The ghost-gluon vertex structure and the coupling at the τ-mass scale

by
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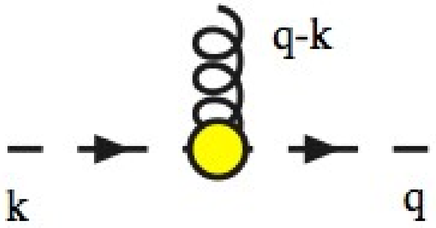
JLab; March 12-15, 2012; Newport (Virginia, USA)

Main characters:

The ghost-gluon coupling

The ghost-gluon vertex:

$$\widetilde{\Gamma}^{abc}_{\nu}(-q,k;q-k) =$$



$$= i g_0 f^{abc}\left(q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k)\right)$$

$$\widetilde{\Gamma}_R = \widetilde{Z}_1 \Gamma$$

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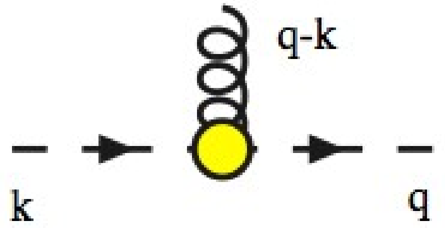

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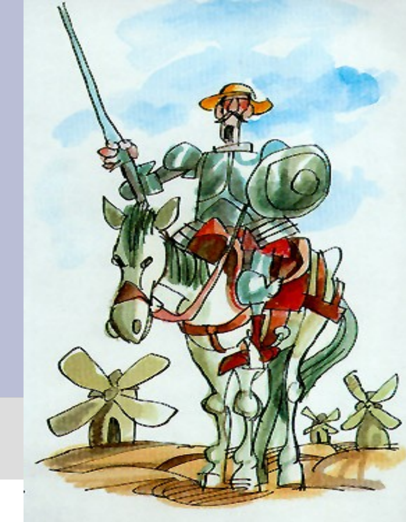

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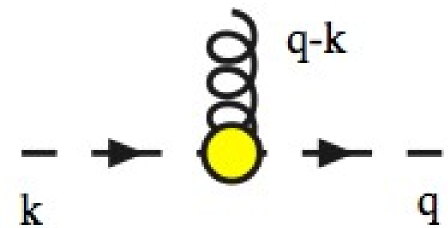

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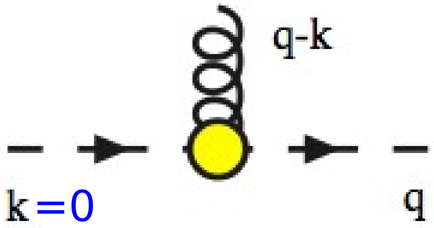

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Main characters:

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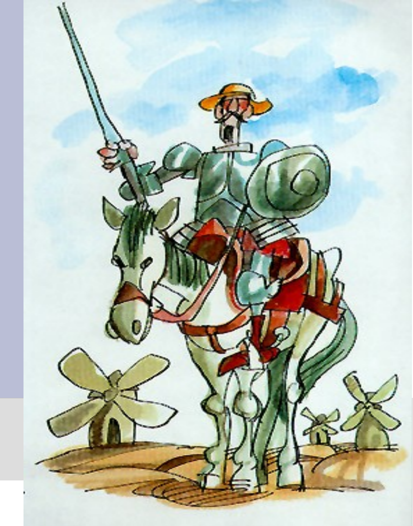

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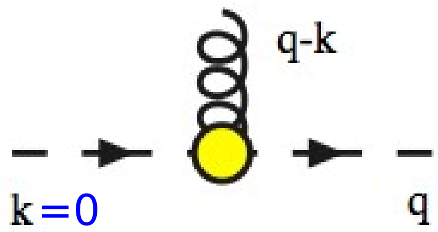

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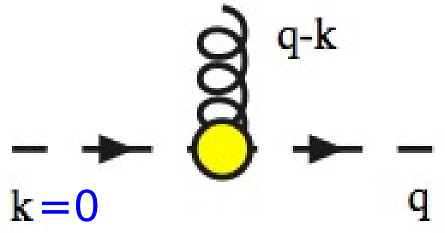

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In Taylor scheme & Landau gauge

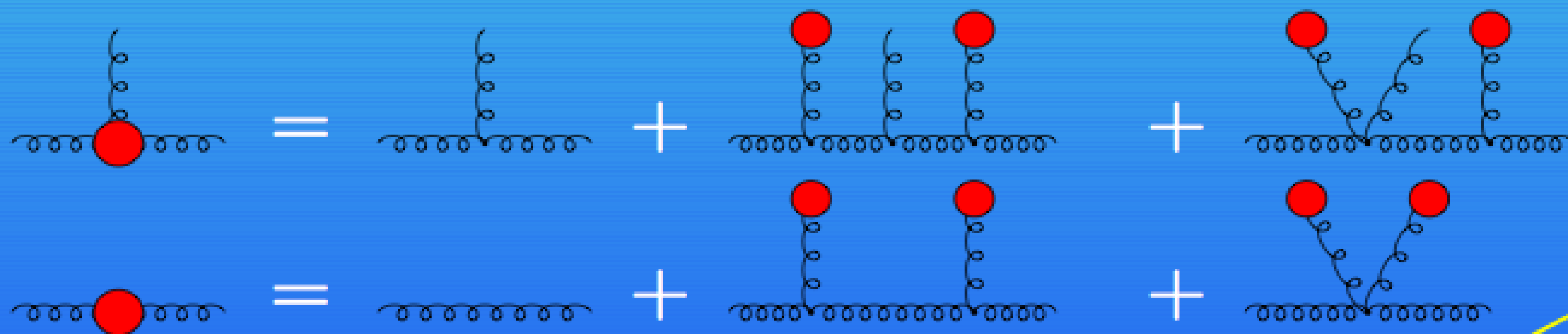
$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda^2)}{4\pi} Z_3(\mu^2,\Lambda^2)\widetilde{Z}_3^2(\mu^2,\Lambda^2)$$

Main characters:

The ghost-gluon coupling & gluon condensate

OPE + SVZ :

(PLB493(2000)315, PRD63(2001)114003)



$$\alpha^{NP}(q,\Lambda_{\overline{MS}}) = q^3 \frac{G_R^{(3)}(q,\mu)}{\left[G_R^{(2)}(q,\mu)\right]^{\frac{3}{2}}} =$$

$$= \boxed{\alpha\left(\ln\frac{q}{\Lambda_{\overline{MS}}}\right)\left(1+\frac{9g_R^2(\mu)\langle A^2\rangle_\mu}{4(N_C^2-1)}\frac{1}{q^2}\right)}$$

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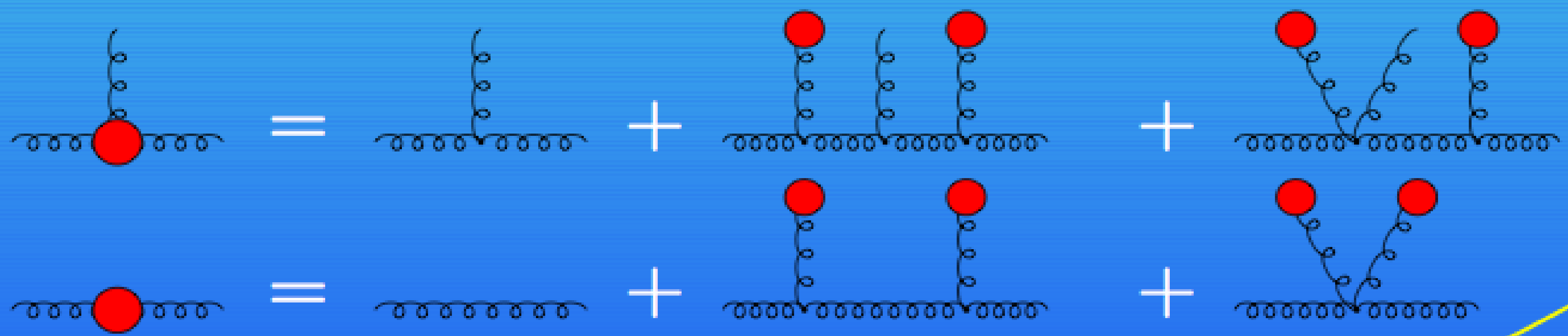

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(JHEP04(2003)005,PRD70(2005)114503)

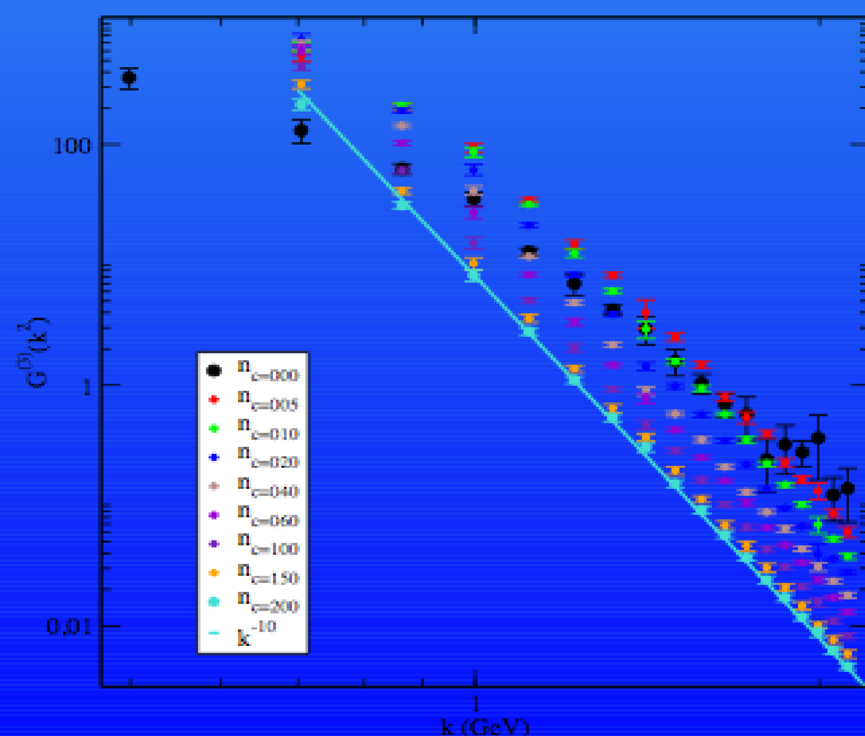
Instanton (BPST) $\rightarrow gA^a_\mu(x) = \dfrac{2\overline{\eta}^a_{\mu\nu} x^\nu \rho^2}{x^2(x^2+\rho^2)} \rightarrow$ $\langle A^2 \rangle \approx n \int d^4x \, A^a_\mu(x) A^a_\mu(x) = \dfrac{12 n \pi^2 \rho^2}{g^2}$

ILM Shuryak,D&P ...

$$\alpha_{\mathrm{MOM}}(q) = \frac{1}{18\pi n} q^4$$

$n = 5.27(4)\ \mathrm{fm}^{-4}$

$\rightarrow g^2_R \langle A^2 \rangle \simeq 2.3\ \mathrm{GeV}^2$

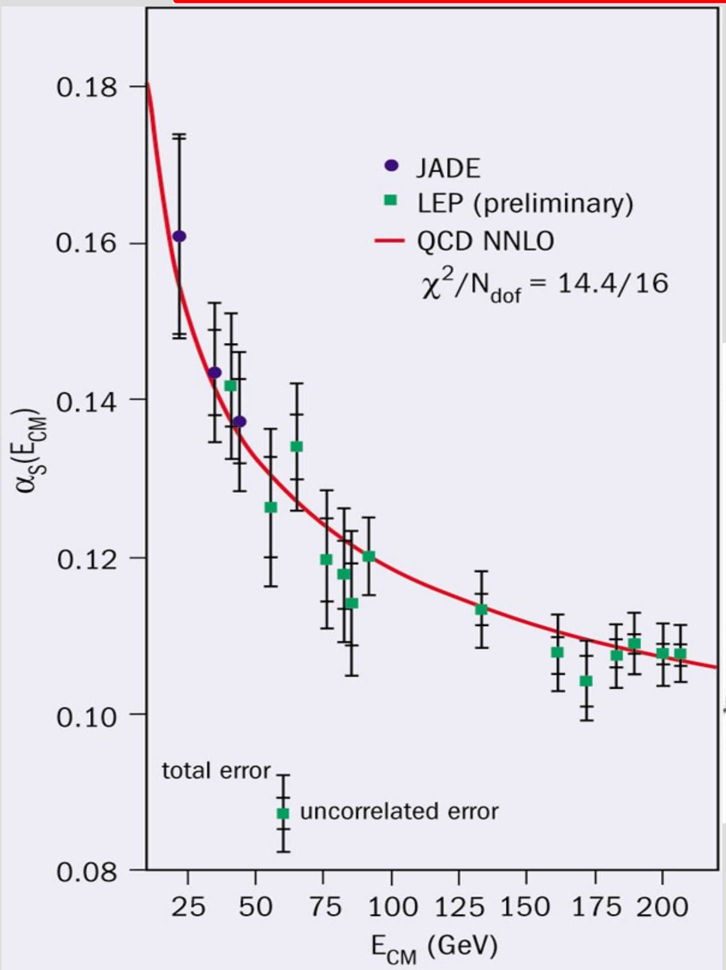


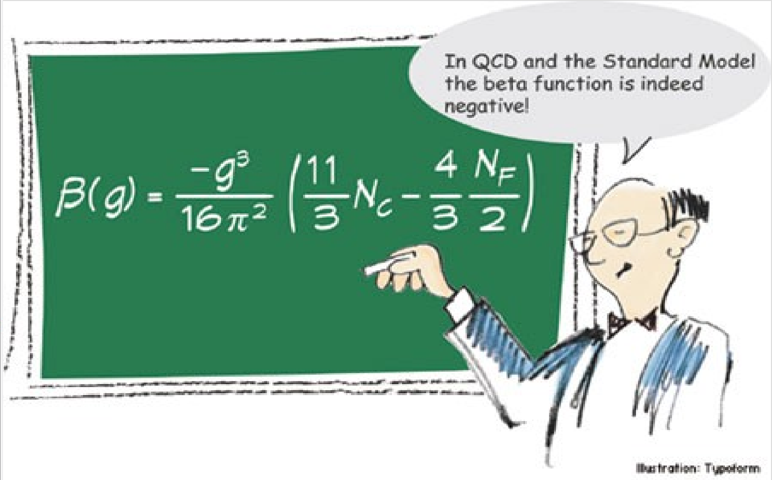
A task:
The running of ALPHA_s

$$\mathcal{L}_{QCD} = \sum_q \left(i\bar{q}\not{D}\, q - m_q \bar{q} q \right) - \frac{1}{4}(D \times A)^2$$

$$(D^\mu \times A^\nu)_a \equiv \partial^\mu A^\nu_a - \partial^\nu A^\mu_a + g \sum f_{abc}\, A^\mu_b A^\nu_c$$

$$\left[\lambda^a, \lambda^b\right] = i f_{abc}\, \lambda^c$$





A task:
The running of ALPHA$_s$

4-loops perturbation theory[3]: $p \gg \Lambda_{QCD}$

$$\alpha_T(\mu^2) = \frac{4\pi}{\beta_0 t}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\log(t)}{t} + \frac{\beta_1^2}{\beta_0^4}\frac{1}{t^2}\left(\left(\log(t) - \frac{1}{2}\right)^2 + \frac{\widetilde{\beta}_2\beta_0}{\beta_1^2} - \frac{5}{4}\right)\right)$$

$$+ \frac{1}{(\beta_0 t)^4}\left(\frac{\widetilde{\beta}_3}{2\beta_0} + \frac{1}{2}\left(\frac{\beta_1}{\beta_0}\right)^3\left(-2\log^3(t) + 5\log^2(t) + \left(4 - 6\frac{\widetilde{\beta}_2\beta_0}{\beta_1^2}\right)\log(t) - 1\right)\right), \; t = \ln\frac{\mu^2}{\Lambda_T^2}$$

$$\beta_T(\alpha_T) = \frac{d\alpha_T}{d\ln\mu^2} = -4\pi\sum_{i=0}\widetilde{\beta}_i\left(\frac{\alpha_T}{4\pi}\right)^{i+2}, \quad \frac{\Lambda_{\overline{MS}}}{\Lambda_T} = e^{-\frac{c_1}{2\beta_0}} = e^{-\frac{507-40N_f}{792-48N_f}} = 0.541449$$

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K.G. Chetyrkin, Nucl. Phys. B710 (2005) 499

A task:
The running of ALPHA$_s$
… from the lattice!!!

