Meson PDFs, Form Factors & Condensates in Confining QCD

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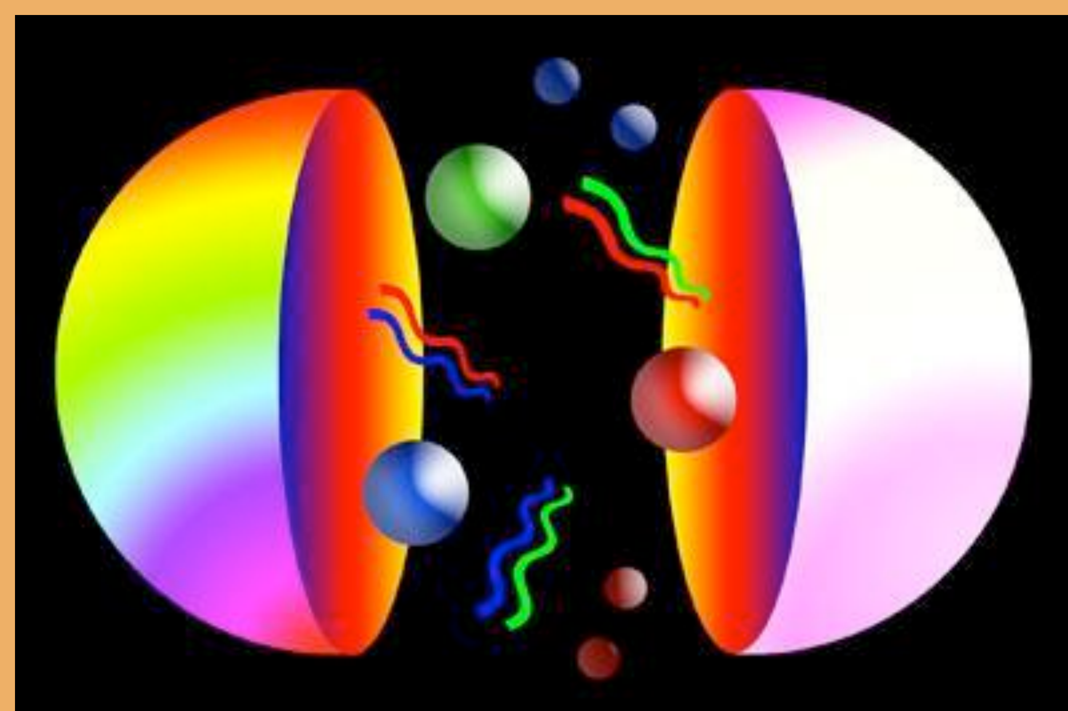
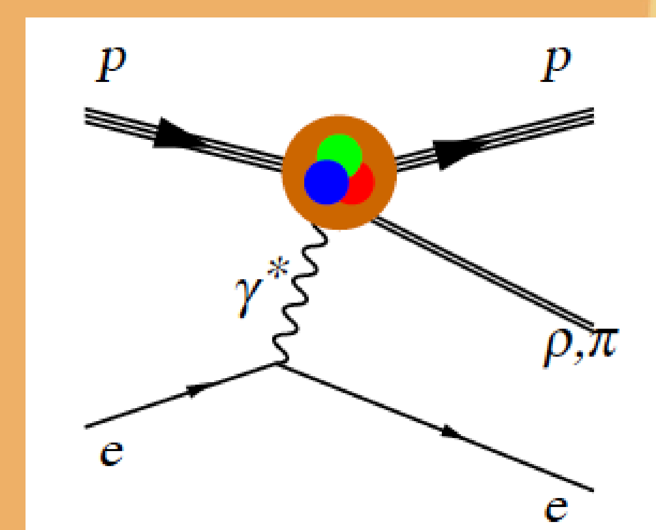




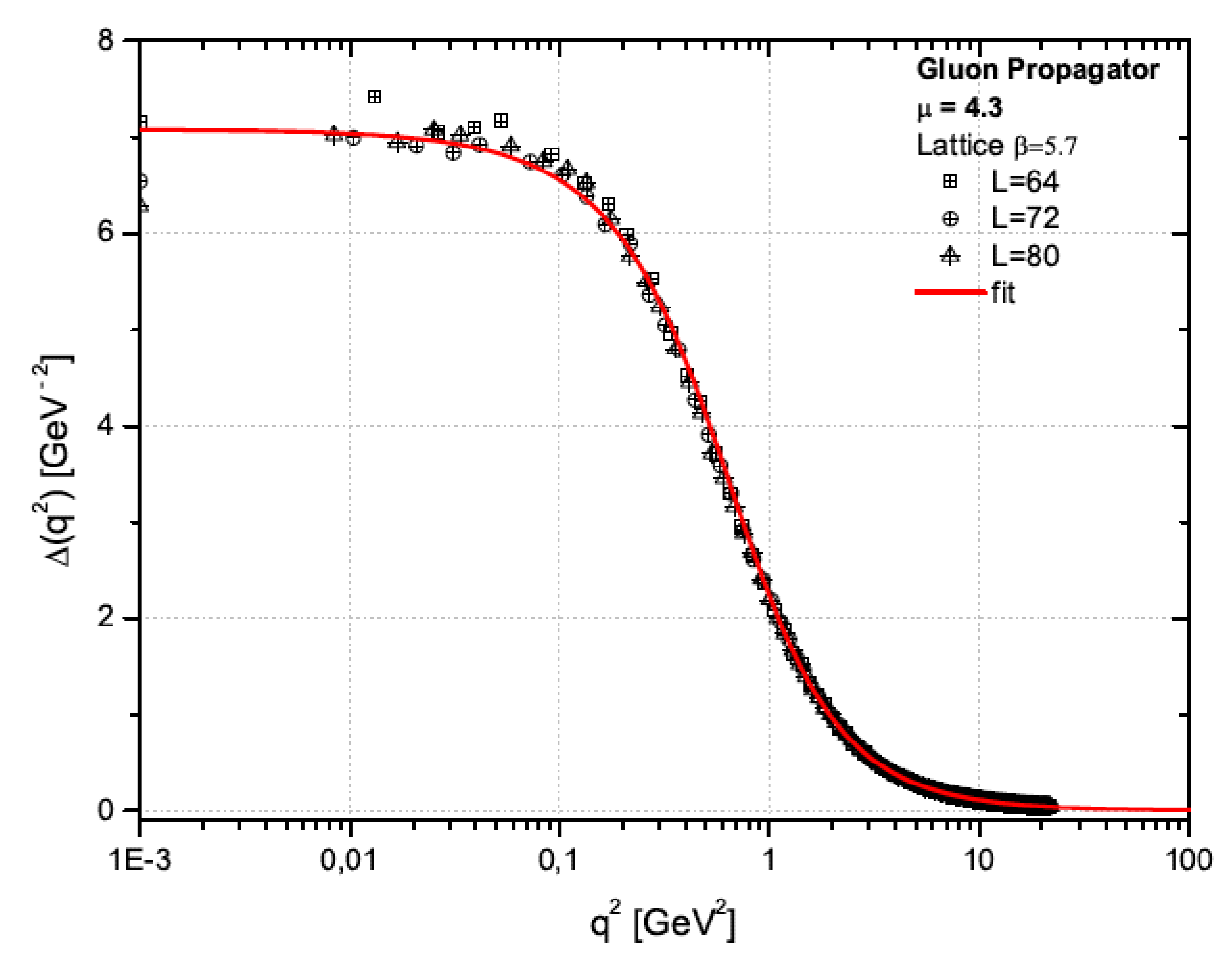
Topics

- DSE modeling of QCD: rainbow-ladder truncation
- Extension to valence quark PDFs in pion, kaon
- Feynman Integral Representation to allow more and faster applications
- Euclidean DSE method to get many PDF moments, and thence the PDFs
- Extending DSE form factor approach to large Q^2

Lattice-QCD and DSE-based modeling

- Lattice: $\langle \mathcal{O} \rangle = \int \, D\bar{q}qG \;\; \mathcal{O}(\bar{q}, q, G) \;\; e^{-\mathcal{S}[\bar{q},q,G]}$
  - Euclidean metric, x-space, Monte-Carlo
  - Issues: lattice spacing and vol, sea and valence m_q, fermion Det
  - Large time limit $\Rightarrow$ nearest hadronic mass pole
- EOMs (DSEs): $0 = \int \, D\bar{q}qG \;\; \frac{\delta}{\delta q(x)} \; e^{-\mathcal{S}[\bar{q},q,G]+(\bar{\eta},q)+(\bar{q},\eta)+(J,G)}$
  - Euclidean metric, p-space, continuum integral eqns
  - Issues: truncation and phenomenology—not full QCD
  - Analtyic contin. $\Rightarrow$ nearest hadronic mass pole
  - Can be quick to identify systematics, mechanisms, $\cdots$

Modern Context for Ladder-Rainbow Kernel



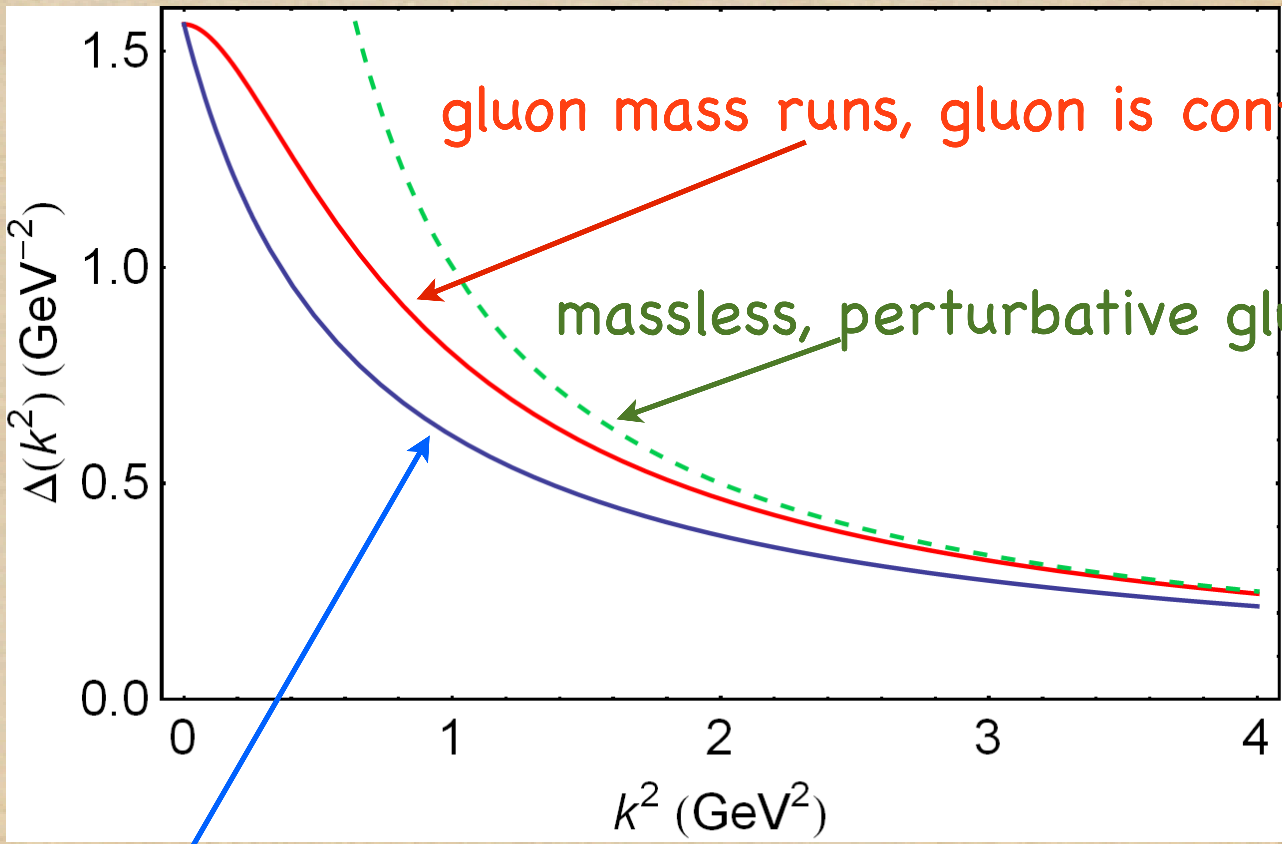
Landau gauge, lattice – QCD gluon propagator,
I.L.Bogolubisky *etal.*, PosLAT2007, 290 (2007)

DSE Studyw/ modern n = pt fns
A.C.Aguilar *etal.*, arXiv : 1010.5815 (2010)
Identified enough stength for physical DCSB

$\Rightarrow m_G(k^2)$ $\quad\quad$ $m_G(0) \sim 0.38$ GeV

$$\mathbf{K}_{\mathrm{BSE}}^{\mathrm{RL}} = \frac{4\pi\hat{\alpha}_{\mathrm{eff}}(\mathbf{q^2})}{\mathbf{m_G^2(q^2) + q^2}}$$

$$\Rightarrow \frac{\hat{\alpha}_{\mathrm{eff}}(\mathbf{0.1})}{\pi} \approx \mathbf{3 - 4}$$



Summary of light meson results

$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle^0_\mu$	$(0.236\text{ GeV})^3$	$(0.241^\dagger)^3$
m_π	0.1385 GeV	$0.138^\dagger$
f_π	0.0924 GeV	$0.093^\dagger$
m_K	0.496 GeV	$0.497^\dagger$
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r^2_π	0.44 fm^2	0.45
$r^2_{K^+}$	0.34 fm^2	0.38
$r^2_{K^0}$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r^2_{\pi\gamma\gamma}$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	$7.6 \cdot 10^6\ s^{-1}$	7.38
$\Gamma(K_{\mu 3})$	$5.2 \cdot 10^6\ s^{-1}$	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
$m_{K^\star}$	0.892 GeV	0.936
$f_{K^\star}$	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^\star K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^\star K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^\star K\gamma}/m_K)^0$	1.28	1.19

Scattering length (PM, Cotanch, PRD66, 116010)

a^0_0	0.220	0.170
a^2_0	0.044	0.045
a^1_1	0.038	0.036

In summary: 31 exptl data @ RMS error of 15%

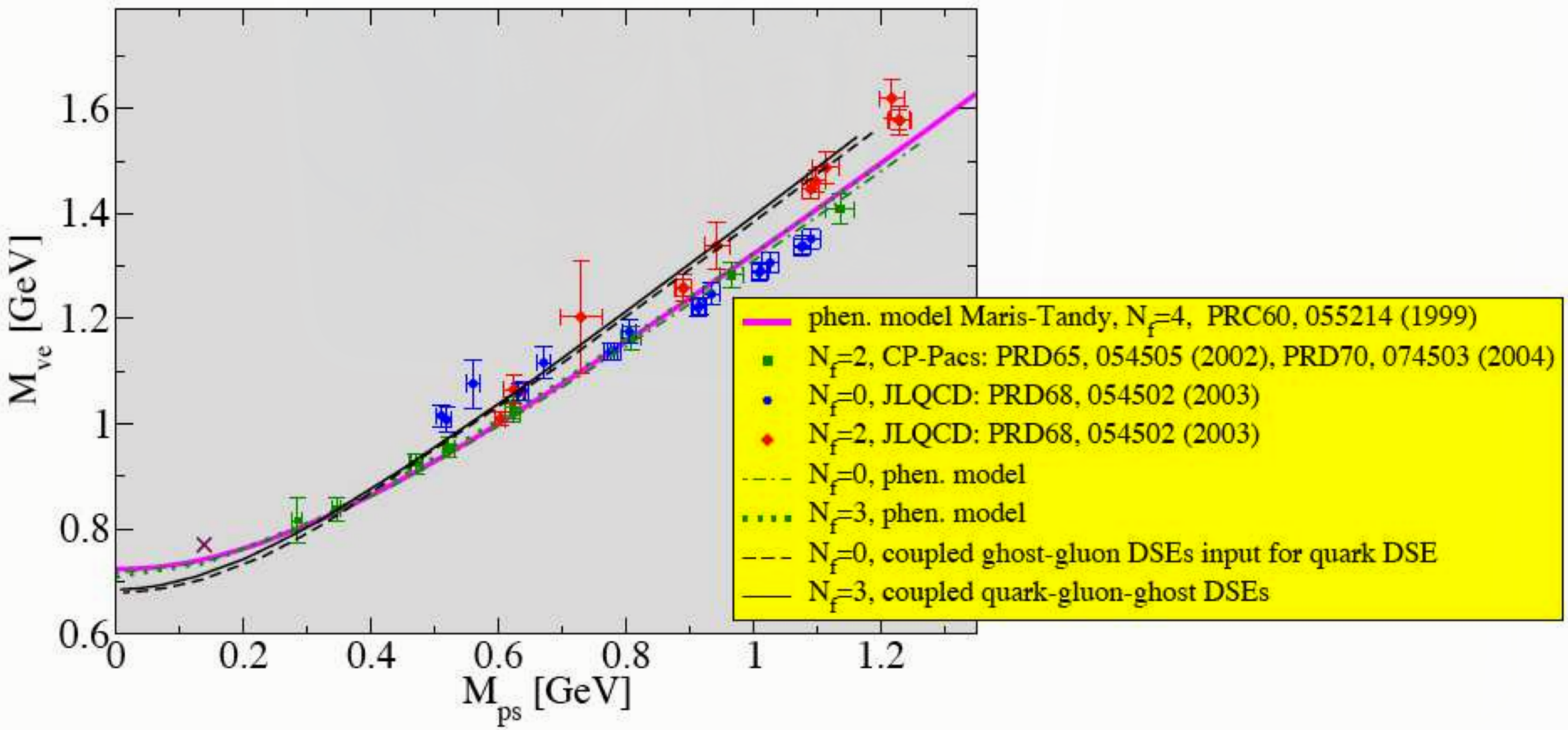
Quarkonia

All GeV	M_{η_c}	f_{η_c}	$M_{J/\psi}$	$f_{J/\psi}$
expt	2.98	0.340	3.09	0.411
calc with $M_c^{\rm cons}$	3.02	0.239	3.19	0.198
calc with $\Sigma_c^{\rm DSE}(p^2)$	3.04	0.387	3.24	0.415

