Hadron properties from Dyson-Schwinger equations

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JLU Giessen

Workshop on Confinement Physics
JLab, USA
March 13, 2012

Hadron physics from QCD

Hadron spectroscopy:

strangeness & charm, glueballs, exotics,
multiquark states, nucleon resonances

Hadron structure:

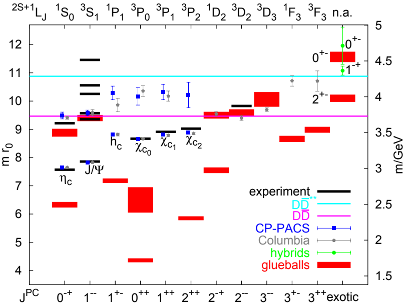
elastic & transition form factors, spin & OAM,
GPDs, factorization & perturbative QCD

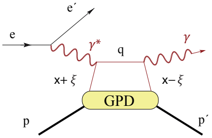
Goal: computation of hadron properties from quark & gluon substructure in QCD.

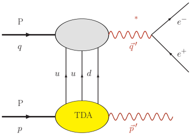
Dyson-Schwinger approach:

From **QCD Green functions** to **hadron wave functions, form factors and reactions.**

Ab-initio, nonperturbative, covariant, continuum,
light and heavy quarks. But: truncations!







(from PANDA Physics Book)

Dyson-Schwinger equations

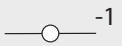
HIC for FAIR
Helmholtz International Center

QCD Lagrangian:
quarks, gluons (+ ghosts)

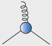
$$\mathcal{L}_{QCD} = \bar{\psi}(x)\left\{\gamma_\mu\left(\partial_\mu + iA_\mu(x)\right) + m\right\}\psi(x) + \tfrac{1}{4}\,F_{\mu\nu}\,F_{\mu\nu}$$

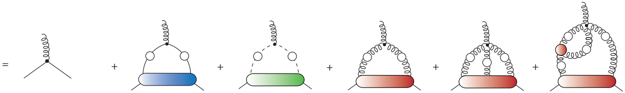
QCD & hadron properties are encoded in **QCD's Green functions.**
Their quantum equations of motion are the **DSEs:**

- **Quark propagator:**

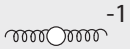


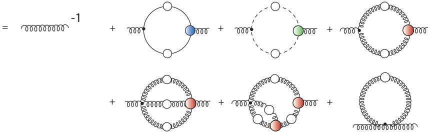
- **Quark-gluon vertex:**





- **Gluon propagator:**





- **Gluon self-interactions, ghosts, . . .**

Dyson-Schwinger equations

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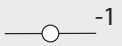

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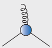
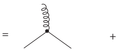
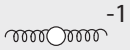
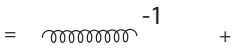




- **Gluon propagator:**





- **Gluon self-interactions, ghosts, . . .**

Relevant physics in small subset ⇒

Truncation: closed system, solvable

All momentum scales:
perturbative & non-perturbative QCD, factorization not required

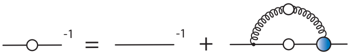
All quark masses:
also chiral limit

Applications:

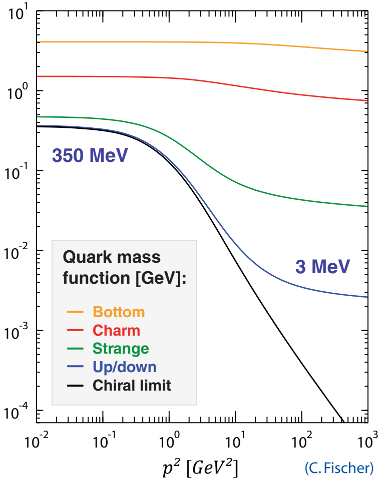
- **Hadron properties**
- **QCD phase diagram**
- **Origin of confinement**