Lattice: $\frac{1}{L^2} \ll p^2 \ll \frac{1}{a^2}$

Propagators in Landau gauge

$$\left(G^{(2)}\right)^{ab}_{\mu\nu}(p^2,\Lambda) = \frac{G(p^2,\Lambda)}{p^2}\,\delta_{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$\left(F^{(2)}\right)^{a,b}(p^2,\Lambda) = -\delta_{ab}\,\frac{F(p^2,\Lambda)}{p^2}$$

Renormalized in non-perturbative MOM-scheme:

$$G_R(p^2,\mu^2) = \lim_{\Lambda\to\infty} Z_3^{-1}(\mu^2,\Lambda)\,G(p^2,\Lambda)$$

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… from the lattice!!!

Ghost and gluon on the lattice

Landau gauge

$$F_U[g] = \mathrm{Re}\left[\sum_x \sum_\mu \mathrm{Tr}\left(1 - \frac{1}{N} g(x) U_\mu(x) g^\dagger(x+\mu)\right)\right]$$

Gluon:

$$A_\mu(x + \hat{\mu}/2) = \frac{U_\mu(x) - U_\mu^\dagger(x)}{2iag_0} - \frac{1}{3}\mathrm{Tr}\left(\frac{U_\mu(x) - U_\mu^\dagger(x)}{2iag_0}\right)$$

$$\left(G^{(2)}\right)^{a_1 a_2}_{\mu_1 \mu_2}(p) = \langle A^{a_1}_{\mu_1}(p) A^{a_2}_{\mu_2}(-p)\rangle$$

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Ghost:

$$\left(F^{(2)}\right)^{ab}(x-y) \equiv \langle \left(M^{-1}\right)^{ab}_{xy} \rangle \,, \quad M(U) = -\frac{1}{N} \nabla \cdot \widetilde{D}(U)$$

$$\widetilde{D}(U)\eta(x) = \frac{1}{2}\left(U_\mu(x)\eta(x+\mu) - \eta(x)U_\mu(x) + \eta(x+\mu)U^\dagger_\mu - U^\dagger_\mu(x)\eta(x)\right)$$

A task:
The running of ALPHA$_s$
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European Twisted Mass Collaboration

Fermions: twisted-mass action

$$S_{\rm tm}^{\rm F} = a^4 \sum_x \left\{ \bar{\chi}_x \left[D_{\rm W} + m_0 + i\gamma_5 \tau_3 \mu_q \right] \chi_x \right\}$$

Gauge fields: tlSym action

$$S_g = \frac{\beta}{3} \sum_x \left(b_0 \sum_{\substack{\mu,\nu=1 \\ 1\le\mu<\nu}}^{4} \{1 - \mathbb{R}\mathrm{eTr}(U_{x,\mu,\nu}^{1\times1})\} + b_1 \sum_{\substack{\mu,\nu=1 \\ \mu\neq\nu}}^{4} \{1 - \mathbb{R}\mathrm{eTr}(U_{x,\mu,\nu}^{1\times2})\} \right) , \quad \beta \equiv 6/g_0^2$$

$(b_0 = 1 - 8b_1, b_1 = -1/12)$ + **Maximal twist :** $\mathcal{O}(a^2)$

$$\begin{array}{llll} V = 24^3 \times 48 & \beta = & 3.9 & \mu = 0.004,\ 0.0064,\ 0.010 \\ V = 32^3 \times 64 & \beta = & 4.05 & \mu = 0.003,\ 0.006,\ 0.008,\ 0.012 \\ & \beta = & 4.2 & \mu = 0.0065 \end{array}$$

Artefacts: $\mathcal{O}(a^2\Lambda_{QCD}^2)$, $\mathcal{O}(a^2p^2)$, $\mathcal{O}(a^2\mu^2)$

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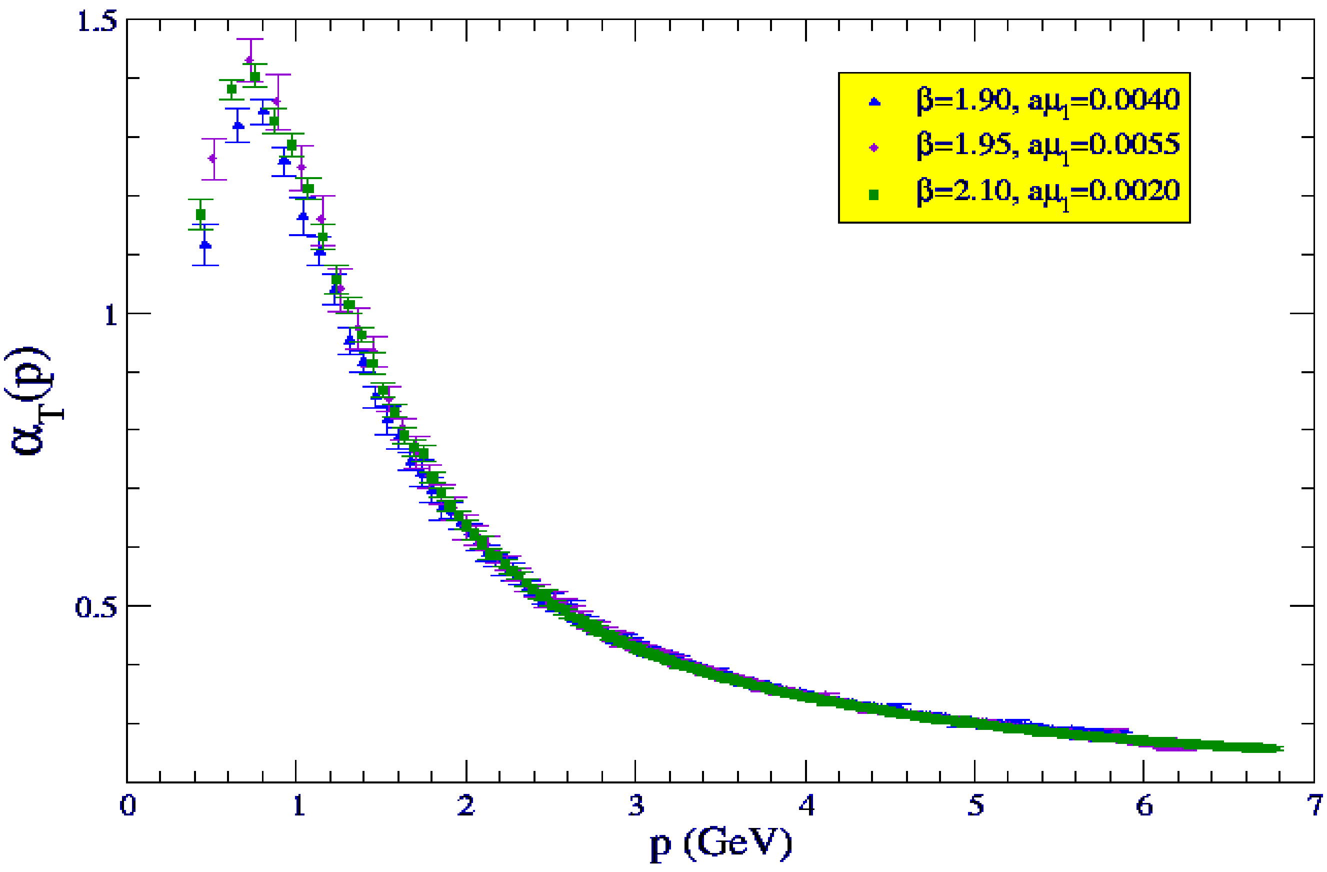
Nf=2+1+1: charmed sea quark included!!!



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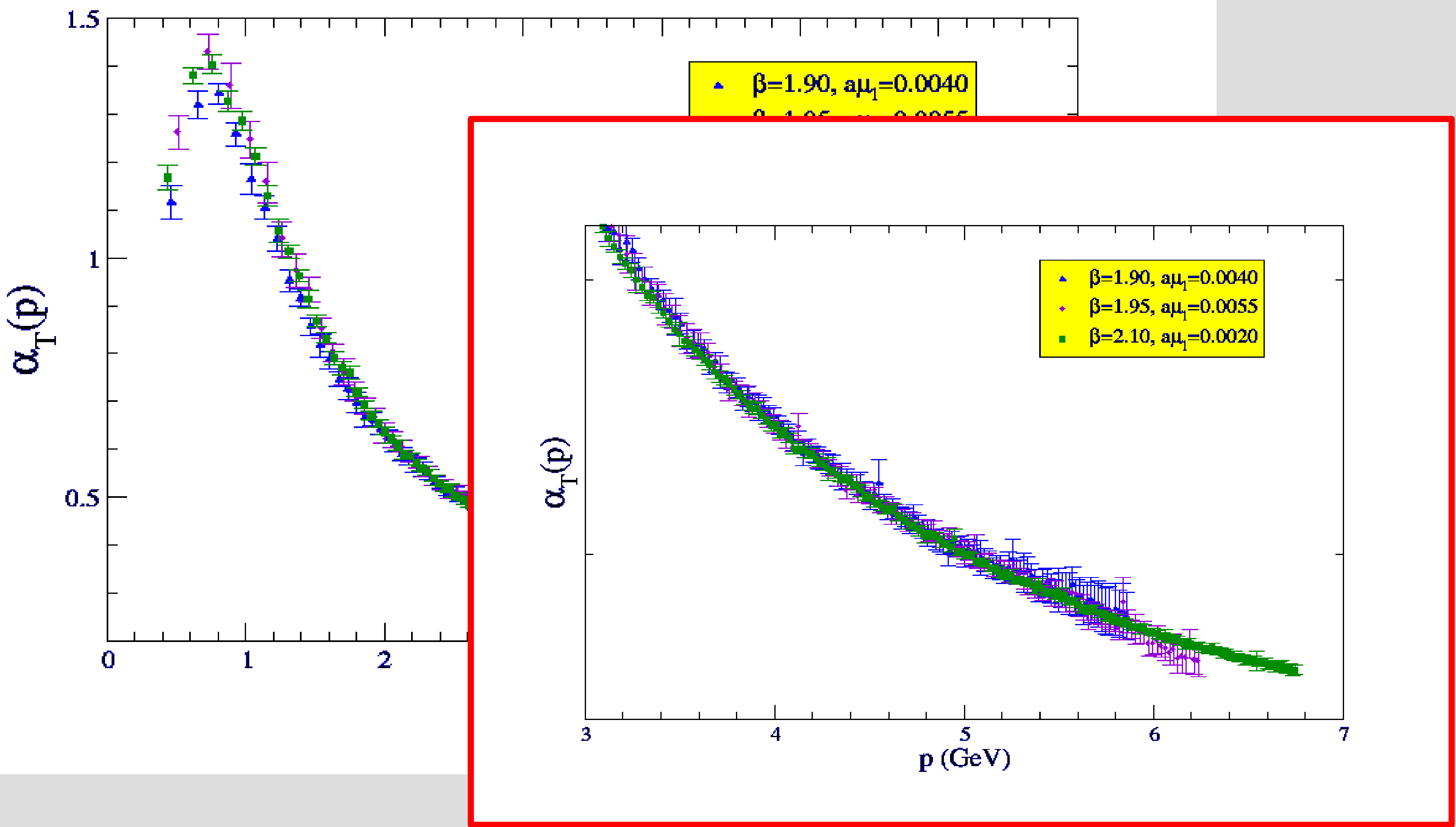

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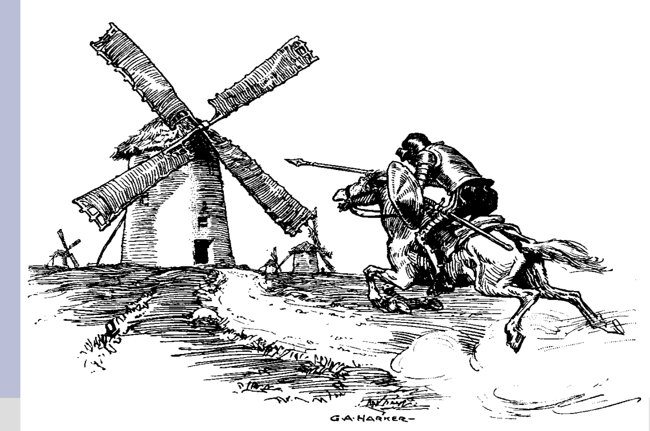