All GeV	M_{η_b}	f_{η_b}	M_{Υ}	f_{Υ}
expt	9.4 ?	?	9.46	0.708
calc with $M_b^{\rm cons}$	9.6	0.244	9.65	0.210
calc with $\Sigma_b^{\rm DSE}(p^2)$	9.59	0.692	9.66	0.682

- QQ and qQ decay constants too low by 30-50% in constituent mass approximation
- Quarkonia decay constants much better for DSE dressed quarks (within 5% of expt.)
- IR sector (gluon k below ~ 0.8 GeV) contribute little for bb or cc quarkonia in DSE, BSEs
- QQ states are more point-like than qq or qQ states

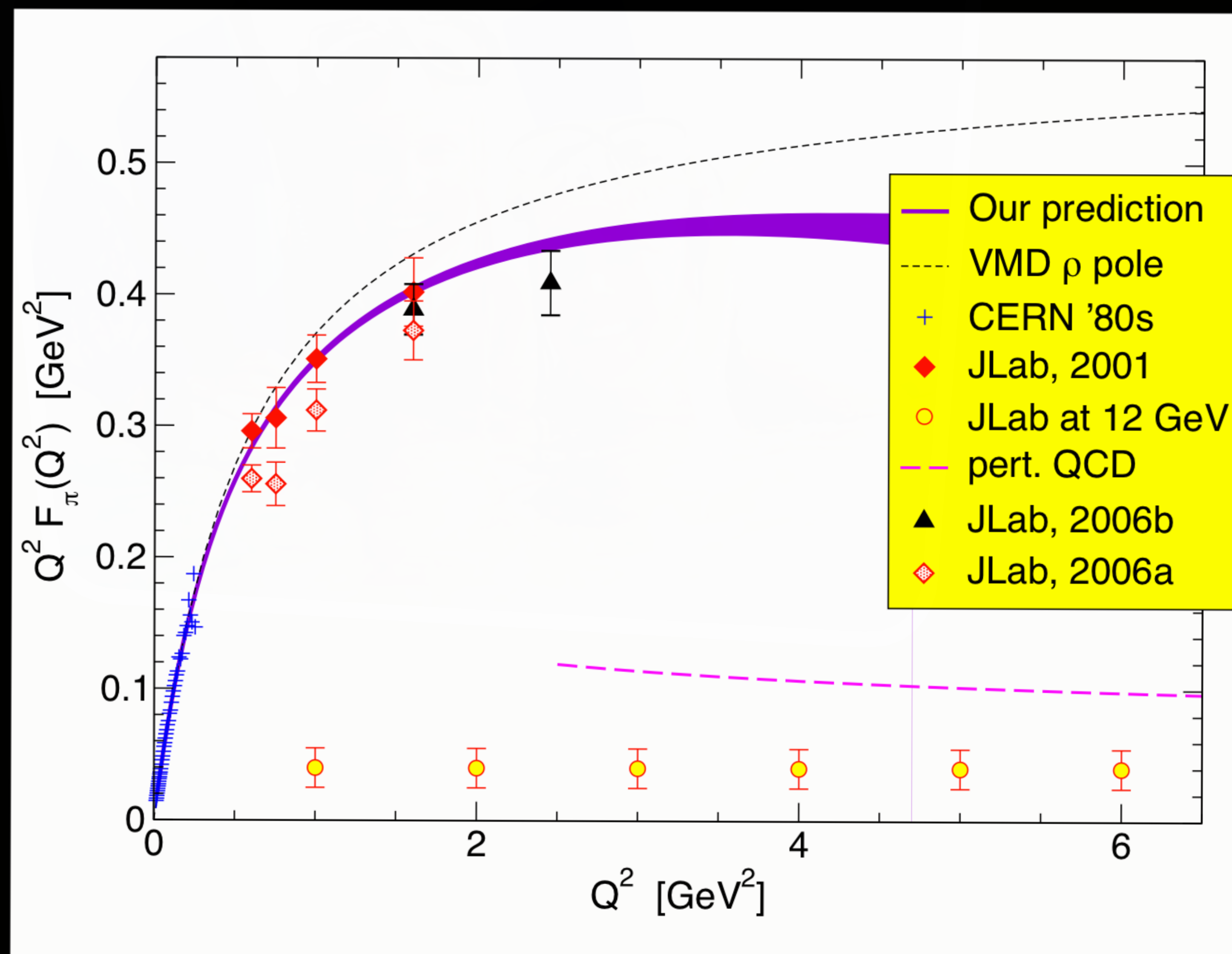
DSE and Lattice results for M_V and M_{ps}



– – – – – C.S.Fischer, P.Watson, W.Cassing, PRD, (2005)

7 *C.S. Fischer, R. Williams*, PRL**103**, 122001 (2009

Pion electromagnetic form factor



PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]
2006a: V. Tadevosyan *et al*, [nucl-ex/0607007], 2006b: T. Horn *et al*, [nucl-ex/0607005]

Beyond LR

- Deficiencies of ladder-rainbow truncation:
- Axial vector (a1, b1) and scalar mesons (L >0) are too light
- This does not bode well for exotic/hybrid hadron states
- Eg, $\pi_1(1^{-+})$ exotic soln of LR-BSE is at 0.9 GeV. Expected to have L >0, so fix that.
- Craig Roberts & Lei Chang have developed a semi-phenomenological extension of LR BSE kernel that is a major advance and performs well for L>0 u/d quark mesons.
- No hadronic decay widths of states--must calc independently
- Any important chiral loop effects have to be added later---(however LR can't be characterized as quenched either)

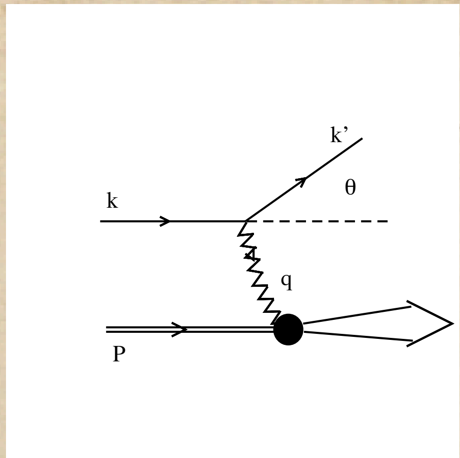
Now Back to Ladder-Rainbow....

PDFs and Form Factors....

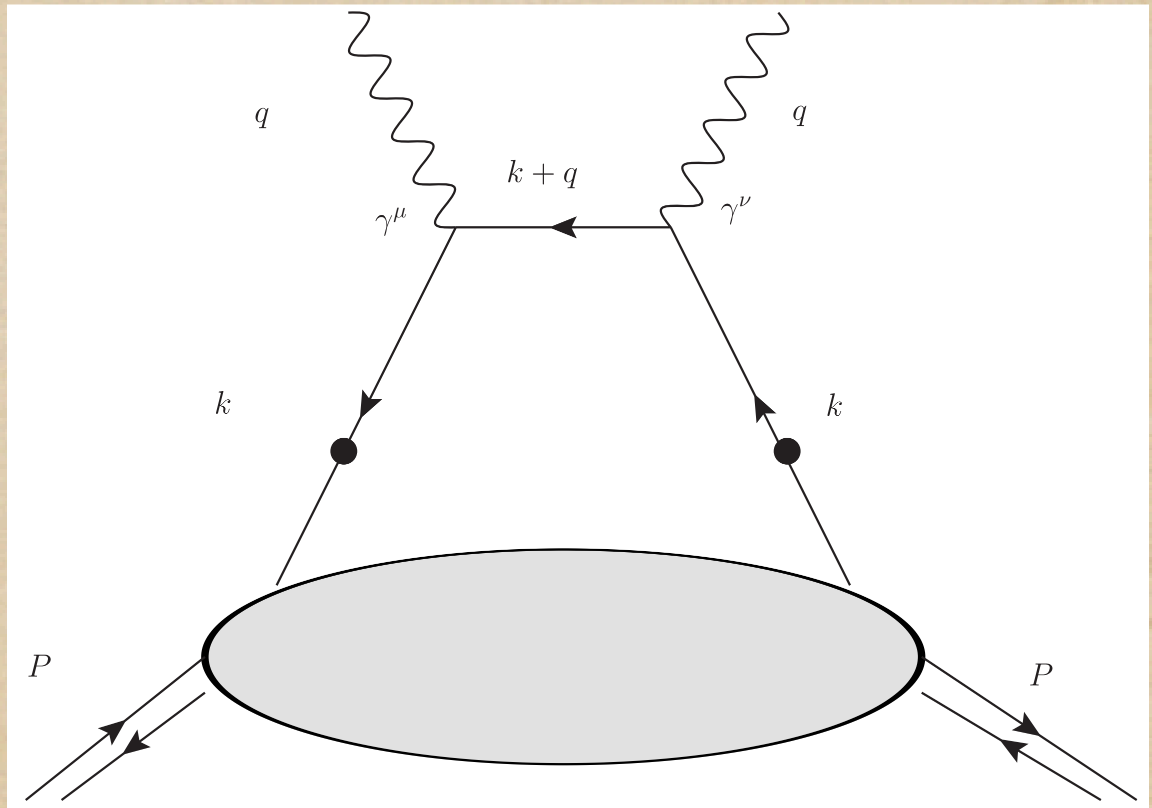
DSE Approach QCD's Parton Distributions

- Unify DSE treatment of PDFs with other aspects of hadron structure: masses, decays, charge form factors, transition form factors.....
- PDFs have their own blend of hard and soft, perturbative and non-perturbative, aspects of QCD.
- E.g. $\langle \mathbf{x^m} \rangle : \ \mathbf{small\ m \sim F_\pi(Q^2 \approx 0),\ large\ m \sim\ uv\ structure\ of\ bound\ state}$
- Can a DSE approach to PDFs compete with a lattice-QCD approach ?   Eg, how difficult is it to calculate a lot of moments, enough to reproduce the distribution?

Deep Inelastic Lepton Scattering



- PDFs: $u_\pi(x)$, $u_K(x)$, $s_K(x)$
- Drell-Yan data exists
- Pion and Kaon/Pion Ratio
- Employ LR DSE model
- Bjorken limit fixes quark $\mathbf{k}^+$



- Covariant formulation, explicitly integrate: $\int \mathbf{dk^-}\ \Gamma(\mathbf{k^2}, \mathbf{k}\cdot\mathbf{P})$
- Evolve from model scale via LO DGLAP

DIS in Bjorken Kinematic Limit

$$\mathbf{T^{\mu\nu} = i \int d^4z \;\; e^{iq\cdot z} \;\; \langle \pi(P) \,|\, T\, J^\mu_H(z)\, J^\nu_H(0)\,|\,\pi(P)\rangle_c}$$

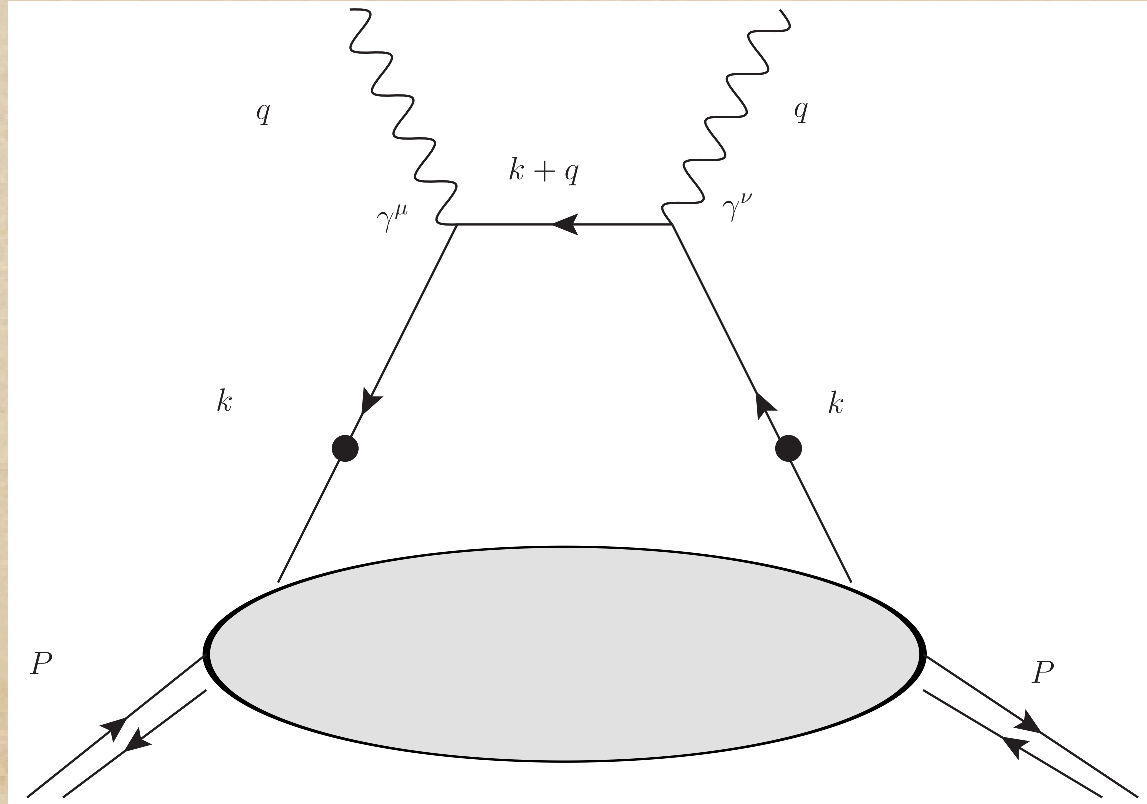
Bjorken limit seeks out dominant singularity : $\mathbf{z^2 = 0^+}$

$$\mathbf{q\cdot n \equiv q^+ = -x\,P\cdot n \quad \Rightarrow \quad z\cdot p \equiv z^- \sim \frac{1}{M_\pi x}}$$

$$\mathbf{q\cdot p \equiv q^- = 2\nu \quad \Rightarrow \quad z\cdot n \equiv z^+ \sim 0}$$

$$\mathbf{S(k+q) \;\to\; \frac{\gamma\cdot n}{2(k\cdot n - xP\cdot n) + i\epsilon}}$$

DIS is hard & fast,
confinement is soft & slow

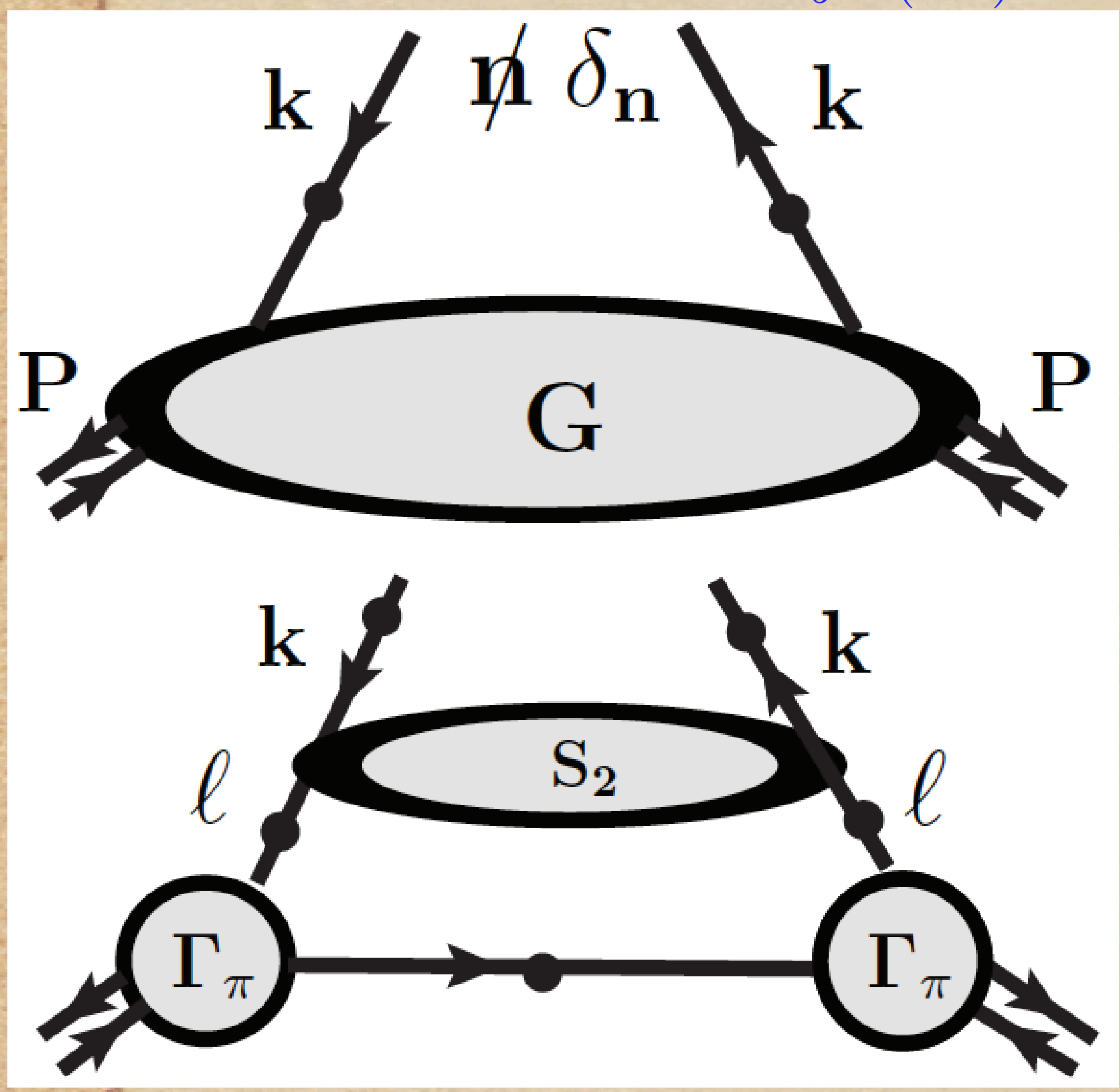


$$W^{\mu\nu} \propto \{T^{\mu\nu}(\epsilon) - T^{\mu\nu}(-\epsilon)\} \Rightarrow \text{Euclidean model elements can be continued}$$