Dynamical quark mass

- **Dynamical chiral symmetry breaking:** explains dynamical generation of “constituent-quark masses”
- Realized in **quark Dyson-Schwinger eq:** momentum-dependent quark mass $M(p^2)$



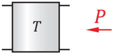
- Mass generation for **light hadrons**



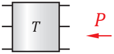
(C. Fischer)

Hadrons

Hadron **wave functions**
encoded in **quark T-matrices:**



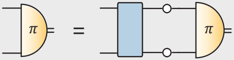
Meson poles
at $P^2 = -m_i^2$



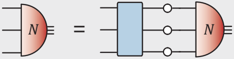
Baryon poles
at $P^2 = -M_i^2$

Covariant **bound-state equations**
determine hadron **masses** and wave functions:

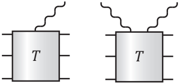
⇒ **Bethe-Salpeter equation:**



⇒ **Faddeev equation:**



Hadron **form factors,**
scattering amplitudes,
structure properties:



Hadrons

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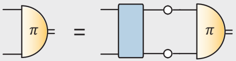
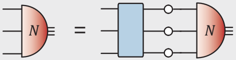
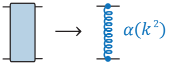


Faddeev equation:



E.g. **“rainbow-ladder” truncation:**
iterated dressed gluon exchange



- **GMOR, em. current conservation, Goldberger-Treiman** at hadron level
- no **pion-cloud effects** (“quark core”), no **decay channels** ($\rho\to\pi\pi$, $\Delta\to N\pi$, ...)

Hadrons

Everything else is determined self-consistently,
no further approximations or model ansätze

- **quark propagator** is DSE solution (rainbow-ladder: complex conjugate poles)
- **quark-antiquark vertices** are BSE solutions, develop meson poles (no widths though)
- full covariant spin-color-flavor structure of **hadron wave functions** implemented

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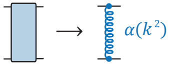
iterated dressed gluon exchange



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Mesons

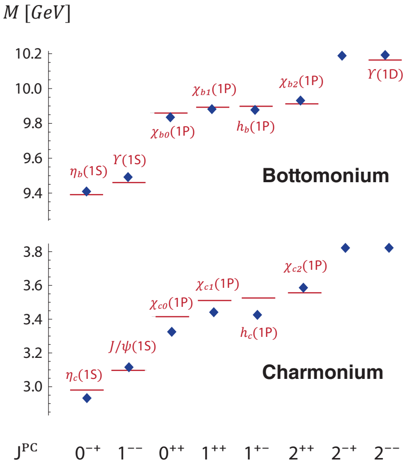
- **Heavy ground-state mesons** √
  Blank, Krassnigg: PRD 84 (2011)

- **Heavy-light mesons**
  Nguyen, Souchlas, Tandy, AIP Conf. Procs 1361 (2011)

- **Light mesons:**
  s waves (RL) √
  Maris, Roberts, Tandy, PLB420 (1998), ...
  p waves (beyond RL) √
  Fischer, Williams, PRL 103 (2009)
  Chang, Roberts, PRL 103 (2009)

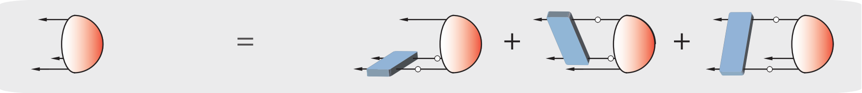
- **Light isoscalars:**
  η, η' (beyond RL) √
  Alkofer, Fischer, Williams, EPJ A38 (2008)

- **Light scalars** (σ meson)
  Tetraquarks?
  Heupel, Eichmann, Fischer, in preparation



Blank, Krassnigg: PRD 84 (2011)

Covariant Faddeev equation



Quark-quark correlations
as dominant structure in baryons

Solved for full Poincaré-covariant wave function:

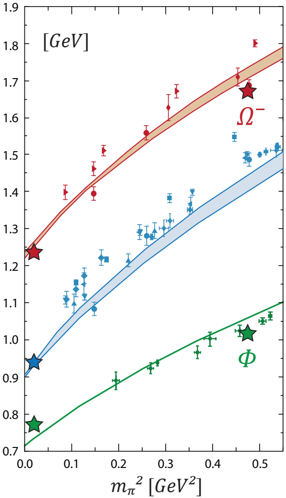
Nucleon:
Eichmann, Alkofer, Krassnigg, Nicmorus: Phys. Rev. Lett. 104 (2010)

Delta:
Sanchis-Alepuz, Eichmann, Villalba-Chavez, Alkofer: PRD 84 (2011)

Hadron masses

Description of **mesons and baryons from a single interaction kernel,** only "active" input parameter is a scale

- **Dynamical chiral symmetry breaking:** mass generation for quarks & hadrons
- **Poincaré covariance:** quark orbital angular momentum in wave functions **(p waves!)**
- **Diquark clustering in baryons:** similar results in quark-diquark model
  Oettel, Alkofer, von Smekal, EPJ A8 (2000)
  Eichmann *et al.*: PRC 79 (2009)



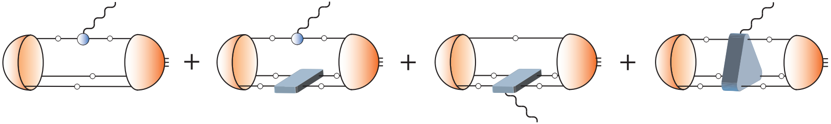
Delta mass:
Sanchis-Alepuz *et al.*, PRD 84 (2011)

Nucleon mass:
Eichmann *et al.*, PRL 104 (2010)
Eichmann, PRD 84 (2011)

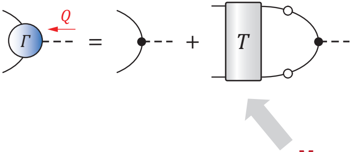
ρ–meson mass:
Maris, Tandy, PRC 60 (1999)

Form factors

Microscopic expression for a baryon's **non-perturbative current:**
Eichmann, PRD 84 (2011)



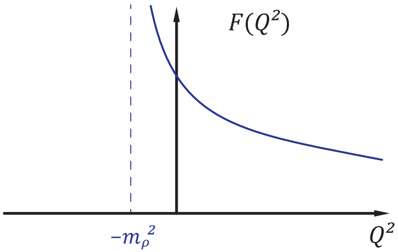
Baryon form factors inherit properties of (pseudoscalar, vector, axial-vector) **quark-antiquark vertices**:



- **Meson bound-state poles:** origin of "vector-meson dominance"
- **Em. current conservation, Goldberger-Treiman relation** automatically satisfied

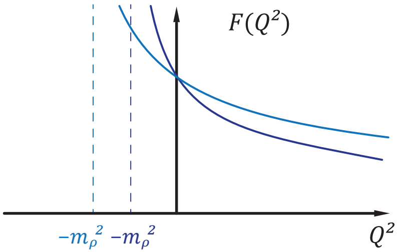
Meson poles at timelike Q^2 in T-matrix
must also appear in vertex,
timelike Q^2 structure in form factors

Form factors



- **FFs dominated by timelike meson poles**
  electromagnetic: ρ, axial: a_1, etc.