S. Bethke et al., arXiv:1110.0016

$$\alpha_{\overline{MS}}(M_\tau)=0.334(14)$$

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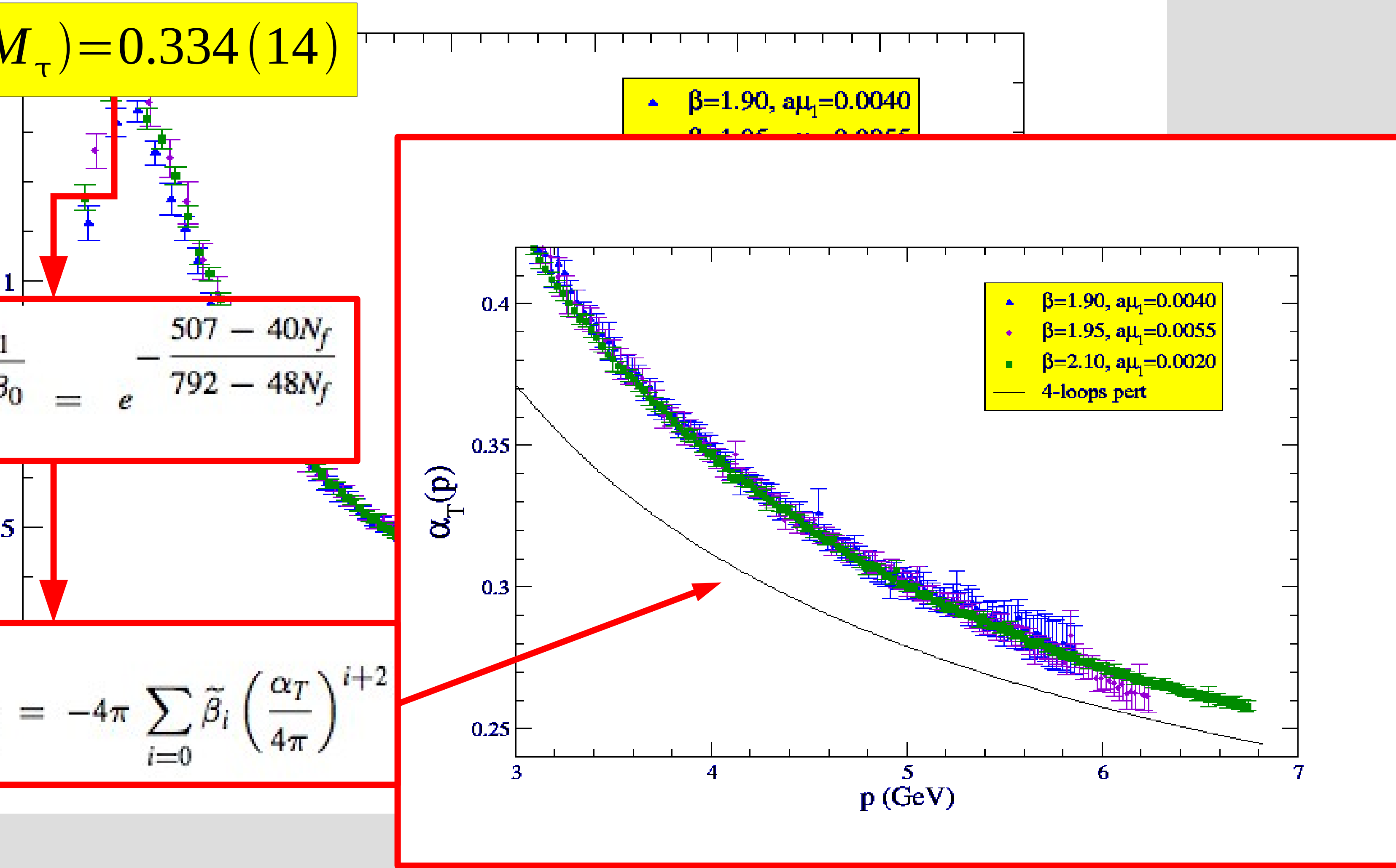

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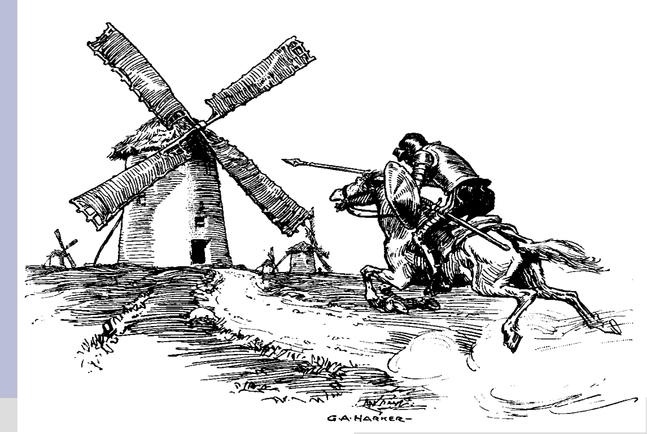

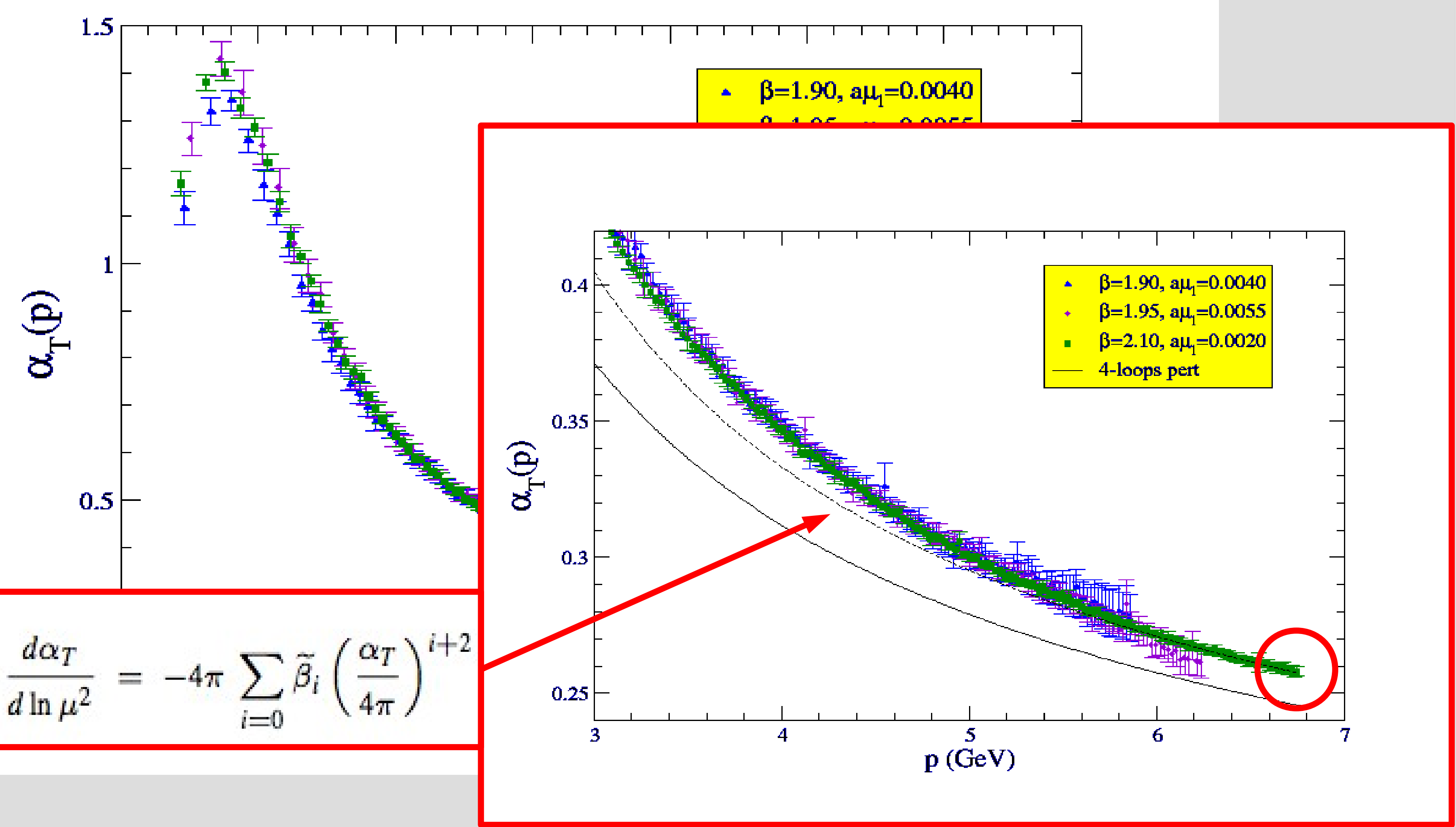

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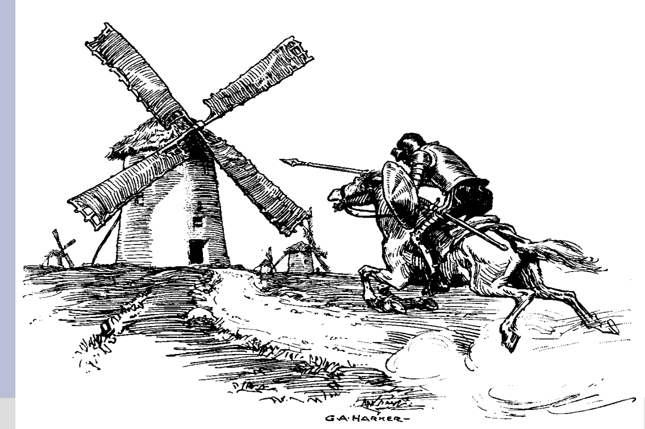
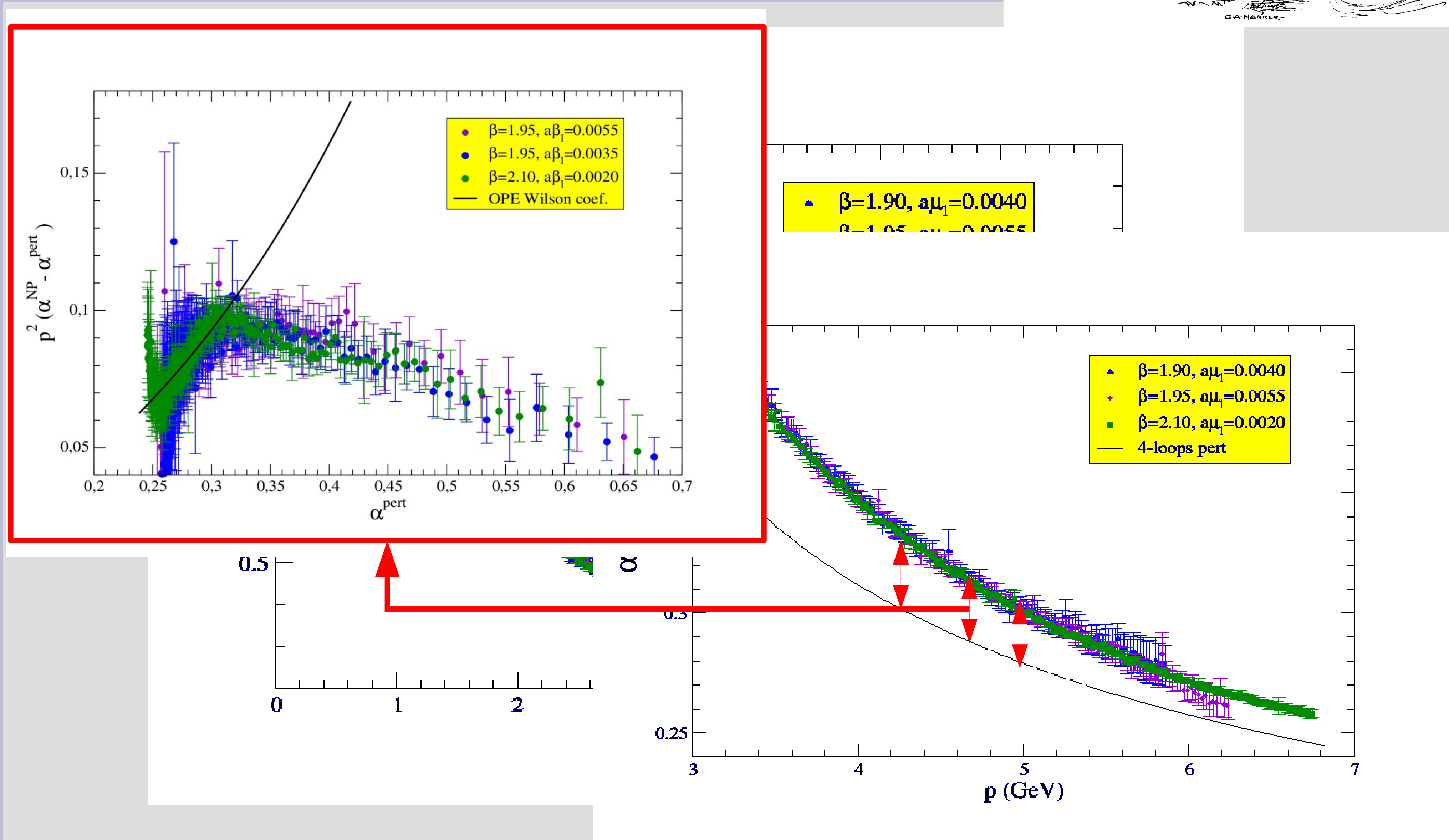


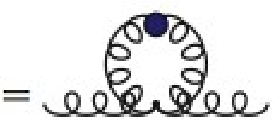
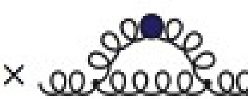


A task:

... then:

OPE power corrections

$$(F^{(2)})^{ab}(q^2) = (F^{(2)}_{\text{pert}})^{ab}(q^2) + w^{ab}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \ldots\,, \quad w^{ab} = 2\times$$ 

$$(G^{(2)})^{ab}_{\mu\nu}(q^2) = (G^{(2)}_{\text{pert}})^{ab}_{\mu\nu}(q^2) + w^{ab}_{\mu\nu}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \ldots\,, \quad w^{ab}_{\mu\nu} =$$  $+ 2\times$ 

$$F_R(q^2,\mu^2) = F_{R,\text{pert}}(q^2,\mu^2)\left(1+\frac{3}{q^2}\frac{g_R^2\langle A^2\rangle_{R,\mu^2}}{4(N_C^2-1)}\right)\,, \quad G_R(q^2,\mu^2) = G_{R,\text{pert}}(q^2,\mu^2)\left(1+\frac{3}{q^2}\frac{g_R^2\langle A^2\rangle_{R,\mu^2}}{4(N_C^2-1)}\right)$$

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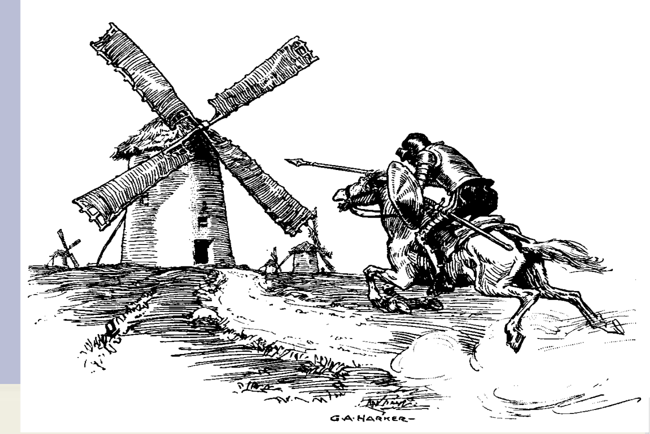