To Leading Order in OPE

$$q_f(x) = \frac{1}{4\pi} \int d\lambda \; e^{-ixP\cdot n\lambda} \, \langle \pi(P) | \, \bar{\psi}_f(\lambda n) \; \not{n} \; \psi_f(0) \; | \pi(P) \rangle_c$$

$$q_f(x) = -\frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \; \delta(k\cdot n - xP\cdot n) \;\; \mathrm{tr}_{cd}[\, i\not{n} \; G(k, P) \,]$$



An infinite subset of Fock space components enter via use of LR dressed objects

$$u_\pi(x) = -\frac{1}{2} \mathrm{tr}_{cd} \int_\ell \bar{\Gamma}_\pi \; S_u(\ell) \, \Gamma^n(\ell; x) \, S_u(\ell) \; \Gamma_\pi \; S_d(\ell - P)$$

$$S_2 \; \otimes \; i\not{n} \; \delta(k\cdot n - xP\cdot n) = S(\ell) \; \Gamma^n(\ell; x) \; S(\ell)$$

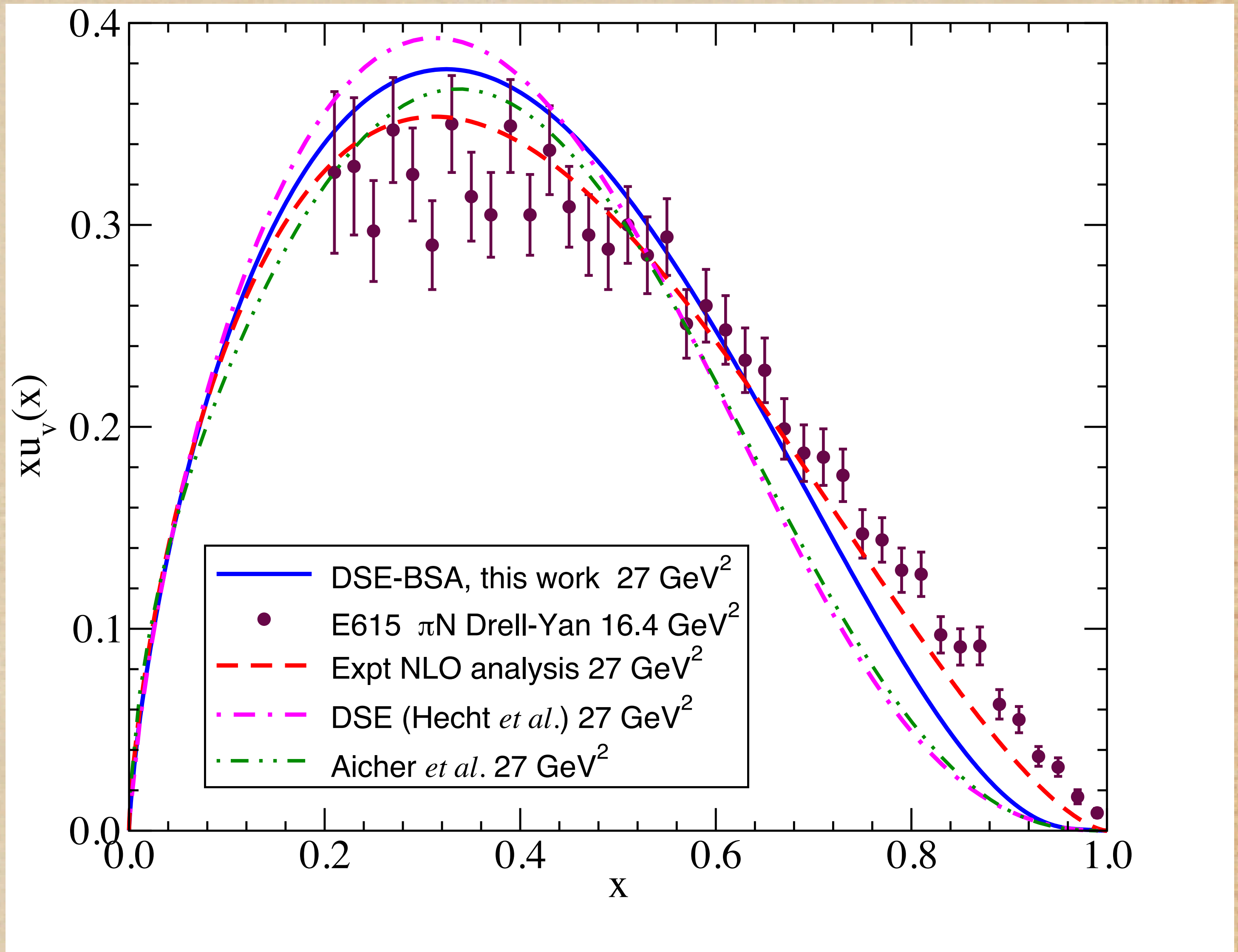
$$\Gamma^n(\ell; x) = i\not{n} \; \delta(\ell\cdot n - xP\cdot n) \; + \cdots$$

$$N_f^v = \int_0^1 dx \, [q_f(x) - q_{\bar{f}}(x)] = \frac{1}{2P^+} \langle \pi(P) | J^+(0) | \pi(P) \rangle_c = 1$$

Valence $u_\pi(x)$ from DSE-BSE solutions

- Valence quarks, handbag diagram
- Data: Conway et al, PRD39, 92 (1989). $M_{l\bar{l}} = 4.05$ GeV
- Prev DSE (phen): Hecht et al, PRC63, 025213 (2001), $\Gamma_\pi(k^2, k\cdot P = 0) \sim i\gamma_5 B_0(k^2)/f_\pi^0 + \cdots$ $S_{phen}(k)$
- Large x behavior: $(1-x)^{2.08}$
- T. Nguyen, PhD 2010, KSU,
- Wijesooriya, Reimer&Holt, PRC72, 065203 (2005)

Nguyen, Bashir, Roberts, PCT, arXiv : 1102.2448 (2011).

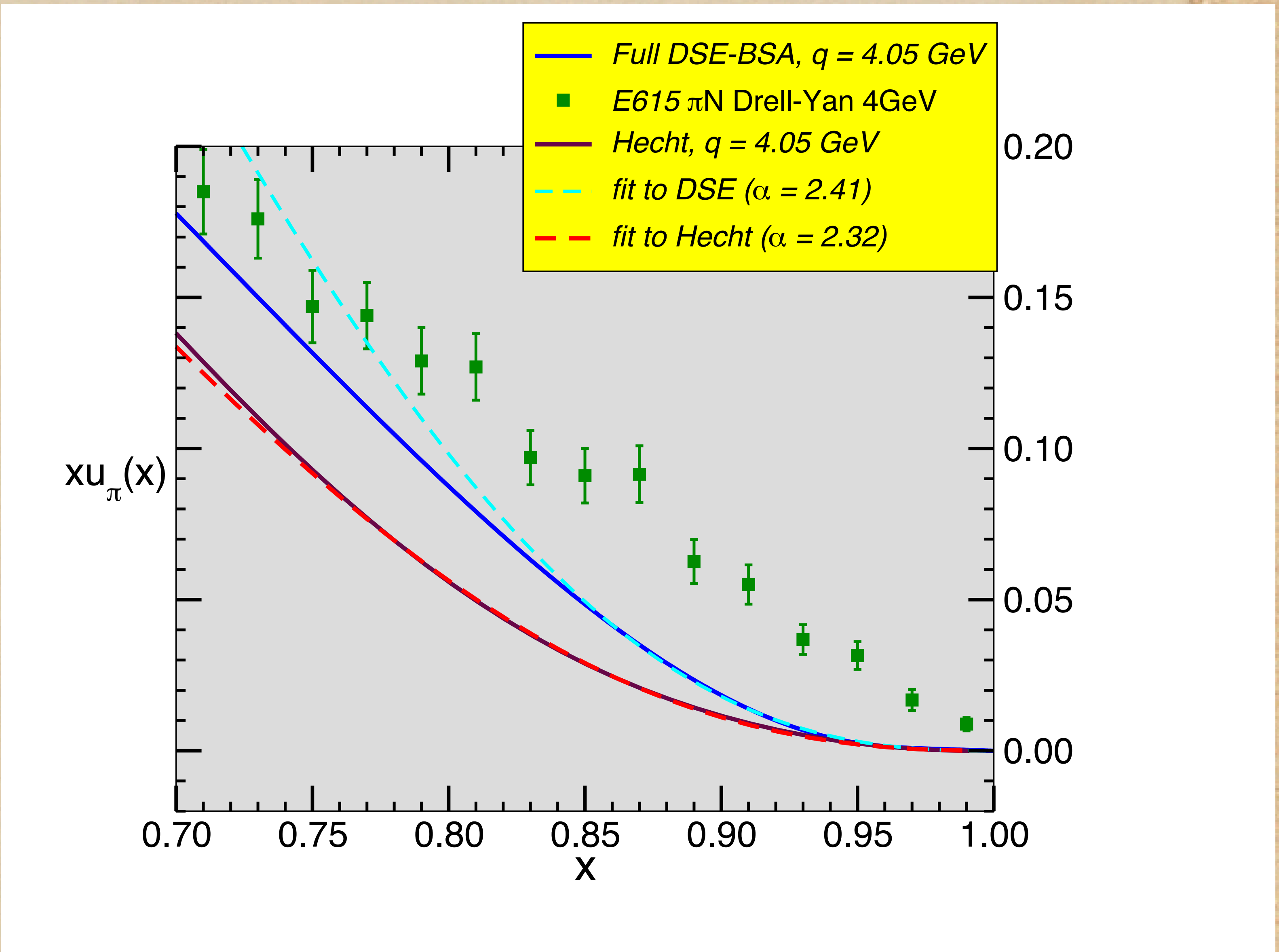


Momentum Sum Rule: $\langle x \rangle_{Q_0^2} = 0.76$

$u_\pi(x)$ at large x; pQCD

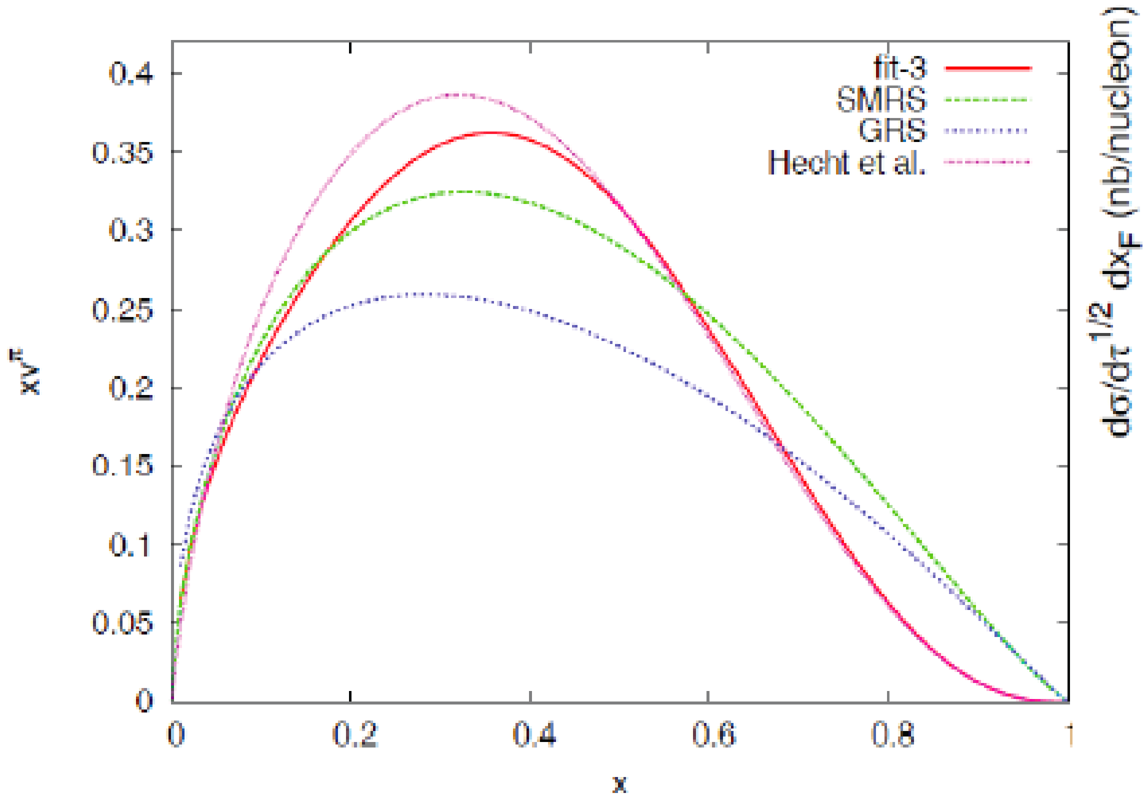
- Scale for pQCD onset is model-depn.
- Global DIS fits: $\alpha \sim 1.5$
- Const. q models, NJL, duality: $\alpha \sim 1$
- pQCD: Farrar-Jackson, Brodsky, Ezawa, DSEs:

$$\alpha = 2 + \gamma(Q^2)$$



High x: Recent Development

Soft Gluon Resummation



Aicher, Schafer, Vogelsang, arXiv:1009.2481

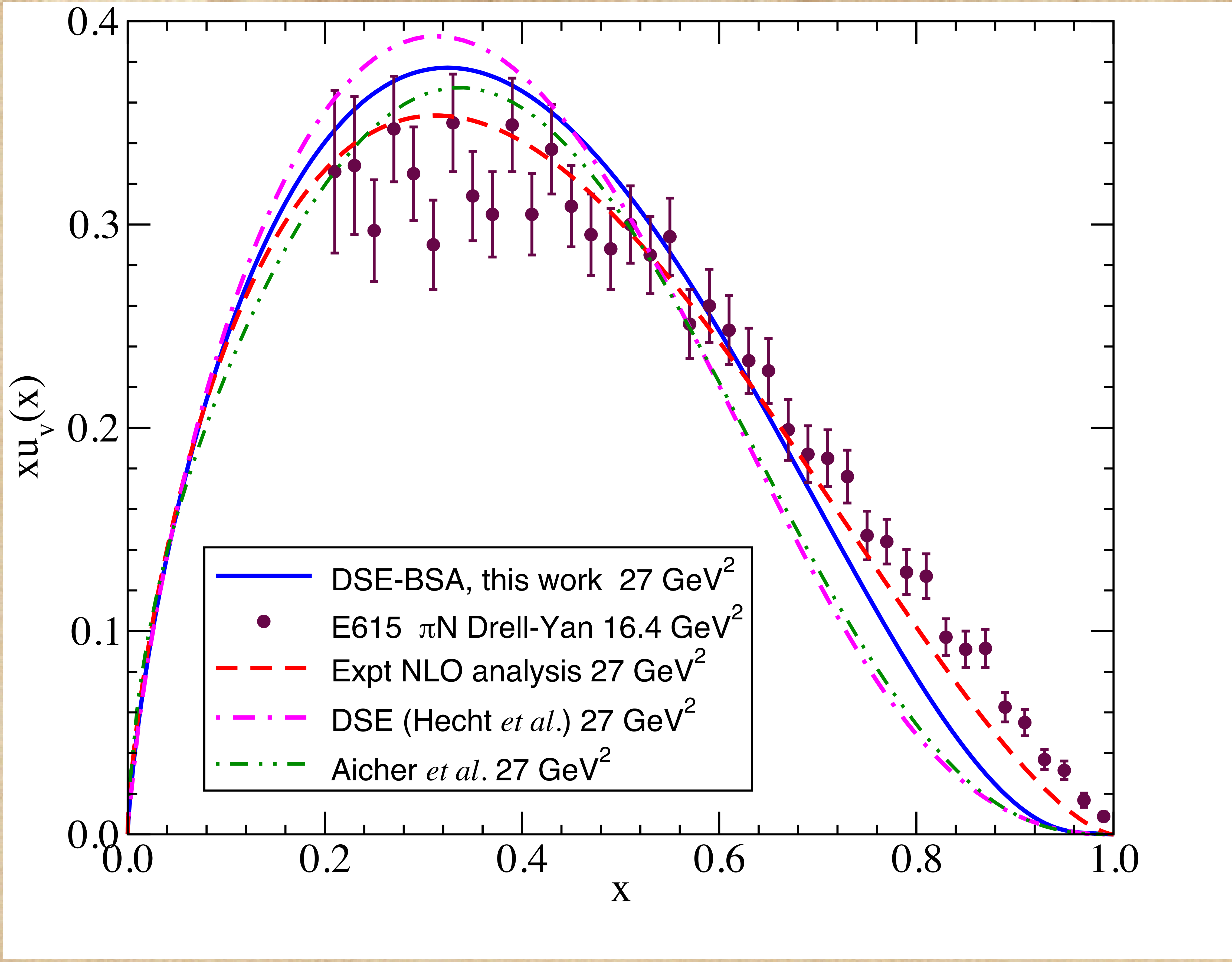
$$xq_V^\pi(x) = A_V^\pi x^\alpha (1-x)^\beta \left(1+\gamma x^\delta\right)$$