Form factors



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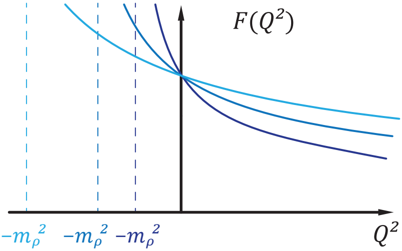

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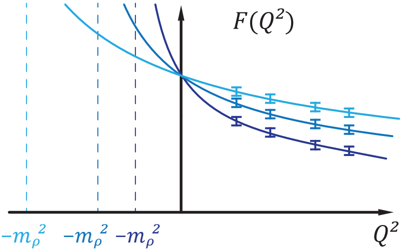

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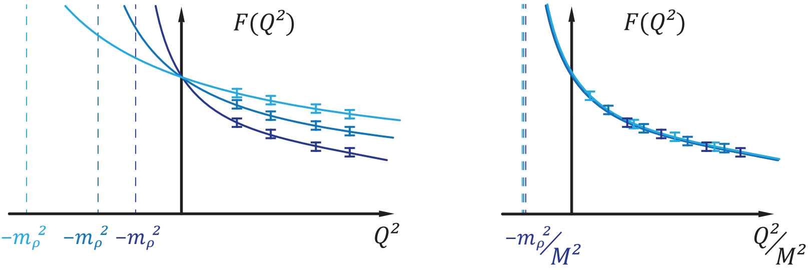

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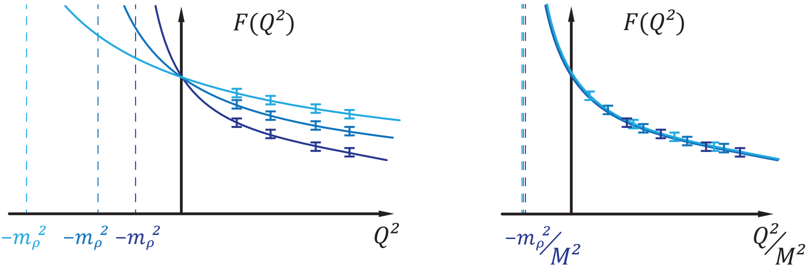
Form factors

FWF
HIC for FAIR
Helmholtz International Center



- **FFs dominated by timelike meson poles**
  electromagnetic: ρ, axial: a_1, etc.
- **FFs(Q^2/M^2) roughly current-mass independent**
  same physics: mostly s wave, p waves fall off slowly,
  no chiral singularities, no pion cloud: **"quark core"**

m_π [GeV]	0.14	0.34	0.75
$s-$wave	0.66	0.67	0.69
$p-$wave	0.33	0.32	0.30
$d-$wave	0.01	0.01	0.01

TABLE I: $s-$, $p-$ and $d-$wave contributions to the nucleon's canonical normalization at three pion masses, expressed as fractions of 1. The first column corresponds to the physical $u/d-$quark mass.

Electromagnetic structure

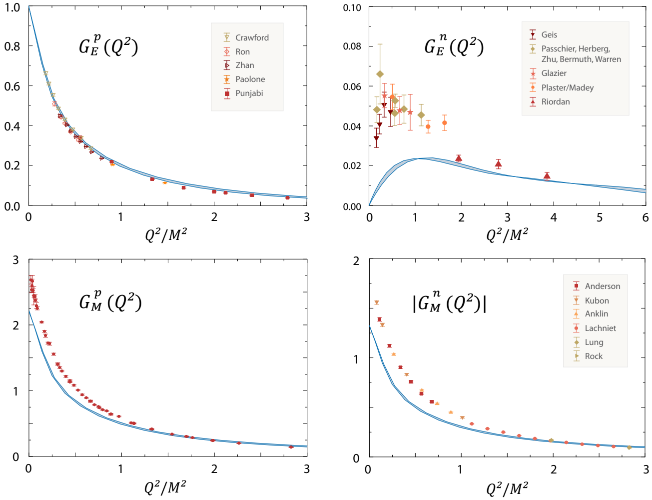
Nucleon em. FFs

vs. momentum transfer

Eichmann, PRD 84 (2011)

- Good agreement with recent **data** at large Q^2
- Good agreement with **lattice** at large quark masses
- **Missing pion cloud** below ~2 GeV^2, in chiral region

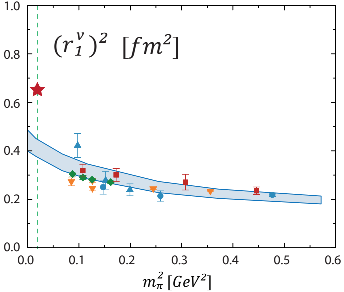
⇒ **nucleon quark core** without pion effects!



