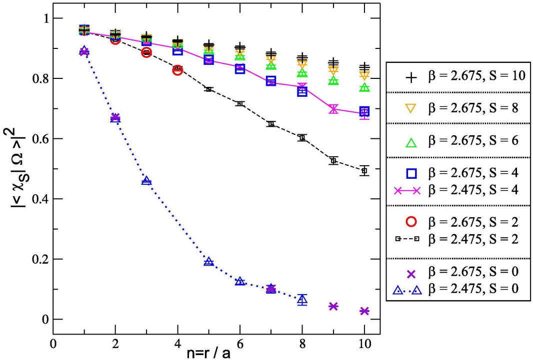
Heinzl, Ilderton, Langfeld, ML, Lutz, McMullan, PRD 08

Axial Trial State, $|\chi\rangle$



- Overlap drops exponentially as n increases
- Continuum limit more sensitive to string UV artefacts

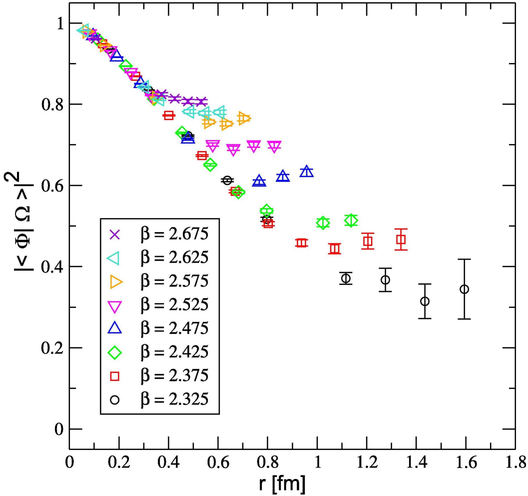
Smeared Results



- For fixed smearing overlap decreases for finer lattices
- To maintain overlap must increase smearing.

Coulomb State Overlap

- Better overlap for finer lattice spacing



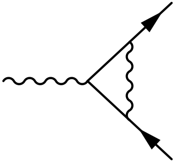
Origin of the IR Problem

- At asymptotic times $\mathcal{H}_{\text{int}} \to 0$ slowly

  Dollard 64, Kulish-Faddeev 70

- Ignoring this leads to IR divergences
- Moral: do not set $e \to 0$ in asymptotic states
- Fermion not gauge invariant at large time: $\psi \to e^{ie\Lambda}\psi$
- UV need renormalised fields
  IR need physical fields
- Dressings help (Bagan, ML, McMullan 2000)
  but will now look at common IR approaches

Coulomb Scattering



Regularised by:
$D = 4 + 2\epsilon_{\mathrm{IR}}$ and $m \neq 0$.

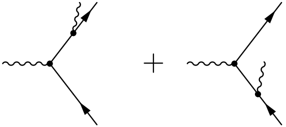
- F_2 IR finite and safe, but…
- soft and collinear divergences in F_1:

$$\frac{1}{\epsilon}\,, \qquad \frac{1}{\epsilon}\ln(m)\,, \qquad \ln^2(m)\,, \qquad \ln(m)\,.$$

- How can it be made IR finite?
- Standard: Bloch-Nordsieck (BN) for soft divergences; Lee-Nauenberg (LN) for collinear.

The Bloch-Nordsieck Response

Include real, soft emission (up to a resolution Δ) but not absorbtion:



- eikonal approximation
- square and add to virtual cross-section

Compare IR divergences in cross-sections:

- Virtual

$$-\frac{A}{\epsilon}\,, \qquad -\frac{B}{\epsilon}\ln(m)\,, \qquad C\ln^2(m)\,, \qquad F\ln(m)\,.$$

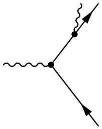
- Emission

$$+\frac{A}{\epsilon}\,, \qquad +\frac{B}{\epsilon}\ln(m)\,, \qquad -C\ln^2(m)\,, \qquad G\ln(m)\,.$$

$F \neq -G$, what kills these logs?

Bloch-Nordsieck Extended to Collinear?