β = 2.03 ±0.06

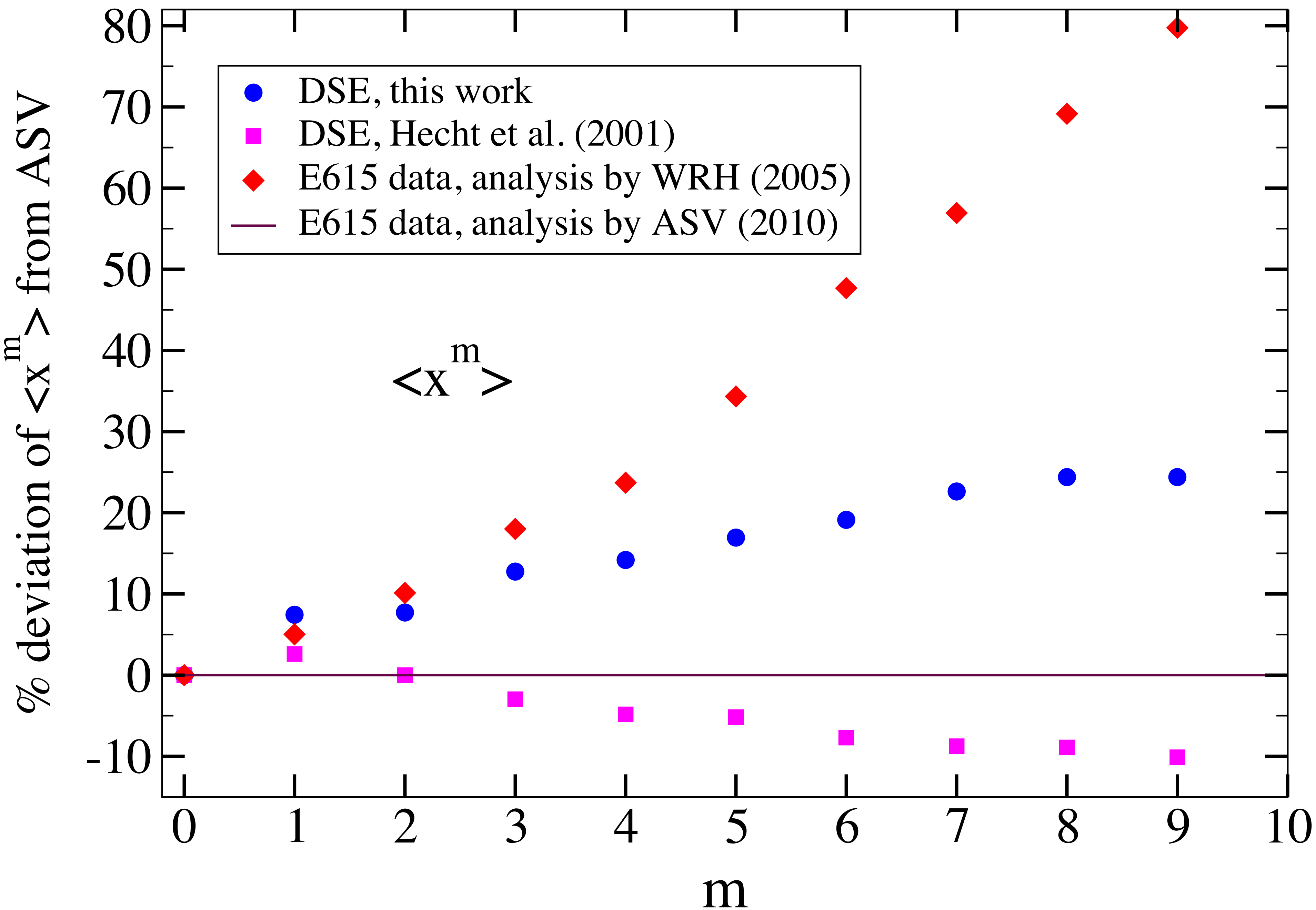
QCD and Dyson-Schwinger survive!
pQCD: xq(x)/ $(1-x)^\beta$ β = 2
DSE: xq(x)/ $(1-x)^\beta$ β ¼ 1.9

---from Paul Reimer, 3rd Int Workshop on Nucleon Structure at High x

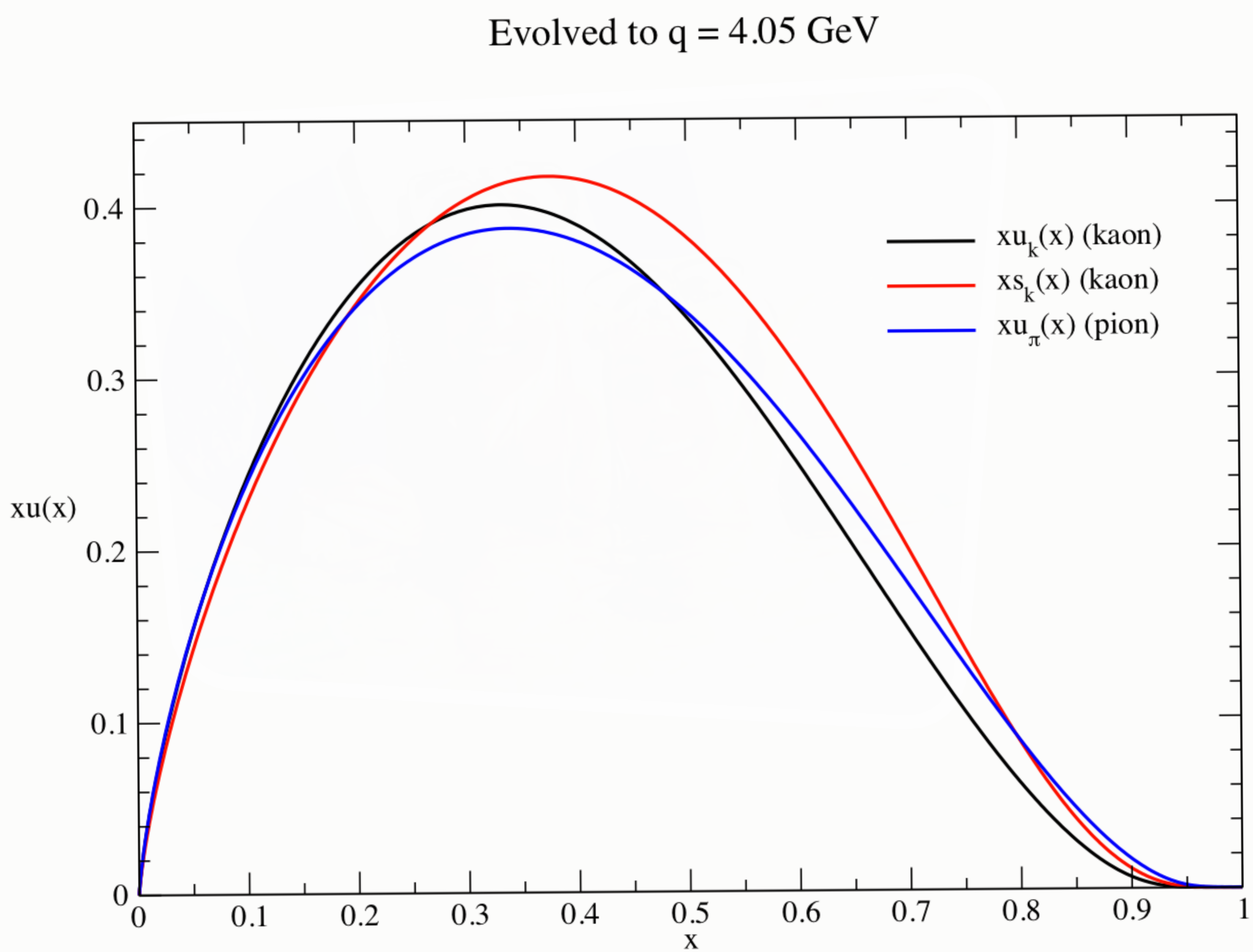
Nguyen, Bashir, Roberts, PCT, arXiv : 1102.2448 (2011).



Valence PDF Moments in Pion



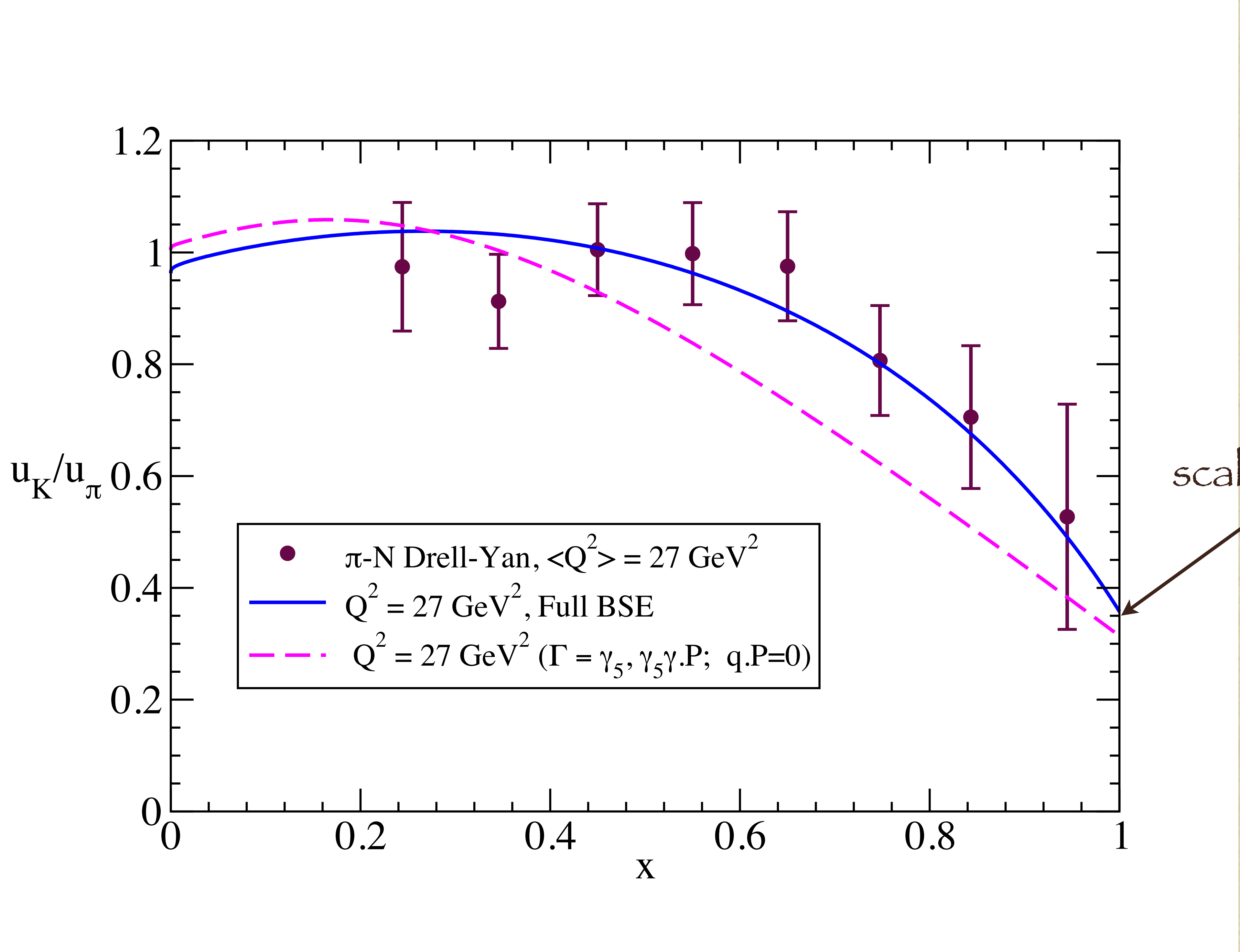
Quark Distributions in π and K



- Environmental depn of $u(x)$ in accordance with effective quark mass
- $u(x), \quad s(x)$ difference in K in accordance with effective quark mass

Environmental Dependence of Valence $u(x)$

Nguyen, Bashir, Roberts, PCT, arXiv : 1102.2448 (2011).



scale invariant

- CERN-SPS data: J. Badier et al, PLB **93**, 354 (1980)   (valence is not isolated)

Large x Estimate of $\frac{u_K(x)}{u_\pi(x)}$

- $\bullet$ Approximations : $\Gamma_{K/\pi}(q^2) \sim \gamma_5 \, N_{K/\pi} \, / \, (q^2 + \Lambda^2_{K/\pi})$ $\quad S(k) \sim 1/(i \not{k} + M_q)$

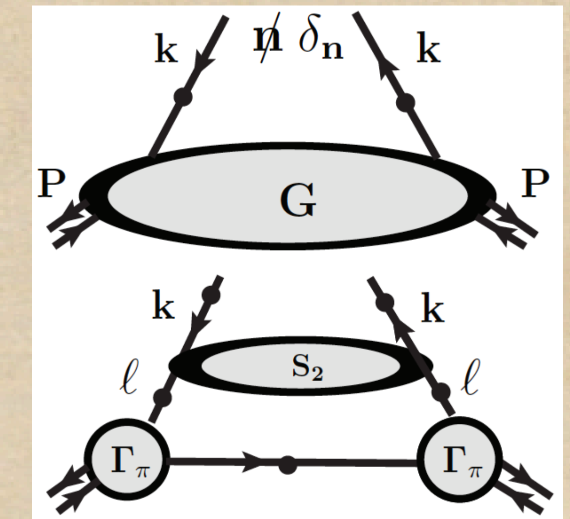
$$\Rightarrow \qquad u_K(x) = N \int_0^\infty d\hat{\mu} \, \frac{\frac{a}{1-x} + b + \hat{\mu}}{[\frac{a}{1-x} + c + \hat{\mu}]^2} \, (\frac{a}{1-x} + d + \hat{\mu})^{-2} \quad , \qquad a = xM_s^2$$

$$\frac{a}{1-x} >> \text{any other mass scale} \Rightarrow \qquad u_{K/\pi}(x) \propto N^2_{K/\pi} \, \frac{(1-x)^2}{M^4_{spect}}$$

In a covariant and properly regularized formulation, $(1-x)^2$ is due totally to the divergence of the relative momentum argument of both $\Gamma_{K/\pi}(q^2)$ ie 1 – gluon exchange binding in pQCD

$$\frac{u_K(1)}{u_\pi(1)} \sim \frac{f_\pi^2}{f_K^2} \, (\frac{M_u}{M_s})^4 \sim 0.3 \qquad \text{cf } 0.35 \text{ from DSE – BSE}$$

PDFs via direct calcn of moments



$\mathbf{Euclidean, ladder - rainbow:}$

$$\langle x^m \rangle = \int_0^1 dx\; x^m\; q(x)$$

$$\langle x^m \rangle = \frac{iN_c}{2\;P\cdot n}\; \mathrm{tr} \int_k \left(\frac{k\cdot n}{P\cdot n}\right)^m \Gamma_\pi(k-\tfrac{P}{2})\; S(k)\; \gamma\cdot n\; S(k)\; \Gamma_\pi(k-\tfrac{P}{2})\; S(k-P)$$

Only trivial analytic continuation needed : $(k\cdot n)^m = (-ik_4 + k_3)^m$

Method can easily exceed the Lattice – QCD practical limit : $m = 3$

Note point limit : $\Gamma_\pi \to \mathrm{const} \;\Rightarrow\; < x^m >_{pt} = \dfrac{1}{1+m}$

Feynman Integral Method/Representation

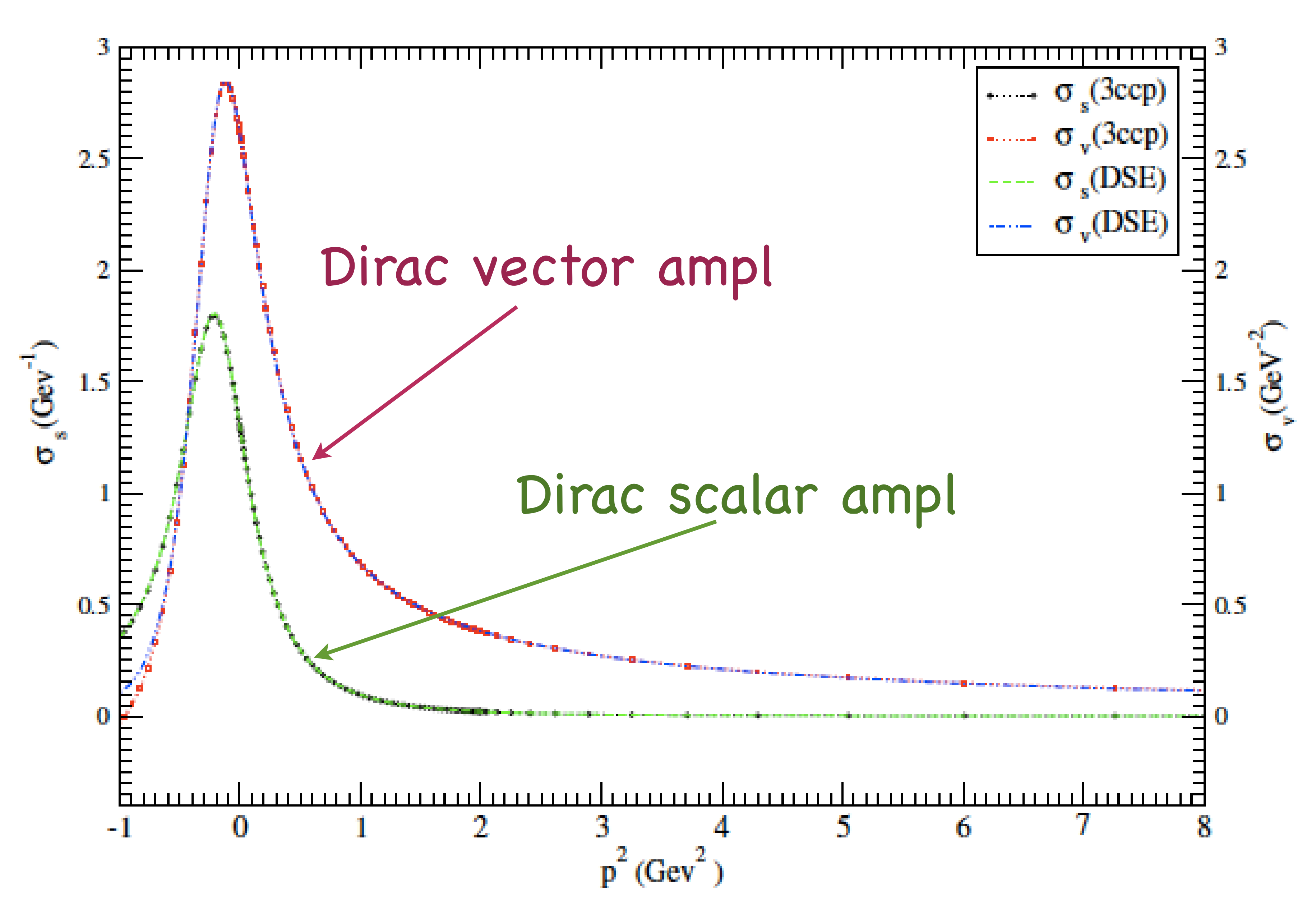
- For triangle diagram, need all momentum integral variables to appear in denominators that are powers of quadratic forms, with necessary finite powers in numerator
- [Could apply to BSE eqn]---see 1960-70s---Perturbation Integral Repn, and Nakanashi Repn of BSE amplitudes
- Here we use it as a convenient fit/representation of existing numerical solns of DSE for q propagtor, and of BSE for meson BSE ampls.
- Momentum integrals done analytically, remaining Feyn parameter integrations done numerically
- Only singularities in the resulting physical quantity are those demanded by unitarity and physical thresholds from open hadronic decays
- Accommodates confining propagators via complex conjugate location of spectral properties--non-positive spectral densities
- Trust in essential content of QFT: analyticity, unitarity, principal mass scales, causality.....etc.