Electromagnetic structure

Nucleon charge radii:

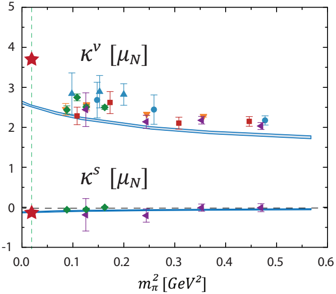
isovector (p-n) Dirac (F1) radius



- **Pion-cloud effects** missing in chiral region (⇒ divergence!), agreement with lattice at larger quark masses.

Nucleon magnetic moments:

isovector (p-n), isoscalar (p+n)

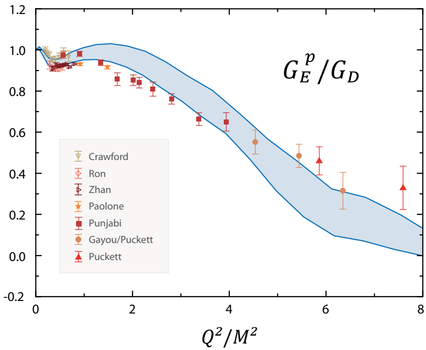


- **But:** pion-cloud **cancels** in κ^s ⟺ **quark core**

  Exp: $\kappa^s = -0.12$

  Calc: $\kappa^s = -0.12(1)$ **!!**

Large Q^2



- Faddeev result consistent with data: **OAM in nucleon amplitude!**
- Soon: investigate **two-photon effects**

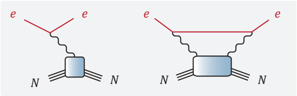
Electric proton form factor

at large momenta Eichmann, PRD 84 (2011)

- **Rosenbluth method** suggested $G_E/G_M =$ const., in agreement with perturbative scaling

  Polarization experiments at JLAB showed **falloff** in G_E/G_M, with possible **zero crossing**

- Difference likely due to **two-photon corrections**
  Guichon, Vanderhaeghen, PRL 91 (2003)



Axial structure

Nucleon axial and pseudoscalar form factors

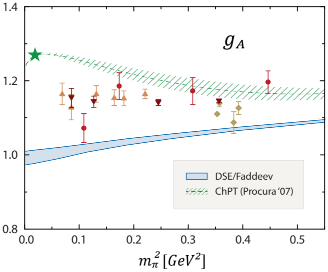
Eichmann & Fischer, 1111.2614 [hep-ph]

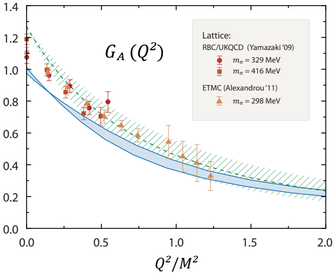
- **Pion-cloud corrections?**
  also missed on the lattice
  if volumes too small

- **Timelike meson poles**
  from quark-antiquark vertices:
  a_1 (1260) in G_A,
  π(138), π(1300) in G_P , $G_{\pi NN}$

- **Goldberger-Treiman relation**
  reproduced for **all** quark masses:

$$G_A(0) = \frac{f_\pi}{M_N}\, G_{\pi NN}(0)$$





Nucleon-Δ-γ transition

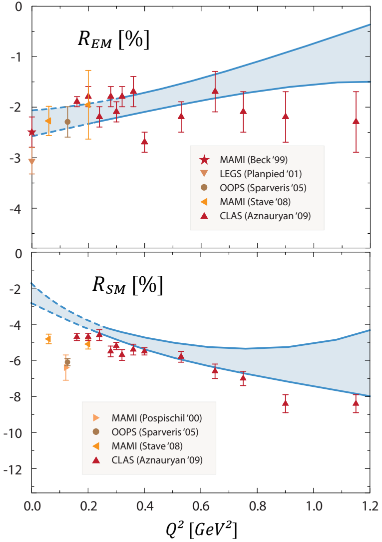
- **Magnetic dipole transition (G_M^*) dominant:** quark spin flip (s wave).