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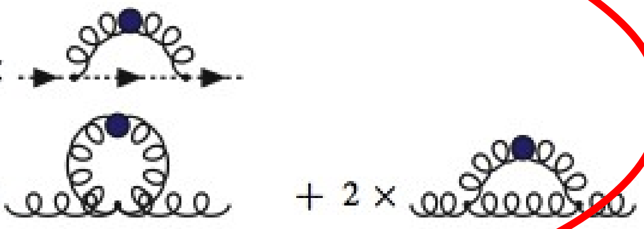
$$(G^{(2)})^{ab}_{\mu\nu}(q^2) = (G^{(2)}_{\rm pert})^{ab}_{\mu\nu}(q^2) + w^{ab}_{\mu\nu}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \dots, \quad w^{ab}_{\mu\nu} = \quad + 2\times$$



$$F_R(q^2,\mu^2) = F_{R,\rm pert}(q^2,\mu^2)\left(1+\frac{3}{q^2}\frac{g_R^2\langle A^2\rangle_{R,\mu^2}}{4(N_C^2-1)}\right), \quad G_R(q^2,\mu^2) = G_{R,\rm pert}(q^2,\mu^2)\left(1+\frac{3}{q^2}\frac{g_R^2\langle A^2\rangle_{R,\mu^2}}{4(N_C^2-1)}\right)$$

A task:

… then:

OPE power corrections

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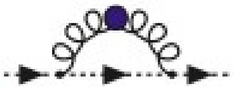

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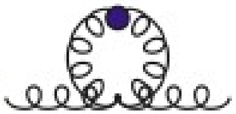
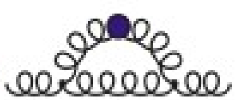


$$+\ 2\times$$



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Leading logarithm [4]: $\alpha_T(\mu^2) = \alpha_T^{\rm pert}(\mu^2)\left(1+\frac{9}{\mu^2}\left(\frac{\alpha_T^{\rm pert}(\mu^2)}{\alpha_T^{\rm pert}(q_0^2)}\right)^{1-\gamma_0^{A^2}/\beta_0}\frac{g_T^2(q_0^2)\langle A^2\rangle_{R,q_0^2}}{4(N_C^2-1)}\right)$

$$1-\gamma_0^{A^2}/\beta_0 = 1-\frac{105-8N_f}{132-8N_f} = \frac{9}{44-\frac{8}{3}N_f}$$

Ph. Boucaud et al, PRD79(2009)014508

A task:

… then:

OPE power corrections

$$(F^{(2)})^{ab}(q^2) = (F^{(2)}_{\rm pert})^{ab}(q^2) + w^{ab}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \ldots\,, \quad w^{ab} = 2\times$$

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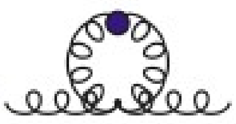
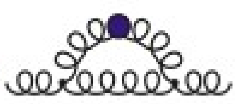


$$+\, 2\times$$



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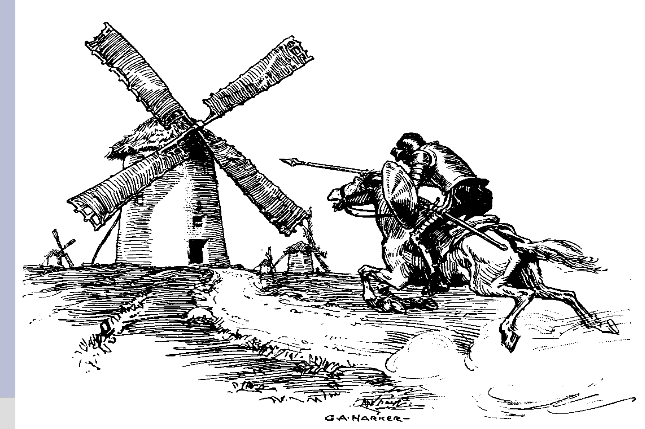
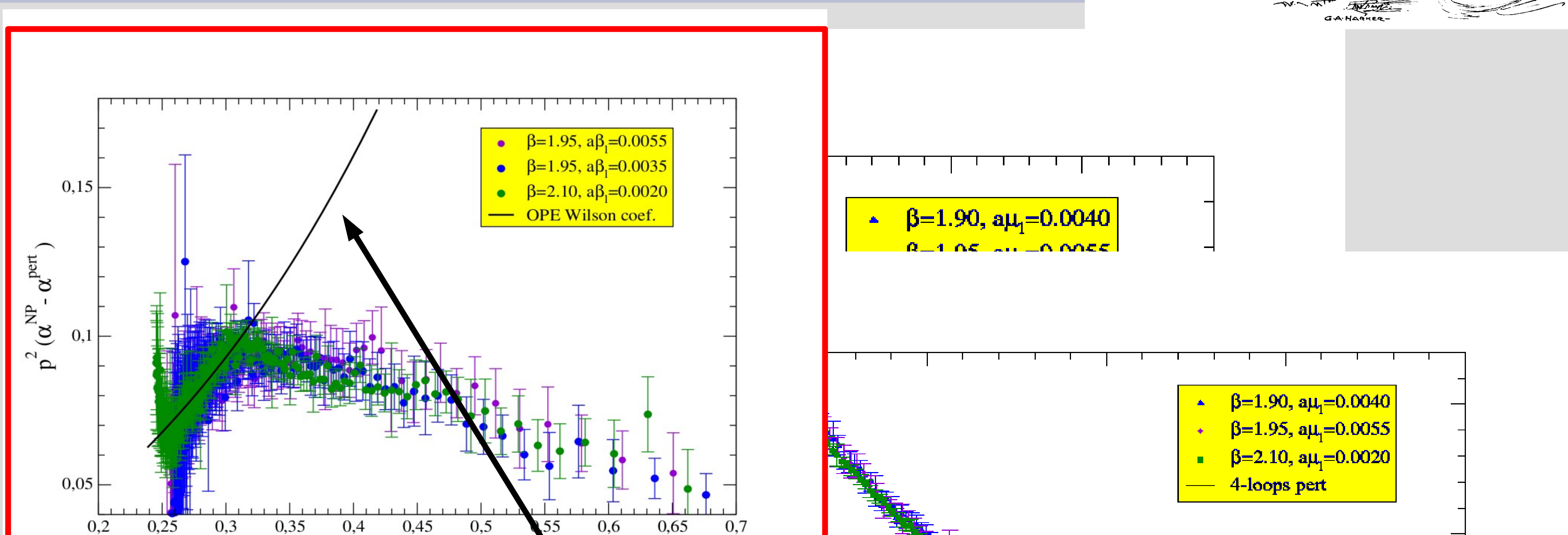
Chetyrkin & Maier, arXiv:0911.0594

J.A. Gracey, PLB552(2003)101 At the three-loop level !!!

A task:
… a gluon condensate is needed!!!





$$p^2\left(\alpha_T(p^2)-\alpha_T^{\text{pert}}(p^2)\right)=\frac{9g_T^2(q_0^2)\langle A^2\rangle_{R,q_0^2}}{4(N_C^2-1)}\,R\left(\alpha_T^{\text{pert}}(p^2),\alpha_T^{\text{pert}}(q_0^2)\right)\,\alpha_T^{\text{pert}}(q_0^2)\left(\frac{\alpha_T^{\text{pert}}(p^2)}{\alpha_T^{\text{pert}}(q_0^2)}\right)^{2-\gamma_0^{A^2}/\beta_0}$$

A task:

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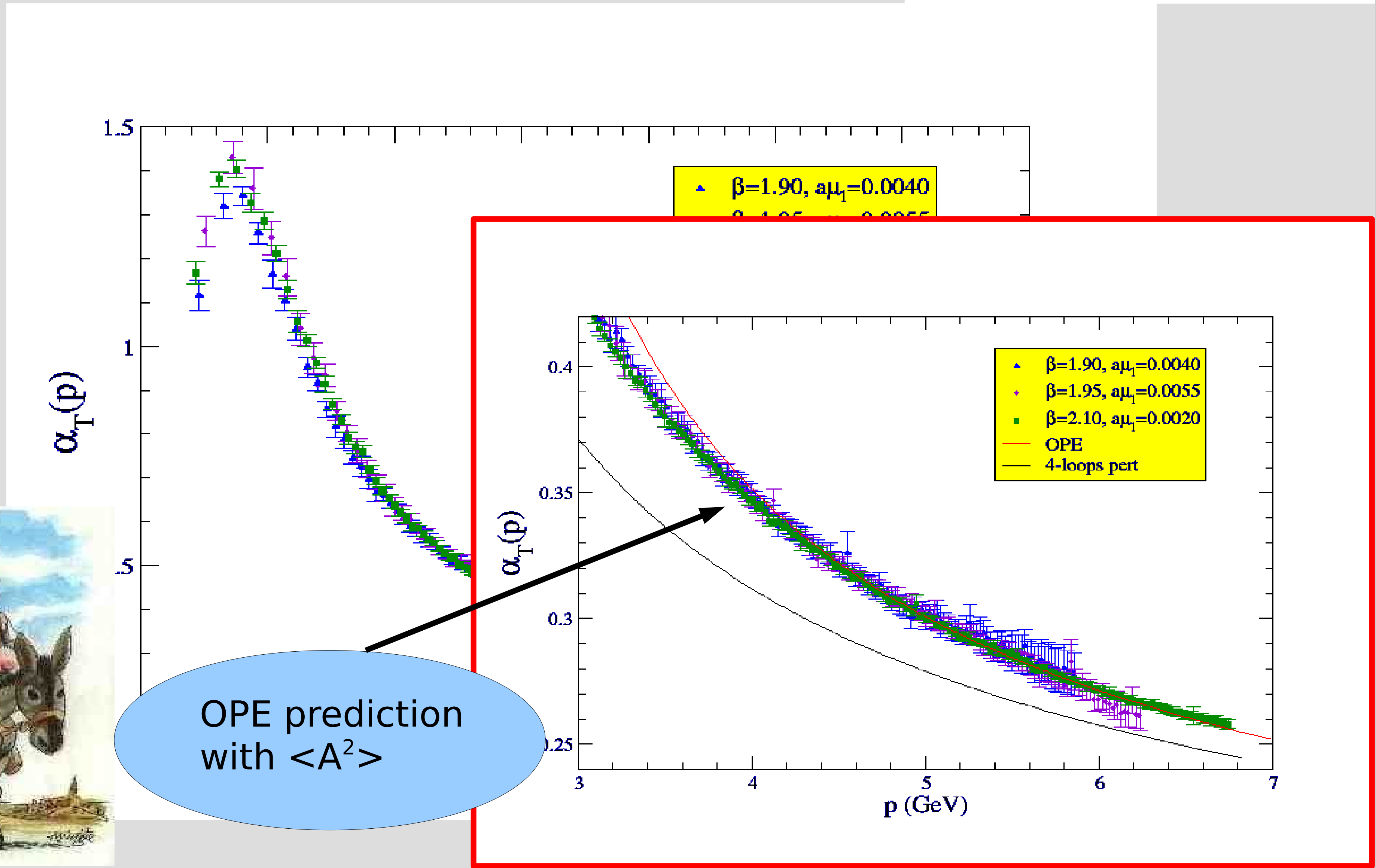
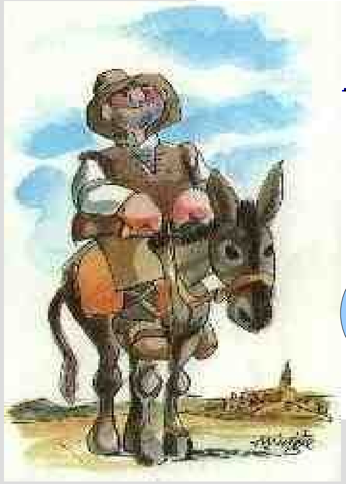
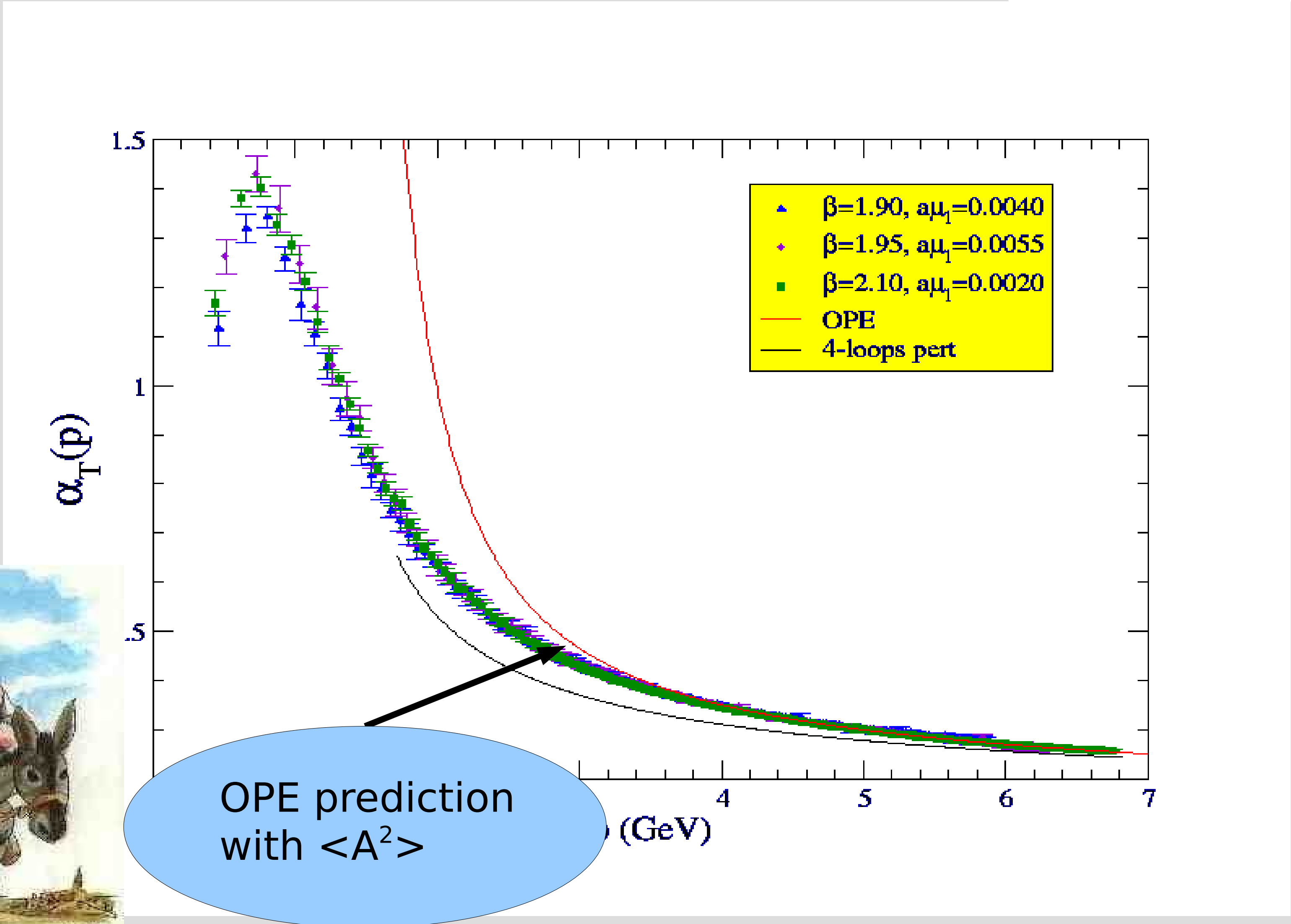
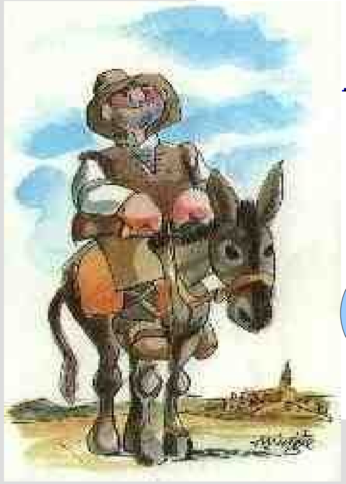




OPE prediction with <A^2>

A task:
… a gluon condensate is needed!!!