Fit Existing DSE Quark Propagators

$$S(q) = \sum_{k=1}^{3} \left(\frac{z_k}{i\, \not{q} + m_k} + \frac{z_k^*}{i\, \not{q} + m_k^*} \right)$$

$$\sigma_s(q^2) = \sum_{k=1}^{3} \left(\frac{z_k m_k}{q^2 + m_k^2} + \frac{z_k^* m_k^*}{q^2 + m_k^{*2}} \right)$$

$$\sigma_v(q^2) = \sum_{k=1}^{3} \left(\frac{z_k}{q^2 + m_k^2} + \frac{z_k^*}{q^2 + m_k^{*2}} \right).$$



Similar repns also work for s-, c-, b-quarks

Fit Existing BSE Amplitudes, for PDF moments

$$\Gamma_\pi(k^2, k.P) = \gamma_5[iE_\pi + \gamma \cdot PF_\pi + \gamma \cdot PG_\pi +]$$

$$E_\pi(k^2, k \cdot P) = \frac{N}{k^2+\Lambda^2}$$

$$E_\pi(k^2, k \cdot P) = N\left\{\frac{1}{k^2+\alpha k\cdot P+\Lambda^2} + \frac{1}{k^2-\alpha k\cdot P+\Lambda^2}\right\}$$

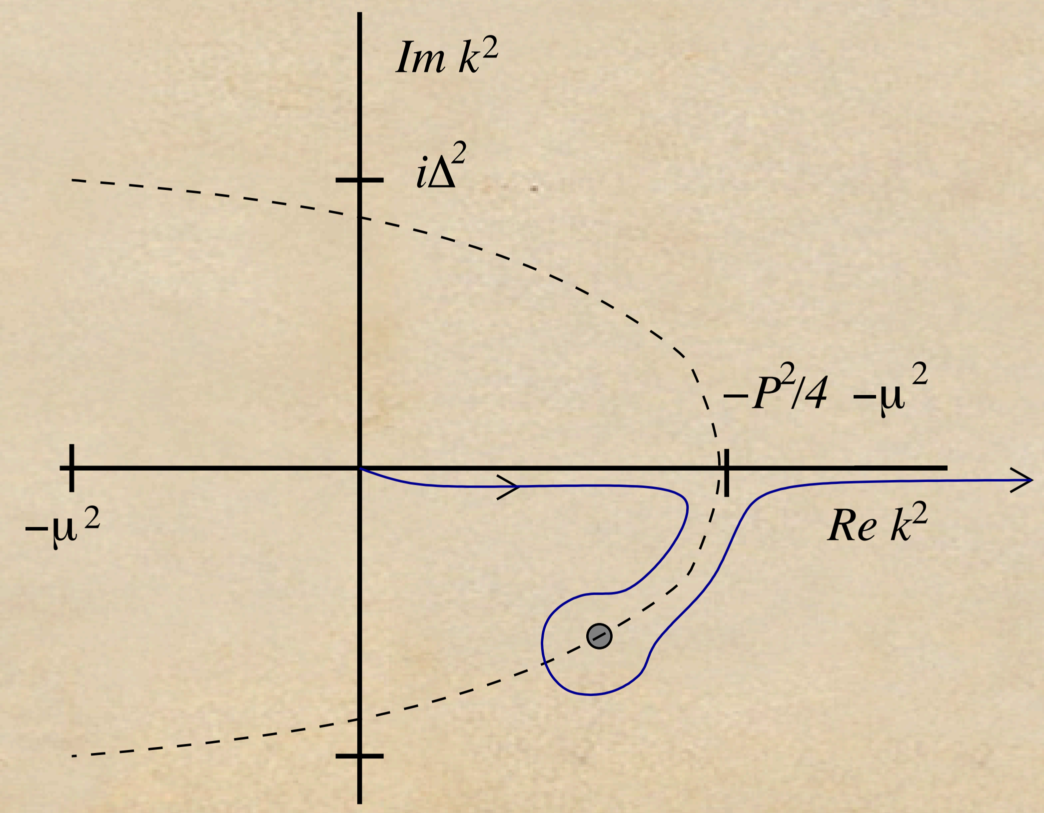
$$E_\pi(k^2, k \cdot P) = N(1 + k^2/\lambda^2)\ \Sigma_i^2\ \left\{\frac{1}{(k^2+\alpha_i k\cdot P+\Lambda_i^2)^2} + \frac{1}{(k^2-\alpha_i k\cdot P+\Lambda_i^2)^2}\right\}$$

Similarly $F_\pi(k^2, k \cdot P)$

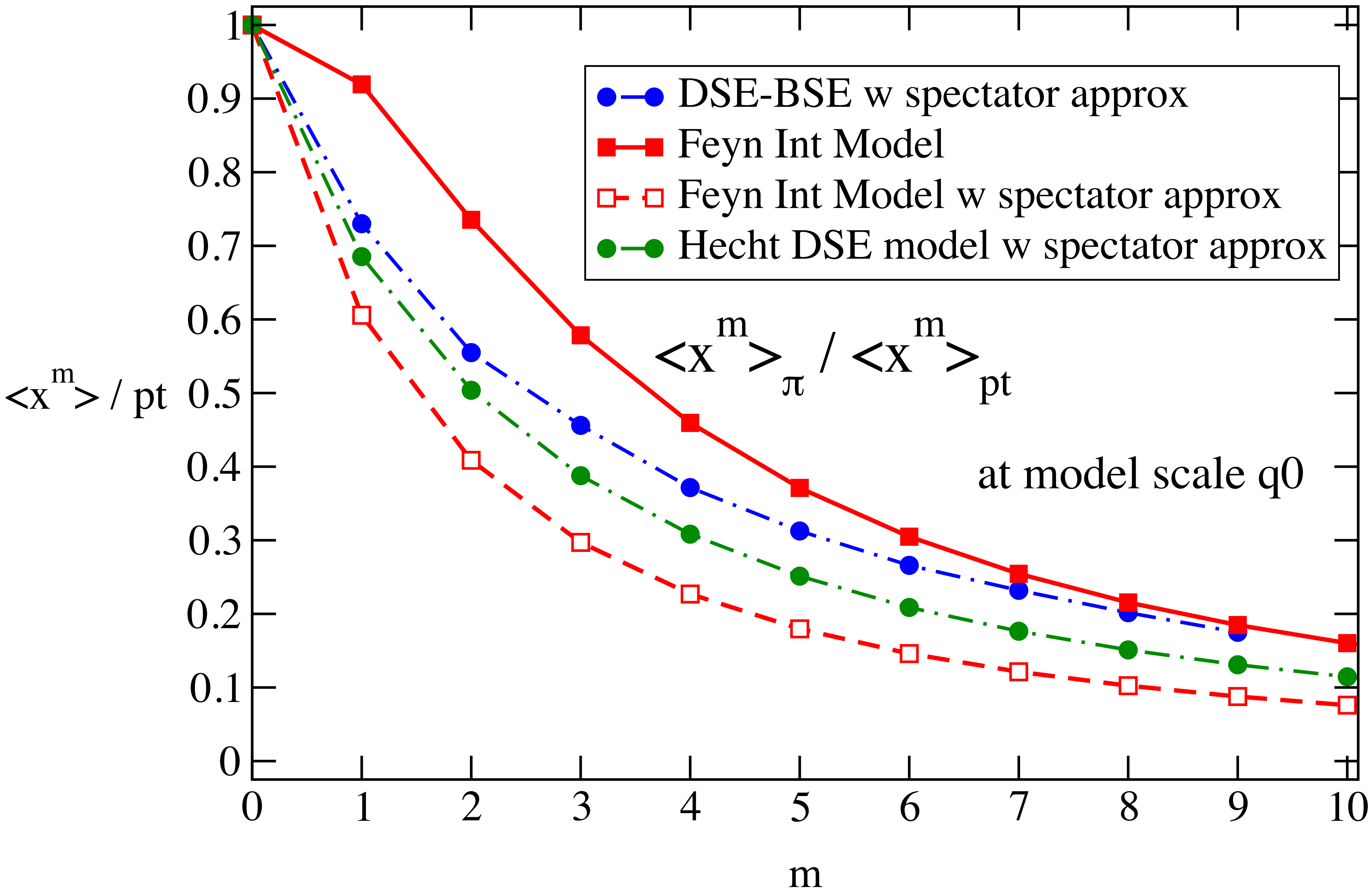
Euclidean loop integral, analytic continuation in external hadron P^2

$$F(Q^2, P^2 = -M^2) = \lim_{P^2 \to -M^2} \int d^4k \; I(k, P, Q)$$

$$\neq \int d^4k \lim_{P^2 \to -M^2} I(k, P, Q) \; , \quad \text{above"threshold"}$$



Pion Valence q PDF Moments at q0: Ratio to Point



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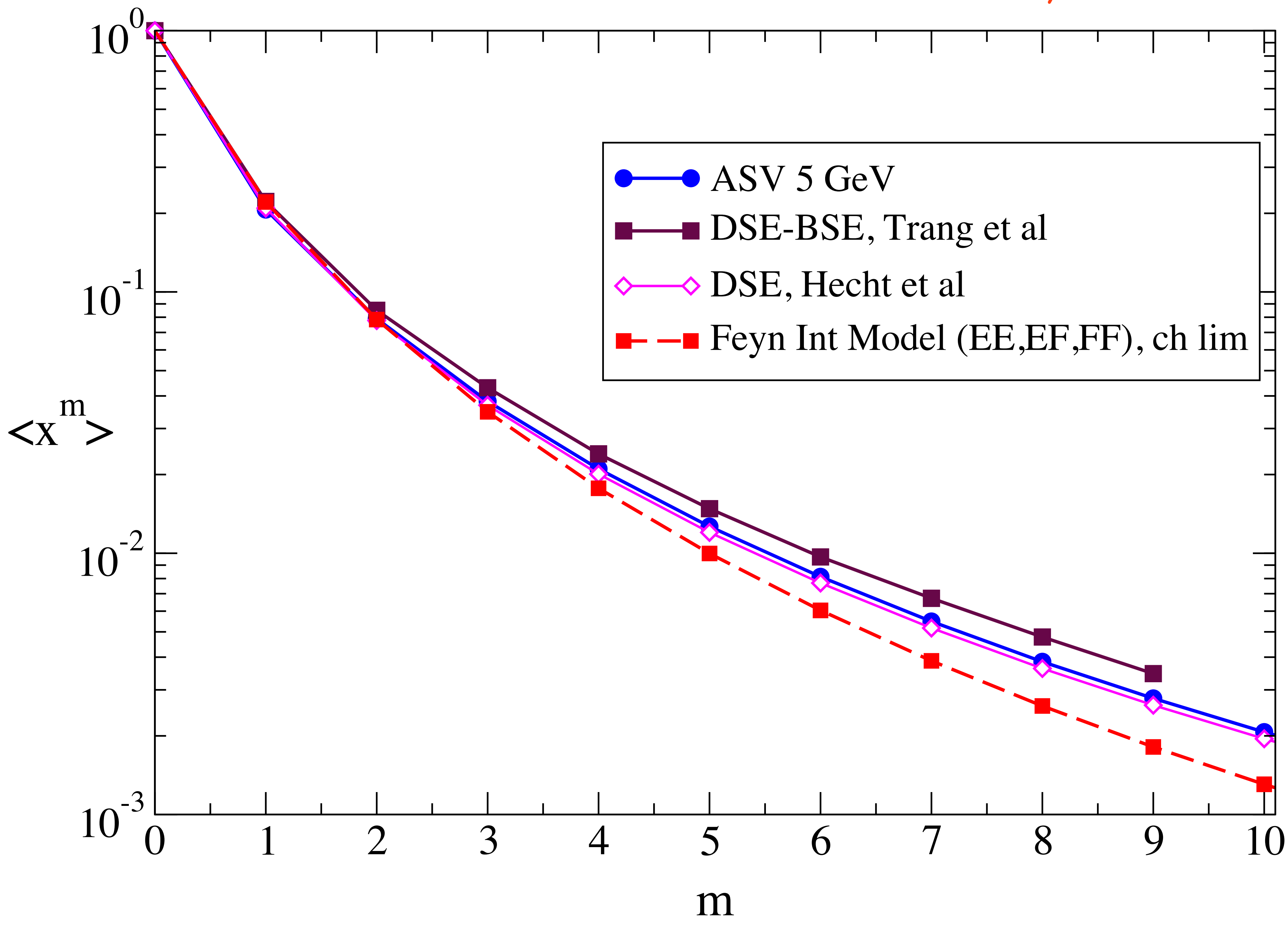
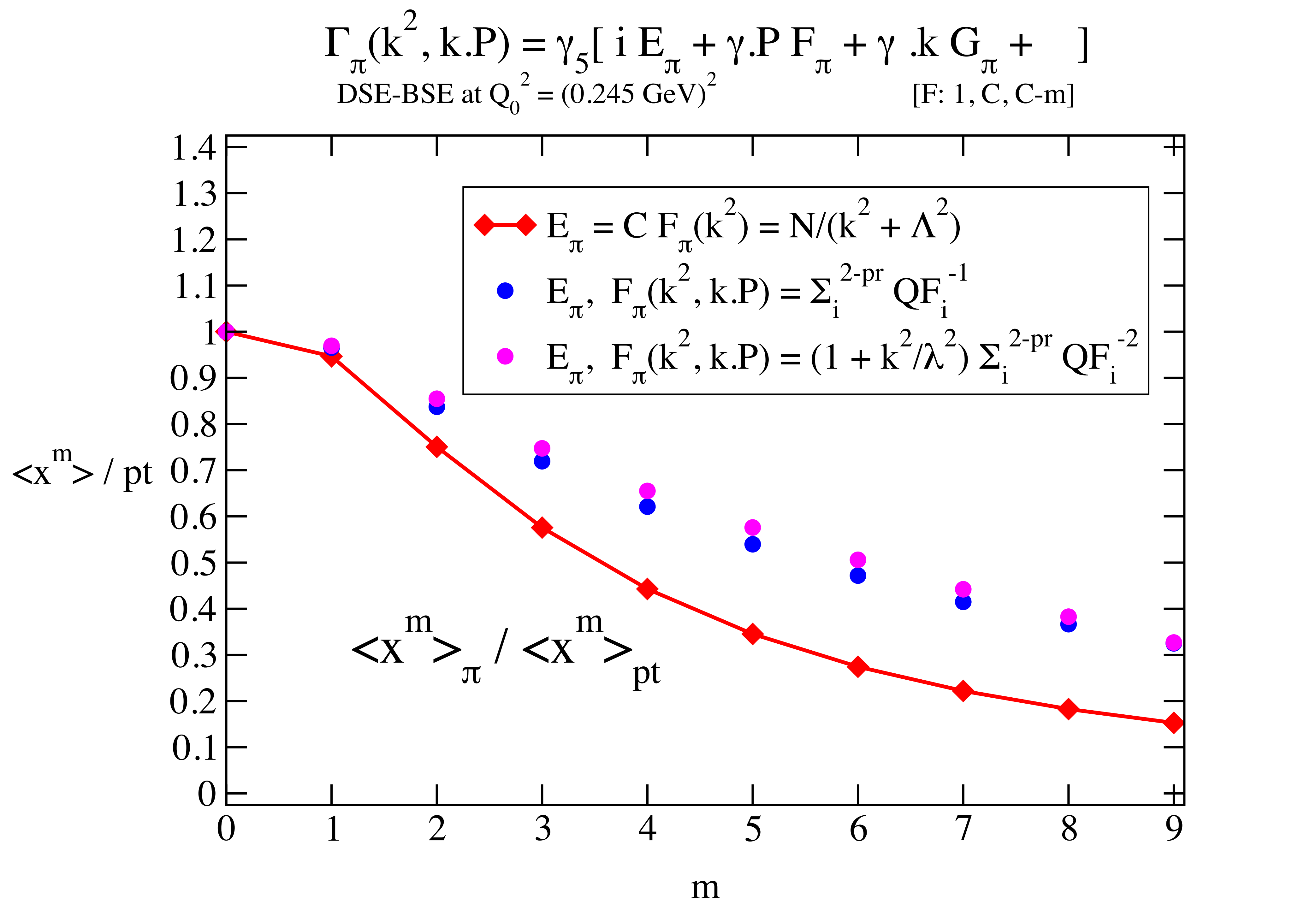
Euclidean DSEs: Chiral Pion Valence PDF Moments