- **Electric & Coulomb quadrupole transitions** small & negative, encode deformation.

  Quark model: need **d waves** or **pion cloud**.
  Perturbative QCD: $R_{EM} \to 1$, $R_{SM} \to$ const.

- **Faddeev calculation** (here: quark-diquark)
  Eichmann & Nicmorus, 1112.2232 [hep-ph]

  Ratios reproduced even without pion cloud?!



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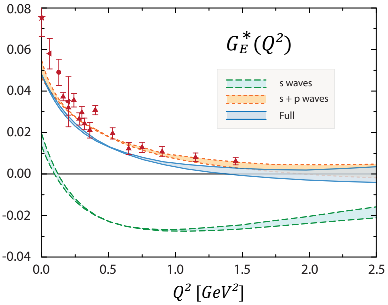
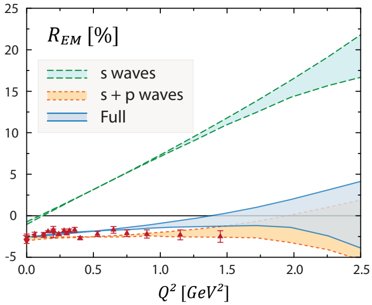


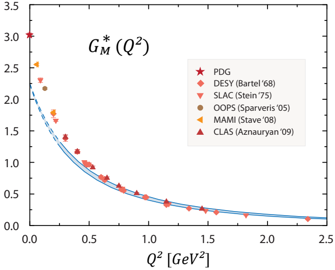


R_{EM} dominated by **p waves!**

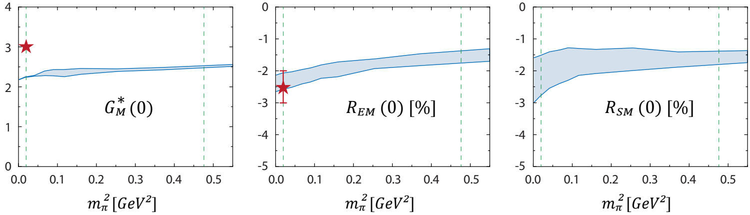
Poincaré covariance essential:
Quark OAM in N and Δ wave functions, p waves much more important than d waves.

Nucleon-Δ-γ transition

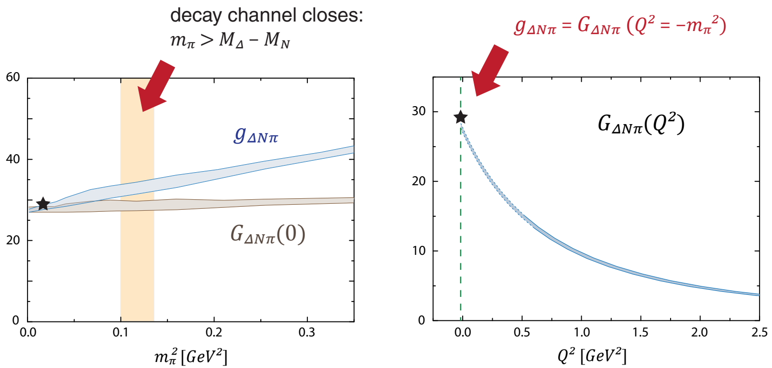
Magnetic dipole form factor:
similar to quark model,
“core + 25% pion cloud”



Flat current-mass dependence (in contrast to ChPT!):



$N\Delta\pi$ transition

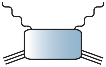


- **Delta decay width** = $g_{\Delta N\pi}$ × phase space (=118 MeV at u/d mass); current-mass dependence governed by phase space.
- $G_{\Delta N\pi}$ determined by pseudoscalar vertex ⇒ **pseudoscalar meson poles:** $\pi(1300)$, ...