Include (semi-hard) emission collinear with outgoing electron (up to an angular resolution δ):



- Virtual + soft emission leave:

$$-\ln(m) \times \left[\frac{3}{4} - \ln\left(\frac{E}{\Delta}\right)\right]$$

- Semi-hard emission fails:

$$+\frac{1}{2}\ln(m) \times \left[\frac{3}{4} - \ln\left(\frac{E}{\Delta}\right)\right]$$

So would it work if add semi-hard absorbtion? Lee-Nauenberg 64

Be Careful: include soft resolution Δ

- Semi-hard emission really generates

$$\frac{1}{2}\ln(m) \times \left[\frac{3}{4} - \ln\left(\frac{E}{\Delta}\right) - \frac{\Delta}{E} + \frac{1}{4}\frac{\Delta^2}{E^2}\right]$$

- However, in eikonal dropped $\not{k}$ in $\not{p} + \not{k} + m$ in numerator.
- Reinstate sub-eikonal: soft finite but kills these collinear logs off.
- They are artefact of energy integral divide:

$$\underbrace{\int_\Delta^E}_{semihard} + \underbrace{\int_0^\Delta}_{soft}$$

- But what about eikonal soft absorbtion?

What Should be Added to the Virtual Cross-Section?

- Cannot separate BN (soft) and LN (collinear) as would include:
  - soft emission;
  - semi-hard emission;
  - semi-hard absorbtion;
  - soft absorbtion – but only sub-eikonal terms in integrals which do not generate a soft divergence! Inconsistent!

- Or, more in spirit of LN, include
  - all degenerate indistinguishable processes!
  - Including initial and final soft and collinear.
  - Soft absorbtion generates soft infra-red divergences (eikonal): what cancels them?

Look at Lee-Nauenberg paper again…

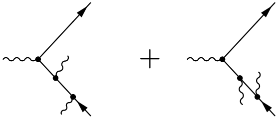
ML, McMullan, JHEP '06

Soft Divergences in the LN Spirit ($m \neq 0$)

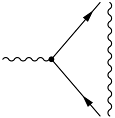
LN: all degeneracies … virtual, emission and absorbtion.

No cancellation: $-\frac{1}{\epsilon} + \frac{1}{\epsilon} + \frac{1}{\epsilon}$.

Add all diagrams with emission and absorbtion



To get cross-section at order e^4 need interference with a disconnected photon!

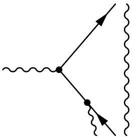


Connected interference terms.

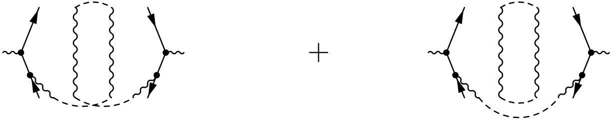
Lee-Nauenberg; Muta-Nelson; de Calan-Valent; Bergere-Szymanowski; Smilga; Ito; Akhoury-Sotiropoulos-Zakharov; …

Still Need a Bit More…

Absorbtion plus a disconnected photon: diagrams like



These yield



Only use connected contribution.

$-1 + 1 + 1 - 2 + 1 = 0$

Soft divergences then sum to zero:

virtual	emit	absorb	emit & abs.	abs. plus disconn.
$-\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$	$-2\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$

- Arbitrary choices? Why not emission plus a disconnected photon line?
- Why stop here? Can have more than one disconnected photon line?

Beyond Truncation

Idea: I. Ito 83, Akhoury-Sotiropoulos-Zakharov 97

Write cross sections as product:

disconnected loops × sum of connected (interference) probabilities

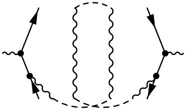
Ito, ASZ argue sum of connected probabilities could be IR finite

$$\sum_{mn} (e + m \text{ soft photons} \to e + n \text{ soft photons})$$

At order e^4 need: virtual loop; 1 emission; 1 absorbtion; 1 emission with 1 absorbtion.