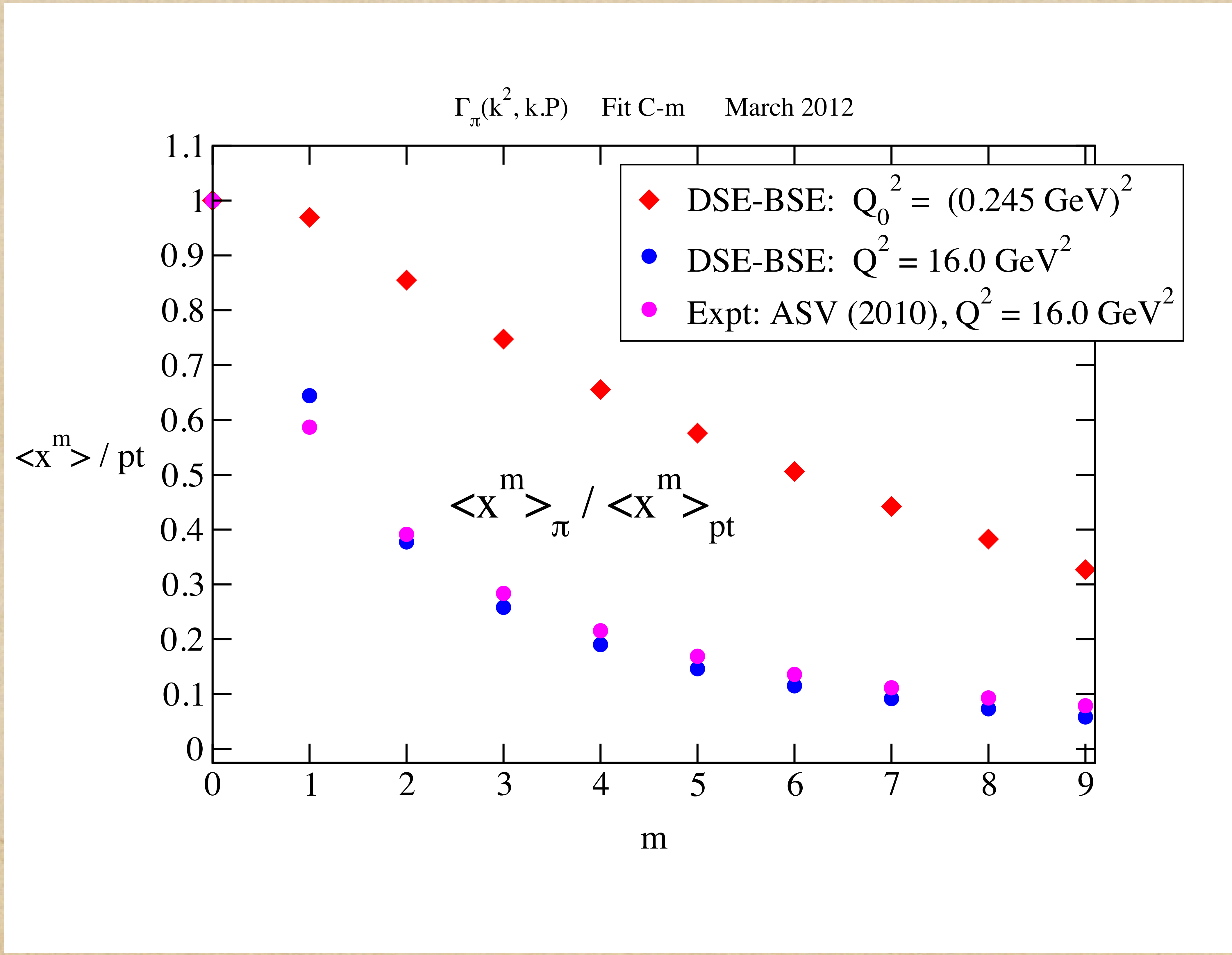
Running quark masses & photon-quark vertex dressing don't really matter for the mostly soft PDF



Feyn Integral Method: Capture Details of BSE Ampls

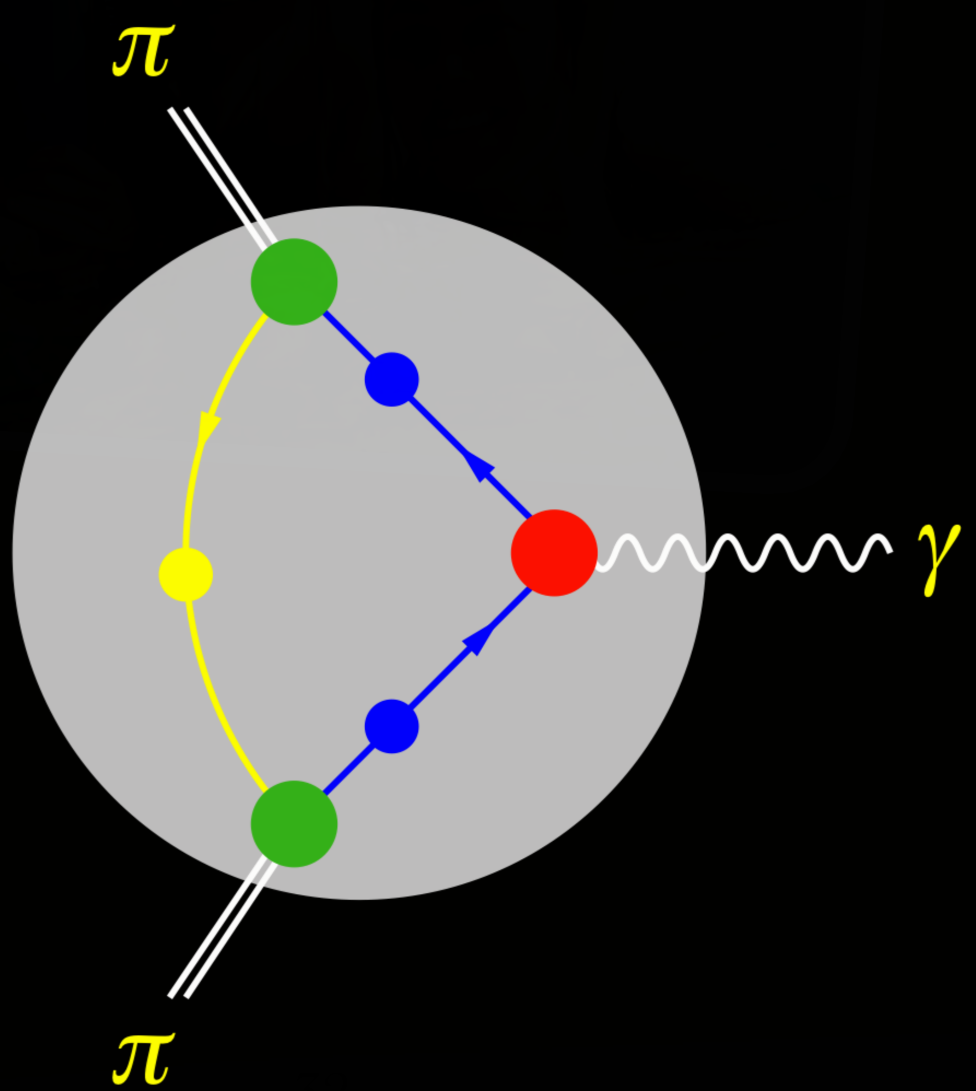


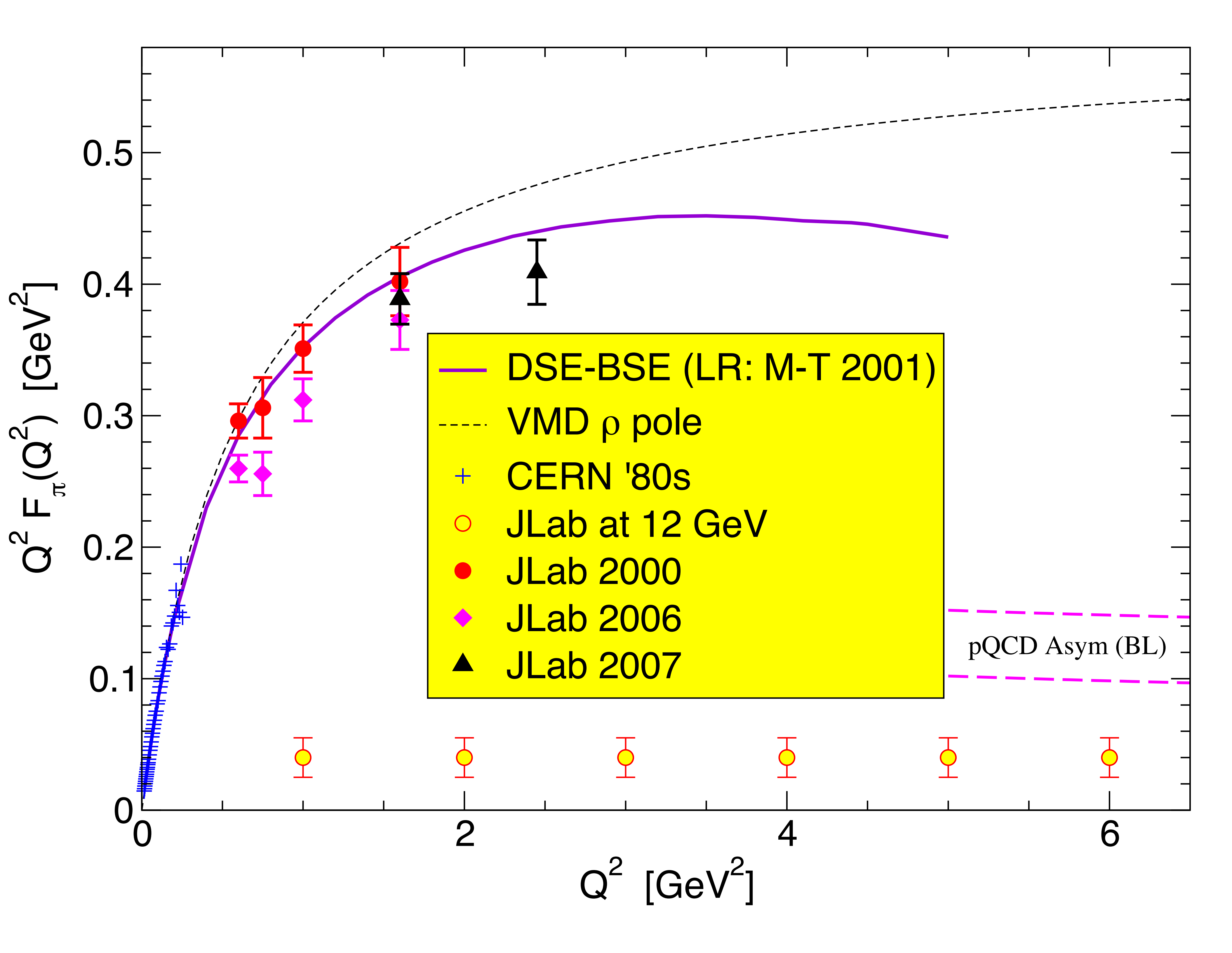
Feyn Integral Method: Best BSE Fit, DGLAP to 16 GeV^2



Pion electromagnetic form factor

$$\Lambda_\mu = (P' + P)_\mu \, F_\pi(Q^2) = N_c \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}\left[\bar{\Gamma}^\pi \, S \, i\Gamma_\mu \, S \, \Gamma^\pi \, S\right]$$





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Asymptotic Pion Transition Form Factor (Brodsky, Lepage...)

$\gamma(Q_1^2)\ \gamma(Q_2^2) \to \pi^0$

$$\mathbf{F(Q_1^2, Q_2^2) \longrightarrow 4\pi^2 f_\pi^2 \{ \frac{J(\omega)}{Q_1^2+Q_2^2} + \mathcal{O}(\frac{\alpha_s}{\pi}\ ,\ \frac{1}{Q^2} \}}$$

$$\mathbf{J(\omega) = \frac{4}{3} \int_0^1 dx \frac{\phi_\pi(x)}{1-\omega^2(2x-1)}} \qquad \omega = \frac{\mathbf{Q_1^2-Q_2^2}}{\mathbf{Q_1^2+Q_2^2}}$$

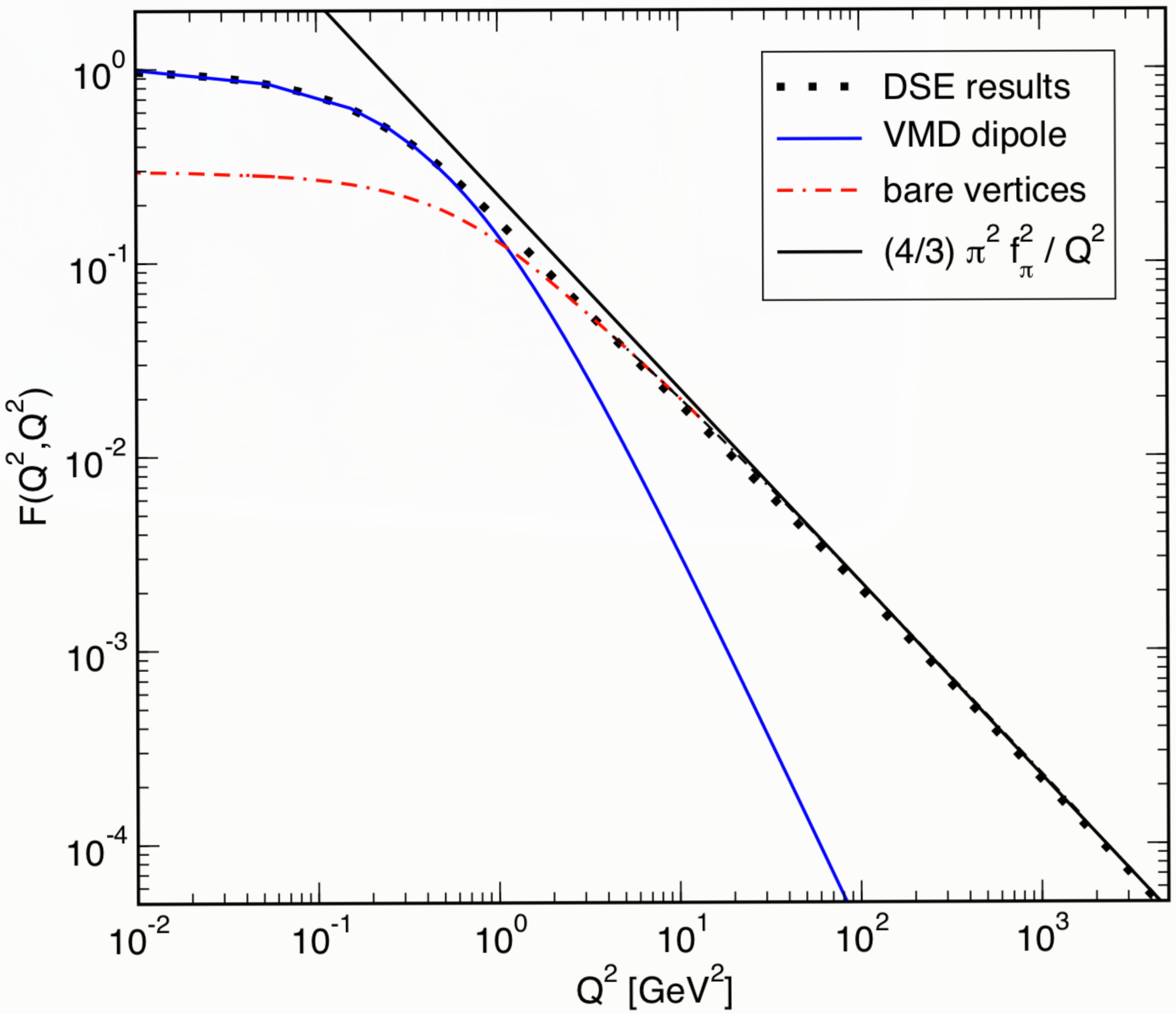
$$\phi_\pi(\mathbf{x}) \propto \int \frac{\mathbf{d}\lambda}{\mathbf{2}\pi}\ \mathrm{e}^{\mathbf{i}\lambda\mathbf{x}}\ \langle \mathbf{0} |\ \bar{\mathbf{q}}(\mathbf{0})\ \gamma\cdot\mathbf{n}\ \gamma_\mathbf{5}\ \mathbf{q}(\lambda\mathbf{n})\ | \pi(\mathbf{P})\rangle \qquad \mathbf{n}^\nu\ \mathbf{n}_\nu = \mathbf{0}$$

$$\phi_\pi^{\mathrm{asym}}(\mathbf{x}) \propto \mathbf{x}(\mathbf{1}-\mathbf{x})$$ **A finite size Bethe – Salpeter bound state**

$$\mathbf{LC\ pQCD\ (BL)} \Rightarrow \mathbf{J(1) = 2} \qquad [\ \mathbf{DSE\ LR} \Rightarrow \tfrac{7}{8} \times \mathbf{2}\ ]$$

$\gamma^\star\pi\gamma^\star$ Asymptotic Limit

Lepage and Brodsky, PRD22, 2157 (1980): LC-QCD/OPE ⇒



Fit Existing BSE Amplitudes, for pion FF

$$\Gamma_\pi(k^2, k.P) = \gamma_5[iE_\pi + \gamma \cdot PF_\pi + \gamma \cdot PG_\pi +]$$