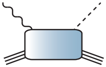
Hadron scattering

Nonperturbative description of hadron-photon and hadron-meson scattering

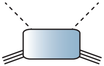
Eichmann & Fischer, PRD 85 (2012)



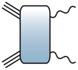
Compton scattering, DVCS, 2γ physics



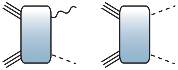
Meson photo- and electroproduction



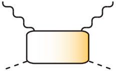
Nucleon-pion scattering



$\bar{p}p \to \gamma\gamma^*$
annihilation



Meson production



Pion Compton scattering

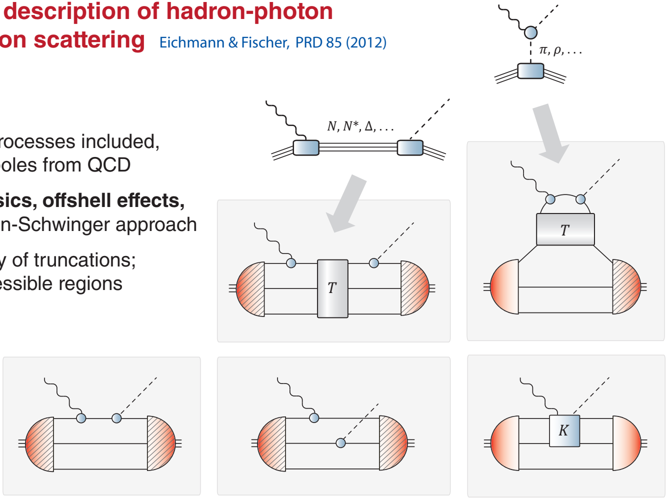
Fully non-perturbative, no factorization into soft and hard physics required!

Hadron scattering

Nonperturbative description of hadron-photon and hadron-meson scattering

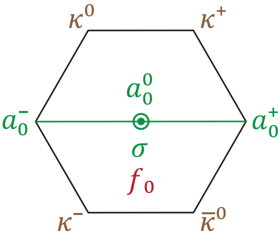
Eichmann & Fischer, PRD 85 (2012)

- Perturbative subprocesses included, s- and t-channel poles from QCD
- **Two-photon physics, offshell effects, GPDs** in the Dyson-Schwinger approach
- **Obstacles:** quality of truncations; kinematically accessible regions



Tetraquarks

Light scalar mesons (0^{++}) don't fit into the conventional meson spectrum:



a_0 (980 MeV) } $u\bar{u}, d\bar{d}, u\bar{d}$
σ (440 MeV) }

κ (660 MeV) $u\bar{s}, d\bar{s}$

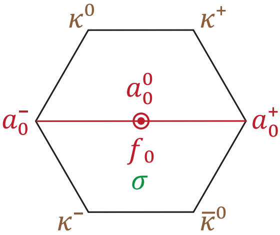
f_0 (980 MeV) $s\bar{s}$

- Why are σ, κ so **light** compared to a_0, f_0?
  Why are the masses of a_0, f_0 degenerate?
- Why do they have so different **decay widths**?
  $\Gamma(\sigma,\kappa) \approx$ 550 MeV
  $\Gamma(a_0, f_0) \approx$ 50–100 MeV
- Why do both f_0 and a_0 couple to $K\bar{K}$?
  (hidden strange-quark content of a_0?)
- Scalar mesons should be predominantly **p-waves** with masses similar to axial-vectors:
  $a_1, f_1 \sim$ 1.3 GeV
- **Lattice:** lightest scalars $\sim$ 1.2–1.4 GeV
  Mathur et al. (2007), Dudek et al. (2010)