A task:

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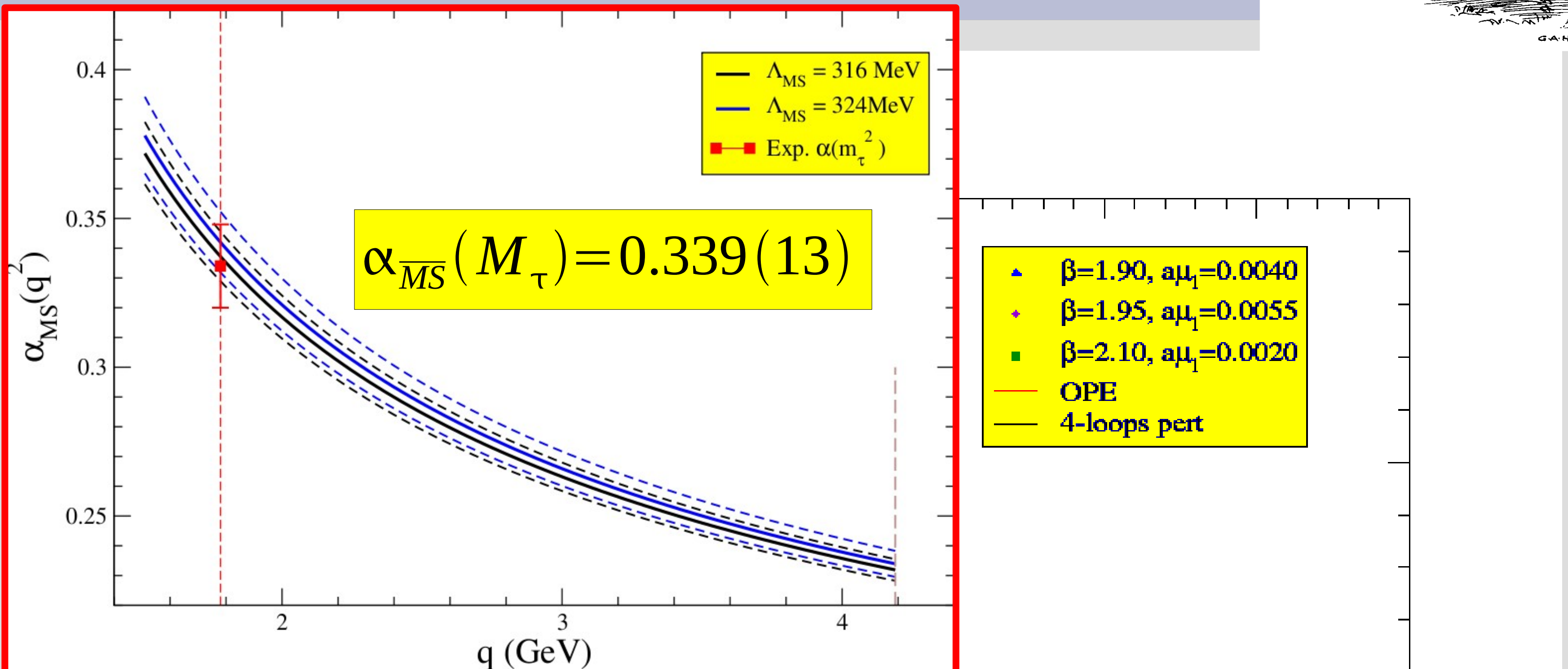

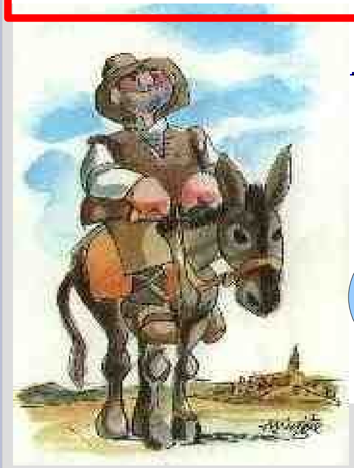

A task:
… a gluon condensate is needed!!!

A lattice regularized QCD action (including a charmed sea quark) with the physical scale fixed by:

$$f_\pi = 130.4(2)\,MeV$$

$$\alpha_{\overline{MS}}(M_\tau) = 0.339(13)$$

$$\alpha_{\overline{\mathrm{MS}}}^{N_f=5}(m_b) = \alpha_{\overline{\mathrm{MS}}}^{N_f=4}(m_b)\left(1 + \sum_n c_{n0}\left(\alpha_{\overline{\mathrm{MS}}}^{N_f=4}(m_b)\right)^n\right)$$

$$\alpha_{\overline{MS}}(M_{Z0}) = 0.1200(14)$$

A task:

… a gluon condensate is needed!!!

$\alpha_{\overline{MS}}(M_\tau)=0.339(13)$

$\alpha_{\overline{MS}}(M_\tau)=0.334(14)$

S. Bethke et al., arXiv:1110.0016 (tau decays)

A lattice regularized QCD action (including a charmed sea quark) with the physical scale fixed by:

$$f_\pi=130.4(2)MeV$$

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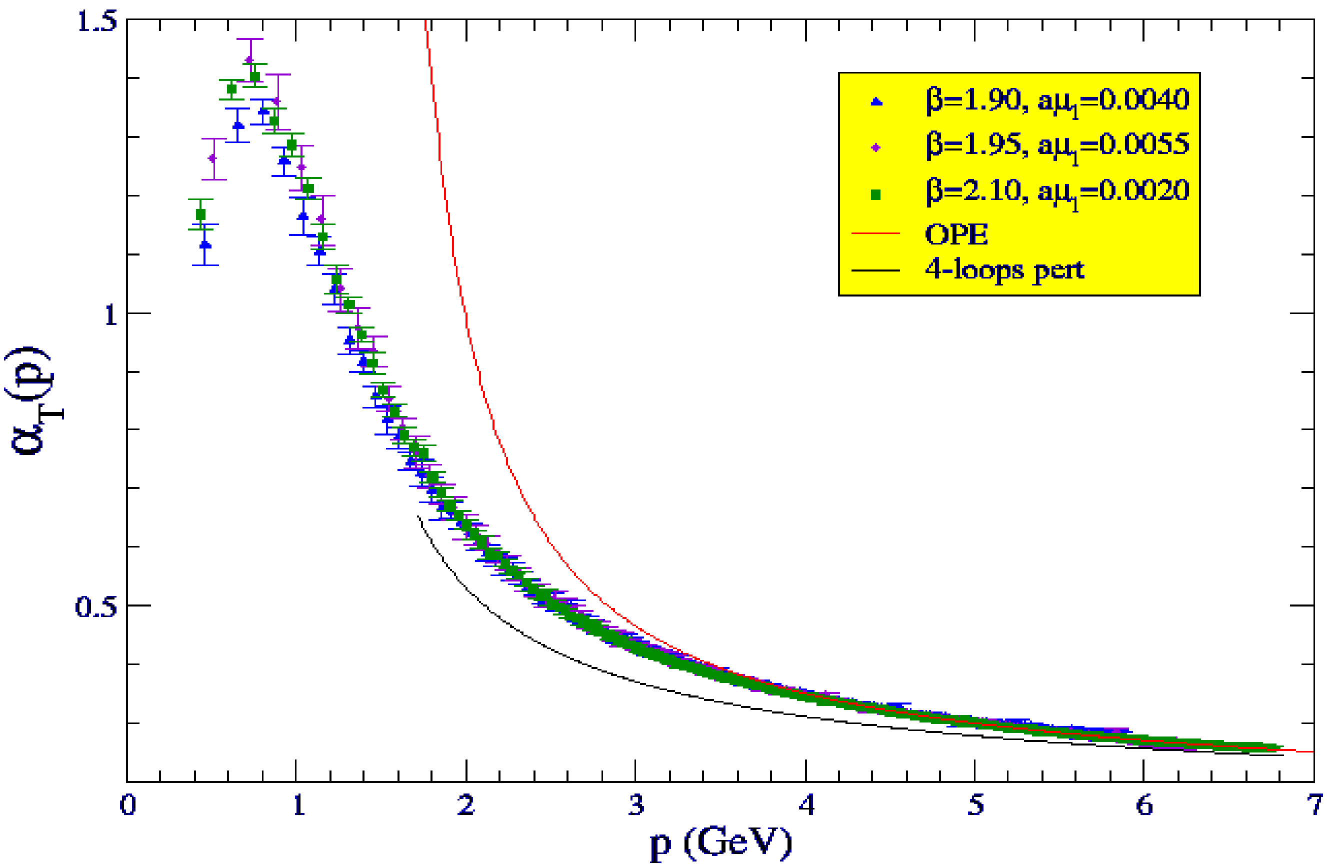
Eur. Phys. J. G 64:689(2009); “World average” (decays, scattering...)

$\alpha_{\overline{MS}}(M_{Z0})=0.1200(14)$

$\alpha_{\overline{MS}}(M_{Z0})=0.1186(11)$

A second task:
How can we describe better the lower momenta region?



A second task:
The GPDSE analysis

GPDSE:

$$\frac{1}{F(k^2)} = 1 + \alpha_T(\mu^2) \int d^4q \, K(k,q) H_1^{\text{bare}}(q,k) F(q^2)$$

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda^2)}{4\pi} Z_3(\mu^2,\Lambda^2)\widetilde{Z}_3^2(\mu^2,\Lambda^2)$$

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The usual approximation
Is to take this to be 1

Lattice inputs!!!

(Cfr. JHEP06(2008)012)

A second task:
The GPDSE analysis

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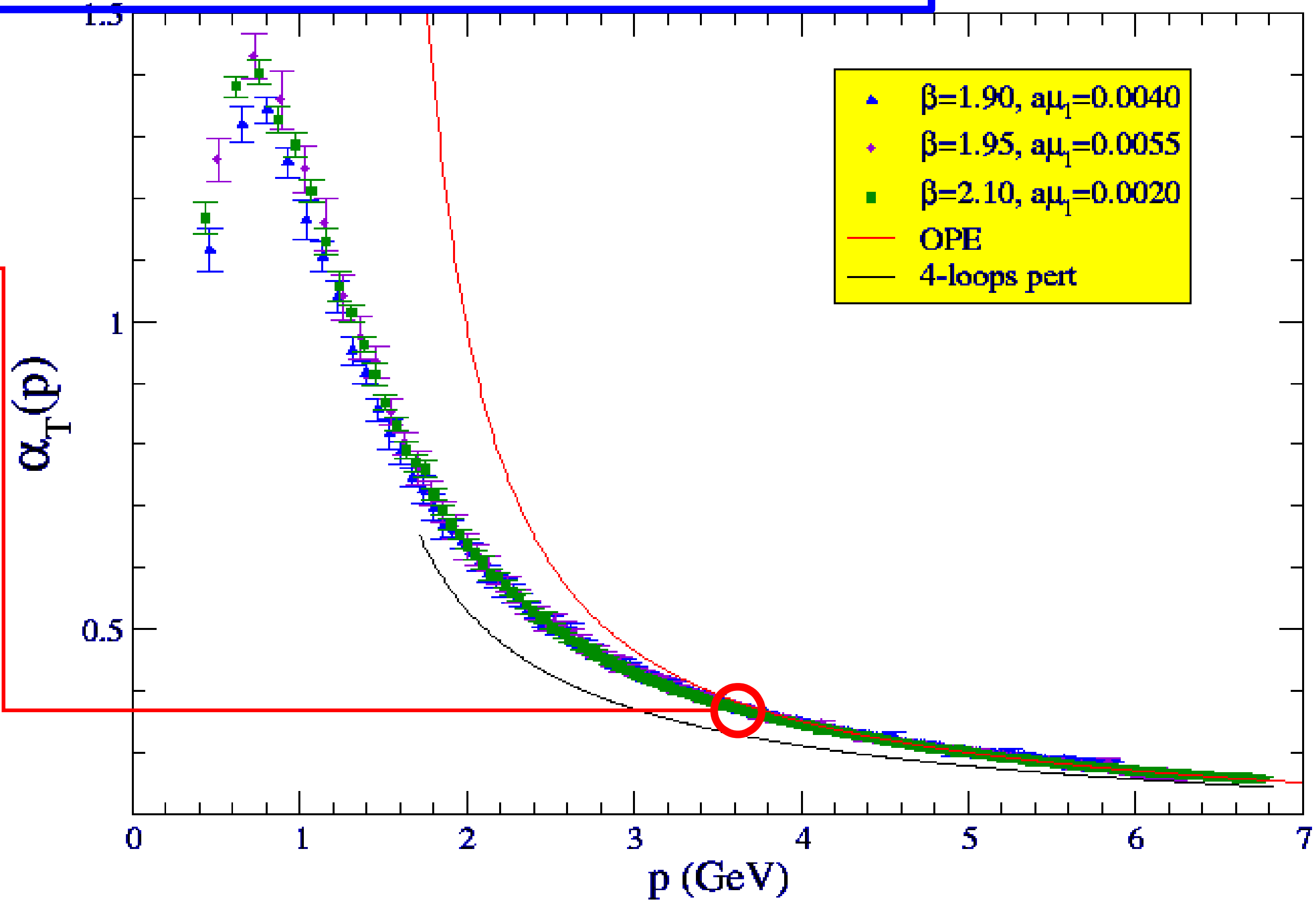
Lattice input