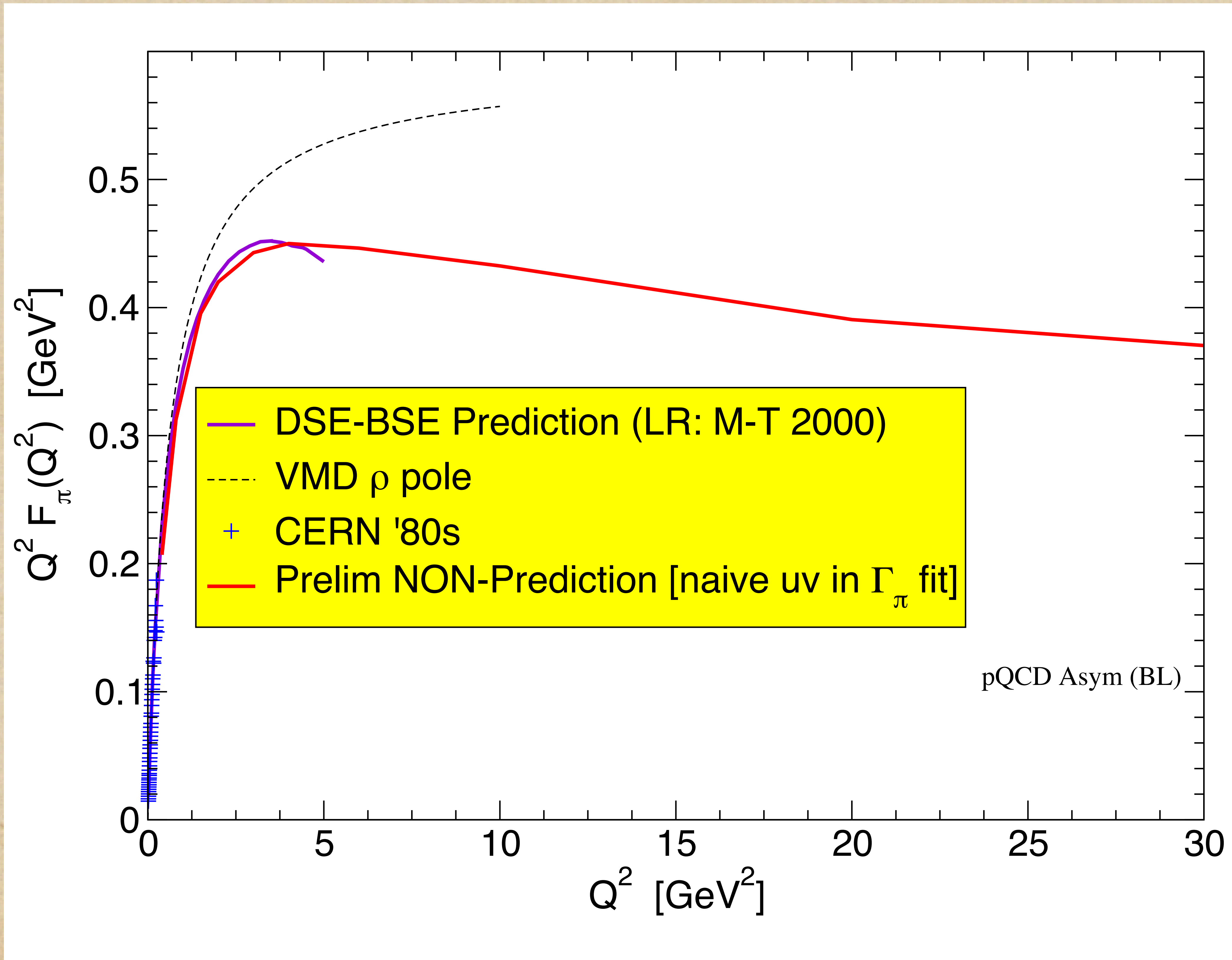
$$E_\pi(k^2, k \cdot P) = \frac{N}{k^2+\Lambda^2}$$

$$E_\pi(k^2, k \cdot P) = N\left\{\frac{1}{k^2+\alpha k\cdot P+\Lambda^2} + \frac{1}{k^2-\alpha k\cdot P+\Lambda^2}\right\}$$

$$E_\pi(k^2, k \cdot P) = N(1 + k^2/\lambda^2)\ \Sigma_i^2\ \left\{\frac{1}{(k^2+\alpha_i k\cdot P+\Lambda_i^2)^2} + \frac{1}{(k^2-\alpha_i k\cdot P+\Lambda_i^2)^2}\right\}$$

Similarly $F_\pi(k^2, k \cdot P)$

Sum of standard analytic Feynman loop integrals



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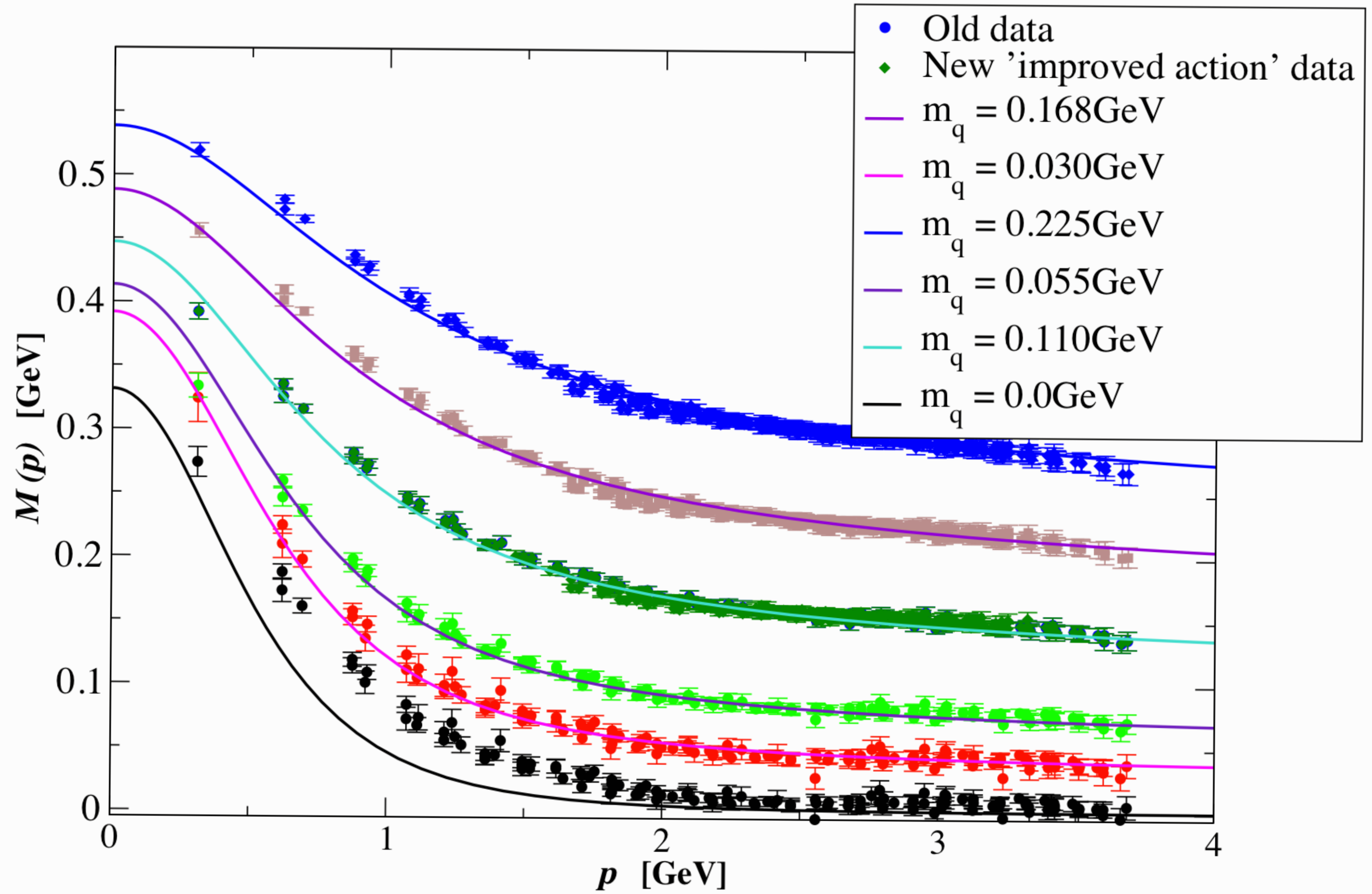
Effect of running quark ampls, eg mass fn?
Recall:

Qu-lattice $S(p), D(q)$ *mapped to a DSE kernel*

$$S(p) = Z(p)\,[i\,\not p + M(p)]^{-1}$$



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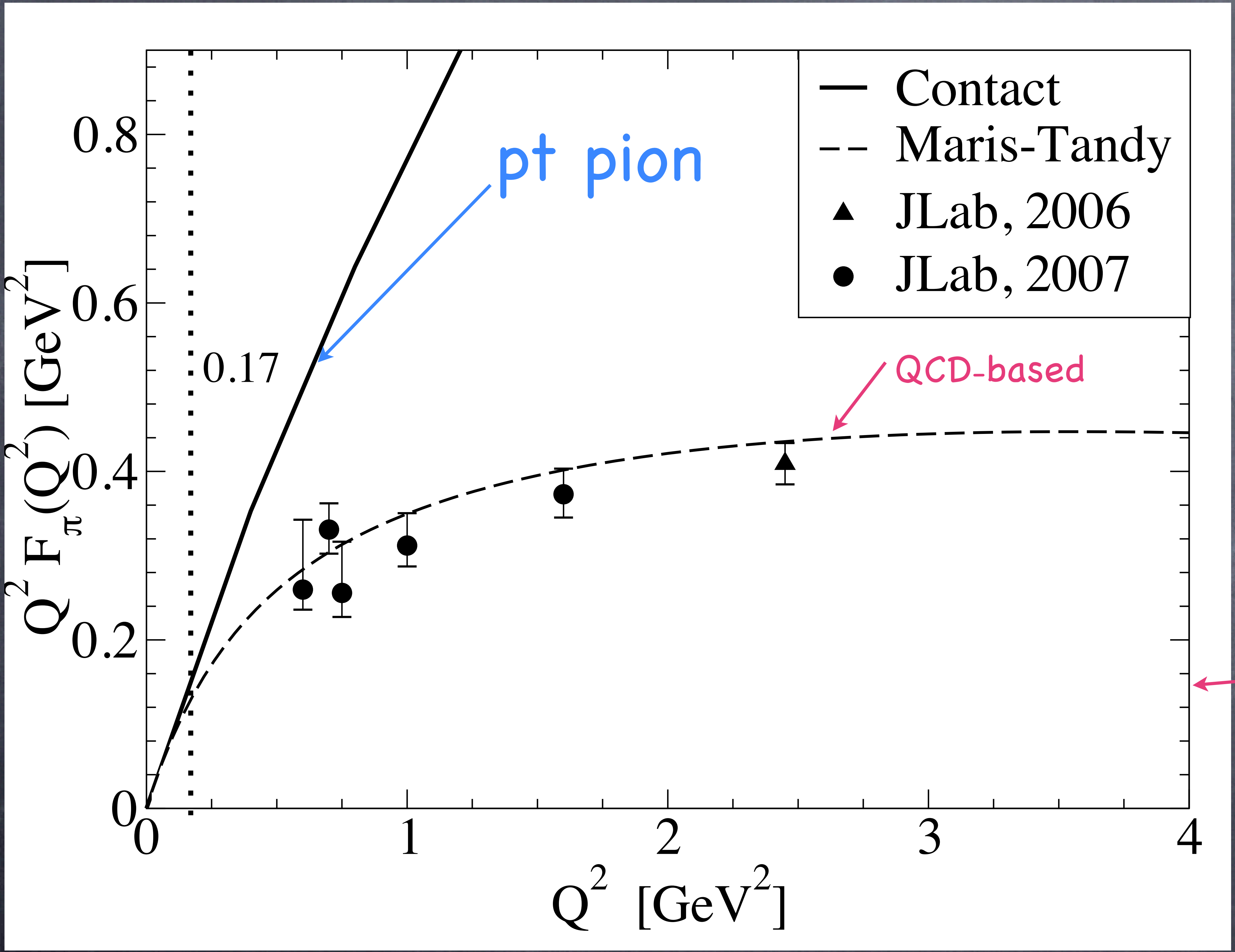
JLab Conf 2012

NSF

Pion Charge Form Factor



asympt pQCD
Farrar–Jackson

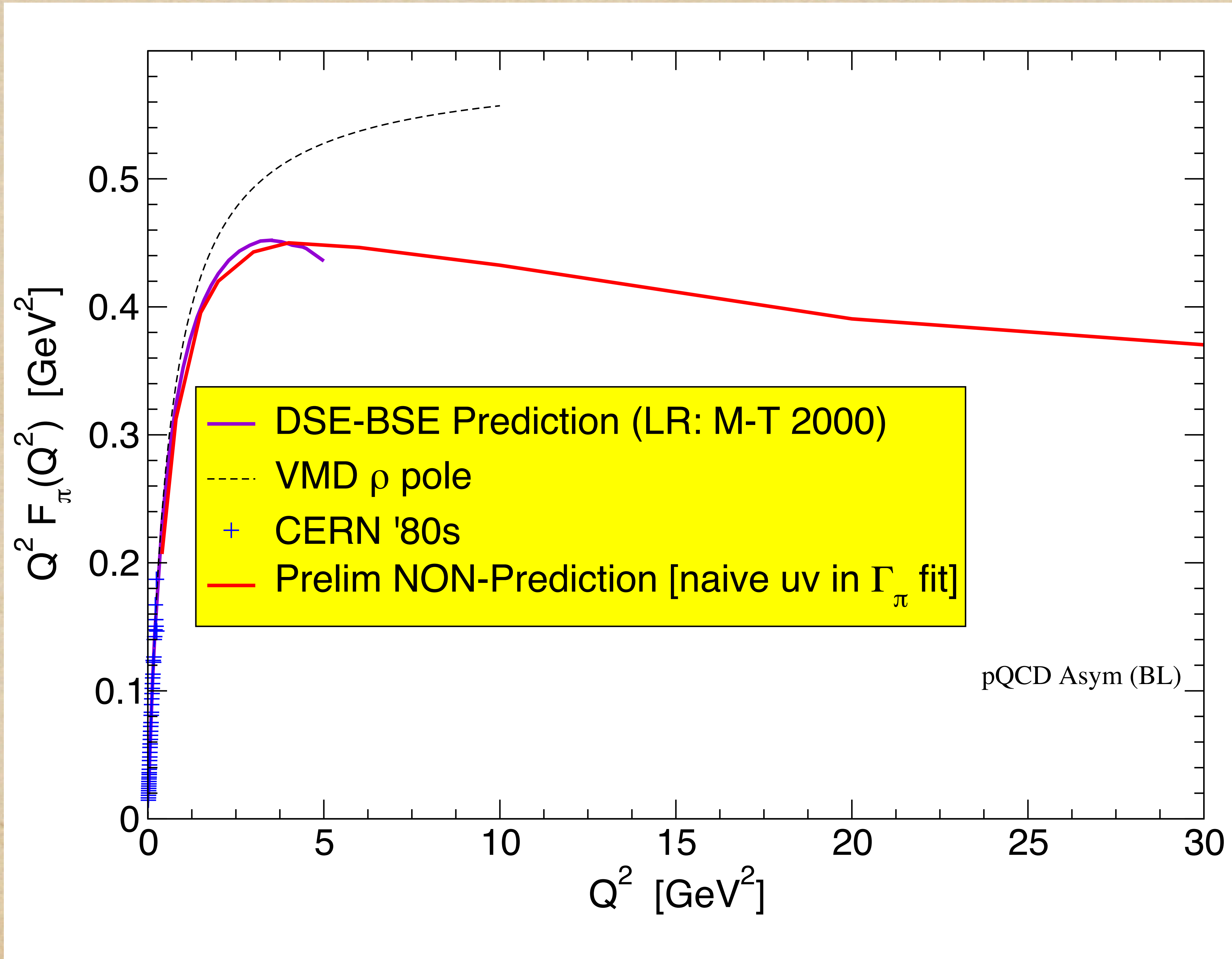
$$16\pi f_\pi^2\ \alpha_s(Q^2)$$

$$\sim 0.15 \text{ at } 10\ \text{GeV}^2$$

$$= \frac{2\ \alpha_s(Q^2)}{\pi}\ [8\pi^2 f_\pi^2 \sim M_\rho^2]$$

$$\text{VMD}: \ F(Q^2) \sim M^2/(Q^2 + M^2) \to M^2/Q^2\ ; \quad M_\rho^2 >> 0.15$$

Sum of standard analytic Feynman loop integrals



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Summary

DSE-modeling of QCD, PDFs and Form Factors

- Unified DSE treatment of PDFs with other aspects of hadron structure: masses, decays, charge form factors, transition form factors.....
- Ladder-rainbow truncated, LR modeling of QCD
- With no free parameters, $u_\pi^v(x)$ agrees quite well with Drell-Yan data
- The ratio $u_K(x)/u_\pi(x)$ also agrees with Drell-Yan data
- Feynman Integral Repn eliminates apparent numerical obstacles from singularities, all mom integrals evaluated analytically, form factors in the uv are accessible, retains essential elements of QFT

Collaborators:

- Craig Roberts, Argonne National Lab
- Adnan Bashir, University of Morelia, Mexico
- Trang Nguyen, PhD thesis, Kent State Univ, Ohio
- Konstantin Khitrin, student, Kent State Univ