  Consistent with 1st radial excitation?
  $a_0, f_0 \sim$ 1.2–1.5 GeV

Tetraquarks

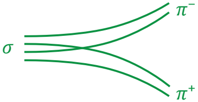
Could these be light **tetraquark** (diquark-antidiquark) states? Jaffe '77



a_0 (980 MeV) } $us\bar{u}\bar{s}$, ...
f_0 (980 MeV) }
κ (660 MeV) $us\bar{u}\bar{d}$, ...
σ (440 MeV) $ud\bar{u}\bar{d}$

- Explains **mass ordering**:
  f_0 and a_0 have same strangeness content

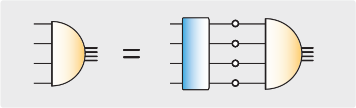
- Explains **decay widths**:
  f_0 and a_0 decay into $K\bar{K}$;
  "OZI-superallowed" mechanism
  leads to large widths for σ, κ :



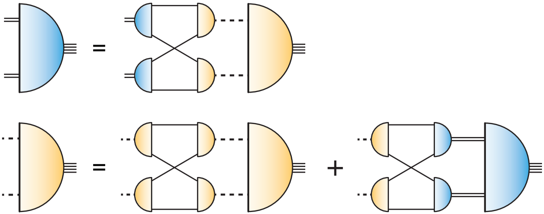
- Lightest scalar $q\bar{q}$ states would be
  $a_0, f_0 \sim$ 1.2–1.5 GeV, consistent with lattice

Tetraquarks

Start from **four-quark bound-state equation:**



Keep only qq, $q\bar{q}$ interactions with separable T-matrix.
Obtain coupled **diquark-antidiquark / meson-meson** equations:
Heupel, Eichmann, Fischer, in preparation



Two-body simplification induces complications...

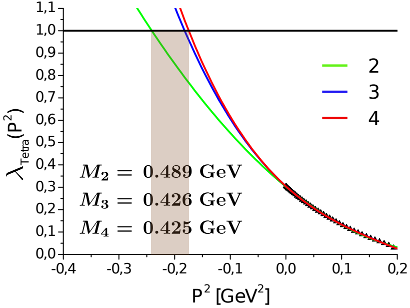
- pion poles in integrand;
- how do pion and diquark behave offshell?

Tetraquarks

Strategy: solve for $P^2 > 0$, extrapolate BSE eigenvalue to timelike $P^2 = -M^2$.

Coupled **diquark-antidiquark / meson-meson** system
yields a light scalar tetraquark ~ 400 MeV

Heupel, Eichmann, Fischer, in preparation



Setup	Mass [GeV]
E	0.43 ± 0.043
E + F	0.44 ± 0.041
E + F*	0.46 ± 0.039
Meson only	0.30 ± 0.042
Prefactors Fit1	0.32 ± 0.038
Prefactors Fit2	0.39 ± 0.038

Next: study four-quark equation directly!

Summary

Systematic description of QCD phenomenology from quark & gluon substructure:

- **Nonperturbative:** factorization property not necessary
- **Dynamical chiral symmetry breaking:** mass generation for quarks and light hadrons
- **Poincaré covariance:** quark orbital angular momentum via p waves
- **Quark-quark interaction** dominant in ground-state baryons
- **Pion cloud** essential for chiral and low-momentum structure
- **Tetraquark** identification for light scalar mesons likely correct

Need to improve **truncations** (pion cloud, decay channels, 3- and 4-quark interactions) and **kinematical coverage**

Interplay between experiment and theory:

- Hadron **masses, wave functions, form factors and scattering amplitudes from QCD**
- Refined tools for understanding fundamental **properties of QCD from experiment**