Non-Convergent Series

- Connected interference from diagram with disconnected line yields same integral as diagram without:



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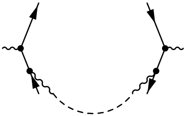
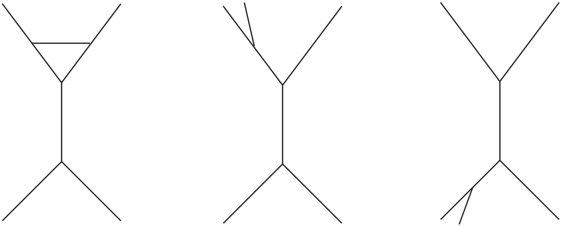


- Combinatorical factors *all* reduce to 1 for connected parts.
- Series do not converge! E.g., for soft absorbtion with n disconnected photons get:

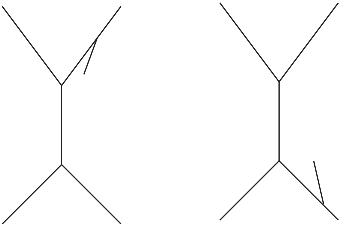
n disconnected photons:	0	1	2	3	...
IR divergence:	$+\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$	...

Consider massless ϕ^3 in $D = 6$: asymptotically free, collinear divergences but no soft divergences. ML, D. McMullan, T.G. Steele, AHEP 2012.



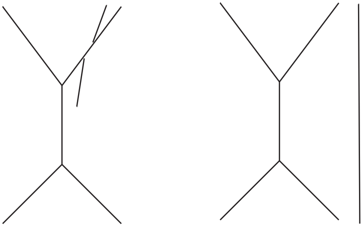
- in $\overline{\text{MS}}$ scheme argued to cancel Srednicki
  but … there are more diagrams

Additional Collinear Divergent Diagrams



- Experiment has energy resolution, Δ
- Soft collinear absorbtion on outgoing lines
- Soft collinear emission from incoming lines
- Generate $\Delta\ln(m)$ divergences
- Not normally considered; cannot cancel with virtual loops.

Disconnected?



- Divergences of form

$$|T_0|^2\left[\alpha\left(\frac{\Delta}{\boldsymbol{k}}\right)^2\ln(m^2)\right].$$

- A further sum of diagrams.
- All too simplistic? E.g., need different initial and final resolutions.
- Need to make sense of divergent series.

Changing Scheme

- Say use $\overline{\text{MS}}$
- Virtual loops IR finite
- Real processes still IR divergent
- What can cancel them in cross-sections?

LSZ tells us we require leg correction factors made from powers of

$$\frac{Z_2^{\overline{\text{MS}}}}{Z_2}$$

where Z_2 is on-shell wave function renormalisation constant.
And Z_2 contains IR divergences. Let's look at QED example

S-Matrix in Off-Shell QED

Emission (Bloch-Nordsieck) contributes IR divergences via:

$$F_1^{\text{emiss}}(v) = -\frac{\alpha}{4\pi}\frac{1}{\tilde{\varepsilon}_{\text{IR}}}\left(2 - \frac{1+v^2}{v}\ln\left(\frac{1+v}{1-v}\right)\right) + \text{IR finite}\,.$$

Wave function renormalisation:

$$\delta Z_2 = -\frac{\alpha}{4\pi}\left(\xi\frac{1}{\tilde{\varepsilon}_{\text{UV}}} + (\xi-3)\frac{1}{\tilde{\varepsilon}_{\text{IR}}} + 4 - 3\ln(m^2/\mu^2)\right)\,.$$

- As Z_2 is gauge dependent cannot cancel (IR finite in Yennie, $\xi = 3$).
- Some IR divergences in F_1 depend upon the relative velocity v (Isgur-Wise; cusp renormalisation)
- This cannot be generated by LSZ leg factors alone (soft vs. collinear).
- Even if were to introduce cusp renormalisation would need to correct gauge dependence.

Summary

- We can talk about quarks … at least perturbatively
- To what extent can we talk non-perturbatively?
- Potential: perturbative and lattice investigations support relevance of these physical states.
- On-shell IR structures support the construction.
  Q. What about emission (IR safety)?
- Identified problems with Lee-Nauenberg (divergent series)
- and with use of LSZ in the off-shell scheme to find S-matrix & cross-section in gauge theories (QED).