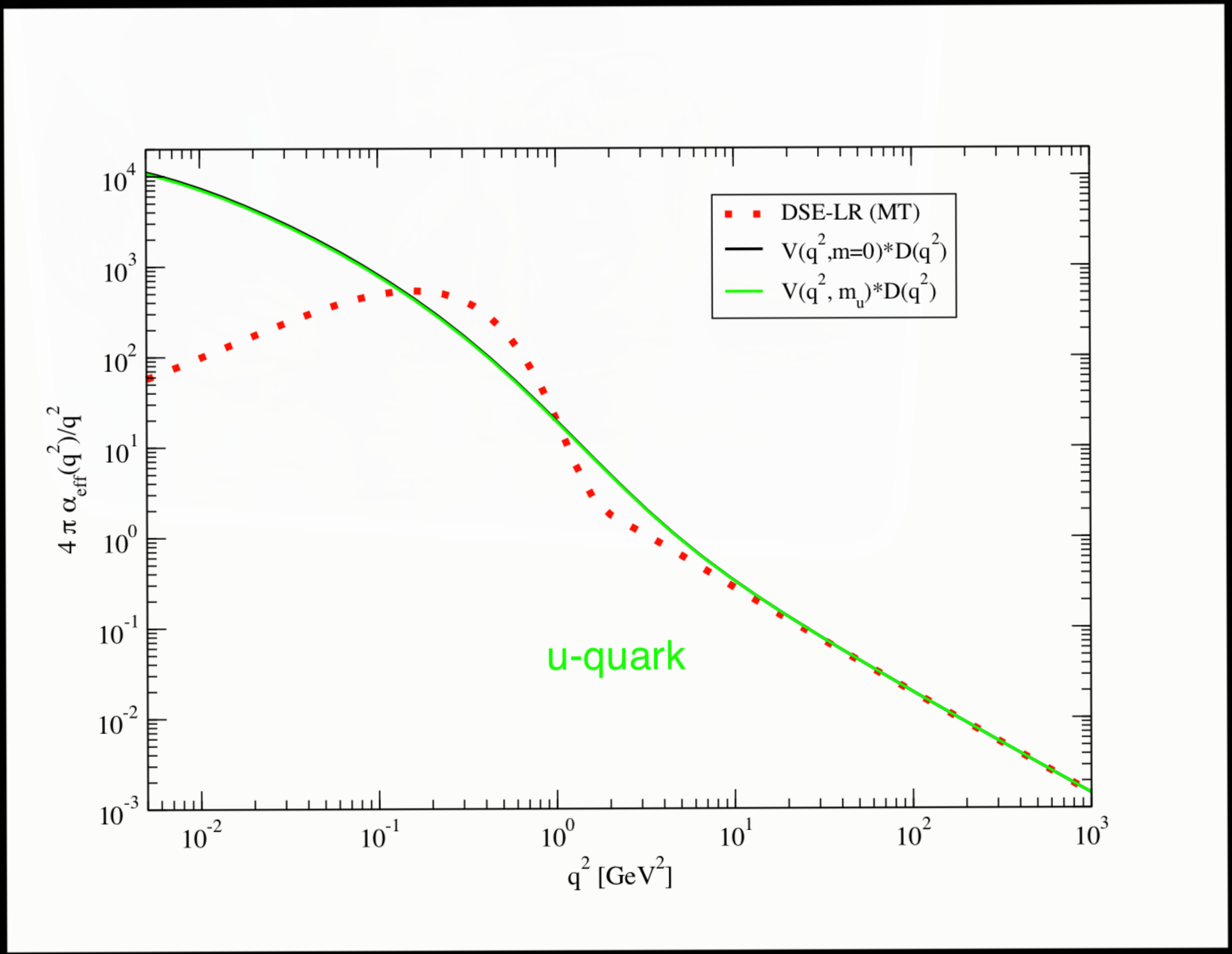
THE END

THANK YOU

Constraints on Modeling

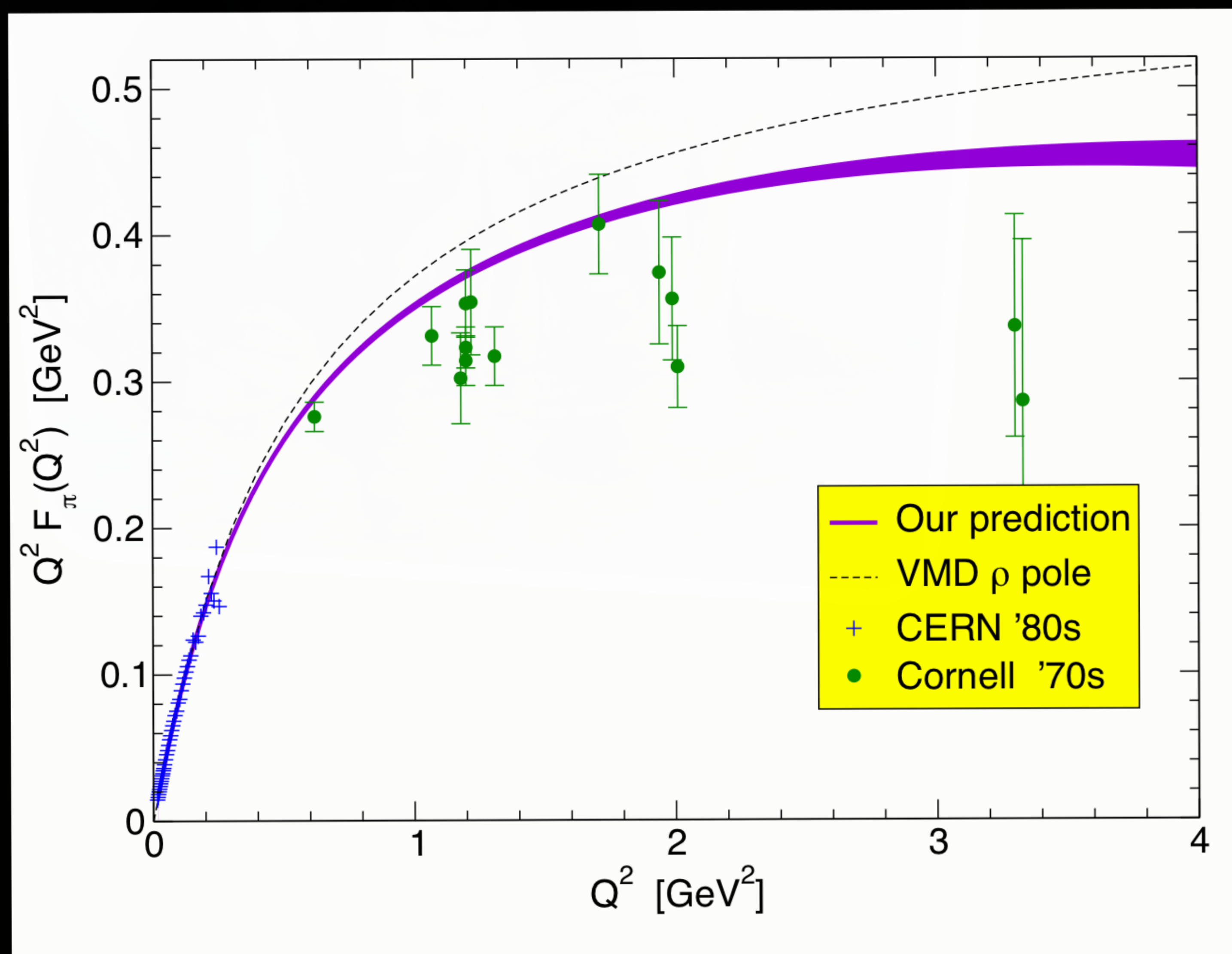
- Preserve vector WTI, and axial vector WTI
  E.g.
  $$-iP_\mu\Gamma_{5\mu}(k;P) = S^{-1}(k_+)\gamma_5\frac{\tau}{2} + \gamma_5\frac{\tau}{2}S^{-1}(k_-) - 2\,m_q(\mu)\,\Gamma_5(k;P)$$
- $\Rightarrow$ kernels of DSE$_q$ and $K_{\rm BSE}$ are related
- Ladder-rainbow is the simplest implementation
- Goldstone Theorem preserved, ps octet masses good, indep of model details
- DCSB $\Rightarrow \pi$: $\Gamma^0_\pi(p^2) = \frac{i\gamma_5}{f^0_\pi}\ [\frac{1}{4}\,{\rm tr}\,S_0^{-1}(p^2)] \ + \cdots$
- Here, 1-body and 2-body systems are the same

Quenched lattice ⇒ m_q Depn of DSE Kernel



Bhagwat, Pichowsky, Roberts, Tandy, PRC68, 015203 (2003)

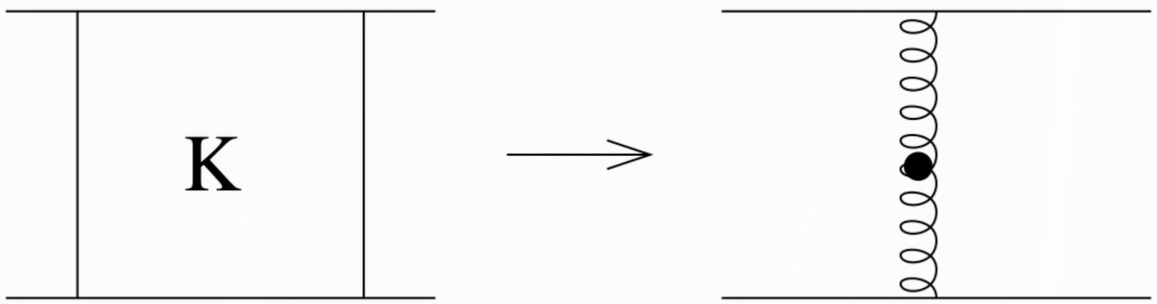
Pion electromagnetic form factor



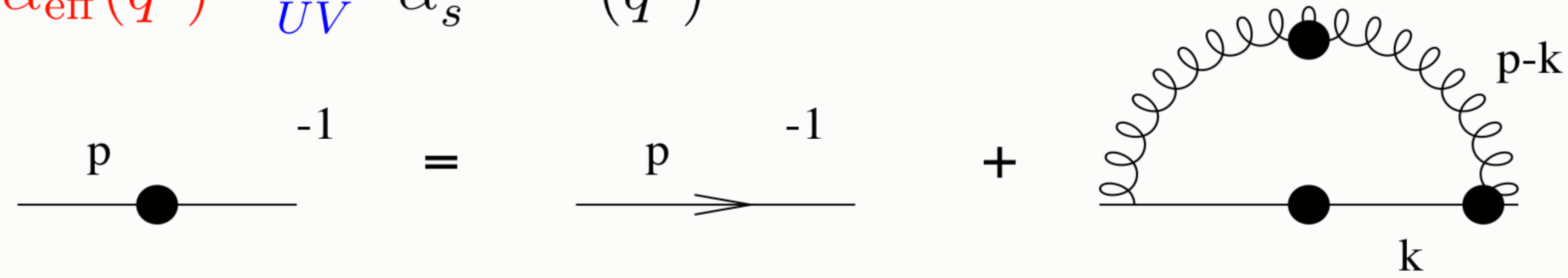
PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]

Ladder-Rainbow Model

Landau gauge only



- $K_{\text{BSE}} \to -\gamma_\mu \frac{\lambda^a}{2} \; 4\pi\alpha_{\text{eff}}(q^2) \; D^{\text{free}}_{\mu\nu}(q) \; \gamma_\nu \frac{\lambda^a}{2}$
- $\alpha_{\text{eff}}(q^2) \; \overrightarrow{IR} \; \langle \bar{q}q \rangle_{\mu=1\ \text{GeV}} = -(240\text{MeV})^3$, incl vertex dressing
- $\alpha_{\text{eff}}(q^2) \; \overrightarrow{UV} \; \alpha_s^{1-\text{loop}}(q^2)$



- P. Maris & P.C. Tandy, PRC60, 055214 (1999)
  $M_\rho, M_\phi, M_{K^\star}$ good to 5%, $f_\rho, f_\phi, f_{K^\star}$ good to 10%

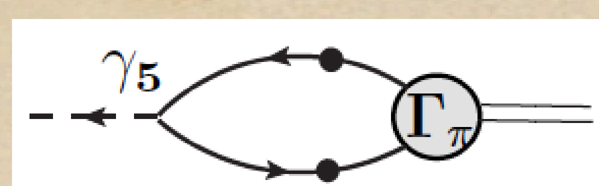
Exact Mass Relation for Flavor Non-Singlet PS Mesons

$$\mathbf{PCAC} \;\Rightarrow\; \langle\, \bar{\mathbf{q}}(\mathbf{x})\mathbf{q}(\mathbf{y})\; (\partial_\mu \mathbf{J}_{5\,\mu} = 2\mathbf{m_q}\, \mathbf{J}_5)\,\rangle \;\Rightarrow \mathbf{AV - WTI}:$$

$$-\mathbf{i}\mathbf{P}_\mu\, \boldsymbol{\Gamma}_{5\mu}(\mathbf{k};\mathbf{P}) = \mathbf{S}^{-1}(\mathbf{k}+\mathbf{P}/2)\; \gamma_5 \frac{\tau}{2} + \gamma_5 \frac{\tau}{2}\; \mathbf{S}^{-1}(\mathbf{k}-\mathbf{P}/2) - 2\mathbf{m_q}\, \boldsymbol{\Gamma}_5(\mathbf{k};\mathbf{P})$$

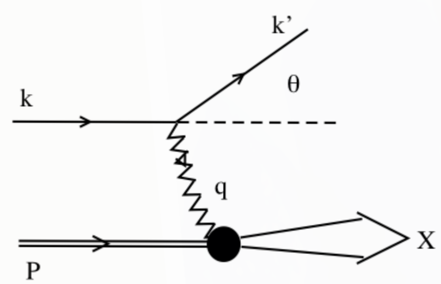
$$\boldsymbol{\Gamma}_\pi(\mathbf{k};\mathbf{P})\, \frac{\mathbf{f}_\pi\, \mathbf{P}_\mu}{\mathbf{P}^2 + \mathbf{m}_\pi^2} \qquad\qquad \boldsymbol{\Gamma}_\pi(\mathbf{k};\mathbf{P})\, \frac{\mathbf{i}\, \rho_\pi}{\mathbf{P}^2 + \mathbf{m}_\pi^2}$$

- $\mathbf{m_q} = 0: \quad \mathbf{S}_0^{-1}(\mathbf{k}) = \mathbf{i}\, \not{k}\mathbf{A}_0(\mathbf{k}^2) + \mathbf{B}_0(\mathbf{k}^2)$
- $\mathbf{m_q} = 0, \mathbf{P} = 0 \;\Rightarrow\; \mathbf{GT_q}: \;\; \boldsymbol{\Gamma}_\pi(\mathbf{k}^2;0) = \mathbf{i}\gamma_5\, \tau\, \frac{\mathbf{B}_0(\mathbf{k}^2)}{\mathbf{f}_\pi^0} + \cdots$ ie, Goldstone Thm
- $\mathbf{m_q} \neq 0: \;\Rightarrow\; \mathbf{f}_\pi\, \mathbf{m}_\pi^2 = 2\,\mathbf{m_q}\, \rho_\pi(\mathbf{m_q})$ [for all $\mathbf{m_q}$, all ps mesons]
- $\rho_{\mathrm{ps}}(\mu) = -\langle 0\,|\,\bar{\mathbf{q}}\,\gamma_5\,\mathbf{q}\,|\,\mathrm{ps}\rangle$



Maris, Roberts, PCT, Phys. Lett. B420, 267(1998) – –an exact result in QCD

Deep Inelastic Lepton Scattering

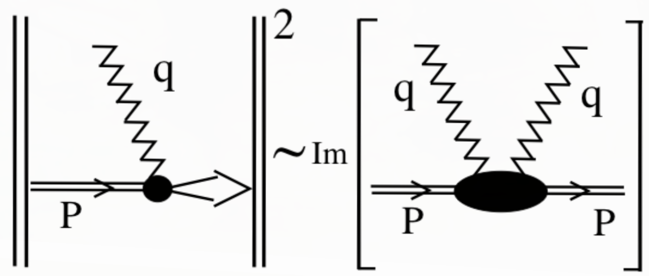


Bjorken limit:

$$\nu = q \cdot P/M \to \infty \;\; ; \;\; -q^2 = Q^2 \to \infty$$

$$0 < x = \frac{Q^2}{2P\cdot q} < 1$$

$W^{\alpha\beta} =$



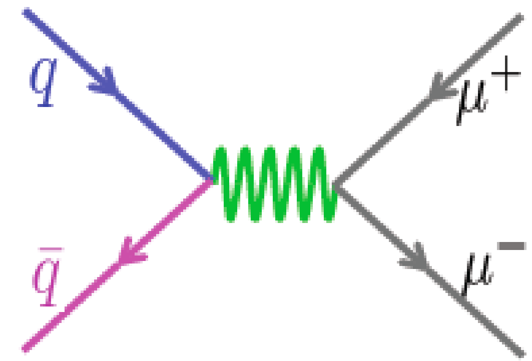
$= \frac{1}{2\pi}\,\mathrm{Disc}\,T^{\alpha\beta}(\nu)$

$$W^{\alpha\beta} = -(g^{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2})\,F_1 + \frac{P_T^\alpha(q)\,P_T^\beta(q)}{P\cdot q}\,F_2$$

$$F_1(x) = \Sigma_q \,\frac{e_q^2}{2} f_q(x) + \cdots$$

High x: Recent Development

Soft Gluon Resummation



$$\frac{d\sigma}{dQ^2 d\eta} = \sigma_0 \sum_{a,b} \int_{x_1^0}^{1} \frac{dx_1}{x_1} \int_{x_2^0}^{1} \frac{dx_2}{x_2} \left[q_a^\pi(x_1) q_b^p(x_2) \right.$$
$$\left. \times\, \omega_{ab}\left(x_1, x_1^0, x_2, x_2^0, Q/\mu\right)\right]$$

- ω_{ab} is hard scattering function
- Resum large logarithmic “soft” gluon contributions which arise as

$$z = \frac{Q^2}{\hat{s}} = \frac{\tau}{x_1 x_2} \to 1$$

- Accomplished with combined Mellin and Fourier transform of the cross section

  Aicher, Schäfer and Vogelsang, arXiv:1009.2481

- Refit of pion Drell-Yan data

---from Paul Reimer, 3rd Int Workshop on Nucleon Structure at High x

Qu-lattice $S(p), D(q)$ mapped to a DSE kernel

$$S(p) = Z(p)\,[i\,\not{p} + M(p)]^{-1}$$