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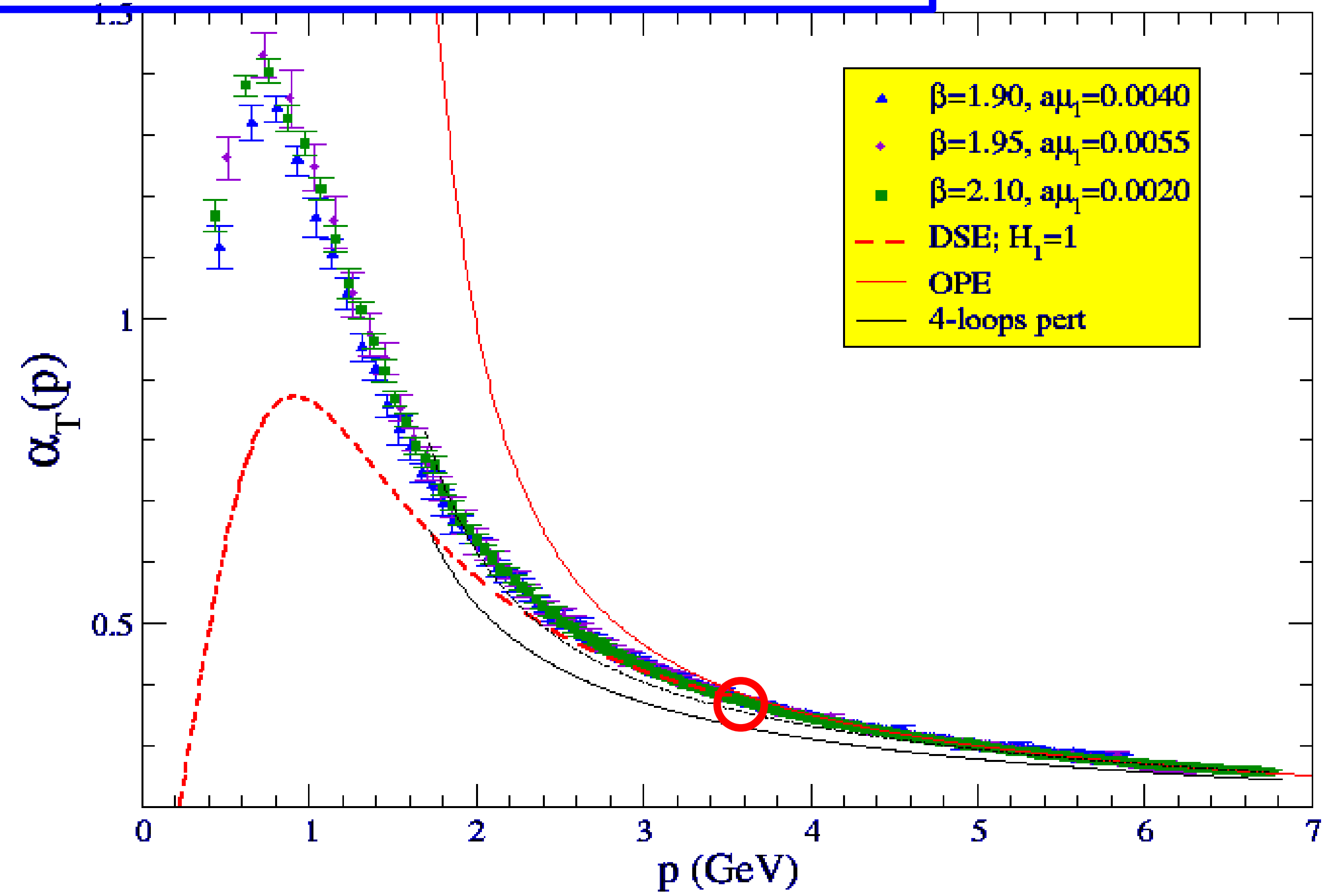
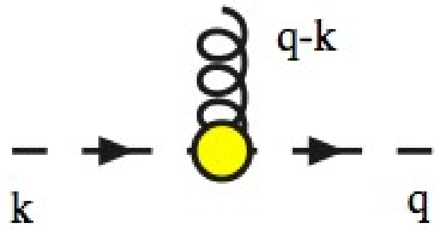


A second task:
OPE+SVZ analysis of the ghost-gluon vertex

$$\widetilde{\Gamma}^{abc}_{\nu}(-q,k;q-k) =$$



$$= ig_0 f^{abc}\left(q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k)\right)$$

$$\widetilde{\Gamma}_R = \widetilde{Z}_1 \Gamma$$

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

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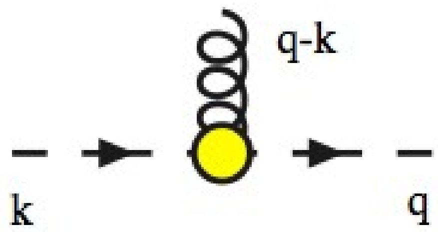

$$= i g_0 f^{abc}\left(q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k)\right)$$

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OPE+SVZ analysis of the ghost-gluon vertex

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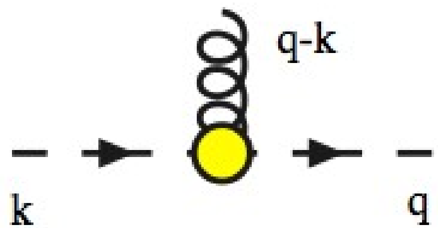

$$= ig_0 f^{abc}\left(q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k)\right)$$

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Ghost Prop. DSE:

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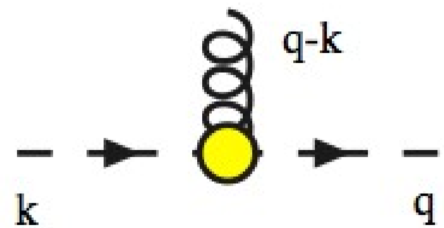

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A second task:
OPE+SVZ analysis of the ghost-gluon vertex

The ghost-antighost-gluon Green function:

$$\begin{aligned} V_\mu^{abc}(-q,k;q-k) &= \Gamma_{\mu'}^{a'b'c'}(-q,k;q-k)\; G_{\mu\mu'}^{bb'}(q-k)\; F^{aa'}(q)\; F^{cc'}(k) \\ &= \int d^4y\; d^4x\; e^{i(q-k)\cdot x}\; e^{ik\cdot y} \langle\, T\left(c^c(y) A_\mu^b(x) \overline{c^a}(0)\right)\, \rangle \end{aligned}$$

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

The ghost-antighost-gluon Green function:

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\begin{aligned}
V_\mu^{abc}(-q,k;q-k) &= \Gamma_{\mu'}^{a'b'c'}(-q,k;q-k)\; G^{bb'}_{\mu\mu'}(q-k)\; F^{aa'}(q)\; F^{cc'}(k) \\
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\end{aligned}
$$

OPE expansion:

$$
\begin{aligned}
V_\mu^{abc}(-q,k;q-k) &= (d_0)^{abc}_{\mu}(q,k) \\
&+ (d_2)^{abc\mu'\nu'}_{\mu a'b'}(q,k) \langle : A^{a'}_{\mu'}(0) A^{b'}_{\nu'}(0) : \rangle + \cdots
\end{aligned}
$$

SVZ sum-rules:

$$
\begin{aligned}
w^{abc}_\mu &= (d_2)^{abc\mu'\nu'}_{\mu a'b'}(q,k)\; \delta^{a'b'} g_{\mu'\nu'} \\
&= 2I^{[1]} + 2I^{[1]}_s + 2I^{[2]} + 4I^{[3]} + I^{[4]} + 2I^{[5]}
\end{aligned}
$$

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

SVZ sum-rules:

$$
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OPE+SVZ analysis of the ghost-gluon vertex

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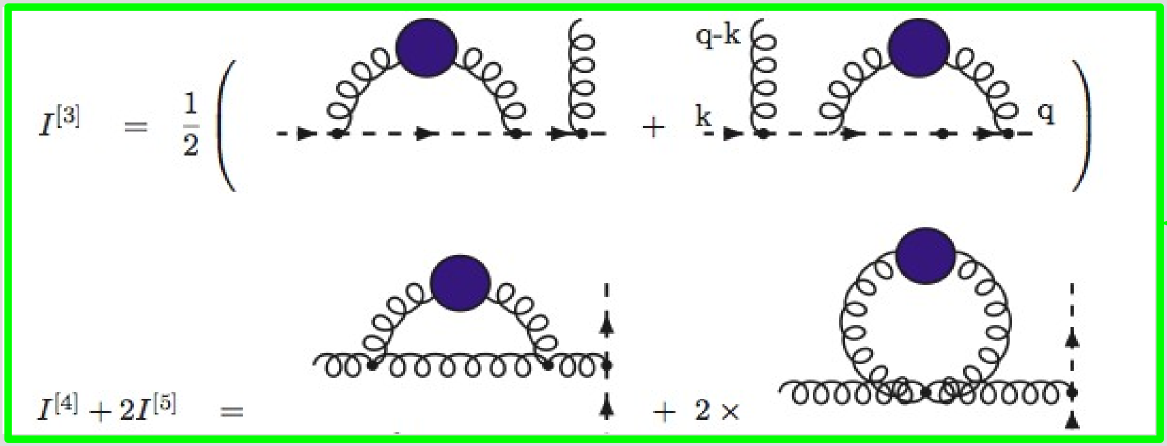

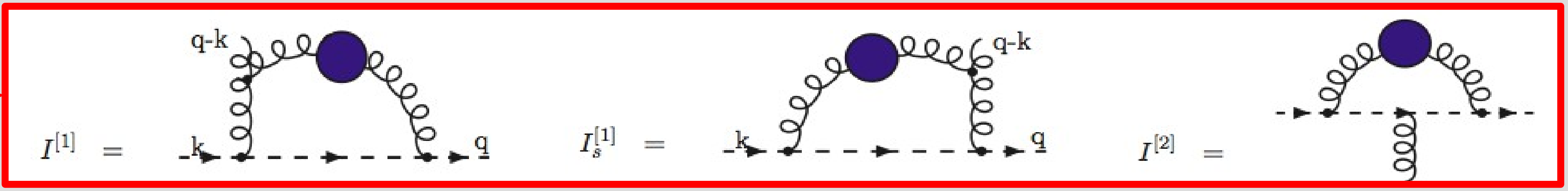

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

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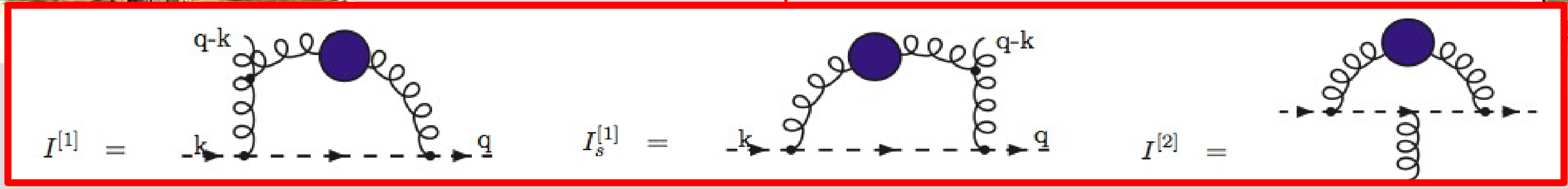
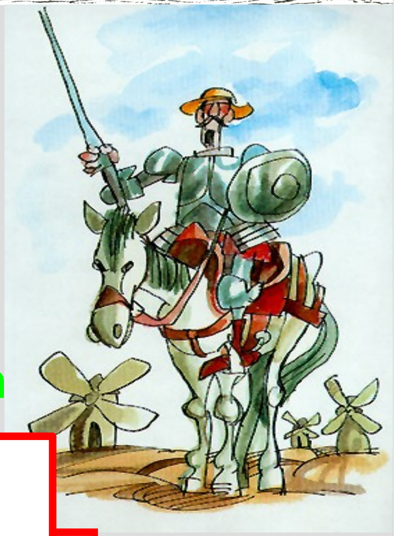
External legs amputation



A second task:
OPE+SVZ analysis of the ghost-gluon vertex



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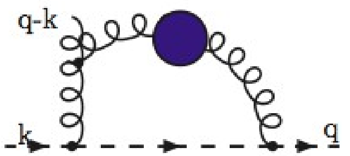
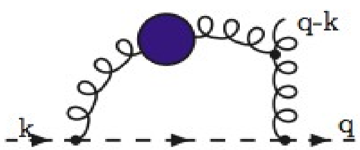
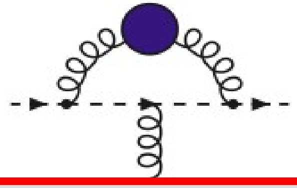
&= 2I^{[1]} + 2I^{[1]}_s + 2I^{[2]} + 4I^{[3]} + I^{[4]} + 2I^{[5]}
\end{aligned}
$$

External legs amputation

$I^{[1]} =$ 

$I^{[1]}_s =$ 

$I^{[2]} =$ 

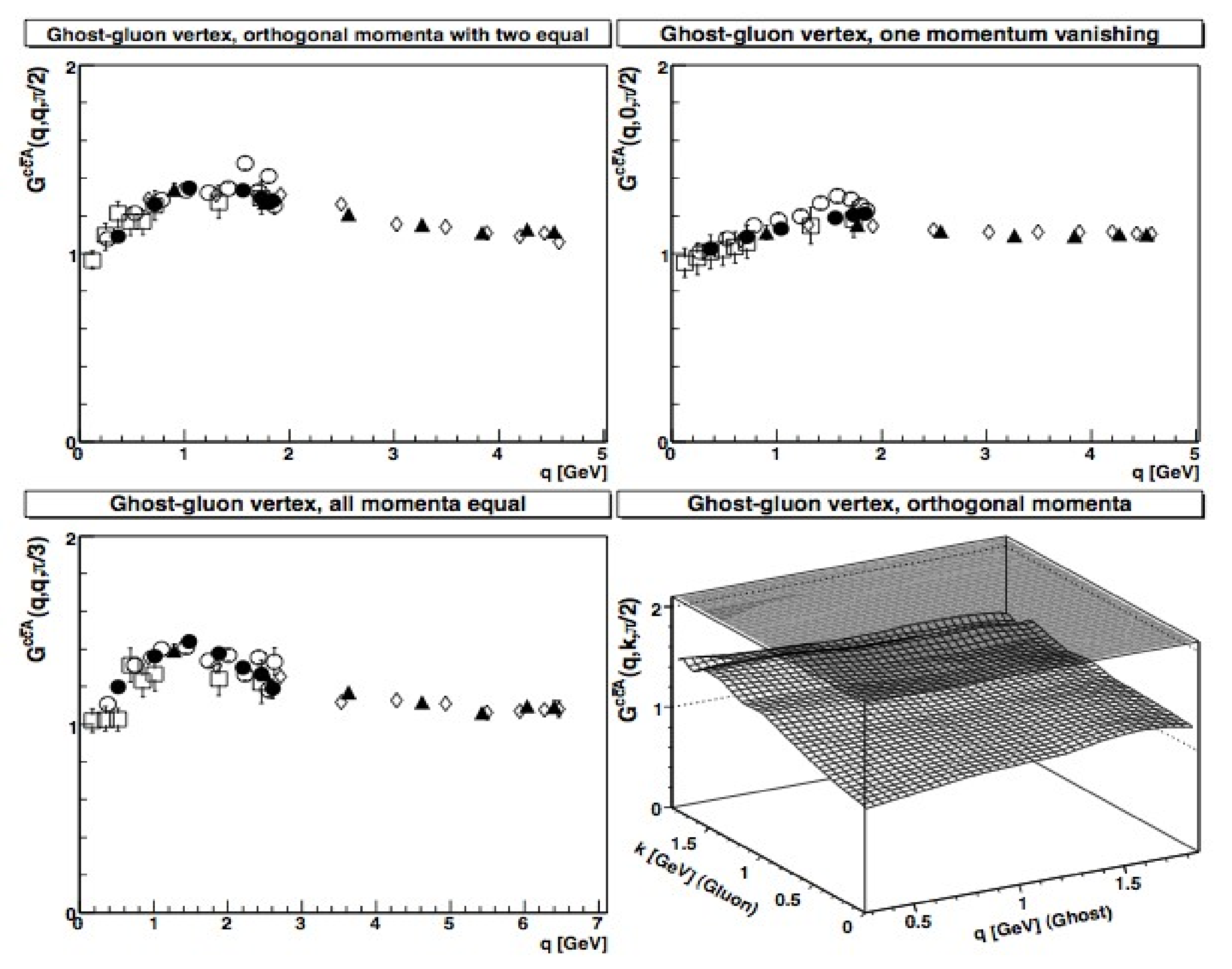
$$
H_1(q,k) = H_1^{\rm pert}(q,k)\left(1 + s_V(q,k)\,\frac{\langle A^2\rangle}{4(N_C^2-1)} + \mathcal{O}(g^4, q^{-4}, k^{-4}, q^{-2}k^{-2})\right)
$$

$$
s_V(q,k) = \frac{N_C}{2}\left(2\,\frac{(q-k)\cdot q}{q^2(q-k)^2} + 2\,\frac{(k-q)\cdot k}{k^2(q-k)^2} + \frac{k\cdot q}{k^2 q^2}\right)
$$

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

SU(2) Lattice results to compare with:

(Phys.Rev.D77:094510,2008)



Three kinematical configurations:

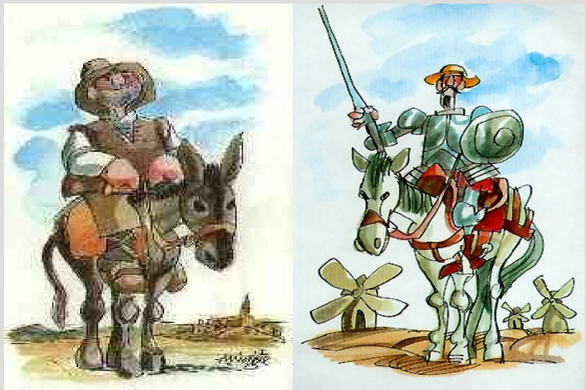
- q-k=0
- $q^2=k^2$; ang(q-k,q)=pi/2
- $q^2=k^2$; ang(q-k,q)=pi/3

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

SU(2) Lattice results to compare with:

(Phys.Rev.D77:094510,2008)

$$
\begin{aligned}
H_1(q^2,k^2,\theta) \;=\; & \widetilde{Z}_1^{-1}\left[1+\frac{N_C g^2\langle A^2\rangle}{8(N_C^2-1)}\right.\\
& \times\left(\frac{\sqrt{k^2q^2}\cos\theta}{k^2q^2+m_{\rm IR}^4}+2\,\frac{q^2-\sqrt{k^2q^2}\cos\theta}{q^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\rm IR}^4}\right.\\
& \left.\left.+\,2\,\frac{k^2-\sqrt{k^2q^2}\cos\theta}{k^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\rm IR}^4}\right)\right],
\end{aligned}
$$



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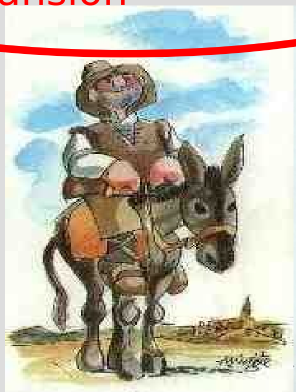
SU(2) Lattice results to compare with:

(Phys.Rev.D77:094510,2008)

$$H_1(q^2,k^2,\theta) = \widetilde{Z}_1^{-1}\left[1 + \frac{N_C g^2\langle A^2\rangle}{8(N_C^2-1)} \times \left(\frac{\sqrt{k^2q^2}\cos\theta}{k^2q^2+m_{\mathrm{IR}}^4} + 2\,\frac{q^2-\sqrt{k^2q^2}\cos\theta}{q^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\mathrm{IR}}^4} + 2\,\frac{k^2-\sqrt{k^2q^2}\cos\theta}{k^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\mathrm{IR}}^4}\right)\right],$$

Infrared mass scale to “regulate” the spurious infinities from the OPE expansion

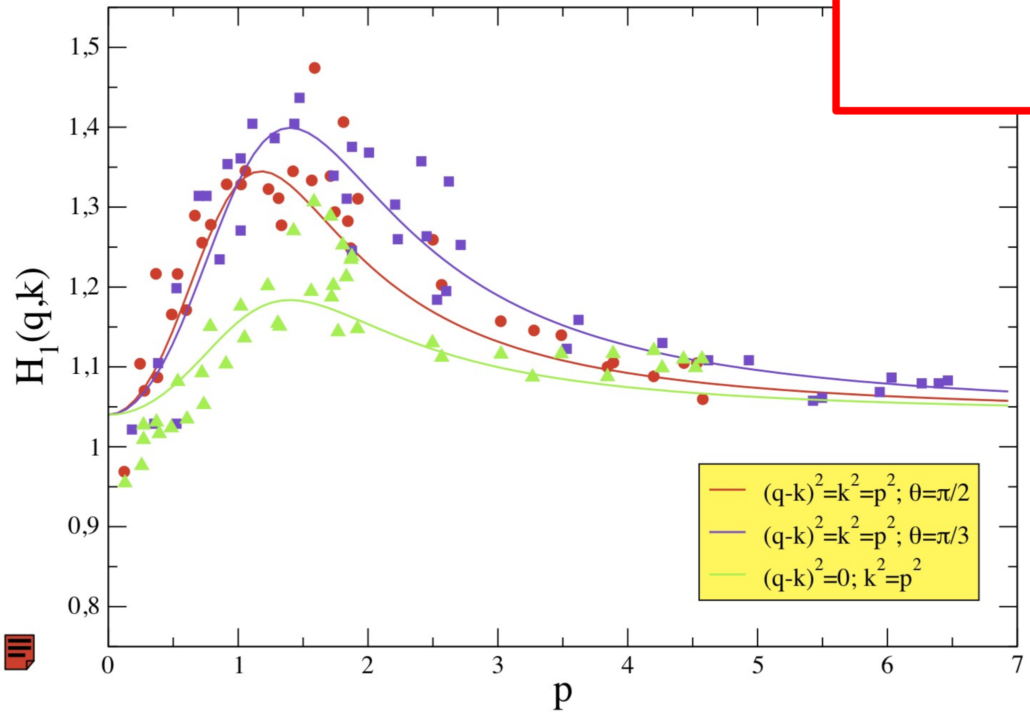


A second task:
OPE+SVZ analysis of the ghost-gluon vertex

SU(2) Lattice results to compare with:

(Phys.Rev.D77:094510,2008)

Very successful description!!!



$$
\begin{aligned}
H_1(q^2,k^2,\theta) &= \widetilde{Z}_1^{-1}\left[1 + \frac{N_C g^2\langle A^2\rangle}{8(N_C^2-1)}\right.\\
&\times\left(\frac{\sqrt{k^2q^2}\cos\theta}{k^2q^2+m_{\rm IR}^4} + 2\,\frac{q^2-\sqrt{k^2q^2}\cos\theta}{q^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\rm IR}^4}\right.\\
&\left.\left.+\,2\,\frac{k^2-\sqrt{k^2q^2}\cos\theta}{k^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\rm IR}^4}\right)\right],
\end{aligned}
$$

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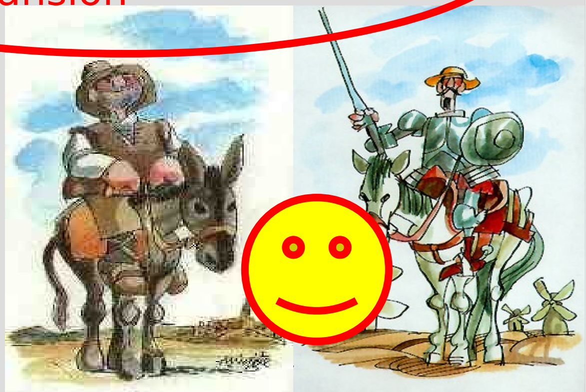

A second task:
… & the Taylor kinematics

$$
\begin{aligned}
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\end{aligned}
$$

$k \to 0$

$$
H_1(q,0) = \widetilde{Z}_1^{-1}\left(1 + N_C\frac{g^2\langle A^2\rangle}{4(N_C^2-1)}\,\frac{q^2}{q^4+m_{IR}^4}\right)
$$

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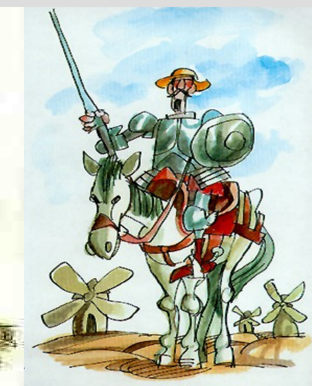
\end{aligned}
$$



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$$

- What about the Taylor's theorem?

$$
\Gamma^{abc}_{{\rm bare},\mu}(-q,0;q) = -g f^{abc} q_\mu
$$

A second task:
… & the Taylor kinematics

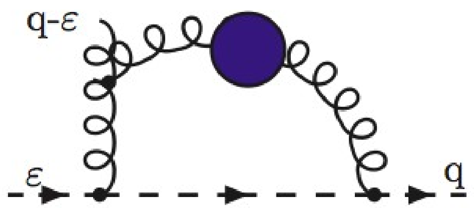
OPE+SVZ for the asymmetric vertex:

$$\Gamma^{abc}_{\mu}(-q,\varepsilon;q-\varepsilon) = \Gamma^{abc}_{\mathrm{pert},\mu}(-q,\varepsilon;q-\varepsilon) + \tilde{v}^{abc}_{\mu}\,\frac{\langle A^2\rangle}{4(N_C^2-1)} + \cdots$$

$$\tilde{v}^{abc}_{\mu} = 2I^{[1]} ,$$

(According to PRD64(2001)114003)

$I^{[1]} =$



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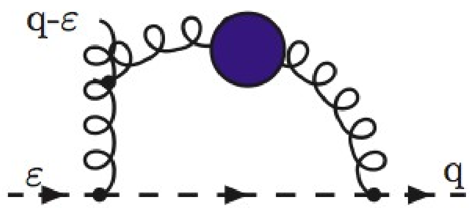
$$H_1(q,\varepsilon) = H_1^{\text{pert}}(q,\varepsilon) + N_C\, g^2\,\frac{(q-\varepsilon)\cdot q}{q^2(q-\varepsilon)^2}\,\frac{\langle A^2\rangle}{4(N_C^2-1)},$$

$$H_2(q,\varepsilon) = H_2^{\text{pert}}(q,\varepsilon) - N_C\, g^2\,\frac{((q-\varepsilon)\cdot q)^2}{q^2(q-\varepsilon)^4}\,\frac{\langle A^2\rangle}{4(N_C^2-1)}.$$

$$I^{[1]} =$$



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A second task:

… & the Taylor kinematics

OPE+SVZ for the asymmetric vertex:

$$\Gamma^{abc}_{\mu}(-q,\varepsilon;q-\varepsilon) \;=\; \Gamma^{abc}_{\mathrm{pert},\mu}(-q,\varepsilon;q-\varepsilon) \;+\; \tilde{v}^{abc}_{\mu}\,\frac{\langle A^2\rangle}{4(N_C^2-1)} \;+\;\cdots$$

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(According to PRD64(2001)114003)

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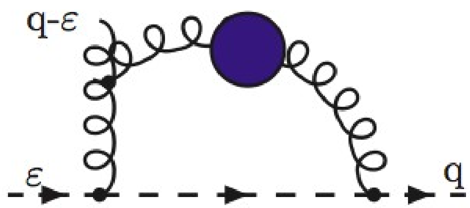
$$H_2(q,\varepsilon) \;=\; H_2^{\mathrm{pert}}(q,\varepsilon) \;-\; N_C\,g^2\,\frac{((q-\varepsilon)\cdot q)^2}{q^2(q-\varepsilon)^4}\,\frac{\langle A^2\rangle}{4(N_C^2-1)}\;.$$

$I^{[1]} \;=$



$$\Gamma^{abc}_{\mu}(-q,0;q) \;=\; -gf^{abc}\,q_{\mu}\left(\,H_1^{\mathrm{pert}}(q,0) \;+\; H_2^{\mathrm{pert}}(q,0)\,\right)$$

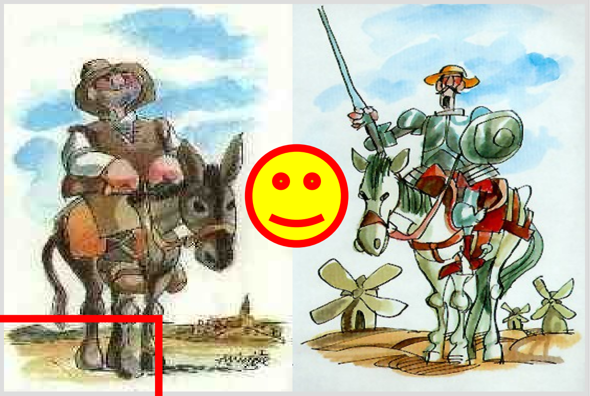
$$\Gamma^{abc}_{\mathrm{bare},\mu}(-q,0;q) \;=\; -gf^{abc}q_{\mu}$$

A second task:
… & the Taylor kinematics

OPE+SVZ for the asymmetric vertex:

$$\Gamma^{abc}_{\mu}(-q,\varepsilon;q-\varepsilon) \;=\; \Gamma^{abc}_{\mathrm{pert},\mu}(-q,\varepsilon;q-\varepsilon) \;+\; \widetilde{v}^{abc}_{\mu} \frac{\langle A^2\rangle}{4(N_C^2-1)} \;+\; \cdots$$

$$\widetilde{v}^{abc}_{\mu} \;=\; 2I^{[1]} \;,$$



(According to PRD64(2001)114003)

$$H_1(q,\varepsilon) \;=\; H_1^{\mathrm{pert}}(q,\varepsilon) \;+\; N_C\, g^2 \frac{(q-\varepsilon)\cdot q}{q^2(q-\varepsilon)^2} \frac{\langle A^2\rangle}{4(N_C^2-1)} \;,$$

$$H_2(q,\varepsilon) \;=\; H_2^{\mathrm{pert}}(q,\varepsilon) \;-\; N_C\, g^2 \frac{((q-\varepsilon)\cdot q)^2}{q^2(q-\varepsilon)^4} \frac{\langle A^2\rangle}{4(N_C^2-1)} \;.$$

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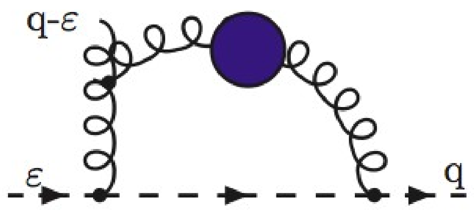


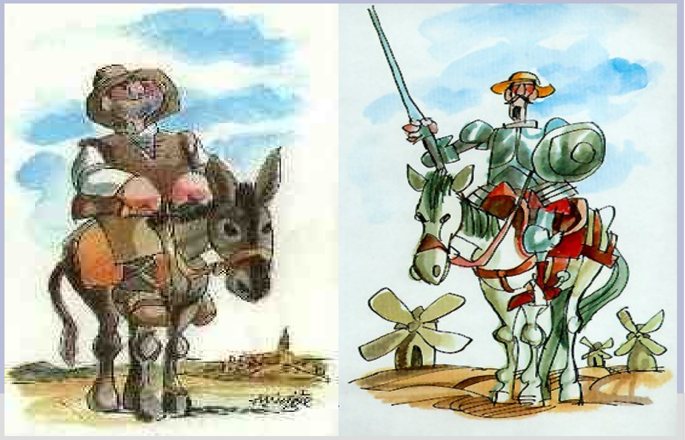
$$\Gamma^{abc}_{\mu}(-q,0;q) \;=\; -g f^{abc}\, q_{\mu} \left(H_1^{\mathrm{pert}}(q,0) \;+\; H_2^{\mathrm{pert}}(q,0) \right)$$

Longitudinal and transverse corrections kill each other!!!

$$\Gamma^{abc}_{\mathrm{bare},\mu}(-q,0;q) \;=\; -g f^{abc} q_{\mu}$$

A second task:

The GPDSE analysis with a gluon condensate



GPDSE:

$$\frac{1}{F(k^2)} = 1 + \alpha_T(\mu^2)\int d^4q\, K(k,q)\, H_1^{\text{bare}}(q,k)\, F(q^2)$$

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda^2)}{4\pi} Z_3(\mu^2,\Lambda^2)\widetilde{Z}_3^2(\mu^2,\Lambda^2)$$

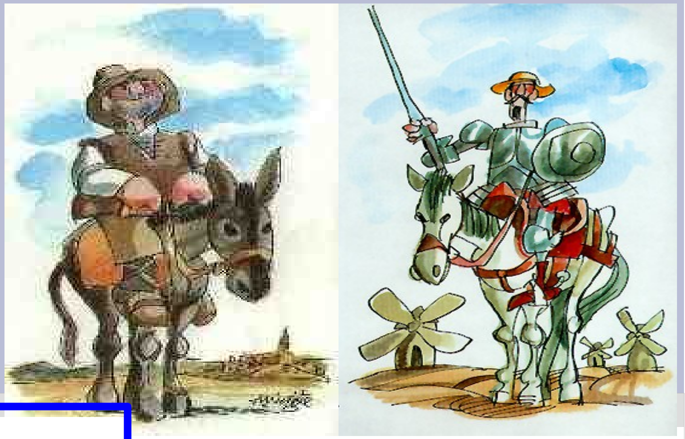
$$H_1(q^2,k^2,\theta) = \widetilde{Z}_1^{-1}\left[1 + \frac{N_C g^2\langle A^2\rangle}{8(N_C^2-1)} \times \left(\frac{\sqrt{k^2q^2}\cos\theta}{k^2q^2+m_{\text{IR}}^4} + 2\frac{q^2-\sqrt{k^2q^2}\cos\theta}{q^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\text{IR}}^4} + 2\frac{k^2-\sqrt{k^2q^2}\cos\theta}{k^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\text{IR}}^4}\right)\right],$$

Lattice inputs!!!

(Cfr. JHEP06(2008)012)

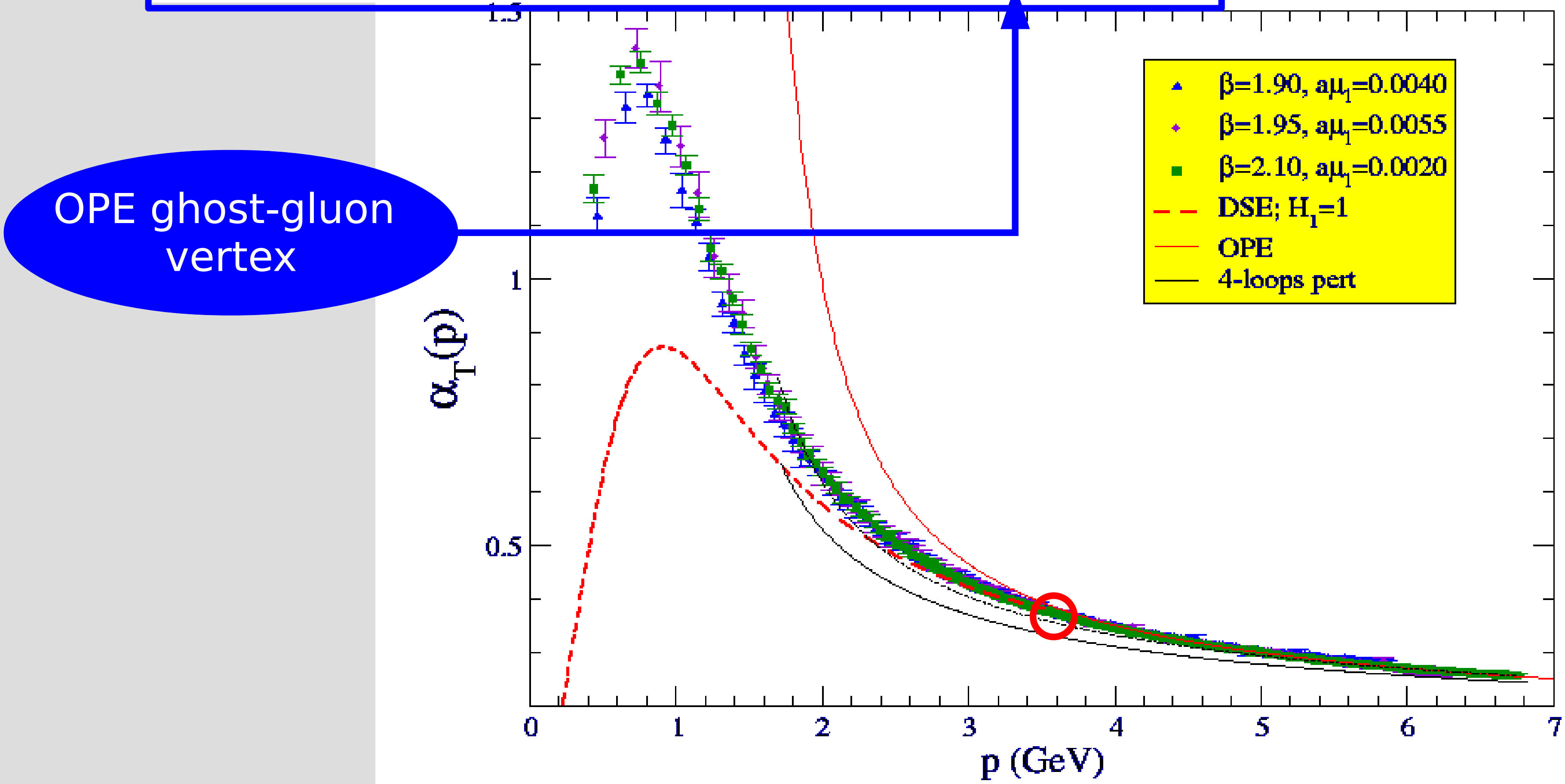
A second task:

The GPDSE analysis with a gluon condensate



GPDSE:

$$\frac{1}{F(k^2)} = 1 + \alpha_T(\mu^2) \int d^4q\, K(k,q) H_1^{\mathrm{bare}}(q,k) F(q^2)$$



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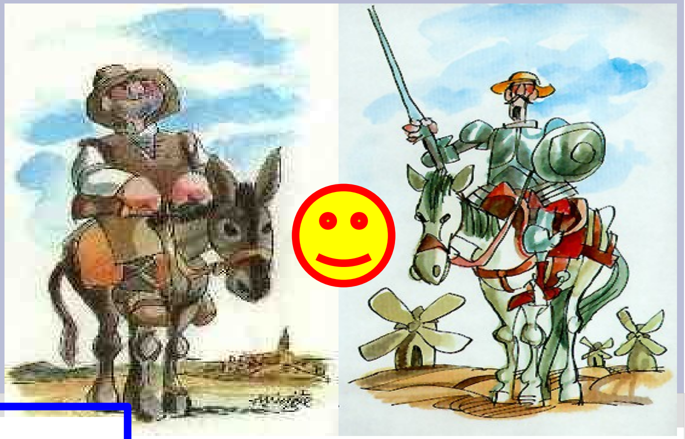
The GPDSE analysis with a gluon condensate



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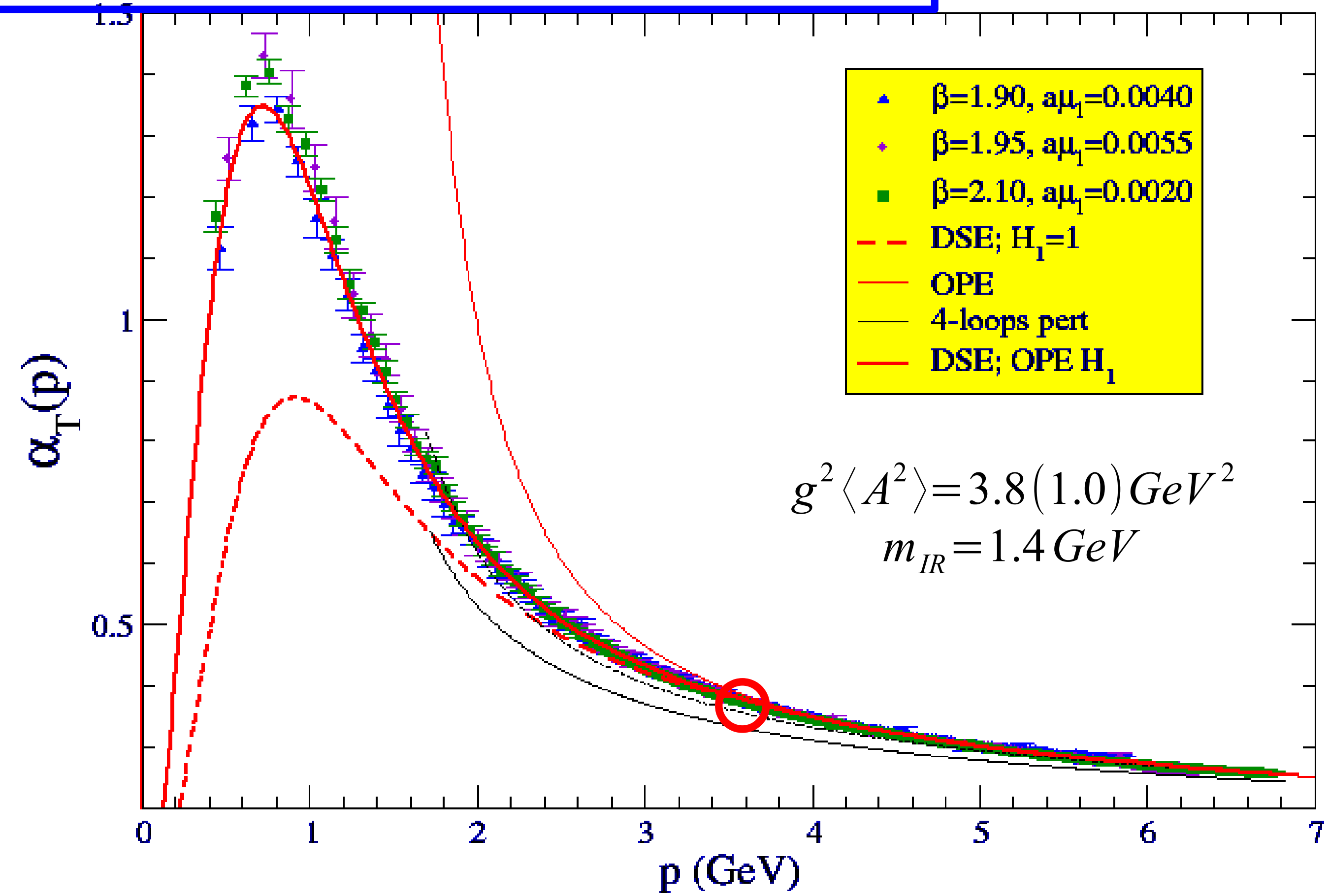
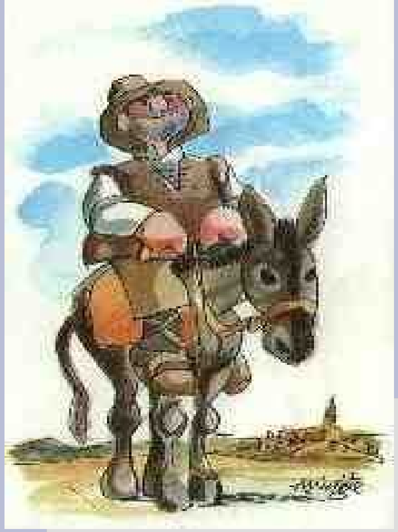




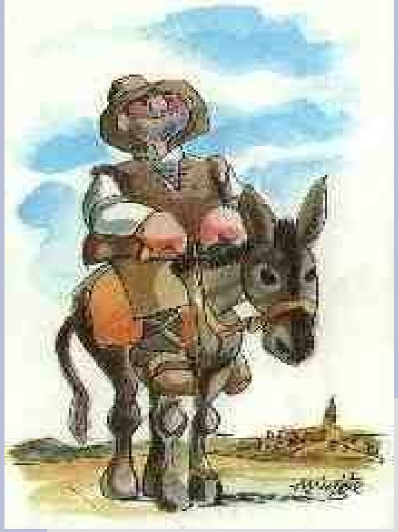
Epilogue:

•We concluded that an OPE contribution including a non-vanishing dimension-two gluon condensate is needed to describe the (experimental) running of alpha in T-scheme.

•The same OPE approach is applied to compute non-perturbative corrections to the ghost-gluon vertex and this inspired a simple model describing its momentum behaviour in pretty good agreement with LQCD.

•We proved that, also in the OPE approach, longitudinal and transverse contributions cancel each other and the Taylor theorem still works (as it should be)

•We show that a quantitative description of the lattice Taylor coupling is possible from the GPDSE with lattice gluon inputs, only when a full ghost-gluon vertex is included.



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•The same OPE+... n-
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•We pro...
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•We preliminary show that ... ption of the lattice Taylor coupling is possible from the GPDSE with lattice gluon inputs, only when a full ghost-gluon vertex is included.

Thank you.